Cycle-based Synthesis and Exact Synthesis

PART 2

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Outline

- Early calculations
- Tool for finding minimal circuits for 4–variable reversible functions
- Extention of the tool
- Extrapolation of reversible functions
- Examples of constructing infinite sequences of gate count rovably minimal reversible circuits

Early exact synthesis

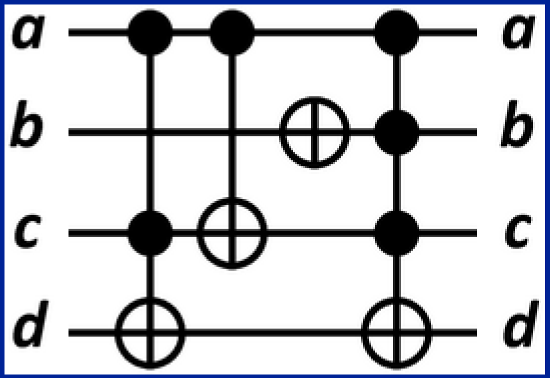
- Exhaustive calculation of one gate minimal reversible circuit for all 3-variable functions (Shende et al., IEEE TCAD 2003)
- Exhaustive calculation of all gate minimal reversible circuit for all 3-variable functions (Kerntopf, RM Workshop 2007)
- Satisfiability equations (using Quantified Boolean Functions) solved using SAT engines (Grosse, Wille, Dueck, Drechsler, IEEE TCAD 2009)

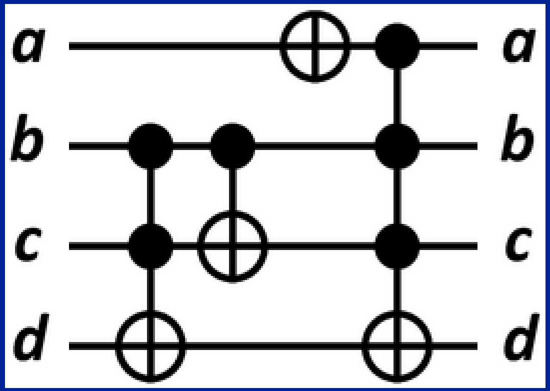
Minimization of 4-input/output circuits

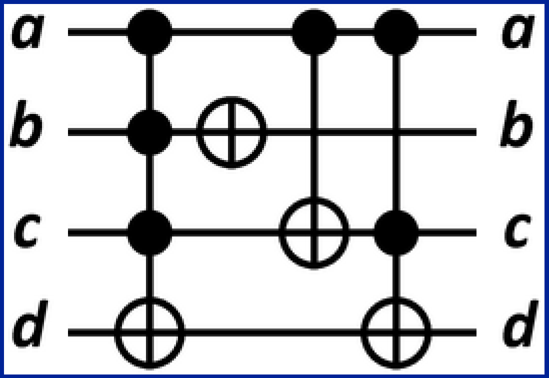
- Golubitsky, Falconer and Maslov (DAC 2010) developed a tool for synthesis of gate count minimal 4-input/output reversible circuits
- Golubitsky and Maslov (IEEE Trans. on Comp. 2012) have shown that optimal 4-input/output circuits require no more than 15 gates

Symmetry in reversible circuits

- Simultaneous input/output permutations (4!=24)
- Inversion (mirror image) (2)







- Canonical representation

Reversible Circuit Database Structure

1-gate circuits

2-gate circuits

3-gate circuits

4-gate circuits

5-gate circuits

6-gate circuits

7-gate circuits

8-gate circuits

9-gate circuits

Minimization tool (Golubitsky, Maslov)

- Library of optimal circuits up to 9 (8) gates
- Circuit compaction under simultaneous input/output relabeling and inversion
- Effective hashmap implementation
- Larger circuits generated by combining all pairs of shorter circuits from the library
- Synthesis speed: less than 0.008s on average for generating a minimal 4x4 circuit (43GB of RAM used)

Constructing optimal circuits

- **A reversible function in the database is stored in hash table with the last gate of its optimal circuit**
- **The GC of the optimal circuit can be quickly checked by a simple lookup of a canonical representative**
- **By combining the information about the gate count optimal circuits up to *n* gates one can construct optimal circuits for functions requiring up to 2*n* gates:**

- an optimal circuit for a function *f* can be partitioned into two optimal circuits such that $f = g \circ r$ and $f \circ r^{-1} = g$

