CE 225: Dynamic of Structures **Fall 2025**

Discussion 1: Equations of Motion and Free Vibration

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1 Logistics

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- Office hours: Wed 3-4pm & Fri 11am-12pm, 504 Davis
- Homework: **bCourses** → **Files**
- Submission: **bCourses** → **Gradescope**
- Class recording: **bCourses** → **Media Gallery**

2 Objective

The purpose of the discussion section is to go over selected topics that accompany but may not have been covered in details during lecture. We may go over problems that are tangentially related to, but not direct copies of, homework assignment problems.

3 Summary of basic concepts

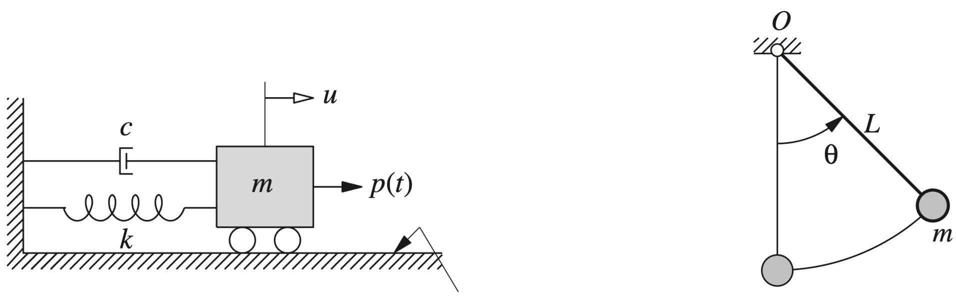


Figure 3.1: Left: Mass-spring-damper system. Right: Simple pendulum.

(a) Inertial forces

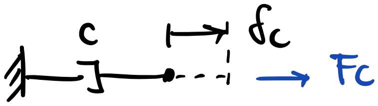
Translational Motion

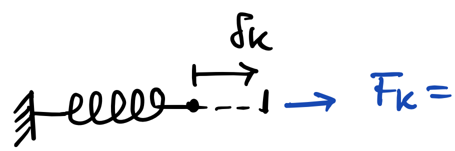
$$f_I = m\ddot{u}_t$$

Rotational Motion

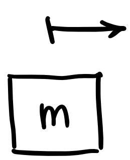
$$M_I = J\cdot\ddot{\theta}$$

(b) Damping and spring forces





(c) Free body diagram



(d) Equation of motion

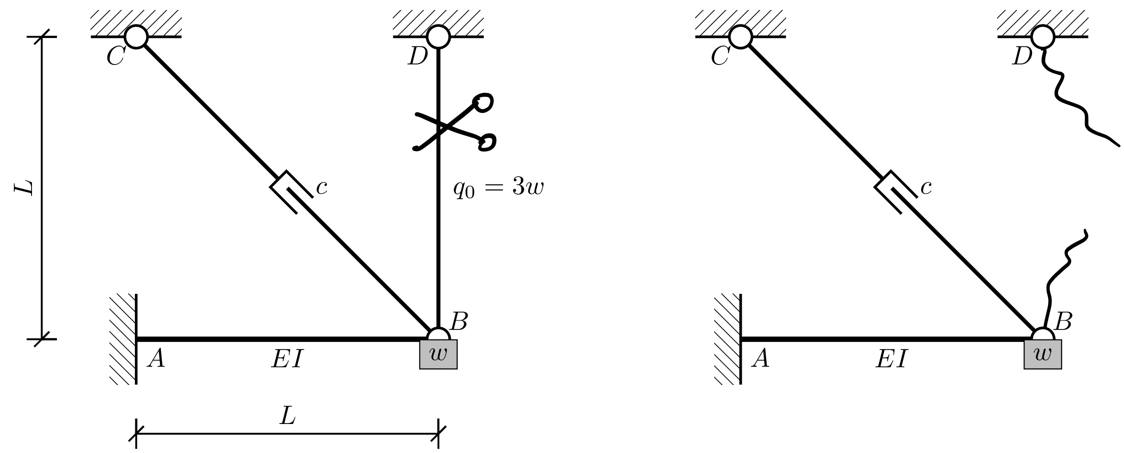
$$\boxed{m\ddot{u} + c\dot{u} + ku = p(t)}$$

Divide by m $\quad \ddot{u} + \frac{c}{m}\cdot\dot{u} + \frac{k}{m}\cdot u = p(t)/m$

Define $\frac{c}{m} = 2\zeta\omega_n$ and $\frac{k}{m} = \omega_n^2$

$$\boxed{\ddot{u} + 2\zeta\omega_n\dot{u} + \omega_n^2 u = p(t)/m}$$

Example 1: The structure shown below consists of a cantilever beam, with a point mass of weight w, attached to its free end. The mass is also attached to a viscous damper of constant c, which is placed in the configuration indicated in the Figure. The structure is initially at rest, connected to a vertical cable, which



is pre-stressed with a force equal to 3 times the weight of the point mass $q_0 = 3w$.

Starting from this position, in an instant, the cable is cut, and the system starts vibrating.

a) Determine the equation of motion for the free vibration of the system.

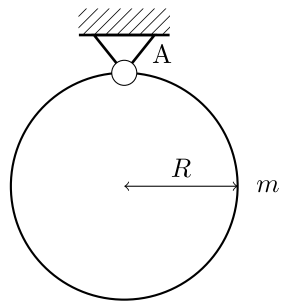
b) Solve the equation of motion, using the provided initial conditions.

c) If after 3 full cycles the amplitude of the motion is 0.5 times what it was initially, determine an expression for c.

