PRINCIPAL COMPONENT ANALYSIS AND VIRTUAL COHERENCE OF THE CROSS-AXIS APPARENT MASS IN WHOLE-BODY VIBRATION

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Abstract

Dynamic forces have been measured at the excitation-subject interface in directions other than the axis of excitation. Quantification of the relative importance of these forces can be achieved by principal component analysis and virtual coherence which has not been previously performed. The present study applied these operations to the inline and cross-axis forces measured with a semi-supine body exposed to longitudinal random (0.25-20 Hz) whole-body vibration at a magnitude of 1.0 ms^{-2} r.m.s. The source identification problem in such a context would be realised by a reversed path where the aim is to identify relative contributions and correlations between the forces in response to a single axis excitation.

1. Introduction

Biodynamic responses, either forces or accelerations, in directions other than the direction of excitation (i.e. cross-axis response) have been extensively reported in seated (e.g. Nawayseh and Griffin, 2003), standing (e.g. Subashi *et al*., 2006), and recumbent persons (e.g. Huang and Griffin, 2008a, b) with single-axis vertical and fore-and-aft excitations, and more recently with simultaneous excitation in orthogonal axes (e.g. Zheng, 2012). Understanding the cross-axis coupling between the axis of excitation and the axes of response is required to predict body movement and forces experienced in vibration environments. Combined with the challenge to understand the mechanism causing the nonlinear magnitude dependency (e.g. Huang and Gunston, 2009), in which the resonance frequency decreases with increasing excitation magnitude, it is imperative to quantify and separate the cross-axis response in each axis. Ordinary frequency response functions (FRFs), such as apparent mass or transmissibilities, and ordinary coherences have been regularly used to compare cross-axis responses with responses inline with the excitation. However, the response in one axis could be caused by, or mathematically 'coherent to', responses in other axes. FRF operation is not able to identify such correlations.

A preliminary study using conditioned reverse path method (CRP) has been conducted in this context to separate the cross-axis forces from the inline response force (referred as 'reverse path multi-input-single-output method', see Huang, 2011). The method was applied to experimental data of a semi-supine human body exposed to multiple magnitudes of random vibration in the longitudinal horizontal direction with response forces measured in the inline longitudinal, cross-axis vertical and cross-axis lateral directions (reported by Huang and Griffin, 2008a). The recumbent position allowed the elimination of forces and voluntary or involuntary movement introduced by the subject. The

longitudinal and cross-axis vertical force dominated the response with minimal forces in the lateral direction. The partial, summed and multiple coherences computed from the uncorrelated (or conditioned) reversed mathematical inputs (i.e. the three response forces) indicated the frequency ranges that the cross-axis vertical force had considerable contribution. However, it was not clear whether the inline longitudinal force and the cross-axis vertical force were correlated, and, over what frequency range. The conditioned spectral analysis was introduced to identify contributions of physical sources, in the case of a reversed path, the response forces (e.g. Bendat and Piersol, 1993).

'Virtual source analysis', more commonly know as principal component analysis (PCA), assumes a new coordinate based on the maximum variation of data sets and offers a different point of view on the frequency-dependent relationship between the inline and cross-axis response. By using the PCA operation alone it is usually difficult to establish relationships between the virtual and physical sources. Based on PCA, the virtual coherence (VC) technique has been used to identify the coherences between virtual and physical sources (e.g. Price and Bernhard, 1986), and more recently to investigate noise sources in a diesel engine (Leclere *et al.*, 2005). The technique effectively identified physical sources that contributed to the virtual source the most over five acquisition channels. With the three orthogonal response forces of the supine body in the present study, the focus is to identify the correlation between the inline longitudinal and the cross-axis vertical forces and their frequency dependencies.

The present study intends to apply the principal component and virtual coherence technique to the inline and cross-axis forces measured with a semi-supine body by Huang and Griffin (2008a) and used previously in a conditioned spectral study by Huang (2011). The source identification problem in such context would be realised by a reversed path where the aim is to identify contributions and correlations between the output forces in response to a single excitation (acceleration).

