# MIE1807

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# multiple regression

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Most datasets have more than one or two columns. The most important statistical model (in my opinion) is the linear regression model with more than one "$x$" variable:

$$y = \beta_0 + \beta_1 x_1 + \cdots + \beta_k x_k + \varepsilon$$

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We treat $y$ as random. The inputs are not random. They can be whatever you like, even functions of one another, with one technical limitation*.

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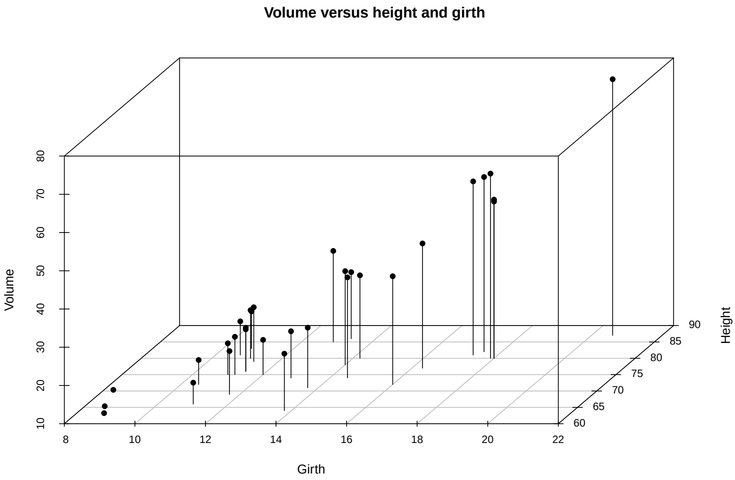
Other special inputs include "indicator variables" (coded 0 and 1) and "interaction terms".

# what is being accomplished in multiple regression?

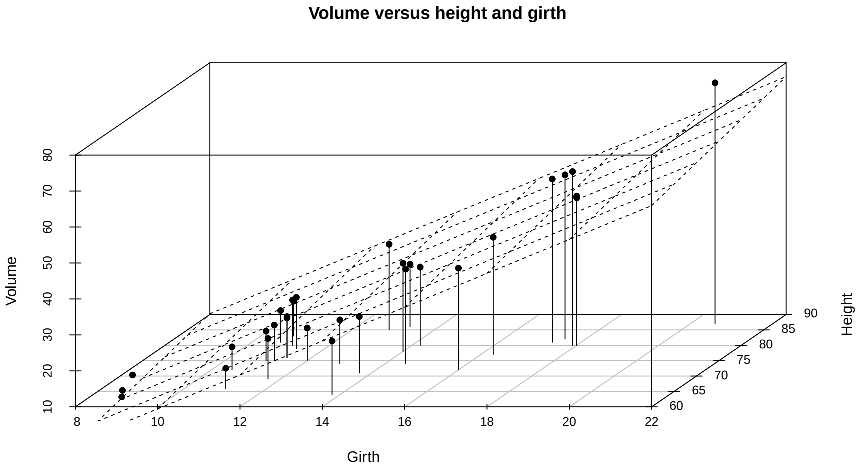
R comes with some sample datasets. One is called `trees` and has variables Girth, Height, and Volume. Here's a peek at the data:

```
## # A tibble: 31 × 3
##   Girth Height Volume
##   <dbl>  <dbl>  <dbl>
## 1   8.3     70   10.3
## 2   8.6     65   10.3
## 3   8.8     63   10.2
## 4  10.5     72   16.4
## 5  10.7     81   18.8
## # ... with 26 more rows
```

# what is being accomplished in multiple regression?



# multiple regression fits a surface to the points



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  - Hard to visualize what is really happening
  - Model selection: which variables?
  - “Multicollinearity” (highly correlated inputs)

# parameter interpretation

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$\beta_0$ is the "intercept"—mainly important to make sure the fitted surface actually goes through the points.

The $\beta_i$ from $i \in \{1, \ldots, k\}$ are the slope parameters, and have a different interpretation than before.

# slope parameter interpretation

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That bold, italic statement should echo in your mind any time you think of anything to do with $\beta_i$.

# trees example

We might want to model $y =$`Volume` (the amount of wood) as a linear model of the input variables $x_1 =$`Girth` and $x_2 =$`Height`, as follows:

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The computer uses the method of "least squares", like before. A full treatment of the analysis requires matrix algebra.

# fitted values | residuals

Here's the first row of the `trees` data:

| Girth | Height | Volume |
|---|---|---|
| 8.3 | 70 | 10.3 |

We could call these values $y_1, x_{11}$, and $x_{21}$

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The residual corresponding to this fitted value is just:

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For a dataset with $n$ rows (the sample size), there is a fitted value and residual for each row.

