Medan Elektromagnetik

HUKUM COULOMB
DAN
INTENSITAS MEDAN LISTRIK

Hukum Coulomb

Coulomb stated that the force between two very small objects separated in a vacuum or free space by a distance, which is large compared to their size, is proportional to the charge on each and inversely proportional to the square of the distance between them, or

$$F = k\frac{Q_1 Q_2}{R^2}$$

$$k = \frac{1}{4\pi\epsilon_0}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \doteq \frac{1}{36\pi}10^{-9} \text{ F/m}$$

$$F = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2}$$

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Hukum Coulomb

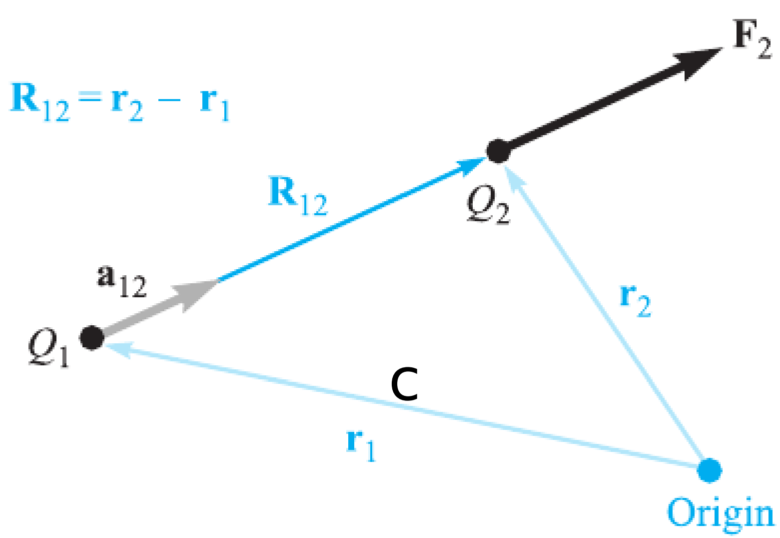

Figure 2.1 If Q_1 and Q_2 have like signs, the vector force F_2 on Q_2 is in the same direction as the vector R_{12}.

The vector form of Coulomb's law is

$$\mathbf{F}_2 = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2}\mathbf{a}_{12}$$

where $\mathbf{a}_{12}$ = a unit vector in the direction of R_{12}, or

$$\mathbf{a}_{12} = \frac{\mathbf{R}_{12}}{|\mathbf{R}_{12}|} = \frac{\mathbf{R}_{12}}{R_{12}} = \frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|}$$

$$\mathbf{F}_1 = -\mathbf{F}_2 = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2}\mathbf{a}_{21} = -\frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2}\mathbf{a}_{12}$$

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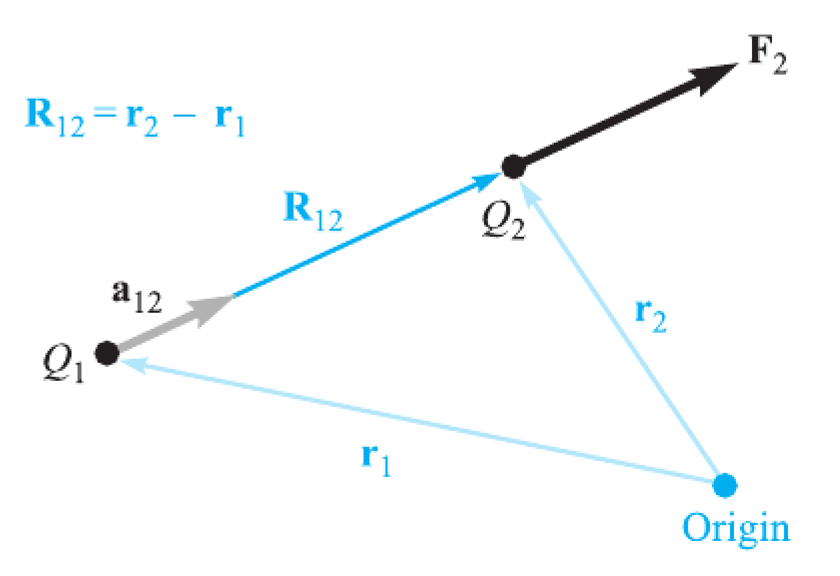
Hukum Coulomb