- the length of the optimal circuit for *f* is found

- the optimal circuit is built recursively

Extended tool for reduction of QC

- *nth_prime4_inc* [Maslov's Benchmarks Page]
- Best known: QC = 51 (GC=15) [Saeedi et al. 2010]
- Our result: QC = 26 (GC=14)

GC	QC	#circuits	time [s]
11	53-55	12	10
12	32-46	2288	591
13	31-93	187945	7282
14	**26**-114	11056332	292578

Library up to 8 gates, 16GB of memory, Power5+ 1,65GHz

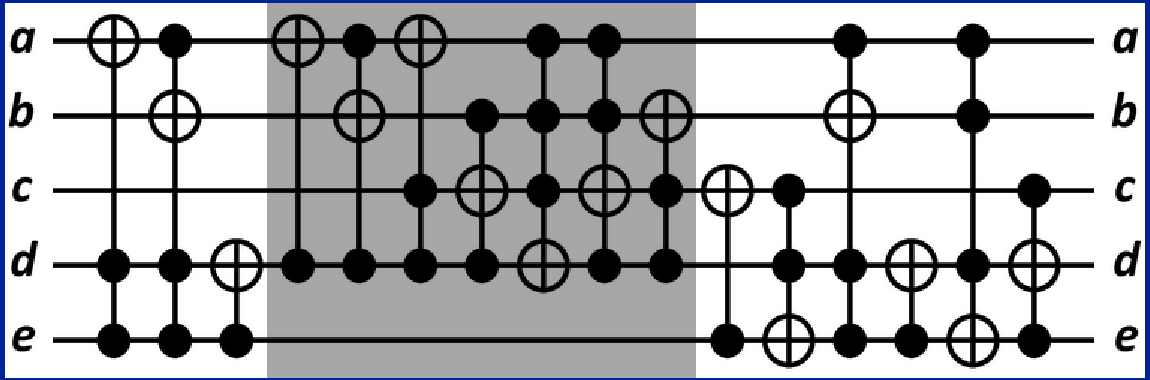
QC reduction algorithm for n>4

1. Identify all 4-input/output subcircuits
2. Perform resynthesis procedure for all such subcircuits
3. Calculate quantum cost reduction for each subcircuit
4. Replace the subcircuit that gives the highest quantum cost reduction
5. Go to 1 and repeat until no reduction can be achieved

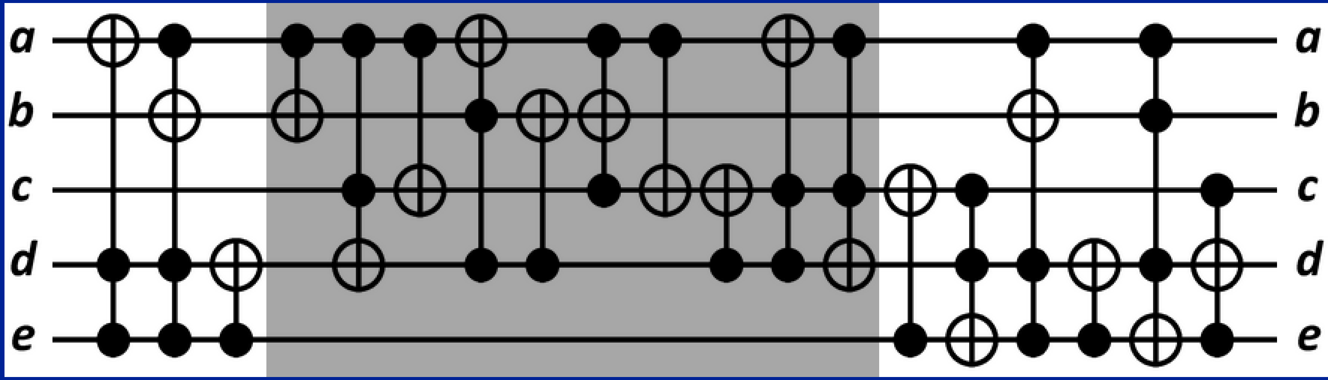
Example: majority5 circuit (step 1)

Gupta, Agrawal, Jha – IEEE Trans. on CAD 2006

GC=16 QC=102



GC=19 QC=79



Final circuit after 4 steps: GC=27 QC=65

Motivation for new benchmarks

- synthesis of optimal reversible circuits has been solved for 3- and 4-variable case only
- for n $\geq$ 5 optimal circuits with small numbers of gates can be found by exhaustive calculations
- infinite sequences of reversible functions have been proposed as benchmarks but no minimal realizations are known for them

  hwb_n,   nth_prime_inc

- large benchmarks with known minimal realizations are needed for better evaluation of synthesis algorithms

