UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, April, 2011

Second Year — Industrial and Mechanical

MIE237H1 S — Statistics & Design of Experiments

Calculator Type: 2

Exam Type: C

Examiner: N. Montgomery

a) There are 17 pages including this page and 4 pages of tables. Please do not detach the tables from the exam—unfortunately this weakens the staple and can lead to an exam falling apart.

b) You can use the backs of the question pages for rough work.

c) There are 60 marks in total. The number of marks available for each part of each question is printed.

Q.	Marks
1.	
2.	
3.	
4.	
5.	
Total	

FAMILY NAME:____

GIVEN NAME:____

STUDENT NUMBER:____________________________________

1. (15 marks total) A company makes packaged soup mixes. The final step of production is a machine that puts some soup mix in a package, seals the package, and puts the packages into boxes for shipping. There are quality checks along the way that can result in some packages not making it into a box for any number of reasons. Also, parts of the machine can break sometimes, and production would stop while they are being fixed.

The company has been experimenting with running the machine at different speeds and seeing how many packages are making it into boxes for shipping. They want to use speed x (measured as "packages per minute filled with mix") to predict output y (measured as "average number of packages per minute boxed in one eight hour shift"). The proposed model is $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$, where the ε_i are independent with a $N(0, \sigma^2)$ distribution.

A sample of 31 pairs of (x, y) measurements is collected and the regression analysis performed. Here is the Minitab output with some parts removed, followed by two plots of the residuals. Some extra information from the data you might need: $\bar{x} = 90$ and $S_{xx} = 9920$.

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The regression equation is
**************************

Predictor      Coef   SE Coef      T      P
Constant     -4.458     3.940  -1.13  0.267
x           0.95959   0.04294  *****  *****

S = *******   R-Sq = *****

Analysis of Variance

Source          DF      SS      MS       F      P
Regression      **  ******  ******  ******  *****
Residual Error  **   530.4  ******
Total           **  9664.9

Unusual Observations

Obs    x        y      Fit  SE Fit  Residual  St Resid
 25  108  107.894   99.178   1.090     8.716     2.11R
 30  118  120.370  108.774   1.427    11.596     2.88R

R denotes an observation with a large standardized residual.
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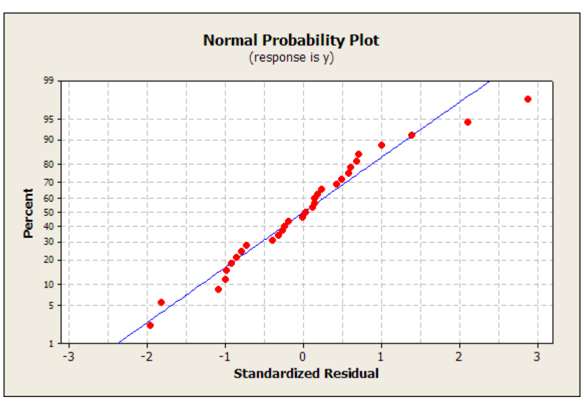
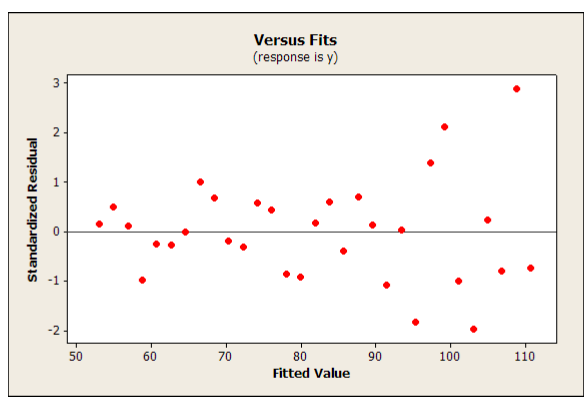




a) **(2 marks)** What is the fitted regression line?

b) **(2 marks)** What is the mean squared error (MSE)?

c) **(3 marks)** Perform the hypothesis test using a t distribution to test if the slope of the regression line is different from 0.

d) **(2 marks)** I always said the value of the intercept term rarely matters and never paid any attention to it. In this case, however, the intercept actually means something, and according to the output there is no evidence that the intercept is non-zero (p-value of 0.267). Explain why an intercept of 0 makes sense in this example.

e) **(3 marks)** Provide a 90% confidence interval for the average number of packages put in boxes per minute when 70 packages are filled per minute.

f) **(3 marks)** If it is possible to produce an accurate 90% interval for the prediction of the number of packages put in boxes per minute when 70 packages per minute are filled, provide the interval. If it is not possible to do so, state why not.

