# Interpolasi

# Definisi interpolasi

- Adalah mencari nilai suatu fungsi yang tidak diketahui diantara beberapa nilai fungsi yang diketahui pada tabel fungsi tersebut

# Definisi interpolasi

| **x** | **f(x)** |
|---|---|
| 0.0 | 0.000 |
| 0.2 | 0.406 |
| 0.4 | 0.846 |
| 0.6 | 1.368 |
| 0.8 | 2.060 |
| 1.0 | 3.114 |
| 1.2 | 5.114 |

- Contoh mencari nilai fungsi f(0.1), f(0.35) dan f(1.11)
- **Interpolasi balik** = mencari nilai x dari variable f(x)
- Contoh mencari nilai x untuk f(x)=3.015 atau f(x)=1.555

# Equispaced vs Non-Equispaced

- **Tabel equispaced** = tabel yang mempunyai nilai beda variabel yang sama ($\Delta x$=konstan)

- **Tabel non-equispaced** = tabel yang **tidak** mempunyai nilai beda variabel yang sama ($\Delta x$ tidak konstan)

# Tabel beda hingga

| x | f(x) | $\Delta f(x)$ | $\Delta^2 f(x)$ | $\Delta^3 f(x)$ | $\Delta^4 f(x)$ | $\Delta^5 f(x)$ | $\Delta^6 f(x)$ |
|---|---|---|---|---|---|---|---|
| 0.0 | 0.000 | | | | | | |
| | | 0.406 | | | | | |
| 0.2 | 0.406 | | 0.034 | | | | |
| | | 0.440 | | 0.048 | | | |
| 0.4 | 0.846 | | 0.082 | | 0.040 | | |
| | | 0.552 | | 0.088 | | 0.064 | |
| 0.6 | 1.368 | | 0.170 | | 0.104 | | 0.254 |
| | | 0.692 | | 0.192 | | 0.318 | |
| 0.8 | 2.060 | | 0.361 | | 0.422 | | |
| | | 1.054 | | 0.614 | | | |
| 1.0 | 3.114 | | 0.976 | | | | |
| | | 2.030 | | | | | |
| 1.2 | 5.114 | | | | | | |

# Penyelesaian Persoalan Interpolasi

- Newton Gregory Forward
- Newton Gregory Backward
- Stirling
- Lagrange
- Hermitte

# Newton Gregory Forward

- Menyelesaikan masalah interpolasi dengan persamaan

$$f(x_s) = f_0 + s\Delta f_0 + \frac{s(s-1)}{2!}\Delta^2 f_0 + \ldots + \frac{s(s-1)(s-2)\ldots(s-n+1)}{n!}\Delta^n f_0$$

dimana

$$s = \frac{x_s - x_0}{x_1 - x_0}$$

# Kelemahan NGF

- Hanya dapat digunakan menyelesaikan persoalan interpolasi equispaced
- Menyelesaikan permasalahan untuk nilai $x_s$ terletak diantara $x_0$ dan $x_1$
- Tidak dapat menyelesaikan persoalan interpolasi balik

# Keuntungan NGF

- Metode yang efektif untuk mencari nilai f(x) di sekitar titik awal

# Tabel beda NGF

| s | x | f(x) | $\Delta f(x)$ | $\Delta^2 f(x)$ | $\Delta^3 f(x)$ | $\Delta^4 f(x)$ | $\Delta^5 f(x)$ |
|---|---|---|---|---|---|---|---|
| 0 | $x_0$ | $\boxed{f_0}$ | | | | | |
| | | | $\boxed{\Delta f_0}$ | | | | |
| 1 | $x_1$ | $f_1$ | | $\boxed{\Delta^2 f_0}$ | | | |
| | | | $\Delta f_1$ | | $\boxed{\Delta^3 f_0}$ | | |
| 2 | $x_2$ | $f_2$ | | $\Delta^2 f_1$ | | $\boxed{\Delta^4 f_0}$ | |
| | | | $\Delta f_2$ | | $\Delta^3 f_1$ | | $\boxed{\Delta^5 f_0}$ |
| 3 | $x_3$ | $f_3$ | | $\Delta^2 f_2$ | | $\Delta^4 f_1$ | |
| | | | $\Delta f_3$ | | $\Delta^3 f_2$ | | |
| 4 | $x_4$ | $f_4$ | | $\Delta^2 f_3$ | | | |
| | | | $\Delta f_4$ | | | | |
| 5 | $x_5$ | $f_5$ | | | | | |

