

RANGKAIAN LISTRIK II

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OUTLINE

- > Frekuensi Kompleks
- > Respon Frekuensi
- ▶ Resonansi

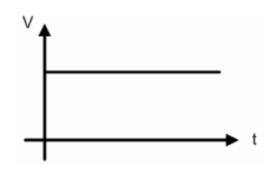
REFERENSI

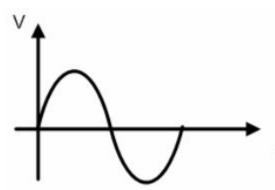
Fundamentals Of Electric Circuits by Alexander Charles K., Sadiku Matthew O. N.

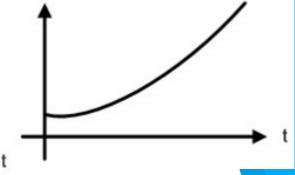
➤ Engineering Circuit Analysis by Hayt

- Frekuensi kompleks = fungsi sinusoidal + konstanta peredam
- ► Fungsi sinusoidal $\rightarrow V_m \cos(\omega t + \theta)$
- ► Konstanta peredam $\rightarrow e^{-\sigma t}$
- σ → faktor peredam/ frekuensi Neper dengan satuan Np/s yang nilainya (-)/0

Fungsi sinusoidal dengan berbagai konstanta peredam dapat digambarkan dalam bentuk kurva.







$$v(t) = Vm$$
 $\sigma = 0$
 $\omega = 0$

$$v(t) = V_m \cos(\omega t + \theta)$$

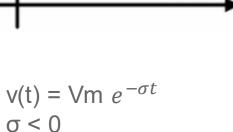
$$\sigma = 0$$

$$v(t) = Vm e^{\sigma t}$$

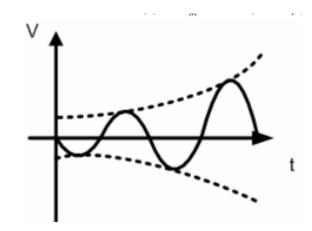
 $\sigma > 0$
 $\omega = 0$

Fungsi sinusoidal dengan berbagai konstanta peredam dapat digambarkan dalam bentuk kurva.



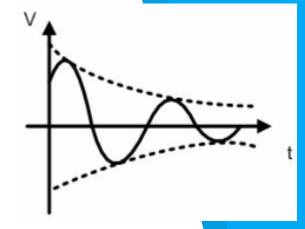


 $\omega = 0$



$$v(t) = V_m e^{\sigma t} \cos(\omega t + \theta)$$

 $\sigma > 0$



$$v(t) = V_m e^{\sigma t} \cos(\omega t + \theta)$$

 $\sigma < 0$

Fasor sinyal AC

Fasor sinyal frekuensi komples

 $\mathbf{v}(t) = V_m \cos(\omega t + \theta)$

 $v(t) = V_m e^{\sigma t} \cos(\omega t + \theta)$

 $V = Re[V_m e^{j(\omega t + \varphi)}]$

 $V = Re [V_m e^{\sigma t} e^{j(\omega t + \varphi)}]$

 $V = Re[V_m e^{j\varphi} e^{j\omega t}]$

 $V = Re[V_m e^{j\varphi} e^{(\sigma+j\omega)t}] \rightarrow s = \sigma + i\omega$

