Fall Semester, 2025 **Instructor**: M.J. DeJong

CE 225 – Dynamics of Structures

Homework #6

Due: Monday, October 13

1) Certain types of near-fault ground motion can be represented by a few cycles of ground acceleration. For example, consider the following ground motion (similar equation as HW#5):

$\ddot{u}_g(t) = 8*\sin(\pi t / 0.4)$ ft/s^2 for $0 \le t \le 1.2$ sec

$\ddot{u}_g(t) = 0$ ft/s^2 for $t > 1.2$ sec

Assuming that the ground velocity and displacement are both zero at time zero, use the constant average acceleration method to numerically determine the pseudo-acceleration response spectrum for $\zeta = 0.05$. Use an appropriate time step and resolution of the natural period, T_n. Plot the spectrum against T_n.

2) a) A full water tank is supported on an 80-ft-high cantilever tower. It is idealized as an SDOF system with weight $w = 100$ kips, lateral stiffness $k = 4$ kips/in., and damping ratio $\zeta = 5\%$. The tower supporting the tank is to be designed for ground motion characterized by the design spectrum of Fig. 6.9.5 (see below), scaled to 0.8g peak ground acceleration. Determine the design values of lateral deformation and base shear.

(b) The deformation computed for the system in part (a) seemed excessive to the structural designer, who decided to stiffen the tower by increasing the size of its cross section. Determine the design values of deformation and base shear for the modified system if its lateral stiffness is 8 kips/in.; assume that the damping ratio is still 5%. Comment on the advantages and disadvantages of stiffening the system?

(c) If the stiffened tower were to support a tank weighing 200 kips, determine the design requirements; assume for purposes of this example that the damping ratio is still 5%. Comment on how the increased weight has affected the design requirements.

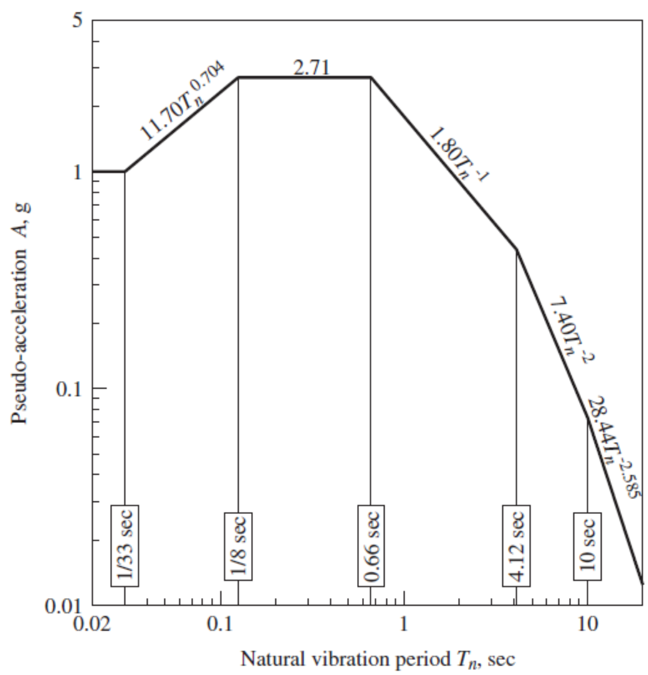
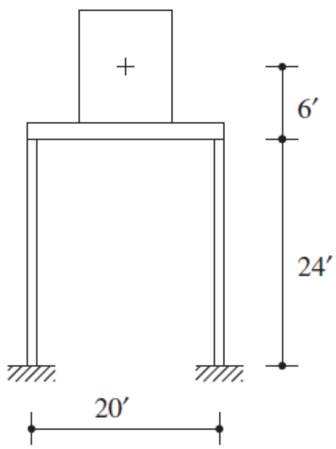


Figure 6.9.5 Elastic pseudo-acceleration design spectrum (84.1th percentile) for ground motions with $\ddot{u}_{go} = 1$g, $\dot{u}_{go} = 48$ in./sec, and $u_{go} = 36$ in.; $\zeta = 5\%$.