EXAMPLE 2.1

We illustrate the use of the vector form of Coulomb's law by locating a charge of $Q_1 = 3 \times 10^{-4}$ C at $M(1, 2, 3)$ and a charge of $Q_2 = -10^{-4}$ C at $N(2, 0, 5)$ in a vacuum. We desire the force exerted on Q_2 by Q_1.



Hukum Coulomb

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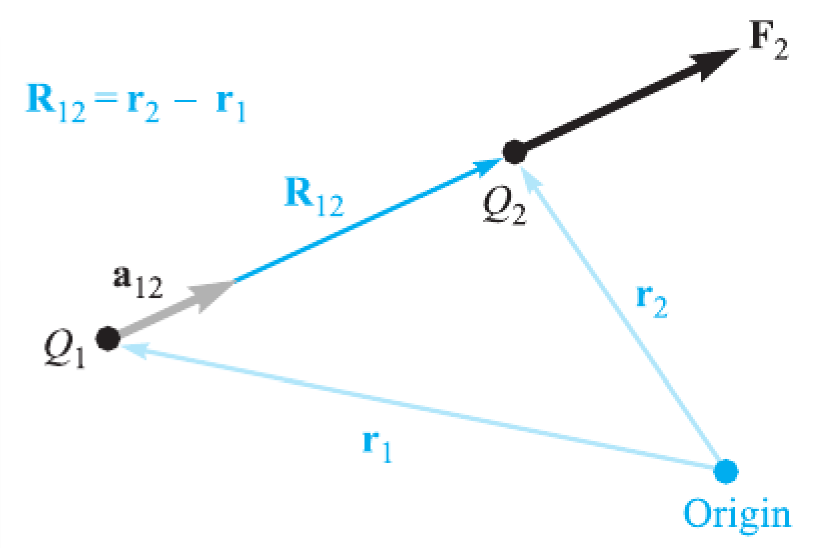

Solution.

The vector $\mathbf{R}_{12}$ is

$$\mathbf{R}_{12} = \mathbf{r}_2 - \mathbf{r}_1 = (2-1)\mathbf{a}_x + (0-2)\mathbf{a}_y + (5-3)\mathbf{a}_z = \mathbf{a}_x - 2\mathbf{a}_y + 2\mathbf{a}_z$$

$$|\mathbf{R}_{12}| = 3 \qquad \mathbf{a}_{12} = \tfrac{1}{3}(\mathbf{a}_x - 2\mathbf{a}_y + 2\mathbf{a}_z)$$

$$\mathbf{F}_2 = \frac{3 \times 10^{-4}(-10^{-4})}{4\pi(1/36\pi)10^{-9} \times 3^2}\left(\frac{\mathbf{a}_x - 2\mathbf{a}_y + 2\mathbf{a}_z}{3}\right)$$

$$= -30\left(\frac{\mathbf{a}_x - 2\mathbf{a}_y + 2\mathbf{a}_z}{3}\right) \text{ N}$$

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$$\mathbf{F}_2 = -10\mathbf{a}_x + 20\mathbf{a}_y - 20\mathbf{a}_z$$

Intensitas Medan Listrik

If we now consider one charge fixed in position, say Q_1, and move a second charge slowly around, we note that there exists everywhere a force on this second charge; in other words, this second charge is displaying the existence of a force *field* that is associated with charge, Q_1. Call this second charge a test charge Q_t. The force on it is given by Coulomb's law,

$$\mathbf{F}_t = \frac{Q_1 Q_t}{4\pi\epsilon_0 R_{1t}^2}\mathbf{a}_{1t}$$

Writing this force as a force per unit charge gives the *electric field intensity*, $\mathbf{E}_1$ arising from Q_1:

$$\mathbf{E}_1 = \frac{\mathbf{F}_t}{Q_1} = \frac{Q_1}{4\pi\epsilon_0 R_{1t}^2}\mathbf{a}_{1t}$$