 $V(j\omega) = V_m e^{j\varphi}$

 $V = Re[V_m e^{j\varphi}e^{st}]$

► $V(j\omega) = V_m \angle \varphi$

 $V(s) = V_m e^{j\varphi}$

 $\blacktriangleright V(j\omega) = V_m \angle \varphi$

Impedansi dalam frekuensi kompleks

$$V(s) = \frac{Z(s)}{I(s)}$$

$$\triangleright Z_R(s) = R$$

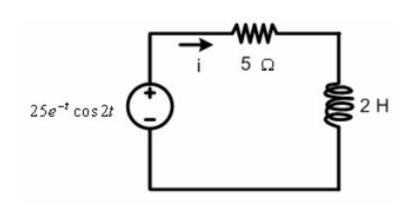
$$Y_R(s) = \frac{1}{R} = G$$

$$Y_L(s) = \frac{1}{sL}$$

$$Y_C(s) = sC$$

Contoh

Tentukan nilai i



$$s = -1 + j2$$

$$Z_{R}(s) = 5$$

$$Z_{L}(s) = sL = 2s$$

$$Z_{T}(s) = 5 + 2s$$

$$V = 25e^{-t}\cos 2t = 25\angle 0^{\circ}$$

$$i(s) = \frac{V(s)}{Z_{T}(s)} = \frac{25\angle 0^{\circ}}{5 + 2s} = \frac{25\angle 0^{\circ}}{5 + 2(-1 + j2)} = 5\angle -53,1^{\circ}$$

$$i(t) = 5e^{-t}\cos(2t - 53,1^{\circ})A$$

- Jika sebuah sumber sinusoidal dengan amplitudo konstan dan frekuensi yang berubah – ubah, akan di dapatkan respon frekuensi.
- ► Respon frekuensi dari rangkaian adalah perubahan sifat rangkaian akibat perubahan frekuensi sinyal
- ► Alat yang digunakan untuk menemukan respon frekuensi dari rangkaian adalah **Transfer Function** \rightarrow **H**(ω)
- ► Respon frekuensi akan digambarkan sebagai **kurva** $\mathbf{H}(\omega)$ vs ω dimana nilai ω dari 0∞ .

 Transfer Function adalah rasio perbandingan dari output dan input sistem, yang bergantung pada frekuensi

$$\mathbf{H}(\omega) = \frac{\mathbf{Y}(\omega)}{\mathbf{X}(\omega)} \qquad \mathbf{X}(\omega) \qquad \text{Linear network} \qquad \mathbf{Y}(\omega)$$
Input
$$\mathbf{H}(\omega) \qquad \mathbf{Output}$$

Dalam rangkaian listrik, input dan output yang akan dicari adalah tegangan dan arus, sehingga kemungkinan dari transfer function ialah:

$$\textbf{H}(\omega) = \text{voltage gain} = \frac{V_o(\omega)}{V_i(\omega)} \qquad \textbf{H}(\omega) = \text{transfer impedance} = \frac{V_o(\omega)}{I_i(\omega)}$$

•
$$H(\omega) = \text{current gain} = \frac{I_0(\omega)}{I_i(\omega)}$$
 $H(\omega) = \text{transfer admittance} = \frac{I_0(\omega)}{V_i(\omega)}$

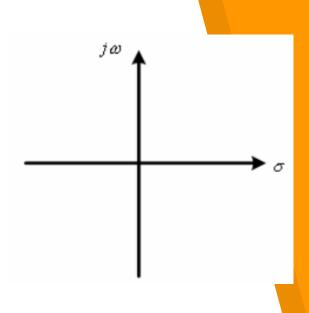
•
$$\mathbf{H}(\omega) = \mathbf{H}(\omega) \angle \theta$$
 — Sudut Fasa
 Magnitude

 Transfer Function dapat di rumuskan sebagai perbandingan numerator polynomial dan denominator polynomial

$$H(\omega) = \frac{N(\omega)}{D(\omega)} \qquad H(s) = \frac{b_m(s - Z_1)(s - Z_2).....(s - Z_m)}{a_n(s - P_1)(s - P_2).....(s - P_n)}$$

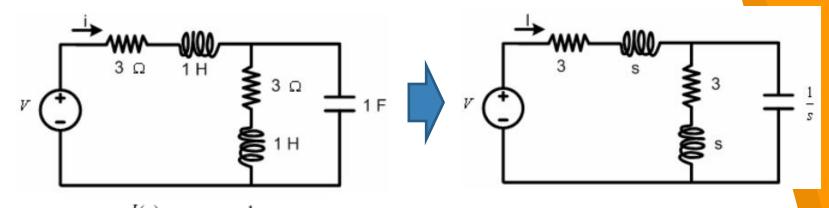
- ► **Zero**, akar dari numerator (Z₁, Z₂,..., Z_m) yang membuat transfer function menjadi nol
- ► **Pole**, akar dari denominator (P₁, P₂,..., P_m) yang membuat fungsi menjadi tak hingga

