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Cooperative synchronization of robots via estimated state feedback

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Abstract—**A controller that solves the problem of position synchronization of two (or more) robot systems, under a cooperative scheme, in the case when only position measurements are available, is presented. The synchronization controller consists of a feedback control law and a set of nonlinear observers. It is shown that the controller yields semi-global exponential convergence of the synchronization closed loop errors. Experimental results show, despite obvious model uncertainties, a good agreement with the predicted convergence.**

I. INTRODUCTION

In processes like manufacturing and automotive applications, the use of integrated and multi-composed systems for tasks such as assembling and transporting, is widely spread. A multi-composed system is a group of individual systems, either identical or different, that work together to execute a task. Many multi-composed systems work either under cooperative or coordinated schemes. Note that coordinated and cooperative systems are nothing else that a requirement of synchronous behavior of the multi-composed system. Synchronization, coordination, and cooperation are intimately linked subjects and very often they are used as synonymous.

In literature several works related with synchronization of rotating bodies and electromechanical systems can be found, [1], [2]. Synchronization is of great importance as soon as two machines have to cooperate. The cooperative behavior gives manoeuvrability that cannot be achieved by an individual system, e.g. multi finger robot-hands, multi robot systems [3], master-slave systems [4]. According to [1] synchronization may be defined as the mutual time conformity of two or more processes. This conformity can be induced through artificial interactions in the system, e.g. input controls, feedback, resulting in what is called *controlled synchronization.* In controlled synchronization distinction should be made between *internal (mutual) synchronization*, when interconnections between all the systems are considered, e.g. cooperative systems, and *external synchronization*, when there are only interconnections from the leader or dominant system to the non-dominant ones e.g. master-slave systems.

In tasks that cannot be carried out by a single robot, either because of the complexity of the task or limitations of the robot, the use of multi-robot systems working in external synchronization or mutual synchronization has proved to be a good alternative.

In this paper a cooperative (mutual) synchronization controller based only on position measurements is proposed. The general setup throughout this paper is as follows. Consider a multi-robot system formed by p ($p \geq 2$) rigid joint robots together with a common desired trajectory for all of them, denoted by $q_d, \dot{q}_d$. Then, the cooperative synchronization control problem consists of designing interconnections and controllers $\tau_i(\cdot)$ for the p robots, such that the positions and velocities $q_i, \dot{q}_i \in \mathbb{R}^n$ of the i-th robot are synchronized with respect to $q_d, \dot{q}_d$ and to the positions of the other robots $q_j, \dot{q}_j \in \mathbb{R}^n$, ($j = 1, \dots, p$, $j \neq i$). It is assumed that the dynamic model of each robot is known and free of uncertainties. The major constraint to design the synchronization controller is that only joint positions q_i of all the robots are measured. This problem is solved by using nonlinear model-based observers, such that the overall cooperative synchronization controller, i.e. feedback controller plus the observers, guarantees cooperative (mutual) synchronization.

The paper is organized as follows. The dynamic model of the robot is presented in Section II. A mutual synchronization controller for frictionless robots, assuming all measurements available is presented in Section III. Section IV presents a modified synchronization controller which considers nonlinear observers. A gain tuning procedure for the observers and feedback controller gains is given in Section V. Section VI presents a modified version of the controller when friction is considered. An experimental study is presented in Section VII. Some concluding remarks are given in Section VIII.

II. DYNAMIC MODEL OF THE ROBOT

Consider p frictionless rigid joint robots with n joints, i.e. with joint coordinates $q_i \in \mathbb{R}^n$, $i = 1, \dots, p$, all the joints are rotational and fully actuated. Hence, applying the Euler-Lagrange formalism [6] the dynamic model of the i-th robot is given by

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i, \qquad i = 1, \dots, p \quad (1)$$

with $M(q_i) \in \mathbb{R}^{n \times n}$ the symmetric, positive definite inertia matrix, $g_i(q_i) \in \mathbb{R}^n$ the gravity forces, $C_i(q_i, \dot{q}_i) \in \mathbb{R}^n$ the Coriolis and centrifugal forces, and τ_i the vector of torques.

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Various mathematical models for friction phenomena, both static and dynamic, have been proposed. A major difficulty in static friction models is the discontinuity that the Coulomb friction represents. A way to deal with the Coulomb discontinuity is to use approximations.

When friction forces $f_i(\dot{q}_i)$ are considered the model (1) changes to

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) + f_i(\dot{q}_i) = \tau_i \qquad (2)$$

where according to [7], friction forces can be modelled as

$$f_i(\dot{q}_i) = B_{v,i}\dot{q}_i + B_{f1,i}\left(1 - \frac{2}{1 + e^{2w_{1,i}\dot{q}_i}}\right) + B_{f2,i}\left(1 - \frac{2}{1 + e^{2w_{2,i}\dot{q}_i}}\right) \qquad (3)$$

where $B_{v,i}$ is the viscous friction coefficient and the remaining terms model the Coulomb and Stribeck friction effects. $w_{1,i}, w_{2,i}$ determine the slope in the approximation of the sgn function in the Coulomb friction and the Stribeck curve.

The dynamic models (1) and (2) possess several structural properties, see [6].