# Contoh soal

| n | **x** | **f(x)** |
|---|---|---|
| 0 | 1.0 | 1.449 |
| 1 | 1.3 | 2.060 |
| 2 | 1.6 | 2.645 |
| 3 | 1.9 | 3.216 |
| 4 | 2.2 | 3.779 |
| 5 | 2.5 | 4.338 |
| 6 | 2.8 | 4.898 |

- Carilah nilai $f(x_s)$ pada $x_s=1.03$ dengan metode NGF

# Solusi

| s | x | f(x) | Δf(x) | $\Delta^2 f(x)$ | $\Delta^3 f(x)$ | $\Delta^4 f(x)$ | $\Delta^5 f(x)$ | $\Delta^6 f(x)$ |
|---|---|---|---|---|---|---|---|---|
| 0 | 1.0 | $\boxed{1.449}$ | | | | | | |
| | | | $\boxed{0.611}$ | | | | | |
| 1 | 1.3 | 2.060 | | $\boxed{-0.026}$ | | | | |
| | | | 0.585 | | $\boxed{0.012}$ | | | |
| 2 | 1.6 | 2.645 | | -0.014 | | $\boxed{-0.006}$ | | |
| | | | 0.571 | | 0.006 | | $\boxed{0.004}$ | |
| 3 | 1.9 | 3.216 | | -0.008 | | -0.002 | | $\boxed{-0.01}$ |
| | | | 0.563 | | 0.004 | | 0.003 | |
| 4 | 2.2 | 3.779 | | -0.004 | | 0.001 | | |
| | | | 0.559 | | 0.005 | | | |
| 5 | 2.5 | 4.338 | | 0.001 | | | | |
| | | | 0.560 | | | | | |
| 6 | 2.8 | 4.898 | | | | | | |

# Solusi

Nilai s diperoleh

$$s = \frac{x_s - x_0}{h} = \frac{1.03 - 1}{1.3 - 1} = 0.1$$

Nilai yang digunakan pada tabel beda digunakan pada persamaan NGF

# Solusi

- Dari hasil tersebut diperoleh

$$f(x_s) = f_0 + s\Delta f_0 + \frac{s(s-1)}{2!}\Delta^2 f_0 + \frac{s(s-1)(s-2)}{3!}\Delta^3 f_0 +$$

$$\frac{s(s-1)(s-2)(s-3)}{4!}\Delta^4 f_0 +$$

$$\frac{s(s-1)(s-2)(s-3)(s-4)}{5!}\Delta^5 f_0 +$$

$$\frac{s(s-1)(s-2)(s-3)(s-4)(s-5)}{6!}\Delta^6 f_0$$

$$= 1.5118136$$

# Newton Gregory Backward

- Menyelesaikan masalah interpolasi dengan persamaan

$$f(x_s) = f_0 + s\Delta f_{-1} + \frac{s(s+1)}{2!}\Delta^2 f_{-2} + \ldots + \frac{s(s+1)(s+2)\ldots(s+n-1)}{n!}\Delta^n f_{-n}$$

dimana

$$s = \frac{x_s - x_0}{h}$$

# Kelemahan NGB

- Hanya dapat digunakan menyelesaikan persoalan interpolasi equispaced
- Menyelesaikan permasalahan untuk nilai $x_s$ terletak diantara $x_0$ dan $x_1$
- Tidak dapat menyelesaikan persoalan interpolasi balik

