UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, April, 2015

Second Year — Industrial and Mechanical

MIE237H1 — Statistics

Calculator Type: 2

Exam Type: B

Examiner: N. Montgomery

a) There are 6 pages including this page.

b) You can use the backs of the question pages for rough work.

c) There are 50 marks in total. The number of marks available for each part of each question is indicated.

d) You will not be evaluated on the quality of your writing. Do not waste time writing lengthy answers when concise ones will do just as well.

FAMILY NAME:____

GIVEN NAME:____

STUDENT NUMBER:____

The Story

A gas distribution company is concerned with the health of the gas meters being used by its industrial customers. There are over 20 000 such customers, which is too many to visit to examine each gas meter. So they select a sample of n=400 meters from the database and send technicians to visit only these meters. The technicians perform an analysis of each meter, which includes some testing, and record the following data (along with the meter ID)

- `age`: age of meter in months
- `max_kpa`: maximum test pressure in kPa
- `min_kpa`: minimum test pressure in kPa
- `tot_gas`: cumulative amount of gas in m^3 that meter has recorded
- `volts`: result of an electric test run through meter in V

- `rust`: amount of corrosion recorded as “low”, “medium”, “high”.
- `brand`: either brand“A” or “B”.

In this exam you will analyse the dataset in various ways.

1. **(20 marks total)** First you will have a look at the relationship between `rust` and `volts`. Here is a table that gives sample mean and sample variance for the `volts` readings within each level of `rust`:

```
Descriptive Statistics: volts

Total
Variable  rust    Count     Mean  Variance
volts     high       93   -5.195     1.031
          medium    197  -5.5652    1.3638
          low       110   -5.788     1.641
```

A normal quantile plot of the `volts` readings shows a perfect straight line, as do each of normal quantile plots of the `volts` readings within each `rust` group.

Denote by μ_1, μ_2, μ_3 the mean values of `volts` within the “low”, “medium”, and “high” rust groups respectively.

a) **(6 marks)** Perform the two-sample t-test for the question “is there any difference in mean `volts` between the “low” and “medium” rust levels?” Include a discussion of any model assumptions you need to make.

```
Two Sample t-test

data:  volts by rust
t = -1.5497, df = 305, p-value = 0.1223
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
-0.50638217  0.06019379
sample estimates:
mean in group low mean in group medium
-5.788334            -5.565240
```

b) **(5 marks)** Given the additional information that the sample variance of the `rust` readings is 1.4011, perform the analysis of variance for the question “are there any differences at all among mean `volts` for the different levels of rust?”. Include a discussion of any model assumptions you need to make.

(Hint: the “MSE” is 1.3629. You need to show how this is calculated to get full marks, but I include this number to let you continue if you get stuck on that point.)

```
Source   DF      SS    MS     F      P
rust      2   17.97  8.99  6.59  0.002
Error   397  541.07  1.36
Total   399  559.05

S = 1.167   R-Sq = 3.21%   R-Sq(adj) = 2.73%
```

c) **(5 marks)** Supposing that you had wanted, even in advance of collecting the data, to estimate $\mu_1 - (\mu_2 + \mu_3)/2$. Do so now using a 95% confidence interval.

$$-11.16851 \pm 2.810941 = [-13.97945, -8.357569]$$

d) **(4 marks)** Perform all pairwise comparisons of the mean `volts` readings within different levels of rust, at the experiment-wise error rate of 0.05. (Lots of space here but not because you need it. . . that's just the way it worked out.)

OMITTED

2. **(15 marks)** The simple regression model with `volts` as the response and `max_kpa` as the input, or "x variable", is fit resulting in the following output with some numbers removed. Recall that the sample variance of the `volts` readings is 1.4011 and the sample size is 400.

```
The regression equation is
volts = - 15.8 + ****** max_kpa

Predictor       Coef   SE Coef       T      P
Constant     -15.761     1.055  -14.94  0.000
max_kpa     ********  0.004506    ****  *****

S = *******   R-Sq = 19.13%

Analysis of Variance

Source           DF     SS      MS      F      P
Regression      ***  ******  ******  *****  *****
Residual Error  ***  ******  ******
Total           ***  ******
```

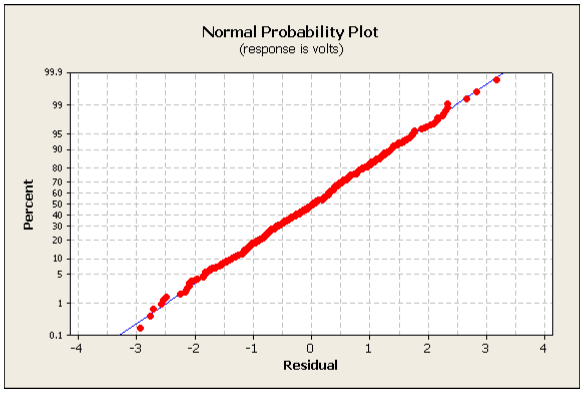
a) **(5 marks)** Show that the correct conclusion to be drawn from the hypothesis test of H_0 : $\beta_1 = 0$ versus $H_1 : \beta_1 \neq 0$ is that there is "overwhelming evidence against the null hypothesis."

