CE 225: Dynamic of Structures **Fall 2025**

Discussion 6: Response of Inelastic Systems - Applications

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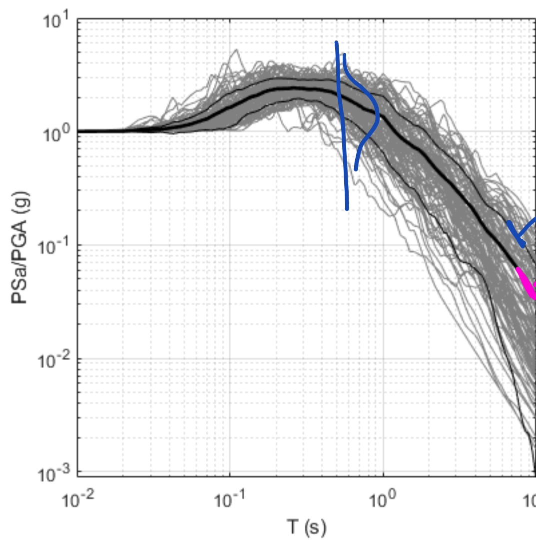
Announcements

- Mid-semester evaluation (scan QR code below).

From Response Spectra to Design Spectra

In last Homework, you had to develop the response spectrum for a specific ground motion. We can do this for any ground motion, as long as we have the recorded accelerations.

In fact, we can compile data from many earthquakes, and compute the response spectrum for all of them to get something that looks like the Figure below. It shows a log-log plot made with multiple earthquake acceleration records available in the PEER NGA-West database.



mean + 1 std

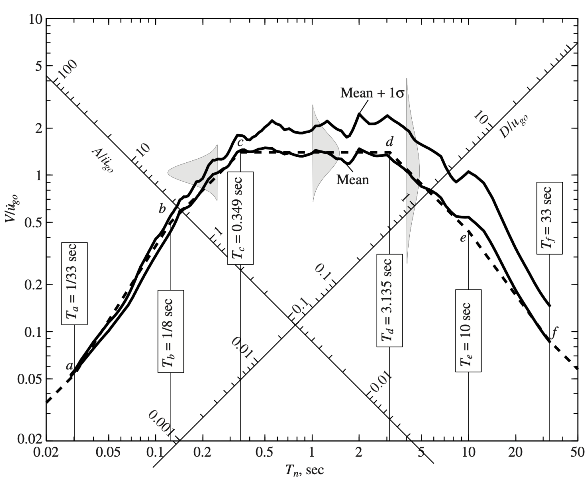
mean

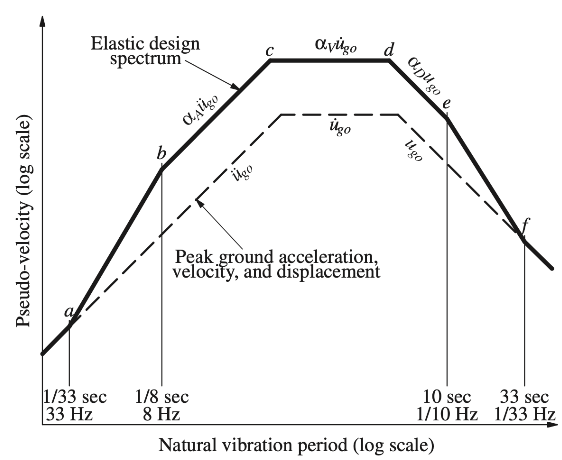
smooth lines in different intervals

Design Spectra

How to go from there to design spectra? Need a simplified version.

V





Design spectrum: 84th percentile

50th percentile (median).

Effect of Damping

more damping = less demands.

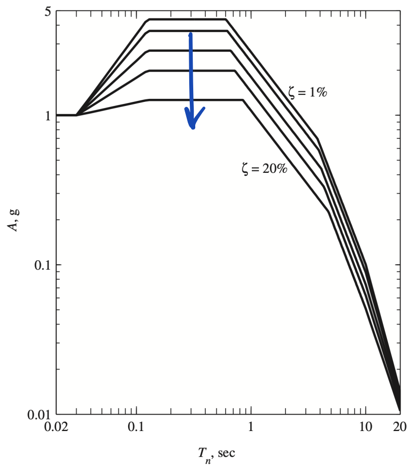


Figure 6.9.8 Pseudo-acceleration design spectrum (84.1th percentile) for ground motions with $\ddot{u}_{go} = 1\text{g}$, $\dot{u}_{go} = 48$ in./sec, and $u_{go} = 36$ in.; $\zeta = 1, 2, 5, 10,$ and 20%.

