CE 225: Dynamic of Structures **Fall 2025**

Discussion 2: Forced Vibration - Harmonic Excitation

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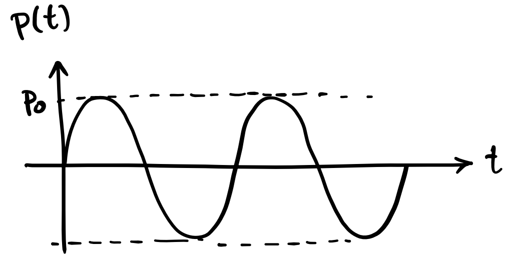
Objectives

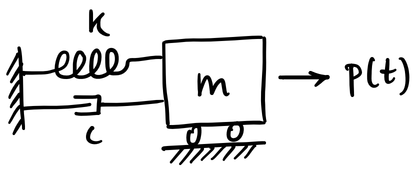
By the end of this discussion we'll be able to:

1. Find the lateral stiffness of a multi-column one-story shear building and its basic dynamic properties.
2. Find the damping coefficient of a structure from a resonance test.
3. Apply the equation for the transmissibility.



Harmonic excitation

$$\boxed{m\ddot{u} + c\dot{u} + ku = p(t) \quad \text{with} \quad p(t) = p_0 \sin \omega t}$$



; $p(t) = p_0 \cdot \sin(\omega t)$.

Cases

$\zeta = 0$ (UNDAMPED)

$$\boxed{m\ddot{u} + ku = p_0 \cdot \sin(\omega t)}$$

Homogeneous:

$m\ddot{u} + ku = 0$

$\rightarrow \ddot{u} + \omega_n^2 u = 0$

$\Rightarrow u_h(t) = A \cdot \cos(\omega_n t) + B \sin(\omega_n t)$

Particular: $\rightarrow$ for $p(t) = p_0 \cdot \sin(\omega_n t)$

$\downarrow$

Depends

$\downarrow$

ω / ω_n

$\frac{\omega}{\omega_n} \neq 1$

$u_p = C \cdot \sin(\omega t)$

$\frac{\omega}{\omega_n} = 1$

(Resonance)

$u_p = Ct \cos(\omega t)$

$\zeta \neq 0$ (DAMPED) $\underline{\underline{\zeta < 1}}$

$$\boxed{m\ddot{u} + c\dot{u} + ku = p_0 \cdot \sin(\omega t).}$$

Homogeneous:

$m\ddot{u} + c\dot{u} + ku = 0$

$\rightarrow \ddot{u} + 2\zeta\omega_n \dot{u} + \omega_n^2 u = 0$

$\Rightarrow u_h(t) = e^{-\zeta\omega_n t}\left(A \cos(\omega_d t) + B \sin(\omega_d t)\right)$

Particular.

$u_p(t) = C \cos(\omega t) + D \sin(\omega t)$

$\downarrow$

Solve for C and D

plugging this into E.O.M.

then do:

$C \cos(\omega t) + D \sin(\omega t) = \sqrt{C^2 + D^2} \sin(\omega t - \phi)$

(*) this can be done with trigonometric identity. –

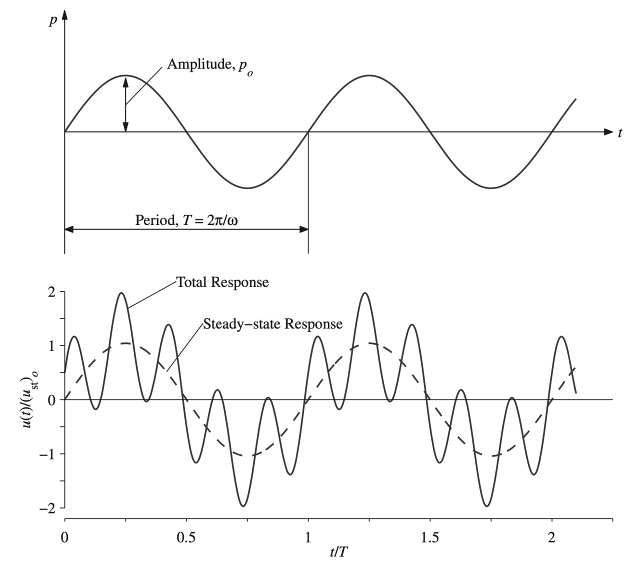
$$\boxed{u_p = (u_{st})_0 \cdot R_d \cdot \sin(\omega t - \phi)}$$

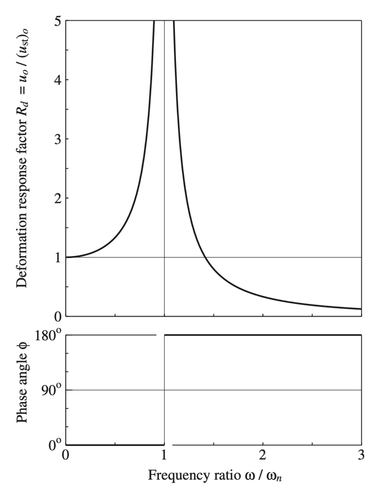
(steady state response).

