Math 307: Problems for section 3.2

February 2, 2011

1. **Review of complex numbers:**
   (a) **Show that** $|zw| = |z||w|$ **for any complex numbers** z **and** w**.**
   (b) **Show that** $\overline{zw} = \bar{z}\bar{w}$ **for any complex numbers** z **and** w**.**
   (c) **Show that** $\bar{z}z = |z|^2$ **for every complex number** z**.**

2. **Calculate the inner products and norms for the following:**
   (a) **the real vectors** $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ **and** $\begin{bmatrix} -3 \\ 5 \\ -1 \end{bmatrix}$,
   (b) **the complex vectors** $\begin{bmatrix} 1+i \\ 3-i \\ 2+2i \\ 6-3i \end{bmatrix}$ **and** $\begin{bmatrix} 2-2i \\ 4+3i \\ 6-i \\ 1 \end{bmatrix}$,
   (c) **the functions** $x-1$ **and** $\cos x$ **on the interval** $[-\pi, \pi]$,
   (d) **the functions** e^{3ix} **and** e^{-ix} **on the interval** $[0, 2\pi]$.

3. **Plot the location of the complex numbers** $z_k = e^{2\pi ik/5}$**,** $k = 0, 1, 2, 3, 4$ **in the complex plane. Show that these numbers are fifth roots of unity, that is, they satisfy** $z^5 = 1$**. What is** z_0**? The numbers** z_k **are the five roots of the polynomial** $z^5 - 1$ **which implies that** $z^5 - 1 = (z - z_0)(z - z_1)(z - z_2)(z - z_3)(z - z_4)$**. Now compute** $(z^5 - 1)/(z - 1)$ **in two ways: by polynomial long division and by dividing the factorization above by** $z - 1$**. Set these expressions equal to find the factorization of** $z^4 + z^3 + z^2 + z + 1$**. Use this factorization to compute** $z_k^4 + z_k^3 + z_k^2 + z_k + 1$ **for** $k = 0, 1, 2, 3, 4$**.**