- Diagram bode pada S-plane
- Dapat menentukan kestabilan sistem, dimana kestabilan BIBO (Bounded Input Bounded Output) terletak di sebelah kiri pole – polenya.
- Jenis kestabilan:
 - Absolutely : ada di sebelah kiri jω axis
 - Conditionally: tidak ada yang di sebelah kanan pole tetapi pada jω axis untuk orde > 1
 - Unstable : ada di sebelah kanan jω axis



Contoh soal

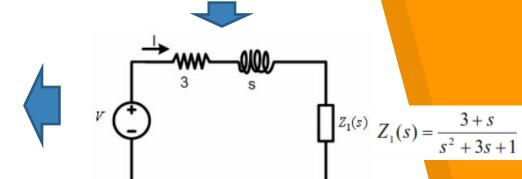
► Tentukan fungsi transfer I terhadap V



$$H(s) = \frac{I(s)}{V(s)} = \frac{1}{3+s+\frac{3+s}{s^2+3s+1}}$$

$$H(s) = \frac{s^2 + 3s + 1}{s^3 + 6s^2 + 11s + 6}$$

$$H(s) = \frac{s^2 + 3s + 1}{(s+2)(s+3)(s+1)}$$



Contoh soal

 Tentukan output tegangan jika diberikan fungsi transfer sebagai berikut

$$H(s) = \frac{4(s+5)}{s^2 + 4s + 5}$$
dimana input $V_i(s) = 2\angle 0^o$ dan $s = -2+j3$

$$Jawaban:$$

$$V_o(s) = H(s).V_i(s) = \frac{4(s+5)}{s^2 + 4s + 5}.2\angle 0^o = \frac{4(-2+j3+5)}{(-2+j3)^2 + 4(-2+j3) + 5}.2\angle 0^o = -3(1+j)$$

$$V_o(s) = 3\sqrt{2}\angle -135^o$$

$$V_o(t) = 3\sqrt{2}e^{-t}\cos(3t - 135^o)$$

Rangkaian RL

Fungsi transfer dalam domain s

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{R}{R + sL} = \frac{1}{1 + \frac{sL}{R}}$$

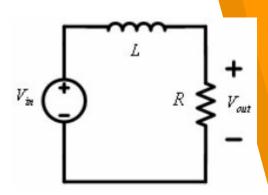
Jika s = jω, fungsi transfernya menjadi

$$H(j\omega) = \frac{1}{1 + \frac{j\omega L}{R}}$$

► Respon frekuensi menjadi

$$|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega L/R)^2}}$$

$$\angle H(j\omega) = -\tan^{-1}\left(\frac{\omega L}{R}\right)$$



Respon Frekuensi Magnitudo

$$\bullet \omega = 0 \rightarrow |H(\omega)| = 1$$

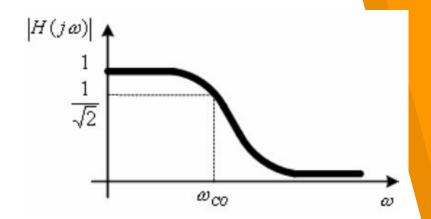
$$\bullet \omega = \infty \rightarrow |H(\omega)| = 0$$

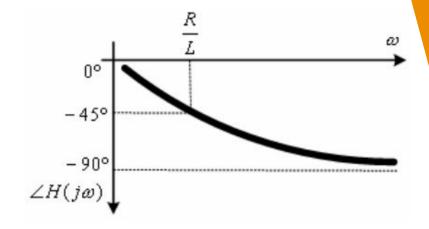
Respon Frekuensi Fasa

$$\bullet \omega = 0 \rightarrow \angle H(\omega) = 0^{\circ}$$

$$\bullet$$
 $\omega = \infty \rightarrow \angle H(\omega) = -90^{\circ}$

$$\omega = \frac{R}{L} \rightarrow \angle H(\omega) = -45^{\circ}$$





Rangkaian RL

Fungsi transfer dalam domain s

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{sL}{sL + R} = \frac{1}{1 + \frac{R}{sL}}$$

