

MATRIKULASI

MATEMATIKA DASAR

Mifta Nur Farid

1. Pertidaksamaan Linier

- ▶ Interval
- ▶ Penyelesaian
Pertidaksamaan

2. Fungsi dan Limit

- ▶ Fungsi
- ▶ Limit

3. Trigonometri

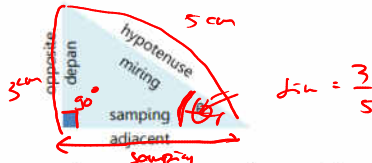
4. Turunan

5. Integral

- ▶ Integral Tak Tentu
- ▶ Integral dengan Substitusi
- ▶ Integral Tentu

- ▶ Trigonometri berasal dari bahasa Yunani.
- ▶ Trigonometri berasal dari dua kata, yaitu *trigono* berarti segitiga dan *metri* berarti ilmu ukur.
- ▶ Trigonometri adalah ilmu matematika yang mempelajari tentang segitiga siku-siku.
- ▶ Pada segitiga siku-siku berlaku teorema Phytagoras dan nilai perbandingan sisi-sisi segitiga siku-siku.
- ▶ Nilai perbandingan trigonometri adalah nilai perbandingan sisi-sisi segitiga siku-siku.

- Macam definisi dari nilai perbandingan trigonometri:



$$\sin = \frac{3}{5}$$

Sin depan

$$\sin \theta = \frac{\text{depan}}{\text{miring}}$$

cosecant

$$\text{cosec } \theta = \frac{1}{\sin \theta} = \frac{\text{miring}}{\text{depan}}$$

cos samping

$$\cos \theta = \frac{\text{samping}}{\text{miring}}$$

secant

$$\sec \theta = \frac{1}{\cos \theta} = \frac{\text{miring}}{\text{samping}}$$

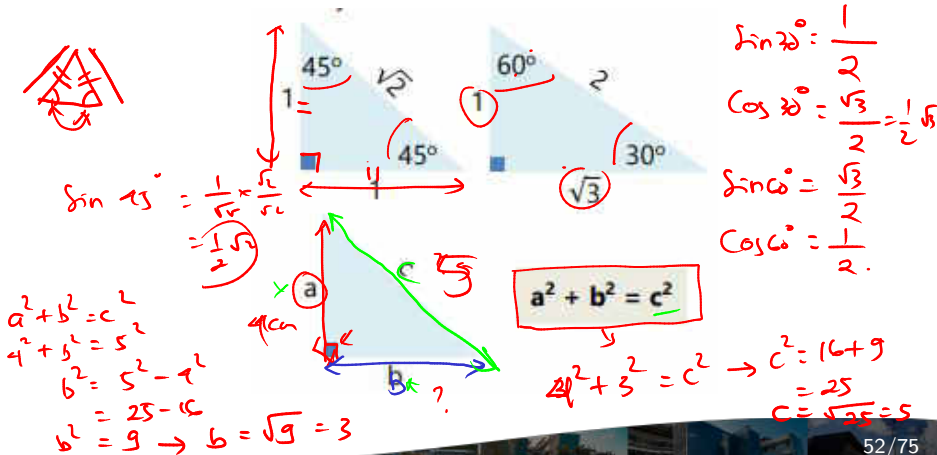
tandesa

$$\tan \theta = \frac{\text{depan}}{\text{samping}} = \frac{\sin \theta}{\cos \theta}$$

Cotangent

$$\cot \theta = \frac{1}{\tan \theta} = \frac{\text{depan}}{\text{samping}} = \frac{\cos \theta}{\sin \theta}$$

- Perbandingan nilai sisi-sisi segitiga istimewa dan sudutnya antara lain:



Hand-drawn diagrams illustrating special right triangles and their trigonometric ratios:

- 45-45-90 Triangle:**
 - Angles: 45° , 45° , 90°
 - Sides: 1 , 1 , $\sqrt{2}$
 - Trigonometric Ratios:
 - $\sin 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
 - $\cos 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
 - $\tan 45^\circ = 1$
- 30-60-90 Triangle:**
 - Angles: 30° , 60° , 90°
 - Sides: 1 , $\sqrt{3}$, 2
 - Trigonometric Ratios:
 - $\sin 30^\circ = \frac{1}{2}$
 - $\cos 30^\circ = \frac{\sqrt{3}}{2}$
 - $\sin 60^\circ = \frac{\sqrt{3}}{2}$
 - $\cos 60^\circ = \frac{1}{2}$
- General Triangle (Pythagorean Theorem):**
 - Diagram of a right triangle with sides a , b , and hypotenuse c .
 - Equation: $a^2 + b^2 = c^2$
 - Example calculation:
 - $4^2 + 3^2 = c^2 \rightarrow c^2 = 16 + 9 = 25$
 - $c = \sqrt{25} = 5$