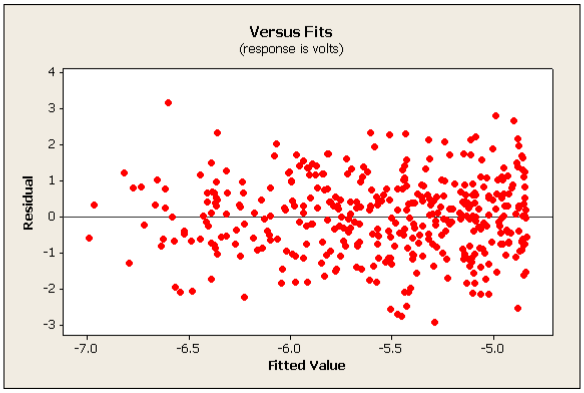
```
The regression equation is
volts = - 15.8 + 0.0437 max_kpa

Predictor      Coef   SE Coef       T      P
```

```
Constant     -15.761      1.055  -14.94  0.000
max_kpa     0.043719  0.004506    9.70  0.000

S = 1.06581   R-Sq = 19.1%   R-Sq(adj) = 18.9%

Analysis of Variance

Source           DF      SS      MS      F      P
Regression        1  106.94  106.94  94.14  0.000
Residual Error  398  452.11    1.14
Total           399  559.05
```

b) **(3 marks)** No plot was given of the data. Draw a simple sketch of what the scatterplot of the data would like like given the correct conclusion to be drawn from a) and the fact that R^2 is "only" 19.13%.

c) **(4 marks)** Here are the normal plot of the residuals and the plot of the residuals versus fitted values. In addition, Minitab gives a list of 27 possible outliers in which the smallest studentized residual is -2.93 and the largest is 3.12. Comment on the extent to which the model assumptions have been satisfied and on the possible existence of outliers.





d) **(3 marks)** A meter outside the sample of 400 being studied here is chosen at random and found to have a `max_kpa` reading that happens (by sheer luck) to be precisely the sample average of the readings in the dataset. Produce a 95% prediction interval for the `volts` reading for this meter.

$$-15.8 + 0.0437\overline{x} \cdot 1.96 \cdot \sqrt{1 + \frac{1}{n}}$$

3. **(10 marks total)** The categorical variables `rust` and `brand` are going to be transformed into so-called dummy variables (or "indicator variables" or "0-1 variables") so that they can be included in a regression model. The new variables called `rust_low`, `rust_medium`, and `brand_A` will be added that take on the values 0 or 1 depending on the actual values of rust and brand in the dataset according to the following tables:

rust	rust_low	rust_medium	brand	brand_ A
low	1	0	A	1
medium	0	1	B	0
high	0	0		

Here are the correlations among the variables that may be included in a regression model in which `volts` remains the output variable:

```
                   age   max_kpa   min_kpa   tot_gas    volts    rust_low  rust_medium
max_kpa          0.077
min_kpa         -0.032    -0.771
tot_gas          0.881     0.077    -0.032
volts            0.070     0.437    -0.488     0.183
rust_low        -0.311     0.004    -0.009    -0.287   -0.129
rust_medium      0.039    -0.040     0.004     0.034   -0.021      -0.607
brand_A          0.010     0.008     0.015    -0.001   -0.031      -0.007       -0.012
```

a) **(4 marks)** What would you expect to see if you were to try to include both `max_kpa` and `min_kpa` at the same time in a regression model with output variable `volts`? Inlude no more than three different possibilities.

b) **(4 marks)** Recall that the sample variance of the `volts` readings is 1.4011 and the sample size is 400. A possible regression model includes the following input variables: `age, max_kpa, min_kpa, tot_gas, rust_low, rust_medium`. The R^2 value is 33.86%. Perform the hypothesis test that answers the question "is there any significant linear relationship between the inputs and the output?"

```
Analysis of Variance

Source           DF       SS      MS      F      P
Regression        6  189.280  31.547  33.53  0.000
Residual Error  393  369.766   0.941
Total           399  559.046
```

c) **(2 marks)** If it possible to do so using the table of correlations, state which of the variables from b) would be the most likely candidate to be removed from the model as part of a general stepwise model selection strategy in which variables can be excluded for having high p-values. If it is not possible, state why not.

4. **(5 marks total)** We'll now consider the variables `rust` and `brand` just by themselves. Here is the contingency table for these two variables with observed cell counts given along with all but one expected cell counts.

```
Rows: rust   Columns: brand
               A       B     All
low           33      77     110
              33.55   *****

medium        59     138     197
              60.09  136.91

high          30      63      93
              28.36   64.64

All           122     278     400
```

a) **(2 marks)** Provide a 95% confidence interval for the proportion of meters with “low” level of rust, being sure to also comment (very briefly!) on any required assumptions for the calculation to be accurate.

```
Variable    X    N  Sample p         95% CI
rust_low  110  400  0.275000  (0.231242, 0.318758)
```

b) **(3 marks)** Perform the hypothesis test that answers the question “are the rows and columns independent?” being sure to also comment (very briefly!) on any required assumptions for the calculation to be accurate.

```
A       B     All

high          30      63      93
28.36   64.64   93.00

low           33      77     110
33.55   76.45  110.00

medium        59     138     197
60.09  136.91  197.00

All          122     278     400
122.00  278.00  400.00

Cell Contents:      Count
Expected count


Pearson Chi-Square = 0.177, DF = 2, P-Value = 0.915
Likelihood Ratio Chi-Square = 0.175, DF = 2, P-Value = 0.916
```