# Keuntungan NGB

- Metode yang efektif untuk mencari nilai f(x) di sekitar titik akhir

# Tabel beda NGB

| s | x | f(x) | Δf(x) | $\Delta^2 f(x)$ | $\Delta^3 f(x)$ | $\Delta^4 f(x)$ | $\Delta^5 f(x)$ |
|---|---|---|---|---|---|---|---|
| -5 | $x_{-5}$ | $f_{-5}$ | | | | | |
| | | | $\Delta f_{-5}$ | | | | |
| -4 | $x_{-4}$ | $f_{-4}$ | | $\Delta^2 f_{-5}$ | | | |
| | | | $\Delta f_{-4}$ | | $\Delta^3 f_{-5}$ | | |
| -3 | $x_{-3}$ | $f_{-3}$ | | $\Delta^2 f_{-4}$ | | $\Delta^4 f_{-5}$ | |
| | | | $\Delta f_{-3}$ | | $\Delta^3 f_{-4}$ | | $\boxed{\Delta^5 f_{-5}}$ |
| -2 | $x_{-2}$ | $f_{-2}$ | | $\Delta^2 f_{-3}$ | | $\boxed{\Delta^4 f_{-4}}$ | |
| | | | $\Delta f_{-2}$ | | $\boxed{\Delta^3 f_{-3}}$ | | |
| -1 | $x_{-1}$ | $f_{-1}$ | | $\boxed{\Delta^2 f_{-2}}$ | | | |
| | | | $\boxed{\Delta f_{-1}}$ | | | | |
| 0 | $x_0$ | $\boxed{f_0}$ | | | | | |

# Contoh soal

| n | **x** | **f(x)** |
|---|---|---|
| -6 | 1.0 | 1.449 |
| -5 | 1.3 | 2.060 |
| -4 | 1.6 | 2.645 |
| -3 | 1.9 | 3.216 |
| -2 | 2.2 | 3.779 |
| -1 | 2.5 | 4.338 |
| 0 | 2.8 | 4.898 |

- Carilah nilai $f(x_s)$ pada $x_s$ =2.67 dengan metode NGB

# Solusi

| s | x | f(x) | $\Delta f(x)$ | $\Delta^2 f(x)$ | $\Delta^3 f(x)$ | $\Delta^4 f(x)$ | $\Delta^5 f(x)$ | $\Delta^6 f(x)$ |
|---|---|---|---|---|---|---|---|---|
| -6 | 1.0 | 1.449 | | | | | | |
| | | | 0.611 | | | | | |
| -5 | 1.3 | 2.060 | | -0.026 | | | | |
| | | | 0.585 | | 0.012 | | | |
| -4 | 1.6 | 2.645 | | -0.014 | | -0.006 | | |
| | | | 0.571 | | 0.006 | | 0.004 | |
| -3 | 1.9 | 3.216 | | -0.008 | | -0.002 | | $\boxed{-0.01}$ |
| | | | 0.563 | | 0.004 | | $\boxed{0.003}$ | |
| -2 | 2.2 | 3.779 | | -0.004 | | $\boxed{0.001}$ | | |
| | | | 0.559 | | $\boxed{0.005}$ | | | |
| -1 | 2.5 | 4.338 | | $\boxed{0.001}$ | | | | |
| | | | $\boxed{0.560}$ | | | | | |
| 0 | 2.8 | $\boxed{4.898}$ | | | | | | |

# Solusi

Nilai s diperoleh

$$s = \frac{x_s - x_0}{h} = \frac{2.67 - 2.8}{1.3 - 1} = -0.4333$$

Nilai yang digunakan pada tabel beda digunakan pada persamaan NGB

# Solusi

- Dari hasil tersebut diperoleh

$$f(x_s) = f_0 + s\Delta f_{-1} + \frac{s(s+1)}{2!}\Delta^2 f_{-2} + \frac{s(s+1)(s+2)}{3!}\Delta^3 f_{-3} +$$

