



1. Para que sea subespacio vectorial:

$$\bullet (x_1, y_1, z_1) + (x_2, y_2, z_2) \in U^3$$

$$(x_1 + x_2, y_1 + y_2, z_1 + z_2) \rightarrow 2x_1 + 2x_2 + 2y_1 + 2y_2 + 2z_1 + 2z_2 =$$

$$\begin{array}{l} 2x_1 + 2y_1 + 2z_1 = 0 \\ 2x_2 + 2y_2 + 2z_2 = 0 \end{array} \quad \Bigg| \quad \begin{array}{l} 2x_1 + 2y_1 + 2z_1 + 2x_2 + 2y_2 + 2z_2 = 0 \\ \parallel \qquad \qquad \parallel \\ 0 \qquad \qquad \qquad 0 \end{array} \quad \checkmark$$

$$\bullet \alpha(x, y, z) \in U^3 \rightarrow 2\alpha x + 2\alpha y + 2\alpha z = \alpha(2x + 2y + 2z) = 0 \rightarrow \alpha \cdot 0 = 0 \quad \checkmark$$

2. Para que sea subespacio vectorial:

$$\bullet (a_0, a_1x, a_2x^2) + (b_0, b_1x, b_2x^2) \quad \text{con dos polinomios que verifiquen:}$$

$$= a_0 + a_1x + a_2x^2 + b_0 + b_1x + b_2x^2 =$$

$$= a_0 + b_0 + (a_1 + b_1)x + (a_2 + b_2)x^2 =$$

$$\Rightarrow a_0 + b_0 + a_2 + b_2 = \underbrace{a_0 + a_2}_{-1} + \underbrace{b_0 + b_2}_{-1} = -1 + (-1) = -2$$

$$a_0 + a_2 = -1$$

$$\text{Para } (1, 0, -2) = a, b = (2, 0, -3),$$

$$3. S = \{ \vec{u}_1 = (-1, 1, 1, 0, -1), \vec{u}_2 = (4, -1, -4, -7, -3), \vec{u}_3 = (4, -1, -1, -1, 0), \vec{u}_4 = (1, 0, 4, -8, 0) \}$$

$$\left(\begin{array}{cccc|c} -1 & 1 & 1 & 0 & -1 \\ 1 & -1 & -1 & 0 & 2 \\ -1 & -1 & -1 & 1 & 8 \\ 0 & -7 & -1 & -9 & 14 \\ -1 & 3 & 0 & 0 & 6 \end{array} \right) \xrightarrow{\substack{F_2' = F_2 + F_1 \\ F_3' = F_3 + F_1 \\ F_5' = F_5 + F_1}} \left(\begin{array}{cccc|c} -1 & 1 & 1 & 0 & -1 \\ 0 & 3 & 3 & 1 & 6 \\ 0 & -8 & -5 & 3 & 16 \\ 0 & -7 & -1 & -8 & 14 \\ 0 & -19 & -16 & -1 & 31 \end{array} \right) \xrightarrow{\substack{F_3' = F_3 - F_2 \\ F_2' = F_2' \\ F_4' = F_2'}}$$

$$\rightarrow \left(\begin{array}{cccc|c} -1 & 1 & 1 & 0 & -1 \\ 0 & 1 & 1 & -11 & 6 \\ 0 & -8 & -5 & 3 & 0 \\ 0 & 3 & 3 & 1 & 0 \\ 0 & -19 & -16 & -1 & 0 \end{array} \right) \xrightarrow{\substack{F_3' = F_3 + 8F_2 \\ F_4' = F_4 + 3F_2 \\ F_5' = F_5 + 19F_2}} \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 8 \\ 0 & 1 & 1 & -11 & 6 \\ 0 & 0 & 27 & -35 & 48 \\ 0 & 0 & -9 & -32 & 18 \\ 0 & 0 & 60 & -213 & 114 \end{array} \right) \xrightarrow{\substack{F_3' = F_3/3 \\ F_4' = F_4/3 \\ F_5' = F_5/3}}$$

$$\rightarrow \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 8 \\ 0 & 1 & 1 & -11 & 6 \\ 0 & 0 & 9 & -35 & 16 \\ 0 & 0 & -3 & -32 & 6 \\ 0 & 0 & 20 & -71 & 0 \end{array} \right) \xrightarrow{\substack{F_1' = F_1 + 3F_2 \\ F_5' = F_5 + 3F_2}} \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 8 \\ 0 & 1 & 1 & -11 & 6 \\ 0 & 0 & 9 & -35 & 16 \\ 0 & 0 & -3 & -32 & 6 \\ 0 & 0 & 0 & -10 & -10 \end{array} \right) \xrightarrow{\substack{\alpha_1, \alpha_2, \alpha_3 \\ \alpha_4, \alpha_5 = 0}} \text{LIBRE}$$