2. (15 marks total) A dataset with sample size 40 consists of a response variable y and four (possible) input variables x_1, x_2, x_3, x_4. So there are 15 possible linear regression models of the type $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \varepsilon$ with $\varepsilon \sim N(0, \sigma^2)$. Excluding an x_i from a model is done by setting its β_i to 0.

Modeling results for all 15 models are summarized in the following table.

The "p-values" entries correspond to p-values for the hypothesis tests $H_0 : \beta_i = 0$ versus $H_1 : \beta_i \neq 0$ for all the β_i included in that row's model, correct to four decimal places. If a p-value entry is 0.0000, it means that p-value is less than 0.00005. If the p-value entry under β_i is '-', it means β_i was set to 0 so that x_i wasn't included in that model. The R^2 for each model is also included, expressed as a percentage.

Number of Inputs	Model #	p-values				R^2
		β_1	β_2	β_3	β_4	
1	1	0.1554	-	-	-	9.3
	2	-	0.0075	-	-	17.8
	3	-	-	0.0009	-	25.6
	4	-	-	-	0.0001	31.9
2	5	0.0635	0.0090	-	-	24.8
	6	0.1859	-	0.0028	-	29.0
	7	0.5719	-	-	0.0010	32.4
	8	-	0.0210	0.0025	-	35.7
	9	-	0.0000	-	0.0000	62.5
	10	-	-	0.1964	0.0269	34.9
3	11	0.1830	0.0220	0.0068	-	38.8
	12	0.7656	0.0000	-	0.0000	62.6
	13	0.5286	-	0.1910	0.0629	35.6
	14	-	0.0000	0.2887	0.0000	63.7
4	15	0.6332	0.0000	0.2664	0.0000	63.9

For example, Model 12 includes x_1, x_2, and x_4, but not x_3 because the p-value for β_3 is given as '-'.

a) **(3 marks)** Which of the two input variables are probably the most highly correlated and why?

b) **(2 marks)** If you were to initially apply the forward regression strategy, which would be the first two models you would select, and why?

c) **(3 marks)** Choose a final model using a general sequential method (possibly with forwards and backwards steps, if necessary) and briefly justify your choice.

d) **(5 marks)** The total sum of squares SST $= \sum_{i=1}^{40} \left(Y_i - \overline{Y}\right)^2$ is 75.36. For the model you chose in (c), perform the hypothesis test using an F distribution that answers the question "is there any significant linear relationship between the inputs and the response".

e) **(2 marks)** In order to check the assumption that the errors are normal you would need to produce the appropriate plot of residuals for the model you chose. Write down the formula for the residuals for the model you chose. (Don't worry about "standardizing" or "studentizing" these residuals, or making any other fancy adjustments.)

3. **(5 marks total)** Consider the basic one-way ANOVA model and analysis setup in the balanced case.

- each of k groups has n observations
- Y_{ij} for the jth observation from the ith group, $\overline{Y}_{i\cdot}$ is the mean response for the ith group, and $\overline{Y}_{\cdot\cdot}$ is the overall mean response.
- The model is $Y_{ij} = \mu_i + \varepsilon_{ij}$, with $\varepsilon_{ij} \sim N(0, \sigma^2)$.

Recall that the basic analysis then proceeds starting with the following "sum of squares decomposition", whose parts are divided by their degrees of freedom and the usual F ratio can be constructed:

$$\sum_{i=i}^{k}\sum_{j=1}^{n_i}\left(Y_{ij} - \overline{Y}_{\cdot\cdot}\right)^2 = \sum_{i=1}^{k} n\left(\overline{Y}_{i\cdot} - \overline{Y}_{\cdot\cdot}\right)^2 + \sum_{i=i}^{k}\sum_{j=1}^{n}\left(Y_{ij} - \overline{Y}_{i\cdot}\right)^2$$

Recall further that the basic formula for the sample variance of $X_1, X_2, \ldots, X_m$ is:

$$S^2 = \frac{\sum_{l=1}^{m}\left(X_l - \overline{X}\right)^2}{m-1}$$

Denote by S_g^2 the sample variance of the k group means (the $\overline{Y}_{i\cdot}$). Denote by S_i^2 the sample variance of the n observations in group k.