$$\frac{s(s+1)(s+2)(s+3)}{4!}\Delta^4 f_{-4} +$$

$$\frac{s(s+1)(s+2)(s+3)(s+4)}{5!}\Delta^5 f_{-5} +$$

$$\frac{s(s+1)(s+2)(s+3)(s+4)(s+5)}{6!}\Delta^6 f_{-6}$$

$$= 4.654783$$

# Stirling Method

- Menyelesaikan masalah interpolasi dengan persamaan

$$
f(x_s) = f_0 + \left|\begin{matrix} s \\ 1 \end{matrix}\right| \frac{\Delta f_{-1} + \Delta f_0}{2} + \frac{\left|\begin{matrix} s+1 \\ 2 \end{matrix}\right| + \left|\begin{matrix} s \\ 2 \end{matrix}\right|}{2} \Delta^2 f_{-1} +
$$

$$
\left|\begin{matrix} s+1 \\ 3 \end{matrix}\right| \frac{\Delta^3 f_{-2} + \Delta^3 f_{-1}}{2} + \frac{\left|\begin{matrix} s+2 \\ 4 \end{matrix}\right| + \left|\begin{matrix} s+1 \\ 4 \end{matrix}\right|}{2} \Delta^4 f_{-2} +
$$

$$
\left|\begin{matrix} s+2 \\ 5 \end{matrix}\right| \frac{\Delta^5 f_{-3} + \Delta^5 f_{-2}}{2} + \frac{\left|\begin{matrix} s+3 \\ 6 \end{matrix}\right| + \left|\begin{matrix} s+2 \\ 6 \end{matrix}\right|}{2} \Delta^6 f_{-3} + \ldots
$$

# Stirling Method

dimana

$$s = \frac{x_s - x_0}{h}$$

sedangkan

$$\begin{vmatrix} s+j \\ k \end{vmatrix} = \frac{(s+j)(s+j-1)(s+j-2)\ldots(s+j-k+1)}{k!}$$

# Kelemahan Stirling

- Hanya dapat digunakan menyelesaikan persoalan interpolasi equispaced
- Menyelesaikan permasalahan untuk nilai $x_s$ terletak diantara $x_0$ dan $x_1$
- Tidak dapat menyelesaikan persoalan interpolasi balik

# Keuntungan Stirling

- Metode yang efektif untuk mencari nilai f(x) di sekitar titik tengah

# Solusi

| s | x | f(x) | Δf(x) | $\Delta^2 f(x)$ | $\Delta^3 f(x)$ | $\Delta^4 f(x)$ | $\Delta^5 f(x)$ | $\Delta^6 f(x)$ |
|---|---|---|---|---|---|---|---|---|
| -3 | $x_{-3}$ | $f_{-3}$ | | | | | | |
| | | | $\Delta f_{-3}$ | | | | | |
| -2 | $x_{-2}$ | $f_{-2}$ | | $\Delta^2 f_{-3}$ | | | | |
| | | | $\Delta f_{-2}$ | | $\Delta^3 f_{-3}$ | | | |
| -1 | $x_{-1}$ | $f_{-1}$ | | $\Delta^2 f_{-2}$ | | $\Delta^4 f_{-3}$ | | |
| | | | $\boxed{\Delta f_{-1}}$ | | $\boxed{\Delta^3 f_{-2}}$ | | $\boxed{\Delta^5 f_{-3}}$ | |
| 0 | $x_0$ | $\boxed{f_0}$ | | $\boxed{\Delta^2 f_{-1}}$ | | $\boxed{\Delta^4 f_{-2}}$ | | $\boxed{\Delta^6 f_{-3}}$ |
| | | | $\boxed{\Delta f_0}$ | | $\boxed{\Delta^3 f_{-1}}$ | | $\boxed{\Delta^5 f_{-2}}$ | |
| 1 | $x_1$ | $f_1$ | | $\Delta^2 f_0$ | | $\Delta^4 f_{-1}$ | | |
| | | | $\Delta f_1$ | | $\Delta^3 f_0$ | | | |
| 2 | $x_2$ | $f_2$ | | $\Delta^2 f_1$ | | | | |
| | | | $\Delta f_2$ | | | | | |
| 3 | $x_3$ | $f_3$ | | | | | | |

