CE 225: Dynamic of Structures **Fall 2025**

Discussion 3: Arbitrary Forcing Functions

Instructor: Matthew DeJong *GSI: Miguel A. Gomez*

Plan for today

The plan for today's discussion includes:

1. Conceptual review of transmissibility (study summary).
2. Example of solving the EOM for an arbitrary forcing function.

1 Transmissibility

$$TR = \frac{\sqrt{1+\left(2\zeta\frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2+\left(2\zeta\frac{\omega}{\omega_n}\right)^2}}$$

This can mean two different things, depending on the type of EOM that we have.

(a) Harmonic Force Case

If our system is subjected to a harmonic force, then the EOM looks like:

$$\boxed{m\ddot{u}+c\dot{u}+ku = p_0\sin\omega t} \tag{1.1}$$

In this case:

$$TR = \frac{f_{T,\max}}{p_o}$$

(b) Harmonic Ground Motion Case

If our system is subjected to a ground motion at its base, then our EOM for the relative displacement of the system w/r to the ground looks like:

$$\boxed{m\ddot{u}+c\dot{u}+ku = -m\ddot{u}_g} \tag{1.2}$$

Where $\ddot{u}_g = \ddot{u}_{go}\sin(\omega t)$

In this case, we can use the Transmissibility equation for two purposes: (1) to find the ratio of the motion transmitted from the ground to the system:

$$TR = \frac{u_o^t}{u_{go}} = \frac{\ddot{u}_o^t}{\ddot{u}_{go}}$$

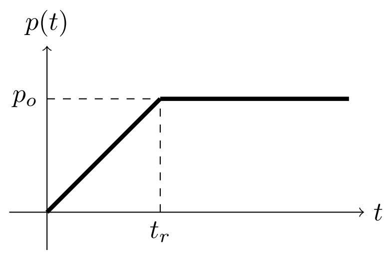
Or, if we compare our equations (1.1) and (1.2), we can define $p_o = -m\ddot{u}_{go}$ and use the transmissibility equation to find a relation between the force transmitted to our system, and the inertial force generated by the ground motion:

$$TR = \frac{f_{T,\max}}{p_o} = \frac{f_{T,\max}}{m\ddot{u}_{go}}$$

2 Arbitrary Forcing Functions

Example

An undamped SDF system, starting from rest, is subjected to a step force with finite rise time, as the one shown in the Figure below.



The response of the system can be shown to be:

$$u(t) = (u_{st})_o \left(\frac{t}{t_r} - \frac{\sin \omega_n t}{\omega_n t_r} \right) \quad \text{for } t \leq t_r \tag{2.3}$$

$$u(t) = (u_{st})_o \left\{ 1 + \frac{1}{\omega_n t_r} \left((1 - \cos(\omega_n t_r)) \sin(\omega_n (t - t_r)) - \sin(\omega_n t_r) \cos(\omega_n (t - t_r)) \right) \right\} \quad \text{for } t > t_r \tag{2.4}$$

Equation (2.4) can be simplified through trigonometric identities to:

$$u(t) = (u_{st})_o \left\{ 1 - \frac{1}{\omega_n t_r} \left[\sin(\omega_n t) - \sin(\omega_n (t - t_r)) \right] \right\}$$

Derive these results with:

(a) Using the convolution integral.

(b) Solving the EOM with analytical procedure.

(c) Using superposition

(a) Convolution Integral

For undamped systems, starting from rest subjected to a force defined by $p(t)$, we can find the response as:

$$\boxed{u(t) = \frac{1}{m\omega_n} \int_0^t p(\tau) \sin\left(\omega_n (t - \tau)\right) d\tau}$$

We also have the damped version of the convolution integral as:

$$u(t) = \frac{1}{m\omega_D} \int_0^t p(\tau) e^{-\zeta\omega_n (t-\tau)} \sin\left(\omega_D (t - \tau)\right) d\tau$$

(b) Solving the EOM

(c) Superposition