LECTURE 12 - EARTHQUAKE RESPONSE OF INELASTIC SYSTEMS

CE 225

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IDEALIZED INELASTIC RESPONSE

ELASTIC - PLASTIC RESPONSE

EQUATION OF MOTION

Define: $z(t) =$ "elastic portion" of $u(t)$

$U(x) =$ heaviside function: $\begin{cases} x > 0 \rightarrow u(x) = 1 \\ x < 0 \rightarrow u(x) = 0 \end{cases}$

$$\longrightarrow \begin{cases} \dot{u} > 0 \text{ and } (z - u_y) > 0 \rightarrow \dot{z}(t) = 0 \\ \dot{u} < 0 \text{ and } (-z - u_y) > 0 \rightarrow \dot{z}(t) = 0 \\ \text{Otherwise } \dot{z}(t) = \dot{u}(t) = \text{elastic deformation.} \end{cases}$$

ELASTIC - PLASTIC RESPONSE

EXAMPLE RESULT #1

ELASTIC - PLASTIC RESPONSE

EXAMPLE RESULT #2

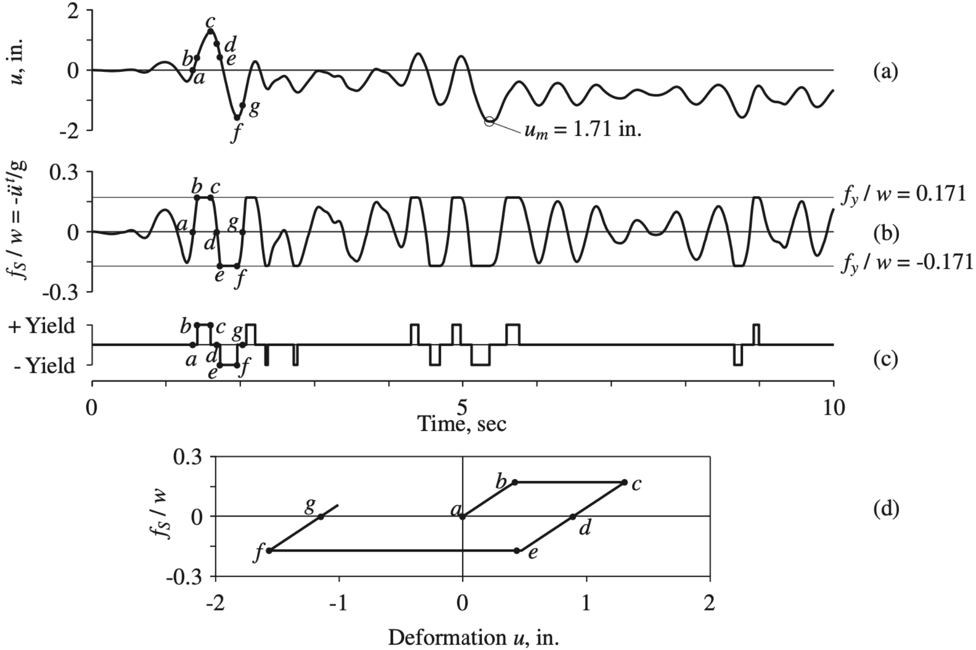

Figure 7.4.2 Response of elastoplastic system with $T_n = 0.5$ sec, $\zeta = 0$, and $\overline{f}_y = 0.125$ to El Centro ground motion: (a) deformation; (b) resisting force and acceleration; (c) time intervals of yielding; (d) force–deformation relation.

TRADE-OFF BETWEEN YIELD STRENGTH f_y AND DUCTILITY μ

All structures are designed for damage in the Maximum Considered Earthquake (MCE) event!

How much can we reduce the design yield strength f_y based on the ductility μ?

Define "Normalized yield strength":

Also, define "yield strength reduction factor":

For design: Take f_{So} and divide by R_y $\rightarrow$ $\boxed{f_y = \dfrac{f_{So}}{R_y}}$ $=$ "design strength"

$\rightarrow$ Then find u_m $\rightarrow$ Is it ok?

