

1. Paso 1: Espacio vectorial \mathbb{R}^4 $PX_{B_1} = X_{B_1}$

Como la antigua base es la base canónica,

$$P = \begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & -1 & -1 \\ 1 & 0 & 0 & 1 \\ -1 & -1 & 0 & 1 \end{pmatrix}$$

Paso 2: Espacio vectorial \mathbb{R}^3 $QY_{B_3} = Y_{B_3}$

$$Q = \begin{pmatrix} 2 & 1 & -2 \\ 0 & 1 & -1 \\ 1 & 2 & -2 \end{pmatrix}, \text{ directamente ya que la base antigua es la canónica.}$$

Paso 3: Aplicación lineal $AX_{B_1} = Y_{B_3}$

$$\text{¿} M X_{B_1} = Y_{B_3} \text{?}$$

$$\downarrow$$

$$APX_{B_1} = QY_{B_3}$$

$$Q^{-1}APX_{B_1} = Y_{B_3}$$

$$M = Q^{-1}AP$$

$$\begin{array}{c} \left(\begin{array}{ccc|ccc} 2 & 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 1 & 2 & -2 & 0 & 0 & 1 \end{array} \right) \xrightarrow[\substack{F_1' = F_3 \\ F_3' = F_1 - F_3}]{\substack{F_2' = F_2 - F_1}} \left(\begin{array}{ccc|ccc} 1 & 2 & -2 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & -3 & 2 & 1 & 0 & -2 \end{array} \right) \\ \downarrow Q \\ \left(\begin{array}{ccc|ccc} 1 & 2 & -2 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 3 & -2 \end{array} \right) \xrightarrow[\substack{F_2' = F_2 + F_3'}]{\substack{F_3' = F_3 \cdot (-1)}} \left(\begin{array}{ccc|ccc} 1 & 2 & -2 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 & 2 & 2 \\ 0 & 0 & 1 & -1 & -3 & 2 \end{array} \right) \end{array}$$

$$\xrightarrow{F_1' = F_1 + 2F_3'} \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & -2 & -6 & 5 \\ 0 & 1 & 0 & -1 & 2 & 2 \\ 0 & 0 & 1 & -1 & -3 & 2 \end{array} \right) \xrightarrow{F_2' = F_2 - F_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & -8 & 1 \\ 0 & 1 & 0 & -1 & 2 & 2 \\ 0 & 0 & 1 & -1 & -3 & 2 \end{array} \right)$$

$$M = \begin{pmatrix} 0 & -2 & 1 \\ -1 & -2 & 2 \\ -1 & -3 & 2 \end{pmatrix} \begin{pmatrix} -3 & -1 & 9 & -2 \\ -2 & -1 & 0 & -2 \\ -5 & -3 & 7 & -5 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & -1 & -1 \\ 1 & 0 & 0 & 1 \\ -1 & -1 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} -1 & -1 & 4 & -1 \\ -3 & -3 & -1 & -4 \\ -1 & -2 & -1 & -2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & -1 & -1 \\ 1 & 0 & 0 & 1 \\ -1 & -1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 0 & 0 & 8 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{aligned}
 f(1,0,0) &= (-2, 4, 3, -1) \\
 f(0,1,0) &= (6, -6, -9, 3) \\
 f(0,0,1) &= (-2, 4, 2, -1)
 \end{aligned}
 \quad A = \begin{pmatrix} -2 & 6 & -2 \\ 4 & -6 & 4 \\ 3 & -9 & 2 \\ -1 & 3 & -1 \end{pmatrix}$$

Para 1. $P X_{B_1} = X'_{B_1}$ $P = \begin{pmatrix} 2 & 0 & -1 & -1 \\ 0 & -1 & -1 & -1 \\ 1 & 0 & 0 & -1 \\ -1 & -1 & 0 & -2 \end{pmatrix}$

Para 2. $Q Y_{B_2} = Y'_{B_2}$ $Q = \begin{pmatrix} 0 & -1 & 1 \\ 0 & 1 & 0 \\ -1 & -1 & 1 \end{pmatrix}$

Para 3. $M = P^{-1} A Q$

~~$$\begin{pmatrix} 2 & 0 & -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & -1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 & 1 & 0 \\ -1 & -1 & 0 & -2 & 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{I} \begin{pmatrix} 1 & 0 & 0 & 0 & -1 & 1 & 2 & -1 \\ 0 & 1 & 0 & 0 & 3 & -3 & -9 & 3 \\ 0 & 0 & 1 & 0 & -2 & 1 & 3 & -1 \\ 0 & 0 & 0 & 1 & -1 & 1 & 1 & -1 \end{pmatrix}$$

$$M = \begin{pmatrix} -1 & 1 & 2 & -1 \\ 3 & -3 & -9 & 3 \\ -2 & 1 & 3 & -1 \\ -1 & 1 & 1 & -1 \end{pmatrix} \begin{pmatrix} -2 & 6 & -2 \\ 4 & -6 & 4 \\ 3 & -9 & 2 \\ -1 & 3 & -1 \end{pmatrix} \begin{pmatrix} 0 & -1 & 1 \\ 0 & 1 & 0 \\ -1 & -1 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} 13 & -33 & 11 \\ -32 & 28 & -28 \\ 18 & -48 & 15 \\ 10 & -24 & 5 \end{pmatrix} \begin{pmatrix} 0 & -1 & 1 \\ 0 & 1 & 0 \\ -1 & -1 & 1 \end{pmatrix} \quad \text{No}$$~~