TUGAS 03 METODE NUMERIK

1. Selesaikan dengan metode determinan dan invers matriks

   $x_1 + 3x_2 + 6x_3 = 17$

   $2x_1 + 8x_2 + 16x_3 = 42$

   $5x_1 + 21x_2 + 45x_3 = 91$

   Answer:

   Step 1 Persamaan disusun dalam bentuk matriks

$$\begin{bmatrix} 1 & 3 & 6 \\ 2 & 8 & 16 \\ 5 & 21 & 45 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 17 \\ 42 \\ 91 \end{bmatrix} \rightarrow A \cdot x = H$$

   Step 2 Mencari determinan matriks, adjoint dan invers kemudian mencari matriks variable

   Determinan matriks A

$$Det\ A = \begin{vmatrix} 1 & 3 & 6 \\ 2 & 8 & 16 \\ 5 & 21 & 45 \end{vmatrix} = 1((8)(45)-(16)(21)) + 3((16)(5)-(2)(45)) + 6((2)(21)-(8)(5)) = 6$$

   Adjoint matriks A

$$AdjA = \begin{bmatrix} \begin{vmatrix} 8 & 16 \\ 21 & 45 \end{vmatrix} & -\begin{vmatrix} 3 & 6 \\ 21 & 45 \end{vmatrix} & \begin{vmatrix} 3 & 6 \\ 8 & 16 \end{vmatrix} \\ -\begin{vmatrix} 2 & 16 \\ 5 & 45 \end{vmatrix} & \begin{vmatrix} 1 & 6 \\ 5 & 45 \end{vmatrix} & -\begin{vmatrix} 1 & 6 \\ 2 & 16 \end{vmatrix} \\ \begin{vmatrix} 2 & 8 \\ 5 & 21 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ 5 & 21 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 2 & 8 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} 24 & -9 & 0 \\ -10 & 15 & -4 \\ 2 & -6 & 2 \end{bmatrix}$$

   Invers matriks A

$$A^{-1} = \frac{adjA}{\det A} = \frac{\begin{bmatrix} 24 & -9 & 0 \\ -10 & 15 & -4 \\ 2 & -6 & 2 \end{bmatrix}}{6} = \begin{bmatrix} 4 & -1.5 & 0 \\ -1.6667 & 2.5 & -0.6667 \\ 0.3333 & -1 & 0.3333 \end{bmatrix}$$

Penyelesaiannya

$$X = A^{-1} \cdot H = \begin{bmatrix} 4 & -1.5 & 0 \\ -1.6667 & 2.5 & -0.6667 \\ 0.3333 & -1 & 0.3333 \end{bmatrix} \begin{bmatrix} 17 \\ 42 \\ 91 \end{bmatrix} = \begin{bmatrix} 5 \\ 16 \\ -6 \end{bmatrix}$$

Sehingga diperoleh penyelesaian dari persamaan linear serentak yaitu $x_1 = 5,\ x_2 = 16,$ dan $x_3 = -6$

2. Selesaikan dengan menggunakan metode determinan & invers matriks, metode dekomposisi L-U, metode iterasi jakobi dan metode Gauss Siedel

$5x_1 - x_2 = 9$

$-x_1 + 5x_2 - x_3 = 4$

$-x_2 + 5x_3 = -6$

Answer:

- **Metode determinan & invers matriks**

Step 1 Persamaan disusun dalam bentuk matriks

$$\begin{bmatrix} 5 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 4 \\ -6 \end{bmatrix} \rightarrow A \cdot x = H$$

Step 2 Mencari determinan matriks, adjoint dan invers kemudian mencari matriks variable

Determinan matriks A

$$Det\ A = \begin{vmatrix} 5 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 5 \end{vmatrix} = 5((5)(5)-(-1)(-1)) - 1((-1)(5)-(-1)(0)) + 0((-1)(-1)-(0)(5)) = 115$$