III. Synchronization Controller Based on Full State Feedback

If the full state of all the robots is available, then the control τ_i, $i = 1, \ldots, p$, can be given by

$$\tau_i = M_i(q_i)\ddot{q}_{ri} + C_i(q_i, \dot{q}_i)\dot{q}_{ri} + g_i(q_i) - K_{d,i}\dot{s}_i - K_{p,i}s_i \quad (4)$$

$K_{p,i}, K_{d,i} \in \mathbb{R}^{n\times n}$ are positive definite gain matrices; $s_i, \dot{s}_i \in \mathbb{R}^n$ are the synchronization errors at position and velocity level

$$s_i := q_i - q_{ri}, \qquad \dot{s}_i := \dot{q}_i - \dot{q}_{ri} \qquad (5)$$

To generate interactions between the robots and to guarantee synchronicity, the reference signals $q_{ri}, \dot{q}_{ri}$ are defined as

$$\begin{aligned} q_{ri} &= q_d - \sum_{j=1, j\neq i}^{p} K_{cp\_i,j}(q_i - q_j) \\ \dot{q}_{ri} &= \dot{q}_d - \sum_{j=1, j\neq i}^{p} K_{cv\_i,j}(\dot{q}_i - \dot{q}_j) \qquad (6) \\ \ddot{q}_{ri} &= \ddot{q}_d - \sum_{j=1, j\neq i}^{p} K_{ca\_i,j}(\ddot{q}_i - \ddot{q}_j) \end{aligned}$$

where $K_{cp\_i,j}$, $K_{cv\_i,j}$, $K_{ca\_i,j} \in \mathbb{R}^{n\times n}$, $i, j = 1, \ldots, p$, are positive semidefinite diagonal matrices that define the interactions between the robots. Therefore information is shared between all the robots. The second term in the right hand side of $q_{ri}, \dot{q}_{ri}$, and $\ddot{q}_{ri}$ represents the "feedback" of cross coupling synchronization errors between the i-th robot and the other robots in the system. Thus (5) and (6) define a trade off. Each robot has to follow the desired common trajectory q_d, at the same time all the robots must mutually synchronize.

Assumption 1: For simplicity it is assumed that the coupling gains $K_{cp\_i,j}$, $K_{cv\_i,j}$, $K_{ca\_i,j}$, $i, j = 1, \ldots, p$, satisfy

$$K_{cp\_i,j} = K_{cv\_i,j} = K_{ca\_i,j} = K_{i,j}$$

Define the coupling (partial) synchronization errors between the i-th and the j-th robots by

$$e_{i,j} = q_i - q_j, \quad \dot{e}_{i,j} = \dot{q}_i - \dot{q}_j, \qquad (7)$$

for all $i, j = 1, \ldots, p, \quad i \neq j$, and for $j = i$ as

$$e_{i,i} = q_i - q_d, \quad \dot{e}_{i,i} = \dot{q}_i - \dot{q}_d \qquad (8)$$

Then $\dot{s}_i, s_i \in \mathbb{R}^n$ defined by (5) can be written as

$$s_i = e_{i,i} + \sum_{j=1, j\neq i}^{p} K_{i,j}e_{i,j}, \qquad \dot{s}_i = \dot{e}_{i,i} + \sum_{j=1, j\neq i}^{p} K_{i,j}\dot{e}_{i,j} \quad (9)$$

A. Stability analysis

Substitution of (4) and (6) in (1) and by considering $s_i, \dot{s}_i$, (5), results in, for $i = 1, \ldots, p$

$$M_i(q_i)\ddot{s}_i = -C_i(q_i, \dot{q}_i)\dot{s}_i - K_{d,i}\dot{s}_i - K_{p,i}s_i \qquad (10)$$

Notice that (10) implies that the synchronization error dynamics is decoupled for every $i = 1, \ldots, p$.

Theorem 1: Consider the closed loop system formed by the controller (4), the reference signals (6) and the p robots (1). Then the synchronization errors $s_i, \dot{s}_i$ are globally asymptotically stable if $K_{d,i}, K_{p,i}$, $i = 1, \ldots, p$ are positive definite.

Proof: Follows from standard Lyapunov theory, see [8], by defining $s = [\ s_1 \ \cdots \ s_p\ ]$, and the Lyapunov function

$$V(s, \dot{s}) = \sum_{i=1}^{p} \left\{ \frac{1}{2}\dot{s}_i^T M_i(q_i)\dot{s}_i + \frac{1}{2}s_i^T K_{p,i}s_i \right\} \qquad (11)$$

■

Note that $s_i, \dot{s}_i$, (9), are linear combinations of the coupling (partial) synchronization errors. Therefore, it is still necessary to prove that $s_i, \dot{s}_i$ being asymptotically stable implies global asymptotic synchronization between the robots.