3) The ash hopper in Fig. 1 consists of a bin mounted on a rigid platform supported by four columns 24 ft long. The weight of the platform is 14 kips and the platform is 1 foot thick. The weight of the bin and its contents is 70 kips and may be taken as a point mass located 6 ft above the bottom of the platform. The columns are braced in the longitudinal direction, that is, normal to the plane of the paper, but are unbraced in the transverse direction. The column properties are: $A = 22$ in^2, $E = 29{,}500$ ksi, $I = 1800$ in^4, and section modulus $S = 140$ in^3.

Taking the damping ratio to be 5%, find the peak lateral displacement and the peak stress in the columns due to gravity and the earthquake characterized by the design spectrum of Fig. 6.9.5 scaled for a PGA of 0.4g acting in the transverse direction. Assume that the columns are clamped (i.e. fixed) at the base and at the rigid platform. Neglect axial deformation of the column and gravity effects on the lateral stiffness.

Figure 1:



In [22]:
```
import numpy as np
import matplotlib.pyplot as plt
```

Problem #1

Certain types of near-fault ground motion can be represented by a full cosine cycle of ground acceleration. For example, consider the following ground motion (similar equation as HW#5):

$$\ddot{u}_g(t) = \begin{cases} 8\sin(\pi t)/0.4 \text{ ft/s}^2 & \text{for } 0 \le t \le 1.1 \text{ sec} \\ 0 & \text{for } 0 \le t \le 1.2 \text{ sec} \end{cases} \tag{1}$$

Assuming that the ground velocity and displacement are both zero at time zero, use the constant average acceleration method to numerically determine the pseudo-acceleration response spectrum for $\zeta = 0.05$. Use an appropriate time step and resolution of the natural period, T_n. Plot the spectrum against T_n.

In [23]:
```
# Let's pull our constant average acceleration method from last HW:

def newmark_beta(m, k, c, uo, udot0, dt, p, beta=1/4, gamma=1/2):
    '''
    Newmark-beta method for solving the equation of motion of a single
    Parameters:
        m : mass
        k : stiffness
        c : damping coefficient
        uo : initial displacement
        udot0 : initial velocity
        dt : time step
        p : external force array (given as np array or list)
        beta : Newmark-beta parameter (default is 1/4)
        gamma : Newmark-gamma parameter (default is 1/2)
    Returns:
        u : displacement array
        v : velocity array
        a : acceleration array
    '''
    n = len(p)  # number of time steps
    u = np.zeros(n)  # displacement array
    v = np.zeros(n)  # velocity array
    a = np.zeros(n)  # acceleration array
    u[0] = uo
    v[0] = udot0

    # Initial acceleration
    a[0] = (p[0] - c * v[0] - k * u[0]) / m
```

```
    # Newmark-beta effective coefficients (Chopra Eq. 16.5.6, 16.5.7)
    a0 = 1.0 / (beta * dt ** 2)
    a1 = gamma / (beta * dt)
    a2 = 1.0 / (beta * dt)
    a3 = 1.0 / (2 * beta) - 1
    a4 = gamma / beta - 1
    a5 = dt * (gamma / (2 * beta) - 1)

    k_hat = k + a0 * m + a1 * c

    # Time-stepping loop
    for i in range(1, n):
        # Effective force (Chopra Eq. 16.5.8)
        p_hat = p[i] + m * (a0 * u[i - 1] + a2 * v[i - 1] + a3 * a[i -
                      + c * (a1 * u[i - 1] + a4 * v[i - 1] + a5 * a[i
        u[i] = p_hat / k_hat
        a[i] = a0 * (u[i] - u[i - 1]) - a2 * v[i - 1] - a3 * a[i - 1]
        v[i] = v[i - 1] + dt * ((1 - gamma) * a[i - 1] + gamma * a[i])

    # print("Newmark-Beta Method Completed")

    return u, v, a
```

