Fall Semester, 2025 **Instructor**: M.J. DeJong
CE 225 – Dynamics of Structures

Homework #2

Due: Friday, September 12

1) A mass m is initially at rest, partially supported by a spring and partially by stops (see Figure 1). In the position shown, the spring force is $mg/4$. At time t = 0 the stops are rotated, suddenly releasing the mass. Determine the motion of the mass.

Main concept: definition of the coordinate system and relation with the form of the equation of motion.

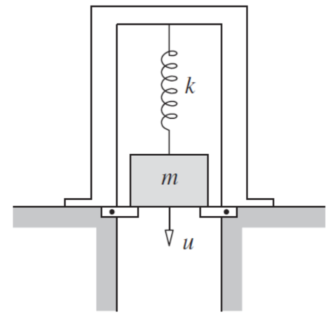


Figure 1

2) The weight of the wooden block shown in Fig. 2 is 20 lb and the spring stiffness is 120 lb/in. The block is initially at rest. A bullet weighing 0.4 lb is fired at a speed of 50 ft/sec into the block and becomes embedded in the block. Determine the resulting motion *u(t)* of the block.

**Hint: Use conservation of momentum.*

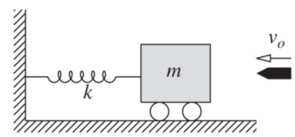


Figure 2

3) The vertical suspension system of an automobile is idealized as a viscously damped SDF system. Under the 3500-lb weight of the car the suspension system deflects 2.1 inches. The suspension is designed to have a damping ratio of $\zeta = 0.7$ with no one in the car.

(a) Calculate the damping and stiffness coefficients of the suspension.

(b) With four 150-lb passengers in the car, what is the effective damping ratio?

(c) Calculate the natural vibration frequency for case (b).

Main concept: static equilibrium position and the vibration properties of a system with damping.

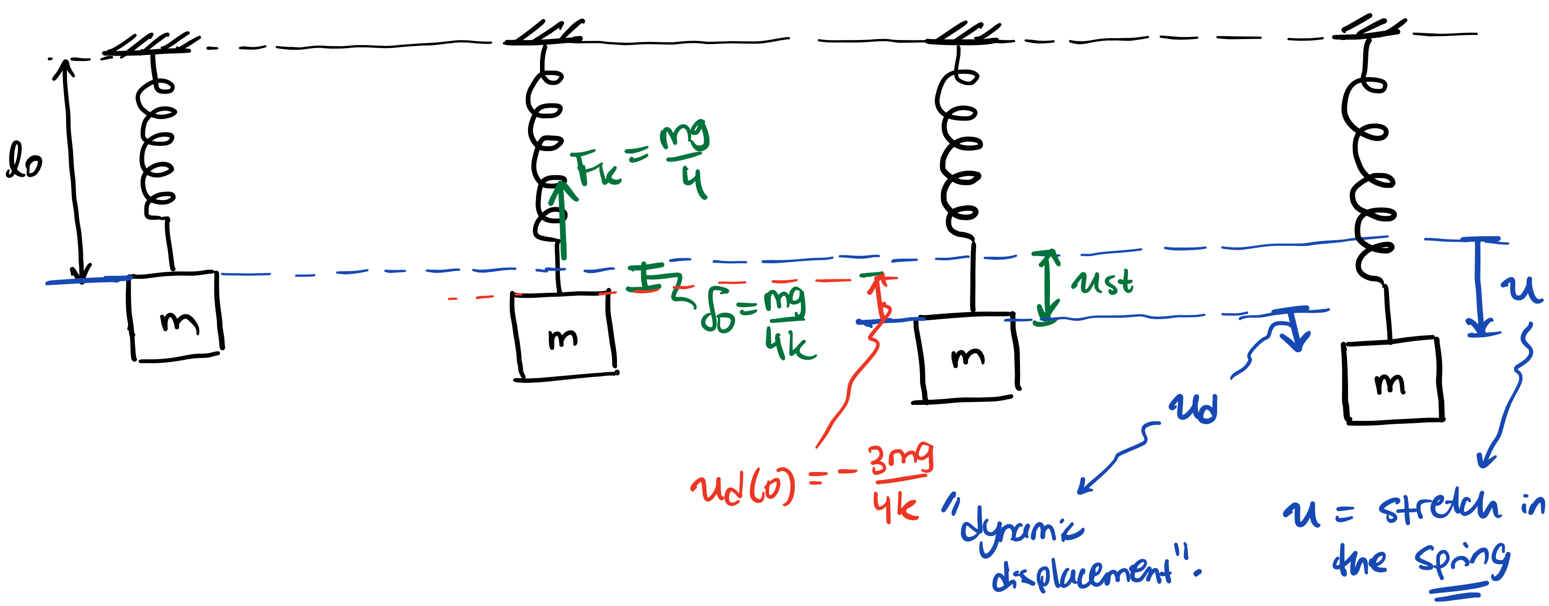
4) Find a linear elastic oscillator, measure its damped natural frequency and determine its percentage of critical damping. Submit a sketch of the system, your measurements, and calculations.