# Contoh soal

| n | **x** | **f(x)** |
|---|---|---|
| -3 | 1.0 | 1.449 |
| -2 | 1.3 | 2.060 |
| -1 | 1.6 | 2.645 |
| 0 | 1.9 | 3.216 |
| 1 | 2.2 | 3.779 |
| 2 | 2.5 | 4.338 |
| 3 | 2.8 | 4.898 |

- Carilah nilai $f(x_s)$ pada $x_s = 1.87$ dengan metode stirling

# Solusi

| s | x | f(x) | $\Delta f(x)$ | $\Delta^2 f(x)$ | $\Delta^3 f(x)$ | $\Delta^4 f(x)$ | $\Delta^5 f(x)$ | $\Delta^6 f(x)$ |
|---|---|---|---|---|---|---|---|---|
| -3 | 1.0 | 1.449 | | | | | | |
| | | | 0.611 | | | | | |
| -2 | 1.3 | 2.060 | | -0.026 | | | | |
| | | | 0.585 | | 0.012 | | | |
| -1 | 1.6 | 2.645 | | -0.014 | | -0.006 | | |
| | | | 0.571 | | 0.006 | | 0.004 | |
| 0 | 1.9 | 3.216 | | -0.008 | | -0.002 | | -0.01 |
| | | | 0.563 | | 0.004 | | 0.003 | |
| 1 | 2.2 | 3.779 | | -0.004 | | 0.001 | | |
| | | | 0.559 | | 0.005 | | | |
| 2 | 2.5 | 4.338 | | 0.001 | | | | |
| | | | 0.560 | | | | | |
| 3 | 2.8 | 4.898 | | | | | | |

# Solusi

Nilai s diperoleh

$$s = \frac{x_s - x_0}{h} = \frac{1.87 - 1.9}{1.3 - 1} = -0.1$$

Nilai yang digunakan pada tabel beda digunakan pada persamaan stirling

# Solusi

$$f\left(x_s\right) = f_0 + \left|\begin{matrix} s \\ 1 \end{matrix}\right| \frac{\Delta f_{-1} + \Delta f_0}{2} + \frac{\left|\begin{matrix} s+1 \\ 2 \end{matrix}\right| + \left|\begin{matrix} s \\ 2 \end{matrix}\right|}{2} \Delta^2 f_{-1} +$$

$$\left|\begin{matrix} s+1 \\ 3 \end{matrix}\right| \frac{\Delta^3 f_{-2} + \Delta^3 f_{-1}}{2} + \frac{\left|\begin{matrix} s+2 \\ 4 \end{matrix}\right| + \left|\begin{matrix} s+1 \\ 4 \end{matrix}\right|}{2} \Delta^4 f_{-2} +$$

$$\left|\begin{matrix} s+2 \\ 5 \end{matrix}\right| \frac{\Delta^5 f_{-3} + \Delta^5 f_{-2}}{2} + \frac{\left|\begin{matrix} s+3 \\ 6 \end{matrix}\right| + \left|\begin{matrix} s+2 \\ 6 \end{matrix}\right|}{2} \Delta^6 f_{-3}$$

$$= 3.159402$$

# Lagrange Method

- Menyelesaikan masalah interpolasi dengan persamaan

$$f(x) = \frac{(x-x_1)(x-x_2)\ldots(x-x_n)}{(x_0-x_1)(x_0-x_2)\ldots(x_0-x_n)} f_0 +$$

$$\frac{(x-x_1)(x-x_2)\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)\ldots(x_1-x_n)} f_1 +$$

$$\ldots + \frac{(x-x_1)(x-x_2)\ldots(x-x_n)}{(x_n-x_1)(x_n-x_2)\ldots(x_n-x_{n-1})} f_n$$

- Bagian denumerator $\neq x_n - x_n$

# Keuntungan Lagrange

- Dapat digunakan menyelesaikan persoalan interpolasi equispaced dan non equispaced
- Dapat menyelesaikan persoalan interpolasi dan interpolasi balik
- Dapat digunakan untuk mencari nilai f(x) di sekitar titik awal, tengah, dan akhir
- Tidak membutuhkan tabel beda dalam penyelesaian masalah

