MATRIKULASI

MATEMATIKA DASAR

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Pokok Bahasan

1. Pertidaksamaan Linier
   - Interval
   - Penyelesaian Pertidaksamaan
2. Fungsi dan Limit
   - Fungsi
   - Limit
3. Trigonometri
4. Turunan
5. Integral
   - Integral Tak Tentu
   - Integral dengan Substitusi
   - Integral Tentu

$f'(x)$

$I : \{ \ldots \}$

Dimisalkan fungsi f terdefinisi pada selang terbuka I yang memuat c. Turunan pertama dari fungsi f di titik c ditulis $f'(c)$ didefinisikan sebagai:

$f'(c)$

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}, \tag{4}$$

jika limitnya ada.

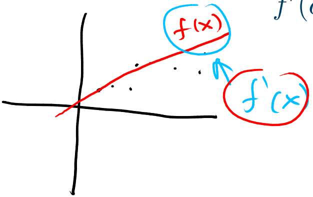
Turunan

$x = c + h$

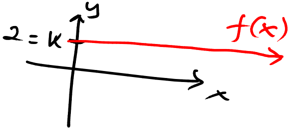
$x \to c$ $\qquad$ $c = c + h$ $\leftarrow$ $h \to 0$

Apabila dilakukan penggantian $x = c + h$, jika $x \to c \leftrightarrow h \to 0$ dan $x - c = h$, turunan fungsi f di c dapat dituliskan dalam bentuk:

$$f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h} \tag{5}$$



← kemiringan dari f(x)



Turunan

① ▶ Jika suatu fungsi konstan, misal $f(x) = k$ untuk sembarang bilangan rill k, maka

$$\frac{d}{dx}[k] = 0 \qquad (6)$$

$f(x) = x^3$
$f'(x) = 3x^2$

② ▶ Jika n suatu bilangan bulat positif, maka:

$f(x) = x^n$

$$\frac{d}{dx}[x^n] = nx^{n-1} \qquad (7)$$

$f(x) = x^2$
$f'(x) = 2x^{2-1}$
$= 2x$

③ ▶ Jika f fungsi yang dapat diturunkan di x dan k sebarang bilangan rill, maka kf juga dapat diturunkan di x, yaitu:

$$\frac{d}{dx}[kf(x)] = k\frac{d}{dx}[f(x)] \qquad (8)$$

$k = 3$
$f(x) = x^2$
$\frac{d}{dx} = k\,f(x) = 3 \cdot \frac{d}{dx}x^2 = 3 \cdot 2x = 6x$

Turunan

$f(x) = x^3 \rightarrow f'(x) = 3x^2$
$g(x) = x^2 \rightarrow g'(x) = 2x$

$\frac{d}{dx}\left(f(x) \pm g(x)\right) = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)$
$= f'(x) \pm g'(x) = 3x^2 \pm 2x$

4
- Jika f dan g fungsi yang dapat diturunkan di x, maka $f+g$ dan $f-g$ juga dapat diturunkan di x dan

$$\frac{d}{dx}[f(x)+g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)] \tag{9}$$

$$\frac{d}{dx}[f(x)-g(x)] = \frac{d}{dx}[f(x)] - \frac{d}{dx}[g(x)] \tag{10}$$

5
- Jika f dan g dapat diturunkan di x, maka

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + \frac{d}{dx}[f(x)]g(x) \tag{11}$$

$= f g' + f' g$

$f(x) = x^2 \rightarrow f'(x) = 2x$
$g(x) = 3x+1 \rightarrow g'(x) = 3$

$\frac{d}{dx}[f(x) \cdot g(x)]$
$= \frac{d}{dx}[x^2(3x+1)]$
$= x^2 \cdot 3 + 2x \cdot (3x+1) = 3x^2 + 6x^2 + 2x$
$= 9x^2 + 2x$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{(g(x))^2}$$

$f(x) = x^2 \rightarrow f'(x) = 2x$

$g(x) = 3x+1 \rightarrow g'(x) = 3$

$$\frac{d}{dx}\left[\frac{x^2}{3x+1}\right] = \frac{2x\cdot(3x+1) - x^2\cdot 3}{(3x+1)^2}$$

$(3x+1)(3x+1)$

$9x^2+3x+3x+1$

$$= \frac{6x^2+2x-3x^2}{9x^2+6x+1}$$

$$= \frac{3x^2+2x}{9x^2+6x+1}$$

6 ▶ Jika f dan g dua fungsi yang dapat diturunkan di x, dan $g(x) \neq 0$ maka $\frac{f}{g}$ juga dapat diturunkan di x, dan

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{\frac{d}{dx}[f(x)]g(x) - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2} \qquad (12)$$

