**CE 225: Dynamic of Structures** **Fall 2025**

# Discussion 5: Response Spectrum - Applications

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## Announcements

- Solution for HW#4 is up on bCourses.

Wrap up: numerical schemes

solving: $m\ddot{u} + c\dot{u} + ku = p(t)$

discretization of $p(t)$, $u(t)$ → approximate response.

3 methods

method based on interpolation of $p(t)$

central difference

Newmark's Method.

Versions for nonlinear systems are also available.

## Summary of Central Difference and Newmark's Methods

| Scheme: | **Central Difference** | **Newmark** |
|---|---|---|
| Algorithm | Define $\Delta t$ (see Stability)<br><br>Initial Calculations<br>$\ddot{u}_0$, $u_{-1}$, $\hat{k}$, $a$, $b$<br><br>For each time step $i = 1, ..., N-1$<br><br>- $\hat{p}_i = p_i + au_{i-1} + bu_i$<br>- $\boxed{u_{i+1} = \frac{\hat{p}_i}{\hat{k}}}$ solving a linear problem @ each time step<br>- If we want: $\dot{u}_i$, $\ddot{u}_i$ | Define $\Delta t$ (See stability)<br>Define $\beta$ and $\gamma$ (See stability)<br><br>Initial Calculations<br>$\ddot{u}_0$, $a_1$, $a_2$, $a_3$, $\hat{k} = k + a_1$<br><br>For each time step $i = 1, ..., N-1$<br><br>- $\hat{p}_{i+1} = p_{i+1} + a_1 u_i + b\dot{u}_i + c\ddot{u}_i$<br>- $u_{i+1} = \frac{\hat{p}_{i+1}}{\hat{k}}$<br>- We need $\dot{u}_{i+1}$, $\ddot{u}_{i+1}$ |
| Type | Explicit<br>- EOM enforced at time $t_i$ | Implicit<br>- EOM enforced at time $t_{i+1}$ |
| Stability | Conditionally stable:<br>$\frac{\Delta t}{T_n} \leq \frac{1}{\pi}$ | It depends on the chosen scheme:<br>$\frac{\Delta t}{T_n} \leq \frac{1}{\pi\sqrt{2}} \frac{1}{\sqrt{\gamma - 2\beta}}$<br><br>In practice:<br>Constant acceleration ($\beta = 1/4$, $\gamma = 1/2$)<br>- Unconditionally stable → less accurate<br><br>Linear acceleration: ($\beta = 1/6$, $\gamma = 1/2$)<br>- Conditionally stable under:<br>$\frac{\Delta t}{T_n} < 0.551$<br>$\frac{\Delta t}{T_n} \sim 0.1$ ok! |
| Nonlinear Systems | Modified $\hat{p}_i$ to account $f_S(u_i)$<br>$\hat{p}_i = p_i - au_{i-1} + \frac{2m}{(\Delta t)^2} u_i - \underline{(f_S)_i}$<br>constitutive behavior of the material | Incremental form of the EOM:<br>$m\Delta\ddot{u}_i + c\Delta\dot{u}_i + \underline{\Delta(f_S)_i} = \Delta p_i$<br>2 Options:<br>- Approximate $(f_S)_{i+1} \approx (k_T)_i u_{i+1}$ no iterations if $\Delta t << 1$<br>- Newton-Raphson Iterations |

Table 0.1: Comparison between the schemes

"state determination" block
↓
typically requires iterations.

State determination, iterations required to get the correct $(f_S)_{i+1}$ @ each time step. –

## Some relevant concepts about the response spectrum

Some questions...

$$m\ddot{u} + c\dot{u} + ku = -m\ddot{u}_g(t).$$

if small $c$:

$$m\ddot{u} + ku = -m\ddot{u}_g(t)$$

$$\Rightarrow \quad ku = -m\left(\ddot{u}_g(t) + \ddot{u}(t)\right)$$

$$ku = -m\,\ddot{u}^t(t).$$

$$\frac{k}{m}u = -\ddot{u}^t(t)$$

$$\Rightarrow \quad \omega_n^2 u = -\ddot{u}^t(t)$$

if we get peak deformation $u_0 = D$ $\Rightarrow$ can get estimate of peak TOTAL acceleration

Definition: Pseudo-acceleration

$$\boxed{A = \omega_n^2 D}$$

estimate of peak total acceleration.

