Solusi

Tugas 1 Metode Numerik dan Teknik Komputasi 2018/2019

1. **Dapatkan deret *Maclaurin* dari** e^x

Deret *Taylor:*

$$f(x) = f(x_0) + \frac{(x–x_0)}{1!}f'(x_0) + \frac{(x–x_0)^2}{2!}f''(x_0) + \cdots + \frac{(x-x_0)^m}{m!}f^{(m)}(x_0) \qquad \textbf{(1)}$$

Sedangkan untuk Deret *Maclaurin*, merupakan Deret *Taylor* dengan kisaran $x_0 = 0$, sehingga Deret *Maclaurin* :

$$\begin{aligned} f(x) &= f(0) + \frac{(x–0)}{1!}f'(0) + \frac{(x–0)^2}{2!}f''(0) + \cdots + \frac{(x-0)^m}{m!}f^{(m)}(0) \\ &= f(0) + \frac{x}{1!}f'(0) + \frac{x^2}{2!}f''(0) + \cdots + \frac{x^m}{m!}f^{(m)}(0) \end{aligned} \qquad \textbf{(2)}$$

kemudian,

$f(x) = e^x \to f(0) = e^0 = 1$

$f'(x) = e^x \to f'(0) = e^0 = 1$

$f''(x) = e^x \to f''(0) = e^0 = 1$

sehingga,

$$\begin{aligned} f(x) = e^x &= 1 + \frac{x}{1!}1 + \frac{x^2}{2!}1 + \cdots \\ &= 1 + x + \frac{x^2}{2} + \cdots \end{aligned}$$

2. Dapatkan deret *Maclaurin* dari $\tan x$

$f(x) = \tan(x) \to f(0) = \tan(0) = 0$

$f'(x) = \sec^2(x) \to f'(0) = \sec^2(0) = 1$

$f''(x) = 2\frac{\sin(x)}{\cos^3(x)} \to f''(0) = 2\frac{\sin(0)}{\cos^3(0)} = 0$

dengan menggunakaan pers. **(2)**

$$\begin{aligned} f(x) = \tan(x) &= f(0) + \frac{x}{1!}f'(0) + \frac{x^2}{2!}f''(0) + \cdots + \frac{x^m}{m!}f^{(m)}(0) \\ &= 0 + x \cdot 0 + \frac{x^2}{2} \cdot 0 + \cdots \\ &= 0 \end{aligned}$$

3. Dapatkan deret *Taylor* dari $\operatorname{cosec} x$ **di sekitar** $x = \frac{\pi}{3}$

karena $x_0 = \frac{\pi}{3}$, maka pers.**(1)** menjadi:

$$f(x) = f(\frac{\pi}{3}) + \frac{(x-\frac{\pi}{3})}{1!} \cdot f'(\frac{\pi}{3}) + \frac{(x-\frac{\pi}{3})^2}{2!} \cdot f''(\frac{\pi}{3}) + \cdots$$

kemudian,

$f(x) = \csc(x) \to f(\frac{\pi}{3}) = \csc(\frac{\pi}{3}) = 1.1547$

$f'(x) = -\csc(x) \cdot \cot(x) \to f'(\frac{\pi}{3}) = -\csc(\frac{\pi}{3}) \cdot \cot(\frac{\pi}{3}) = -0.6667$

$f''(x) = -\csc(x) \cdot \cot^2(x) + \csc^3(x) \to f''(\frac{\pi}{3}) = -\csc(\frac{\pi}{3}) \cdot \cot^2(\frac{\pi}{3}) + \csc^3(\frac{\pi}{3}) = 1.1547$

sehingga,

$f(x) = 1.1547 + \frac{(x - \frac{\pi}{3})}{1!} \cdot (0.6667) + \frac{(x - \frac{\pi}{3})^2}{2!} \cdot 1.1547 + \cdots$

4. Dapatkan deret *Taylor* dari $\sec x$ di sekitar $x = \frac{\pi}{3}$

karena $x_0 = \frac{\pi}{3}$, maka pers.**(1)** menjadi:

$f(x) = f(\frac{\pi}{3}) + \frac{(x - \frac{\pi}{3})}{1!} \cdot f'(\frac{\pi}{3}) + \frac{(x - \frac{\pi}{3})^2}{2!} \cdot f''(\frac{\pi}{3}) + \cdots$

kemudian,

$f(x) = \sec(x) \to f(\frac{\pi}{3}) = \sec(\frac{\pi}{3}) = 2$

$f'(x) = \sec(x) \cdot \tan(x) \to f'(\frac{\pi}{3}) = \sec(\frac{\pi}{3}) \cdot \tan(\frac{\pi}{3}) = 3.4641$

$f''(x) = \sec(x) \cdot \tan^2(x) + \sec^3(x) \to f''(\frac{\pi}{3}) = \sec(\frac{\pi}{3}) \cdot \tan^2(\frac{\pi}{3}) + \sec^3(\frac{\pi}{3}) = 14$

sehingga,

$f(x) = 2 + \frac{(x - \frac{\pi}{3})}{1!} \cdot (3.4641) + \frac{(x - \frac{\pi}{3})^2}{2!} \cdot (14) + \cdots$