Show that n times the sample variance of the group means, divided by the average of the S_i^2, is equal to the usual F ratio used to analyse the basic one-way ANOVA model.

(More space to answer question 3 should you need it.)

4. **(20 marks total)** A company that sells frozen fruits and vegetables wants to test the ability of three preservatives to prevent discoloration in frozen strawberries. A quantity of fresh strawberries was randomly divided into four groups: I, II, III, and IV. Group I was the control group, and received no preseratives. Groups II, III, and IV each received a different preservative. The strawberries from each group were divided into 8 plastic bags and frozen. After 6 months, each bag of strawberries was rated on a scale of 1 to 10 points for discoloration (High score = high discoloration).

The results are summarized in the following table. The last two columns are the sample averages and sample standard deviations for each group.

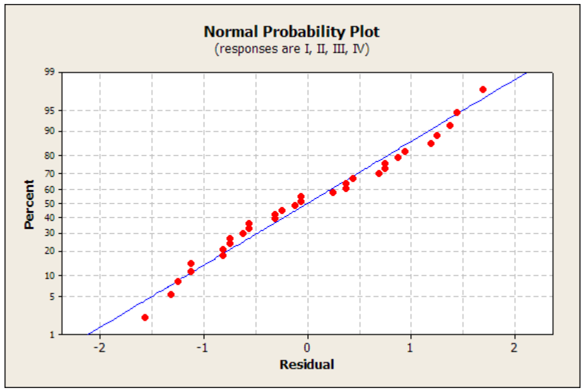
Group	Scores								$\overline{x}_i$	s_i
I	10	8	7.5	8	9.5	9	7.5	7	8.3125	1.0670
II	6	7.5	8	7	6.5	6	7.5	5.5	6.7500	0.8864
III	3	5.5	4	4.5	3	5.0	4.5	3.5	4.1250	0.9161
IV	2	1	2.5	3	4	3.5	2	2.5	2.5625	0.9425

a) **(6 marks)** The total sum of squares is 174.84 and the error sum of squares is 40.69. Fill in the entries in the following ANOVA table:

Source	DF	SS	MS	F
Factor	______	________	________	________
Error	______	________	________	
Total	______	________		

b) **(2 marks)** Perform a test of H_0: there is no difference between the four groups vs. H_1: there is some difference.

c) **(4 marks)** Check to see if the model assumptions have been satisfied. Here is a normal probability plot of the residuals:



d) **(4 marks)** Since the Group III preservative is made of organic ingredients and Group I got no preservative at all (organic by default), the company was interested (in advance of the experiment) to compare Groups I and III versus Groups II and IV. Perform the appropriate hypothesis test for this comparison.

e) **(4 marks)** After looking at the data they were surprised by the difference between I and II and wanted to see if they are "significantly different". Perform the appropriate procedure that allows them to make this comparison, controlling for a 0.05 experiment-wise error rate.

5. (5 marks) In this question I will describe a study whose results were published in the Canadian Medical Association Journal in 2003, and whose results were reported widely in the popular media.

In this question, don't worry about the design or the execution of the study. You can assume these were done as well as they possibly could have been. This question concerns itself with the results themselves.

The researchers wanted to find out if the average waiting time for surgery is different for people who live in rich neighborhoods and people who live in poor neighborhoods. They studied 23 different types of surgery. Among other things, they conducted a two-sample t test for each of the 23 types of surgery, to evaluate the evidence against the hypothesis that there is no difference in average waiting time between rich and poor people.

The study reported that for two of the 23 types of surgery, the p-value obtained was below 0.05. All other p-values were greater than 0.05. The two p-values were 0.026 and 0.037. The Toronto Star newspaper reported there was a significant difference in waiting times for those two types of surgery, listed the surgery types (prostatectomy and tubal ligation), and even went so far as to state whether rich people or poor people had to wait longer, on average, for each of the two surgeries.

Do you think the article in the Toronto Star was correct to give so much attention to these two p-values out of the 23 that were reported?

