Fall Semester, 2025 **Instructor**: M.J. DeJong
CE 225 – Dynamics of Structures

Homework #4

Due: Monday, September 29

1. (a) Show that the motion of an undamped system starting from rest due to a suddenly applied force p_o that decays exponentially with time (Fig. 1) is:

$$\frac{u(t)}{(u_{\mathrm{st}})_o} = \frac{1}{1 + a^2/\omega_n^2}\left[\frac{a}{\omega_n}\sin\omega_n t - \cos\omega_n t + e^{-at}\right]$$

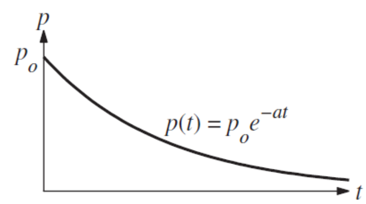
Notes:
- You may solve this using whatever method you prefer.
- The variable a has the same units as ω_n.
- I strongly suggest that you do this derivation without using AI. You will not have AI on the exam.

(b) Plot this motion for a/ω_n = 0.02, 0.2, and 2.0.

(c) Show that the amplitude for large values of t is:

$$\frac{u_o}{(u_{\mathrm{st}})_o} = \frac{1}{\sqrt{1 + a^2/\omega_n^2}}$$

Figure 1:



2. An elevator is idealized as a weight of mass m supported by a spring of stiffness k. If the upper end of the spring begins to move upward with a constant velocity v, show that the total distance u^t that the mass has risen in time t is governed by the equation:

$$m\ddot{u}^t + ku^t = kvt$$

If the elevator starts from rest, show that the motion is: $u^t(t) = vt - \dfrac{v}{\omega_n}\sin\omega_n t$

Sketch a plot of this result. I suggest you try sketching by hand first, and then check your work by plotting.

3. Using Duhamel's integral, determine the response $u(t)$ of an undamped SDOF system to a rectangular pulse force of amplitude p_o and duration t_d . You should have one solution for $u(t)$ for $t < t_d$, and another solution for $t > t_d$.

4. The elevated water tank of Fig. 2 weighs 360 kips when full with water. The tower has a lateral stiffness of 10 kips/in. Treating the water tower as an SDOF system, estimate the maximum lateral displacement due to each of the two dynamic forces shown without any “exact” dynamic analysis. Instead, use your understanding of how the maximum response depends on the ratio of the rise time of the applied force to the natural vibration period of the system; neglect damping.

Figure 2: