CE 225: Dynamic of Structures **Fall 2025**

Midterm Review Notes

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Introduction

This document is a quick guide to help you review the main concepts for the first midterm. It is not meant as a comprehensive list of topics or concepts you need to know for the exam, but it might help your review process.

1 Equations of motion

You are expected to be able to write down the equations of motion for single degree of freedom systems with multiple springs, dashpots and masses, for translational or rotational motions. Remember that we have three types of forces that arise:

$$f_S : \text{Spring forces}$$
$$f_D : \text{Damping forces}$$
$$f_I : \text{Inertial forces}$$

- Spring forces are proportional to the **deformation** of the spring or elastic element. So make sure the u you're plugging into your $f = ku$ is a deformation (relative displacement between the two ends of the element).
- Damping forces are proportional to the rate of change of the deformation $\dot{u}$ (relative velocity). Again, you need to be careful to use the relative velocity in the case of systems subjected to ground motions for instance.
- Inertial forces are proportional to the translational or rotational inertia (depending on the type of motion on the system) and the **total acceleration**.

All of these forces **oppose** the motion or deformation or rate of deformation.

Before drawing the free body diagram with the forces, try to figure out how the system will move, what is the degree of freedom that you're studying, and how deformations will occur. Define your coordinate system and draw the system in a deformed configuration. This will help you figure out the direction of the forces on your FBD and the correct deformations to use to plug into the spring and dashpot forces.

For rotational systems, it might be better to use a rotational degree of freedom θ. How does a translational ground motion affect a rotational problem?

2 Free vibration

A system is on free vibration if there are no applied external forces or ground motion (except maybe gravity, and we can get rid of the effects of gravity in our EOM by using the "dynamic displacement" concept. See Lecture 1 and Discussion 4 for details).

You are given the solutions to the EOM:

$$m\ddot{u} + c\dot{u} + ku = 0 \tag{2.1}$$

on your formula sheet in terms of the initial conditions for the cases of small damping ($\zeta < 1$) and zero damping. However, you are expected to know the meaning of the parameters there (ω_n, ζ...), and their effect in the response.

If necessary, write down the EOM of the system you're analyzing, to make sure you're using the right coefficients. If something is confusing, go back and draw the FBD, and get the EOM, write it down as Equation 2.1 so you can interpret the coefficients in the formula depending on what they are for your specific problem (e.g., building with multiple columns where k comes from flexural stiffness coefficients).

For the initial conditions, think of how the selection of your coordinate system might affect the initial conditions you have to plug into the solution of the EOM.

If your system is on free vibration, you can also use the logarithmic decrement equations, which are given to you in the cheat sheet. You should know how to use them, and what are the assumptions you're making when using the simplified formula (the one on the right hand side).

3 Harmonic excitation

Here, our system is subjected to a **permanent harmonic force** in the form:

$$p(t) = p_o \sin \omega t$$

And you EOM should look like:

$$m\ddot{u} + c\dot{u} + ku = p_o \sin \omega t \tag{3.2}$$

before applying the formulas on the cheat sheet.

You have the steady-state response solution in the formula sheet $u(t)$, in terms of R_d and the phase angle for $\zeta < 1$. Steady-state is what happens when your system has been exposed to the force for a while, and then it vibrates at the frequency of the forcing function ω. Can you **sketch a plot of the steady-state response** of a system if given the forcing function, i.e., physically interpret the meaning of R_d and ϕ. How about the transient response?

The R_d factor modifies the static solution $u_{st} = p(t)/k$ depending on the ratio of the frequencies. R_d is a **deformation factor**, so it will give you how much is the peak deformation on your system, compared to the static response (it won't give you total displacement for a ground motion, for that you have T_R). The phase shift ϕ represents the time lag of the peak deformations with respect to the peaks of $p(t)$. How damping affect these two? Can you sketch the plots of the ratio of the frequencies vs R_d and ϕ?

Transmissibility

Transmissibility can mean two things: for problems with an applied harmonic force, transmissibility is the ratio of the peak transmitted force (through the damper and the spring $f_T = c\dot{u} + ku$) to the support of our system and the peak applied force p_o.

When we are dealing with systems subjected to harmonic ground motion, we can also use the definition above, with $p_o = -m\ddot{u}_{go}$. But in this case, you can also use T_R to find the motion transmition from the ground to your system, in this case as peak total motion in displacement u_o^t or acceleration $\ddot{u}_o^t$ (see formula sheet).

Energy concepts

You should be familiar with the concept of equivalent viscous damping, and on what context we use the formula given on the cheat sheet (see section 3.9 on the textbook). How much energy is dissipated by viscous damping in one cycle of harmonic motion? How much energy is "stored" in the springs at peak deformation?

