LECTURE 17 - EQUATIONS OF MOTION - MDOF SYSTEMS

CE 225

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DERIVING EQUATIONS OF MOTION - OPTION 1

EQUILIBRIUM APPROACH

Equivalent System:

Derive EOM:

FBD I:

FBD II:

DERIVING EQUATIONS OF MOTION - OPTION 1

EQUILIBRIUM APPROACH

Apply Equilibrium:

DERIVING EQUATIONS OF MOTION - OPTION 2

DISPLACEMENT METHOD FOR STIFFNESS MATRIX

Stiffness

- Apply unit displacement to a single DOF.
- Determine required forces f_{ij}: (i = DOF of the force, j = DOF of displacement)

$$\underline{f}_s = \underline{\underline{k}}\, \underline{u} \longrightarrow \begin{bmatrix} f_{s1} \\ f_{s2} \\ \vdots \\ f_{sN} \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} & \cdots & k_{1N} \\ k_{21} & k_{22} & \cdots & k_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ k_{N1} & k_{N2} & \cdots & k_{NN} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{bmatrix}$$

If $u_1 = 1$ and all other u values are 0 $\longrightarrow \begin{bmatrix} f_{s1} \\ f_{s2} \\ \vdots \\ f_{sN} \end{bmatrix} = \begin{bmatrix} k_{11} \\ k_{21} \\ \vdots \\ k_{N1} \end{bmatrix}$

**Therefore, the forces required to enforce the displacements $u_1 = 1$ and $u_2 = u_3 = ... = u_N = 0$ actually provide the first column of the stiffness matrix $\underline{\underline{k}}$

Deriving Equations of Motion - Option 2

Displacement Method for Stiffness Matrix

Plot:

$$f_{11} = k_1 u_1 + K_2(u_1 - 0) = (k_1 + k_2)u_1$$

$$f_{11} = \boxed{k_1 + k_2 = k_{11}}$$

$$f_{21} = -k_2 u_1 \rightarrow \boxed{k_{21} = -k_2}$$

Plot:

$$f_{22} = k_2 u_2 \rightarrow \boxed{k_{22} = k_2}$$

$$f_{12} = -k_2 u_2 \rightarrow \boxed{k_{12} = -k_2}$$

$$\boxed{\underline{\underline{k}} = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}}$$

DERIVING EQUATIONS OF MOTION - OPTION 2

ACCELERATION METHOD FOR MASS MATRIX

- Apply unit acceleration to a single DOF.
- Determine external forces required for equilibrium f_{ij}: (i = DOF of force, j = DOF of the acceleration)

$$\underline{f}_I = \underline{\underline{m}}\,\underline{\ddot{u}} \longrightarrow \begin{bmatrix} f_{I1} \\ f_{I2} \\ \vdots \\ f_{IN} \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & \cdots & m_{1N} \\ m_{21} & m_{22} & \cdots & m_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ m_{N1} & m_{N2} & \cdots & m_{NN} \end{bmatrix} \begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \vdots \\ \ddot{u}_N \end{bmatrix}$$

If $\ddot{u}_1 = 1$ and all other $\ddot{u}$ values = 0 $\longrightarrow \begin{bmatrix} f_{I1} \\ f_{I2} \\ \vdots \\ f_{IN} \end{bmatrix} = \begin{bmatrix} m_{11} \\ m_{21} \\ \vdots \\ m_{N1} \end{bmatrix}$

**Therefore, the forces required to cause the accelerations $\ddot{u}_1 = 1$ and $\ddot{u}_2 = \ddot{u}_3 = ... = \ddot{u}_N = 0$ actually provide the first column of the mass matrix $\underline{\underline{m}}$

DERIVING EQUATIONS OF MOTION - OPTION 2

ACCELERATION METHOD FOR MASS MATRIX

Plot:

$$f_{11} = m_1 \ddot{u}_1 \rightarrow \boxed{m_{11} = m_1}$$

$$f_{21} = m_2 \ddot{u}_2 = 0 \rightarrow \boxed{m_{21} = 0}$$

Skip DOF 2 by inspection

$$\boxed{\underline{\underline{m}} = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}}$$

DERIVING EQUATIONS OF MOTION - OPTION 2

EXAMPLE: BAR ON SPRINGS

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DERIVING EQUATIONS OF MOTION - OPTION 2

EXAMPLE: BAR ON SPRINGS - ALTERNATE COORDINATE SYSTEM

Deriving Equations of Motion - Option 2

Example: Bar On Springs - Alternate Coordinate System

EXAMPLE: LUMPED MASS WITH ROTATIONAL INERTIA