Proposed approach

- constructing sequences of large reversible functions by extrapolation of functional representations of known minimal circuits

- finding minimal circuits for such functions and proving their minimality

Papers on extrapolation of reversible functions

- J. Jegier, P. Kerntopf, M. Szyprowski: An approach to constructing reversible multi-qubit benchmarks with provably minimal implementations, 13th IEEE International Conference on Nanotechnology, August 2013
- J. Jegier, P. Kerntopf: Progress towards constructing sequences of benchmarks for quantum Boolean circuits synthesis, 14th IEEE International Conference on Nanotechnology, August 2014
- J. Jegier, P. Kerntopf: Gate count minimal reversible circuits for two infinite sequences of self-inverse functions, 11th International Workshop on Boolean Problems, September 2014
- J. Jegier, P. Kerntopf: Gate count minimal reversible circuits, Problems and New Solutions in the Boolean Domain, 2016, in press

Reversible functions and gates

Definition

A reversible *n*x*n* function is called

- **self-inverse** if $f^{-1} = f$, i.e. it is its own inverse.
- **conservative** if the number of 1's at the outputs is always equal to the number of 1's at the inputs.
- **parity-preserving** if the number of 1's at the outputs has the same parity as the number of 1's at the inputs.

A self-inverse function can be described by the list of transpositions.

The class of conservative functions is included in the class of parity-preserving functions.

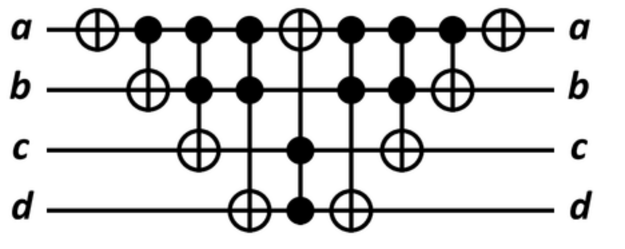
Assumptions

- binary reversible circuits
- NCT library of gates (NOT, CNOT and Toffoli gates)
- gate count (GC) optimization
- no additional lines

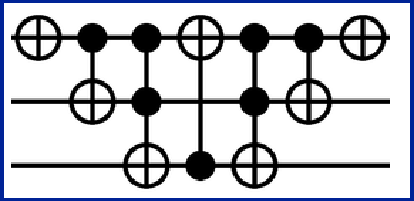
Constructing sequences of functions

- Step 1 Select a 4-variable reversible function *F* and find all minimal citcuits implementing it
- Step 2 Among the circuits found in Step 1 look for a circuit with a kind of structural regularity
- Step 3 Once such a circuit *C* has been found calculate the function implemented by it
- Step 4 Search the database for a 3x3 circuit *C'* with a similar structure to the circuit *C*
- Step 5 Find the function *F'* implemented by *C'*
- Step 6 If *F* and *F'* have similar cycle structure try to extrapolate both circuits and functions

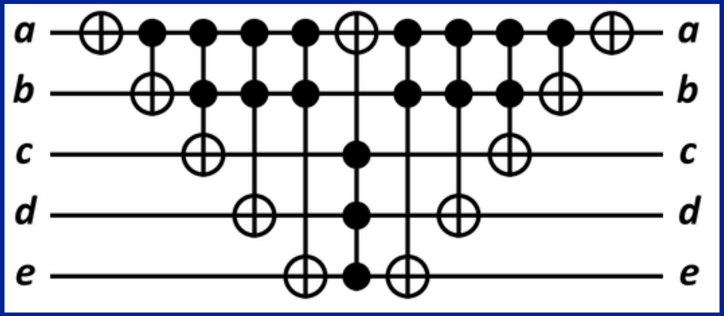
Example 1



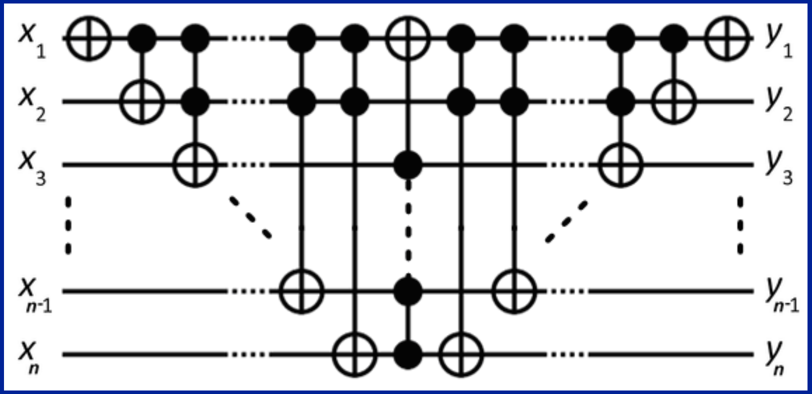
Cycles: (0,15) (13,14)



