TE201416: SINYAL DAN SISTEM

SINYAL

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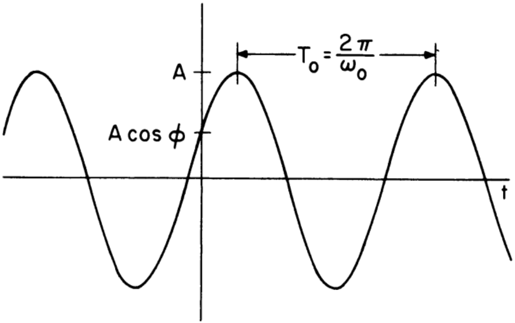
Sinyal sinusoidal waktu kontinu

$$x(t) = A\cos(\omega_0 t + \phi)$$



Sinyal sinusoidal waktu kontinu

- **Periodic:**

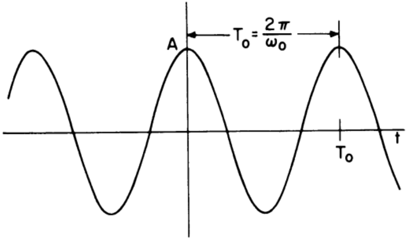
$$x(t) = x(t + T_o) \qquad \text{period} \triangleq \text{smallest } T_o$$

$$A\cos[\omega_o t + \phi] = A\cos[\omega_o t + \underbrace{\omega_o T_o}_{2\pi m} + \phi]$$

$$T_o = \frac{2\pi m}{\omega_o} \Rightarrow \text{period} = \frac{2\pi}{\omega_o}$$

Sinyal sinusoidal waktu kontinu

$\phi = 0 \qquad x(t) = A\cos\omega_0 t$

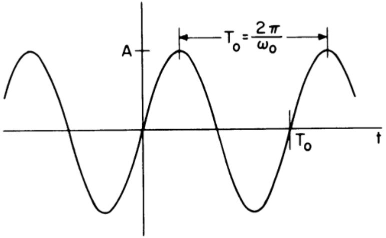


Periodic: $x(t) = x(t + T_0)$

Even: $x(t) = x(-t)$

Sinyal sinusoidal waktu kontinu

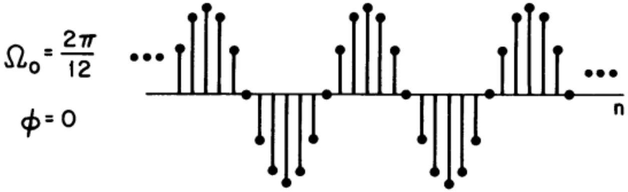
$$\phi = -\frac{\pi}{2} \qquad x(t) = \begin{cases} A\cos(\omega_0 t - \frac{\pi}{2}) \\ A\sin \omega_0 t \\ A\cos[\omega_0(t - \frac{T_0}{4})] \end{cases}$$

