

Persamaan Diferensial

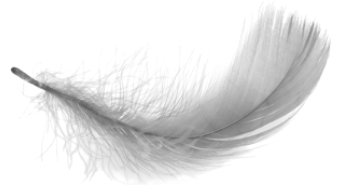
Pendahuluan

- Persamaan diferensial biasa = persamaan diferensial dengan satu perubah

$$F\left(x, y, \frac{dy}{dx}, \frac{d^2 y}{dx^2}, \dots, \frac{d^n y}{dx^n}\right) = 0$$

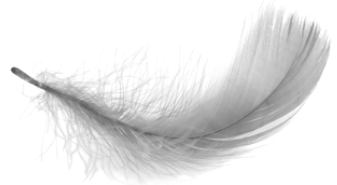
- Persamaan diferensial ordo tinggi => persamaan diferensial ordo satu dengan mendefinisikan variable baru

$$x^3 \frac{d^3 y}{dx^3} + x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + (x^2 - p^2)y = 0$$



Metode Penyelesaian

- Metode Taylor
- Metode Euler
- Metode Euler yang dimodifikasi
- Metode Runge Kutta orde 4
- Metode Adam
- Metode Milnie
- Metode Adam-Moulton

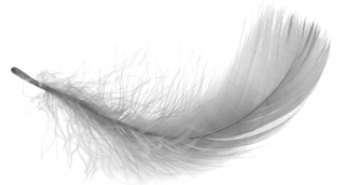


Metode Taylor

- Proses mencari nilai fungsi $y(x)$ pada titik x tertentu

$$y(x_m) = y(x_{m-1}) + hy'(x_{m-1}) + \frac{h^2 y''(x_{m-1})}{2!} + \frac{h^3 y'''(x_{m-1})}{3!} + \dots + \frac{h^n y^n(x_{m-1})}{n!}$$

dimana $h = x_m - x_{m-1}$



Metode Taylor

$$y' = (x_{m-1}) = f(x_{m-1}, y_{m-1})$$

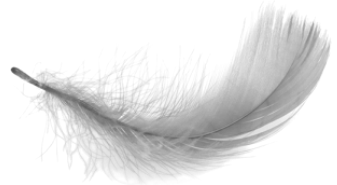
$$y'' = (x_{m-1}) = f'(x_{m-1}, y_{m-1})$$

\vdots

$$y^n = (x_{m-1}) = f^{n-1}(x_{m-1}, y_{m-1})$$

$$n = 1, 2, 3, 4, \dots$$

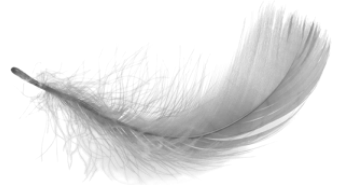
$$m = 0, 1, 2, 3, \dots$$



Contoh

Carilah nilai $y(0.1)$ dari persamaan diferensial biasa berikut dengan metode Taylor

$$f(x, y) = \frac{dy}{dx} = x + y, \quad y(0) = y_0 = 1.5$$



Solusi

- **Step 1** Mencari turunan PD yang diketahui

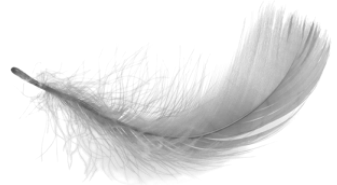
$$y'(x) = f(x, y) = x + y$$

$$y''(x) = f'(x, y) = 1 + y'(x) = 1 + f(x, y)$$

$$y'''(x) = f''(x, y) = y''(x) = f'(x, y)$$

⋮

$$y^n(x) = f^{n-1}(x, y) = y^{n-1} = f^{n-2}(x, y)$$



Solusi

sehingga

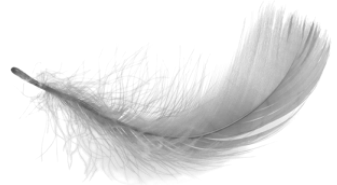
$$y'(x_0) = f(x_0, y_0) = x_0 + y_0 = 0 + 1.5 = 1.5$$