Cycles: (0,7) (5,6)

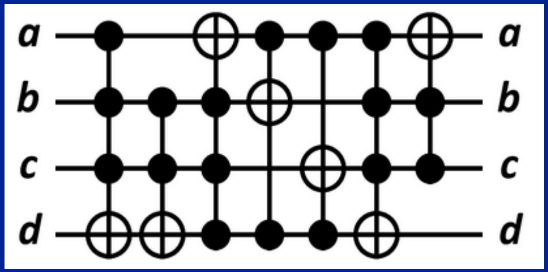


Cycles: (0,31) (29,30)

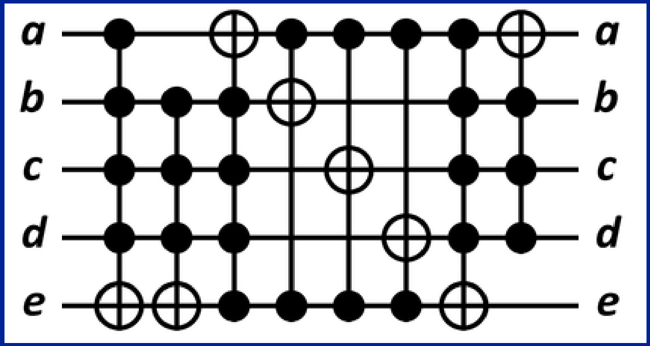


Cycles: $(0,2^n-1)$ $(2^n-3, 2^n-2)$

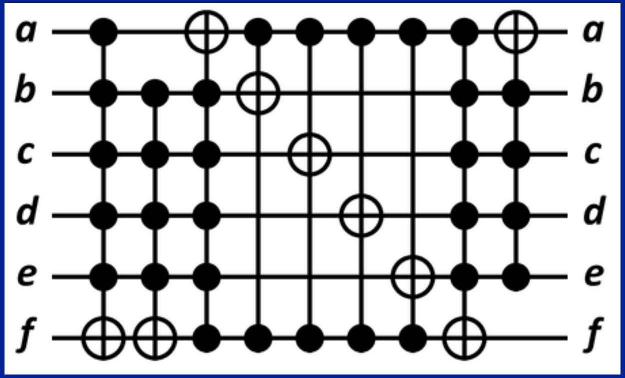
Example 2



(6,9) (7,14) (11,13)



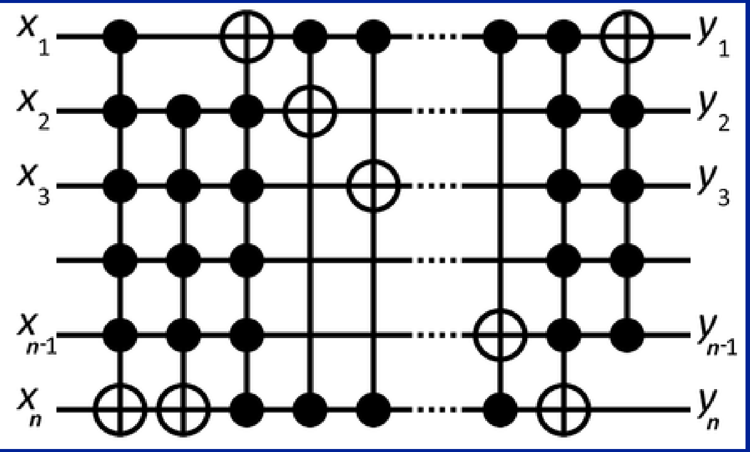
(14,17) (15,30) (19,29)
(21,27) (23,25)



(30,33) (31,62) (35,61)

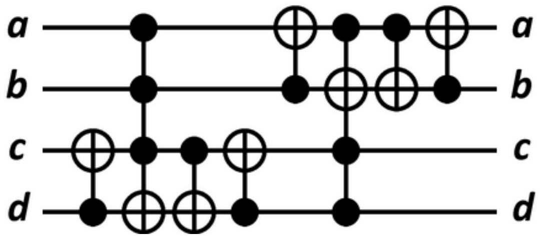
(37,59) (39,57) (41,55)