A (linear scale).

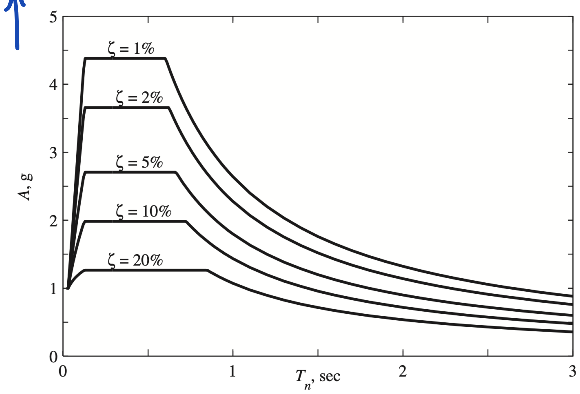
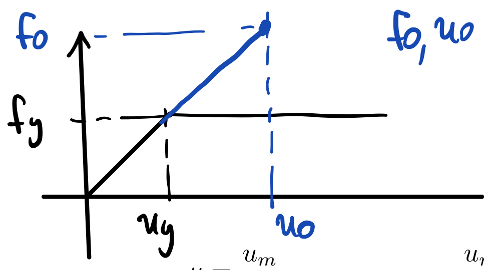


Figure 6.9.9 Pseudo-acceleration design spectrum (84.1th percentile) for ground motions with $\ddot{u}_{go} = 1\text{g}$, $\dot{u}_{go} = 48$ in./sec, and $u_{go} = 36$ in.; $\zeta = 1, 2, 5, 10,$ and 20%.

Effect of Inelastic Behavior



fo, uo : peak elastic responses.

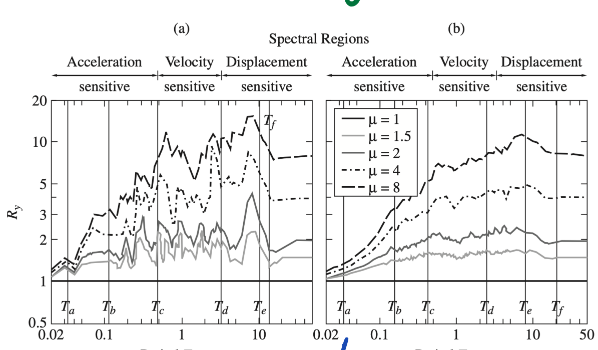
$$\overline{f}_y = \frac{f_y}{f_o} = \frac{u_y}{u_o} = \frac{1}{R_y} \qquad \mu = \frac{u_m}{u_y} \qquad \frac{u_m}{u_o} = \mu \overline{f}_y = \frac{\mu}{R_y}$$

Formulas above does not establish a relation between μ and R_y directly. So, need additional formulas to define:

$\mu \rightarrow f_y, u_m$

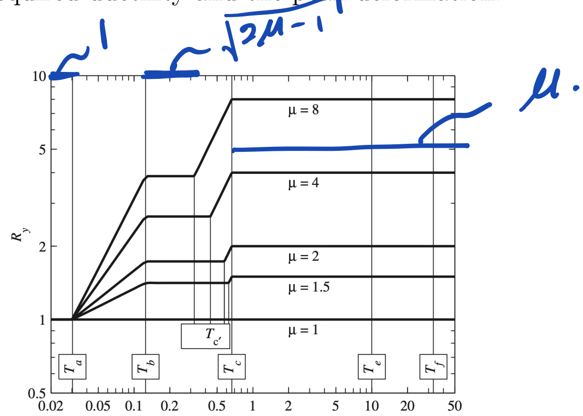
- Case (1): Given an allowed ductility μ compute the required yield strength f_y and the design peak deformation u_m.
- Case (2): Given some yield strength f_y compute the required ductility and the peak deformation.

$f_y \rightarrow \mu, u_m$



one ground motion

median of many grand motions.



smooth design curves.

$$R_y = \begin{cases} 1 \\ \sqrt{2\mu - 1} \\ \mu. \end{cases}$$

Elastic and Inelastic Design

Example 1

Consider a one-story frame with lumped weight W and natural vibration period in the linear elastic range $T_n = 0.25$ s. Determine the maximum lateral deformation and maximum lateral force (in terms of W) for which the frame should be designed if:

(a) The system is required to remain elastic.

(b) The allowable ductility factor is $\mu = 4$.

(c) The allowable ductility factor is $\mu = 8$. Assume that $\zeta = 5\%$ and elastoplastic force-deformation behavior. The design earthquake has a PGA=0.5g, and the elastic design spectrum is shown below.