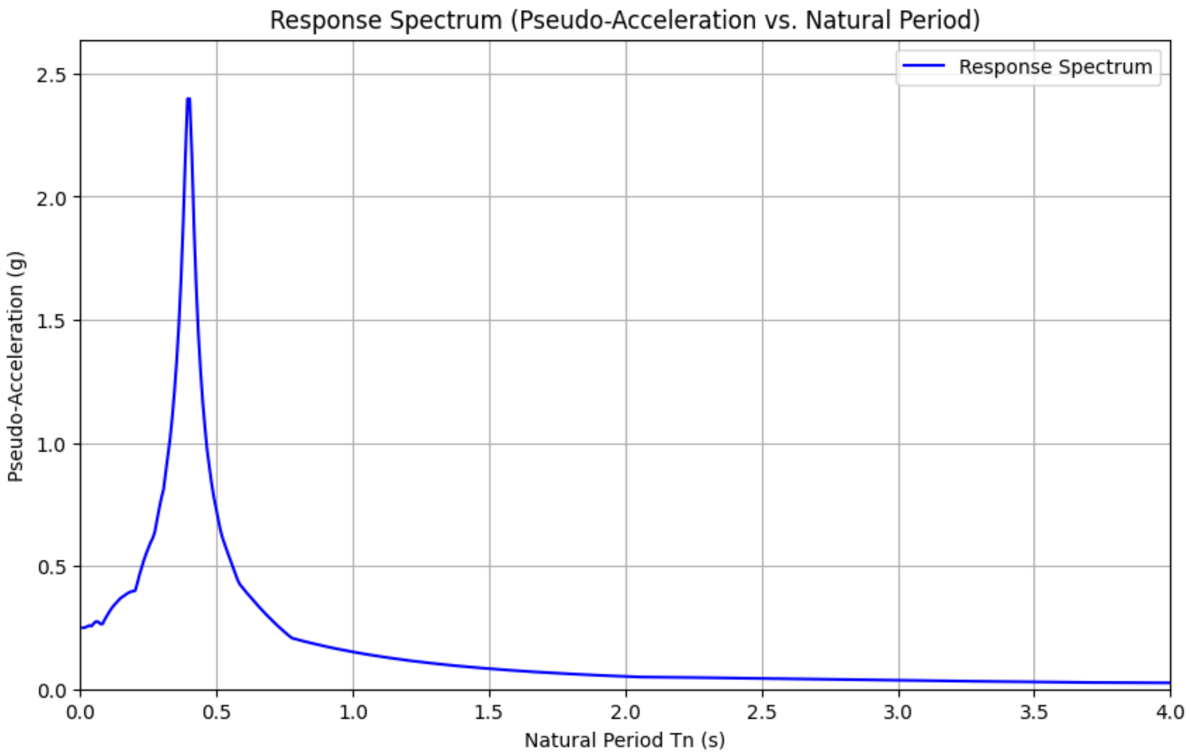
In [24]:

```
# Now, let's create a loop to generate the response spectrum

m = 1.0  # mass
zeta = 0.05  # damping ratio
g = 32.2  # gravitational acceleration in ft/s^2

A = []  # to store pseudo-acceleration
Tn_vec = np.linspace(0.01, 4, 500)  # natural periods

for Tn in Tn_vec:
    wn = 2 * np.pi / Tn  # natural frequency
    k = m * wn ** 2  # stiffness
    c = 2 * m * wn * zeta  # damping coefficient

    # Define time parameters
    dt = min(0.01, Tn / 20)  # time step (picking an appropriate dt fo

    t = np.arange(0, 5, dt)  # time array (up to 5 seconds)
    ug_ddot = 8.0 * np.sin(2 * np.pi / 0.4 * t)  # ground acceleration
    p = -m * ug_ddot  # effective force

    u, v, a = newmark_beta(m, k, c, 0, 0, dt, p)

    U = np.max(np.abs(u))  # maximum absolute displacement
    A.append(wn**2 * U / g)  # pseudo-acceleration (Chopra Eq. 16.5.1)
```