Adjoint matriks A

$$AdjA = \begin{bmatrix} \begin{vmatrix} 5 & -1 \\ -1 & 5 \end{vmatrix} & -\begin{vmatrix} -1 & 0 \\ -1 & 5 \end{vmatrix} & \begin{vmatrix} -1 & 0 \\ 5 & -1 \end{vmatrix} \\ -\begin{vmatrix} -1 & -1 \\ 0 & 5 \end{vmatrix} & \begin{vmatrix} 5 & 0 \\ 0 & 5 \end{vmatrix} & -\begin{vmatrix} 5 & 0 \\ -1 & -1 \end{vmatrix} \\ \begin{vmatrix} -1 & 5 \\ 0 & -1 \end{vmatrix} & -\begin{vmatrix} 5 & -1 \\ 0 & -1 \end{vmatrix} & \begin{vmatrix} 5 & -1 \\ -1 & 5 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} 24 & 5 & 1 \\ 5 & 25 & 5 \\ 1 & 5 & 24 \end{bmatrix}$$

Invers matriks A

$$A^{-1} = \frac{adjA}{\det A} = \frac{\begin{bmatrix} 24 & 5 & 1 \\ 5 & 25 & 5 \\ 1 & 5 & 24 \end{bmatrix}}{115} = \begin{bmatrix} 0.2087 & 0.0435 & 0.0087 \\ 0.0435 & 0.2174 & 0.0435 \\ 0.0087 & 0.0435 & 0.2087 \end{bmatrix}$$

Penyelesaiannya

$$X = A^{-1} \cdot H = \begin{bmatrix} 0.2087 & 0.0435 & 0.0087 \\ 0.0435 & 0.2174 & 0.0435 \\ 0.0087 & 0.0435 & 0.2087 \end{bmatrix} \begin{bmatrix} 9 \\ 4 \\ -6 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

Sehingga diperoleh penyelesaian dari persamaan linear serentak dengan metode determinan & invers matriks yaitu $x_1 = 2,\ x_2 = 1,$ dan $x_3 = -1$

- **Metode Determinan L-U**

Step 1 Menyusun persamaan ke dalam bentuk matriks

$$\begin{bmatrix} 5 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 4 \\ -6 \end{bmatrix} \rightarrow A \cdot x = H$$

Step 2 Mencari matriks L dan matriks U dari matriks koefisien A

$$\begin{bmatrix} 5 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 5 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

Diagonal utama matriks U, semuanya bernilai 1

Pada $j = 1$, didapatkan

$$l_{11} = a_{11} = 5$$

$$l_{21} = a_{21} = -1$$

$$l_{31} = a_{31} = 0$$

Pada $i = 1$, didapatkan

$$u_{12} = \frac{a_{12}}{a_{11}} = \frac{-1}{5} = -0.2$$

$$u_{13} = \frac{a_{13}}{a_{11}} = \frac{0}{5} = 0$$

$$l_{22} = a_{22} - \sum_{k=1}^{1} l_{ik} \cdot u_{kj} = a_{22} - (l_{21} \cdot u_{12}) = 5 - ((-1)(-0.2)) = 5 - 0.2 = 4.8$$

$$l_{32} = a_{32} - \sum_{k=1}^{1} l_{ik} \cdot u_{kj} = a_{32} - (l_{31} \cdot u_{12}) = -1 - ((0)(-0.2)) = -1$$

$$u_{23} = \frac{a_{23} - \sum_{k=1}^{1} l_{ik} \cdot u_{kj}}{l_{22}} = \frac{a_{23} - (l_{21} \cdot u_{13})}{l_{22}} = \frac{-1 - ((-1)(0))}{4.8} = -0.2083$$

$$l_{33} = a_{33} - \sum_{k=1}^{2} l_{ik} \cdot u_{kj} = a_{33} - (l_{31} \cdot u_{13} + l_{32} \cdot u_{23}) = -1 - ((0)(0) + (-1)(-1)) = -2$$

Jadi matriks L dan U adalah

$$L = \begin{bmatrix} 5 & 0 & 0 \\ -1 & 4.8 & 0 \\ 0 & -1 & -2 \end{bmatrix} \qquad U = \begin{bmatrix} 1 & -0.2 & 0 \\ 0 & 1 & -0.2083 \\ 0 & 0 & 1 \end{bmatrix}$$

Matriks H' diperoleh dengan

$$h'_1 = \frac{h_1}{l_{11}} = \frac{9}{5} = 1.8$$

$$h'_2 = \frac{h_2 - l_{21}h'_1}{l_{22}} = \frac{4 - (-1)(1.8)}{4.8} = 1.2083$$

$$h'_3 = \frac{h_3 - (l_{31}h'_1 + l_{32}h'_2)}{l_{33}} = \frac{-6 - ((0)(1.8) + (-1)(1.2083))}{-2} = 2.3959$$

