

$$1. A = \begin{pmatrix} 1 & 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & -2 & 1 \\ -4 & -4 & -4 & 16 & -4 \\ 3 & 35 & 3 & 6 & 35 \end{pmatrix} \begin{array}{l} F_3' = F_3 + 7F_1 \\ F_2' = F_2 - F_1 \\ F_4' = F_4 - 3F_1 \end{array} \rightarrow \begin{pmatrix} 1 & 1 & 1 & 2 & 1 \\ 0 & 0 & 0 & -4 & 0 \\ 0 & 0 & 0 & 24 & 0 \\ 0 & 32 & 0 & 0 & 32 \end{pmatrix} \begin{array}{l} F_2' = F_2/4 \\ F_4' = F_4/32 \\ F_3' = F_3/24 \end{array}$$

$$\rightarrow \begin{pmatrix} 1 & 1 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{array}{l} F_2' = F_2 \\ F_4' = F_4 - F_3 \end{array} \rightarrow \begin{pmatrix} 1 & 1 & 1 & 2 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & 0 & 1 & 2 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{array}{l} F_1' = F_1 - 2F_3 \\ F_4' = F_4 - F_3 \end{array} \rightarrow \begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$2. A(x=-2) = \begin{pmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & -12 & 12 \end{pmatrix} \begin{array}{l} F_3' = F_3 - F_2 \\ F_4' = F_4/12 \end{array} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{array}{l} F_3' = F_3 - F_2 \\ F_4' = F_4 - F_2 \end{array}$$

$$\rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 & -3 \end{pmatrix} \begin{array}{l} F_1' = F_1 + F_3 \\ F_2' = F_2 + 2F_3 \\ F_4' = F_4 - \frac{3}{2}F_3 \end{array} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{array}{l} F_2' = F_2/(-2) \\ F_3' = F_3/(-2) \end{array}$$

$$\rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$3. A = \begin{pmatrix} 1 & i & 1 \\ 1 & -1+2i & 2+2i \\ 1 & -2+3i & 3+4i \end{pmatrix} \begin{array}{l} F_2' = F_2 - F_1 \\ F_3' = F_3 - F_1 \end{array} \rightarrow \begin{pmatrix} 1 & i & 1 \\ 0 & -1+i & 1+2i \\ 0 & -2+2i & 2+4i \end{pmatrix} \begin{array}{l} F_3' = F_3/2 \\ F_2' = F_2 - 2F_1 \end{array}$$

$$\rightarrow \begin{pmatrix} 1 & i & 1 \\ 0 & -1+i & 1+2i \\ 0 & 0 & 0 \end{pmatrix} \begin{array}{l} F_2' = F_2/(1+i) \\ F_3' = F_3/(1+i) \end{array} \rightarrow \begin{pmatrix} 1 & i & 1 \\ 0 & 1 & \frac{1+2i}{-1+i} \\ 0 & 0 & 0 \end{pmatrix} \begin{array}{l} F_2' = F_2 - iF_1 \\ F_3' = F_3 - iF_1 \end{array}$$

$$\rightarrow \begin{pmatrix} 1 & 0 & \frac{1}{5} - \frac{3}{5}i \\ 0 & 1 & \frac{3}{5} - \frac{1}{5}i \\ 0 & 0 & 0 \end{pmatrix}$$

$$\frac{1+2i}{-1+i} = \frac{1+2i}{1-2i} = \frac{(1+2i)^2}{(1+4)} = \frac{1-4+4i}{5} = \frac{-3+4i}{5} = -\frac{3}{5} + \frac{4}{5}i$$