Math 307: Problems for section 3.4

November 19, 2009

1. **Calculate the Fourier coefficients (c_n's, a_n's and b_n's) for the triangle function**

$$f(t) = \begin{cases} 2t & \textbf{if} \quad 0 \le t \le 1/2 \\ 2 - 2t & \textbf{if} \quad 1/2 \le t \le 1 \end{cases}$$

and show that the Fourier series decomposition of $f(t)$ may be written

$$f(t) = \frac{1}{2} - \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} \frac{4}{\pi^2 n^2} \cos(2\pi n t) = \frac{1}{2} - \sum_{n=0}^{\infty} \frac{4}{\pi^2 (2n+1)^2} \cos(2\pi(2n+1)t)$$

What does Parseval's formula say in this case?

2. **Modify the file `ftdemo1.m` so that it plots the partial sums of the Fourier series in the previous question. Hand in the code and a plot of the partial sums with 1, 2, 5 and 10 non-zero terms.**

3. **Calculate the Fourier coefficients (c_n's, a_n's and b_n's) for the half sine wave**

$$f(t) = \sin(\pi t) \quad \textbf{for } 0 \le t \le 1$$

and show that the Fourier series for $f(t)$ can be written

$$f(t) = \frac{2}{\pi} + \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{1 - 4n^2} \cos(2\pi n t)$$

4. **Calculate the Fourier coefficients (c_n's, a_n's and b_n's) for the function**

$$f(t) = t^2 - 1 \quad \textbf{for } -1 \le t \le 1$$

and show that the Fourier series for $f(t)$ can be written

$$f(t) = -\frac{2}{3} + \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos(n\pi t).$$

5. **Show that the Fourier series of $f(t) = e^t$ on the interval $-\pi \le t \le \pi$ is**

$$f(t) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \frac{1}{1 - in} (e^{(1-in)\pi} - e^{-(1-in)\pi}) e^{int}.$$

Deduce that

$$\sum_{n=1}^{\infty} \frac{1}{1+n^2} = \frac{1}{2}(\pi \coth \pi - 1).$$