

Math 307: Problems for section 1.2

Many problems in this homework make use of a few MATLAB/Octave .m files that are provided on the website. In order to use them, make sure that the files are in the same directory that you are running MATLAB/Octave from (to see which directory this is, type `pwd` in MATLAB/Octave).

1. Compute the determinant of a 4×4 Vandermonde matrix. Bonus: show that the general formula for the determinant of a Vandermonde matrix is correct.
2. Let V_n be the Vandermonde matrix for n equally spaced points between 0 and 1. Do you think the condition number of V_n is increasing exponentially in n ? To make an informed guess, use MATLAB/Octave to make a plot of $\log(\text{cond}(V_n))$ against n . You will need to use relatively small values of n (say $n < 20$ or so) to get a reasonable looking plot. What do you think is happening when you use larger values of n ?
3. Use MATLAB/Octave to plot the Lagrange interpolating function through the points $(1, 2.3)$, $(2, 5)$, $(2.4, 9)$, $(2.5, 5)$, $(3, 0)$ and $(5, -1)$.
4. Derive the matrix equation to solve in order to find the cubic spline passing through the three points $(0, 1)$, $(0.5, 2)$ and $(1, 4)$. Plot the resulting spline (you may use the file `plotspline.m`).
5. What happens to the condition number of the matrix S used in cubic spline interpolation as the size n becomes large (you may use the file `splinemat.m`)?
6. A parabolic runout spline is the interpolating function you get by changing the condition $f''(x_1) = f''(x_n) = 0$ to the condition that $p_1(x)$ and $p_{n-1}(x)$ should be quadratic polynomials (that is, $a_1 = a_{n-1} = 0$). Modify the file `splinemat.m` so that it computes the matrix relevant to this modified problem. Call the modified file `splinematpr.m`. (Hand in a description of your changes, or a print-out of the modified file.) Use your new file to graph the parabolic runout spline for the points $(1, 1)$, $(2, 1)$, $(3, 2)$, $(4, 4)$ and $(5, 3)$. (The easiest way to do this is to change `splinemat` to `splinematpr` inside the file `plotspline.m` and call the modified file `plotsplinepr.m`. Use this new file to plot the modified spline.) Hand in a plot of both the parabolic runout spline and the cubic spline on the same graph.
7. Consider the problem of interpolating four points (x_1, y_1) , (x_2, y_2) , (x_3, y_3) and (x_4, y_4) with a function $f(x)$ that is given by a quadratic polynomial in each interval x_i, x_{i+1} , (i.e., $p_i(x) = a_i(x - x_i)^2 + b_i(x - x_i) + c_i$) and whose first derivative $f'(x)$ is continuous across the points x_i . Write down the system of equations for this problem. Is there a unique solution to this problem?