7 ▶ Untuk turunan fungsi trigonometri sebagai berikut

$\frac{d}{dx}[\sin x] = \cos x$	$\frac{d}{dx}[\operatorname{cosec} x] = -\operatorname{cosec} x \cot x$
$\frac{d}{dx}[\cos x] = -\sin x$	$\frac{d}{dx}[\sec x] = \sec x \tan x$
$\frac{d}{dx}[\tan x] = \sec^2 x$	$\frac{d}{dx}[\tan x] = -\operatorname{cosec}^2 x$

$$\frac{2x(3x+1)-x^2\cdot 3}{(3x+1)^2}$$

$$= \frac{\boxed{2x(\cancel{3x+1})-x^2\cdot 3}\,(3x+1)}{(3x+1)(3x+1)}$$

$$\frac{(\cancel{3x+1})\cdot\left[2x-x^2\cdot 3\right]}{(\cancel{3x+1})(3x+1)}$$

Contoh 1

$$\frac{d}{dx}(f(x)) \leftarrow \boxed{f'(c) = \lim_{h\to 0} \frac{f(c+h)-f(c)}{h}}$$

Menggunakan definisi turunan, tentukan turunan terhadap x dari $f(x) = \sqrt{x}$

$f(x) = \sqrt{x}$

$f(x+h) = \sqrt{x+h}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \left[\frac{\sqrt{x+h} - \sqrt{x}}{h} \times \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \right]$$

$$= \lim_{h\to 0} \left[\frac{\cancel{x}+h-\cancel{x}}{h\sqrt{x+h} + h\sqrt{x}} \right] = \lim_{h\to 0} \left[\frac{h}{h\sqrt{x+h} + h\sqrt{x}} \right]$$

$$f'(x) = \lim_{h\to 0}\left[\frac{h}{h\sqrt{x+h}+h\sqrt{x}}\right] = \lim_{h\to 0}\left[\frac{\cancel{h}\,1}{\cancel{h}\left(\sqrt{x+h}+\sqrt{x}\right)}\right]$$

$$= \lim_{h\to 0}\left[\frac{1}{\sqrt{x+h}+\sqrt{x}}\right]$$

$$= \frac{1}{\sqrt{x+0}+\sqrt{x}}$$

$$= \frac{1}{\sqrt{x}+\sqrt{x}}$$

$$f'(x) = \frac{1}{2\sqrt{x}} \quad //$$

Contoh 2

$$f'(x) = g'(x)\,h(x) + g(x) \cdot h'(x)$$

$$g(x) = 2x \rightarrow g'(x) = 2$$

$$h(x) = x^2 + 1 \rightarrow h'(x) = 2x$$

Dapatkan turunan dari fungsi $f(x) = \underbrace{2x}_{g(x)}\underbrace{(x^2+1)}_{h(x)}$

$$f'(x) = 2 \cdot (x^2+1) + 2x \cdot 2x$$

$$= 2x^2 + 2 + 4x^2$$

$$f'(x) = 6x^2 + 2$$

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Aturan Rantai

$$y' = \frac{d}{dx} y(x) \quad \Big\} \quad \frac{d}{\cancel{du}} \cdot y \cdot \frac{\cancel{d}u}{dx}$$

$$y = u(x)$$

Contoh 3

Dapatkan turunan dari fungsi $f(x) = \sin 5x$ → $\sin(5x)$

$u = 5x \rightarrow \sin(5x) = \sin(u)$

$$f'(x) = \boxed{\frac{d}{du} \sin(u)} \cdot \frac{d}{dx} \cdot 5x$$

$$= \cos(u) \cdot 5$$

$$= \cos(5x) \cdot 5$$

$$f'(x) = 5\cos(5x)$$

Soal Latihan

1. Carilah turunan fungsi berikut menggunakan definisi turunan
   a. $f(x) = x^2 + 5x$
   b. $f(x) = \sqrt{x+1}$
   c. $f(x) = \sin x$ → PR
2. Tentukan $f'(x)$ dan $f'(1)$ jika
   a. $f(x) = \sqrt{5}$
   b. $f(x) = 5\sqrt{x}$
   c. $f(x) = \left(2x^2 + 5\right)^3$
   d. $f(x) = \frac{3x^3 - 5x}{x^2}$
   e. $f(x) = x^2 \tan(2x + 1)$

$$\boxed{f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}}$$

1.a) $f(x) = x^2 + 5x \rightarrow f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$f(x+h) = (x+h)^2 + 5(x+h)$$
$$= x^2 + \underline{2hx} + h^2 + \underline{5x} + 5h \Leftarrow$$
$$= x^2 + (2h+5)x + h^2 + 5h.$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{x^2} + 2hx + h^2 + \cancel{5x} + 5h - (\cancel{x^2} + \cancel{5x})}{h}$$

$$= \lim_{h \to 0} \frac{h^2 + 2hx + 5h}{h} = \lim_{h \to 0} \frac{\cancel{h}(h + 2x + 5)}{\cancel{h}}$$

$$= (0 + 2x + 5) = 2x + 5 \,//$$

1.b) $f(x) = \sqrt{x+1}$

$$\boxed{f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}}$$

$f(x+h) = \sqrt{(x+h)+1}$

$= \sqrt{x+h+1}$

$$f'(x) = \lim_{h \to 0} \left[\frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \times \frac{\sqrt{x+h+1} + \sqrt{x+1}}{\sqrt{x+h+1} + \sqrt{x+1}} \right]$$

