# COMP590: Homework 3

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Assigned: April 11, 2016
Due: April 21, 2016

**Problem 1:** What is the relation between the eigenvalues of $A$ and the eigenvalues of its powers? Conclude that if $\lim_{k\to\infty} A^k \neq \infty$, then all eigenvalues of $A$ belong to the unit circle in the complex plane. What can you say about the eigenvalues of the matrix $e^{-L(\mathcal{D})}$ when $\mathcal{D}$ contains a rooted out-branching?

**Problem 2:** Examine the argument following Corollary 4.2 in Mesbahi and Egerstedt. Provide an analysis for why the value of the Lyapunov function

$$V(x) = max_i x_i - min_i x_i \tag{1}$$

has to decrease at every iteration if $x$ does not belong to the agreement subspace, that is, when $V(x) > 0$. Plot $V(x)$ for a representative digraph on five nodes containing a rooted out-branching running the agreement protocol.

**Problem 3:** Consider the system

$$\dot{\theta}_i(t) = \omega_i + \sum_{j\in\mathcal{N}(i)} \sin(\theta_j(t) - \theta_i(t)), \quad \text{for } i = 1, 2, \ldots, n \tag{2}$$

which resembles the agreement protocol with the linear term $x_j - x_i$ replaced by the nonlinear term $\sin(x_j - x_i)$. For $\omega_i = 0$, simulate (**??**) for $n = 5$ and various connected graphs on five nodes. Do the trajectories of (**??**) always converge for any initialization? How about for $\omega_i \neq 0$? (This is a "simulation-inspired question" so it is okay to conjecture!)