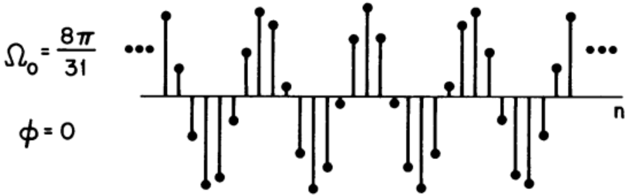


Periodic: $x(t) = x(t + T_0)$

Odd: $x(t) = -x(-t)$

Sinyal sinusoidal waktu diskret





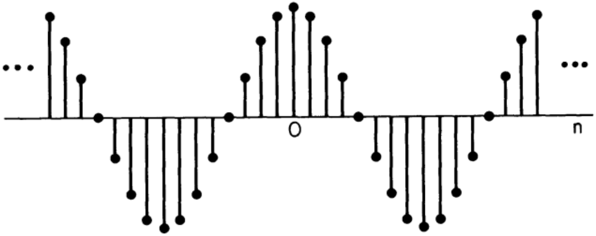
Sinyal sinusoidal waktu diskret

Time Shift => Phase Change

$$A \cos [\Omega_o(n + n_o)] = A \cos [\Omega_o n + \Omega_o n_o]$$

Sinyal sinusoidal waktu diskret

$\phi = 0 \qquad x[n] = A \cos \Omega_0 n$



$\Omega_0 = \frac{\pi}{8}$

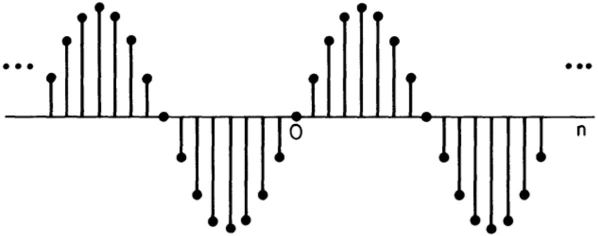
even: $\qquad x[n] = x[-n]$

Sinyal sinusoidal waktu diskret

$\phi = -\frac{\pi}{2}$

$$x[n] = \begin{cases} A\cos(\Omega_o n - \frac{\pi}{2}) \\ A\sin \Omega_o n \\ A\cos[\Omega_o(n - n_o)] \end{cases}$$

$n_o = ?$



$\Omega_o = \frac{\pi}{8}$

odd: $x[n] = -x[-n]$

Sinyal sinusoidal waktu diskret

Time Shift => Phase Change

$$A \cos\left[\Omega_o(n + n_o)\right] = A \cos\left[\Omega_o n + \Omega_o n_o\right]$$

Time Shift $\stackrel{?}{<=}$ Phase Change

$$A \cos\left[\Omega_o(n + n_o)\right] \stackrel{?}{=} A \cos\left[\Omega_o n + \phi\right]$$

Sinyal sinusoidal waktu diskret

$$x[n] = A\cos(\Omega_o n + \phi)$$

Periodic?

$x[n] = x[n+N]$ smallest integer $N \triangleq$ period

$$A\cos[\Omega_o(n+N)+\phi] = A\cos[\Omega_o n + \underbrace{\Omega_o N}_{} + \phi]$$

integer multiple of 2π ?

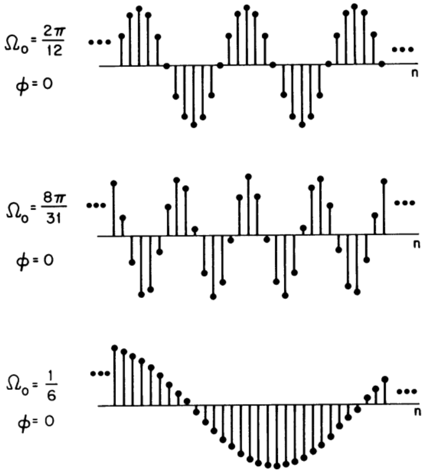
Periodic $=> \Omega_o N = 2\pi m$

$$N = \frac{2\pi m}{\Omega_o}$$

N,m must be integers

smallest N (if any) = period

Sinyal sinusoidal waktu diskret



Sinyal sinusoidal waktu diskret

$A\cos(\omega_0 t+\phi)$	$A\cos(\Omega_0 n+\phi)$
Distinct signals for distinct values of ω_0	Identical signals for values of Ω_0 separated by 2π
Periodic for any choice of ω_0	Periodic only if $\Omega_0 = \frac{2\pi m}{N}$ for some integers $N > 0$ and m

Sinyal sinusoidal saat frekuensinya berbeda

Continuous time:

$$x_1(t) = A\cos(\omega_1 t + \phi) \quad \text{If} \quad \omega_2 \neq \omega_1$$

$$x_2(t) = A\cos(\omega_2 t + \phi) \quad \text{Then } x_2(t) \neq x_1(t)$$

Discrete time:

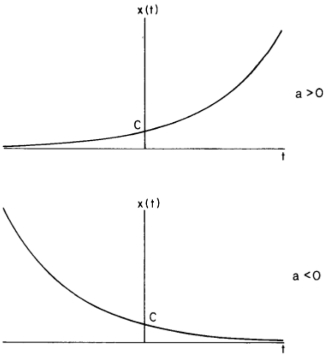
$$x_1[n] = A\cos[\Omega_1 n + \phi] \quad \text{If} \quad \Omega_2 = \Omega_1 + 2\pi m$$