# trees data fitted model

Here's what R produces:

```
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -57.9877     8.6382  -6.713 2.75e-07
## Girth         4.7082     0.2643  17.816  < 2e-16
## Height        0.3393     0.1302   2.607   0.0145
##
## Residual standard error: 3.882 on 28 degrees of freedom
## Multiple R-squared:  0.948,  Adjusted R-squared:  0.9442
## F-statistic:   255 on 2 and 28 DF,  p-value: < 2.2e-16
```

# individual slope parameter hypothesis testing

The usual hypothesis test for a single parameter:

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***given all the other $x$'s in the model***

# the overall hypothesis test

"Is there any linear relationship between $y$ and the input variables?"

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Null hypothesis can be expressed as:

$$H_0 : \beta_1 = \beta_2 = \cdots = \beta_k = 0$$

# estimating $\sigma$

This works the same as with simple regression, in which we used $\sqrt{MSE}$ where:

$$MSE = \frac{\sum_{j=1}^{n} \left(y_j - \hat{y}_j\right)^2}{n-2}$$

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There was only one input variable, so another way to think of this was "sample size minus the number of input variables, then minus 1."

# estimating $\sigma$

In multiple regression, nothing changes. Use $\sqrt{MSE}$, where:

$$MSE = \frac{\sum_{j=1}^{n} \left(y_j - \hat{y}_j\right)^2}{n - (k+1)}$$

# hypothesis testing for $\beta_i$

The computer produces the estimate $b_i$, which has these properties:

$$E(b_i) = \beta_i$$
$$\text{Var}(b_i) = \sigma^2 \cdot c_i$$

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Just like before, we get:

$$\frac{b_i - \beta_i}{\sqrt{MSE}\sqrt{c_i}} \sim t_{n-k+1}$$

# hypothesis testing for $\beta_i$ in the trees example

```
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# the overall $F$ test

"Is there any linear relationship between $y$ and the input variables?"

Based on the same, original SS decomposition.

$$\text{variation in the } y \;=\; \text{variation due to the model} \;+\; \text{variation due to error}$$

$$\sum (y_i - \overline{y})^2 = \qquad\qquad\qquad +$$

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$$\sum (y_i - \overline{y})^2 = \sum (\hat{y}_i - \overline{y})^2 + \sum (y_i - \hat{y}_i)^2$$
$$SS_{Total} = SS_{Regression} + SS_{Error}$$

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The p-value then comes from:

$$\frac{SS_{Regression}/k}{SS_{Error}/(n-k-1)} = \frac{MSR}{MSE} \sim F_{k,n-k-1}$$

# the overall $F$ test - trees example

The information is in the usual `R` output:

```
##
## Residual standard error: 3.882 on 28 degrees of freedom
## Multiple R-squared:  0.948,  Adjusted R-squared:  0.9442
## F-statistic:   255 on 2 and 28 DF,  p-value: < 2.2e-16
```

# model assumptions and calculation requirements

Model:

$$y = \beta_0 + \beta_1 x_1 + \cdots + \beta_k x_k + \varepsilon, \qquad \varepsilon \sim N(0, \sigma)$$

Pretty much the same as with simple regression.

The main ones to worry about are:

1. The linear model is appropriate (fatal if violated).

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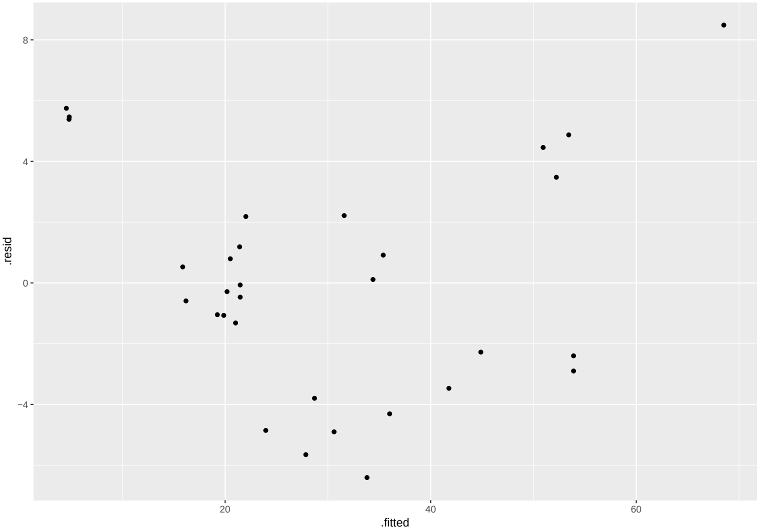
1. and 2. are verified with a plot of residuals versus fitted values, and 3. is verified with a normal quantile plot of the residuals.

# residuals versus fitted values - trees example (fatal)



# not surprising, since the model was obviously wrong

If you really wanted to model the $y =$`Volume` of wood using $x_1 =$`Girth` and $x_2 =$`Height`, you need to include the square of `Girth`, because of the volume-of-a-cylinder formula $V = \pi r^2 h$.

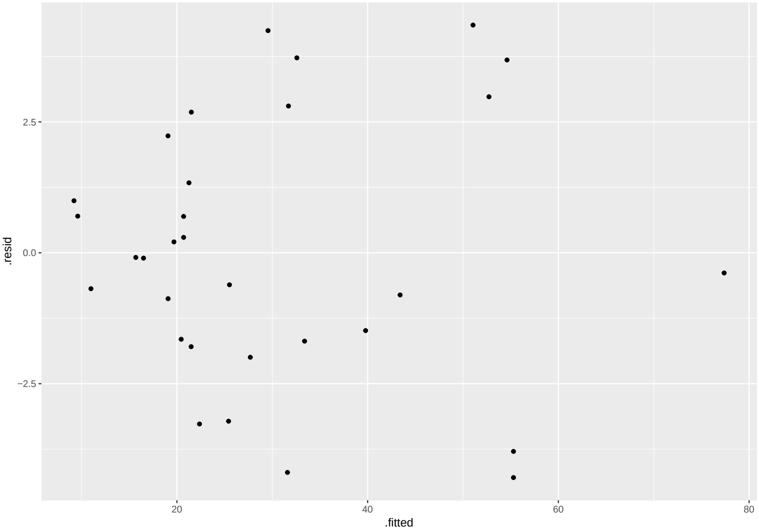
So let's fit the model:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1^2 + \varepsilon$$

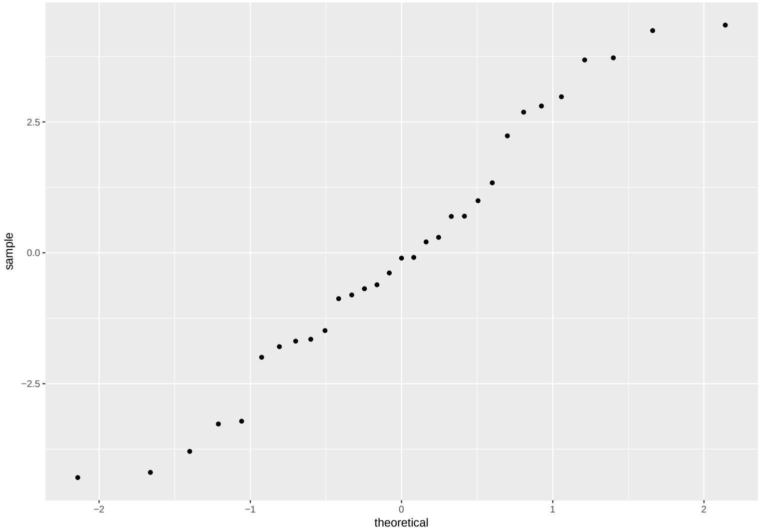
# new trees model fit