Lemma 1: Consider the diagonally dominant matrix $M_c(K_{i,j}) \in \mathbb{R}^{(n\cdot p)\times(n\cdot p)}$, given by

$$M_c(K_{i,j}) = \qquad (12)$$

$$\begin{bmatrix} (I_n + \sum_{j=1,j\neq 1}^{p} K_{1,j}) & -K_{1,2} & \cdots & -K_{1,p} \\ -K_{2,1} & (I_n + \sum_{j=1,j\neq 2}^{p} K_{2,j}) & \cdots & -K_{2,p} \\ \vdots & & \ddots & \\ -K_{p,1} & -K_{p,2} & \cdots & (I_n + \sum_{j=1,j\neq p}^{p} K_{p,j}) \end{bmatrix}$$

with $K_{i,j}$, $i, j = 1, \ldots, p$, (6), thus $M_c(K_{i,j})$ can be considered as a coupling matrix between the robots. The matrix $M_c(K_{i,j})$ is nonsingular for all positive semidefinite diagonal matrices $K_{i,j}$, $i,j=1,\dots,p$. Moreover,

$$M_c(K_{i,j})\begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_p \end{bmatrix} = \begin{bmatrix} q_d \\ q_d \\ \vdots \\ q_d \end{bmatrix} \Leftrightarrow \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_p \end{bmatrix} = \begin{bmatrix} q_d \\ q_d \\ \vdots \\ q_d \end{bmatrix} \quad (13)$$

holds for all positive semidefinite diagonal matrices $K_{i,j}$.

Proof: By definition of q_{ri}, (6), $K_{i,j}$, $i,j=1,\dots,p$ are diagonal positive semidefinite, therefore, all their diagonal entries are greater than or equal to zero. Then from Gerschgorin's theorem about location of eigenvalues [9] it can be proved that zero is not an eigenvalue of $M_c(K_{i,j})$ for all diagonal $K_{i,j}\geq 0$, $i,j=1,\dots,p$. Then (13) follows from non singularity of $M_c(K_{i,j})$. ■

Theorem 2: Global asymptotic stability of the synchronization errors $s_i, \dot{s}_i$ implies global asymptotic synchronization of all the robots in the multi-composed synchronization system, i.e. for all $i,j=1,\dots,p$, it follows that $q_i\to q_d$, $\dot{q}_i\to\dot{q}_d$ as $t\to\infty$, so that $q_i\to q_j$, $\dot{q}_i\to\dot{q}_j$ as $t\to\infty$.

Proof: It follows from (9) and the synchronization errors (7), (8), by considering the limit $t\to\infty$ and Lemma 1. ■

Remark 1: Master-slave synchronization can be achieved by taking some $K_{i,j}=0$.

IV. Synchronization Controller Based on Estimated Variables

If it is assumed that only joint positions q_i, $i=1,\dots,p$, are measured, τ_i (4), $\dot{q}_{ri}$, $\ddot{q}_{ri}$, (6), and $\dot{s}_i$, (5), cannot be implemented. As an option τ_i, (4), can be modified as follows

$$\tau_i = M_i(q_i)\widehat{\ddot{q}}_{ri} + C_i(q_i,\widehat{\dot{q}}_i)\widehat{\dot{q}}_{ri} + g_i(q_i) - K_{d,i}\widehat{\dot{s}}_i - K_{p,i}s_i \quad (14)$$

$K_{p,i}, K_{d,i}\in\mathbb{R}^{n\times n}$ are positive definite gain matrices; $\widehat{\dot{s}}_i$, $\widehat{\dot{q}}_i, \widehat{\ddot{q}}_{ri}$ and $\widehat{\dot{q}}_{ri}$ are estimates of $\dot{s}_i$, (5), $\dot{q}_i$ and $\dot{q}_{ri}$, $\ddot{q}_{ri}$, (6), and given by

$$\widehat{\dot{q}}_{ri} = \dot{q}_d - \sum_{j=1,j\neq i}^{p} K_{cv\_i,j}(\widehat{\dot{q}}_i - \widehat{\dot{q}}_j) \quad (15)$$

$$\widehat{\ddot{q}}_{ri} = \ddot{q}_d - \sum_{j=1,j\neq i}^{p} K_{ca\_i,j}(\widehat{\ddot{q}}_i - \widehat{\ddot{q}}_j) \quad (16)$$

$$\widehat{\dot{s}}_i = \widehat{\dot{q}}_i - \widehat{\dot{q}}_{ri}$$

or as function of the coupling synchronization errors

$$\widehat{\dot{s}}_i := \widehat{\dot{e}}_{i,i} + \sum_{j=1,j\neq i}^{p} K_{cv\_i,j}\widehat{\dot{e}}_{i,j} \quad (17)$$

$$\begin{aligned} \widehat{\dot{e}}_{i,j} &= \widehat{\dot{q}}_i - \widehat{\dot{q}}_j, && \text{for all } i,j=1,\dots,p, \quad i\neq j \\ \widehat{\dot{e}}_{i,i} &= \widehat{\dot{q}}_i - \dot{q}_d, && \text{for } i=j \end{aligned} \quad (18)$$