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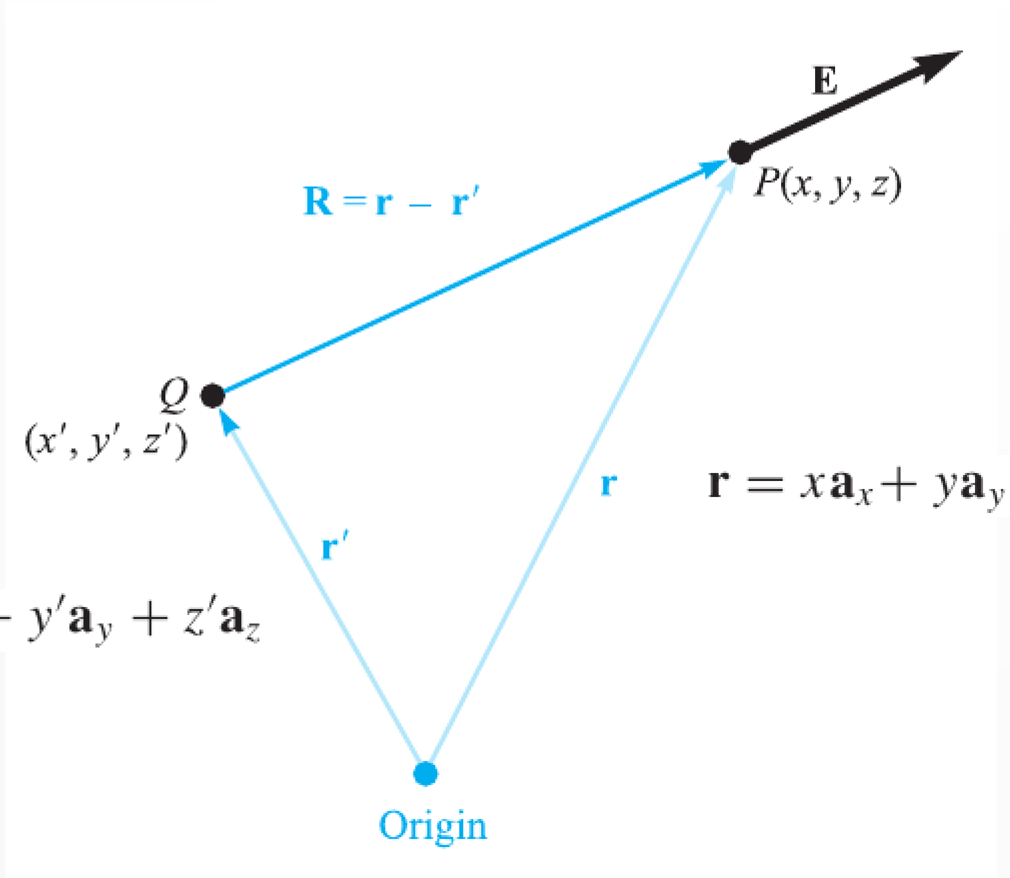
Intensitas Medan Listrik



$$\mathbf{r} = x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z$$

$$\mathbf{r}' = x'\mathbf{a}_x + y'\mathbf{a}_y + z'\mathbf{a}_z$$

$$\mathbf{E}(\mathbf{r}) = \frac{Q}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}'|^2}\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|} = \frac{Q(\mathbf{r}-\mathbf{r}')}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}'|^3}$$

$$= \frac{Q[(x-x')\mathbf{a}_x + (y-y')\mathbf{a}_y + (z-z')\mathbf{a}_z]}{4\pi\epsilon_0[(x-x')^2 + (y-y')^2 + (z-z')^2]^{3/2}}$$