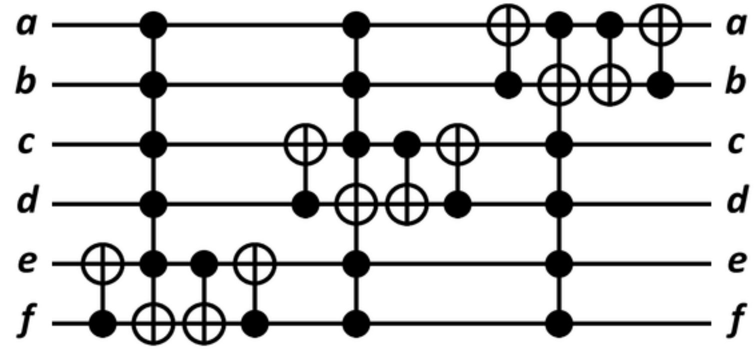
(43,53) (45,51) (47,49)

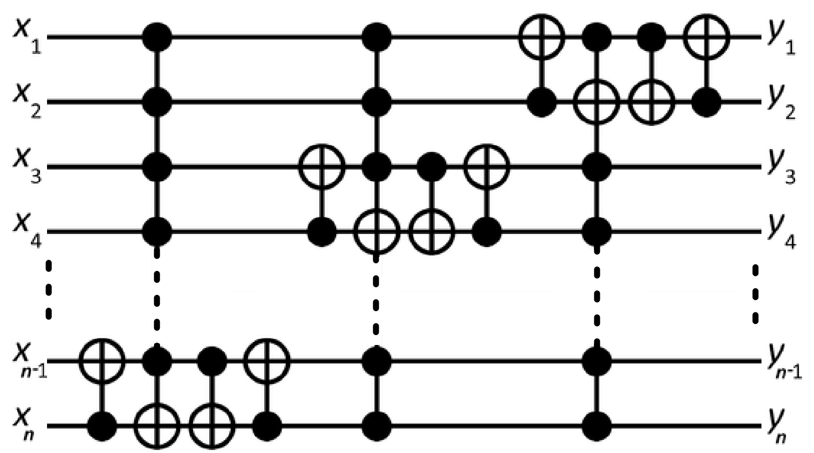


$(2^{n-1}-2,\ 2^{n-1}+1)$ $(2^{n-1}-1,\ 2^{n}-2)$…

Example 3

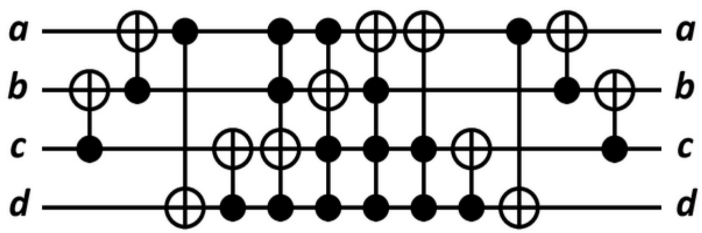




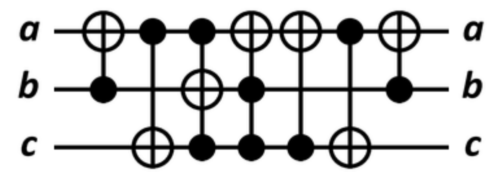


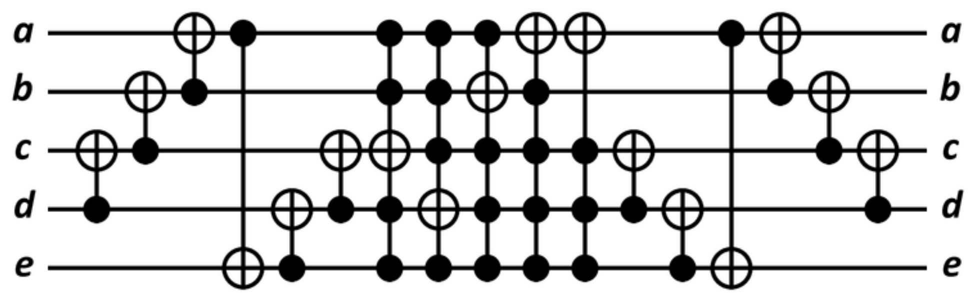
Example 4

(1,2,4,8)



(1,2,4)





(1,2,4,8,16)