The estimates $\widehat{q}_i, \widehat{\dot{q}}_i, \frac{d}{dt}\widehat{\dot{q}}_i$ can be obtained from the observer

$$\begin{aligned} \frac{d}{dt}\widehat{q}_i &= \widehat{\dot{q}}_i + \mu_{i,1}\widetilde{q}_i \\ \frac{d}{dt}\widehat{\dot{q}}_i &= -M_i(q_i)^{-1}\left[C_i(q_i,\widehat{\dot{q}}_i)\widehat{\dot{q}}_i + g_i(q_i) - \tau_i\right] + \mu_{i,2}\widetilde{q}_i \end{aligned} \quad (19)$$

where the estimation errors $\widetilde{q}_i$ and $\widetilde{\dot{q}}_i$, $\widetilde{\ddot{q}}_i$ are defined by

$$\widetilde{q}_i := q_i - \widehat{q}_i, \quad \widetilde{\dot{q}}_i := \dot{q}_i - \widehat{\dot{q}}_i, \quad \widetilde{\ddot{q}}_i = \frac{d}{dt}\left(\dot{q}_i - \widehat{\dot{q}}_i\right) \quad (20)$$

and $\mu_{i,1}, \mu_{i,2}\in\mathbb{R}^{n\times n}$ are positive definite gain matrices.

The estimated reference signals $\widehat{\ddot{q}}_{ri}$ (16) depend on $\widehat{\ddot{q}}_i$, $i=1,\dots,p$. Therefore when τ_i, (14), is substituted in (19), it generates an algebraic loop between the set of p observers. But this loop can be solved by pure algebraic manipulation.

Lemma 2: The observers given by (19) can be written as

$$\begin{aligned} \frac{d}{dt}\widehat{q}_i &= \widehat{\dot{q}}_i + \mu_{i,1}\widetilde{q}_i \qquad i=1,\dots,p \\ \frac{d}{dt}\widehat{\dot{q}}_i &= f(\ddot{q}_d, q_j, \widehat{\dot{q}}_j, \widetilde{q}_j, e_j, \widehat{\dot{e}}_j) \quad j=1,\dots,p \end{aligned} \quad (21)$$

with $f(\cdot)$ a known nonlinear function that is Lipschitz in $\ddot{q}_d$, q_j, $\widehat{\dot{q}}_j$, $\widetilde{q}_j$, e_j, $\widehat{\dot{e}}_j$.

Proof: Define the vectors $x,y\in\mathbb{R}^{n\cdot p}$ as

$$\begin{aligned} x &= \left[\ \tfrac{d}{dt}\widehat{\dot{q}}_1^T \ \ \tfrac{d}{dt}\widehat{\dot{q}}_2^T \ \cdots \ \tfrac{d}{dt}\widehat{\dot{q}}_p^T\ \right]^T \\ y &= \left[\ y_1^T \ \ y_2^T \ \cdots \ y_p^T\ \right]^T \end{aligned}$$

$$y_i = \ddot{q}_d + \mu_{i,2}\widetilde{q}_i - M_i(q_i)^{-1}\left[C_i(q_i,\widehat{\dot{q}}_i)\widehat{\dot{s}}_i + K_{d,i}\widehat{\dot{s}}_i + K_{p,i}s_i\right]$$

After substitution of τ_i (14) in (19), the second equation of (19) can be written as $M_c(K_{i,j})x=y$, $i=1,\dots,p$, with $M_c(K_{i,j})$ given by (12). Then the conclusion follows from non singularity of $M_c(K_{i,j})$, see Lemma 1. ■

A. Stability analysis

Assumption 2: The gains in the controller (14) and the observers (19) are a positive multiple of the unit matrix, i.e. of the form $K=kI$ with k a positive scalar and I the identity matrix of appropriate dimensions. It is also assumed that the gain matrices in the observers are such that $\mu_{i,1}=\mu_1 I$, $\mu_{i,2}=\mu_2 I$ for all $i=1,\dots,p$.

Assumption 3: The velocity of the common desired trajectory $\dot{q}_d(t)$ is bounded, i.e. there exist $V_M>0$ such that

$$\sup_t \|\dot{q}_d(t)\| = V_M < \infty \quad (22)$$

Based on the Assumptions 1, 2, and 3 the main result of the paper is formulated as follows.

Theorem 3: Consider a multi-robot system formed by p rigid joint robots with dynamic models given by (1). Each robot in closed loop with the controller (14), the reference signals (15, 16) and the observers (19). Introduce a positive scalar parameter η_0, defined throughout the proof. Then the p robots are semiglobally exponentially synchronized, i.e. for $i,j=1,\dots,p$, $q_i\to q_j$, $\dot{q}_i\to\dot{q}_j$ exponentially in a region that can be made arbitrarily large, if the scalar in the gains $K_{p,i}$ $K_{d,i}$, μ_1, μ_2 are chosen such that for $i = 1, \ldots, p$

$$K_{p,i} > 0, \qquad K_{d,i} > 0, \qquad \eta_0 > 0 \tag{23}$$

$$\mu_1 > \max\left\{\mu_{1,1}^*, \ldots, \mu_{1,p}^*\right\} \tag{24}$$

$$\mu_2 > \max\{\frac{1}{M_{i,m}^2}\left(\eta_0^2 - \eta_0\mu_1 - 2V_M C_{i,M}\left(\mu_1 + \eta_0 M_{i,m}^{-1}\right)\right), \mu_{2,1}^*, \ldots, \mu_{2,p}^*\} \tag{25}$$

where $\mu_{i,1}^*, \mu_{i,2}^*$ are scalars given in Section V, $\star_m$, $\star_M$ stand for the minimum and maximum eigenvalue of the matrix $\star$.