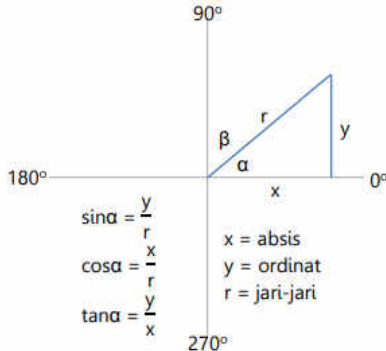
- Nilai perbandingan trigonometri pada sudut-sudut istimewa:

θ°	$\sin \theta$	$\cos \theta$	$\tan \theta$	$\operatorname{cosec} \theta$	$\sec \theta$	$\cot \theta$
0°	0	1	0	∞	1	∞
30° $\left(\frac{\pi}{6}\right)$	$\frac{1}{2}$	$\frac{1}{2}\sqrt{3}$	$\frac{1}{3}\sqrt{3}$	2	$\frac{2}{3}\sqrt{3}$	$\sqrt{3}$
45° $\left(\frac{\pi}{4}\right)$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60° $\left(\frac{\pi}{3}\right)$	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2}{3}\sqrt{3}$	2	$\frac{1}{3}\sqrt{3}$
90° $\left(\frac{\pi}{2}\right)$	1	0	∞	1	∞	0

Handwritten notes: $\frac{\pi}{6}$ radians points to 30° . Red circles highlight $\sin 30^\circ$, $\cos 30^\circ$, $\tan 45^\circ$, and $\sin 60^\circ$. Red arrows indicate the relationships between these values and the values in the 45-degree row.

Trigonometri

- Nilai perbandingan trigonometri suatu sudut yang besarnya $< 90^\circ$ dapat dijelaskan melalui kuadran koordinat kartesius.



- Tanda nilai perbandingan trigonometri berbeda di masing-masing kuadrannya.

$$\cos x = \frac{1}{2}$$

$$x = 45^\circ$$

$$\cos x = -\frac{1}{2}$$

$$x = \{135^\circ, 225^\circ\}$$

II $90^\circ \leq \alpha \leq 180$		I $0 \leq \alpha \leq 90$	
$\sin +$	$\csc +$	$\sin +$	$\csc +$
$\cos -$	$\sec -$	$\cos +$	$\sec +$
$\tan -$	$\cot -$	$\tan +$	$\cot +$
180°			0°
$\sin -$	$\csc -$	$\sin -$	$\csc -$
$\cos -$	$\sec -$	$\cos +$	$\sec +$
$\tan +$	$\cot +$	$\tan -$	$\cot -$
180 ≤ α ≤ 270		270 ≤ α ≤ 360	
III		IV	
	270°		

$$\sin(135^\circ) = \sin(180^\circ - 45^\circ) = \sin(45^\circ) = \frac{1}{2}\sqrt{2}$$

