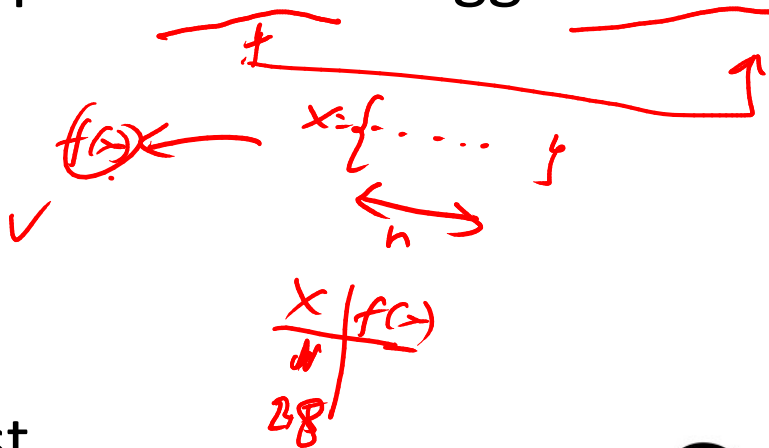# Integrasi Numerik

Pendahuluan

- Penyelesaian lebih mudah untuk mencari nilai integral suatu fungsi yang cukup kompleks

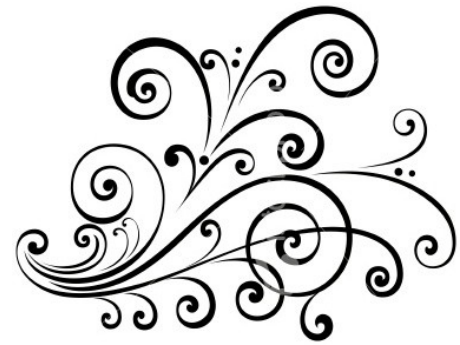- Misal mencari integral pada x=1.0 hingga x=2.8 dari fungsi berikut

$$f(x) = \frac{x^2 \ln(x) + e^{-x}}{5 x \sin x}$$

$$f(x) = \frac{x^2 \cos x}{e^{-x}} \qquad \checkmark \quad \text{dst}$$

Metode Integral Numerik

1. Metode Trapezoida
2. Metode Simpson 1/3

Metode Trapezoida

① equispaced ?
② formula ↑

- Metode mencari nilai integral fungsi f(x) dengan batas tertentu (dari x=x_0 ke x_n)

x | $f(x)$
c_1
c_2
c_3
c_4

- Kondisi non equispaced

$$\int_{x_0}^{x_n} f(x)\,dx = \frac{(x_1-x_0)}{2}(f_1+f_0)+\ldots+\frac{(x_n-x_{n-1})}{2}(f_n+f_{n-1})$$

- Kondisi equispaced

data terakhir

$$\int_{x_0}^{x_n} f(x)\,dx = \frac{h}{2}\left[f_0+2(f_1+f_2+\ldots+f_{n-1})+f_n\right]$$

data 2 terakhir

$$h = x_1-x_0 = x_2-x_1 = \ldots = x_n-x_{n-1}$$

Contoh soal

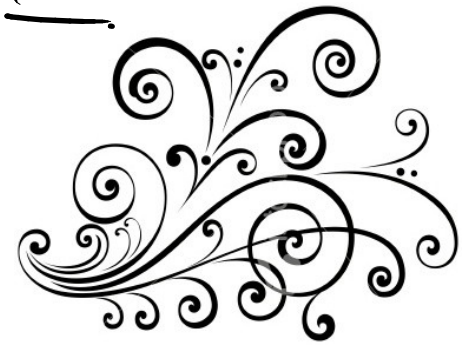| n	x	f(x)
0	1.0	1.449
1	1.3	2.060
2	1.6	2.645
3	1.9	3.216
4	2.2	3.779
5	2.5	4.338
6	2.8	4.898

0,3 (
0,3 (
0,3 (
0,3 (
0,3 (
0,3 (

- Carilah nilai integral dengan batas x=1.0 hingga x=2.8 dengan metode trapezoida

$$\int_{1.0}^{2.8} f(x) = ?$$

① equispaced ? yes

②

$$\int_{1.0}^{2.8} f(x)\, dx = \frac{h}{2} \left\{ f_0 + 2(f_1 + \ldots + f_5) + f_6 \right\}$$

$$\int_{1.0}^{2.8} f(x)\,dx = \frac{0.3}{2}\left\{1.449 + 2\left(2.06 + 2.645 + 3.216 \right.\right.$$
$$\left.\left. + 3.779 + 4.338\right)\right.$$
$$\left. + 4.898\right\}$$