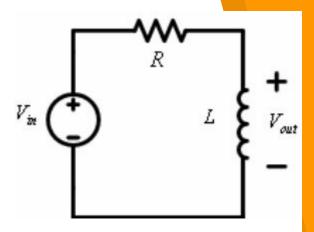
Jika s = jω, fungsi transfernya menjadi

$$H(j\omega) = \frac{1}{1 + R/j\omega L} = \frac{1}{1 - \frac{jR}{\omega L}}$$

► Respon frekuensi menjadi

$$|H(j\omega)| = \frac{1}{\sqrt{1 + (R/\omega L)^2}}$$

$$\angle H(j\omega) = -\tan^{-1}\left(-\frac{R}{\omega L}\right)$$



Respon Frekuensi Magnitudo

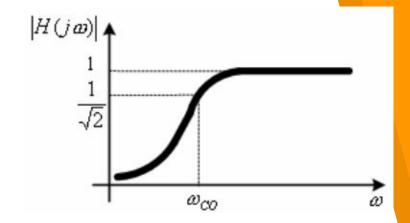
$$\bullet \omega = 0 \rightarrow |H(\omega)| = 0$$

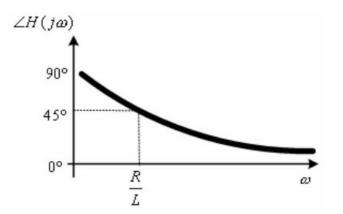
$$\bullet \omega = \infty \rightarrow |H(\omega)| = 1$$

Respon Frekuensi Fasa

$$\sim \omega = 0 \rightarrow \angle H(\omega) = 90^{\circ}$$

$$\bullet \omega = \infty \rightarrow \angle H(\omega) = 0^{\circ}$$





Rangkaian RC

Fungsi transfer dalam domain s

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{\frac{1}{sC}}{\frac{1}{sC} + R} = \frac{1}{1 + sCR}$$

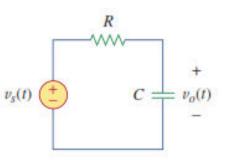
Jika s = jω, fungsi transfernya menjadi

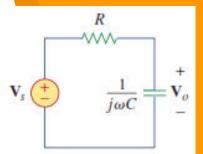
$$H(j\omega) = \frac{1}{1 + j\omega CR}$$

Respon frekuensi menjadi

$$|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega CR)^2}}$$

$$\angle H(j\omega) = -\tan^{-1}(\omega CR)$$





Respon Frekuensi Magnitudo

$$\bullet \omega = 0 \rightarrow |H(\omega)| = 1$$

$$\bullet \omega = \infty \rightarrow |H(\omega)| = 0$$

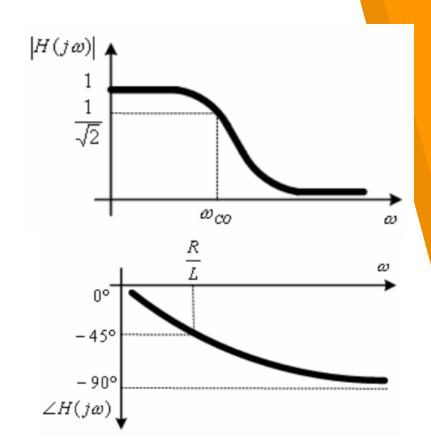
•
$$\omega = \frac{1}{CR} \to |H(\omega)| = \frac{1}{\sqrt{2}}$$

Respon Frekuensi Fasa

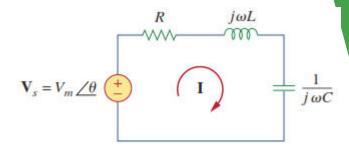
$$\bullet \omega = 0 \Rightarrow \angle H(\omega) = 0^{\circ}$$

•
$$\omega = \infty \rightarrow \angle H(\omega) = -90^{\circ}$$

$$\omega = \frac{1}{CR} \rightarrow \angle H(\omega) = -45^{\circ}$$



- Resonansi adalah keadaan dimana nilai magnitude dari reaktansi kapasitif dan induktif sama sehingga impedansi rangkaian hanya berupa resistif murni
- Rangkaian resonansi dibuat untuk membentuk filter, karena Transfer function dari rangkaian dapat memilih frekuensi
- Resonansi dapat dibentuk dari rangkaian seri ataupun paralel