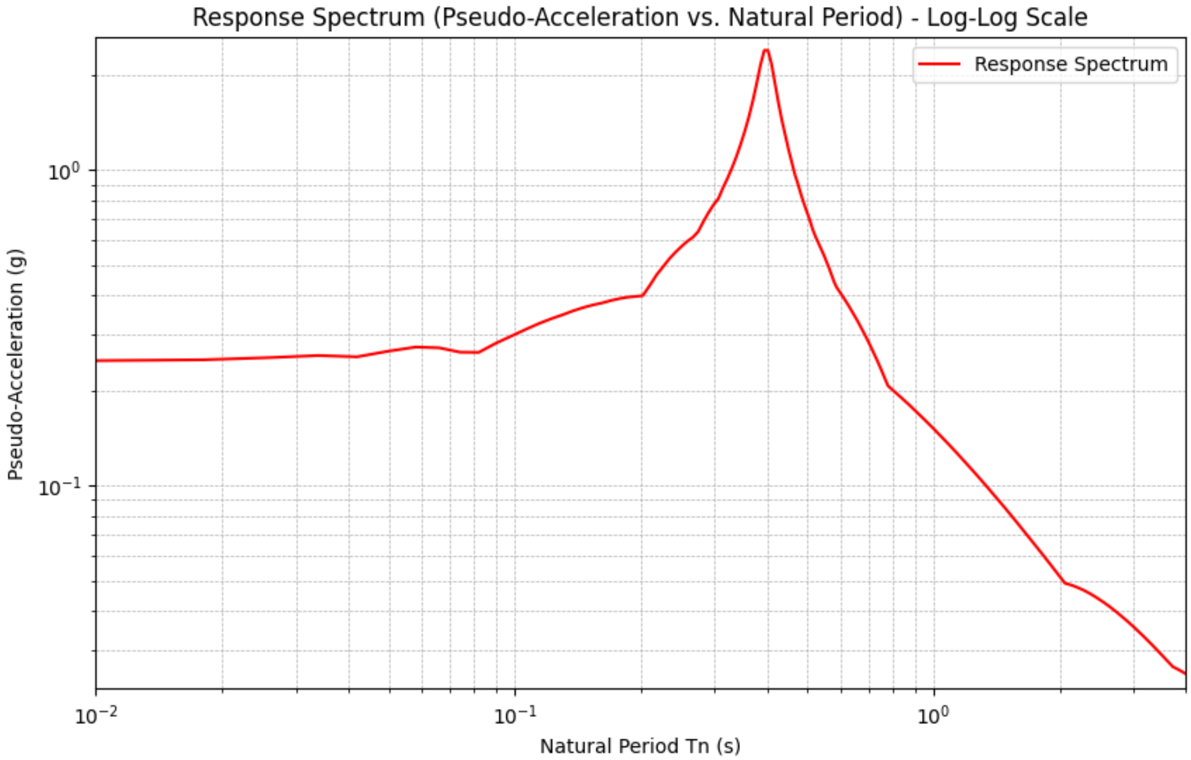
In [25]:

```
# Plotting the response spectrum

plt.figure(figsize=(10, 6))
```

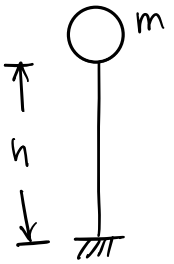
```
plt.plot(Tn_vec, A, label='Response Spectrum', color='blue')
plt.title('Response Spectrum (Pseudo-Acceleration vs. Natural Period)'
plt.xlabel('Natural Period Tn (s)')
plt.ylabel('Pseudo-Acceleration (g)')
plt.grid()
plt.legend()
plt.xlim(0, 4)
plt.ylim(0, max(A)*1.1)
plt.show()
```



In [26]:
```
# In log-log scale
plt.figure(figsize=(10, 6))
plt.loglog(Tn_vec, A, label='Response Spectrum', color='red')
plt.title('Response Spectrum (Pseudo-Acceleration vs. Natural Period)
plt.xlabel('Natural Period Tn (s)')
plt.ylabel('Pseudo-Acceleration (g)')
plt.grid(which='both', linestyle='--', linewidth=0.5)
plt.legend()
plt.xlim(0.01, 4)
plt.ylim(min(A)*0.9, max(A)*1.1)
plt.show()
```


Response Spectrum (Pseudo-Acceleration vs. Natural Period) - Log-Log Scale
Response Spectrum
Pseudo-Acceleration (g)
10^0
10^{-1}
10^{-2}
10^{-1}
10^0
Natural Period Tn (s)

Problem #2.



$h = 80$ (ft).
$w = 100$ (kips).
$k = 4$ kips/in.
$\zeta = 5\%$.

$PGA = 0.8g$.