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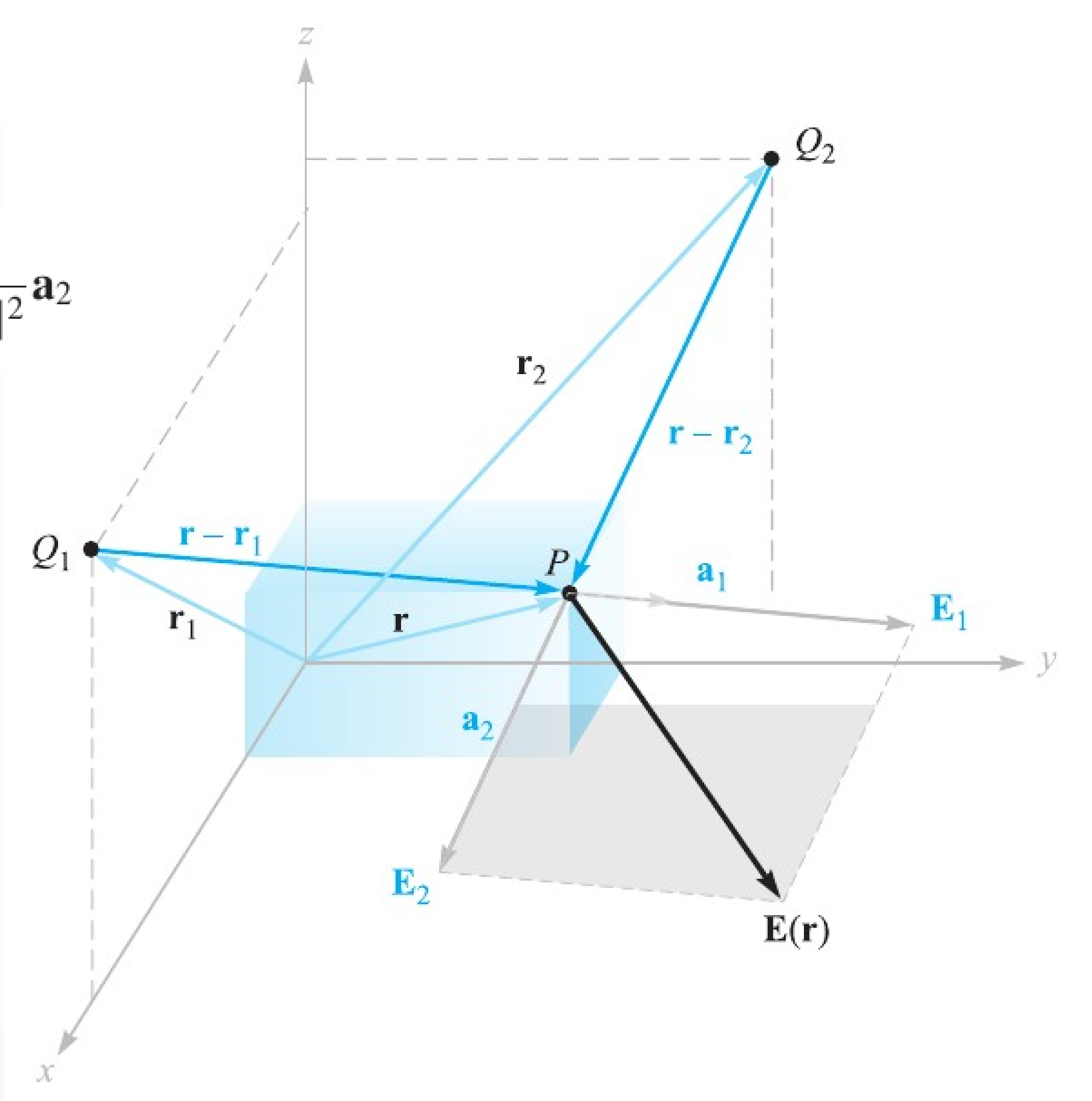
$$\mathbf{E}(\mathbf{r}) = \frac{Q_1}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}_1|^2}\mathbf{a}_1 + \frac{Q_2}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}_2|^2}\mathbf{a}_2$$

