Diferensiasi Numerik

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Pendahuluan

- Penyelesaian lebih mudah untuk mencari nilai diferensial suatu fungsi yang cukup kompleks
- Misal mencari diferensial pada x=1.6 dari fungsi berikut

$$f(x) = \frac{x^2 \ln(x) + e^{-x}}{5x \sin x}$$

$$f(x) = \frac{x^2 \cos x}{e^{-x}} \qquad \text{dst}$$

Metode Diferensiasi Numerik

1. Metode Newton-Gregory Forward
2. Metode Newton Gregory Backward
3. Metode Stirling
4. Metode Lagrange

Metode Newton-Gregory Forward

- Penurunan persamaan Newton-Gregory Forward pada interpolasi

$$f'(x) = \frac{1}{h}\left[\Delta f_0 + \frac{2s-1}{2!}\Delta^2 f_0 + \frac{3s^2-6s+2}{3!}\Delta^3 f_0 + \ldots\right]$$

Tahap Penyelesaian

1. Mencari nilai beda dan membuat tabel beda hingga
2. Mencari nilai s dan mencari nilai diferensial pada titik yang diketahui

Contoh soal

n	**x**	**f(x)**
0	1.0	1.449
1	1.3	2.060
2	1.6	2.645
3	1.9	3.216
4	2.2	3.779
5	2.5	4.338
6	2.8	4.898

- Carilah nilai f’(x) pada x=1.03 dengan metode NGF

Solusi

Step 1

s	x	f(x)	Δf(x)	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$	$\Delta^5 f(x)$	$\Delta^6 f(x)$
0	1.0	$\boxed{1.449}$						
			$\boxed{0.611}$					
1	1.3	2.060		$\boxed{-0.026}$				
			0.585		$\boxed{0.012}$			
2	1.6	2.645		-0.014		$\boxed{-0.006}$		
			0.571		0.006		$\boxed{0.004}$	
3	1.9	3.216		-0.008		-0.002		$\boxed{-0.01}$
			0.563		0.004		0.003	
4	2.2	3.779		-0.004		0.001		
			0.559		0.005			
5	2.5	4.338		0.001				
			0.560					
6	2.8	4.898						

Solusi

Step 2

Nilai s diperoleh

$$s = \frac{x - x_0}{h} = \frac{1.03 - 1}{1.3 - 1} = 0.1$$

Nilai diferensial dapat diperoleh dengan menggunakan persamaan yang ada dan nilai dari Δf

Solusi

Step 2

Nilai diferensial saat x=1.03

$$f'(x) = \frac{1}{h}\left[\Delta f_0 + \frac{2s-1}{2!}\Delta^2 f_0 + \frac{3s^2-6s+2}{3!}\Delta^3 f_0 + \right.$$

$$\frac{4s^3-18s^2+22s-6}{4!}\Delta^4 f_0 + \frac{5s^4-40s^3+105s^2-100s+24}{5!}\Delta^5 f_0 +$$

$$\left. \frac{6s^5-75s^4+340s^3-675s^2+548s-120}{6!}\Delta^6 f_0 \right] = 2.088647$$

Metode Newton-Gregory Backward

- Penurunan persamaan Newton-Gregory Backward pada interpolasi

$$f'(x) = \frac{1}{h}\left[\Delta f_{-1} + \frac{2s+1}{2!}\Delta^2 f_{-2} + \frac{3s^2+6s+2}{3!}\Delta^3 f_{-3} + \dots\right]$$

Tahap Penyelesaian

1. Mencari nilai beda dan membuat tabel beda hingga
2. Mencari nilai s dan mencari nilai diferensial pada titik yang diketahui

Contoh soal

n	**x**	**f(x)**
-6	1.0	1.449
-5	1.3	2.060
-4	1.6	2.645
-3	1.9	3.216
-2	2.2	3.779
-1	2.5	4.338
0	2.8	4.898

- Carilah nilai f'(x) pada x=2.67 dengan metode NGB

Solusi

s	x	f(x)	Δf(x)	$Δ^2$f(x)	$Δ^3$f(x)	$Δ^4$f(x)	$Δ^5$f(x)	$Δ^6$f(x)
-6	1.0	1.449						
			0.611					
-5	1.3	2.060		-0.026				
			0.585		0.012			
-4	1.6	2.645		-0.014		-0.006		
			0.571		0.006		0.004	
-3	1.9	3.216		-0.008		-0.002		$\boxed{-0.01}$
			0.563		0.004		$\boxed{0.003}$	
-2	2.2	3.779		-0.004		$\boxed{0.001}$		
			0.559		$\boxed{0.005}$			
-1	2.5	4.338		$\boxed{0.001}$				
			$\boxed{0.560}$					
0	2.8	$\boxed{4.898}$						