Trigonometri

II	I
III	IV

$$\sin(60^\circ) = \sin(90^\circ - 30^\circ) = \cos 30^\circ$$

► Perbandingan trigonometri sudut berelasi sebagai berikut

1. Sudut berelasi $(90^\circ - \theta)$

a. $\sin(90^\circ - \theta) = \cos \theta$

b. $\cos(90^\circ - \theta) = \sin \theta$

c. $\tan(90^\circ - \theta) = \cot \theta$

$$\begin{array}{l} -\cos 60^\circ \\ -\sin 30^\circ \end{array}$$

2. Sudut berelasi $(180^\circ - \theta)$

a. $\sin(180^\circ - \theta) = \sin \theta$

b. $\cos(180^\circ - \theta) = -\cos \theta$

c. $\tan(180^\circ - \theta) = -\tan \theta$

3. Sudut berelasi $(270^\circ - \theta)$

a. $\sin(270^\circ - \theta) = -\cos \theta$

b. $\cos(270^\circ - \theta) = -\sin \theta$

c. $\tan(270^\circ - \theta) = \cot \theta$

4. Sudut berelasi $(-\theta)$

a. $\sin(-\theta) = -\sin \theta$

b. $\cos(-\theta) = \cos \theta$

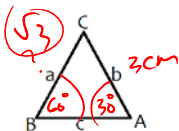
c. $\tan(-\theta) = -\tan \theta$

-30, 120, 210, 240

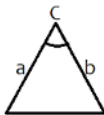
Trigonometri

$$\frac{3}{\sin 60^\circ} = \frac{x}{\sin 30^\circ} \rightarrow x = \frac{3}{\sin 60^\circ} \cdot \sin 30^\circ$$

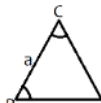
$$= \frac{3}{\frac{1}{2}\sqrt{3}} \cdot \frac{1}{2} \cdot \sqrt{3} = \sqrt{3}$$



gambar 5



gambar 6



gambar 7

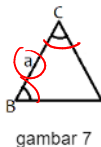
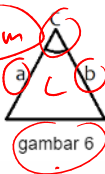
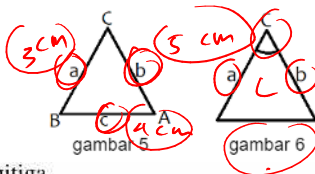
- Aturan sinus: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ (lihat gambar 5)

- Aturan cosinus:

$$I. a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$II. b^2 = a^2 + c^2 - 2ac \cdot \cos B$$

$$III. c^2 = a^2 + b^2 - 2ab \cdot \cos C$$



- Luas segitiga

a) $L = \sqrt{s(s-a)(s-b)(s-c)}, s = \frac{1}{2}(a+b+c)$: gambar 5

b) $L = \frac{1}{2} a \cdot b \sin C$: gambar 6

c) $L = \frac{a^2 \sin B \sin C}{2 \sin(B+C)}$: gambar 7

- Jumlah dan Selisih Dua Sudut

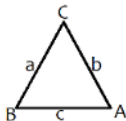
a. $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

b. $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

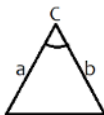
c. $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$$\sin(30^\circ + 60^\circ)$$

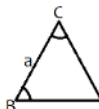
$$\rightarrow \sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ$$



gambar 5



gambar 6



gambar 7

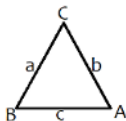
- Perkalian Sinus dan Kosinus

a. $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$

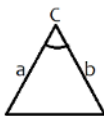
b. $2 \cos A \sin B = \sin(A + B) - \sin(A - B)$

c. $2 \cos A \cos B = \cos(A + B) + \cos(A - B)$

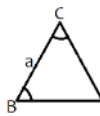
d. $-2 \sin A \sin B = \cos(A + B) - \cos(A - B)$



gambar 5



gambar 6



gambar 7

- Penjumlahan dan Pengurangan Sinus, Kosinus dan Tangen

a. $\sin A + \sin B = 2 \sin \frac{1}{2}(A + B) \cdot \cos \frac{1}{2}(A - B)$

b. $\sin A - \sin B = 2 \cos \frac{1}{2}(A + B) \cdot \sin \frac{1}{2}(A - B)$

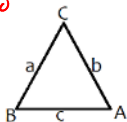
c. $\cos A + \cos B = 2 \cos \frac{1}{2}(A + B) \cdot \cos \frac{1}{2}(A - B)$

d. $\cos A - \cos B = -2 \sin \frac{1}{2}(A + B) \cdot \sin \frac{1}{2}(A - B)$

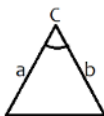
e. $\tan A + \tan B = \frac{\sin(A+B)}{\cos A \cos B}$

f. $\tan A - \tan B = \frac{\sin(A-B)}{\cos A \cos B}$

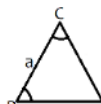
$\sin 30^\circ = \frac{1}{2}$
 $\sin 30^\circ = 2 \sin 15^\circ \cdot \cos 15^\circ$



gambar 5



gambar 6



gambar 7

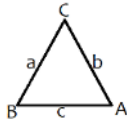
- Sudut Rangkap

a. $\sin 2A = 2 \sin A \cdot \cos A$

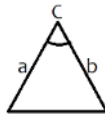
b. $\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$