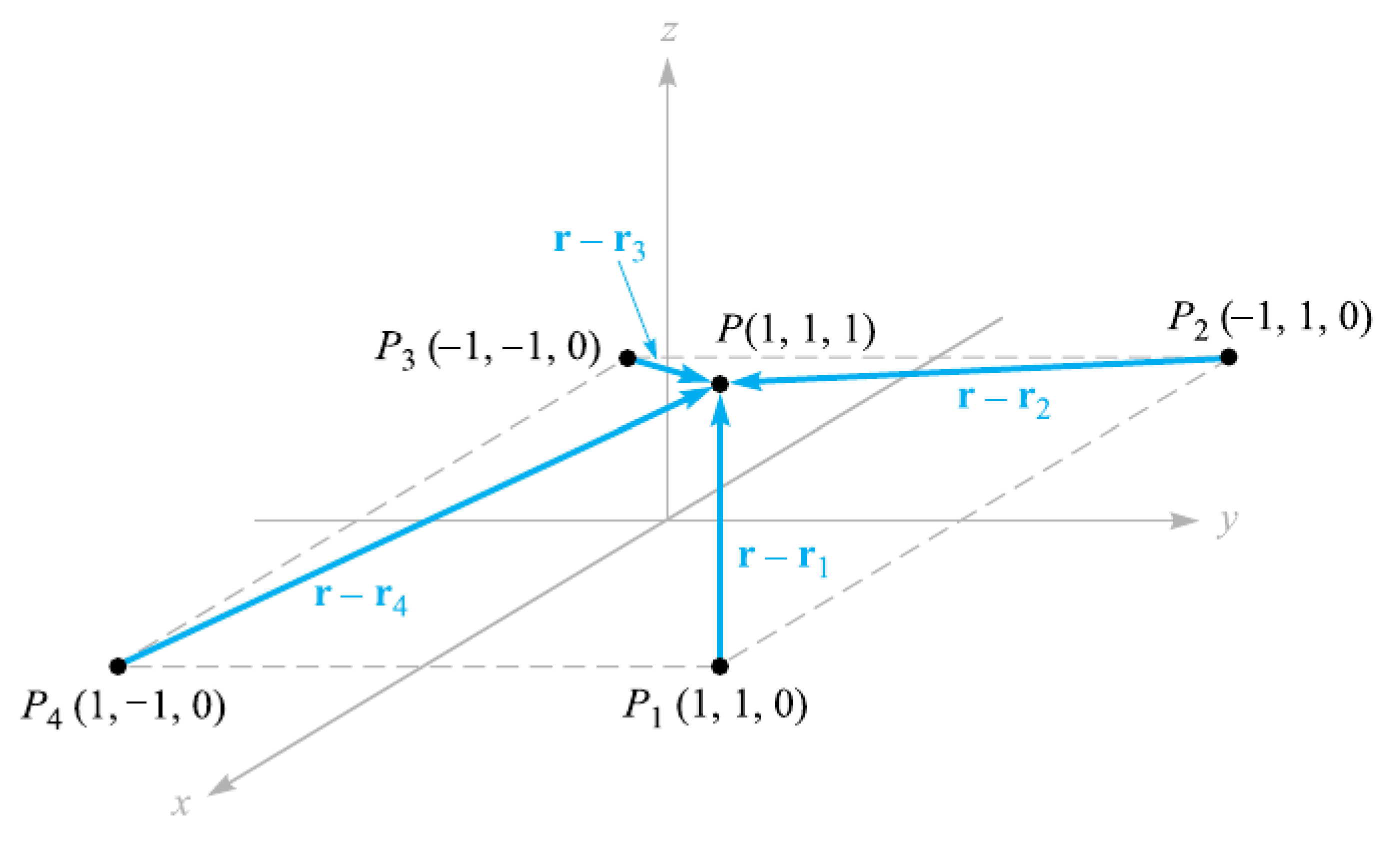
$$\mathbf{E}(\mathbf{r}) = \sum_{m=1}^{n} \frac{Q_m}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}_m|^2}\mathbf{a}_m$$



Intensitas Medan Listrik

EXAMPLE 2.2

In order to illustrate the application of (11), we find $\mathbf{E}$ at $P(1, 1, 1)$ caused by four identical 3-nC (nanocoulomb) charges located at $P_1(1, 1, 0)$, $P_2(-1, 1, 0)$, $P_3(-1, -1, 0)$, and $P_4(1, -1, 0)$, as shown in Figure 2.4.



