BirdVision

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1 Physics Engine

1.1 Setup

BirdVision is a classical dynamics package for computing the trajectory of tennis balls. We model a tennis ball as a perfectly spherical object with radius R which has a centre-of-mass position $\mathbf{r}$, velocity $\mathbf{v}$, and angular momentum ω. For our coordinate system we choose the court to lie in the xy-plane with the origin being the very centre of the court. The ball has a mass m and the air density is ρ. For our system we will assume $R = 0.0343\,\mathrm{m}$, $m = 0.00575\,\mathrm{kg}$ and $\rho = 1.293\,\mathrm{kg\,m^{-3}}$.

1.2 Forces

The forces experience by a tennis ball include. The gravitational force is

$$\mathbf{F}_{\mathrm{grav}} = -mg\hat{\mathbf{z}}.$$

The drag force is given by the formula

$$\mathbf{F}_{\mathrm{drag}} = -\frac{1}{2}\rho A C_{\mathrm{drag}} v^2 \hat{\mathbf{v}},$$

and acts in the opposite direction to the motion of the ball. C_{drag} is the coefficient of drag, which we will assume to be 0.507. The Magnus force due to the spin is

$$\mathbf{F}_{\mathrm{spin}} = \frac{1}{2}\rho A C_{\mathrm{lift}} v^2 \left(\hat{\omega} \times \hat{\mathbf{v}}\right).$$

C_{lift} varies as a function of the spin rate $S = R\omega/v$ as (Cross, Lindsey)

$$C_{\mathrm{lift}}\left(S\right) \approx 0.6S.$$

ω is in units of radians per second. One radian per second is equivalent to 9.5492968 rotations per minute (rpm).

1.3 Integrator

Trajectories are propagated through the velocity-Verlet integrator that for a discrete time-step Δt, reads

$$\begin{aligned} \mathbf{r}\left(t+\Delta t\right) &\simeq \mathbf{r}\left(t\right) + \mathbf{v}\left(t\right)\Delta t + \frac{\mathbf{f}\left(t\right)}{2m}\Delta t^2 + \mathcal{O}\left(\Delta t^3\right), && (1) \\ \mathbf{v}\left(t+\Delta t\right) &\simeq \mathbf{v}\left(t\right) + \frac{\mathbf{f}\left(t\right) + \mathbf{f}\left(t+\Delta t\right)}{2m}\Delta t + \mathcal{O}\left(\Delta t^3\right). && (2) \end{aligned}$$

Hence we may deduce the position and momentum of the ball at any time via knowledge of the forces experienced by the ball.

The equations of motion is

$$\begin{aligned} m\dot{\mathbf{v}} &= -mg\hat{\mathbf{z}} - \frac{1}{2}\rho A C_{\mathrm{drag}} v^2 \hat{\mathbf{v}} + \frac{1}{2}\rho A C_{\mathrm{lift}} v^2 \left(\hat{\omega} \times \hat{\mathbf{v}}\right) && (3) \\ \dot{\mathbf{x}} &= \mathbf{v}. && (4) \end{aligned}$$

1.4 Net

A single net has a height of 1.065m and a height at the centre of 0.915m. The equation for a rope under tension and gravity follows the equation

$$z = z_0 + a\left(\cosh\left(\frac{x}{a}\right) - 1\right).$$

Using the boundary conditions $z(0) = 0.915$ and $z(5.02) = 1.065$, we may determine that $a = 84.026$, and so the equation of the net is

$$z = 0.915 + 84.026\left(\cosh\left(\frac{x}{84.026}\right) - 1\right).$$

1.5 File Formats, Input/Output

- Initial configuration file: starting positions and orientation of the ball
- Trajectory file: positions of the ball at every printed time-step
- Event file: registers if the ball hit the net, cleared the net and if so, if it landed in
- Energy file: logs the kinetic and potential energy of the system

Once compiled, the BirdVision executable takes one argument, and input file. The input file is of the form:

```
steps = 100000 # number of time-steps
step_print = 10 # frequency of printing values
dt = 0.001 # size of time-step in seconds
conf = init.conf # absolute path to initial condition
traj = trajectory.dat # name of outputted trajectory
event = event.dat # name of outputted event file
```

Configuration files (initial and trajectories) are lines consisting of 9 floating point numbers corresponding to the 3 Cartesian components of the position $\mathbf{r}$, velocity $\mathbf{v}$ and angular velocity ω.

$$r_x \quad r_y \quad r_z \quad v_x \quad v_y \quad v_z \quad \omega_x \quad \omega_y \quad \omega_z. \tag{5}$$

The event file has the following format: The first entry is the Net? boolean, which is one if the ball hits the net and zero otherwise. The second entry is the Out? boolean, which is 1 if the ball is out and 0 if it is in. If the ball hits the net then the Out? boolean is automatically set to 1. The final entry is the Impact Position, if the ball hits the net then is the position where the ball and net come into contact. If the ball clears the net then the impact position is where the ball hits the floor or "bounces". For example, a ball that clears the net and lands in would have a event file which looks like:

```
Net? Out? Impact Position

0 0 0.5976 1.5488 -0.0066
```