Lecture 1 - Equations of Motion

CE 225

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DEFINITIONS

RESPONSE QUANTITIES

Displacement

$u(t)$

Velocity

$$v(t) = \frac{du}{dt} =$$

Acceleration

$$a(t) = \frac{d^2u}{dt^2} =$$

FORCES IN LINEAR SYSTEMS

DERIVED FOR SIMPLE FRAME

(1) Stiffness force (elastic resisting force):

- Rigid beam ($EI_b = \infty$)

- Flexible beam ($EI_b = 0$)

FORCES IN LINEAR SYSTEMS

DERIVED FOR SIMPLE FRAME

(2) Damping force: c = viscous damping coefficient

(3) Inertial force (fictitious): = D'Alembert Force (always opposes the motion)

In rotational terms:

(4) External Forces:

Equations of Motion

Example 1: Frame

$$\boxed{m\ddot{u} + c\dot{u} + ku = p(t)}$$

EQUATIONS OF MOTION

EXAMPLE 2: CART ON SLOPE

$$\boxed{m\ddot{u} + c\dot{u} + ku = mg\sin\theta + p(t)} \tag{1}$$

EQUATIONS OF MOTION

EXAMPLE 2: CART ON SLOPE

Alternatively, define u_{st} as the static equilibrium position:

Define "dynamic" displacement:

$$\begin{cases} u = \\ \dot{u} = \\ \ddot{u} = \end{cases}$$

$$m\ddot{u}_d + c\dot{u}_d + ku_d = -ku_{st} + mg\sin\theta + p(t)$$

$$\boxed{m\ddot{u}_d + c\dot{u}_d + ku_d = p(t)} \qquad (2)$$

EQUATIONS OF MOTION

EXAMPLE 3: GROUND MOTION (NO OTHER EXTERNAL FORCE)

$$\boxed{m\ddot{u} + c\dot{u} + ku = -m\ddot{u}_g}$$

$$\boxed{m\ddot{u}_t + c\dot{u}_t + ku_t = c\dot{u}_g + ku_g}$$

EQUATIONS OF MOTION

EXAMPLE 4: ROTATIONAL SYSTEMS

$$mh^2\ddot{\theta} + k_s\theta - mgh\sin\theta = -m\ddot{u}_g(h\cos\theta)$$

$$\boxed{mh^2\ddot{\theta} + (k_s - mgh)\theta = -mh\ddot{u}_g}$$

Equations of Motion

Example 5: Rotational Systems with Rotational Inertia

$$\rightarrow (J_c + mR^2)\ddot{\theta} + mg(R\sin\theta) = 0$$

$$\boxed{(J_c + mR^2)\ddot{\theta} + mg(R\theta) = 0}$$