Critical Values of the t-distribution

	α						
df	0.40	0.30	0.20	0.15	0.10	0.05	0.025
1	0.324	0.726	1.376	1.962	3.077	6.313	12.706
2	0.288	0.617	1.060	1.386	1.885	2.919	4.302
3	0.276	0.584	0.978	1.249	1.637	2.353	3.182
4	0.270	0.568	0.940	1.189	1.533	2.131	2.776
5	0.267	0.559	0.919	1.155	1.475	2.015	2.570
6	0.264	0.553	0.905	1.134	1.439	1.943	2.446
7	0.263	0.549	0.896	1.119	1.414	1.894	2.364
8	0.261	0.545	0.888	1.108	1.396	1.859	2.306
9	0.260	0.543	0.883	1.099	1.383	1.833	2.262
10	0.260	0.541	0.879	1.093	1.372	1.812	2.228
11	0.259	0.539	0.875	1.087	1.363	1.795	2.200
12	0.259	0.538	0.872	1.083	1.356	1.782	2.178
13	0.258	0.537	0.870	1.079	1.350	1.770	2.160
14	0.258	0.536	0.868	1.076	1.345	1.761	2.144
15	0.257	0.535	0.866	1.073	1.340	1.753	2.131
16	0.257	0.535	0.864	1.071	1.336	1.745	2.119
17	0.257	0.534	0.863	1.069	1.333	1.739	2.109
18	0.257	0.533	0.862	1.067	1.330	1.734	2.100
19	0.256	0.533	0.860	1.065	1.327	1.729	2.093
20	0.256	0.532	0.859	1.064	1.325	1.724	2.085
21	0.256	0.532	0.859	1.062	1.323	1.720	2.079
22	0.256	0.532	0.858	1.061	1.321	1.717	2.073
23	0.256	0.531	0.857	1.060	1.319	1.713	2.068
24	0.256	0.531	0.856	1.059	1.317	1.710	2.063
25	0.256	0.531	0.856	1.058	1.316	1.708	2.059
26	0.255	0.530	0.855	1.057	1.314	1.705	2.055
27	0.255	0.530	0.855	1.056	1.313	1.703	2.051
28	0.255	0.530	0.854	1.055	1.312	1.701	2.048
29	0.255	0.530	0.854	1.055	1.311	1.699	2.045
30	0.255	0.530	0.853	1.054	1.310	1.697	2.042
31	0.256	0.530	0.853	1.054	1.309	1.696	2.040
32	0.255	0.530	0.853	1.054	1.309	1.694	2.037
33	0.255	0.530	0.853	1.053	1.308	1.692	2.035
34	0.255	0.529	0.852	1.052	1.307	1.691	2.032
35	0.255	0.529	0.852	1.052	1.306	1.690	2.030
36	0.255	0.529	0.852	1.052	1.306	1.688	2.028
37	0.255	0.529	0.851	1.051	1.305	1.687	2.026
38	0.255	0.529	0.851	1.051	1.304	1.686	2.024
39	0.255	0.529	0.851	1.050	1.304	1.685	2.023
40	0.255	0.528	0.850	1.050	1.303	1.683	2.021
60	0.254	0.527	0.847	1.045	1.295	1.670	2.000
120	0.253	0.525	0.844	1.040	1.288	1.657	1.979
∞	0.253	0.524	0.841	1.036	1.281	1.644	1.959

$\alpha = 0.05$ Critical Values of the F-Distribution

$df2$	$df1$ = 1	2	3	4	5	6	7	8	9
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
31	4.16	3.30	2.91	2.68	2.52	2.41	2.32	2.25	2.20
32	4.15	3.29	2.90	2.67	2.51	2.40	2.31	2.24	2.19
33	4.14	3.28	2.89	2.66	2.50	2.39	2.30	2.23	2.18
34	4.13	3.28	2.88	2.65	2.49	2.38	2.29	2.23	2.17
35	4.12	3.27	2.87	2.64	2.49	2.37	2.29	2.22	2.16
36	4.11	3.26	2.87	2.63	2.48	2.36	2.28	2.21	2.15
37	4.11	3.25	2.86	2.63	2.47	2.36	2.27	2.20	2.14
38	4.10	3.24	2.85	2.62	2.46	2.35	2.26	2.19	2.14
39	4.09	3.24	2.85	2.61	2.46	2.34	2.26	2.19	2.13
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88

$\alpha = 0.01$ Critical Values of the F-Distribution

$df2$	$df1$ = 1	2	3	4	5	6	7	8	9
1	4052.18	4999.50	5403.35	5624.58	5763.65	5858.99	5928.36	5981.07	6022.47
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07
31	7.53	5.36	4.48	3.99	3.67	3.45	3.28	3.15	3.04
32	7.50	5.34	4.46	3.97	3.65	3.43	3.26	3.13	3.02
33	7.47	5.31	4.44	3.95	3.63	3.41	3.24	3.11	3.00
34	7.44	5.29	4.42	3.93	3.61	3.39	3.22	3.09	2.98
35	7.42	5.27	4.40	3.91	3.59	3.37	3.20	3.07	2.96
36	7.40	5.25	4.38	3.89	3.57	3.35	3.18	3.05	2.95
37	7.37	5.23	4.36	3.87	3.56	3.33	3.17	3.04	2.93
38	7.35	5.21	4.34	3.86	3.54	3.32	3.15	3.02	2.92
39	7.33	5.19	4.33	3.84	3.53	3.30	3.14	3.01	2.90
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41