$$x_2[n] = A\cos[\Omega_2 n + \phi] \quad \text{Then } x_2[n] = x_1[n]$$

Sinyal eksponensial riil waktu kontinu

$x(t) = Ce^{at}$

C and a are real numbers



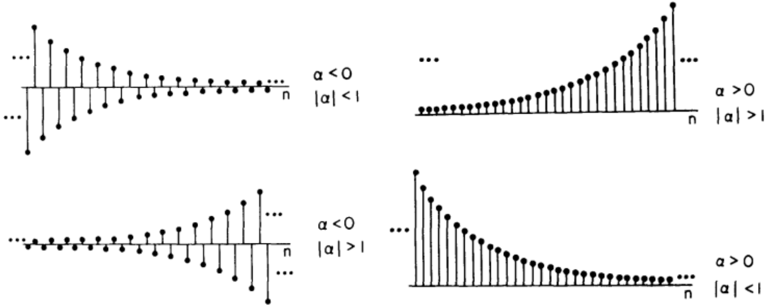
Time Shift <=> Scale Change

$Ce^{a(t+t_0)} = Ce^{at_0}e^{at}$

Sinyal eksponensial riil waktu diskret

$$x[n] = Ce^{\beta n} = C\alpha^n$$

C, α are real numbers



Sinyal eksponensial kompleks waktu kontinu

$$x(t) = Ce^{at}$$

C and a are complex numbers

$$C = |C|\, e^{j\theta}$$

$$a = r + j\omega_o$$

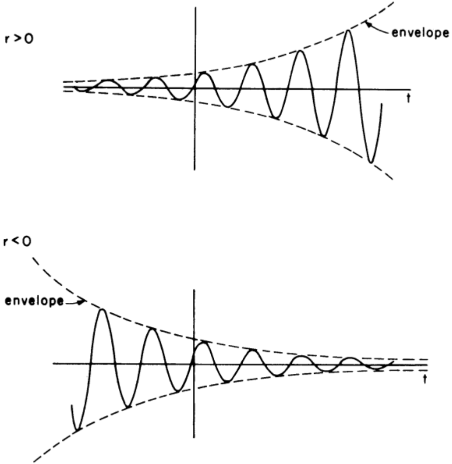
$$x(t) = |C|\, e^{j\theta}\, e^{(r + j\omega_o)t}$$

$$= |C|\, e^{rt}\, \underbrace{e^{j(\omega_o t + \theta)}}$$

Euler's Relation: $\cos(\omega_o t + \theta) + j\sin(\omega_o t + \theta) = e^{j(\omega_o t + \theta)}$

$$x(t) = |C|\, e^{rt} \cos(\omega_o t + \theta) + j\,|C|\, e^{rt} \sin(\omega_o t + \theta)$$

Sinyal eksponensial kompleks waktu kontinu



Sinyal eksponensial kompleks waktu diskret

$$x[n] = C\alpha^n$$

C and α are complex numbers

$$C = |C|\, e^{j\theta}$$

$$\alpha = |\alpha|\, e^{j\Omega_o}$$

$$x[n] = |C|\, e^{j\theta} \left(|\alpha|\, e^{j\Omega_o}\right)^n$$

$$= |C|\, |\alpha|^n \underbrace{e^{j(\Omega_o n + \theta)}}$$

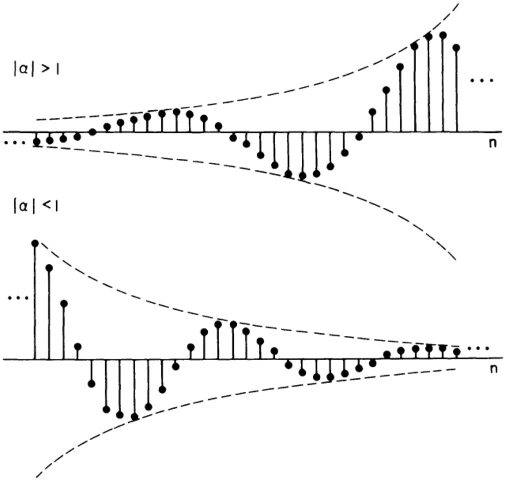
Euler's Relation: $\cos(\Omega_o n + \theta) + j \sin(\Omega_o n + \theta)$

$$x[n] = |C|\, |\alpha|^n \cos(\Omega_o n + \theta) + j\, |C|\, |\alpha|^n \sin(\Omega_o n + \theta)$$

$|\alpha| = 1$ => sinusoidal real and imaginary parts

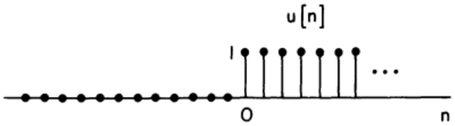
$C e^{j\Omega_o n}$ periodic ?