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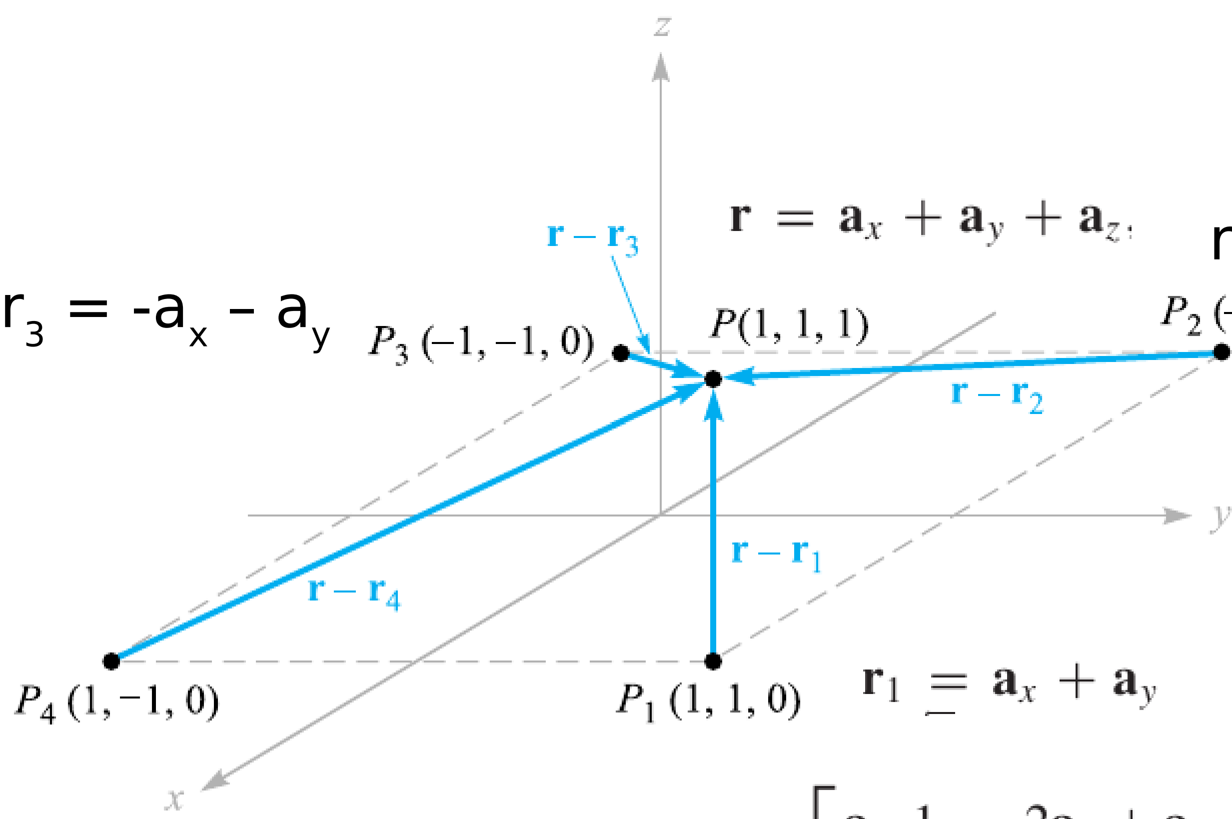
EXAMPLE 2.2



