1. $\begin{pmatrix} 0 & 0 & 0 & -3 & -2 \\ 0 & 0 & -2 & -5 & 1 \\ 0 & 0 & -3 & -9 & 0 \\ 1 & 0 & 0 & -6 & -3 \end{pmatrix} \xrightarrow[F_4'=F_1]{F_1'=F_4} \begin{pmatrix} 1 & 0 & 0 & -6 & -3 \\ 0 & 0 & -2 & -5 & 1 \\ 0 & 0 & -3 & -9 & 0 \\ 0 & 0 & 0 & -3 & -2 \end{pmatrix}$

$(0, 1, 0, 0, 0)$

2. $B'\vec{V}_{B'} = B\vec{V}_B \rightarrow P = B^{-1}B'$

$B = \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 1 \end{array}\right) \xrightarrow[F_3'=F_3-F_1]{F_2'=F_2+F_1} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & -1 & 0 & 1 \end{array}\right)$

$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & -1 \end{array}\right)$
$\quad I \qquad\qquad B^{-1}$

$P = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 2 \\ 1 & -1 & 1 \end{pmatrix}$

2) $B_2 \cdot \vec{\omega_2} = I \cdot \vec{\omega_c}$

$\vec{\omega_2} = B_2^{-1} \cdot I \cdot \vec{\omega_c}$

$\vec{\omega_2} = B_2^{-1} \cdot \vec{\omega_c}$ siendo B_2^{-1} B'^{-1}

$B' = \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{array}\right) \xrightarrow[F_3'=F_3-2F_2]{F_1'=F_1-F_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & -2 & 1 \end{array}\right)$

$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 3 & -2 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & -2 & -1 \end{array}\right)$ $\quad \vec{\omega_2} = \begin{pmatrix} 1 & 3 & -2 \\ 0 & 1 & 0 \\ 0 & -2 & -1 \end{pmatrix}\begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix} = \begin{pmatrix} -12 \\ -2 \\ -1 \end{pmatrix}$
$\qquad\qquad B'^{-1}$

3. de la misma forma, $\vec{\omega_1} = B_1^{-1} \cdot \vec{\omega_c}$

$\vec{\omega_1} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & -1 \end{pmatrix}\begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$

4. $P\vec{\omega_2} = \vec{\omega_1}$

$P \begin{pmatrix} -12 \\ -2 \\ -1 \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix}$

$$U_1 \rightarrow \begin{pmatrix} -3 & -4 & 0 & 0 & 1 \\ -12 & -16 & 1 & 0 & 7 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} -3 & -4 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$B_{U_1} = [(-3,-4,0,0,1),(0,0,1,0,3),(0,0,0,0,1)]$

Como es ~~[illegible]~~ linealmente independiente, $\dim(U_1)=3$

$$U_2 \rightarrow \begin{pmatrix} -3 & -2 & 2 & 2 & 4 \\ -3 & -6 & 12 & 5 & 3 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & 0 & 2 & 1 & 1 \end{pmatrix} \xrightarrow{F_2' = F_2 - F_1} \begin{pmatrix} -3 & -2 & 2 & 2 & 4 \\ 0 & -4 & 10 & 3 & -1 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & 0 & 2 & 1 & 1 \end{pmatrix} \begin{matrix} F_2' = F_3 \\ F_3' = F_2 \end{matrix}$$

$$\rightarrow \begin{pmatrix} -3 & -2 & 2 & 2 & 4 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & -4 & 10 & 3 & -1 \\ 0 & 0 & 2 & 1 & 1 \end{pmatrix} \xrightarrow{F_3' = F_3 + 4F_2} \begin{pmatrix} -3 & -2 & 2 & 2 & 4 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & 0 & 14 & 7 & 7 \\ \cancel{0} & \cancel{0} & \cancel{2} & \cancel{1} & \cancel{1} \end{pmatrix}$$

$B_{U_2} = [(-3,-2,2,2,4),(0,1,1,1,2),(0,0,2,1,1)]$

$\dim(U_2)=3$

$$U_1+U_2: \begin{pmatrix} -3 & 0 & 0 & -3 & 0 & 0 \\ -4 & 0 & 0 & -2 & 1 & 0 \\ 0 & 1 & 0 & 2 & 1 & 2 \\ 0 & 0 & 0 & 2 & 1 & 1 \\ 1 & 3 & 1 & 4 & 2 & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} -3 & -3 & 0 & 0 & 0 & 0 \\ -4 & -2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & \text{[illegible]} & \text{[illegible]} & 0 \\ 0 & 2 & 1 & \text{[illegible]} & \text{[illegible]} & 0 \\ 1 & 4 & 2 & \text{[illegible]} & \text{[illegible]} & 1 \end{pmatrix}$$

$$\longrightarrow \cancel{\begin{pmatrix} -3 & 1 & 0 & 0 & 0 & 0 \\ -4 & -2 & 1 & 0 & 0 & 0 \\ -2 & -10 & -3 & -5 & 0 & 0 \\ -1 & -2 & -1 & -3 & 0 & 0 \\ 1 & 4 & 2 & 3 & 1 & 1 \end{pmatrix}} \sim \overset{C_2' = C_2 + C_1}{\begin{pmatrix} -3 & 0 & 0 & 0 & 0 & 0 \\ -4 & 2 & 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 1 & 2 & 0 \\ 0 & 2 & 1 & 0 & 1 & 0 \\ 1 & 3 & 2 & 3 & 1 & 1 \end{pmatrix}} \begin{matrix} C_3' = C_3 (\\ \sim \end{matrix}$$

$$\begin{pmatrix} -3 & 0 & \cancel{0} & 0 & 0 & 0 \\ -4 & 2 & \cancel{0} & 0 & 0 & 0 \\ 0 & 2 & \cancel{0} & 2 & 1 & 0 \\ 0 & 2 & \cancel{0} & 1 & 0 & 0 \\ 1 & 3 & -1 & 1 & 3 & 1 \end{pmatrix} \sim \overset{C_4' = -2C_4 + C_2}{\begin{pmatrix} -3 & 0 & 0 & 0 & 0 \\ -4 & 2 & 0 & 0 & 0 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 2 & 1 & 1 & 0 \\ 1 & 3 & 1 & -5 & 1 \end{pmatrix}}$$

$B_{U_1+U_2} = ((-3,-4,0,0,1),(0,2,2,2,3),(0,0,2,1,1),(0,0,0,1,-5),$
$(0,0,0,0,1)$ $\quad \dim(U_1+U_2)=5$

$\dim(U_1 \cap U_2) = \cancel{[illegible]} \dim(U_1)+\dim(U_2)-\dim(U_1+U_2) = 1$

$\alpha(-3,-4,0,0,1)+\beta(0,0,1,0,3)+\gamma(0,0,0,0,1) = \mu(-3,-2,2,2,4)+\lambda(0,1,1,1,2)+\sigma$

$$\left.\begin{array}{l} -3\alpha = -3\mu \\ -4\alpha = -2\mu \\ \beta = 2\mu+\lambda+2\sigma \\ 0 = 2\mu+\lambda+\sigma \\ \alpha+3\beta+\gamma = 4\mu+2\lambda+\sigma \end{array}\right\} \begin{array}{l} \alpha=\mu \\ \lambda=-2\mu \\ \sigma=0 \end{array} \left| \begin{array}{l} \mu=-1 \\ \vec{v}=\mu(-3,-4,0,0,0) \\ \boxed{\vec{v}=(3,4,0,0,0)} \end{array}\right.$$