Proof: A sketch of the proof is presented in this section, for details see [2].

1) Lyapunov function: Consider the coupled synchronization error dynamics obtained from a state space representation of (1) and substitution of (14), (19), and the partial synchronization errors (8). Define the vectors s, $\widetilde{q}$ as

$$s = \left[\begin{array}{ccc} s_1 & \cdots & s_p \end{array}\right], \qquad \widetilde{q} = \left[\begin{array}{ccc} \widetilde{q}_1 & \cdots & \widetilde{q}_p \end{array}\right] \tag{26}$$

and take as a Lyapunov function

$$V(\dot{s}, \dot{\widetilde{q}}, s, \widetilde{q}) = \sum_{i=1}^{p} V_i(\dot{s}_i, \dot{\widetilde{q}}_i, s_i, \widetilde{q}_i) \tag{27}$$

$$V_i(\dot{s}_i, \dot{\widetilde{q}}_i, s_i, \widetilde{q}_i) = V_{i,1}(\dot{s}_i, s_i) + V_{i,2}(\dot{\widetilde{q}}_i, \widetilde{q}_i) \tag{28}$$

$$V_{i,1}(\dot{s}_i, s_i) = \frac{1}{2}\dot{s}_i^T M_i(q_i)\dot{s}_i + \frac{1}{2}s_i^T K_{p,i} s_i \tag{29}$$

$$V_{i,2}(\dot{\widetilde{q}}_i, \widetilde{q}_i) = \frac{1}{2}\left[\begin{array}{cc} \dot{\widetilde{q}}_i^T & \widetilde{q}_i^T \end{array}\right]\left[\begin{array}{cc} M_i(q_i) & \eta_i(\widetilde{q}_i)I_n \\ \eta_i(\widetilde{q}_i)I_n & \mu_2 + \beta_i I_n \end{array}\right]\left[\begin{array}{c} \dot{\widetilde{q}}_i \\ \widetilde{q}_i \end{array}\right] \tag{30}$$

$$\eta_i(\widetilde{q}_i) = \frac{\eta_0}{1 + \|\widetilde{q}_i\|} \tag{31}$$

$$\beta_i = \eta_0\mu_1 + 2V_M C_{i,M}\left(\mu_1 + \eta_0 M_{i,m}^{-1}\right) - \mu_2\left(1 - M_{i,m}\right) \tag{32}$$

where the scalars $M_{i,m}, C_{i,M}$ are the bounds of $M_i(q_i)$, $C_i(q_i, \dot{q}_i)$, η_o is a positive scalar to be determined.

$V_{i,1}(\dot{s}_i, s_i)$ is positive definite for all $K_{p,i} > 0$. A sufficient condition for positive definiteness of $V_{i,2}(\dot{\widetilde{q}}_i, \widetilde{q}_i)$ is given by

$$\mu_2 > \frac{1}{M_{i,m}^2}\left(\eta_0^2 - \left(\eta_0 + 2V_M C_{i,M}\right)\mu_1 - 2V_M C_{i,M}\eta_0 M_{i,m}^{-1}\right) \tag{33}$$

with $M_{i,m}$ the minimum eigenvalue of the matrix $M_i(q_i)$.

Define the coupled synchronization error s_c as

$$s_c = \left[\begin{array}{cccc} \dot{s} & \dot{\widetilde{q}} & s & \widetilde{q} \end{array}\right] \tag{34}$$

then for some positive scalar P_m, P_M, (27) satisfies

$$P_m \|s_c(t)\|^2 \leq V(s_c(t)) \leq P_M \|s_c(t)\|^2 \tag{35}$$

2) Time derivative of the Lyapunov function: Consider $\widetilde{q}$ defined by (26). Then along the closed loop system the time derivative of (27) has an upper bound given by

$$\dot{V}_i(\dot{s}_i, \dot{\widetilde{q}}_i, s_i, \widetilde{q}_i) \leq \|\Phi_{3,i}\| - \left[\begin{array}{ccc} \|\dot{s}_i\| & \|\dot{\widetilde{q}}\| & \|\widetilde{q}\| \end{array}\right] M_{vi} \left[\begin{array}{ccc} \|\dot{s}_i\| & \|\dot{\widetilde{q}}\| & \|\widetilde{q}\| \end{array}\right]^T \tag{36}$$

with $\|\Phi_{3,i}\|$ the upper bound of non quadratic terms, see [2]; and the matrix M_{vi} given by

$$M_{vi} = \left[\begin{array}{ccc} M_{vi,11} & M_{vi,12} & \mu_1 M_{vi,12} \\ M_{vi,12} & M_{vi,22} & 0 \\ \mu_1 M_{vi,12} & 0 & M_{vi,33} \end{array}\right] \tag{37}$$

$$M_{vi,11} = K_{di} \tag{38}$$

$$M_{vi,12} = -\frac{1}{2}[K_{di} - V_M C_{i,M} + 2V_M \sum_{j=1, j\neq i}^{p} K_{i,j}\left(M_{i,m}M_{j,m}^{-1}C_{j,M} - C_{i,M}\right)] \tag{39}$$

$$M_{vi,22} = M_{i,m}\mu_1 - 2\eta_0 - \frac{1}{2}M_{i,pM} + 2V_M C_{i,M} \tag{40}$$

$$M_{vi,33} = \eta_0\left(\mu_2 + 2\mu_1 V_M C_{i,M} M_{i,M}^{-1}\right) \tag{41}$$

