TE201416: SINYAL DAN SISTEM

KONVOLUSI

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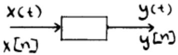
Maret 5, 2020

Pengantar

System Properties



- Memory
- Invertibility
- Causality
- Stability
- Time Invariance
- Linearity

Time-Invariance

C-T:

$x(t) \to y(t)$

Then

$x(t-t_0) \to y(t-t_0)$ any t_0

D-T:

$x[n] \to y[n]$

$x[n-n_0] \to y[n-n_0]$ any n_0

Linearity

$\phi_k \to \psi_k$

Then

$a_1\phi_1 + a_2\phi_2 + \ldots \to a_1\psi_1 + a_2\psi_2 + \ldots$

STRATEGY:

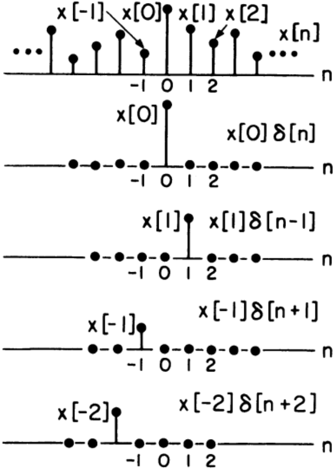
- decompose input signal into a linear combination of basic signals
- choose basic signals so that response easy to compute

LTI Systems:

delayed impulses $\Longleftrightarrow$ Convolution

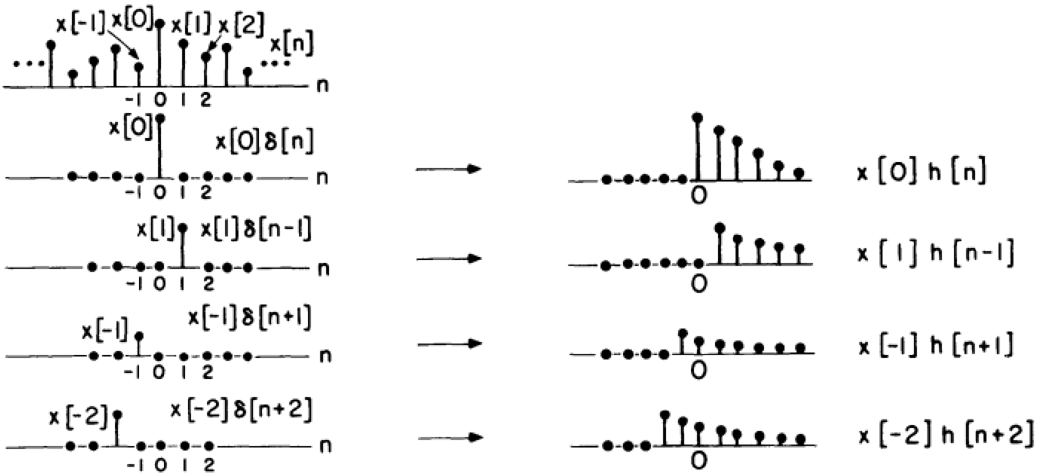
complex exponentials $\Longleftrightarrow$ Fourier Analysis

Superposisi sinyal waktu diskrit



$$x[n] = x[0]\delta[n] + x[1]\delta[n-1] + x[-1]\delta[n+1] + \cdots = \sum_{k=-\infty}^{+\infty} x[k]\delta[n-k]$$

Konvolusi penjumlahan sistem LTI waktu diskrit



Konvolusi penjumlahan sistem LTI waktu diskrit

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k]\ \delta[n-k]$$

Linear System:

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]\ h_k[n]$$

$$\delta[n-k] \rightarrow h_k[n]$$

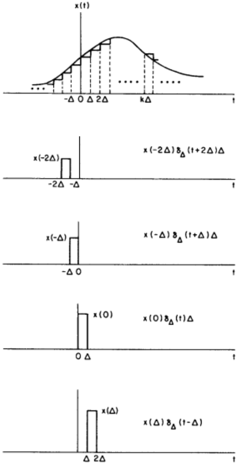
If Time-Invariant:

$$h_k[n] = h_o[n-k]$$

$$\text{LTI: } \boxed{y[n] = \sum_{k=-\infty}^{+\infty} x[k]\ h[n-k]}$$

Convolution Sum

Sinyal waktu kontinu dari kombinasi rectangular pulse



Sinyal waktu kontinu dari kombinasi rectangular pulse

$$x(t) \cong x(0)\,\delta_\Delta(t)\,\Delta + x(\Delta)\,\delta_\Delta(t-\Delta)\,\Delta + x(-\Delta)\,\delta_\Delta(t+\Delta)\,\Delta + \ldots$$

$$x(t) \cong \sum_{k=-\infty}^{+\infty} x(k\Delta)\,\delta_\Delta(t-k\Delta)\,\Delta$$

$$x(t) = \lim_{\Delta \to 0} \sum_{k=-\infty}^{+\infty} x(k\Delta)\,\delta_\Delta(t-k\Delta)\,\Delta$$

$$= \int_{-\infty}^{+\infty} x(\tau)\,\delta(t-\tau)\,d\tau$$

Penurunan konvolusi integral

$$x(t) = \lim_{\Delta \to 0} \sum_{k=-\infty}^{+\infty} x(k\Delta)\, \delta_\Delta(t - k\Delta)\, \Delta$$