```
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -9.92041   10.07911  -0.984 0.333729
## Girth       -2.88508    1.30985  -2.203 0.036343
## I(Girth^2)   0.26862    0.04590   5.852 3.13e-06
## Height       0.37639    0.08823   4.266 0.000218
##
## Residual standard error: 2.625 on 27 degrees of freedom
## Multiple R-squared:  0.9771, Adjusted R-squared:  0.9745
## F-statistic: 383.2 on 3 and 27 DF,  p-value: < 2.2e-16
```

# new trees model resids v. fits



# normal quantile plot of residuals



# towards an "adjusted" $R^2$

$R^2$ comes from dividing $SS_{Total}$ through the SS decomposition:

$$SS_{Total} = SS_{Regression} + SS_{Error}$$

The definition $R^2 = SSR/SST = 1 - SSE/SST$ is the same no matter how many input variables there are.

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One use of $R^2$ is to compare two different regression models...

...but the problem is that $R^2$ always goes up when you add any new input variable to the model. This is because

$$SS_{Error}$$

always goes down with a new variable added.

# adjusting $R^2$ for the number of input variables

A more fair (but still not perfect) single-number-summary of a multiple regression fit is:

$$R^2_{adj} = 1 - \frac{MS_{Error}}{MS_{Total}}$$

where $MS_{Total}$ is just another name for the sample variance of the output $y$ values:

$$MS_{Total} = \frac{SS_{Total}}{n-1} = \frac{\sum_{i=1}^{n} (y_i - \overline{y})^2}{n-1}$$

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$$MS_{Total} = \frac{SS_{Total}}{n-1} = \frac{\sum\limits_{i=1}^{n} (y_i - \overline{y})^2}{n-1}$$

The adjustment works on the basis of this trade-off: while $SS_{Error}$\$ goes down, the error degrees of freedom also goes down.

$R^2_{adj}$ will play more of a role in the next topic—model selection

# model selection preview

A Body Fat % dataset.

