Math 307: Problems for section 4.5

1. **If $V_1 = -5$, and all the other V_n's are zero in the tight binding model, compute the probability that the bound state electron is located at the nth site for $n = 1, 2, 3$.**

   The probabilities are
   $$p_n = |\psi_n|^2/N^2$$
   where $N^2 = \sum_{n=0}^{\infty} |\psi_n|^2$. We have $\psi_0 = 0$ and $\psi_1 = 1$. To compute the rest of the ψ_n we use the fact that $a_1 = 0$ for $E = -5.2$. Thus, for this value of E, $\mathbf{x}_1 = a_2\mathbf{v}_2$ so that
   $$\begin{bmatrix} \psi_{n+1} \\ \psi_n \end{bmatrix} = \mathbf{x}_n = A^{n-1}a_2\mathbf{v}_2 = a_2\lambda_2^{n-1}\mathbf{v}_2$$
   Since
   $$\mathbf{v}_2 = \begin{bmatrix} -1 \\ E + \lambda_2 \end{bmatrix}$$
   we find that for $n \geq 1$
   $$\psi_n = a_2\lambda_2^{n-1}(E + \lambda_2).$$
   Using the formula for the sum of the geometric series, we find
   $$N^2 = \sum_{n=1}^{\infty} |a_2(E + \lambda_2)|^2|\lambda_2|^{2(n-1)} = |a_2(E + \lambda_2)|^2(1 - |\lambda_2|^2)^{-1}$$
   Now we can substitute the numerical values to obtain $\lambda_1 = 5$, $\lambda_2 = 0.2$, $a_2 = -0.2$, $N^2 = 1.041666667$ and finally
   $$\begin{aligned} p_1 &= 0.9599999997 \\ p_2 &= 0.0383999999 \\ p_3 &= 0.0015360000 \end{aligned}$$