Linear System:

$$y(t) = \lim_{\Delta \to 0} \sum_{k=-\infty}^{+\infty} x(k\Delta)\, h_{k\Delta}(t)\, \Delta$$

$$= \int_{-\infty}^{+\infty} x(\tau)\, h_\tau(t)\, d\tau$$

If Time-Invariant:

$$h_{k\Delta}(t) = h_o(t - k\Delta)$$

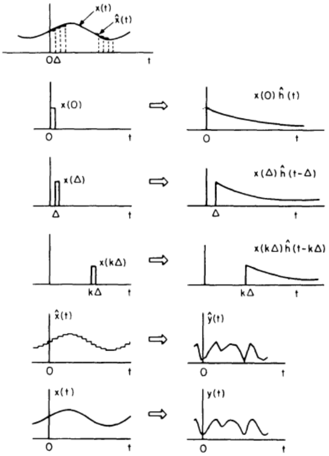
$$h_\tau(t) = h_o(t - \tau)$$

LTI:

$$\boxed{y(t) = \int_{-\infty}^{+\infty} x(\tau)\, h(t - \tau)\, d\tau}$$

Convolution Integral

Interpretasi konvolusi integral



Perbandingan konvolusi penjumlahan dan integral

Convolution Sum:

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k]\,\delta[n-k]$$

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]\,h[n-k] = x[n] * h[n]$$

Convolution Integral:

$$x(t) = \int_{-\infty}^{+\infty} x(\tau)\,\delta(t-\tau)\,d\tau$$

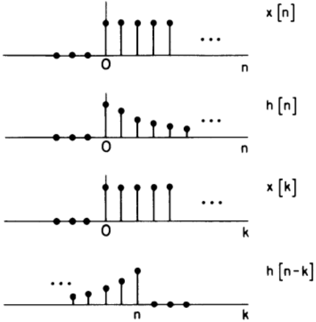
$$y(t) = \int_{-\infty}^{+\infty} x(\tau)\,h(t-\tau)\,d\tau = x(t) * h(t)$$

Evaluasi konvolusi penjumlahan

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]\,h[n-k]$$

$$x[n] = u[n]$$

$$h[n] = \alpha^n u[n]$$

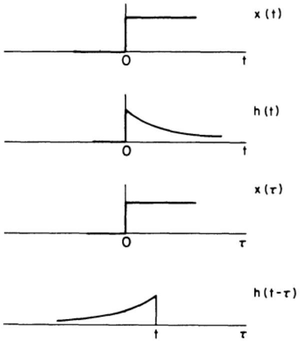


Evaluasi konvolusi integral

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$x(t) = u(t)$

$h(t) = e^{-at}\, u(t)$

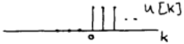


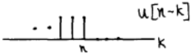
Evaluasi konvolusi integral

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]\, h[n-k]$$

$$= \sum_{k=-\infty}^{+\infty} u[k]\, \alpha^{n-k} u[n-k]$$

Interval 1: $n<0$

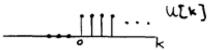


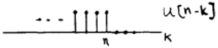


No overlap $\Longrightarrow$

$y[n]=0 \quad n<0$

Interval 2: $n>0$





overlap for $k=0,1,\ldots n$

$$y[n] = \sum_{k=0}^{n} \alpha^{n-k}$$

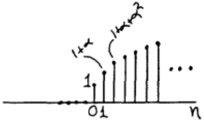
$$= \alpha^{n} \sum_{k=0}^{n} (\alpha^{-1})^{k}$$

$$\sum_{k=0}^{r} \beta^{k} = \frac{1-\beta^{r+1}}{1-\beta}$$

$$y[n] = \alpha^{n} \underbrace{\sum_{k=0}^{n} (\alpha^{-1})^{k}}_{\frac{1-(\alpha^{-1})^{n+1}}{1-\alpha^{-1}}}$$

$\vdots$

$$y[n] = \frac{1-\alpha^{n+1}}{1-\alpha} \qquad n>0$$



Evaluasi konvolusi integral

$$y(t)=\int_{-\infty}^{+\infty} x(\tau)\,h(t-\tau)\,d\tau$$

$$=\int_{-\infty}^{+\infty} u(\tau)\,e^{-a(t-\tau)}\,u(t-\tau)\,d\tau$$

Interval 1: $t<0$

No overlap between

$u(\tau)$ & $u(t-\tau) \Longrightarrow$

$y(t)=0 \quad t<0$

Interval 2: $t>0$

$$y(t)=\int_{-\infty}^{+\infty} u(\tau)\,e^{-a(t-\tau)}\,u(t-\tau)\,d\tau$$

$u(\tau)u(t-\tau) = 1$

for $\quad 0\leq\tau\leq t$

$$y(t)=\int_{0}^{t} e^{-a(t-\tau)}\,d\tau$$

$$=e^{-at}\underbrace{\int_{0}^{t} e^{a\tau}\,d\tau}_{\frac{1}{a}\left[e^{at}-1\right]}$$

$$=\frac{1}{a}\left[1-e^{-at}\right] \quad t>0$$

$$y(t)=\begin{cases} 0 & t<0 \\ \frac{1}{a}\left[1-e^{-at}\right] & t>0 \end{cases}$$