Example 1, continued...

Example 2: A uniform ring with mass m and radius R is connected to the ceiling with a pin at point A. As shown in the lecture, the equation of motion of the system is:

$$(mR^2 + J_c)\ddot{\theta} + mgR\sin(\theta) = 0$$



Take $R = 10$ in, $g = 386$ in/s^2 and:

a) Compute the natural frequency ω_n of the ring.

b) A small rock with mass $2m$ is thrown towards the ring in horizontal direction with a velocity of $v_0 = 10$ in/s. The impact happens at the center of the ring, at a vertical distance R from point A. Right after the impact, the rock's horizontal component of the velocity is zero, and it just falls to the ground. Write the equation that describes the rotation of the ring at time t after the impact. Assume no damping.

c) (Extra) Solve (2) again, but now considering that the ring has a damping ratio of 2% of the critical damping.

Example 2, continued...

4 Free vibration

Free vibration (with $\zeta < 1$):

$$\boxed{u(t) = e^{-\zeta\omega_n t}\left[A\cos(\omega_D t) + B\sin(\omega_D t)\right]} \qquad \omega_D = \omega_n\sqrt{1-\zeta^2}$$

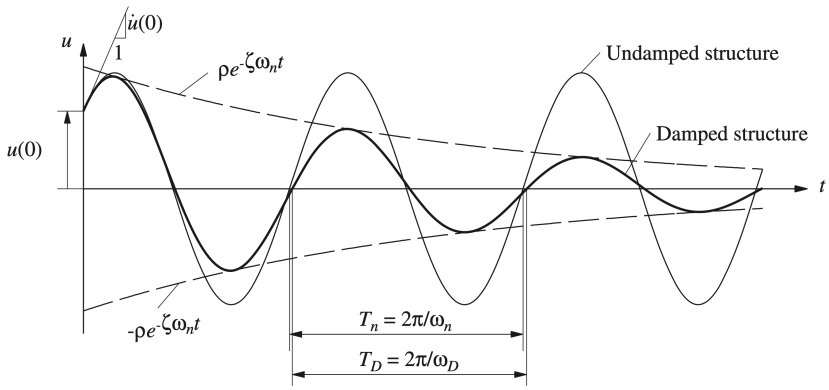


Figure 4.2: Effects of damping on free vibration

5 Rotational Inertia

Rotational Inertia is the counterpart of **mass** when an object is subject to rotational motion instead of translational motion. The rotational inertia is a scalar value that describes the resistance to change of rotational (angular) velocity of an object around a given axis.

Fig. 5.3 shows the rotational inertia for some simple shapes.

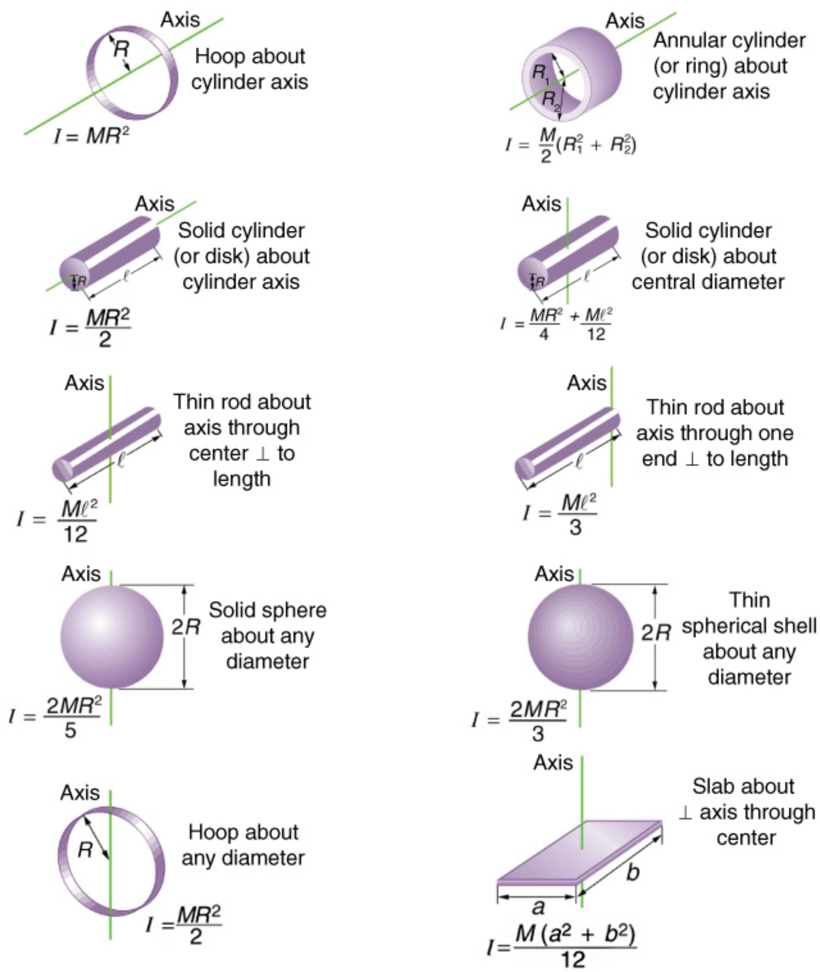


Figure 5.3: Rotational inertia of some simple shapes under rotation

The parallel axis theorem (Eq. 5.1) allows us to find the rotational inertia of an object about a point o as long as we know the rotational inertia of the shape around its centroid c, mass m and distance d between points o and c.

$$I_o = I_c + md^2 \tag{5.1}$$