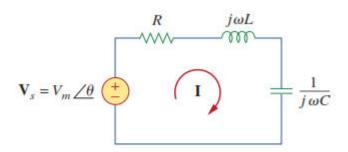


Transfer function

$$\mathbf{Z} = \mathbf{H}(\omega) = \frac{\mathbf{V}_s}{\mathbf{I}} = R + j\omega L + \frac{1}{j\omega C}$$

Total impedansi rangkaian

$$\mathbf{Z} = R + j \left(\omega L - \frac{1}{\omega C} \right)$$



Keadaan resonansi akan tercapai jika total reaktansi indukstif dan kapasitif adalah nol

$$Im(\mathbf{Z}) = \omega L - \frac{1}{\omega C} = 0$$

Nilai ω yang dapat memenuhi kondisi di atas di sebut frekuensi resonansi (ω_0)

$$\omega_0 L = \frac{1}{\omega_0 C}$$

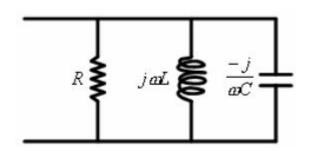


$$\omega_0 L = \frac{1}{\omega_0 C}$$
 $\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s}$

$$\omega_0 = 2 \times \pi \times f_0$$

$$f_0 = \frac{1}{2\pi \sqrt{LC}} \text{Hz}$$

- Impedansi menjadi resistif murni, sehingga LC seri akan menjadi SC dan seluruh tegangan melewati R
- Vs dan I berada satu fasa, pf menjadi unity
- ► $H(\omega) = Z(\omega)$ -> minimum



Admitansi total:

$$\frac{1}{\mathbf{Z}_{tot}} = \frac{1}{R} + \frac{1}{j\omega L} + \frac{1}{-j/\omega C} = \frac{1}{R} - \frac{j}{\omega L} + j\omega C$$

$$\frac{1}{\mathbf{Z}_{tot}} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

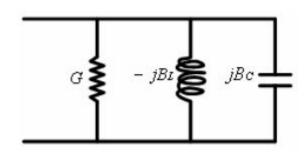
saat resonansi:

$$\omega C - \frac{1}{\omega L} = 0 \to \omega C = \frac{1}{\omega L}$$

$$\omega^2 = \frac{1}{LC}$$

$$f_o = \frac{1}{2\pi} \frac{1}{\sqrt{LC}}$$

Pada saat resonansi impedansi Z maksimum, sehingga arusnya minimum.



$$Y = G + jB_C - jB_L$$

$$Y = G + j(\omega C - \frac{1}{\omega L})$$
saat resonansi:
$$\omega C - \frac{1}{\omega L} = 0 \to \omega C = \frac{1}{\omega L}$$

$$\omega^2 = \frac{1}{LC}$$

$$f_o = \frac{1}{2\pi} \frac{1}{\sqrt{LC}}$$

$$\frac{1}{\mathbf{Z}_{tot}} = \frac{1}{R_L + j\omega L} + \frac{1}{R_C - \frac{j}{\omega C}}$$

$$\frac{1}{\mathbf{Z}_{tot}} = \frac{1}{R_L + j\omega L} \left(\frac{R_L - j\omega L}{R_L - j\omega L} \right)$$

$$R_{c}$$

$$\frac{1}{\mathbf{Z}_{tot}} = \frac{1}{R_L + j\omega L} \left(\frac{R_L - j\omega L}{R_L - j\omega L} \right) + \frac{1}{R_C - \frac{j}{\omega C}} \left(\frac{R_C + \frac{j}{\omega C}}{R_C + \frac{j}{\omega C}} \right)$$

$$\frac{1}{\mathbf{Z}_{tot}} = \frac{R_L - j\omega L}{{R_L}^2 + (\omega L)^2} + \frac{R_C + \frac{j}{\omega C}}{{R_C}^2 + \left(\frac{1}{\omega C}\right)^2}$$