Note: It is up to you how you measure the response. One option is to use your cell phone as an accelerometer.

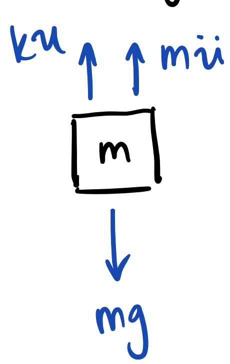
Solution.

Problem #1.

Position of no force in the spring. @ $t=0$ static equilibrium position @ time $= t$



A free body diagram gives the E.O.M @ time t.



$$\Rightarrow \quad m\ddot{u} + ku = mg$$

$$u_{st} = \frac{mg}{k}.$$

(see picture above)

if we introduce $u_d = u - u_{st}$

$$\ddot{u}_d = \ddot{u}$$

for u_d the EOM is:

$$m\ddot{u}_d + ku_d = 0$$

and the solution to this EOM is:

$$u_d(t) = A \cdot \cos(\omega_n t) + B \cdot \sin(\omega_n t)$$

where A and B can be found based on the initial conditions.

@ $t=0$ (see picture above)

$$u_d(0) = -\frac{3mg}{4k} \qquad \dot{u}_d(0) = 0 \quad \text{(starts from rest)}.$$

and:

$$\dot{u}_d(t) = -A \cdot \omega_n \cdot \sin(\omega_n t) + B \cdot \omega_n \cdot \cos(\omega_n t)$$

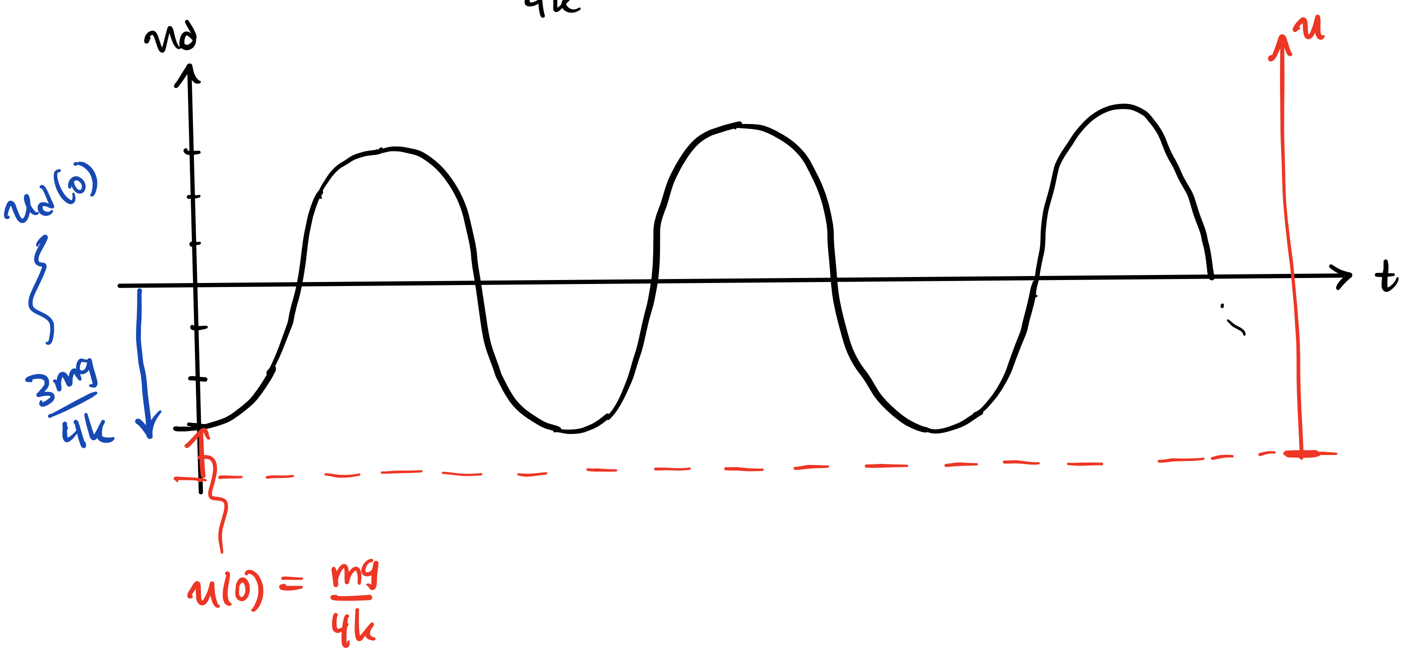
$$\dot{u}_d(0) = 0 \Rightarrow \qquad B \cdot \omega_n \cdot (1) = 0 \rightarrow \boxed{B = 0}$$