```
## # A tibble: 250 × 15
##   `Pct BF`   Age Weight Height  Neck Chest Abdomen    waist   Hip
##      <dbl> <int>  <dbl>  <dbl> <dbl> <dbl>   <dbl>    <dbl> <dbl>
## 1     12.3    23 154.25  67.75  36.2  93.1    85.2 33.54331  94.5
## 2      6.1    22 173.25  72.25  38.5  93.6    83.0 32.67717  98.7
## 3     25.3    22 154.00  66.25  34.0  95.8    87.9 34.60630  99.2
## 4     10.4    26 184.75  72.25  37.4 101.8    86.4 34.01575 101.2
## 5     28.7    24 184.25  71.25  34.4  97.3   100.0 39.37008 101.9
## # ... with 245 more rows, and 6 more variables: Thigh <dbl>,
## #   Knee <dbl>, Ankle <dbl>, Bicep <dbl>, Forearm <dbl>, Wrist <dbl>
```

# model selection preview

We could these two simple regression models:

```
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -14.69314    2.76045  -5.323 2.29e-07
## Weight        0.18938    0.01533  12.357  < 2e-16
```

```
##
## Coefficients:
##            Estimate Std. Error t value Pr(>|t|)
## (Intercept) 25.58078   14.15400   1.807   0.0719
## Height      -0.09316    0.20119  -0.463   0.6438
```

# model selection preview

Model with both. Is this a contradiction?

```
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 76.78100   10.04121   7.647 4.59e-13
## Weight       0.26326    0.01536  17.136  < 2e-16
## Height      -1.48829    0.15873  -9.376  < 2e-16
##
## Residual standard error: 5.626 on 247 degrees of freedom
## Multiple R-squared:  0.5435, Adjusted R-squared:  0.5398
## F-statistic: 147.1 on 2 and 247 DF,  p-value: < 2.2e-16
```

relationships among the inputs

# "multicollinearity"

I stated the following fact about the $b_i$ estimates for $\beta_i$:

$$\frac{b_i - \beta_i}{\sqrt{MSE}\sqrt{c_i}} \sim t_{n-k-1}$$

where $c_i$ is a number that reflects the relationships between $x_i$ and the other inputs.

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For example, when $x_i$ and some other $x_j$ are highly “correlated”, it means they are close to linear functions of one another.

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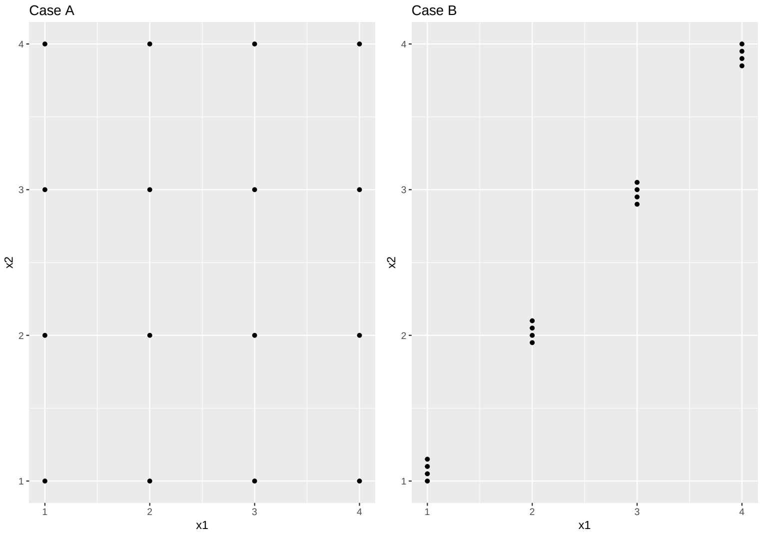
For example, when $x_i$ and some other $x_j$ are highly “correlated”, it means they are close to linear functions of one another.

What happens when $c_i$ is large?

# illustration of the problem - two pairs of inputs



# illustration of the problem

I'll generate some data from the same model in each case:

$$y = 1 + 2x_1 + 3x_2 + \varepsilon, \quad \varepsilon \sim N(0,1)$$

Then fit the two datasets to regression models...

# Case A

```
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   1.5331     1.0177   1.506    0.156
## x1            1.9401     0.2744   7.069 8.43e-06
## x2            2.8854     0.2744  10.513 1.00e-07
##
## Residual standard error: 1.227 on 13 degrees of freedom
## Multiple R-squared:  0.9251, Adjusted R-squared:  0.9135
## F-statistic: 80.25 on 2 and 13 DF,  p-value: 4.843e-08
```

# Case B

```
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   1.5331     1.0177   1.506    0.156
## x1            4.1181     5.2218   0.789    0.444
## x2            0.7074     5.4890   0.129    0.899
##
## Residual standard error: 1.227 on 13 degrees of freedom
## Multiple R-squared:  0.9591, Adjusted R-squared:  0.9528
## F-statistic: 152.3 on 2 and 13 DF,  p-value: 9.506e-10
```

Note the small p-value for the overall $F$ test.

# Note that multicollinearity is merely a *possible* problem

Case C: same model fit to the Case B situation but with $n = 288$

```
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   1.0510     0.1888   5.565 6.03e-08
## x1            2.1419     0.9690   2.210  0.02787
## x2            2.8299     1.0186   2.778  0.00583
##
## Residual standard error: 0.9663 on 285 degrees of freedom
## Multiple R-squared:  0.9693, Adjusted R-squared:  0.9691
## F-statistic:  4502 on 2 and 285 DF,  p-value: < 2.2e-16
```

# bodyfat correlation matrix

