TE201416: SINYAL DAN SISTEM

SINYAL

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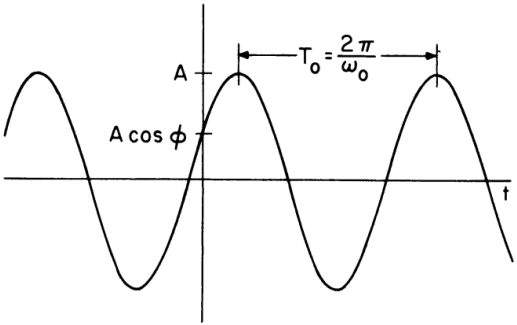
Bahan Kajian

1. Sinyal sinusoidal waktu kontinu
2. Sinyal sinusoidal waktu diskret
3. Sinyal sinusoidal saat frekuensinya berbeda
4. Sinyal eksponensial riil waktu kontinu
5. Sinyal eksponensial riil waktu diskret
6. Sinyal eksponensial kompleks waktu kontinu
7. Sinyal eksponensial kompleks waktu diskret

Sinyal sinusoidal waktu kontinu

$$x(t) = A\cos(\omega_o t + \phi)$$



Sinyal sinusoidal waktu kontinu

- **Periodic:**

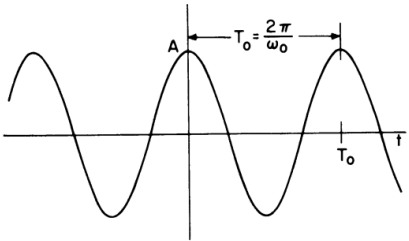
$$x(t) = x(t + T_o) \qquad \text{period} \triangleq \text{smallest } T_o$$

$$A\cos[\omega_o t + \phi] = A\cos[\omega_o t + \underbrace{\omega_o T_o}_{2\pi m} + \phi]$$

$$T_o = \frac{2\pi m}{\omega_o} \Rightarrow \text{period} = \frac{2\pi}{\omega_o}$$

Sinyal sinusoidal waktu kontinu

$\phi = 0$ $\quad x(t) = A \cos \omega_o t$

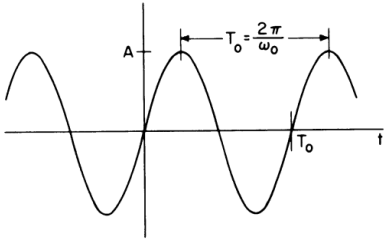


Periodic: $x(t) = x(t + T_o)$

Even: $x(t) = x(-t)$

Sinyal sinusoidal waktu kontinu

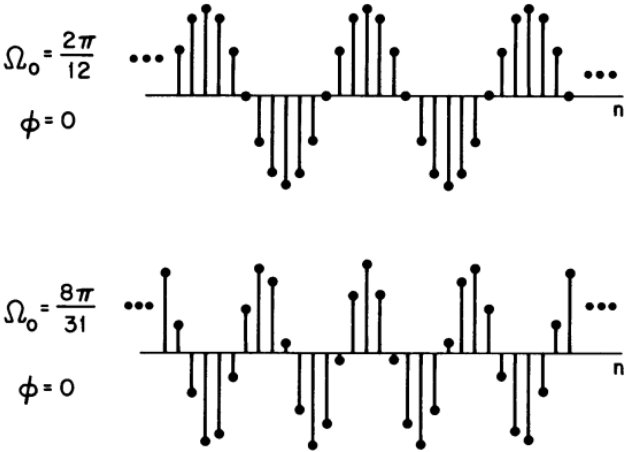
$$\phi = -\frac{\pi}{2} \qquad x(t) = \begin{cases} A\cos(\omega_o t - \frac{\pi}{2}) \\ A\sin\omega_o t \\ A\cos[\omega_o(t - \frac{T_o}{4})] \end{cases}$$