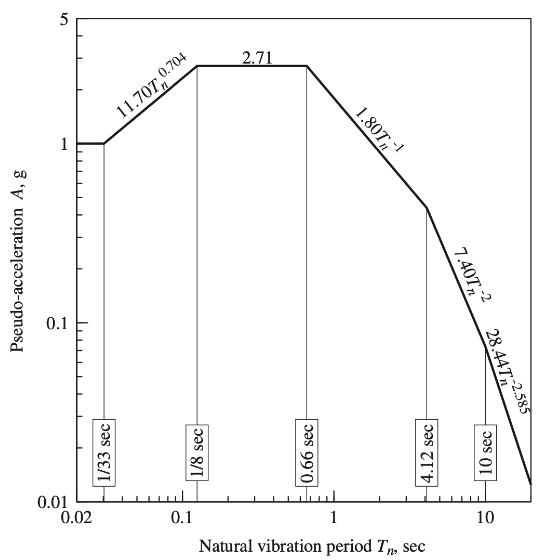


Figure 6.9.5 Elastic pseudo-acceleration design spectrum (84.1th percentile) for ground motions with $\ddot{u}_{go} = 1$g, $\dot{u}_{go} = 48$ in./sec, and $u_{go} = 36$ in.; $\zeta = 5\%$.

(a) Elastic system.

$$T_n = 0.25 \text{ (sec)} \rightarrow \frac{A}{PGA} = 2.71 \Rightarrow A = 2.71 \cdot 0.5g = 1.355g.$$

$$f_o = m \cdot A = \frac{w}{g} \cdot 1.355g = 1.355\,w$$

$$u_o = D = \frac{A}{\omega_n^2} = \frac{1.36 \cdot 386 \text{ (in/s}^2)}{(2\pi / 0.25)^2} = 0.831''.$$

(b) if $\mu = 4$

for $T_n = 0.25$ (sec) $\quad R_y = \sqrt{2\mu - 1} = 2.646$

$$\Rightarrow f_y = \frac{f_o}{R_y} = \frac{1.355\,w}{2.656} = 0.512\,w$$

$$u_m = \mu \cdot u_y \quad \text{but} \quad \frac{u_y}{u_o} = \frac{1}{R_y} \Rightarrow u_y = \frac{u_o}{R_y}$$

$$u_m = \mu \cdot \frac{u_o}{R_y} = 4 \cdot \frac{0.831 \text{ (in)}}{2.646} = 1.256 \text{ (in)}$$

(c) if $\mu = 8 \Rightarrow R_y = \sqrt{2\mu - 1} = 3.873.$

$$f_y = \frac{f_o}{R_y} = \frac{1.355\,w}{3.873} = 0.350\,w$$

$$u_m = \mu \cdot \frac{u_o}{R_y} = \frac{8 \cdot 0.831 \text{ (in)}}{3.873} = 1.733 \text{ (in)}$$

larger μ $\Rightarrow$ larger u_m

$\downarrow$

larger R $\Rightarrow$ smaller f_y

"weaker" more ductile structure.

Example 2

Consider a one-story frame with lumped weight W, $T_n = 0.25$ s, and $f_y = 0.512W$. Assume that $\zeta = 4\%$ and elasto-plastic force-deformation behavior. Determine the lateral deformation for the design earthquake defined in the previous example.

if $T_n = 0.25$ (sec)

$$f_0 = 1.355\,w \Rightarrow R_y = \frac{f_0}{f_y} = \frac{1.355\,w}{0.512\,w} = 2.646$$

can compute ductility:

$$R_y = \sqrt{2\mu - 1} \Rightarrow R_y^2 = 2\mu - 1 \quad \mu = \frac{R_y^2 - 1}{2} = 4.0$$

and $$u_m = \mu \cdot \frac{u_0}{R_y} = 4 \cdot \frac{0.831 \text{ (in)}}{2.646} = 1.256 \text{ (in)}.$$

$E = 29{,}000$ (ksi)
$I = 800$ (in^4)
PGA = 0.5g.

