LECTURE 4 - HARMONIC FORCING (PART 2)

CE 225

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DYNAMIC AMPLIFICATION

TRIPARTITE SPECTRA - PLOT

$$R_v = \frac{\omega}{\omega_n} R_d \quad \longrightarrow \quad \log R_v = \log \frac{\omega}{\omega_n} + \log R_d$$

$$R_v = \frac{1}{\omega/\omega_n} R_a \quad \longrightarrow \quad \log R_v = -\log \frac{\omega}{\omega_n} + \log R_a$$

DYNAMIC AMPLIFICATION

TRIPARTITE SPECTRA - PLOT

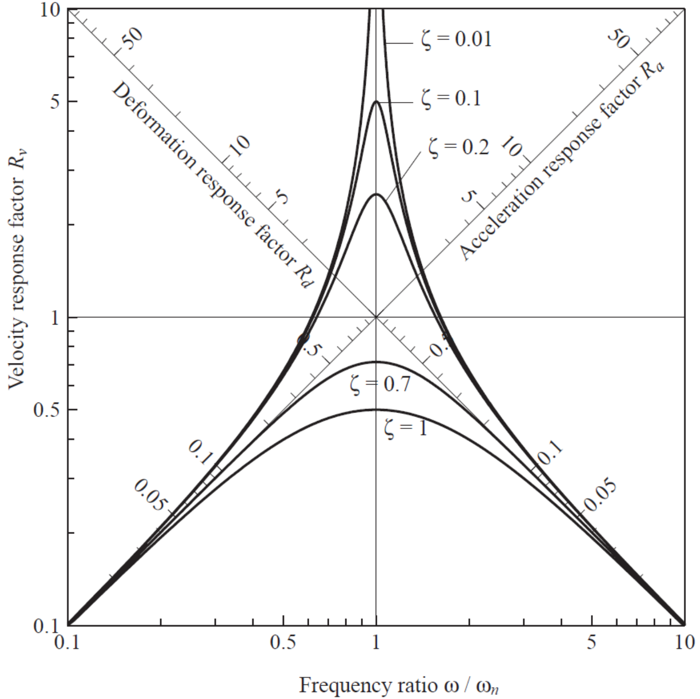

Figure 3.2.8 Four-way logarithmic plot of deformation, velocity, and acceleration response factors for a damped system excited by harmonic force.

DYNAMIC AMPLIFICATION

HALF-POWER BANDWIDTH METHOD

$\longrightarrow$ Find damping in "real" structure

1. Plot data
2. Estimate $R_{d,max}$
3. Find $\omega_a, \ \omega_b$ at $\frac{R_{d,max}}{\sqrt{2}}$
4. Damping $\rightarrow$ $\boxed{\zeta = \frac{\omega_b - \omega_a}{2\omega_n}}$

Narrow = low damping

Wide = high damping

DYNAMIC AMPLIFICATION

PROOF

$$R_d\left(\frac{\omega}{\omega_n}\right) = \frac{R_{d,max}}{\sqrt{2}} \longrightarrow \text{solve for } \omega \text{ to find } \omega_a,\ \omega_b$$

$$\frac{1}{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\left(\frac{\omega}{\omega_n}\right)\right]^2} = \frac{1}{\sqrt{2}}\,\frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

Solution: $\dfrac{\omega}{\omega_n} \approx 1 \pm \zeta \quad \rightarrow \quad \dfrac{\omega_a}{\omega_n} \approx 1-\zeta \quad ; \quad \dfrac{\omega_b}{\omega_n} \approx 1+\zeta \quad \rightarrow$

$$\boxed{\frac{\omega_b - \omega_a}{\omega_n} = 2\zeta}$$

VIBRATION GENERATOR (ECCENTRIC MASS)

FBD:

$$\longrightarrow \begin{cases} F_x = (m_e e\omega^2)\cos\omega t \\ F_y = (m_e e\omega^2)\sin\omega t \end{cases}$$

Two masses:

$$\longrightarrow \begin{cases} F_x = 2\left(\dfrac{m_e}{2} e\omega^2\right)\cos\omega t \\ F_y = 0 \end{cases}$$

VIBRATION GENERATOR (ECCENTRIC MASS)

TWO MASSES

Essentially creates harmonic forcing of magnitude: $p_0 = m_e e \omega^2$

$$\therefore \quad (u_{st})_0 = \frac{p_0}{k} =$$

$$\longrightarrow \begin{cases} \text{Max disp. } u_{max} = (u_{st})_0 R_d = \\ \\ \text{Max acc. } \ddot{u}_{max} = \dfrac{p_0}{m} R_a = \dfrac{m_e e \omega^2}{m} R_a \left(\dfrac{\omega_n}{\omega_n}\right)^2 = \end{cases}$$

$$\boxed{\frac{\ddot{u}_{max}}{\frac{m_e}{m} e \omega_n^2} = \left(\frac{\omega}{\omega_n}\right)^2 R_a}$$

VIBRATION GENERATOR (ECCENTRIC MASS)

FIDGET SPINNER EXAMPLE

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FIDGET SPINNER EXAMPLE

VIBRATION GENERATOR (ECCENTRIC MASS)

STEPS TO FIND R_d AND ζ

1. Test $\longrightarrow$ Measure max response @ multiple frequencies:

   Recall: $$R_a = \frac{\ddot{u}_{max}}{\left(\frac{m_e}{m_{\text{eff}}}\right) e\omega_n^2 \left(\frac{\omega}{\omega_n}\right)^2}$$

2. Divide y-axis by $\frac{m_e}{m} e\omega^2$ and multiply x-axis by $\frac{2\pi}{\omega_n}$ $\longrightarrow$

3. Divide R_a by $\left(\frac{\omega}{\omega_n}\right)^2$ to get R_d $\longrightarrow$

4. Apply Half-power bandwidth $\longrightarrow$

HARMONIC FORCING

FORCE TRANSMISSION

Recall Solution: $u(t) = (u_{st})_0 R_d \sin(\omega t - \phi)$

$$\therefore \quad \dot{u}(t) = (u_{st})_0 R_d \omega \cos(\omega t - \phi)$$

$$f_T = ku(t) + c\dot{u}(t)$$

$$\rightarrow$$

$$\therefore \quad f_{T\text{max}} = (u_{st})_0 R_d \sqrt{k^2 + c\omega^2}$$

Transmissibility (TR) $= \dfrac{f_{T,max}}{p_0} = R_d \sqrt{1 + \left(\dfrac{c}{k}\omega\right)^2}$

$$\longrightarrow \boxed{TR = R_d \sqrt{1 + \left(2\zeta \frac{\omega}{\omega_n}\right)^2}}$$