Sehingga matriks H' adalah

$$H' = \begin{bmatrix} 1.8 \\ 1.2083 \\ 2.3959 \end{bmatrix}$$

Step 3 Menyusun augmented matriks

$$UH' = \begin{bmatrix} 1 & -0.2 & 0 & 1.8 \\ 0 & 1 & -0.2083 & 1.2083 \\ 0 & 0 & 1 & 2.3959 \end{bmatrix}$$

Penyelesaiannya adalah

$$x_3 = h'_3 = 2.3959$$

$$x_2 = h'_2 - u_{23} \cdot x_3 = 1.2083 - (-0.2083)(2.3959) = 1.7073$$

$$x_1 = h'_1 - (u_{12} \cdot x_2 + u_{13} \cdot x_3) = 1.8 - ((-0.2)(1.7073) + (0)(2.3959)) = 2.1415$$

Jadi penyelesaian persamaan linear serentak dengan metode determinan & invers matriks adalah $x_1 = 2.1415$, $x_2 = 1.7073$, dan $x_3 = 2.3959$

- **Metode iterasi Jakobi**

Step 1 Memeriksa apakah memenuhi syarat

$5x_1 - x_2 = 9$

$-x_1 + 5x_2 - x_3 = 4$

$-x_2 + 5x_3 = -6$

Dari persamaan diatas diketahui telah memenuhi syarat dimana

$|a_{ii}| > \sum_{j=1}^{n} |a_{ij}|$ untuk $i = 1,2,...,N$ dan $i \neq j$

Step 2 Mencari matriks koefisien, matriks variable dan matriks hasil

$$\begin{bmatrix} 5 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 4 \\ -6 \end{bmatrix} \rightarrow A \cdot x = H$$

Step 3 Menentukan titik variable dan melakukan iterasi

Jika diambil nilai pertama $x_1^{(1)} = x_2^{(1)} = x_3^{(1)} = 0$

Iterasi pertama, $n = 1$

$$x_1^{(2)} = \frac{h_1}{a_{11}} - \sum_{j \neq i}^{3} \frac{a_{ij}}{a_{ii}} x_j^{(1)} = \frac{9}{5} - \left(\frac{a_{12}}{a_{11}} x_2^{(1)} + \frac{a_{13}}{a_{11}} x_3^{(1)} \right) = \frac{9}{5} - (0+0) = 1.8$$

$$x_2^{(2)} = \frac{h_2}{a_{22}} - \sum_{j \neq i}^{3} \frac{a_{ij}}{a_{ii}} x_j^{(1)} = \frac{4}{5} - \left(\frac{a_{21}}{a_{22}} x_1^{(1)} + \frac{a_{23}}{a_{22}} x_3^{(1)} \right) = \frac{4}{5} - (0+0) = 0.8$$

$$x_3^{(2)} = \frac{h_3}{a_{33}} - \sum_{j \neq i}^{3} \frac{a_{ij}}{a_{ii}} x_j^{(1)} = -\frac{6}{5} - \left(\frac{a_{31}}{a_{33}} x_1^{(1)} + \frac{a_{32}}{a_{33}} x_2^{(1)} \right) = -\frac{6}{5} - (0+0) = -1.2$$

Iterasi kedua, $n = 2$

$$x_1^{(3)} = \frac{h_1}{a_{11}} - \sum_{j \neq i}^{3} \frac{a_{ij}}{a_{ii}} x_j^{(2)} = \frac{9}{5} - \left(\frac{a_{12}}{a_{11}} x_2^{(2)} + \frac{a_{13}}{a_{11}} x_3^{(2)} \right) = \frac{9}{5} - \left(\frac{(-1)}{5}(0.8) + \frac{0}{5}(-1.2) \right) = 1.96$$

$$x_2^{(3)} = \frac{h_2}{a_{22}} - \sum_{j \neq i}^{3} \frac{a_{ij}}{a_{ii}} x_j^{(2)} = \frac{4}{5} - \left(\frac{a_{21}}{a_{22}} x_1^{(2)} + \frac{a_{23}}{a_{22}} x_3^{(2)} \right) = \frac{4}{5} - \left(\frac{(-1)}{5}(1.8) + \frac{(-1)}{5}(-1.2) \right) = 0.92$$

$$x_3^{(3)} = \frac{h_3}{a_{33}} - \sum_{j \neq i}^{3} \frac{a_{ij}}{a_{ii}} x_j^{(2)} = -\frac{6}{5} - \left(\frac{a_{31}}{a_{33}} x_1^{(2)} + \frac{a_{32}}{a_{33}} x_2^{(2)} \right) = -\frac{6}{5} - \left(\frac{0}{5}(1.8) + \frac{(-1)}{5}(0.8) \right) = -1.04$$