```
##          Pct BF   Age Weight Height Neck Chest Abdomen waist
## Pct BF     1.00  0.30   0.62  -0.03 0.49  0.70    0.82  0.82
## Age        0.30  1.00  -0.02  -0.25 0.12  0.18    0.24  0.24
## Weight     0.62 -0.02   1.00   0.51 0.81  0.89    0.87  0.87
## Height    -0.03 -0.25   0.51   1.00 0.32  0.22    0.19  0.19
## Neck       0.49  0.12   0.81   0.32 1.00  0.77    0.73  0.73
## Chest      0.70  0.18   0.89   0.22 0.77  1.00    0.91  0.91
## Abdomen    0.82  0.24   0.87   0.19 0.73  0.91    1.00  1.00
## waist      0.82  0.24   0.87   0.19 0.73  0.91    1.00  1.00
```

# forwards, backwards, and step-wise model selection

A very simple method is to just fit all possible models and see which one is the best (with small p-values and a nice $R^2_{adj}$ (or any number of other single-number-summaries you might like). But there may be too many models to consider.

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These are accessible strategies for novices, but they are known to have issues, *especially when input variables are highly "correlated".*

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These are accessible strategies for novices, but they are known to have issues, *especially when input variables are highly "correlated".*

There are (significantly) more sophisticated strategies also, which are worth it if you are serious about model selection.

# backwards selection

Consider interactions or powers of terms when there is a rational basis for doing so.

Then, start with all input variables and remove the one with the highest p-value.

Repeat until all the p-values are small.

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Known problems specific to this procedure:

- sample size may not sensibly suppose "all" input variables

# backwards selection

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Then, start with all input variables and remove the one with the highest p-value.

Repeat until all the p-values are small.

Known problems specific to this procedure:

- sample size may not sensibly suppose “all” input variables
- p-values for variables involved in correlations may be artifically high.

# backwards with bodyfat - full model F test

```
##
## Residual standard error: 4.255 on 236 degrees of freedom
## Multiple R-squared:  0.7505, Adjusted R-squared:  0.7368
## F-statistic: 54.61 on 13 and 236 DF,  p-value: < 2.2e-16
```

# backwards with bodyfat - full model all p-values

```
##
## Coefficients: (1 not defined because of singularities)
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.68516   23.37412   0.072 0.942587
## Age          0.07189    0.03217   2.234 0.026389
## Weight      -0.01762    0.06714  -0.263 0.793153
## Height      -0.24675    0.19114  -1.291 0.197989
## Neck        -0.38682    0.23486  -1.647 0.100887
## Chest       -0.11919    0.10825  -1.101 0.272004
## Abdomen      0.90452    0.09140   9.897  < 2e-16
## waist             NA         NA      NA       NA
## Hip         -0.15878    0.14586  -1.089 0.277446
## Thigh        0.17299    0.14683   1.178 0.239926
## Knee        -0.04580    0.24560  -0.186 0.852230
## Ankle        0.18502    0.21985   0.842 0.400862
## Bicep        0.17968    0.17039   1.054 0.292732
## Forearm      0.27605    0.20692   1.334 0.183454
```

# what's up with waist and Abdomen?

```
## # A tibble: 250 × 3
##       waist Abdomen ratio
##       <dbl>   <dbl> <dbl>
## 1 33.54331    85.2  2.54
## 2 32.67717    83.0  2.54
## 3 34.60630    87.9  2.54
## 4 34.01575    86.4  2.54
## 5 39.37008   100.0  2.54
## # ... with 245 more rows
```

# backwards with bodyfat - full model all p-values

| term | estimate | std.error | statistic | p.value |
|---|---|---|---|---|
| (Intercept) | 1.685 | 23.374 | 0.072 | 0.943 |
| Age | 0.072 | 0.032 | 2.234 | 0.026 |
| Weight | -0.018 | 0.067 | -0.263 | 0.793 |
| Height | -0.247 | 0.191 | -1.291 | 0.198 |
| Neck | -0.387 | 0.235 | -1.647 | 0.101 |
| Chest | -0.119 | 0.108 | -1.101 | 0.272 |
| Abdomen | 0.905 | 0.091 | 9.897 | 0.000 |
| Hip | -0.159 | 0.146 | -1.089 | 0.277 |
| Thigh | 0.173 | 0.147 | 1.178 | 0.240 |
| Knee | -0.046 | 0.246 | -0.186 | 0.852 |
| Ankle | 0.185 | 0.220 | 0.842 | 0.401 |
| Bicep | 0.180 | 0.170 | 1.054 | 0.293 |
| Forearm | 0.276 | 0.207 | 1.334 | 0.183 |
| Wrist | -1.802 | 0.533 | -3.380 | 0.001 |

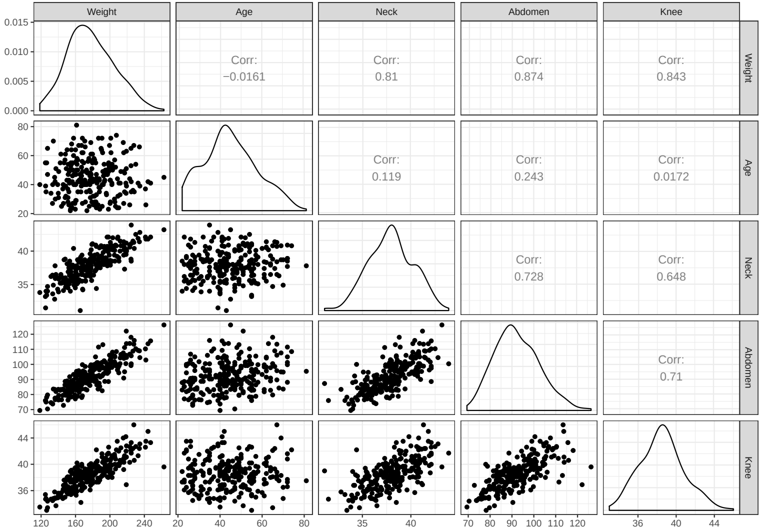
# interlude - possibly doesn't mean Knee, Weight, and Ankle are actually useless