2. Experimental data and ordinary frequency response functions

The analytical study presented is based on the longitudinal inline (z-axis), cross-axis vertical (x-axis) and cross-axis lateral (y-axis) forces of one semi-supine subject (S9) exposed to longitudinal (z-axis) broadband random (nominally flat from 0.25 to 20 Hz) vibration at 1.0 ms^{-2} r.m.s. (Huang and Griffin, 2008a). The forces are measured at the excitation-subject interface, or driving point, of the back support rigidly mounted on the longitudinal moving vibration platform (Figure 1). Driving point apparent masses in the inline and cross axis are computed using cross spectral density (CSD) method, all performed in MATLAB 7.10:

$$H(f) = G_{fa}(f) / G_{aa}(f) \qquad (1)$$

where H denotes the apparent mass FRFs; subscript 'a' the measured z-axis excitation acceleration; subscript 'f' the measured driving point forces – either the inline force or the cross-axis forces; f the frequency in Hz; G_{fa} the cross spectral density (CSD) function between the response force and the excitation acceleration; G_{aa} the power spectral density (PSD) function of the excitation acceleration. All acceleration and force signals were acquired for 90 seconds at 200 samples per second, through an anti-aliasing filter low-passed at 67 Hz. The spectral analysis was performed with a FFT length of

2048, a Hamming window with 50% overlap (or 100% use of data) giving a frequency resolution of about 0.1 Hz.

The ordinary coherence function is defined as:

$$coh_{af}(f) = |\, G_{af}(f)\, |^2 \,/\, (\, G_{aa}(f)\; G_{ff}(f)\,) \tag{2}$$

A coherence of less than unity indicates an output (e.g. response force) that is not purely caused by a linear function of input (e.g. excitation acceleration). The frequency range at which the inline force coherence drops is around 10 to 16 Hz (Figure 1b). This shows relatively high coherence between the cross-axis vertical force and the excitation (Figure 1d). With the low inline apparent mass (Figure 1a) comparing to the cross-axis vertical apparent mass (Figure 1c) over this frequency range, the ordinary coherences tend to suggest the cross axis vertical force dominates the response between 10 and 16 Hz. But it is not clear whether the cross-axis vertical force is correlated to the inline force, and if so by how much. The cross-axis lateral force was minimal and again the ordinary coherence did not tell much on its relative contribution of this response (Figure 1e, f).
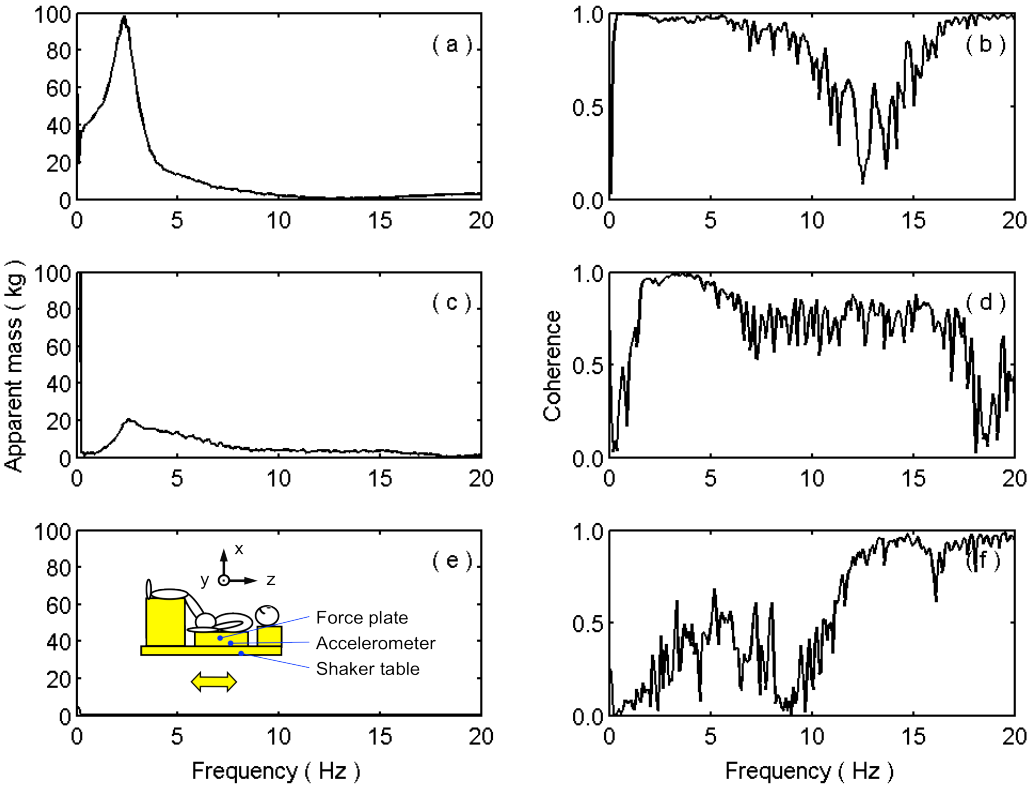

