MIE237 Tutorial

Week of February 29, 2016

F and R^2

First let's practice how the F and R^2 business fits in with the rest of the numbers, with the data from question 11.49 from the book. Here is the usual regression output:

```
##
## Call:
## lm(formula = lm(`Heart-Weight` ~ `Body-Weight`, data = wc))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.1669 -1.1364 -0.2301  0.4459  4.0386
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)   12.2546451  1.2076527  10.147 1.25e-08 ***
## `Body-Weight` -0.0001109  0.0003702  -0.299    0.768
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.619 on 17 degrees of freedom
## Multiple R-squared:  0.005248,   Adjusted R-squared:  -0.05327
## F-statistic: 0.08968 on 1 and 17 DF,  p-value: 0.7682
```

The R^2 value is after `Multiple R-squared:`.

Question: construct the ANOVA table for this regression analysis.

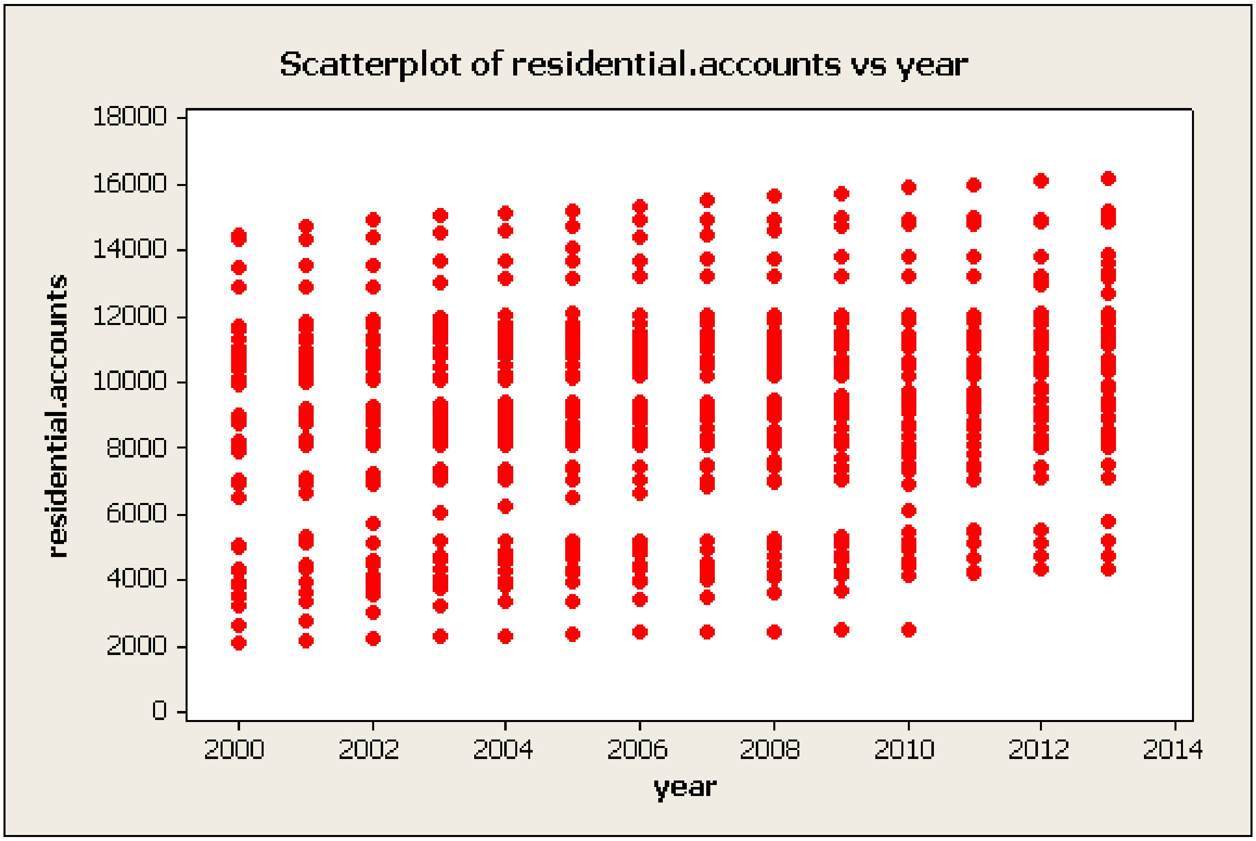
A question from a 2015 midterm

The City of Toronto is divided into 44 areas called `wards''. (Each ward elects one member to the City Council, which is irrelevant to this question, but is mentioned to give you an idea as to the meaning of`ward".)

The City of Toronto provides fresh water to its residents, who pay a fee based on the amount of water used. Some residents of Toronto, such as people who live in houses or condos, have their own water account direct with the city. The accounts are called "residential accounts."

The City of Toronto published a dataset with the number of residential accounts in each of the 44 wards for the 14 years from 2000 up to 2013. In other words, the sample size is 616.

Here is a scatterplot of the data:



It will be inconvenient to work with numbers in the thousands, so I divided those all the residential account count numbers by 1000 to make a new variable called `residential.accounts.000`. Here is the computer output from fitting the simple regression model with `residential.accounts.000` as the response variable and `year` as the input variable.

Some of the numbers have been obscured. For this dataset $S_{xx} = 10026.28$.

The output is not from R, but it is similar.

```
The regression equation is
residential.accounts.000 = ***************************

Predictor      Coef  SE Coef       T      P
Constant    -264.18    62.49   -4.23  0.000
year        *******  *******   *****  *****

S = 3.11591   R-Sq = ***%

Analysis of Variance

Source           DF       SS      MS      F      P
Regression      ***  *******  ******  *****  *****
Residual Error  ***  *******  ******
Total           ***  6146.85
```

1. Provide the missing entries for the Analysis of Variance table. Use the 0.05 and 0.01 tables for F distributions to estimate the p-value. You'll only be able to say one of "less than 0.01", "between 0.01 and 0.05", and "greater than 0.05" using those tables.

2. Estimate β_1 and perform the hypothesis test for $H_0 : \beta_1 = 0$ versus $H_1 : \beta_1 \neq 0$ using a t distribution and basing your conclusion on a p-value.
3. Can you now give a better estimate for the p-value from 1.?
4. R^2 is a measure of the "strength" of a linear model. Calculate its value in this case and explain how it is possible for R^2 to be so low and yet the slope parameter is also significantly different from 0.
5. Estimate the average number of thousands of residential water accounts for a City of Toronto ward for the year 2014. Produce a 95% confidence interval for this estimate.
6. Predict the number of thousands of residential water accounts in a randomly selected City of Toronto ward for the year 2014. Produce a 95% prediction interval for this prediction.
7. The regression analysis using the original data (before dividing the counts of residential accounts by 1000) results in exactly the same p-values. Explain why.