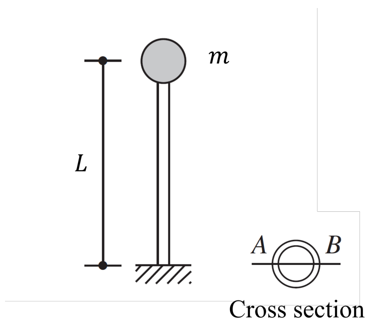
# Response Spectrum Applications

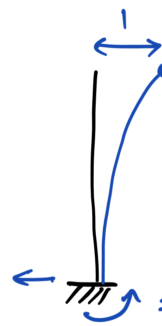
## Example 1 - Peak response

A 10-ft long vertical cantilever made of a steel pipe supports a 3000-lb weight attached at the tip, as shown in the Figure below. The properties of the pipe are: outside diameter = 6.625 in, inside diameter = 6.065 in, thickness = 0.280 in, second moment of cross-sectional area $I = 28.1$ in$^4$, Young's modulus $E = 29,000$ ksi, and weight per unit length = 18.97 lb/ft. Determine the peak deformation and the bending stress in the cantilever due to the El Centro ground motion. Assume $\zeta = 5\%$. The peak ground acceleration (PGA) of the El Centro Ground motion is 0.319g, and the response spectrum is shown below.

$u_0 = D$ (for earthquake loading)

need $T_n$ to enter response spectrum.




→ $\frac{3EI}{L^3}$

$\frac{3EI}{L^2}$ → moment @ the base due to unit displacement @ the tip.

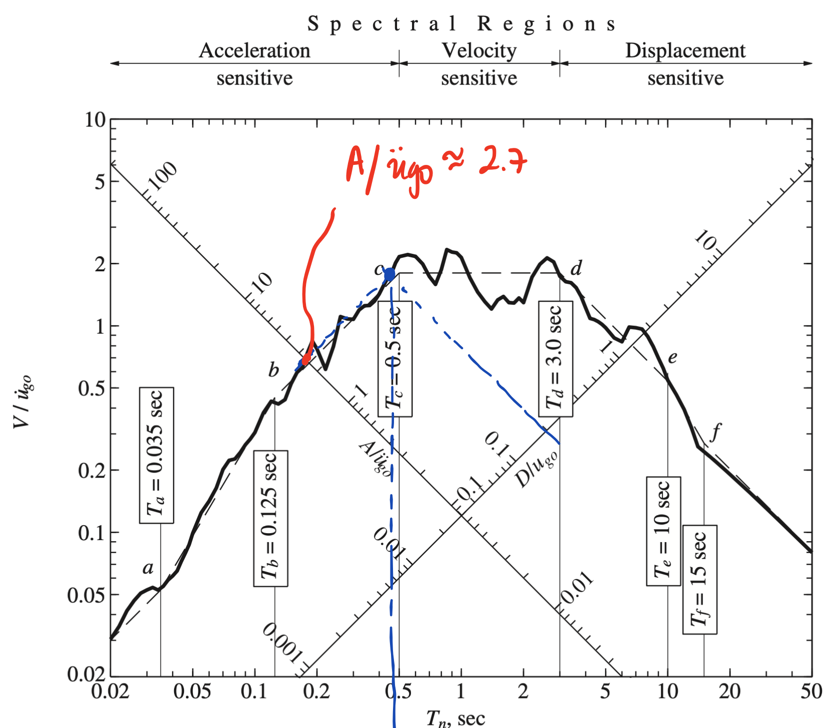



**Figure 6.8.3** Response spectrum for El Centro ground motion shown by a solid line together with an idealized version shown by a dashed line; $\zeta = 5\%$.

0.47 (sec)

$$k = \frac{3EI}{L^3} = \frac{3 \cdot 29{,}000\ (\text{ksi}) \cdot 28.1\ (\text{in}^4)}{(10 \cdot 12\ \text{in})^3} = 1.41\ (\text{Kips/in})$$

$$\omega_n = \sqrt{\frac{k}{m}}\ ; \qquad w = 18.97\ (\text{lb/ft}) \cdot 5(\text{ft}) + 3{,}000\ (\text{lb}) = 3{,}095\ (\text{lb})$$

$$\Rightarrow \quad \omega_n = \sqrt{\frac{1{,}410\ (\text{lb/in}) \cdot 386\ (\text{in/s}^2)}{3{,}095\ (\text{lb})}} = 13.26\ (\text{rad/sec}) \Rightarrow T_n = \frac{2\pi}{13.26} = 0.47\ (\text{sec})$$

From the response spectrum, with $T_n = 0.47$ (sec).

