Math 307: Problems for section 4.2

November 14, 2012

1. **(i) What can you say about the diagonal elements of a Hermition matrix?**

   (ii) Show that if A is an $n \times n$ matrix such that $\langle \mathbf{v}, A\mathbf{w}\rangle = \langle A\mathbf{v}, \mathbf{w}\rangle$ then A is Hermitian.

2. **Show that if A is any matrix then A^*A and AA^* are Hermitian with non-negative eigenvalues.**

3. **Follow the procedure in the notes to find an orthogonal matrix V such that V^TAV is upper triangular when $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$.**

4. **Explain why the Laplacian matrix L for a resistor network has non-negative eigenvalues.**

5. **Redo the calculation of the effective resistance between nodes 1 and 7 of the resistor cube in section II.2.12. For this problem the Laplacian is defined by**

   ```
   >L=[3 -1 0 -1 -1 0 0 0;-1 3 -1 0 0 -1 0 0;
   0 -1 3 -1 0 0 -1 0;-1 0 -1 3 0 0 0 -1;
   -1 0 0 0 3 -1 0 -1;0 -1 0 0 -1 3 -1 0;
   0 0 -1 0 0 -1 3 -1;0 0 0 -1 -1 0 -1 3];
   ```

   and the answer is $R = 5/6 = 0.83333$

6. **Which of these two resistor networks do you think has a lower effective resistance between the indicated nodes? Check your guess using MATLAB/Octave and the eigenvalue/vector formula for the effective resistance.**