and $u_d(0) = u_0 \rightarrow \quad A \cdot (1) = u_0 \quad \rightarrow \boxed{A = u_0}$

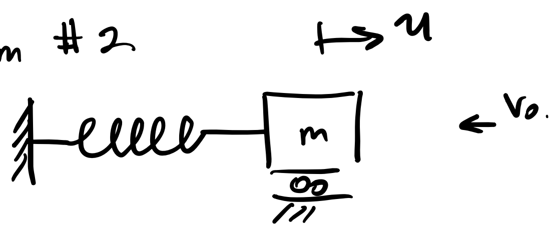
thus, the motion:

$$u_d(t) = -\frac{3mg}{4k} \cdot \cos(\omega_n t)$$

and $u(t) = -\frac{3mg}{4k} \cdot \cos(\omega_n t) + \frac{mg}{k}$ (because $u(t) = u_{st} + u_d(t)$)



Problem # 2



E.O.M: $m\ddot{u} + ku = 0$

$\Rightarrow \quad u(t) = A\cdot\cos(\omega_n t) + B\cdot\sin(\omega_n t)$

where $u(0) = 0 \rightarrow A\cdot(1) + B\cdot(0) = 0 \rightarrow A = 0$

$\dot{u}(0) = \dot{u}_0$

$\hookrightarrow \quad \dot{u}(t) = B\cdot\omega_n\cdot\cos(\omega_n t)$

$\dot{u}(0) = \dot{u}_0 \rightarrow B\cdot\omega_n = \dot{u}_0 \rightarrow B = \dfrac{\dot{u}_0}{\omega_n}$

and $u(t) = \dfrac{\dot{u}_0}{\omega_n}\cdot\sin(\omega_n t)$

So, need to get $\dot{u}_0 \rightarrow$ conservation of linear momentum!

(m_b is the mass of the bullet).

$$\underbrace{m\cdot(0) + m_b\cdot v_0}_{\text{before impact}} = \underbrace{(m+m_b)\,\dot{u}_0}_{\text{after impact.}}$$

Bullet now is part of the block.

$$\Rightarrow \quad \dot{u}_0 = -\frac{m_b}{m+m_b}\cdot v_0$$

Plugging the values: $\dot{u}_0 = -\dfrac{0.4}{20.4\ lb}\cdot 80\ ft/sec = -0.98\ (ft/sec)$