(a) Design values of lateral deformation and base shear.

$$\omega_n = \sqrt{k/m} = \sqrt{\frac{4\ \text{kips/in} \cdot 386\ \text{in/s}^2}{100\ (\text{kips})}} = 3.93\ (\text{rad/s}) \Rightarrow T_n = \frac{2\pi}{\omega_n} = 1.6\ (\text{sec}).$$

From the design spectrum $\frac{A}{PGA} = \frac{1.8}{T_n} \Rightarrow A = \frac{1.8 \cdot 0.8g}{1.6} = 0.9g$

So the base shear:

$$V_0 = m \cdot A = \frac{100\,(\text{kips})}{g} \cdot 0.9g \Rightarrow \boxed{V_0 = 90.0\ \text{kips.}}$$

And the peak deformation:

$$D = \frac{A}{\omega_n^2} = \frac{0.9 \cdot 386\ (\text{in/s}^2)}{(3.93)^2} = 22.5\ (\text{in})$$

Note: $V_0 = k \cdot D = 22.5\ (\text{in}) \cdot 4\ (\text{kip/in}) = 89.9\ (\text{kips})$ always good to check.

(b) if $k = 8$ kips/in:

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{8\ (\text{kips/in}) \cdot 386\ (\text{in/s}^2)}{100\ (\text{kips})}} = 5.56\ (\text{rad/sec}) \Rightarrow T_n = 1.13\ (\text{sec}).$$

$$\Rightarrow \frac{A}{PGA} = \frac{1.8}{1.11} \Rightarrow A = \frac{1.8 \cdot 0.8g}{1.13} = 1.27g$$

$$V_0 = m \cdot A = \frac{100\ (\text{kips})}{g} \cdot 1.27\ g \Rightarrow \boxed{V_0 = 127\ (\text{kips}).}$$

and $D = \frac{A}{\omega_n^2} = \frac{1.27 \cdot 386}{(5.56)^2} = 15.86''$ → observe that 2x stiffer does not get 1/2 deformation. —

D is 30% smaller for the stiffer structure..
V_0 is 41% larger for the stiffer structure. —

↝ lower deformations come at the expense of larger design forces. Need to make a stronger column.

(c) Now $W = 200$ (kips).

$$\omega_n = \sqrt{\frac{8\,(\text{kips}) \cdot 386\,(\text{in}/s^2)}{200\,(\text{kips})}} = 3.93\ (\text{rad}/s) \Rightarrow T_n = 1.6\ (\text{sec}).$$

$$\frac{A}{PGA} = \frac{1.8}{T_n} \Rightarrow A = \frac{1.8 \cdot 0.8g}{1.6} = 0.9g$$

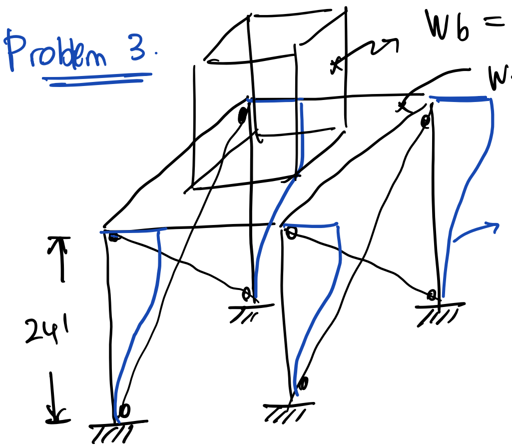
$$\Rightarrow V_0 = m \cdot A = \frac{200\,(\text{kips})}{g} \cdot 0.9g \Rightarrow \boxed{V_0 = 180\ (\text{kips})}$$

and $$D = \frac{A}{\omega_n^2} = \frac{0.9 \cdot 386}{(3.93)^2} \Rightarrow \boxed{D = 22.5\ (\text{in})}$$

Observations: For similar T_n → V_0 scales linearly with weight, but D is insensitive to the weight.
D only depends on the period T_n (or what is the ratio of k/m).

In this last case we see that V_0 is almost twice as in part (a), which comes from W being almost twice as big. Since the T_n is similar, we get similar D values.

Problem 3.