$\mathbf{r} = \mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z$

$r_3 = -a_x - a_y$

$r_2 = -a_x + a_y$

$\mathbf{r}_1 = \mathbf{a}_x + \mathbf{a}_y$

$r_4 = a_x - a_y$

$\mathbf{r}-\mathbf{r}_1 = \mathbf{a}_z$

$r - r_2 = 2a_x + a_z$

$r - r_3 = 2a_x + 2a_y + a_z$

$r - r_4 = 2a_y + a_z$

$|\mathbf{r}-\mathbf{r}_1| = 1,\ |\mathbf{r}-\mathbf{r}_2| = \sqrt{5},$

$|\mathbf{r}-\mathbf{r}_3| = 3$, and $|\mathbf{r}-\mathbf{r}_4| = \sqrt{5}.$

$$Q/4\pi\epsilon_0 = 3\times10^{-9}/(4\pi\times8.854\times10^{-12}) = 26.96\ \mathrm{V}$$

$$\mathbf{E} = 26.96\left[\frac{\mathbf{a}_z}{1}\frac{1}{1^2} + \frac{2\mathbf{a}_x+\mathbf{a}_z}{\sqrt{5}}\frac{1}{(\sqrt{5})^2} + \frac{2\mathbf{a}_x+2\mathbf{a}_y+\mathbf{a}_z}{3}\frac{1}{3^2} + \frac{2\mathbf{a}_y+\mathbf{a}_z}{\sqrt{5}}\frac{1}{(\sqrt{5})^2}\right]$$

$$\mathbf{E} = 6.82\mathbf{a}_x + 6.82\mathbf{a}_y + 32.8\mathbf{a}_z\ \mathrm{V/m}$$

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Muatan Volume

denote volume charge density by ρ_v

The small amount of charge ΔQ in a small volume Δv is $\Delta Q = \rho_v \Delta v$

$$\rho_v = \lim_{\Delta v \to 0} \frac{\Delta Q}{\Delta v} \quad \longleftrightarrow \quad Q = \int_{\text{vol}} \rho_v \, dv$$

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Muatan Volume

EXAMPLE 2.3

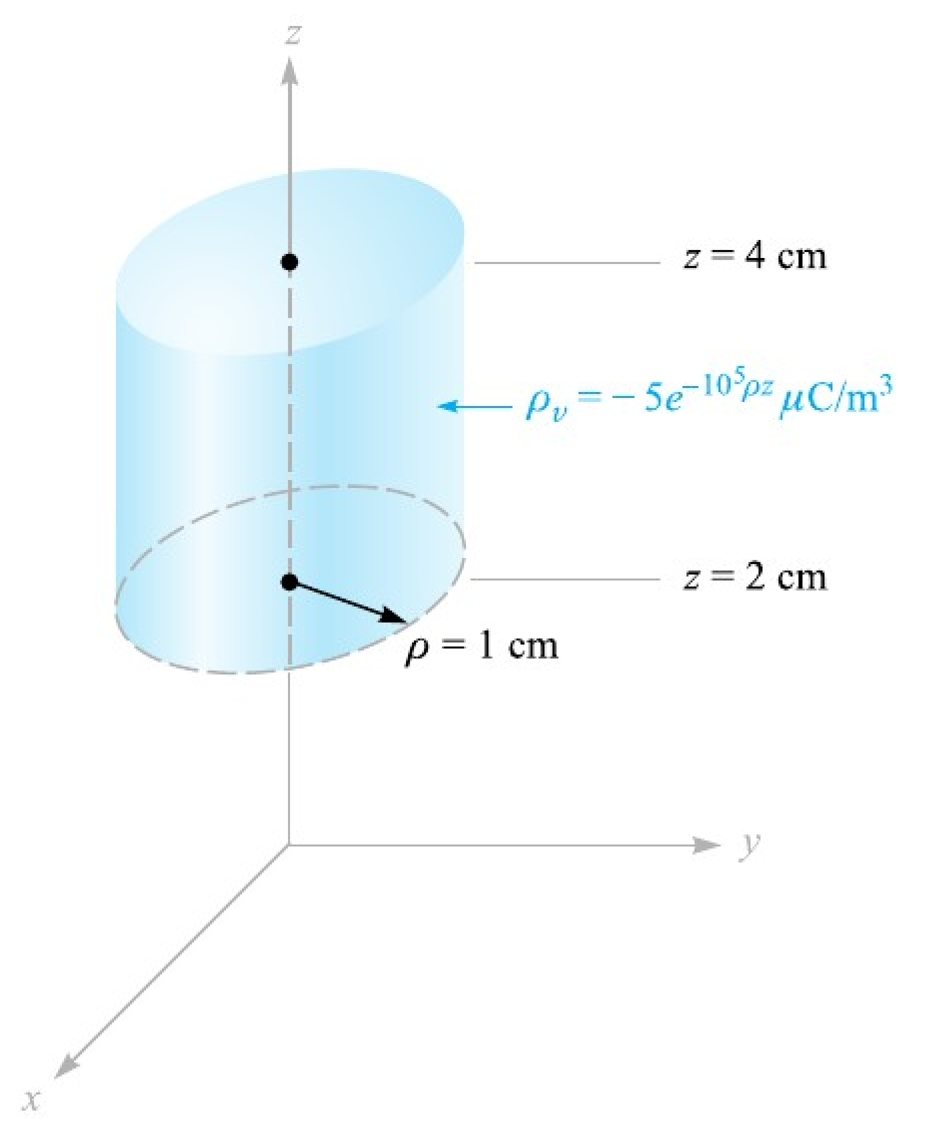
$$\rho_v = -5 \times 10^{-6} e^{-10^5 \rho z} \ \mathrm{C/m^2}$$