Figure 1   Inline longitudinal apparent mass peaked at 2.3 Hz (a) and its coherence (b), cross-axis vertical apparent mass peaked at 2.5 Hz (c) and coherence (d), and cross-axis lateral apparent mass (e) and coherence (f) of a semi-supine subject exposed to longitudinal (z-axis) random vibration (0.25 to 20 Hz flat) at 1.0 ms^{-2} r.m.s. (Huang and Griffin, 2008a).

3. Principal components and virtual coherences

The principal component analysis assumes that the dynamic system of the human body is excited by n uncorrelated principal components (PCs), or virtual sources, which is formed by a linear combination of m different physically acquired sources where $n \leq m$. In the present application $m = n = 3$. For m measured forces, the cross spectral density matrix takes the form:

$$G(f) = \begin{bmatrix} G_{11}(f) & G_{12}(f) & ... & G_{1m}(f) \\ G_{21}(f) & G_{22}(f) & ... & \\ ... & & & \\ G_{m1}(f) & & ... & G_{mm}(f) \end{bmatrix} \quad \text{or} \quad G(f) = \begin{bmatrix} G_{11}(f) & G_{12}(f) & G_{13}(f) \\ G_{21}(f) & G_{22}(f) & G_{23}(f) \\ G_{31}(f) & G_{32}(f) & G_{33}(f) \end{bmatrix} \quad (3)$$

where for the first subscript '1' is the measured longitudinal horizontal inline response force, '2' the vertical cross-axis force, and '3' the lateral cross-axis force. So for example G_{11} is the PSD for inline response force.

Eigen decomposition of the spectral density matrix $G(f)$ at each frequency yields:

$$G(f) = U(f)\ \Lambda(f)\ U^{\mathrm{T}}(f) \quad (4)$$

where $\Lambda(f)$ is a diagonal array containing the eigenvalues of $G(f)$ with their values in descending order. $\Lambda(f)$ represents cross spectral density matrix of a set of uncorrelated 'principal components' or 'virtual sources'. The physical sources, i.e. the three response forces, can be considered as a linear combination of these principle components. The eigenvectors $U(f)$ (and T for transpose) define the linear relationship between the virtual and physical sources. For example, the corresponding column in $U(f)$ at each frequency for each eigenvalue (in the diagonal array $\Lambda(f)$) comprises the three coefficients for each of the three principal components, i.e. PC1, PC2 and PC3. The number of 'significant' high-value eigenvalues at each frequency is the number of independent contributors or significant principle components.