c. $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$

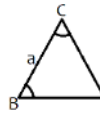
d. $\sin 3A = 3 \sin A - 4 \sin^3 A$



gambar 5



gambar 6



gambar 7

- Persamaan Trigonometri

a. $\sin x^\circ = \sin p$

$$x_1 = p + 360k$$

$$x_2 = (180 - p) + 360k$$

b. $\cos x^\circ = \cos p$

$$x_{1,2} = \pm p + 360k$$

c. $\tan x^\circ = \tan p$

$$x_1 = p + 180k$$

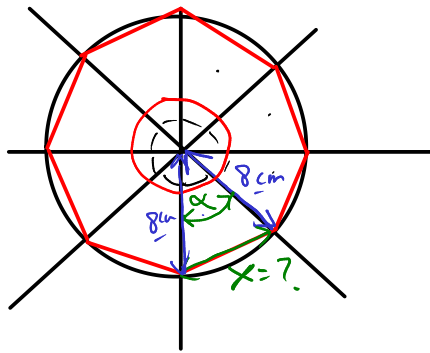
$$x_2 = (180 + p) + 180k$$

d. Bentuk: $A \text{ trig}^2 + B \text{ trig} + C = 0$ diselesaikan seperti menyelesaikan persamaan kuadrat

Soal Latihan

1. Dalam suatu lingkaran yang berjari-jari 8 cm, dibuat segi-8 beraturan. Panjang sisi segi-8 tersebut adalah ... cm
2. Jika luas segi delapan beraturan adalah $200\sqrt{2} \text{ cm}^2$, maka panjang jari-jari lingkaran luarnya adalah ... cm
3. Diketahui segitiga ABC dengan panjang sisi $AB = 3 \text{ cm}$, $AC = 4 \text{ cm}$, dan $\angle CAB = 60^\circ$. CD adalah tinggi segitiga ABC. Panjang CD = ... cm
4. Himpunan penyelesaian dari persamaan $\cos(x + 210)^\circ + \cos(x - 210)^\circ = \frac{1}{2}\sqrt{3}$ untuk $0 \leq x \leq 360^\circ$ adalah ...
5. Pada segitiga ABC lancip, diketahui $\cos A = \frac{4}{5}$ dan $\sin B = \frac{12}{13}$, maka $\sin C = \dots$

①



Sudut dlm lingkaran = 360°

Sudut $\alpha = \frac{360^\circ}{8} = 45^\circ$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$x = ?$

$$x^2 = r^2 + r^2 - 2 \cdot r \cdot r \cdot \cos \alpha$$

$$x^2 = 8^2 + 8^2 - 2 \cdot 8 \cdot 8 \cdot \cos 45^\circ$$

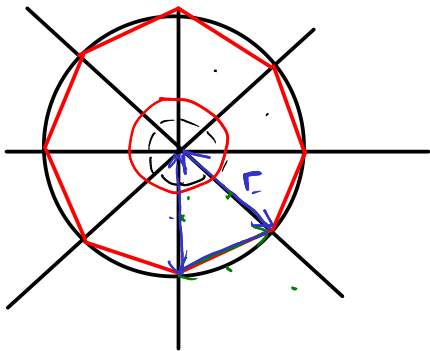
$$x^2 = 64 + 64 - 128 \cdot \frac{1}{2} \sqrt{2}$$

$$x = \sqrt{128 - 64\sqrt{2}} \rightarrow$$

$$x = \sqrt{37,5}$$

$$x = 6,12 \text{ cm}$$

②



Luas Segi delapan = $200\sqrt{2} \text{ cm}^2$
 $\alpha = 45^\circ$

① $L_{\Delta} = \frac{\text{Luas Segi delapan}}{8} = \frac{200\sqrt{2}}{8} = 25\sqrt{2} \text{ cm}^2$