$= -11.76\ (in/sec).$

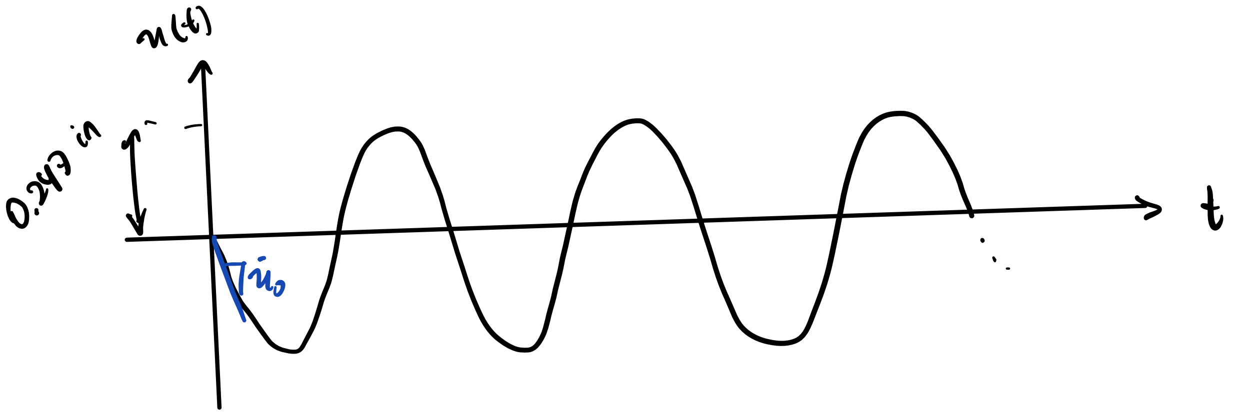
Now, $\omega_n = \sqrt{k/M} \longrightarrow$ here $M = 20.4\ lb$!

$$= \sqrt{\frac{120\ lb/in}{20.4\ lb}\cdot 386\ \frac{in}{s^2}} = 47.65\ (rad/sec).$$

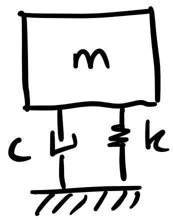
So :
$$\frac{\dot{u}_0}{\omega_n} = \frac{11.76}{47.65}\left(\frac{in/s}{1/s}\right) = -0.247 \quad (in)$$

and the motion is :

$$u(t) = 0.247 \cdot \sin\left(47.65t\right) \quad (in).$$



Problem # 3.



(a)

We are given: $\delta_{st} = 2.1 \text{ (in)}$

$$\delta_{st} = \frac{mg}{k} \Rightarrow \quad k = \frac{mg}{\delta_{st}} = \frac{3{,}500 \text{ (lb)}}{2.1 \text{ (in)}} = 1667 \text{ (lb/in)}.$$

$$\boxed{k = 1667 \text{ (lb/in)}}$$

for damping c: $c = 2m \cdot \omega_n \cdot \zeta$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1667 \text{ (lb/in)}}{3{,}500 \text{ (lb)}} \cdot 386 \text{ (in/s}^2\text{)}} = 13.56 \text{ (rad/s)}$$

mass pound.

force pound.

$$c = 2 \cdot 3500 \text{ (lb)} \cdot 13.56 \left(\frac{\text{rad}}{\text{s}}\right) \cdot 0.7 = 66{,}444 \text{ (lb/sec)}$$

however, if we want $[c \cdot \dot{x}] = [\text{Force}]$ we can get c in $\frac{\text{lb} \cdot \text{s}}{\text{in.}}$

$$c = 64{,}444 \left(\frac{\text{lb}}{\text{s}^2}\right) \cdot \frac{1}{386} \cdot \left(\frac{\text{s}^2}{\text{in}}\right) = 172.13 \left(\text{lb} \cdot \frac{\text{s}}{\text{in}}\right).$$

(b) New mass, same stiffness, same c.

$$\omega_n^{NEW} = \sqrt{\frac{k}{M}} = \sqrt{\frac{1667 \text{ (lb/in)}}{150.4 + 3500 \text{ (lb)}} \cdot 386 \text{ (in/s}^2\text{)}} = 12.53 \text{ (rad/s)}$$

now $c = 2m\omega_n \zeta \Rightarrow \zeta = \frac{c}{2m\omega_n}$

for dimensionless ζ.