$$\frac{A}{\ddot{u}_{go}} \approx 2.7 \Rightarrow A = 2.7 \cdot \ddot{u}_{go} = 2.7 \cdot 0.319\,g$$

$$\Rightarrow \boxed{A = 0.861\,g}$$

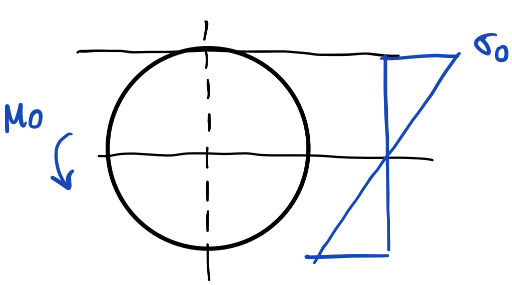
but we want $D = A / \omega_n^2$

$$\Rightarrow \quad D = \frac{0.861 \cdot 386\ (\text{in/s}^2)}{(13.26)^2} = 1.9\ (\text{in}).$$

To get peak bending stress: $\sigma = \dfrac{M \cdot y}{I} \rightarrow M = \dfrac{3EI}{L^2} \cdot D$

$$\Rightarrow \quad M_o = \frac{3EI}{L^3} \cdot D = \frac{3 \cdot 29{,}000\ (\text{ksi}) \cdot 28.1\ (\text{in}^4)}{(10 \cdot 12\ \text{in})^2} \cdot 1.9\ (\text{in}) = 320\ (\text{kip} \cdot \text{in}).$$

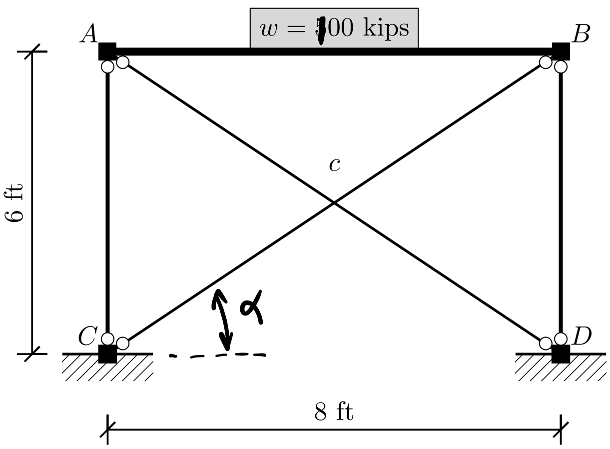
And the peak bending stress: $\sigma_o = \dfrac{M \cdot y}{I} = \dfrac{320 \cdot (6.625/2)}{28.1} = 37.31\ (\text{ksi}).$



Example 2

$0.153\ (in^2)$ $\quad$ $20\ (kips)$

The braced frame in the Figure below is made of two hinged columns and two wires. The columns do not participate in the lateral stiffness. The wires are prestressed to an initial tension of $p_o =$ [illegible] kips. The properties of each wire are: cross-section area $A =$ [illegible] and $E = 29,000$ ksi. The structure supports a rigid beam, which supports a weight of [illegible]00 kips. The structure is subjected to the El Centro ground motion. Will the wires become loose during the earthquake?




Recall: $\quad k_{lat} = 2\,\frac{EA}{L}\cdot\cos^2(\alpha)$ $\quad$ and $\quad \Delta_{lat} = \frac{\delta}{\cos(\alpha)}$ ; $\delta$: deformation on the wires.

From the prestress $\quad q_0 = 20\ (kips)$

$$q_0 = \frac{EA}{L}\cdot\delta_0 \Rightarrow \delta_0 = \frac{q_0\cdot L}{EA} = \frac{20\,(kips)\cdot(10\cdot 12\ in)}{29{,}000\ (kips/in^2)\cdot 0.153\ (in^2)} = 0.541\ (in).$$

How much lateral displacement this represents?

$$\Delta_{lat} = \frac{\delta}{\cos(\alpha)} = \frac{0.541}{0.8} = 0.676\ (in).$$

if $D > 0.676''$ $\Rightarrow$ lose tension on the wires. -

$$\omega_n = \sqrt{\frac{k}{m}} \rightarrow k = 47.33\ (kips/in) \qquad \omega_n = \sqrt{\frac{47.33\cdot 386}{100}} = 13.52\ (rad/sec)$$

$$T_n = \frac{2\pi}{\omega_n} = 0.465\ (sec).$$

go to spectrum:

$$\frac{A}{\ddot{u}_{go}} = 2.7 \Rightarrow A = 0.861g \Rightarrow D = \frac{0.861\cdot 386}{(13.52)^2} = 1.91'' > 0.676''$$

$\Rightarrow$ yes! lose tension.

## Mid-semester evaluation

I'd like to get your input on how you feel discussions and office hours are going so far. Your feedback is very much appreciated! Here's the link to my mid-semester evaluation. Your responses will remain anonymous.

https://tinyurl.com/ce225-gsi-eval