Solusi

Nilai s diperoleh

$$s = \frac{x_s - x_0}{h} = \frac{2.67 - 2.8}{1.3 - 1} = -0.4333$$

Nilai yang digunakan pada tabel beda digunakan pada persamaan NGB

Solusi

Step 2

Nilai diferensial saat x=2.67

$$f'(x) = \frac{1}{h}\left[\Delta f_{-1} + \frac{2s+1}{2!}\Delta^2 f_{-2} + \frac{3s^2+6s+2}{3!}\Delta^3 f_{-3} + \right.$$

$$\frac{4s^3+18s^2+22s+6}{4!}\Delta^4 f_{-4} + \frac{5s^4+40s^3+105s^2+100s+24}{5!}\Delta^5 f_{-5} +$$

$$\left. \frac{6s^5+75s^4+340s^3+675s^2+548s+120}{6!}\Delta^6 f_{-6} \right] = 1.8711214$$

Metode Stirling

- Penurunan persamaan stirling pada interpolasi

$$f'(x) = \frac{1}{h}\left[\frac{\Delta f_{-1} + \Delta f_0}{2} + s\Delta^2 f_{-1} + \frac{3s^2 - 1}{3!}\frac{\left(\Delta^3 f_{-1} + \Delta^3 f_{-2}\right)}{2} + \ldots\right]$$

Contoh soal

n	**x**	**f(x)**
-3	1.0	1.449
-2	1.3	2.060
-1	1.6	2.645
0	1.9	3.216
1	2.2	3.779
2	2.5	4.338
3	2.8	4.898

- Carilah nilai f(x) pada x=1.87 dengan metode stirling

Solusi

s	x	f(x)	Δf(x)	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$	$\Delta^5 f(x)$	$\Delta^6 f(x)$
-3	1.0	1.449						
			0.611					
-2	1.3	2.060		-0.026				
			0.585		0.012			
-1	1.6	2.645		-0.014		-0.006		
			$\boxed{0.571}$		$\boxed{0.006}$		$\boxed{0.004}$	
0	1.9	$\boxed{3.216}$		$\boxed{-0.008}$		$\boxed{-0.002}$		$\boxed{-0.01}$
			$\boxed{0.563}$		$\boxed{0.004}$		$\boxed{0.003}$	
1	2.2	3.779		-0.004		0.001		
			0.559		0.005			
2	2.5	4.338		0.001				
			0.560					
3	2.8	4.898						

Solusi

Nilai s diperoleh

$$s = \frac{x_s - x_0}{h} = \frac{1.87 - 1.9}{1.3 - 1} = -0.1$$

Nilai yang digunakan pada tabel beda digunakan pada persamaan stirling

Solusi

Step 2

Nilai diferensial saat x=1.87

$$f'(x) = \frac{1}{h}\left[\frac{\Delta f_{-1} + \Delta f_0}{2} + s\Delta^2 f_{-1} + \frac{3s^2 - 1}{3!}\frac{\left(\Delta^3 f_{-1} + \Delta^3 f_{-2}\right)}{2} + \right.$$

$$\frac{4s^3 - 2s}{4!} + \Delta^4 f_{-2} + \frac{5s^4 - 15s^2 + 4}{5!}\frac{\left(\Delta^5 f_{-2} + \Delta^5 f_{-3}\right)}{2} +$$

$$\left. \frac{6s^5 - 20s^3 + 8s}{6!}\Delta^6 f_{-3} \right] = 1.890292$$

Metode Lagrange

$$f'(x) = \sum_{m=1}^{n+1} \frac{f_m - 1}{\prod\limits_{\substack{k=1 \\ k \neq m}}^{n+1} \left(x_{m-1} - x_{k-1} \right)} \left(\sum_{\substack{j=1 \\ j \neq m}}^{n+1} \frac{\prod\limits_{\substack{i=1 \\ i \neq 1}}^{n+4} \left(x - x_{i-1} \right)}{\left(x - x_{j-1} \right)} \right)$$

Contoh soal

n	**x**	**f(x)**
0	1.0	0.00000
1	1.2	0.26254
2	1.5	0.91230
3	1.9	2.31709
4	2.1	3.27194
5	2.5	5.72682
6	3.0	9.88751

- Carilah nilai f(x) pada x=2.25 dengan metode lagrange

Solusi

$$f'(x)=\sum_{\substack{m=1}}^{n+1}\frac{f_m-1}{\prod\limits_{\substack{k=1\\k\neq m}}^{n+1}\left(x_{m-1}-x_{k-1}\right)}\left(\sum_{\substack{j=1\\j\neq m}}^{n+1}\frac{\prod\limits_{\substack{i=1\\i\neq 1}}^{n+4}\left(x-x_{i-1}\right)}{\left(x-x_{j-1}\right)}\right)$$

Solusi

- Untuk x=2.25

$$\frac{(x-x_2)(x-x_3)(x-x_4)+(x-x_1)(x-x_3)(x-x_4)+(x-x_1)(x-x_2)(x-x_4)+(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)}f_0+$$

$$\frac{(x-x_2)(x-x_3)(x-x_4)+(x-x_0)(x-x_3)(x-x_4)+(x-x_0)(x-x_2)(x-x_4)+(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)}f_1+$$

$$\frac{(x-x_1)(x-x_3)(x-x_4)+(x-x_0)(x-x_3)(x-x_4)+(x-x_0)(x-x_1)(x-x_4)+(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)}f_2+$$

dst