TRADE-OFF BETWEEN YIELD STRENGTH f_y AND DUCTILITY μ

EXAMPLE RESPONSE

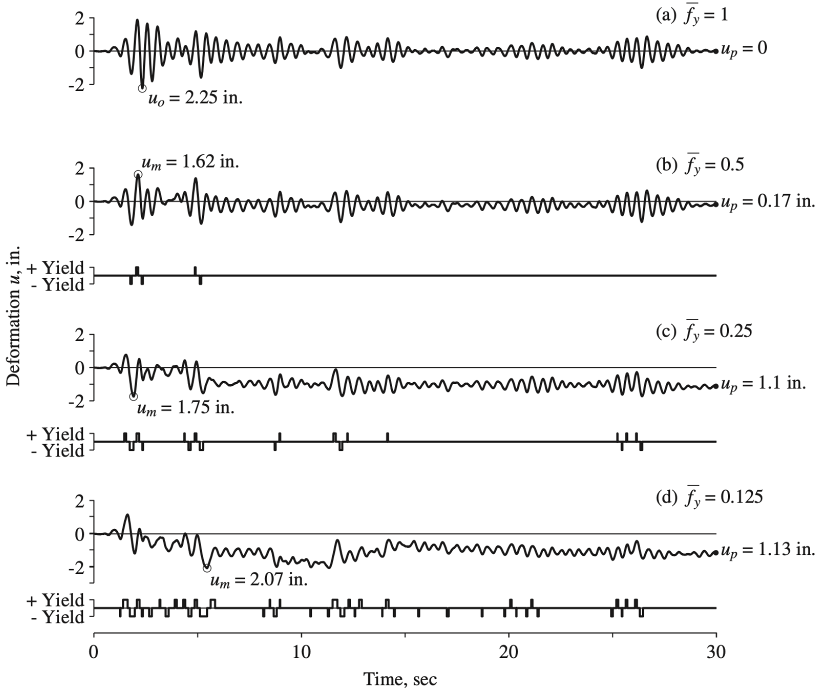

Figure 7.4.3 Deformation response and yielding of four systems due to El Centro ground motion; $T_n = 0.5$ sec, $\zeta = 5\%$; and $\bar{f}_y = 1$, 0.5, 0.25, and 0.125.