Lemma 3: The matrix $M_{v,i}$, (37), is positive definite if

$$\eta_0 > 0, \qquad K_{di} > 0 \tag{42}$$

$$\mu_{1,i}^* > M_{i,m}^{-1}\left(\frac{M_{vi,12}^2}{M_{vi,11}} + 2\eta_0 + \frac{1}{2}M_{i,pM} - 2V_M C_{i,M}\right) \tag{43}$$

$$\mu_{2,i}^* > \frac{\mu_1^2 M_{vi,12}^2 M_{vi,22}}{\eta_0\left(M_{vi,11}M_{vi,22} - M_{vi,12}^2\right)} - 2\mu_1 V_M C_{i,M} M_{i,M}^{-1} \tag{44}$$

Proof: Follows from Sylvester's criterion. ∎

Remark 2: In (43, 44) $\mu_{1,i}^*$, $\mu_{2,i}^*$, are equivalent to μ_1, μ_2, but the notation is used to emphasize that for each matrix M_{vi}, $i = 1, \ldots, p$, a different μ_1, μ_2 will be obtained.

Consider s_c, (34), then $\dot{V}_i(\dot{s}_i, \dot{\widetilde{q}}_i, s_i, \widetilde{q}_i)$, (36), results in

$$\dot{V}_i(\dot{s}_i, \dot{\widetilde{q}}_i, s_i, \widetilde{q}_i) \leq \|s_c\|^2\left(-M_{vi,m} + \alpha\|s_c\|\right) \tag{45}$$

with $M_{vi,m}$ the minimum eigenvalue of M_{vi}, $i = 1, \ldots, p$, so, $M_{vi,m}$ is positive if (42), (43), and (44) are satisfied.

The coefficient α is determined by $\|\Phi_{3,i}\|$, see [2]. The minimum eigenvalue of M_{vi}, i.e. $M_{vi,m}$, is proportional to μ_2, while α is independent of μ_2. Therefore by increasing μ_2 it can be ensured that for $i = 1, \ldots, p$

$$\dot{V}_i(\dot{s}_i, \dot{\widetilde{q}}_i, s_i, \widetilde{q}_i) \leq \|s_c\|^2\left(-M_{vi,m} + \alpha\|s_c\|\right) < 0$$

So, for $\dot{V}(\dot{s}, \dot{\widetilde{q}}, s, \widetilde{q})$, (36), there exist $\kappa > 0$, such that

$$\dot{V} \leq -\kappa\|s_c\|^2 \qquad \text{for all} \quad t \geq 0$$

thus from (35), there exist constants $m^*, \rho > 0$, such that

$$\|s_c(t)\|^2 \leq m^* e^{-\rho t} \|s_c(0)\|^2 \qquad \text{for all} \quad t \geq 0$$

thus by s_c, (34), the synchronization errors are semi-globally exponentially stable with convergence region β_c

$$\beta_c = \left\{ s_c \quad | \quad \|s_c\| < \frac{M_{vi,m}}{\alpha} \sqrt{\frac{P_m}{P_M}} \right\} \tag{46}$$

Since s_c, (34), is exponentially stable, then s, (26), and s_i, $i = 1, \ldots, p$, are exponentially stable. The rest of the proof follows in the same lines as in the proof of Theorem 2. ∎

V. Gain tuning procedure

The gain tuning procedure to ensure the stability results stated in Theorem 3 can be summarized as follows

1) Determine the bounds of the physical parameters $M_i(q_i)$, $C_i(q_i, \dot{q}_i)$, $\dot{M}_i(q_i)$.
2) Determine the bound of the common desired trajectory at velocity level $\dot{q}_d$, i.e. V_M.
3) Choose positive semidefinite coupling gains $K_{i,j}$ for $i, j = 1, \ldots, p$, $j \neq i$.
4) Choose the scalars on the gains $K_{p,i}$, and $K_{d,i}$, for $i = 1, \ldots, p$, and the auxiliary scalar η_0, to be positive.
5) For $i = 1, \ldots, p$ determine the value $\mu_{1,i}^*$, that is given by (43), and take μ_1 as the maximum of all $\mu_{1,i}^*$.
6) For $i = 1, \ldots, p$ determine the value $\mu_{2,i}^*$, that is given by (44), and take μ_2 as the maximum of all $\mu_{2,i}^*$ and

$$\frac{1}{M_{i,m}^2} \left(\eta_0^2 - \eta_0 \mu_1 - 2 V_M C_{i,M} \left(\mu_1 + \eta_0 M_{i,m}^{-1} \right) \right)$$