② $L_{\Delta} = \frac{1}{2} \cdot r \cdot r \cdot \sin(\alpha)$

$$25\sqrt{2} = \frac{1}{2} \cdot r^2 \cdot \sin(45^\circ)$$

$$2 \cdot 25\sqrt{2} = r^2 \cdot \frac{1}{2} \sqrt{2}$$

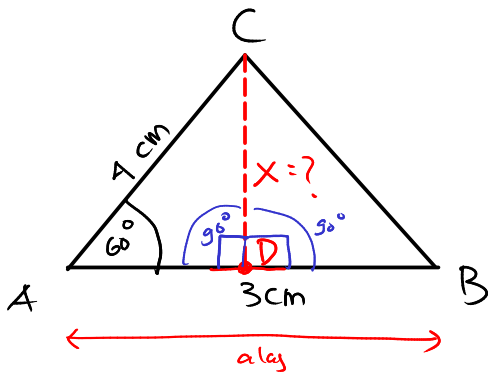
$$2 \cdot 50 = r^2 \rightarrow r = \sqrt{100}$$

$$r = 10 \text{ cm} //$$

$$L = \frac{1}{2} \cdot a \cdot b \cdot \sin C$$

L_{Δ}

③



$$210 = (90 + 30^\circ)$$



$$\frac{x}{\sin 60^\circ} = \frac{4 \text{ cm}}{\sin 90^\circ}$$

$$\frac{x}{\frac{1}{2}\sqrt{3}} = \frac{4 \text{ cm}}{1}$$

$$x = 4 \cdot \frac{1}{2} \sqrt{3}$$

$$x = 2\sqrt{3} \text{ cm}$$

$$\overline{CD} = 2\sqrt{3} \text{ cm} //$$

④ $\cos(x + 210^\circ) + \cos(x - 210^\circ) = \frac{1}{2}\sqrt{3} ; 0^\circ \leq x \leq 360^\circ$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\cos x \cdot \cos 210^\circ - \sin x \sin 210^\circ + \cos x \cos 210^\circ + \sin x \sin 210^\circ = \frac{1}{2}\sqrt{3}$$

$$\cos x \cdot \left(-\frac{1}{2}\sqrt{3}\right) + \cos x \cdot \left(-\frac{1}{2}\sqrt{3}\right) = \frac{1}{2}\sqrt{3}$$

$$2 \cos x \cdot \left(-\frac{1}{2}\sqrt{3}\right) = \frac{1}{2}\sqrt{3}$$

$$-2 \cos x = \frac{1}{2}$$

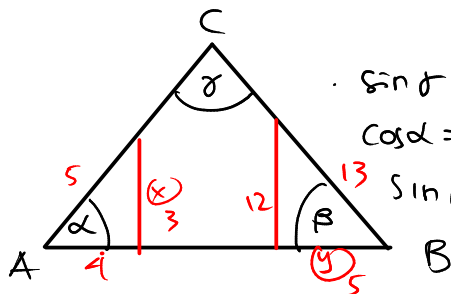
$$\cos x = -\frac{1}{4}$$

$$\begin{cases} \cos 210^\circ = \cos(270^\circ - 60^\circ) = -\sin 60^\circ = -\frac{1}{2}\sqrt{3} \\ \sin 210^\circ = \sin(270^\circ - 60^\circ) = -\cos 60^\circ = -\frac{1}{2} \end{cases}$$

$$-30^\circ, 120^\circ, 210^\circ, 240^\circ$$

$$x = \{120^\circ, 210^\circ, 240^\circ\} //$$

(5)



$$\sin \gamma = ?$$

$$\cos \alpha = \frac{4}{5} \rightarrow \sin \alpha = \frac{3}{5}$$

$$\sin \beta = \frac{12}{13} \rightarrow \cos \beta = \frac{5}{13}$$

(1)

$$180^\circ = \alpha + \beta + \gamma$$

$$180^\circ - (\alpha + \beta) = \gamma$$

$$\sin(180^\circ - (\alpha + \beta)) = \sin \gamma$$

$$\sin(180^\circ - \theta) = \sin \theta$$

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin(\alpha + \beta) = \sin \gamma$$

$$\sin \alpha \cos \beta + \cos \alpha \sin \beta = \sin \gamma$$

$$\frac{3}{5} \cdot \frac{5}{13} + \frac{4}{5} \cdot \frac{12}{13} = \sin \gamma$$

$$\frac{15 + 48}{65} = \sin \gamma$$

$$\frac{63}{65} = \sin \gamma$$

// 😊

$$x^2 = 5^2 - 4^2$$

$$x^2 = 25 - 16$$

$$= 9$$

$$x = \sqrt{9}$$

$$x = 3$$

$$y^2 = 13^2 - 12^2$$

$$y^2 = 169 - 144$$

$$y^2 = 25$$

$$y = \sqrt{25}$$

$$y = 5$$