Iterasi kedua, $n = 3$

$$x_1^{(4)} = \frac{h_1}{a_{11}} - \sum_{j \neq i}^{3} \frac{a_{ij}}{a_{ii}} x_j^{(3)} = \frac{9}{5} - \left(\frac{a_{12}}{a_{11}} x_2^{(3)} + \frac{a_{13}}{a_{11}} x_3^{(3)} \right) = \frac{9}{5} - \left(\frac{(-1)}{5}(0.92) + \frac{0}{5}(-1.04) \right) = 1.984$$

$$x_2^{(4)} = \frac{h_2}{a_{22}} - \sum_{j \neq i}^{3} \frac{a_{ij}}{a_{ii}} x_j^{(3)} = \frac{4}{5} - \left(\frac{a_{21}}{a_{22}} x_1^{(3)} + \frac{a_{23}}{a_{22}} x_3^{(3)} \right) = \frac{4}{5} - \left(\frac{(-1)}{5}(1.96) + \frac{(-1)}{5}(-1.04) \right) = 0.984$$

$$x_3^{(4)} = \frac{h_3}{a_{33}} - \sum_{j \neq i}^{3} \frac{a_{ij}}{a_{ii}} x_j^{(3)} = -\frac{6}{5} - \left(\frac{a_{31}}{a_{33}} x_1^{(3)} + \frac{a_{32}}{a_{33}} x_2^{(3)} \right) = -\frac{6}{5} - \left(\frac{0}{5}(1.96) + \frac{(-1)}{5}(0.92) \right) = -1.016$$

Jadi penyelesaian persamaan linear serentak dengan metode iterasi jakobi adalah $x_1 = 1.984$, $x_2 = 0.984$, dan $x_3 = -1.016$

- **Metode Iterasi Gauss Siedel**

Step 1 Memeriksa apakah memenuhi syarat

$5x_1 - x_2 = 9$

$-x_1 + 5x_2 - x_3 = 4$

$-x_2 + 5x_3 = -6$

Dari persamaan diatas diketahui telah memenuhi syarat dimana

$|a_{ii}| > \sum_{j=1}^{n} |a_{ij}|$ untuk $i = 1,2,\ldots,N$ dan $i \neq j$

Step 2 Mencari matriks koefisien, matriks variable dan matriks hasil

$$\begin{bmatrix} 5 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 4 \\ -6 \end{bmatrix} \rightarrow A \cdot x = H$$

Step 3 Menentukan titik variable dan melakukan iterasi

Jika diambil nilai pertama $x_1^{(1)} = x_2^{(1)} = x_3^{(1)} = 0$

Iterasi pertama, $n = 1$

$$x_1^{(2)} = \frac{h_1}{a_{11}} - \sum_{j=1}^{0} \frac{a_{1j}}{a_{11}} x_j^{(n+1)} - \sum_{j=2}^{3} \frac{a_{1j}}{a_{11}} x_j^{(1)} = \frac{9}{5} - 0 - \left(\frac{a_{12}}{a_{11}} x_2^{(1)} + \frac{a_{13}}{a_{11}} x_3^{(1)} \right) = \frac{9}{5} - 0 - (0 + 0) = 1.8$$

$$x_2^{(2)} = \frac{h_2}{a_{22}} - \sum_{j=1}^{1} \frac{a_{2j}}{a_{22}} x_j^{(n+1)} - \sum_{j=3}^{3} \frac{a_{2j}}{a_{22}} x_j^{(1)} = \frac{4}{5} - \frac{a_{21}}{a_{22}} x_1^{(2)} - \frac{a_{23}}{a_{22}} x_3^{(1)}$$

$$= \frac{4}{5} - \frac{(-1)}{5}(1.8) - \frac{(-1)}{5}(0) = 1.16$$

$$x_3^{(2)} = \frac{h_3}{a_{33}} - \sum_{j=1}^{2} \frac{a_{3j}}{a_{33}} x_j^{(n+1)} - \sum_{j=4}^{3} \frac{a_{3j}}{a_{33}} x_j^{(1)} = -\frac{6}{5} - \left(\frac{a_{31}}{a_{33}} x_1^{(2)} + \frac{a_{32}}{a_{33}} x_2^{(2)} \right) - 0$$

$$= -\frac{6}{5} - \left(\frac{0}{5}(1.8) + \frac{(-1)}{5}(1.16) \right) - 0 = -0.968$$