```
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   3.1215     9.4771   0.329  0.74216
## Knee         -0.1489     0.3366  -0.442  0.65870
## Weight        0.2297     0.0287   8.003  4.8e-14
## Ankle        -0.8348     0.3121  -2.675  0.00798
```

# interlude - correlations of Weight with all others

```
##         Pct BF         Age    Height      Neck     Chest   Abdomen
## [1,] 0.6172994 -0.01605487 0.512913 0.8100143 0.8912862 0.8737351
##          waist       Hip    Thigh      Knee     Ankle    Bicep   Forear
## [1,] 0.8737351 0.9326905 0.852116 0.8427445 0.5809059 0.785214 0.683333
##          Wrist
## [1,] 0.7251042
```

# interlude - scatterplots of Weight versus some others



# backwards with bodyfat: -Knee

| term | estimate | std.error | statistic | p.value |
|---|---|---|---|---|
| (Intercept) | 1.393 | 23.274 | 0.060 | 0.952 |
| Age | 0.070 | 0.031 | 2.266 | 0.024 |
| Weight | -0.019 | 0.066 | -0.290 | 0.772 |
| Height | -0.253 | 0.188 | -1.349 | 0.179 |
| Neck | -0.383 | 0.233 | -1.640 | 0.102 |
| Chest | -0.118 | 0.108 | -1.096 | 0.274 |
| Abdomen | 0.905 | 0.091 | 9.922 | 0.000 |
| Hip | -0.161 | 0.145 | -1.107 | 0.270 |
| Thigh | 0.165 | 0.140 | 1.176 | 0.241 |
| Ankle | 0.178 | 0.216 | 0.823 | 0.411 |
| Bicep | 0.181 | 0.170 | 1.067 | 0.287 |
| Forearm | 0.274 | 0.206 | 1.329 | 0.185 |
| Wrist | -1.808 | 0.531 | -3.407 | 0.001 |

# backwards with bodyfat: -Knee -Weight

| term | estimate | std.error | statistic | p.value |
|---|---|---|---|---|
| (Intercept) | 7.665 | 8.523 | 0.899 | 0.369 |
| Age | 0.072 | 0.031 | 2.359 | 0.019 |
| Height | -0.293 | 0.127 | -2.299 | 0.022 |
| Neck | -0.399 | 0.226 | -1.767 | 0.078 |
| Chest | -0.135 | 0.090 | -1.502 | 0.134 |
| Abdomen | 0.895 | 0.085 | 10.575 | 0.000 |
| Hip | -0.179 | 0.131 | -1.368 | 0.173 |
| Thigh | 0.156 | 0.136 | 1.142 | 0.255 |
| Ankle | 0.164 | 0.210 | 0.781 | 0.436 |
| Bicep | 0.172 | 0.166 | 1.033 | 0.303 |
| Forearm | 0.266 | 0.204 | 1.305 | 0.193 |
| Wrist | -1.837 | 0.521 | -3.527 | 0.001 |

# backwards with bodyfat: -Knee -Weight -Ankle

| term | estimate | std.error | statistic | p.value |
|---|---|---|---|---|
| Abdomen | 0.892 | 0.085 | 10.560 | 0.000 |
| Wrist | -1.713 | 0.496 | -3.456 | 0.001 |
| Age | 0.070 | 0.030 | 2.293 | 0.023 |
| Height | -0.280 | 0.126 | -2.218 | 0.027 |
| Neck | -0.415 | 0.225 | -1.850 | 0.066 |
| Chest | -0.130 | 0.090 | -1.447 | 0.149 |
| Hip | -0.174 | 0.131 | -1.335 | 0.183 |
| Forearm | 0.270 | 0.204 | 1.325 | 0.186 |
| Thigh | 0.165 | 0.136 | 1.214 | 0.226 |
| Bicep | 0.170 | 0.166 | 1.020 | 0.309 |
| (Intercept) | 7.685 | 8.516 | 0.902 | 0.368 |

# backwards with bodyfat: -Knee -Weight -Ankle -Bicep(s)

| term | estimate | std.error | statistic | p.value |
|---|---|---|---|---|
| Abdomen | 0.885 | 0.084 | 10.511 | 0.000 |
| Wrist | -1.679 | 0.495 | -3.395 | 0.001 |
| Age | 0.070 | 0.030 | 2.324 | 0.021 |
| Height | -0.279 | 0.126 | -2.207 | 0.028 |
| Neck | -0.388 | 0.223 | -1.739 | 0.083 |
| Forearm | 0.335 | 0.194 | 1.726 | 0.086 |
| Thigh | 0.205 | 0.130 | 1.581 | 0.115 |
| Hip | -0.176 | 0.131 | -1.345 | 0.180 |
| Chest | -0.114 | 0.088 | -1.287 | 0.199 |
| (Intercept) | 6.251 | 8.400 | 0.744 | 0.458 |

# backwards with bodyfat: -Knee -Weight -Ankle -Bicep(s) -Chest

| term | estimate | std.error | statistic | p.value |
|---|---|---|---|---|
| Abdomen | 0.823 | 0.069 | 11.958 | 0.000 |
| Wrist | -1.731 | 0.494 | -3.506 | 0.001 |
| Age | 0.073 | 0.030 | 2.396 | 0.017 |
| Height | -0.268 | 0.126 | -2.125 | 0.035 |
| Neck | -0.451 | 0.218 | -2.073 | 0.039 |
| Thigh | 0.224 | 0.129 | 1.735 | 0.084 |
| Forearm | 0.296 | 0.192 | 1.542 | 0.124 |
