COMP590: Midterm 1

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Problem 1

I first state the agreement protocol for this digraph:

$$\dot{x} = -L(\mathcal{D})x$$

If I suppose that $x(t) \neq 0$, then compare against the symmetric algorithm

$$L(\mathcal{D}) = L(\mathcal{D})^T$$

Now,

$$L(\mathcal{D}) = \Delta(\mathcal{D}) - A(\mathcal{D}) \text{ and } \Delta(\mathcal{D}) = \Delta(\mathcal{D})^T$$

And for any two matricies of the same dimensions

$$(A + B)^T = A^T + B^T$$

So

$$A = A^T$$

Which is to say, when this digraph is bidirectional it is identical to our traditional convergence algorithm. However, we can take this reasoning one step farther. If $\mathcal{D}$ is balanced (has the same in-degree as out-degree) and weakly connected,

$$L(\mathcal{D}) + L(\mathcal{D})^T$$

will correspond to a graph laplacian, and so the algorithm will converge in the agreement subspace.

Problem 2

I first note that for any matrix M with eigenvalue λ and eigenvector $\vec{v}$, and all $\alpha, \beta \in \mathbb{R}$, we see that

$$\alpha M + \beta I$$

has eigenvector

$$\alpha\lambda + \beta$$

since

$$(\alpha M + \beta I)\vec{v} = (\alpha\lambda + \beta)\vec{v}$$

This implies that if

$$\dot{x}(t) = (-\alpha L + I\beta)x(t)$$

the new eigenvalues are now $\alpha\lambda_i + \beta$ where λ_i is the eigenvalue of L. From this, I propose the following

$$\dot{x}(t) = (-\alpha L + \mathrm{diag}(d))L$$

and the following matlab code.

```
function [ s, x ] = p2agreement( N, initial_x, alpha, d )
%UNTITLED4 Summary of this function goes here
%   Detailed explanation goes here
L = diag(sum(N)) - N;
x = initial_x;
TOLORANCE = 1e-3;
s = 0;
plot(ones(size(x))*s, x, '*')
hold on
while norm(x - (alpha*ones(size(x))+d)) > TOLORANCE && s < 2e2 %norm(x-mean(x)) > TOLORANCE
    %x = -L*x*1e-3 + x;
    x = (-alpha.*L)*x+d.*x;
    s = s+1;
    plot(ones(size(x))*s, x, '*')
    hold on
end
hold off
```

Problem 3

$$u_i(t)$$

We want

$$p_i + (v_i + u_i(t))dt = \frac{\sum_{j=1,i\neq j}^{n} p_i(t) + v_i(t)dt}{n}$$

$$u_i(t) = \frac{\frac{\sum_{j=1,i\neq j}^{n} p_i(t)+v_i(t)dt}{n} - p_i}{dt} - v_i$$

Problem 4