VI. Friction compensation

Consider the dynamic model of the robot, (2), then τ_i, (14), can be modified as

$$\begin{aligned} \tau_{if} &= \tau_i + f_i(\hat{\dot{q}}_i) \\ &= M_i(q_i) \hat{\ddot{q}}_{ri} + C_i(q_i, \hat{\dot{q}}_i) \hat{\dot{q}}_{ri} + g_i(q_i) \\ &\quad - K_{d,i} \hat{s}_i - K_{p,i} s_i + f_i(\hat{\dot{q}}_i) \end{aligned} \tag{47}$$

$$\begin{aligned} f_i(\hat{\dot{q}}_i) &= B_{v,i} \hat{\dot{q}}_i + B_{f1,i} \left(1 - \frac{2}{1 + e^{2 w_{1,i} \hat{\dot{q}}_i}} \right) \\ &\quad + B_{f2,i} \left(1 - \frac{2}{1 + e^{2 w_{2,i} \hat{\dot{q}}_i}} \right) \end{aligned} \tag{48}$$

with $\hat{\dot{q}}_i$ the estimated of $\dot{q}_i$, obtained by the observer (21).

The stability analysis of the closed loop system formed by the synchronization controller (47), the observers (21), and the robots in the multi-composed system, follows in the same way as for frictionless rigid joint robots, Theorem 3. To support this note that the friction model (3) implies that

$$\left\| f_i(\hat{\dot{q}}_i) - f_i(\dot{q}_i) \right\| \leq B_{v,iM} \left\| \tilde{\dot{q}}_i \right\| + 2 B_{f1,iM} + 2 B_{f2,iM} \tag{49}$$

with $B_{v,iM}$, $B_{f1,iM}$, $B_{f2,iM}$ the maximum eigenvalue of the coefficient matrices $B_{v,i}$, $B_{f1,i}$, $B_{f2,i}$.

VII. Experimental case study

The controller τ_{if}, (47), has been implemented on a four degree of freedom (d.o.f.) multi-robot system formed by two transposer robots fabricated by the Centre for Manufacturing Technology (CFT) Philips Laboratory. The CFT robot is a Cartesian robot which can move up and down, rotate and translate, see Figure 1. It has 4 Cartesian d.o.f., denoted by x_{ci} ($i = 1, \ldots, 4$), and 7 d.o.f. in the joint space, denoted by q_j ($j = 1, \ldots, 7$), and is actuated by 4 DC brushless servomotors, 3 of the 7 d.o.f. in joint space, $\{q_3, q_6, q_7\}$, are kinematically constrained, so, the robot can be represented by 4 d.o.f. $\{q_1, q_2, q_4, q_5\}$ actuated by 4 servomotors. For a description of the experimental setup see [2].

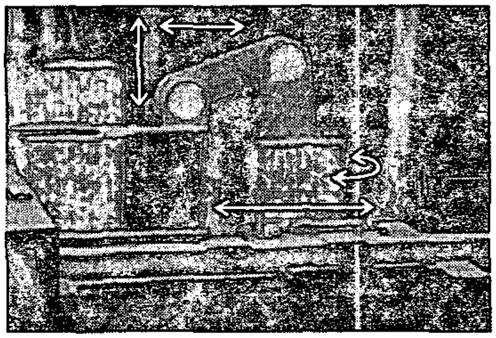

Fig. 1. The CFT-transposer robot

Experimental results: The synchronization controllers τ_{1f}, τ_{2f} are given by (47), with τ_1, τ_2 as in (14). The friction compensation term $f_i(\hat{\dot{q}}_i)$ is a function of $\hat{\dot{q}}_i$, that is obtained by (19), but rewritten according to Lemma 2. All the data, such as gain values, initial values, for the run of experiments can be found in [2]. For the brevity of space only results for the joint $q_{i,5}$ of robots R1 and R2 $i = 1, 2$ are shown in Figure 2. Nevertheless the other joints present similar results, see [2]. Figure 2 shows for the robots R1, R2 the position trajectories desired $q_{d,5}$ and $q_{i,5}$, and the coupling errors $e_{1,1} = q_{1,5} - q_d$ (dashed), $e_{2,2} = q_{2,5} - q_d$ (dotted), and $e_{1,2} = q_{1,5} - q_{2,5}$ (solid), after the transient period has finished, $j = 1, 2, 4, 5$.

From Figure 2 it is evident that synchronization between the robots is achieved. Figure 2 shows that $e_{1,1}, e_{2,2}$ are penalized to minimize the coupling error $e_{1,2}$. Thus mutual synchronization between the robots is favored over the tracking between the robots and the desired common trajectory q_d.

