Seperable potentials

$$V(q',q) = g\, f(q') f(q) \quad \cdots\cdots ①$$

$$f(q) = \exp\left(-\frac{q^4}{\Lambda^4}\right)$$

The solution to the LSE

$$R(q',q) = V(q',q) + P\int_0^\infty dk\, k^2 \frac{M}{q^2-k^2}\, V(q',k) R(k,q)$$

$R(q',q)$ has the form

$$R(q',q) = r(q) f(q') f(q) \quad \cdots ②$$

You can easily verify that the above ansatz satisfies the LSE. Substitute Eq. 2 back to the LSE, you will find $r(q)$; thus, $R(q,q)$. On the other hand, you can solve for $R(q,q)$ numerically with Eq. ①.

Compare both.