Math 307: Problems for section 4.1

November 14, 2012

1. **For the following matrices find**
   (a) **all eigenvalues**
   (b) **linearly independent eigenvectors for each eigenvalue**
   (c) **the algebraic and geometric multiplicity for each eigenvalue**

   and state whether the matrix is diagonalizable.

$$A = \begin{bmatrix} 3 & 7 \\ 2 & -2 \end{bmatrix} \quad \textbf{(calculate by hand)}$$

$$B = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix} \quad \textbf{(calculate using Matlab/Octave or otherwise)}$$

$$C = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 0 & -2 \\ -1 & 2 & 3 \end{bmatrix} \quad \textbf{(calculate using Matlab/Octave or otherwise)}$$

2. **Find a 3×3 real, non-zero (*i.e.* not all entries zero) matrix which has all three eigenvalues zero.**

3. (a) **By hand find a matrix with eigenvalues $\lambda_1 = 1$ and $\lambda_2 = 2$ and corresponding eigenvectors**

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

   (b) **Using Matlab/Octave or otherwise, find a matrix with eigenvalues $\lambda_1 = 1$, $\lambda_2 = 2$ and $\lambda_3 = 3$ and corresponding eigenvectors**

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 9 \\ 4 \\ 4 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}.$$

4. **Show that if A is an $n \times n$ square matrix and each column sums to c, then c is an eigenvalue of A. *Hint: if you cannot show this in a few lines, try another approach.***

5. **If $p(\lambda)$ is the characteristic polynomial of an $n \times n$ invertible matrix A, find an expression for the characteristic polynomial of A^{-1} in terms of the characteristic polynomial of A.**

6. **Find the Jordan canonical form of the matrix**

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$