The inline longitudinal force dominated up to about 5 Hz, and the cross-axis vertical force exhibited similar levels as inline response from around 5 to 10 Hz and exceeded the inline between 10 and 16 Hz (G_{11} and G_{22} in Figure 2a). The first two principal components (PC1 and PC2 in Figure 2b) showed dominance over the frequency range up to 20 Hz, indicating that two independent contributors dominated but with PC1 20 dB more important than PC2 across the frequency range. A closer look at the coefficients for the linear combination of the three physical sources contributed to PC1 in Figure 3 confirms that the inline response is dominant up to 5 Hz, and then the cross-axis vertical force contributed marginally more than the inline between 5 and 10 Hz and the cross-axis vertical dominated from 10 to 16 Hz. Such observation can be appreciated qualitatively from comparing the inline and cross-axis apparent masses and/or the PSDs at these frequency bands (Figure 1a, c and Figure 2a). However, the PCA operation was able to identify first, the virtual source or coordinate that yields the maximum independent variation, and second, quantify the proportional contribution, or relative importance, of each physical source to the most important virtual source in this application PC1. An alternative point of view in understanding this relationship is to consider how much each virtual source (PC) has contributed to each physical source through virtual coherences.
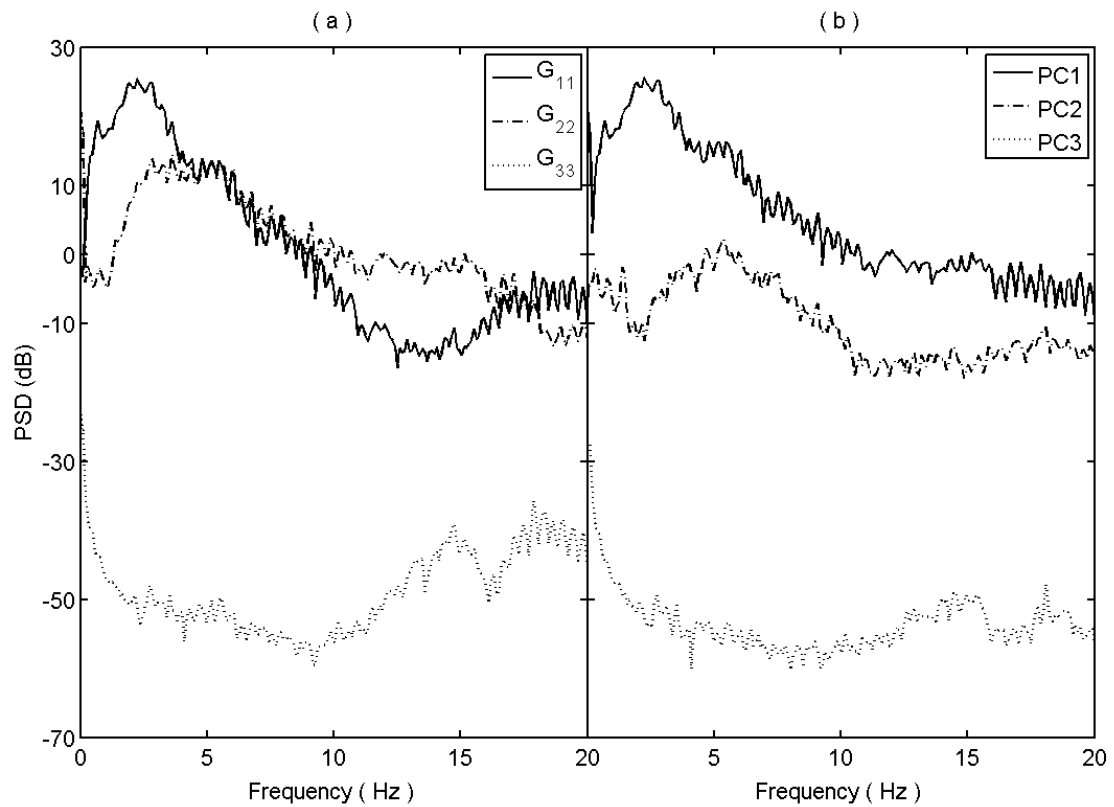
Figure 2   Relative power spectral density functions of (a) the inline longitudinal force (G_{11}), cross-axis vertical force (G_{22}) and cross-axis lateral force (G_{33}), and (b) eigenvalues of principal components (PC1, PC2 and PC3, i.e. the eigenvalues of G) of which the three orthogonal forces are a linear combination
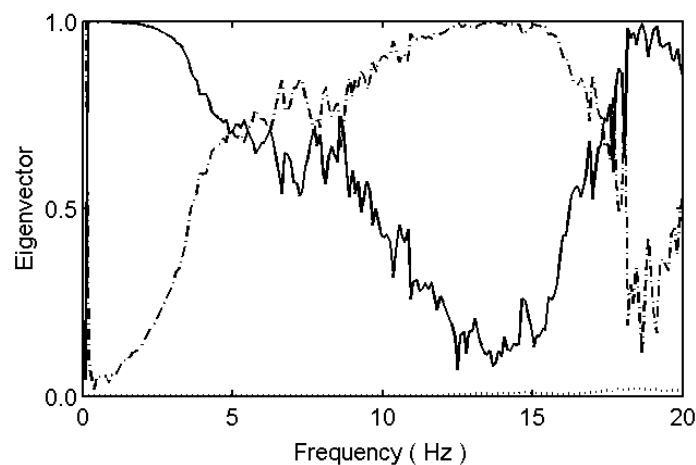