Miscellaneous

Make sure you understand the concepts behind the formulas given on the cheat sheet:

- **Half power bandwidth**. What are the terms in the equation? How do we conduct a test to measure them? How can you use the formula?
- **Vibration generator** Make sure you understand how to use the formula, and what are m_e and e.

4 Arbitrary forcing and pulses

If the force applied to your system is **not a permanent harmonic force**, then you need to find the solution using one of the three methods discussed. Here are some pros and cons for each of them:

(a) Duhamel's Integral

$$u(t) = \frac{1}{m\omega_D} \int_0^t p(\tau) e^{-\zeta\omega_n(t-\tau)} \sin\left[\omega_D(t-\tau)\right] d\tau$$

It gets very complicated to solve the problem this way if the system has damping, because you're integral term is ugly. If there's no damping, the response is found "simply" integrating using the formula.

Be careful with the limits of integration! You may want to draw the plot we used in Discussion to remind yourself to integrate up to the correct t for each interval of your forcing function. Remember that this solution is only for systems starting from rest.

(b) Solving the ODE

Solving the ODE involves finding a particular $u_p(t)$ and a complementary solution $u_h(t)$.

The particular is always in the shape of the forcing function, and it has to be a complete set of functions (complete 3rd degree polynomial for instance). You find the coefficients of the particular solution by plugging your u_p into the EOM, and solving the system of equations that arise from that.

The complementary solution is the solution to the homogeneous equation, which is on the shape of the free vibration response. You then find the coefficients of this $u_h(t)$ using the initial conditions on your total solution:

$$u(t) = u_h(t) + u_p(t)$$

So, you do it only after you found your $u_p(t)$.

(c) Superposition

You can solve by superposition decomposing the force applied to your system into multiple “simpler” forces that add up to the total applied force. You need to make sure that you know the solutions to the proposed decomposed individual forces.

On the formula sheet you are given the solutions for unit impulse, a step force and the ramp force. With these you can create any force that is linear within a certain interval, like a rectangular pulse or a triangular pulse. Think of combinations of these.

(d) Response to pulses

On top of familiarizing yourself with the pulse responses given on the cheat sheet, you should also check the response plots to different pulse shapes given in Lecture 7. Make sure you understand these, especially the rectangular pulse, and how to use the shock spectra (Figure 4.10.1 in the Textbook).

(e) Short pulse approximation

If a pulse can be considered to be short (this depends on the ratio t_d/T_n (duration/natural period $< 1/4$), you may consider applying the simplified equation with the impulse, and your solution is given by:

$$u(t) = I\left(\frac{1}{m\omega_n}\sin\omega_n t\right)$$

This equation only applies to systems without damping starting from rest.

5 Numerical schemes

There are a few questions that can come in the exam about this. You should know how to do the time step iterations if given the formulas, but you should also be familiar with some concepts regarding numerical methods:

- What it means for a method to be **implicit** vs **explicit**. Remember this has to do with the instant where you are enforcing equilibrium. Explicit schemes enforce equilibrium at your current time step, while implicit schemes do it at the next time step.

- **Conditional** vs **unconditional stability**. Check the formulas in the book regarding limits on the time step. You are not expected to memorize the complicated ones, but you should know these limits are meant for our numerical errors to be bounded, and therefore for our solution to be stable in time (not blow up, as that one case you had in the homework!).
- Check what are the **assumptions** made when developing the equations for each method (e.g., constant acceleration method means you assume that the acceleration during a time step is equal to the average of the current and the next time step accelerations).

6 Linear earthquake response

For earthquake response, the main concept is the response/design spectrum. You are expected to know how to generate a response spectrum, and conceptually understand how to get to the design spectrum, and the multiple alternatives available (PSHA and the Uniform Hazard Spectrum vs Design Spectra coming from Response Spectra). You should also be familiar with what happens to response spectra when modifying the damping ratio.

Make sure you're familiar with the response spectrum and you can read it in any of its shapes (log-log, linear-linear, or combinations, normalized and not-normalized), and how to get pseudo-accelerations pseudo-velocities and deformations.

Remember: pseudo-acceleration is an approximation to the total acceleration. The deformation spectrum (D vs T_n) gives you deformation values (or relative displacement values). You should know the definitions of these terms, and why they are used.

For design problems, make sure you understand where the forces are acting (inertial forces that arise from the earthquake) and how to use static analysis procedures to come up with the forces and stresses on the structural elements.

7 Nonlinear earthquake response

Make sure you know what all the terms in the formula sheet mean, and you can compute inelastic responses from elastic responses and vice-versa (R_y, μ, f_o, u_o, f_y and u_m). Conceptually understand the tradeoffs between elastic and inelastic design.

Also, conceptually understand how to generate inelastic response spectra. Where the engineering relations between μ and R_y come from and the regions of periods the formulas apply for (e.g., what is the equal displacement rule). These equations may or may not be given to you in the exam, so memorizing them would not be a bad idea.

HGP/MGF