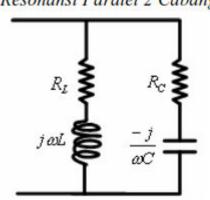
$$\begin{array}{c|c}
R_{L} & R_{C} \\
\hline
j \omega L & -\frac{j}{\omega C}
\end{array}$$

$$\frac{1}{\mathbf{Z}_{tot}} = \frac{R_L}{R_L^2 + (\omega L)^2} + \frac{R_C}{R_C^2 + \left(\frac{1}{\omega C}\right)^2} + j \left(\frac{\frac{1}{\omega C}}{R_C^2 + \left(\frac{1}{\omega C}\right)^2} - \frac{\omega L}{R_L^2 + (\omega L)^2}\right)$$

saat resonansi:

$$\frac{\frac{1}{\omega C}}{R_C^2 + \left(\frac{1}{\omega C}\right)^2} - \frac{\omega L}{R_L^2 + (\omega L)^2} =$$

Resonansi Paralel 2 Cabang



$$\frac{\frac{1}{\omega C}}{R_C^2 + \left(\frac{1}{\omega C}\right)^2} = \frac{\omega L}{R_L^2 + (\omega L)^2}$$

$$R_L^2 + (\omega L)^2 = \omega^2 LC \left(R_C^2 + \left(\frac{1}{\omega C} \right)^2 \right)$$

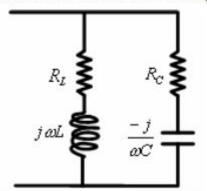
$$R_L^2 + \omega^2 L^2 = \omega^2 L C R_C^2 + \frac{L}{C}$$

$$\omega^2 L C R_C^2 - \omega^2 L^2 = R_L^2 - \frac{L}{C}$$

$$\omega^2 LC \left(R_C^2 - \frac{L}{C} \right) = R_L^2 - \frac{L}{C}$$

$$f_o = \frac{1}{2\pi\sqrt{LC}} \sqrt{\frac{{R_L}^2 - \frac{L}{C}}{{R_C}^2 - \frac{L}{C}}}$$

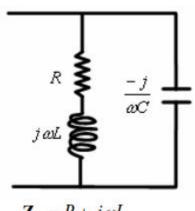
Resonansi Paralel 2 Cabang



Perlu diingat bahwa :
$$\sqrt{\frac{R_L^2 - \frac{L}{C}}{R_C^2 - \frac{L}{C}}}$$
 harus positif real sehingga syarat :

$$R_1^2 > \frac{L}{C} \operatorname{dan} R_C^2 > \frac{L}{C} \operatorname{atau} R_L^2 < \frac{L}{C} \operatorname{dan} R_C^2 < \frac{L}{C}$$

Ketika nilai
$$R_L^2 = R_C^2 = \frac{L}{C}$$
, maka rangkaian beresonansi untuk semua frekuensi.



$$\mathbf{Z}_1 = R + j\omega L$$

$$\mathbf{Z}_2 = \frac{1}{j\omega C}$$

$$\frac{1}{\mathbf{Z}_{tot}} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} = \frac{1}{R + j\omega L} + \frac{1}{\frac{1}{j\omega C}} = \frac{1}{R + j\omega L} + j\omega C$$

$$\frac{-j}{\omega C} \frac{1}{\mathbf{Z}_{tot}} = j\omega C + \frac{1}{R + j\omega L} \left(\frac{R - j\omega L}{R - j\omega L} \right)$$