$\alpha = 0.05$ critical values of the studentized range distribution

Error d.f. v	Number of groups k								
	2	3	4	5	6	7	8	9	10
2	6.08	8.33	9.80	10.88	11.73	12.43	13.03	13.54	13.99
3	4.50	5.91	6.82	7.50	8.04	8.48	8.85	9.18	9.46
4	3.93	5.04	5.76	6.29	6.71	7.05	7.35	7.60	7.83
5	3.64	4.60	5.22	5.67	6.03	6.33	6.58	6.80	6.99
6	3.46	4.34	4.90	5.30	5.63	5.90	6.12	6.32	6.49
7	3.34	4.16	4.68	5.06	5.36	5.61	5.82	6.00	6.16
8	3.26	4.04	4.53	4.89	5.17	5.40	5.60	5.77	5.92
9	3.20	3.95	4.41	4.76	5.02	5.24	5.43	5.59	5.74
10	3.15	3.88	4.33	4.65	4.91	5.12	5.30	5.46	5.60
11	3.11	3.82	4.26	4.57	4.82	5.03	5.20	5.35	5.49
12	3.08	3.77	4.20	4.51	4.75	4.95	5.12	5.27	5.39
13	3.06	3.73	4.15	4.45	4.69	4.88	5.05	5.19	5.32
14	3.03	3.70	4.11	4.41	4.64	4.83	4.99	5.13	5.25
15	3.01	3.67	4.08	4.37	4.59	4.78	4.94	5.08	5.20
16	3.00	3.65	4.05	4.33	4.56	4.74	4.90	5.03	5.15
17	2.98	3.63	4.02	4.30	4.52	4.70	4.86	4.99	5.11
18	2.97	3.61	4.00	4.28	4.49	4.67	4.82	4.96	5.07
19	2.96	3.59	3.98	4.25	4.47	4.65	4.79	4.92	5.04
20	2.95	3.58	3.96	4.23	4.45	4.62	4.77	4.90	5.01
21	2.94	3.56	3.94	4.21	4.42	4.60	4.74	4.87	4.98
22	2.93	3.55	3.93	4.20	4.41	4.58	4.72	4.85	4.96
23	2.93	3.54	3.91	4.18	4.39	4.56	4.70	4.83	4.94
24	2.92	3.53	3.90	4.17	4.37	4.54	4.68	4.81	4.92
25	2.91	3.52	3.89	4.15	4.36	4.53	4.67	4.79	4.90
26	2.91	3.51	3.88	4.14	4.35	4.51	4.65	4.77	4.88
27	2.90	3.51	3.87	4.13	4.33	4.50	4.64	4.76	4.86
28	2.90	3.50	3.86	4.12	4.32	4.49	4.62	4.74	4.85
29	2.89	3.49	3.85	4.11	4.31	4.47	4.61	4.73	4.84
30	2.89	3.49	3.85	4.10	4.30	4.46	4.60	4.72	4.82
31	2.88	3.48	3.84	4.09	4.29	4.45	4.59	4.71	4.81
32	2.88	3.48	3.83	4.09	4.28	4.45	4.58	4.70	4.80
33	2.88	3.47	3.83	4.08	4.28	4.44	4.57	4.69	4.79
34	2.87	3.47	3.82	4.07	4.27	4.43	4.56	4.68	4.78
35	2.87	3.46	3.81	4.07	4.26	4.42	4.56	4.67	4.77
36	2.87	3.46	3.81	4.06	4.25	4.41	4.55	4.66	4.76
37	2.87	3.45	3.80	4.05	4.25	4.41	4.54	4.66	4.76
38	2.86	3.45	3.80	4.05	4.24	4.40	4.53	4.65	4.75
39	2.86	3.45	3.79	4.04	4.24	4.39	4.53	4.64	4.74
40	2.86	3.44	3.79	4.04	4.23	4.39	4.52	4.63	4.73