Sinyal eksponensial kompleks waktu diskret



Unit Step & Unit Impulse

UNIT STEP FUNCTION: DISCRETE-TIME

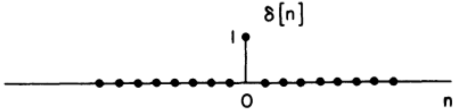


$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

UNIT IMPULSE FUNCTION: DISCRETE-TIME

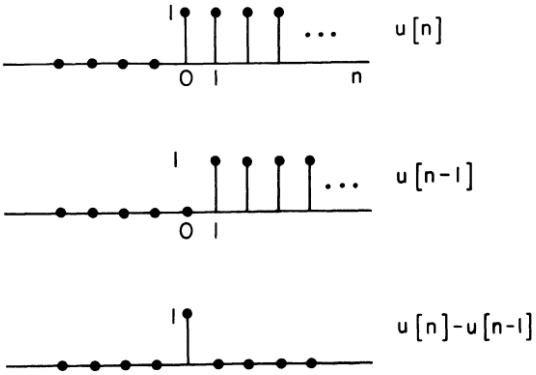
(Unit Sample)

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$



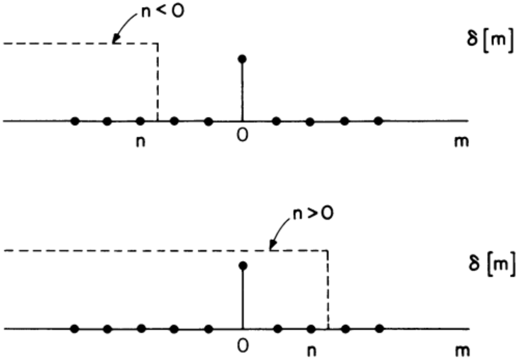
Unit Impulse Sequence

$$\delta[n] = u[n] - u[n-1]$$



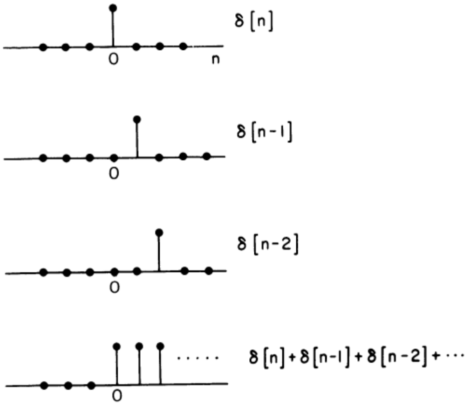
Unit Step Sequence

$$u[n] = \sum_{m=-\infty}^{n} \delta[m]$$



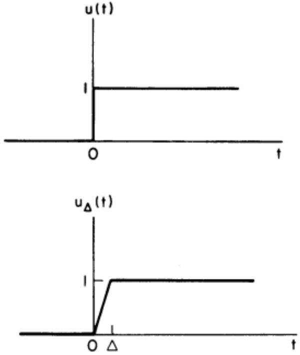
Unit Step Sequence

$$u[n] = \sum_{k=0}^{\infty} \delta[n-k]$$



Unit Step Function Waktu Kontinu

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$



$u(t) = u_\Delta(t)$ as $\Delta \to 0$

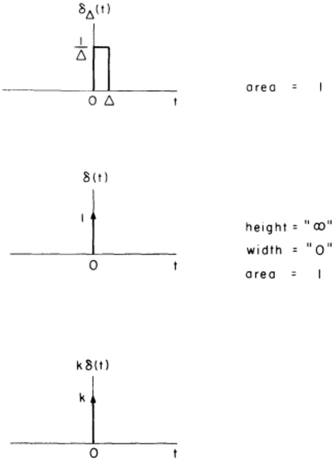
Unit Impulse Function

$$\delta(t) = \frac{du(t)}{dt}$$

$$\delta_{\triangle}(t) = \frac{du_{\triangle}(t)}{dt}$$

$$\delta(t) = \delta_{\triangle}(t) \text{ as } \triangle \to 0$$

Unit Impulse Waktu Kontinu



Unit Step Waktu Kontinu

$$\delta(t) = \frac{du(t)}{dt}$$

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$$