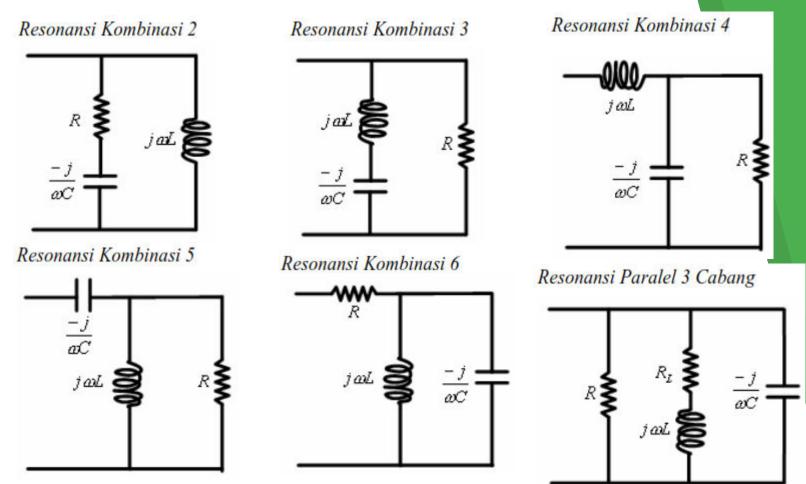
$$\frac{1}{\mathbf{Z}_{-}} = j\omega C + \frac{R - j\omega L}{R^2 + \omega^2 L^2}$$

$$\frac{1}{\mathbf{Z}_{tot}} = \frac{R}{R^2 + \omega^2 L^2} + j \left(\omega C - \frac{\omega L}{R^2 + \omega^2 L^2} \right)$$

saat resonansi : $\omega_C = \frac{\omega_L}{R^2 + \omega^2 I^2}$, sehingga :

$$R^{2} + \omega^{2}L^{2} = \frac{L}{C} \rightarrow \omega^{2}L^{2} = \frac{L}{C} - R^{2} \rightarrow \omega^{2} = \frac{1}{L^{2}} \left(\frac{L}{C} - R^{2} \right) = \frac{1}{LC} - \frac{R^{2}}{L^{2}} = \frac{1}{LC} \left(1 - \frac{R^{2}C}{L^{2}} \right)$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{\left(1 - \frac{R^2C}{L^2}\right)}$$



Suatu rangkaian seri RLC dengan R = 50Ω, L = 0,05H, C = 20μF terpasang pada V = 100∠0° dengan frekuensi variabel. Pada frekuensi berapa tegangan induktor mencapai maksimum? Berapakah tegangan induktor tersebut?
 Jawaban:

Tegangan induktor maksimum jika arus maksimum, arus maksimum jika Z minimum, Z minimum terjadi saat resonansi.

$$f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.05.20.10^{-6}}} = 159.1$$
Hz

$$Z_{resonansi} = R \rightarrow i_{maks} = \frac{V}{Z_{res}} = \frac{100 \angle 0^{\circ}}{50} = 2 \angle 0^{\circ}$$

$$V_L = i_{maks}.X_L = i_{maks}.j\omega L = 2\angle 0^{\circ}.2\pi J L \angle 90^{\circ} = 2\angle 0^{\circ}.2\pi.159,1.0,05\angle 90^{\circ} = 100\angle 90^{\circ}$$

Pada saat terjadi resonansi tegangan terpasang pada rangkaian seri RLC adalah $v = 70.7 \sin(500t + 30^{\circ})V$ menghasilkan arus sebesar $i = 2.83 \sin(500t + 30^{\circ})A$, jika L = 0.5H. Tentukan nilai R dan C!

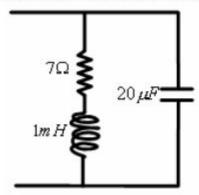
Jawaban:

$$Z = \frac{V}{I} = \frac{70.7 \angle 30^{\circ}}{2.83 \angle 30^{\circ}} = 25 \rightarrow R = 25\Omega$$

$$f_o = \frac{1}{2\pi\sqrt{LC}} \to \omega^2 = \frac{1}{LC}$$

$$C = \frac{1}{\omega^2 L} = \frac{1}{500^2.0.5} = 8\mu F$$

3. Tentukan frekuensi resonansi pada gambat berikut :



Jawaban:

$$\frac{1}{20\,\mu^{R}} = j\omega C + \frac{1}{R + j\omega L} \left(\frac{R - j\omega L}{R - j\omega L} \right)$$

$$\frac{1}{\mathbf{Z}_{...}} = j\omega C + \frac{R - j\omega L}{R^2 + \omega^2 L^2}$$