Periodic: $x(t) = x(t + T_o)$

Odd: $x(t) = -x(-t)$

Sinyal sinusoidal waktu diskret



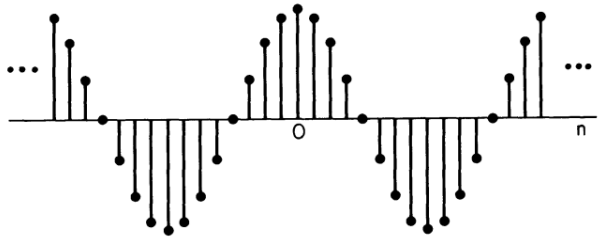
Sinyal sinusoidal waktu diskret

Time Shift => Phase Change

$$A \cos [\Omega_o(n + n_o)] = A \cos [\Omega_o n + \Omega_o n_o]$$

Sinyal sinusoidal waktu diskret

$\phi = 0$ $\quad x[n] = A \cos \Omega_o n$



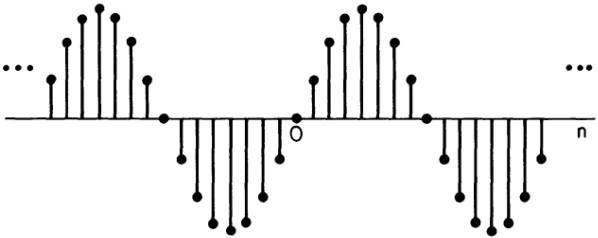
$\Omega_o = \frac{\pi}{8}$

even: $\quad x[n] = x[-n]$

Sinyal sinusoidal waktu diskret

$$\phi = -\frac{\pi}{2} \qquad x[n] = \begin{cases} A\cos\left(\Omega_o n - \frac{\pi}{2}\right) \\ A\sin \Omega_o n \\ A\cos\left[\Omega_o (n - n_o)\right] \end{cases}$$

$n_o = ?$



$\Omega_o = \frac{\pi}{8}$

odd: $x[n] = -x[-n]$

Sinyal sinusoidal waktu diskret

Time Shift => Phase Change

$$A \cos [\Omega_o(n + n_o)] = A \cos [\Omega_o n + \Omega_o n_o]$$

Time Shift $\overset{?}{<=}$ Phase Change

$$A \cos [\Omega_o(n + n_o)] \overset{?}{=} A \cos [\Omega_o n + \phi]$$

Sinyal sinusoidal waktu diskret

$$x[n] = A \cos(\Omega_o n + \phi)$$

Periodic?

$x[n] = x[n + N]$ smallest integer $N \triangleq$ period

$$A \cos[\Omega_o(n + N) + \phi] = A \cos[\Omega_o n + \underbrace{\Omega_o N + \phi}]$$

integer multiple of 2π ?

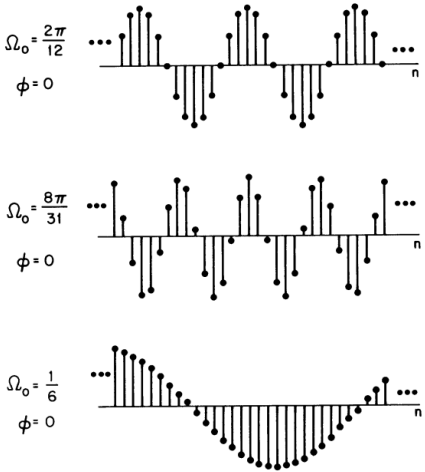
Periodic $=>$ $\Omega_o N = 2\pi m$

$$N = \frac{2\pi m}{\Omega_o}$$

N,m must be integers

smallest N (if any) = period

Sinyal sinusoidal waktu diskret



Sinyal sinusoidal waktu diskret

$A\cos(\omega_o t + \phi)$	$A\cos(\Omega_o n + \phi)$
Distinct signals for distinct values of ω_o	Identical signals for values of Ω_o separated by 2π
Periodic for any choice of ω_o	Periodic only if $\Omega_o = \frac{2\pi m}{N}$ for some integers $N > 0$ and m

Sinyal sinusoidal saat frekuensinya berbeda

Continuous time:

$$x_1(t) = A\cos(\omega_1 t + \phi) \quad \text{If} \quad \omega_2 \neq \omega_1$$

$$x_2(t) = A\cos(\omega_2 t + \phi) \quad \text{Then } x_2(t) \neq x_1(t)$$

Discrete time:

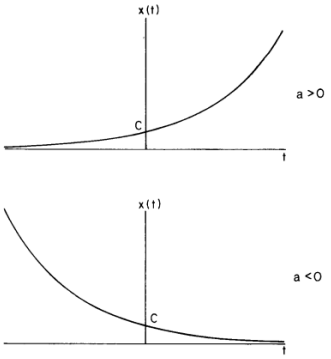
$$x_1[n] = A\cos[\Omega_1 n + \phi] \quad \text{If} \quad \Omega_2 = \Omega_1 + 2\pi m$$