$$= \frac{0.3}{2}\left\{1.449 + 32.076 + 4.898\right\}$$

$$= \frac{0.3}{2} \cdot 38.423$$

$$\int_{1.0}^{2.8} f(x)\,dx = 5.76345$$

Solusi

- Karena merupakan tabel equispaced, maka integral f(x) dengan batas x=1.0 hingga x=2.8

$$\int f(x)\,dx = \frac{h}{2}\left[f_0 + 2\left(f_1 + f_2 + f_3 + f_4 + f_5 \right) + f_6 \right]$$

$$= \frac{(1.3-1.0)}{2}\left[1.449 + 2\left(2.060 + 2.645 + 3.216 + \right.\right.$$

$$\left.\left. 3.779 + 4.338 \right) + 4.898 \right)$$

$$= 5.76345$$

② Metode Simpson 1/3

- Metode mencari nilai integral fungsi f(x) dengan batas tertentu (dari x=x_0 ke x_n)

- Hanya untuk kondisi equispaced

$$\int_{x_0}^{x_n} f(x)\,dx = \frac{h}{2}\left[f_0 + 4\left(f_1 + f_3 + f_5 + \ldots + f_{n-1}\right) + 2\left(f_2 + f_4 + f_6 + \ldots + f_{n-2}\right) + f_n\right]$$

$$h = x_1 - x_0 = x_2 - x_1 = \ldots = x_n - x_{n-1}$$

- Lebih efektif jika n genap

Contoh soal

n	x	f(x)
0	1.0	1.449
1	1.3	2.060
2	1.6	2.645
3	1.9	3.216
4	2.2	3.779
5	2.5	4.338
6	2.8	4.898

- Carilah nilai integral dengan batas x=1.0 hingga x=2.8 dengan metode simpson 1/3

Solusi

$$\int f(x)\,dx = \overbrace{\frac{h}{3}}^{x_1-x_0}\left[f_0 + 4\left(f_1 + f_3 + f_5\right) + 2\left(f_2 + f_4\right) + f_6\right]$$

$$= \overbrace{\frac{\left(1.3-1.0\right)}{3}}^{x_1-x_0}\left[1.449 + 4\left(2.060 + 3.216 + 4.338\right) + \right.$$

$$\left. 2\left(2.645 + 3.779\right) + 4.898\right)$$

$$= 5.7651$$

$x =$ [0.   , 0.26179939, 0.52359878, 0.78539816, 1.04719755,
       1.30899694]

$f(x) =$ 0.   , 0.25881905, 0.5   , 0.70710678, 0.8660254 ,
       0.96592583]

$$\int_{0}^{\pi/2} f(x)\, dx = \int_{0}^{\pi/2} \sin(x)\, dx = -\cos(x)\Big|_{0}^{\pi/2}$$

$$= -\cos(\pi/2) - (-\cos(0))$$
$$= -0 - (-1)$$
$$= 1$$

$$\int f(x)\,dx = \frac{h}{2}\left[f_0 + 2(f_1 + f_2 + f_3 + f_4 + \cancel{f_5}) + f_5\right]$$

$$= \frac{0.26}{2}\left[0 + 2(0.26 + 0.5 + 0.71 + 0.87) + 0.96\right]$$

$$\int_0^{\pi/2} f(x)\,dx = 0.73 \longrightarrow \boxed{\text{(0.27)}} \text{ (error)}$$

$$\int f(x)\,dx = \frac{\breve{h}}{3}\left[f_0 + 4(f_1 + f_3 + \cancel{f_5}) + 2(f_2 + f_4) + f_5\right]$$

$$= \frac{0.26}{3}\left\{0 + 4(0.26 + 0.71) + 2(0.5 + 0.87) + 0.96\right\}$$

$$\int f(x)\,dx = 0.66 \sim \text{error: } 0.34$$

① **Regresi** dari data

$x = [0. \quad , 0.26179939, 0.52359878, 0.78539816, 1.04719755,$
$\qquad 1.30899694]$

$f(x) = \quad 0. \quad , 0.25881905, 0.5 \quad , 0.70710678, 0.8660254 ,$
$\qquad 0.96592583]$

polinom orde 2.

② $f(x) = ax^2 + bx + c. \quad \rightarrow \quad \left[\dfrac{2x+b}{} \right] \Big|_0^{\pi/2}$

③ $\displaystyle\int_0^{\pi/2} f(x)\, dx = \boxed{\quad \dots \quad ? \quad}$

dibandingkan dg hasil analitik

④ kesimpula. mana yg lebih dekat dg ↗