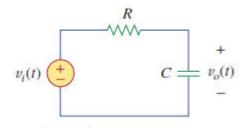
$$\frac{1}{\mathbf{Z}_{tot}} = \frac{R}{R^2 + \omega^2 L^2} + j \left(\omega C - \frac{\omega L}{R^2 + \omega^2 L^2} \right)$$

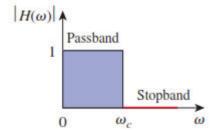
saat resonansi : $\omega_C = \frac{\omega_L}{R^2 + \omega^2 I^2}$, sehingga :

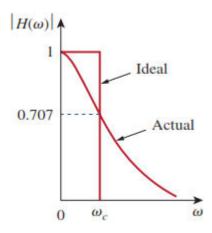
$$R^{2} + \omega^{2}L^{2} = \frac{L}{C} \rightarrow \omega^{2}L^{2} = \frac{L}{C} - R^{2} \rightarrow \omega^{2} = \frac{1}{L^{2}} \left(\frac{L}{C} - R^{2} \right) = \frac{1}{LC} - \frac{R^{2}}{L^{2}} = \frac{1}{LC} \left(1 - \frac{R^{2}C}{L^{2}} \right)$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{\left(1 - \frac{R^2C}{L^2}\right)} = \frac{1}{2\pi\sqrt{10^{-3}.20.10^{-6}}}\sqrt{\left(1 - \frac{7^2.20.10^{-6}}{10^{-3}}\right)} = 159,2Hz$$

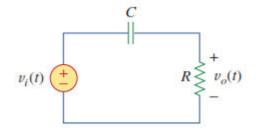
Type of Filter	H(0)	$H(\infty)$	$H(\omega_c)$ or $H(\omega_0)$
Lowpass	1	0	$1/\sqrt{2}$
Highpass	0	1	$1/\sqrt{2}$
Bandpass	0	0	1
Bandstop	1	1	0

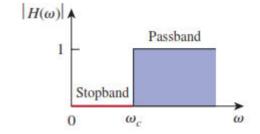


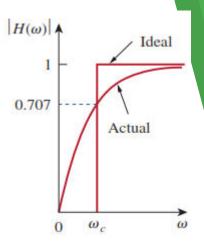




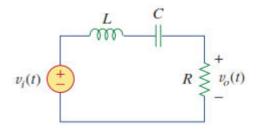
Type of Filter	H(0)	$H(\infty)$	$H(\omega_c)$ or $H(\omega_0)$
Lowpass	1	0	$1/\sqrt{2}$
Highpass	0	1	$1/\sqrt{2}$
Bandpass	0	0	1
Bandstop	1	1	0

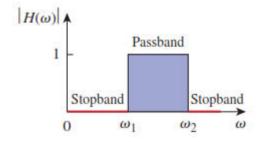


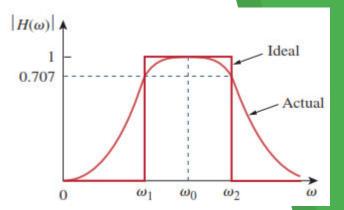




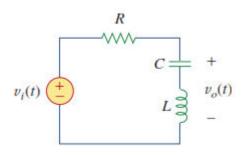
Type of Filter	H(0)	$H(\infty)$	$H(\omega_c)$ or $H(\omega_0)$
Lowpass	1	0	$1/\sqrt{2}$
Highpass	0	1	$1/\sqrt{2}$
Bandpass	0	0	1
Bandstop	1	1	0

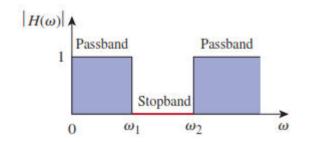


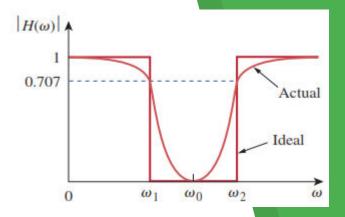




Type of Filter	H(0)	$H(\infty)$	$H(\omega_c)$ or $H(\omega_0)$
Lowpass	1	0	$1/\sqrt{2}$
Highpass	0	1	$1/\sqrt{2}$
Bandpass	0	0	1
Bandstop	1	1	0









THANKS!

Any questions?