| Hip | -0.195 | 0.130 | -1.501 | 0.135 |
| (Intercept) | 5.040 | 8.359 | 0.603 | 0.547 |

# backwards with bodyfat: -Knee -Weight -Ankle -Bicep(s) -Chest -Hip

| term | estimate | std.error | statistic | p.value |
|---|---|---|---|---|
| Abdomen | 0.756 | 0.052 | 14.408 | 0.000 |
| Wrist | -1.851 | 0.488 | -3.791 | 0.000 |
| Age | 0.081 | 0.030 | 2.718 | 0.007 |
| Height | -0.322 | 0.121 | -2.657 | 0.008 |
| Neck | -0.418 | 0.217 | -1.926 | 0.055 |
| Forearm | 0.288 | 0.192 | 1.499 | 0.135 |
| Thigh | 0.120 | 0.109 | 1.099 | 0.273 |
| (Intercept) | 2.541 | 8.212 | 0.309 | 0.757 |

# backwards with bodyfat: -Knee -Weight -Ankle -Bicep(s) -Chest -Hip -Thigh (could stop here)

| term | estimate | std.error | statistic | p.value |
|---|---|---|---|---|
| Abdomen | 0.793 | 0.040 | 19.703 | 0.000 |
| Wrist | -1.789 | 0.485 | -3.686 | 0.000 |
| Height | -0.315 | 0.121 | -2.601 | 0.010 |
| Age | 0.063 | 0.025 | 2.532 | 0.012 |
| Neck | -0.391 | 0.216 | -1.813 | 0.071 |
| Forearm | 0.315 | 0.191 | 1.653 | 0.100 |
| (Intercept) | 3.607 | 8.159 | 0.442 | 0.659 |

# backwards with bodyfat: -Knee -Weight -Ankle -Bicep(s) -Chest -Hip -Thigh -Forearm (could stop here)

| term | estimate | std.error | statistic | p.value |
|---|---|---|---|---|
| Abdomen | 0.801 | 0.040 | 20.011 | 0.000 |
| Wrist | -1.587 | 0.471 | -3.367 | 0.001 |
| Height | -0.314 | 0.122 | -2.582 | 0.010 |
| Age | 0.052 | 0.024 | 2.152 | 0.032 |
| Neck | -0.287 | 0.207 | -1.384 | 0.168 |
| (Intercept) | 4.621 | 8.164 | 0.566 | 0.572 |

# backwards with bodyfat: -Knee -Weight -Ankle -Bicep(s) -Chest -Hip -Thigh -Neck (rather than forearm) (could stop here)

| term | estimate | std.error | statistic | p.value |
|---|---|---|---|---|
| Abdomen | 0.758 | 0.035 | 21.361 | 0.000 |
| Wrist | -2.129 | 0.450 | -4.735 | 0.000 |
| Height | -0.326 | 0.121 | -2.684 | 0.008 |
| Age | 0.065 | 0.025 | 2.595 | 0.010 |
| Forearm | 0.214 | 0.183 | 1.167 | 0.244 |
| (Intercept) | 1.786 | 8.134 | 0.220 | 0.826 |

# backwards with bodyfat: -Knee -Weight -Ankle -Bicep(s) -Chest -Hip -Thigh -Forearm -Neck (could stop here)

| term | estimate | std.error | statistic | p.value |
|---|---|---|---|---|
| Abdomen | 0.771 | 0.034 | 22.932 | 0.000 |
| Wrist | -1.911 | 0.410 | -4.667 | 0.000 |
| Height | -0.323 | 0.122 | -2.657 | 0.008 |
| Age | 0.056 | 0.024 | 2.351 | 0.020 |
| (Intercept) | 2.900 | 8.084 | 0.359 | 0.720 |

# backwards with bodyfat: -Knee -Weight -Ankle -Bicep(s) -Chest -Hip +Thigh -Forearm -Neck -Wrist (trying a few things)

| term | estimate | std.error | statistic | p.value |
|---|---:|---:|---:|---:|
| Abdomen | 0.693 | 0.052 | 13.412 | 0.000 |
| Height | -0.554 | 0.117 | -4.715 | 0.000 |
| Age | 0.028 | 0.029 | 0.960 | 0.338 |
| (Intercept) | -6.286 | 8.357 | -0.752 | 0.453 |
| Thigh | -0.017 | 0.108 | -0.157 | 0.876 |

# backwards with bodyfat: -Knee -Weight -Ankle -Bicep(s) +Chest -Hip -Thigh -Forearm -Neck -Wrist (trying a few things)

| term | estimate | std.error | statistic | p.value |
|---|---|---|---|---|
| Abdomen | 0.852 | 0.067 | 12.700 | 0.000 |
| Height | -0.523 | 0.114 | -4.569 | 0.000 |
| Chest | -0.228 | 0.083 | -2.735 | 0.007 |
| Age | 0.027 | 0.024 | 1.115 | 0.266 |
| (Intercept) | -1.069 | 8.291 | -0.129 | 0.898 |

| term | estimate | std.error | statistic | p.value |
|---|---|---|---|---|
| Abdomen | 0.771 | 0.034 | 22.932 | 0.000 |
| Wrist | -1.911 | 0.410 | -4.667 | 0.000 |
| Height | -0.323 | 0.122 | -2.657 | 0.008 |
| Age | 0.056 | 0.024 | 2.351 | 0.020 |
| (Intercept) | 2.900 | 8.084 | 0.359 | 0.720 |

# backwards with bodyfat: previous two models compared with $R^2_{adj}$