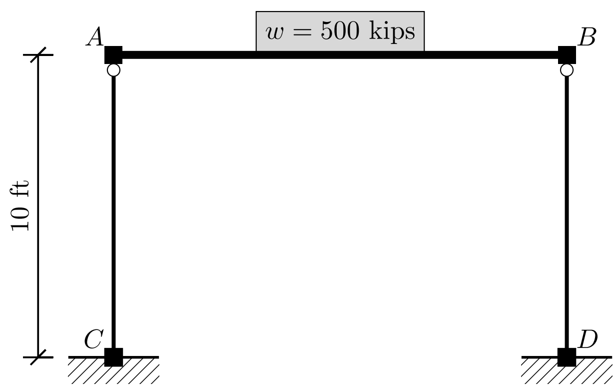
Example 3

For the following structural configuration, the corresponding lateral yield strengths are given as a function of the moment strength of the columns M_y. Here we are assuming elastoplastic behavior of the columns.

These two structures are to be designed with an $R_y = 6$. Compute the design yield strength for each structure, the corresponding required ductility, and the deformation the structures are to be designed for. Which of the two configurations would you recommend?

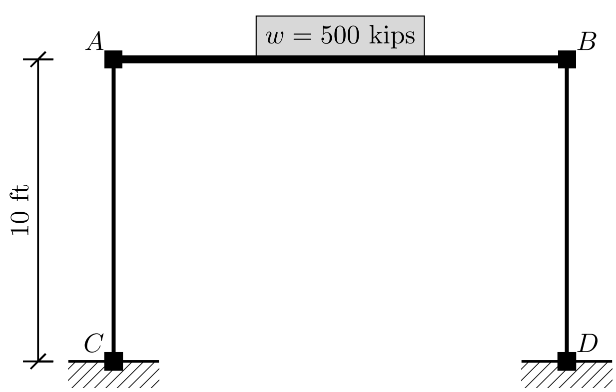
Configuration #1

$$f_y = \frac{2M_y}{L} \qquad u_y = \frac{L^2}{3EI}M_y \qquad k = \frac{6EI}{L^3}$$



Configuration #2

$$f_y = \frac{4M_y}{L} \qquad u_y = \frac{L^2}{6EI}M_y \qquad k = \frac{24EI}{L^3}$$



Structure #1

$$k_1 = \frac{6EI}{L^3} = \frac{6 \cdot 29{,}000\,(ksi) \cdot 800\,(in^4)}{(10 \cdot 12\ in)^3} = 80.56\ (kips/in)$$

$$\omega_{n1} = \sqrt{\frac{k}{m}} = \sqrt{\frac{80.56\,(kip/in) \cdot 386\,(in/s^2)}{500\ kips}} = 7.85\ (rad/s)$$

$$\Rightarrow T_1 = \frac{2\pi}{\omega_{n1}} = 0.8\ (sec)$$

Structure #2:

$$k_2 = \frac{24EI}{L^3} = 322.22\ (kips/in) \Rightarrow \omega_{n2} = \sqrt{\frac{k}{m}} = 15.71\ (rad/sec)$$

$$T_2 = \frac{2\pi}{\omega_{n2}} = 0.4\ (sec).$$

From design spectrum:

– Structure #1: $\frac{A}{PGA} = \frac{1.8}{T_n} \Rightarrow A = \frac{1.8 \cdot 0.5g}{0.8} = 1.125g$

$$f_0 = mA = \frac{w}{g} \cdot 1.125g = 1.125\ w$$

$$f_y = \frac{1.125}{6}w = 0.1875w = 93.75\ (kips) = \frac{2M_y}{L}$$

$$\Rightarrow \boxed{M_y = 5{,}625\ (kip \cdot in)}$$

For this T_n range:

$$\mu = R_y \Rightarrow \mu = 6 \quad \text{and} \quad u_m = \mu \cdot \frac{u_0}{R_y} = u_0 = D = \frac{A}{\omega_n^2}$$

$$\Rightarrow u_m = \frac{1.125 \cdot 386}{(7.85^2)} = 7.05\ (in).$$

For structure 2:

$$A = 2.71 \cdot 0.5g = 1.355g \Rightarrow u_0 = D = 2.12\ (in)$$

$$f_0 = m \cdot A = 1.355w \Rightarrow f_y = 0.226w = 112.9\ (kips \cdot in)$$

$$f_y = \frac{4 M_y}{L} \Rightarrow M_y = \frac{112.9 \cdot 10 \cdot 12}{4} \Rightarrow \boxed{M_y = 3{,}387.5 \text{ (kip} \cdot \text{in)}}$$

For this T_n : $R_y = \sqrt{2\mu - 1} \Rightarrow \mu = \frac{R^2 - 1}{2} = 18.5$.

$$u_m = \mu \cdot \frac{u_0}{R_y} = 18.5 \cdot \frac{2.12}{6} = 6.53 \text{ (in)}$$

Q: is it realistic to have $\mu = \underline{\underline{18.5}}$?

what about residual deformations?