Iterasi kedua, $n = 2$

$$x_1^{(3)} = \frac{h_1}{a_{11}} - \sum_{j=1}^{0} \frac{a_{1j}}{a_{11}} x_j^{(n+1)} - \sum_{j=2}^{3} \frac{a_{1j}}{a_{11}} x_j^{(2)} = \frac{9}{5} - 0 - \left(\frac{a_{12}}{a_{11}} x_2^{(2)} + \frac{a_{13}}{a_{11}} x_3^{(2)} \right)$$

$$= \frac{9}{5} - 0 - \left(\frac{(-1)}{5}(1.16) + \frac{0}{5}(-0.968) \right) = 2.032$$

$$x_2^{(3)} = \frac{h_2}{a_{22}} - \sum_{j=1}^{1} \frac{a_{2j}}{a_{22}} x_j^{(n+1)} - \sum_{j=3}^{3} \frac{a_{2j}}{a_{22}} x_j^{(2)} = \frac{4}{5} - \frac{a_{21}}{a_{22}} x_1^{(3)} - \frac{a_{23}}{a_{22}} x_3^{(2)}$$

$$= \frac{4}{5} - \frac{(-1)}{5}(2.032) - \frac{(-1)}{5}(-0.968) = 1.0128$$

$$x_3^{(3)} = \frac{h_3}{a_{33}} - \sum_{j=1}^{2} \frac{a_{3j}}{a_{33}} x_j^{(n+1)} - \sum_{j=4}^{3} \frac{a_{3j}}{a_{33}} x_j^{(2)} = -\frac{6}{5} - \left(\frac{a_{31}}{a_{33}} x_1^{(3)} + \frac{a_{32}}{a_{33}} x_2^{(3)} \right) - 0$$

$$= -\frac{6}{5} - \left(\frac{0}{5}(2.032) + \frac{(-1)}{5}(1.0128) \right) = -0.99744$$

Iterasi kedua, $n = 3$

$$x_1^{(4)} = \frac{h_1}{a_{11}} - \sum_{j=1}^{0} \frac{a_{1j}}{a_{11}} x_j^{(n+1)} - \sum_{j=2}^{3} \frac{a_{1j}}{a_{11}} x_j^{(3)} = \frac{9}{5} - 0 - \left(\frac{a_{12}}{a_{11}} x_2^{(3)} + \frac{a_{13}}{a_{11}} x_3^{(3)} \right)$$

$$= \frac{9}{5} - 0 - \left(\frac{(-1)}{5}(1.0128) + \frac{0}{5}(-0.99744) \right) = 2.00256$$

$$x_2^{(4)} = \frac{h_2}{a_{22}} - \sum_{j=1}^{1} \frac{a_{2j}}{a_{22}} x_j^{(n+1)} - \sum_{j=3}^{3} \frac{a_{2j}}{a_{22}} x_j^{(3)} = \frac{4}{5} - \frac{a_{21}}{a_{22}} x_1^{(4)} - \frac{a_{23}}{a_{22}} x_3^{(3)}$$

$$= \frac{4}{5} - \frac{(-1)}{5}(2.00256) - \frac{(-1)}{5}(-0.99744) = 1.001024$$

$$x_3^{(4)} = \frac{h_3}{a_{33}} - \sum_{j=1}^{2} \frac{a_{3j}}{a_{33}} x_j^{(n+1)} - \sum_{j=4}^{3} \frac{a_{3j}}{a_{33}} x_j^{(3)} = -\frac{6}{5} - \left(\frac{a_{31}}{a_{33}} x_1^{(4)} + \frac{a_{32}}{a_{33}} x_2^{(4)} \right) - 0$$

$$= -\frac{6}{5} - \left(\frac{0}{5}(2.00256) + \frac{(-1)}{5}(1.001024) \right) = -0.9997952$$

Jadi penyelesaian persamaan linear serentak dengan metode iterasi jakobi adalah $x_1 = 2.00256$, $x_2 = 1.001024$, dan $x_3 = -0.9997952$