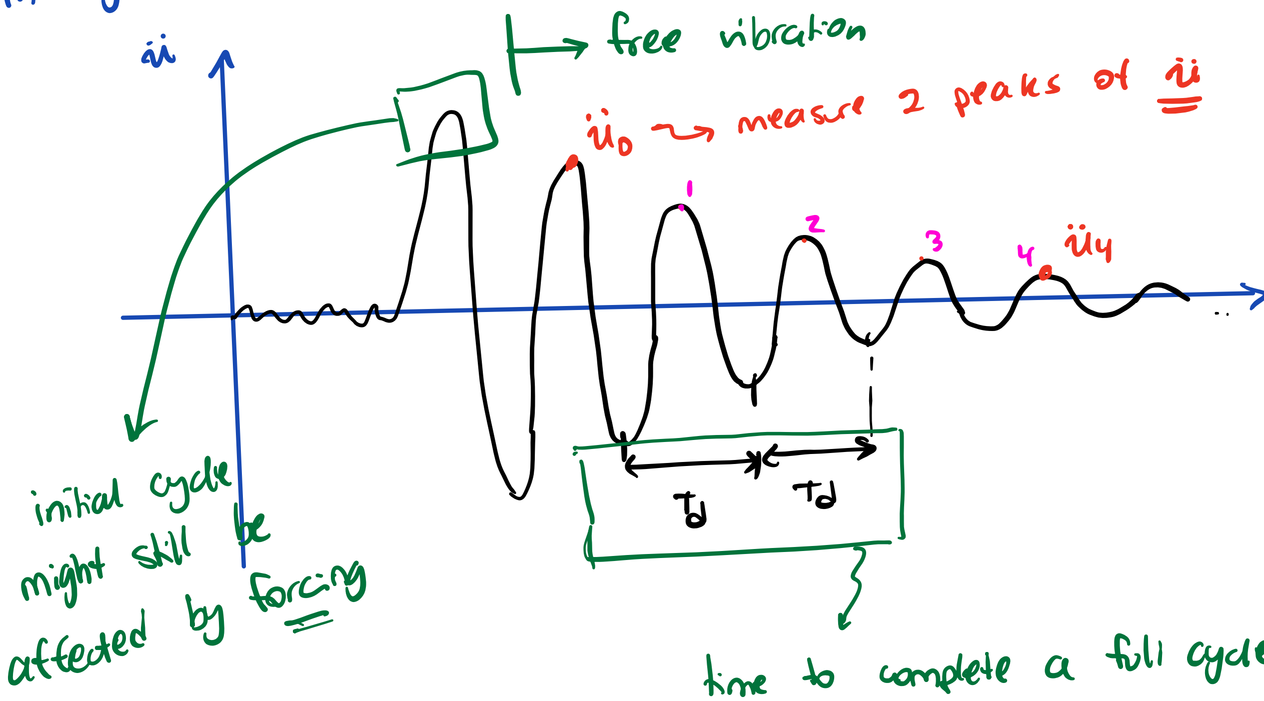
$$\Rightarrow \zeta^{NEW} = \frac{c}{2M \cdot \omega_n^{NEW}} = \frac{172.13 \text{ (lb}\cdot\text{s/in)}}{2 \cdot (3500 + 150.4 \text{ lb}) \cdot 12.53 \text{ (1/s)}} \cdot 386 \text{ (in/s}^2\text{)}$$

$$\Rightarrow \zeta^{NEW} = 0.647.$$

(c) $\omega_n^{NEW} = 12.53 \text{ (rad/sec)}.$

Problem #4.

The general idea was to measure $\ddot{u}(t)$ for free vibration.



time to complete a full cycle = Td

then $\omega_d = \dfrac{2\pi}{T_d}$.

From the two peaks of $\ddot{u}$, get ζ using log decrement:

$$\ln\left(\frac{u_i}{u_{i+j}}\right) = 2\pi\zeta j \rightarrow \text{ for my example plot:}$$

$$\ln\left(\frac{u_0}{u_4}\right) = 8\pi\zeta \Rightarrow \zeta = \frac{\ln(u_0/u_4)}{8\pi}.$$