Figure 3   Absolute values of eigenvectors of inline longitudinal force (solid line), cross-axis vertical force (dash-dot line), and cross-axis lateral force (dotted line) to the first principal component (PC1) showing the unified (to one) linear coefficients of the physical sources that contributed to PC1

The virtual coherence identifies contribution of each virtual source, or principal component, to individual physical source. The virtual source between the jth virtual source and the ith physical source is the ratio between the contribution of virtual source j and the PSD of physical source i:

$$vcoh_{ij}(f) = |\ U_{ij}(f)^*\ (\varLambda_{jj}(f))^{1/2}\ |^2\ /\ G_{ii}(f) \tag{5}$$

where $U_{ij}(f)^*$ is the conjugate of the eigenvector coefficient of the ith physical source contributing to the jth virtual source; $\varLambda_{jj}(f)$ is the jth eigenvalue of the virtual source.

The virtual coherence of the inline longitudinal force showed that it contributed primarily to the first two PCs: for PC1 at up to 13 Hz and 15 – 20 Hz, and for PC2 5 – 18 Hz (Figure 4a). PC1 is the most important component. Below 10 Hz and above 16 Hz, the inline longitudinal force is the most important contributor. In Figure 4b, cross-axis vertical force showed the strongest influence on PC1 over the frequency range 2 – 16 Hz. The 'coherent' contribution of the first two physical sources to PC1 implies that the inline longitudinal force and the cross-axis vertical force are correlated to each other between about 2 and 10 Hz. At frequencies higher than 10 Hz, the two physical sources had more independent contribution to the most important principal component PC1.
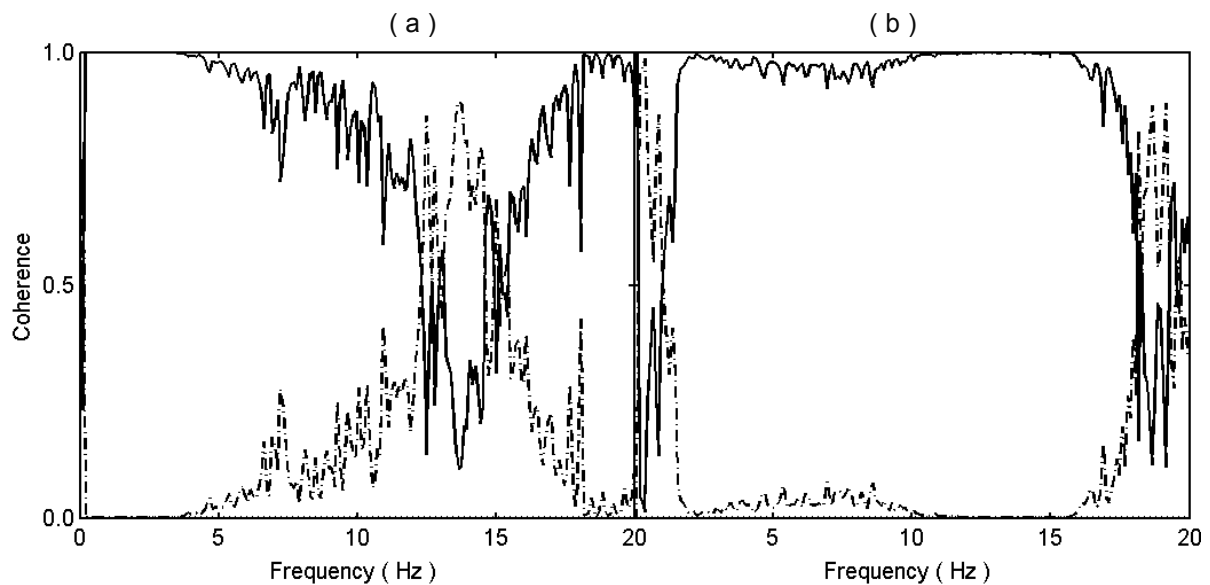