TRADE-OFF BETWEEN YIELD STRENGTH f_y AND DUCTILITY μ

EXAMPLE RESPONSE

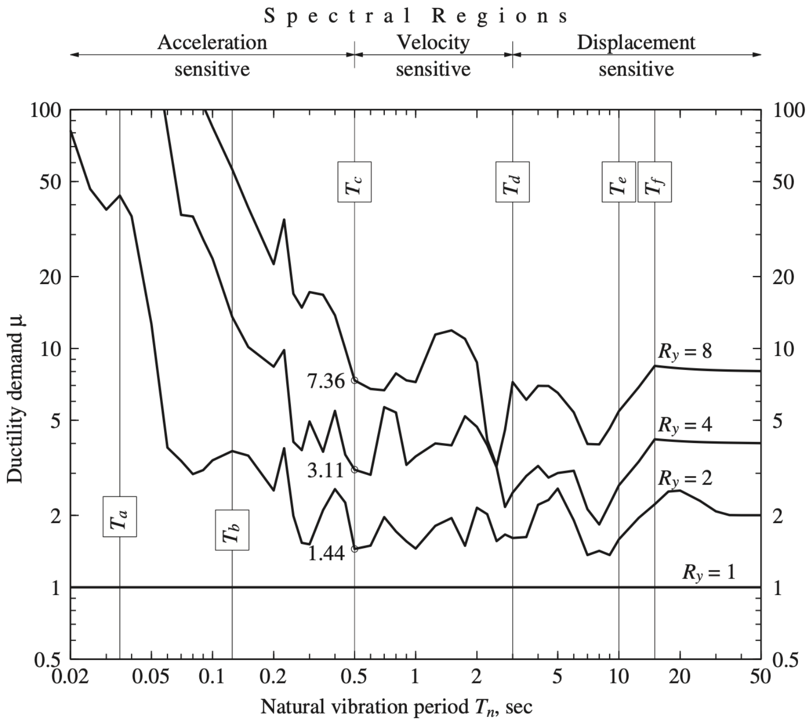

Figure 7.4.5 Ductility demand for elastoplastic system due to El Centro ground motion; $\zeta = 5\%$ and $\overline{f}_y = 1, 0.5, 0.25,$ and 0.125, or $R_y = 1, 2, 4,$ and 8.

Trade-off Between Yield Strength f_y and Ductility μ

Alternatively

Specify ductility, $\mu \rightarrow$ find required design strength, f_y

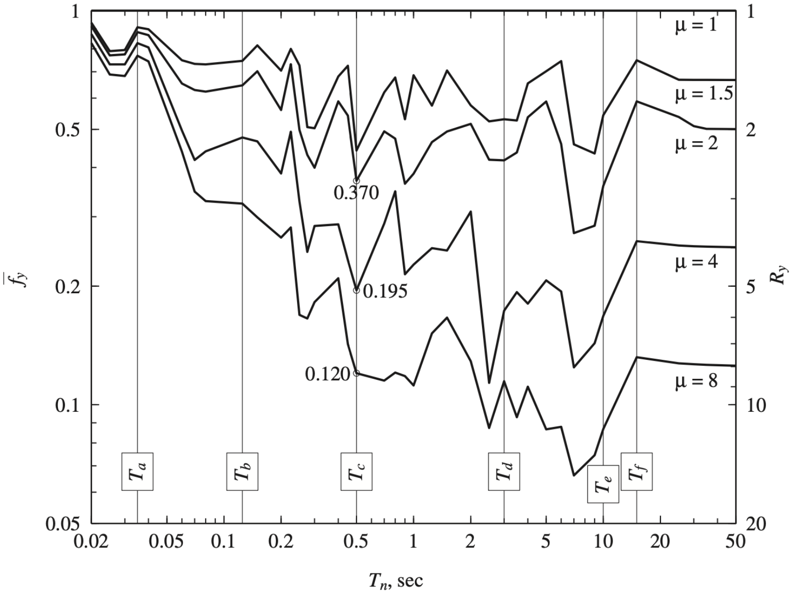

Figure 7.7.1 Normalized strength $\overline{f}_y$ of elastoplastic systems as a function of natural vibration period T_n for $\mu = 1, 1.5, 2, 4,$ and 8; $\zeta = 5\%$; El Centro ground motion.

TRADE-OFF BETWEEN YIELD STRENGTH f_y AND DUCTILITY μ

ALTERNATIVELY

Can also plot yield acceleration response spectra for different values of ductility:

Define: $A_y = \omega_n^2 D_y$

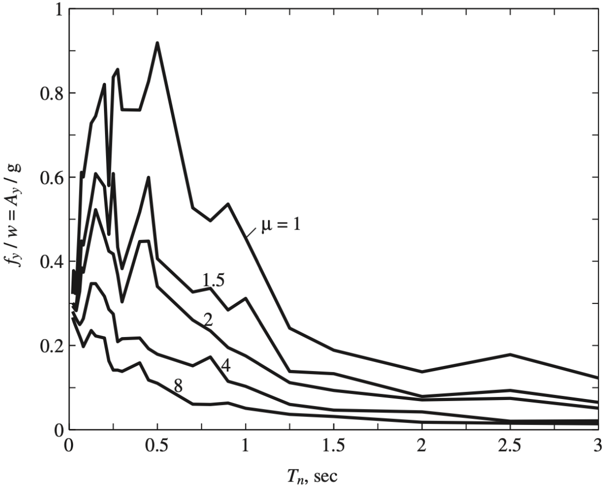

Figure 7.5.2 Constant-ductility response spectrum for elastoplastic systems and El Centro ground motion; $\mu = 1, 1.5, 2, 4,$ and 8; $\zeta = 5\%$.

