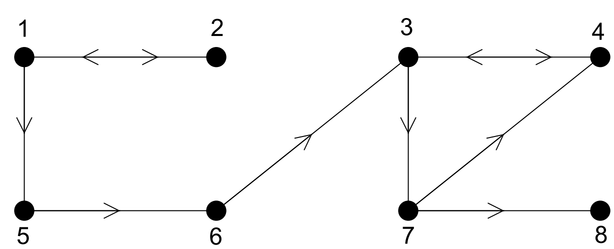
Math 307: Problems for section 4.6

1. **Suppose that there is a fixed population of cola drinkers each with a favourite among Coke, Pepsi and Thums Up. Every month 3% of the Coke drinkers switch to Pepsi while 5% switch to Thums Up. Every month 2% of the Pepsi drinker switch to Coke while 3% switch to Thums Up. Every month 1% of the Thums Up drinkers switch to Coke and 1% switch to Pepsi. What are the eventual market shares of these drinks? (You can use MATLAB/Octave to compute things, but explain what you are doing in your solution.)**

2. **A flea hops randomly on vertices of a triangle, hopping to each of the other vertices with equal probability (never remaining at the same vertex). The flea starts at vertex 1. What is the probability that the flea is at vertex 1 again after n hops?**

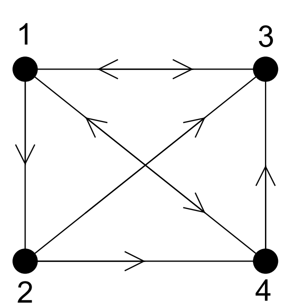
3. **Show that the product of two $n \times n$ stochastic matrices is also stochastic.**

4. **Find the rank for each site with $\alpha = 1$ and 0.85 for an internet that is linked in the following way**



You can use `eig` to find the relevant eigenvector, but for $\alpha = 0.85$, check that the power method gives the same answer (give the MATLAB/Octave commands you used for the power method).

5. **Consider the following internet:**



If $\alpha = 1$, show that the rankings of the pages are such that page 1 is highest.

The owners of page 3 are furious that their page doesn't rank highest. They try to increase the rank of page 3 by creating a new page, page 5, to which page 3 links and

which links to page 3. Does this increase the rank of page 3 above that of page 1 (with $\alpha = 1$)?

6. **Prove that for an internet, the rankings of a page having no links to it from any other page is always zero assuming $\alpha = 1$.**

7. **Write a MATLAB/Octave function metropolis.m such that `metropolis(P,p)` produces a stochastic matrix whose invariant distribution is propotional to `p`. (You may assume that the entries `p` in the input are strictly positive.**

8. **Using the function in the previous question or by hand, compute the matrix Q given by the Metropolis algorithm when**

$$P = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}$$

and

$$p = [1, 2, 3]$$

9. **Compute the matrix Q given by the Metropolis algorithm when**

$$P = \begin{bmatrix} 1/2 & 0 & 1/3 \\ 1/2 & 1 & 1/3 \\ 0 & 0 & 1/3 \end{bmatrix}$$

and p is anything.