$$y''(x_0) = f'(x_0, y_0) = 1 + y'(x_0) = 1 + 1.5 = 2.5$$

$$y'''(x_0) = f''(x_0, y_0) = y''(x_0) = f'(x_0, y_0) = 2.5$$

\vdots

$$y^n(x_0) = f^{n-1}(x_0, y_0) = y^{n-1} = f^{n-2}(x_0, y_0) = 2.5$$

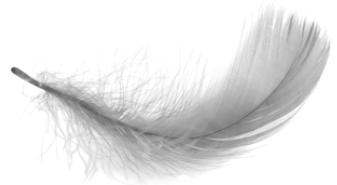


Solusi

- **Step 2** Mencari nilai $y(x_1)$ dimana $x_1=0.1$

$$h = x_1 - x_0 = 0.1 - 0 = 0.1$$

$$\begin{aligned} y(0.1) &= y(0) + hy'(0) + \frac{h^2 y''(0)}{2!} + \dots + \frac{h^n y^{(n)}(0)}{n!} \\ &= 1.5 + (0.1)(1.5) + \frac{(0.1)^2 (2.5)}{2} + \frac{(0.1)^2 (2.5)}{6} + \dots \\ &= 1.6629273 \end{aligned}$$



Metode Euler

- Proses mencari nilai fungsi $y(x)$ pada titik x tertentu

$$\Delta y_{n-1} = f(x_{n-1}, y_{n-1}) \Delta x$$

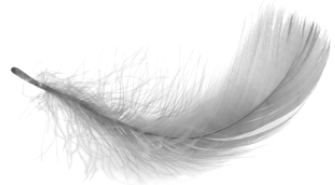
$$y_n = y_{n-1} + \Delta y_{n-1}$$

$$\Delta x = \frac{x_n - x_0}{n}$$

x_n = x yang ditanya nilai fungsinya

x_0 = x awal

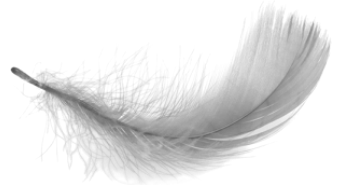
n = bilangan asli sembarang



Contoh

Carilah nilai $y(0.1)$ dari PD dibawah ini

$$f(x, y) = \frac{dy}{dx} = \frac{y - x}{y + x} \quad , y(0) = 2$$

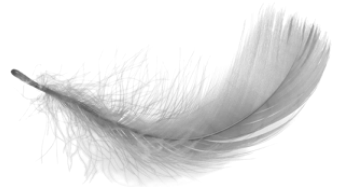


Solusi

- **Step 1** Mencari nilai x

$x_n=0.1$ dan $x_0=0$. Jika diambil $n=5$

$$\Delta x = \frac{x_n - x_0}{n} = \frac{0.1 - 0}{5} = 0.02$$



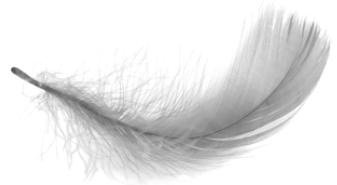
Solusi

- **Step 2** Mencari penyelesaian dengan iterasi

Iterasi pertama, n=1

$$\begin{aligned}\Delta y_0 &= f(x_0, y_0) \Delta x \\ &= \frac{y_0 - x_0}{y_0 + x_0} \Delta x = \frac{2 - 0}{2 + 0} (0.02) = 0.02\end{aligned}$$

$$y_1 = y(0.02) = y_0 + \Delta y_0 = 2 + 0.02 = 2.02$$



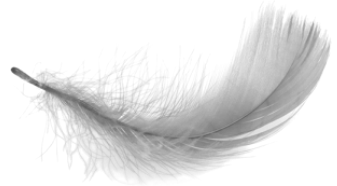
Solusi

Iterasi pertama, n=2

$$x_1 = x_0 + \Delta x = 0 + 0.02 = 0.02$$

$$\begin{aligned}\Delta y_1 &= f(x_1, y_1) \Delta x \\ &= \frac{y_1 - x_1}{y_1 + x_1} \Delta x = \frac{2.02 - 0.02}{2.02 + 0.02} (0.02) = 0.01961\end{aligned}$$

$$y_2 = y(0.04) = y_1 + \Delta y_1 = 2.02 + 0.01961 = 2.03961$$



Solusi

Iterasi pertama, n=5

$$x_4 = x_3 + \Delta x = 0.06 + 0.02 = 0.08$$

$$\begin{aligned}\Delta y_4 &= f(x_4, y_4) \Delta x \\ &= \frac{y_4 - x_4}{y_4 + x_4} \Delta x = \frac{2.07771 - 0.08}{2.07771 + 0.08} (0.02) = 0.01852\end{aligned}$$

$$y_5 = y(0.1) = y_4 + \Delta y_4 = 2.07771 + 0.01852 = 2.09623$$