$$x_2[n] = A\cos[\Omega_2 n + \phi] \quad \text{Then } x_2[n] = x_1[n]$$

Sinyal eksponensial riil waktu kontinu

$x(t) = Ce^{at}$

C and a are real numbers



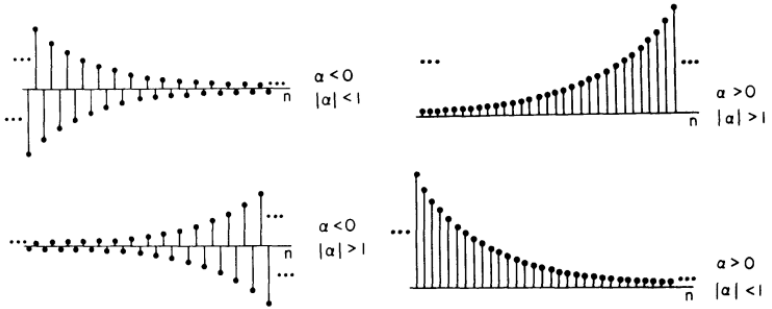
Time Shift <=> Scale Change

$Ce^{a(t + t_0)} = Ce^{at_0} e^{at}$

Sinyal eksponensial riil waktu diskret

$$x[n] = Ce^{\beta n} = C\alpha^{n}$$

C, α are real numbers



Sinyal eksponensial kompleks waktu kontinu

$$x(t) = Ce^{at}$$

C and a are complex numbers

$$C = |C|\, e^{j\theta}$$

$$a = r + j\omega_o$$

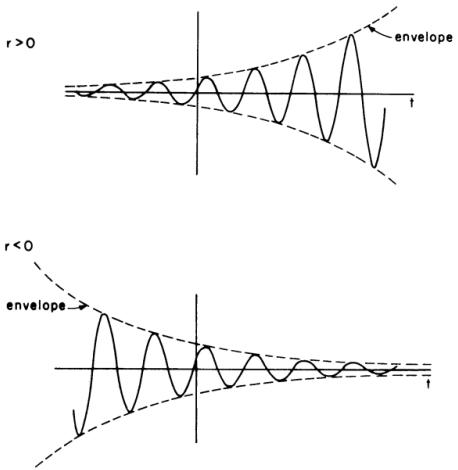
$$x(t) = |C|\, e^{j\theta}\, e^{(r + j\omega_o)t}$$

$$= |C|\, e^{rt}\, e^{j(\omega_o t + \theta)}$$

Euler's Relation: $\cos(\omega_o t + \theta) + j \sin(\omega_o t + \theta) = e^{j(\omega_o t + \theta)}$

$$x(t) = |C|\, e^{rt} \cos(\omega_o t + \theta) + j\, |C|\, e^{rt} \sin(\omega_o t + \theta)$$

Sinyal eksponensial kompleks waktu kontinu



Sinyal eksponensial kompleks waktu diskret

$$x[n] = C\alpha^n$$

C and α are complex numbers

$$C = |C|\, e^{j\theta}$$

$$\alpha = |\alpha|\, e^{j\Omega_o}$$

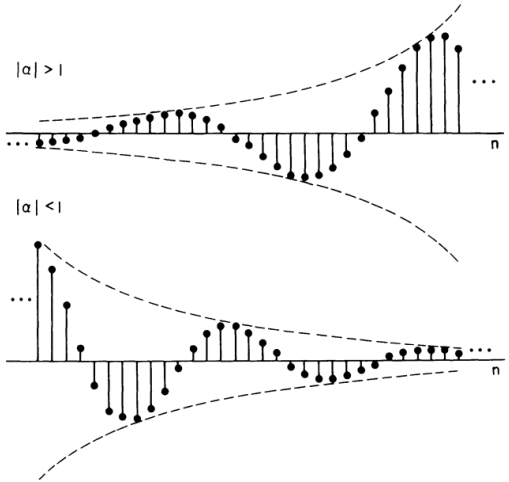
$$x[n] = |C|\, e^{j\theta}\, (|\alpha|\, e^{j\Omega_o})^n$$

$$= |C|\, |\alpha|^n\, e^{j(\Omega_o n + \theta)}$$

Euler's Relation: $\cos(\Omega_o n + \theta) + j \sin(\Omega_o n + \theta)$

$$x[n] = |C|\, |\alpha|^n \cos(\Omega_o n + \theta) + j\, |C|\, |\alpha|^n \sin(\Omega_o n + \theta)$$

$|\alpha| = 1 \Rightarrow$ sinusoidal real and imaginary parts

$Ce^{j\Omega_o n}$ periodic ?

Sinyal eksponensial kompleks waktu diskret