LECTURE 15 - GENERALIZED SDOF SYSTEMS (PART 2)

CE 225

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RAYLEIGH'S QUOTIENT

Rayleigh's Quotient: $$\omega_n^2 = \frac{\widetilde{k}}{\widetilde{m}} = \frac{\int_0^L EI[\psi''(x)]^2 dx}{\int_0^L m(x)[\psi(x)]^2 dx}$$

Two Properties:

- Actual $\omega_n \leq$ approximation using $\psi(x)$ (assuming geometric BC's are entered).
- ω_n is relatively insensitive to shape function (for reasonable shapes).

RESPONSE TO GROUND MOTION

$$F_{eq} = \widetilde{p} = \int_0^L p(x,t)\psi(x)dx$$

EOM for response of $\psi(x)$ to ground motion:

RESPONSE TO GROUND MOTION

CHIMNEY EXAMPLE

**Design chimney using the design spectrum in Fig. 6.9.5

Given:

$A = 373\ ft^3$ (cross-section)

$m = 1.75\ \text{kip} - \text{s}^2/\text{ft}^2$

$I = 105,000\ \text{ft}^4$

$EI = 5.47 * 10^{10}\ \text{k} - \text{ft}^2$

$\zeta = 5\%$

$PGA = 0.4g$

(i) Find ω_n

Pick static deflected shape. For this example: $\psi(x) = 2\left(\frac{x}{L}\right)^2 - \frac{4}{3}\left(\frac{x}{L}\right)^3 + \frac{1}{3}\left(\frac{x}{L}\right)^4$

RESPONSE TO GROUND MOTION

CHIMNEY EXAMPLE

(ii) Find peak response of $z(t)$

From Graph $\rightarrow$

(6.9.5)

(Note: Response spectrum is derived for $\ddot{u} + 2\zeta\omega_n\dot{u} + \omega_n^2 u = -\ddot{u}_g$)

RESPONSE TO GROUND MOTION

CHIMNEY EXAMPLE

(iii) Find peak response over the height

(iv) Find equivalent static force for the design

RESPONSE TO GROUND MOTION

CHIMNEY EXAMPLE

(v) Find base shear V_{base}

(vi) Find base overturning moment

GENERALIZED SDOF SYSTEMS

EXAMPLE: LUMPED MASS PLUS DISTRIBUTED MASS

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