Persamaan Diferensial

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Pendahuluan

- Persamaan diferensial biasa = persamaan diferensial dengan satu perubah

$$F\left(x, y, \frac{dy}{dx}, \frac{d^2y}{dx^2}, \ldots, \frac{d^ny}{dx^n}\right) = 0$$

- Persamaan diferensial ordo tinggi => persamaan diferensial ordo satu dengan mendefinisikan variable baru

$$x^3 \frac{d^3y}{dx^3} + x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + \left(x^2 - p^2\right)y = 0$$

Metode Penyelesaian

- Metode Taylor
- Metode Euler
- Metode Euler yang dimodifikasi
- Metode Runge Kutta orde 4
- Metode Adam
- Metode Milnie
- Metode Adam-Moulton

Metode Taylor

- Proses mencari nilai fungsi y(x) pada titik x tertentu

$$y(x_m) = y(x_{m-1}) + hy'(x_{m-1}) + \frac{h^2 y''(x_{m-1})}{2!} + \frac{h^3 y'''(x_{m-1})}{3!} +$$

$$\ldots + \frac{h^n y^n (x_{m-1})}{n!}$$

dimana $h = x_m - x_{m-1}$

Metode Taylor

$$y' = \left(x_{m-1}\right) = f\left(x_{m-1}, y_{m-1}\right)$$

$$y'' = \left(x_{m-1}\right) = f'\left(x_{m-1}, y_{m-1}\right)$$

$$\vdots$$

$$y^{n} = \left(x_{m-1}\right) = f^{\,n-1}\left(x_{m-1}, y_{m-1}\right)$$

$$n = 1,2,3,4,\ldots$$

$$m = 0,1,2,3,\ldots$$

Contoh

Carilah nilai y(0.1) dari persamaan diferensial biasa berikut dengan metode Taylor

$$f(x, y) = \frac{dy}{dx} = x + y \quad , y(0) = y_0 = 1.5$$

Solusi

- **Step 1** Mencari turunan PD yang diketahui

$$
\begin{aligned}
y'(x) &= f(x,y) = x + y \\
y''(x) &= f'(x,y) = 1 + y'(x) = 1 + f(x,y) \\
y'''(x) &= f''(x,y) = y''(x) = f'(x,y) \\
&\vdots \\
y^{n}(x) &= f^{\,n-1}(x,y) = y^{n-1} = f^{\,n-2}(x,y)
\end{aligned}
$$

Solusi

sehingga

$$
\begin{aligned}
y'(x_0) &= f(x_0, y_0) = x_0 + y_0 = 0 + 1.5 = 1.5 \\
y''(x_0) &= f'(x_0, y_0) = 1 + y'(x_0) = 1 + 1.5 = 2.5 \\
y'''(x_0) &= f''(x_0, y_0) = y''(x_0) = f'(x_0, y_0) = 2.5 \\
&\vdots \\
y^{n}(x_0) &= f^{\,n-1}(x_0, y_0) = y^{n-1} = f^{\,n-2}(x_0, y_0) = 2.5
\end{aligned}
$$

Solusi

- **Step 2** Mencari nilai $y(x_1)$ dimana x_1=0.1

$$h = x_1 - x_0 = 0.1 - 0 = 0.1$$

$$y(0.1) = y(0) + hy'(0) + \frac{h^2 y''(0)}{2!} + \cdots + \frac{h^n y''(0)}{n!}$$

$$= 1.5 + (0.1)(1.5) + \frac{(0.1)^2 (2.5)}{2} + \frac{(0.1)^2 (2.5)}{6} + \cdots$$

$$= 1.6629273$$

Metode Euler

- Proses mencari nilai fungsi y(x) pada titik x tertentu

$$\Delta y_{n-1} = f\left(x_{n-1}, y_{n-1}\right)\Delta x$$

$$y_n = y_{n-1} + \Delta y_{n-1}$$

$$\Delta x = \frac{x_n - x_0}{n}$$

x_n = x yang ditanya nilai fungsinya

x_0 = x awal

n =bilangan asli sembarang

Contoh

Carilah nilai y(0.1) dari PD dibawah ini

$$f(x,y)=\frac{dy}{dx}=\frac{y-x}{y+x} \qquad ,y(0)=2$$

Solusi

- **Step 1** Mencari nilai x

x_n=0.1 dan x_0=0. Jika diambil n=5

$$\Delta x = \frac{x_n - x_0}{n} = \frac{0.1-0}{5} = 0.02$$

Solusi

- **Step 2** Mencari penyelesaian dengan iterasi

Iterasi pertama, n=1

$$\Delta y_0 = f(x_0, y_0)\Delta x$$

$$= \frac{y_0 - x_0}{y_0 + x_0}\Delta x = \frac{2-0}{2+0}(0.02) = 0.02$$

$$y_1 = y(0.02) = y_0 + \Delta y_0 = 2 + 0.02 = 2.02$$

Solusi

Iterasi pertama, n=2

$$x_1 = x_0 + \Delta x = 0 + 0.02 = 0.02$$

$$\begin{aligned}\Delta y_1 &= f\left(x_1, y_1\right)\Delta x \\ &= \frac{y_1 - x_1}{y_1 + x_1}\Delta x = \frac{2.02 - 0.02}{2.02 + 0.02}\left(0.02\right) = 0.01961\end{aligned}$$

$$y_2 = y\left(0.04\right) = y_1 + \Delta y_1 = 2.02 + 0.01961 = 2.03961$$

Solusi

Iterasi pertama, n=5

$$x_4 = x_3 + \Delta x = 0.06 + 0.02 = 0.08$$

$$\Delta y_4 = f\left(x_4, y_4\right)\Delta x$$

$$= \frac{y_4 - x_4}{y_4 + x_4}\Delta x = \frac{2.07771 - 0.08}{2.07771 + 0.08}\left(0.02\right) = 0.01852$$

$$y_5 = y\left(0.1\right) = y_4 + \Delta y_4 = 2.07771 + 0.01852 = 2.09623$$