$W_b = 70$ (kips)

$W_p = 14$ (kips)

columns have

$A = 22\ (in^2)$

$E = 29{,}500$ (ksi)

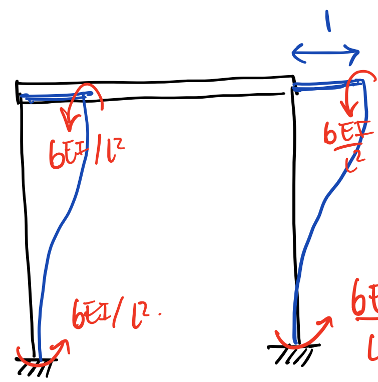
$I = 1800\ (in^4)$.

$S = 140\ (in^3)$.

$\zeta = 5\%$

$PGA = 0.4g$.

Scheme of the deflections:



combined stiffness from the 4 columns.

$\rightarrow k = 4\left(\frac{12EI}{L^3}\right)$.

$\frac{6EI}{L^2}$ for an individual column.

$\Rightarrow \left(M_{max} = \frac{6EI}{L^2}\cdot\Delta\ ,\ \Delta:\text{ lateral deformation}\right)$.

$$k = \frac{4\cdot 12\cdot 29{,}500\ (ksi)\cdot 1{,}800\ (in^4)}{(24'\cdot 12\ (in))^3} = 106.7\ (kip/in).$$

$W_{total} = W_{bin} + W_{platform} = 14 + 70 = 84$ (kips).

$$\Rightarrow \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{106.7\ (kip/in)\cdot 386\ (in/s^2)}{84\ (kips)}} = 22.14\ (rad/sec)$$

$$\Rightarrow T_n = \frac{2\pi}{\omega_n} = 0.284\ (sec).$$

From the design spectrum $\rightarrow \frac{A}{PGA} = 2.71 \Rightarrow A = 2.71\cdot 0.4g$

$$\Rightarrow \boxed{A = 1.08g}$$

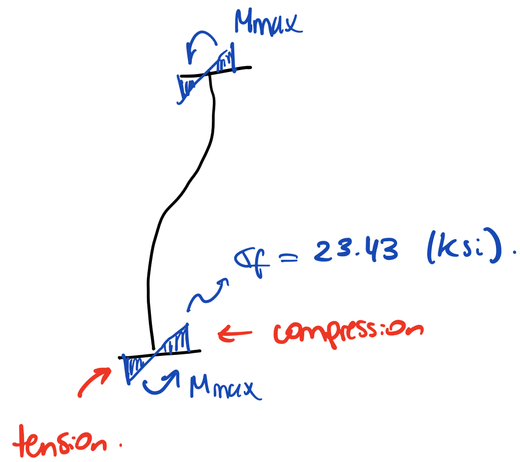
And $D = \frac{A}{\omega_n^2} = \frac{1.08\cdot 386}{(22.14)^2} \Rightarrow \boxed{D = 0.854''}$

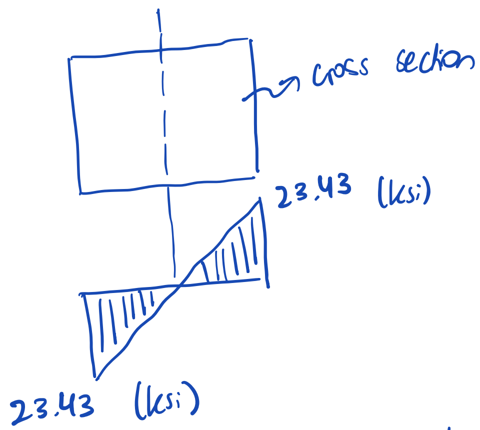
(1) Stresses due to flexure: $\sigma_f = \dfrac{M_{max}}{S}$

where $M_{max} = \dfrac{6EI}{L^2}\cdot \underbrace{\Delta}_{D}$ → here we want stresses in a single column (they are all the same).

$$\Rightarrow M_{max} = \frac{6\cdot 29{,}500\ (ksi)\cdot 1800\ (in^4)\cdot 0.854\ (in)}{(24\cdot 12\ in)^2} = 3{,}280\ (kip\text{-}in).$$