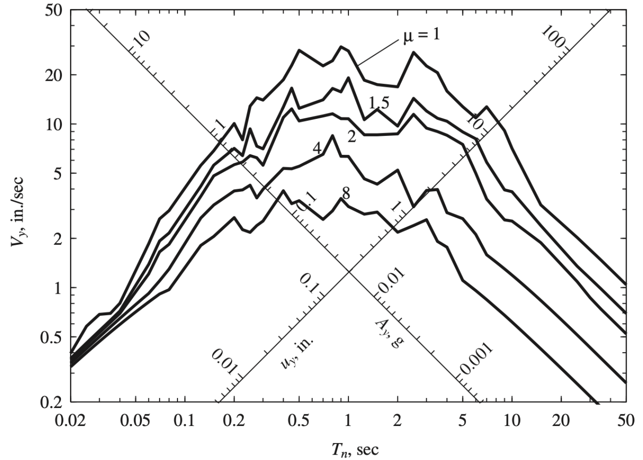

Figure 7.5.3 Constant-ductility response spectrum for elastoplastic systems and El Centro ground motion; $\mu = 1, 1.5, 2, 4,$ and 8; $\zeta = 5\%$.

ENERGY DISSIPATION BY YIELDING

DERIVATION

At time t:

Kinetic Energy $= E_k(t) = \frac{1}{2}m(\dot{u}(t))^2$

Strain Energy $= E_S(t) = \frac{1}{2}k(z(t))^2 = \frac{1}{2}k\left(\frac{f_S(t)}{k}\right)^2 = \frac{1}{2k}(f_S(t))^2$

Up to time t:

Cumulative Viscous Dissipated Energy $=$

Cumulative Yield Dissipated Energy $=$

ENERGY DISSIPATION BY YIELDING

EXAMPLE RESPONSE

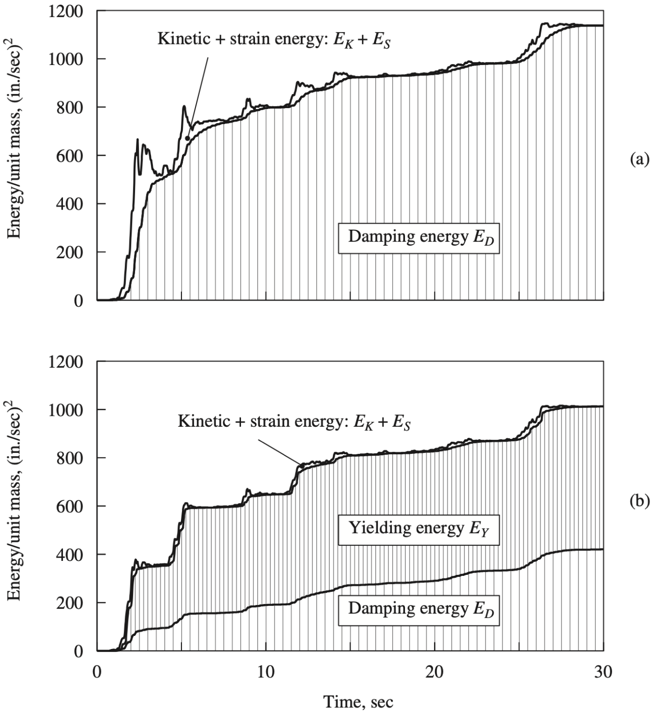

Figure 7.9.1 Time variation of energy dissipated by viscous damping and yielding, and of kinetic plus strain energy; (a) linear system, $T_n = 0.5$ sec, $\zeta = 5\%$; (b) elastoplastic system, $T_n = 0.5$ sec, $\zeta = 5\%$, $\overline{f}_y = 0.25$.