$$Q = \int_{0.02}^{0.04} \int_0^{2\pi} \int_0^{0.01} -5 \times 10^{-6} e^{-10^5 \rho z} \rho \, d\rho \, d\phi \, dz$$

$$Q = \int_{0.02}^{0.04} \int_0^{0.01} -10^{-5} \pi e^{-10^5 \rho z} \rho \, d\rho \, dz$$

$$Q = \int_0^{0.01} \left(\frac{-10^{-5}\pi}{-10^5 \rho} e^{-10^5 \rho z} \rho \, d\rho \right)_{z=0.02}^{z=0.04}$$

$$= \int_0^{0.01} -10^{-5} \pi (e^{-2000\rho} - e^{-4000\rho}) d\rho$$



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Muatan & Medan Volume

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$$Q = -10^{-10}\pi \left(\frac{e^{-2000\rho}}{-2000} - \frac{e^{-4000\rho}}{-4000} \right)_0^{0.01}$$

$$Q = -10^{-10}\pi \left(\frac{1}{2000} - \frac{1}{4000} \right)$$

$$= \frac{-\pi}{40} = 0.0785 \text{ pC}$$

$$\Delta \mathbf{E}(\mathbf{r}) = \frac{\Delta Q}{4\pi\epsilon_0 |\mathbf{r} - \mathbf{r}'|^2} \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|}$$

$$= \frac{\rho_v \Delta v}{4\pi\epsilon_0 |\mathbf{r} - \mathbf{r}'|^2} \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|}$$

$$\mathbf{E}(\mathbf{r}) = \int_{\text{vol}} \frac{\rho_v(\mathbf{r}')\, dv'}{4\pi\epsilon_0 |\mathbf{r} - \mathbf{r}'|^2} \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|}$$

Latihan Soal

D2.2. A charge of $-0.3\,\mu$C is located at $A(25, -30, 15)$ (in cm), and a second charge of $0.5\,\mu$C is at $B(-10, 8, 12)$ cm. Find $\mathbf{E}$ at: (a) the origin; (b) $P(15, 20, 50)$ cm.

Ans. $92.3\mathbf{a}_x - 77.6\mathbf{a}_y - 94.2\mathbf{a}_z$ kV/m; $11.9\mathbf{a}_x - 0.519\mathbf{a}_y + 12.4\mathbf{a}_z$ kV/m

D2.4. Calculate the total charge within each of the indicated volumes: (a) $0.1 \leq |x|, |y|, |z| \leq 0.2$: $\rho_v = \dfrac{1}{x^3y^3z^3}$; ($b$) $0 \leq \rho \leq 0.1, 0 \leq \phi \leq \pi, 2 \leq z \leq 4$; $\rho_v = \rho^2 z^2 \sin 0.6\phi$; ($c$) universe: $\rho_v = e^{-2r}/r^2$.

Ans. 0; 1.018 mC; 6.28 C

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