When the coupling gains are set to zero, i.e. $K_{i,j} = 0$ the controller (14) becomes the tracking controller proposed in [10] but based on estimated velocities. Figure 3 shows comparative results between the mutual synchronization controller - coupled case $K_{1,2} = K_{2,1} = 100$ - and the tracking controller - uncoupled case $K_{1,2} = K_{2,1} = 0$ - . It is obvious that in the coupled case the cross error $e_{1,2}$

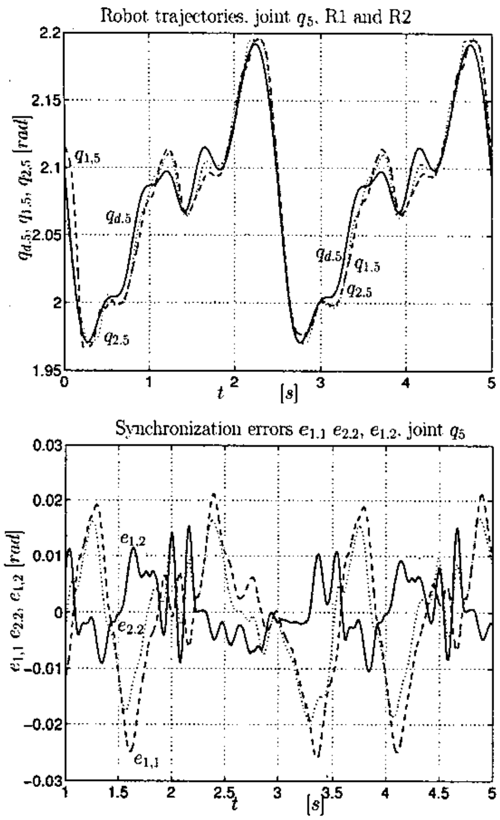

Fig. 2. Position trajectories and synchronization errors for joint q_5.

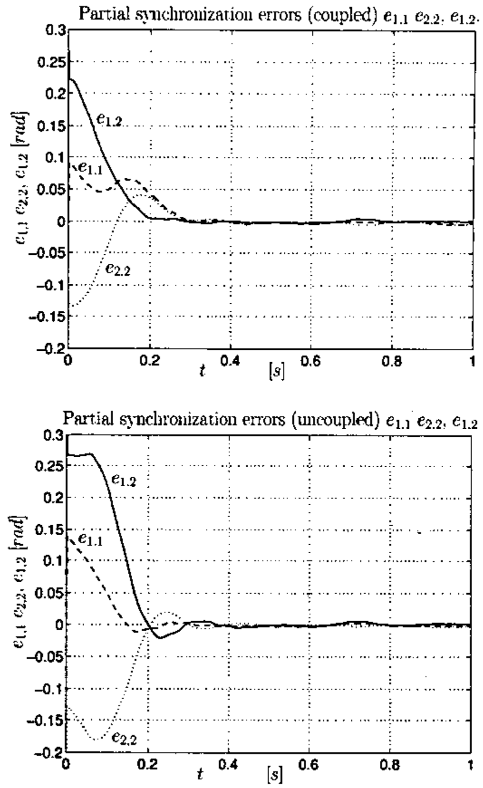

Fig. 3. Synchronization errors: $e_{1,1}$ (dashed), $e_{2,2}$ (dotted), $e_{1,2}$ (solid).

converges faster than the errors $e_{1,1}$, $e_{2,2}$, what proves mutual synchronization.

VIII. Concluding remarks

The proposed synchronizing controller yields semi-global exponential cooperative (mutual) synchronization of a multi-robot system. The controller provides a systematic way of proving synchronization. Nevertheless, even without knowledge of the bounds in (24 - 25), synchronization can be achieved, by selecting the control gains large enough. Although high gains are not desirable since they may amplified the noise in the position measurements.

The advantage of the proposed mutual synchronization scheme over traditional tracking controllers lies in the ability to control the relationships between the position and velocities of all the robots in the system.

IX. References

[1] I. I. Blekhman, P. S. Landa, and M. G. Rosenblum, Synchronization and chaotization in interacting dynamical systems, *ASME Applied Mechanical Review*, vol. 48, pp. 733–752, 1995.

[2] H. Nijmeijer, A. Rodriguez-Angeles, *Synchronization of mechanical systems*, World Scientific Publishing, Singapore, 2003.

[3] Y. H. Liu, Y. Xu, and M. Bergerman, Cooperation control of multiple manipulators with passive joints, *IEEE Trans. Robot. Automat.*, vol. 15, pp. 258–267, 1999.

[4] A. Rodriguez-Angeles and H. Nijmeijer, Coordination of two robot manipulators based on position measurements only, *International Journal of Control*, vol. 74, pp. 1311–1323, 2001.

[5] D. Sun and J. Mills, "Adaptive synchronized control for coordination of two robot manipulators," in *Proceedings of the 2002 IEEE International Conference on Robotics and Automation*, 2002, pp. 976–981.

[6] M. W. Spong and M. Vidyasagar, *Robot Dynamics and Control*, Wiley, New York; 1989.

[7] R. H. A. Hensen, G. Z. Angelis, M. J. G. v. d. Molengraft, A. G. de Jager, and J. J. Kok, Grey-box modeling of friction: An experimental case-study, *European Journal of Control*, vol. 6, pp. 258–267, 2000.

[8] F. T. Lewis, C. T. Abdallah, and D. M. Dawson, *Control of Robot Manipulators*, MacMillan, New York; 1993.

[9] G. Stewart and J.-G. Sun, *Matrix Perturbation Theory*, Academic Press, London; 1990.

[10] B. Paden and R. Panja,Globally asymptotically stable 'PD+' controller for robot manipulators, *Int. J. Control*, vol. 47, pp. 1697–1712, 1988.