```
##
## Residual standard error: 4.397 on 245 degrees of freedom
## Multiple R-squared:  0.7235, Adjusted R-squared:  0.719
## F-statistic: 160.3 on 4 and 245 DF,  p-value: < 2.2e-16
```

```
##
## Residual standard error: 4.277 on 245 degrees of freedom
## Multiple R-squared:  0.7383, Adjusted R-squared:  0.7341
## F-statistic: 172.8 on 4 and 245 DF,  p-value: < 2.2e-16
```

# backwards with bodyfat: perspectives

I could try seeing if anything outperforms `Wrist`, for example.

Backwards strategy is a "greedy" method (follows the best path on each short step), which isn't guaranteed to get a "best" model in the end.

The "rankings" of the variables change quite a bit.

Everything is affected by correlations among the inputs.

# forwards with bodyfat

This is a little more tedious:

1. Start with the “best” one-term model.

# forwards with bodyfat

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2. Look at all two-term models (including the step 1 “winner”), and choose the best.

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3. Look at all three-term models (including step 2 “winner”)...

# forwards with bodyfat

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# forwards with bodyfat

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1. Start with the "best" one-term model.
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...until you stop, because adding more terms doesn't seem to accomplish anything.

The "best" could be highest $R_a^2dj$, smallest new p-value, etc.

# forwards with bodyfat - step 1

You can easily find the "best" first model just by finding the input most highly correlated with the output.

| rowname | r |
|---|---|
| Height | -0.029 |
| Ankle | 0.245 |
| Age | 0.295 |
| Wrist | 0.339 |
| Forearm | 0.365 |
| Bicep | 0.482 |
| Neck | 0.489 |
| Knee | 0.492 |
| Thigh | 0.549 |
| Weight | 0.617 |
| Hip | 0.633 |
| Chest | 0.701 |
| Abdomen | 0.824 |
| waist | 0.824 |

# forwards with bodyfat: +Abdomen

The two-term model "winner" (by $R^2_{adj}$) is Weight:

```
##   adj.r.squared
## 1     0.7205176
```

Here's for, say Height:

```
##   adj.r.squared
## 1     0.7108945
```

# perspectives on forwards

Forwards strategy is also a “greedy” method (follows the best path on each short step), which isn’t guaranteed to get a “best” model in the end.

We can immediately see it will result in a different model from the backwards strategy.

The “rankings” of the variables change quite a bit.

Everything is affected by correlations among the inputs.

It is actually the greedy method I tend to use most often.