$$= \lim_{h \to 0} \frac{(x+h+1) - (x+1)}{h\left(\sqrt{x+h+1} + \sqrt{x+1}\right)} = \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}\left(\sqrt{x+h+1} + \sqrt{x+1}\right)}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h+1} + \sqrt{x+1}} = \frac{1}{\sqrt{x+0+1} + \sqrt{x+1}}$$

$$f'(x) = \frac{1}{\sqrt{x+1} + \sqrt{x+1}} = \frac{1}{2\sqrt{x+1}}$$

1.c) $f(x) = \sin(x)$

$$\boxed{f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}}$$

$$f(x+h) = \underline{\sin(x+h)} = \sin x \cos h + \cos x \sin h$$

$$\boxed{\sin(a+b) = \sin a \cos b + \cos a \sin b}$$

$$f'(c) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sin(x+h) - \sin(x)}{h} \cdot \frac{\sin(x+h) + \sin(x)}{\sin(x+h) + \sin(x)}$$

$$= \lim_{h \to 0} \frac{\boxed{\sin^2(x+h) - \sin^2(x)}}{h(\sin(x+h) + \sin(x))}$$

$$f'(x) = \lim_{h \to 0} \frac{\boxed{\sin^2(x+h) - \sin^2(x)}}{h(\sin(x+h) + \sin(x))}$$

$$\frac{h[\quad]}{h(} \qquad \frac{1}{0}$$

2.a) $f(x) = \sqrt{5}$; $f'(x) = ?$ $\quad f'(1) = ?$

$$f(x) = \sqrt{5} \rightarrow f'(x) = 0$$

2.b) $f(x) = 5\sqrt{x}$

$$f'(x) = \frac{d}{dx}\left[5\sqrt{x}\right] = 5 \cdot \frac{d}{dx}\left[\sqrt{x}\right]$$

$$= 5 \cdot \frac{d}{dx}\left[x^{\frac{1}{2}}\right]$$

$$= 5 \cdot \left[\frac{1}{2} \cdot x^{\frac{1}{2}-1}\right] = 5 \cdot \frac{1}{2} x^{-\frac{1}{2}}$$

$$f'(x) = 5 \cdot \frac{1}{2} x^{-\frac{1}{2}} = \frac{5}{2} \cdot \frac{1}{x^{\frac{1}{2}}} = \frac{5}{2} \cdot \frac{1}{\sqrt{x}} = \frac{5}{2\sqrt{x}}$$

$$f'(1) = \frac{5}{2\sqrt{1}} = \frac{5}{2}$$

2.6) $f(x) = (2x^2+5)^3 \rightarrow f'(x)$

$$u = \underline{2x^2+5} \rightarrow f(x) = u^3$$

$$f' = \frac{df}{du} \cdot \frac{du}{dx} = \underline{\frac{d}{du} \cdot u^3} \cdot \underline{\frac{d}{dx} \cdot 2x^2+5}$$

$$= 3u^2 \cdot 4x$$

$$= 3(2x^2+5)^2 \cdot 4x$$

$$f'(1) = 3(2(1)^2+5)^2 \cdot 4(1) = 3(7)^2 \cdot 4 = 3 \cdot 49 \cdot 4 = 588$$

2.d) $f(x) = \dfrac{3x^3 - 5x}{x^2} \begin{matrix} \leftarrow f \\ \leftarrow g \end{matrix}$ $\quad \dfrac{d}{dx}\left[\dfrac{f}{g}\right] = \dfrac{f'g - fg'}{g^2}$

$$f'(x) = \frac{(9x^2 - 5)\,x^2 - (3x^3 - 5x)\,2x}{(x^2)^2}$$

$$= \frac{9x^4 - 5x^2 - 6x^4 + 10x^2}{x^4}$$

$$= \frac{3x^4 + 5x^2}{x^4} = \frac{(3x^2 + 5)\cancel{x^2}}{x^2 \cdot \cancel{x^2}} = \frac{3x^2 + 5}{x^2}$$

$$f'(1) = \frac{3(1)^2 + 5}{(1)^2} = 8$$

2.e) $f(x) = x^2 \tan(2x+1)$

$$\frac{d}{dx}[g \cdot h] = g'h + gh'$$

$$\frac{d}{dx} f = \frac{d}{du} f(u) \cdot \frac{d}{dx} u(x)$$

$g(x) = x^2 \rightarrow g'(x) = 2x$

$h(x) = \tan(2x+1)$

$u = 2x+1$

$h(u) = \tan(u)$

$$h'(x) = \frac{d}{du} \cdot \tan(u) \cdot \frac{d}{dx} 2x+1$$

$$= \sec^2(u) \cdot 2$$

$$= 2\sec^2(2x+1).$$

$$f'(x) = 2x \cdot \tan(2x+1) + x^2 \cdot 2\sec^2(2x+1)$$

$$f'(1) = 2(1) \cdot \tan(2 \cdot 1+1) + (1)^2 \cdot 2\sec^2(2 \cdot 1+1)$$

$$= 2\tan(3) + 2\sec^2(3)$$

$$= 2\tan(3) + 2 \cdot \frac{1}{\cos^2(3)}$$

$f'(1) = 1,75$ //