# Kelemahan Lagrange

- Jika nilai variabel dan nilai fungsi terlalu banyak, maka perhitungan menjadi kompleks

# Contoh soal

| n | **x** | **f(x)** |
|---|---|---|
| 0 | 1.0 | 0.00000 |
| 1 | 1.2 | 0.26254 |
| 2 | 1.5 | 0.91230 |
| 3 | 1.9 | 2.31709 |
| 4 | 2.1 | 3.27194 |
| 5 | 2.5 | 5.72682 |
| 6 | 3.0 | 9.88751 |

- Carilah nilai f(x) pada x=1.03 dengan metode lagrange

# Solusi

- Untuk x=1.03

$$f(1.03)=\frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)(x-x_5)(x-x_6)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)(x_0-x_5)(x_0-x_6)}f_0+$$

$$\frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)(x-x_5)(x-x_6)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)(x_1-x_5)(x_1-x_6)}f_1+$$

$$\ldots+\frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)(x-x_5)(x-x_6)}{(x_6-x_0)(x_6-x_1)(x_6-x_2)(x_6-x_3)(x_6-x_4)(x_6-x_5)}f_6$$

$$f(1.03)=0.031352$$

# Hermitte Method

- Menyelesaikan masalah interpolasi dengan persamaan

$$f(x)=\frac{\sin(x-x_1)\sin(x-x_2)\ldots\sin(x-x_n)}{\sin(x_0-x_1)\sin(x_0-x_2)\ldots\sin(x_0-x_n)}f_0+$$

$$\frac{\sin(x-x_1)\sin(x-x_2)\ldots\sin(x-x_n)}{\sin(x_1-x_0)\sin(x_1-x_2)\ldots\sin(x_1-x_n)}f_1+$$

$$\ldots+\frac{\sin(x-x_1)\sin(x-x_2)\ldots\sin(x-x_n)}{\sin(x_n-x_1)\sin(x_n-x_2)\ldots\sin(x_n-x_{n-1})}f_n$$

- Bagian denumerator $\neq x_n - x_n$

# Keuntungan Hermitte

- Dapat digunakan menyelesaikan persoalan interpolasi equispaced dan non equispaced
- Dapat menyelesaikan persoalan interpolasi dan interpolasi balik
- Dapat digunakan untuk mencari nilai f(x) di sekitar titik awal, tengah, dan akhir
- Tidak membutuhkan tabel beda dalam penyelesaian masalah
- Hanya efektif untuk persoalan dengan metode periodik

# Kelemahan Lagrange

- Jika nilai variabel dan nilai fungsi terlalu banyak, maka perhitungan menjadi kompleks

# Contoh soal

| n | **x** | **f(x)** |
|---|---|---|
| 0 | 2.823 | 0.31323 |
| 1 | 3.016 | 0.12526 |
| 2 | 3.458 | -0.31115 |
| 3 | 4.398 | -0.95099 |
| 4 | 5.655 | -0.58768 |

- Carilah nilai f(x) pada x=3.535 dengan metode hermitte

# Solusi

- Untuk x=3.535

$$f(3.535)=\frac{\sin(x-x_1)\sin(x-x_2)\sin(x-x_3)\sin(x-x_4)}{\sin(x_0-x_1)\sin(x_0-x_2)\sin(x_0-x_3)\sin(x_0-x_4)}f_0+$$

$$\frac{\sin(x-x_1)\sin(x-x_2)\sin(x-x_3)\sin(x-x_4)}{\sin(x_1-x_0)\sin(x_1-x_2)\sin(x_1-x_3)\sin(x_1-x_4)}f_1+$$

$$\ldots+\frac{\sin(x-x_1)\sin(x-x_2)\sin(x-x_3)\sin(x-x_4)}{\sin(x_4-x_0)\sin(x_4-x_1)\sin(x_4-x_2)\sin(x_4-x_3)}f_4$$

$$f(3.535)=-0.20365$$