LECTURE 16 - GENERALIZED SDOF SYSTEMS (PART 3)

CE 225

Prof DeJong

UC Berkeley

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GENERALIZED SDOF SYSTEMS

EXAMPLE: LUMPED MASS PLUS DISTRIBUTED MASS

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DISCRETE SYSTEMS

DERIVE EQUIVALENT SDOF

Lateral Stiffness: $k_j = 2\left(\frac{12EI_j}{L_j^3}\right)$

$$\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} = z(t) \begin{bmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_n \end{bmatrix} \longrightarrow$$

Derive Equivalent SDOF for shape:

$$KE = \sum_{j=1}^{n} \frac{1}{2} m_j (\dot{u}_j)^2 = \frac{1}{2} \sum_{j=1}^{n} m_j (\dot{z}(t)\psi_j)^2 = \frac{1}{2} [\dot{z}(t)]^2 \sum_{j=1}^{n} m_j \psi_j^2$$

DISCRETE SYSTEMS

DERIVE EQUIVALENT SDOF

$$PE =$$

$$PE = \frac{1}{2}\sum_{j=1}^{n} k_j \left(z(t)\psi_j - z(t)\psi_{j-1}\right)^2 = \frac{1}{2}(z(t))^2 \sum_{j=1}^{n} k_j(\psi_j - \psi_{j-1})^2$$

Work done by force:

Side note for ground motion:

DISCRETE SYSTEMS

EXAMPLE: TWO STORY FRAME

$$m_1 = m_2 = m$$

$$k_1 = k_2 = \frac{24EI}{L^3} = k$$

(i) Find Natural Frequency

DISCRETE SYSTEMS

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(ii) Find the maximum response of the top floor under horizontal harmonic ground motion with an amplitude of 1 inch and a frequency of $\omega = 0.5\omega_n$.
Assume $\zeta = 0.05$.