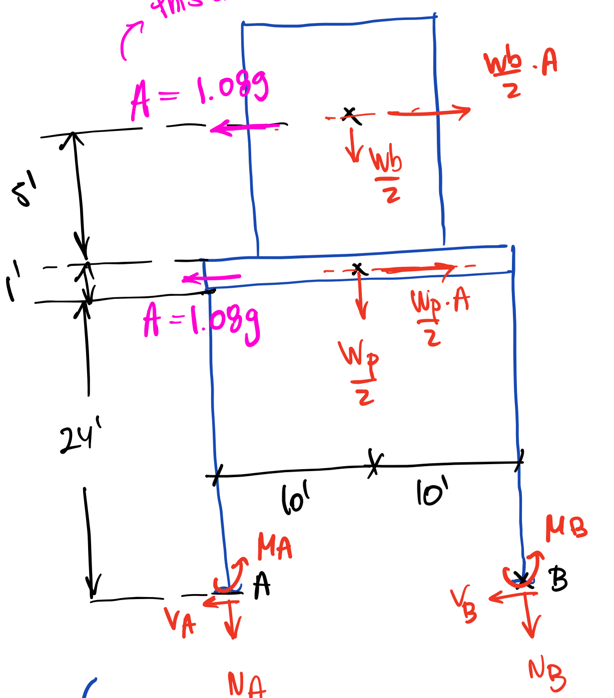
And $\sigma_f = \dfrac{3{,}280\ (kip\text{-}in)}{140\ (in^3)} = 23.43\ (ksi).$





Now, need to add the axial force in the column. For this, we use A as the total acceleration experienced by the system, and we draw a F.B.D.:

this direction of A is compatible with the direction of the analyzed D above. – recall that $f_I = m\cdot A$, in the direction opposite to A.



we also know what are M_A and M_B, from our stiffness coefficients:

$$M_A = M_B = M_0 = 3{,}280\ (kip\text{-}in).$$
$$= 273.3\ (kip\text{-}ft)$$

N_A and N_B are normal (axial) forces acting on each column.

By intuition, largest peak stress will be compressive, so we'll compute N_B which is most likely a compression.

Also, we'll do the analysis on one frame, that's why all W's are divided by 2.

$\Sigma M_A = 0 \Rightarrow$

$$M_A + M_B - \left(\frac{w_p}{2}+\frac{w_b}{2}\right)\cdot(10') - N_B\cdot 20' - \frac{w_b}{2}\cdot A\,(24+1+5) - \frac{w_p}{2}\cdot A\,(24'+0.5)$$

$$\Rightarrow -N_B\cdot 20' = \left(\frac{w_p}{2}+\frac{w_b}{2}\right)\cdot 10' + \frac{w_b}{2}\cdot A\,(30') + \frac{w_p}{2}\cdot A\cdot 24.5' - 2M_{max}$$

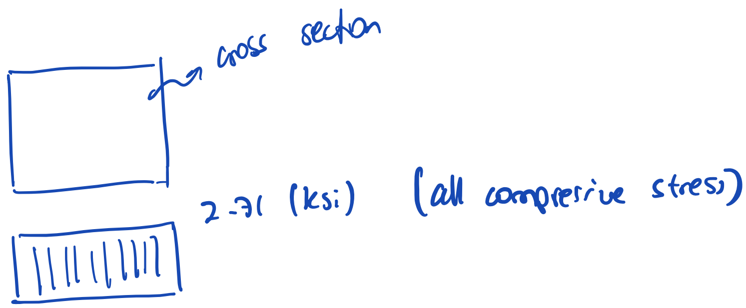
$$\Rightarrow -N_B = \frac{14+70}{2}\cdot\frac{10}{20} + \frac{70}{2}\cdot 1.08\cdot\frac{30}{20} + \frac{14}{2}\cdot 1.08\cdot\frac{24.5}{20} - 2\cdot\frac{273.3}{20}$$

$$\boxed{N_B = -59.63 \text{ (kips)}}$$

the stresses in column B due to the axial force:

$$\sigma_A = -\frac{59.63 \text{ (kips)}}{22 \text{ (in}^2\text{)}} = -2.71 \text{ (ksi)}$$

in a diagram:



And the total stresses: