LECTURE 14 - GENERALIZED SDOF SYSTEMS

CE 225

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STRUCTURES WITH DISTRIBUTED MASS (E.G. BEAM)

- Continuous system $\rightarrow \infty$ modes.
- Create SDOF "system" for each mode by defining a shape function: $\psi(x)$

ENERGY APPROACH TO DERIVE EQUIVALENT SDOF SYSTEM

KINETIC ENERGY

System I:

System II:

$$\therefore \boxed{M_{eq} = \int_0^L m(\psi(x))^2 dx = \widetilde{m}}$$

Energy Approach to Derive Equivalent SDOF System

Potential Energy

- System I:

  $PE =$ Strain energy caused by bending moment (M) causing a rotation of: $d\theta$

  Beam Theory:

ENERGY APPROACH TO DERIVE EQUIVALENT SDOF SYSTEM

POTENTIAL ENERGY

- System I (cont.)

- System II:

$$\therefore \boxed{K_{eq} = \int_0^L EI \left(\frac{d^2\psi}{dx^2}\right)^2 dx = \widetilde{k}}$$

ENERGY APPROACH TO DERIVE EQUIVALENT SDOF SYSTEM

EXTERNAL WORK

- System I:

- System II:

$$\therefore \boxed{F_{eq} = \int_0^L f(x,t)\psi(x)dx = \widetilde{p}}$$

CHOOSING A SHAPE FUNCTION: $\psi(x)$

- Exact solution: Structural Mode (later in the course)... math can be complicated.
- Approximate solution: Choose $\psi(x)$
  - Must satisfy: Geometric boundary conditions (displacement, rotation).
  - Optional, but useful:
    - Satisfy "force" boundary conditions.
    - Choose $\psi(x)$ based on static deflection from a given loading.

GENERALIZED SDOF SYSTEM

EXAMPLE 1

Cantilever Beam: Find SDOF equivalent and the natural frequency ω_n.

GENERALIZED SDOF SYSTEM

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EXAMPLE 1 - APPLY TO METER STICK

Given:

$E = 1.75$ x 10^6 lb/in^2 ; $I = 0.0011$ in^4

$\therefore\ EI = 1925$ lb-in^2

$$m_{total} = mL = \frac{0.180 \text{ lb}}{386 \text{ in/s}^2}$$

$L = 95$ cm $= 37.4$ in

GENERALIZED SDOF SYSTEM

EXAMPLE 2 - BETTER SHAPE FUNCTION?

**Assume $\psi(x)$ is the deflected shape under self-weight: $\dfrac{d^4\psi}{dx^4} = \dfrac{w}{EI} =$ constant

GENERALIZED SDOF SYSTEM

EXAMPLE 2 - BETTER SHAPE FUNCTION?

Solve from BC's: $\psi(x) = 2\left(\frac{x}{L}\right)^2 - \frac{4}{3}\left(\frac{x}{L}\right)^3 + \frac{1}{3}\left(\frac{x}{L}\right)^4$

$$\left.\begin{aligned} \widetilde{m} &= 0.257\ mL \\ \widetilde{k} &= 3.20\ EI/L^3 \\ \widetilde{p} &= 0.4\ P_0 L \end{aligned}\right\} \longrightarrow \omega_n = 3.53\sqrt{\frac{EI}{mL^4}}$$

Exact solution (Euler-Bernoulli Beam Theory): $\omega_n = 3.518\sqrt{\frac{EI}{mL^4}}$

*Note: As long as all solutions obey BC's (displacement), the solution with the lowest natural frequency gives the most accurate approximation.

NOTE ON SHAPE FUNCTIONS TO APPROXIMATE MODE SHAPES

Mode #	Cantilever Beam	S.S. Beam

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