The University of British Columbia

27 February 2014

Midterm for MATH 307 Section 202 Winter 2014

Closed book examination Time: 50 minutes

Last Name ______________________ **First** ______________

Signature ______________

Student Number ______________

Special Instructions:

No memory aids are allowed. No communication devices. Show all your work; little or no credit will be given for a numerical answer without the correct accompanying work. If you need more space than the space provided, use the back of the previous page. **Where boxes are provided for answers, put your final answers in them.**

Rules governing examinations

• Each candidate must be prepared to produce, upon request, a UBCcard for identification.

• Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

• Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

(a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.

(b) Speaking or communicating with other candidates.

(c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.

• Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

• Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1		0
2		0
3		0
4		0
Total		0

1. Option I Consider the MATLAB/Octave command

```
> A = vander([1 2 3 4])
```

(a) What matrix is output by this command?

(b) Matrix A is a (circle one):

i. Vandermonde matrix
ii. Hilbert matrix
iii. Tridiagonal matrix
iv. Triangular matrix

(c) Suppose we have the following output:

```
> cond(A,1)
  ans = 2000.0
> cond(A,2)
  ans = 1171.0
> cond(A,inf)
  ans = 1926.7
```

Why is the output from these different calls to the `cond()` function different?

(d) Suppose we have a vector **b** whos 2-norm is known to a relative accuracy of 1×10^{-7}. What is the worst relative accuracy for vector **x** we can expect when solving:

```
> x = A\b
```

(e) Matrix A is often used to compute interpolation parameters. How do the relatively large values of `cond(A)` suggest an advantage of cubic spline interpolation over Lagrange interpolation?

1. Option II Consider the MATLAB/Octave command

```
> A = diag([1 1 1],-1)+4*diag([1 1 1 1])+diag([1 1 1],1)
```

(a) What matrix is output by this command?

(b) Matrix A is a (circle one):

i. Vandermonde matrix
ii. Hilbert matrix
iii. Tridiagonal matrix
iv. Triangular matrix

(c) Suppose we have the following output:

```
> cond(A,1)
  ans = 2.7273
> cond(A,2)
  ans = 2.3586
> cond(A,'fro')
  ans = 4.8947
```

Why is the output from these different calls to the `cond()` function different?

(d) Suppose we have a vector **b** whos 2-norm is known to a relative accuracy of 1×10^{-7}. What is the worst relative accuracy for vector **x** we can expect when solving:

```
> x = A\b
```

(e) Matrix A is often used to compute interpolation parameters. How do the relatively small values of `cond(A)` suggest an advantage of cubic spline interpolation over Lagrange interpolation?

2. Option I Consider the sparse matrix equation

$$\frac{1}{h^2}\begin{bmatrix} -2 & 1 & & \\ 1 & -2 & 1 & \\ & \ddots & \ddots & \ddots \\ & & 1 & -2 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_{n-1} \end{bmatrix} = \begin{bmatrix} e^{-h}\cos(\pi h) \\ e^{-2h}\cos(2\pi h) \\ \vdots \\ e^{-(n-1)h}\cos(\pi(n-1)h) \end{bmatrix}, \quad n = 1/h. \tag{1}$$

(a) Write an ODE that this matrix equation approximates, including boundary conditions.

(b) Modify the matrix equation in two ways:

i. change the left-hand boundary condition to $u(0) = 1$;

ii. change the left-hand boundary condition to $u'(0) = 1$.

(c) Give a reason why you might not want to choose an extremely small value for h. [Ans: condition number grows as $\mathcal{O}(n^2)$ for large n]

2. Option II Consider the sparse matrix equation

$$\frac{1}{h^2}\begin{bmatrix} -2 & 1 & & \\ 1 & -2 & 1 & \\ & \ddots & \ddots & \ddots \\ & & 1 & -2 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_{n-1} \end{bmatrix} = \begin{bmatrix} \log(\pi h + 1) \\ \log(2\pi h + 1) \\ \vdots \\ \log(\pi(n-1)h + 1) \end{bmatrix}, \quad n = 1/h. \tag{2}$$

(a) Write an ODE that this matrix equation approximates, including boundary conditions.

(b) Modify the matrix equation in two ways:

i. change the left-hand boundary condition to $u(0) = -1$;

ii. change the left-hand boundary condition to $u'(0) = -1$.

(c) Give a reason why you might want to choose a small value for h. [Ans: this FD approximation gives $\mathcal{O}(h^2)$ accuracy for small h]