Figure 4   Virtual coherences of (a) PC1 (solid line), PC2 (dash-dot line), and PC3 (dotted line) to the first physical source (inline longitudinal force), and (b) PC1, PC2 and PC3 to the second physical source (cross-axis vertical force) showing linear contribution of each PC to the physical sources

4. Discussion

Principal component analysis is able to separate out independent or uncorrelated contributors of (physical) vibration sources and rank them in order. The present study adapted this source identification method in a reversed fashion where the 'output' response forces were considered as the 'sources', and the aim was to identify the relationship and relative importance of each force. However, this technique cannot tell whether all important physical sources are included. For example, pitch or roll of the semi-supine body could also have influenced and/or correlated to the three response forces. The fundamental assumption was that the number of physical sources acquired needed to be larger than the number of potentially important principal components.

The analysis was performed with all twelve subjects whose apparent masses were originally reported by Huang and Griffin (2008a). The presented results from one subject showed typical characteristics of the relative contributions in each axis. In general, the inline longitudinal and the cross-axis vertical forces were correlated to each other from a low (e.g. 2 Hz for the present subject) to a medium frequency (e.g. 10 Hz). Above the medium frequency where the apparent mass is much lower than at lower frequencies, the two forces tended to be independent in their contribution to the overall response. The exact frequency at which these bands switch varied but the twelve subjects all conformed to the pattern identified by the above bands. This was consistent with the multiple coherence analysis performed with the same subject using conditioned reverse path (CRP) method where after removing 'correlated' contributions of cross-axis forces the inline force dominated below 10 Hz (Huang, 2011). However, multiple coherence could not quantify the linear combination of coefficients identified by the eigenvectors of the dominant principal component (Figure 3).

The cause of cross-axis coupling in whole-body vibration is usually speculated to be the geometric arrangement of the skeletal structure and the soft tissue at the excitation-subject interface (e.g. Kitazaki and Griffin, 1998; Matsumoto and Griffin, 1998). With the recumbent position and an excitation in the shearing direction of the soft tissue at the contact interface, the cross-axis response would not be caused by bending of the spine or pitching of the pelvis. But it was more likely to be caused by the 'weight transfer' of the sprung mass, i.e. the bony and soft constructions of the body above the interface soft tissue, as illustrated schematically in Figure 5. Linear combinations identified by the PCA operation would assist development of analytical models like this.
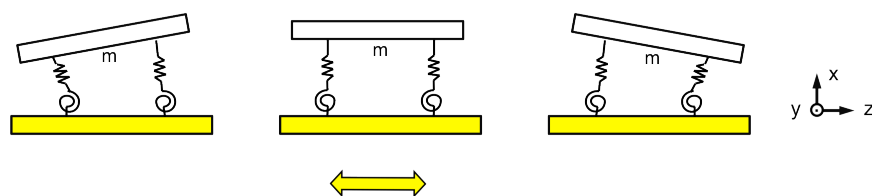

Figure 5   Schematic of a simplified model representing weight transfer of the torso of a recumbent person exposed to longitudinal horizontal excitation at the equilibrium and the two extreme positions during a cycle. Z-x cross-axis coupling is enabled by a combination of translational and rotational springs between the sprung mass m (usually nonhomogeneous) and the excitation base. Damping components accompanying the springs are ignored for simplicity.

The present study demonstrated a relatively simple example with different physical sources that have high, medium and low relative contributions to the overall dominant virtual response (i.e. PC1). With acceleration (or transmissibility) measured in multiple translational and rotational directions between the base of excitation and a location on the body or between two locations on the body, one can derive the correlation between the axes of response and their relative contributions. A comparison between the principal components (and virtual coherences) of these physical vibration sources obtained from single-axis excitation and multi-axis excitation would provide a more quantitative insight into the effect of additional axes of simultaneous excitation on individual responses.

5. Conclusion

Principle component analysis is demonstrated in the present study with a relatively simple example of three acquired dynamic forces that contributed in different proportions to the dominant virtual source or principal component. The introduction of the principal components serves as a new virtual coordinate that points to the maximum variation of the overall response described by the three forces in the inline and cross-axis directions. The degree of correlation established from the eigenvectors of principal components and virtual coherences of each physical source could help understand the mechanisms causing the cross-axis coupling commonly seen in biodynamic responses of whole-body vibration.

6. References

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