(a) Undamped system ($\zeta = 0$)

Case i: $\omega/\omega_n \neq 1$

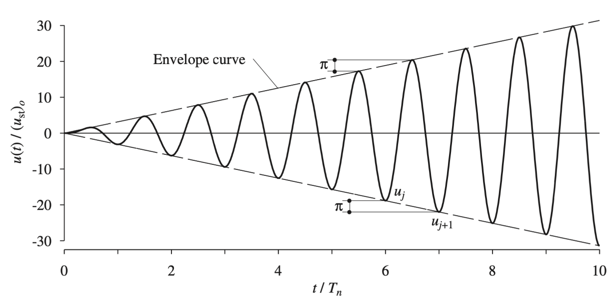
$$\boxed{u(t) = A\sin\omega_n t + B\cos\omega_n t + \frac{p_0}{k}R_d\sin(\omega t - \phi)} \quad \text{where} \quad R_d = \left|\frac{1}{1-\left(\dfrac{\omega}{\omega_n}\right)^2}\right|$$





Case ii: Resonance $\omega/\omega_n = 1$

$$\boxed{u(t) = A\cos\omega_n t + B\sin\omega_n t - \frac{p_0}{2k}\omega_n t\cos\omega_n t}$$

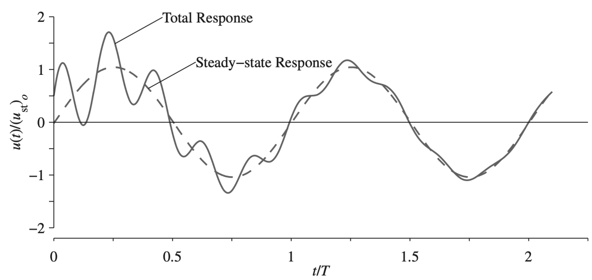


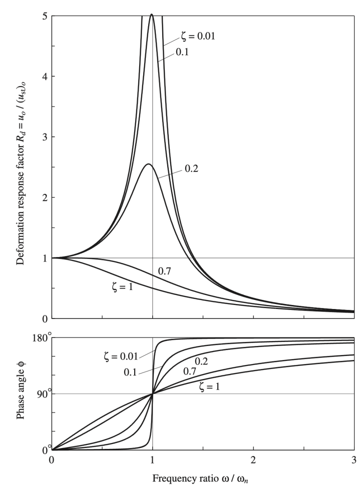
(b) Damped system with $\zeta < 1$

Case i: $\omega/\omega_n \neq 1$

$$\boxed{u(t) = e^{-\zeta\omega_n t}\left(A\cos\omega_n t + B\sin\omega_n t\right) + \frac{p_0}{k}R_d\sin\left(\omega t - \phi\right)}$$

$$\text{where} \quad R_d = \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2}} \quad \text{and} \quad \phi = \tan^{-1}\left[\frac{2\zeta\frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right]$$





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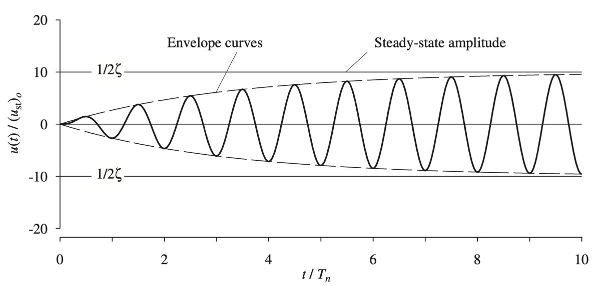
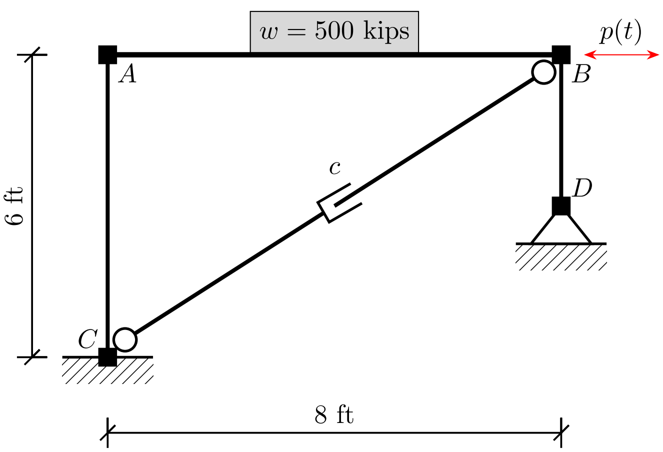


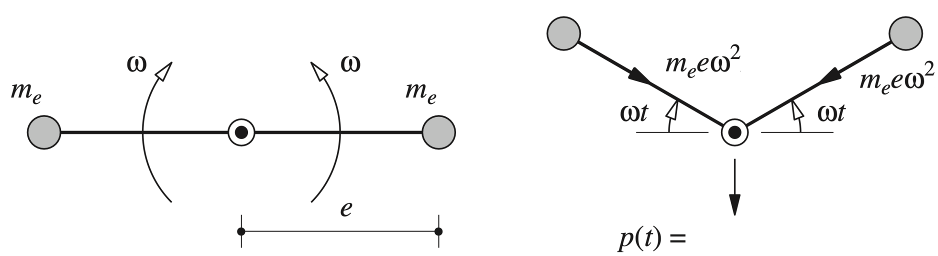
Vibration Generator

A one story reinforced concrete building has a roof weighing 500 kips, supported by two columns with $I = 448$ in^4 and $E = 29,000$ ksi (W16 × 36). The roof can be considered infinitely rigid ($EI = \infty$).

The building is excited by a vibration generator with two weights, each 50 lb, rotating about a vertical axis at an eccentricity of 12 in. When the vibration generator runs at the natural frequency of the building, the amplitude of roof acceleration at steady-state is measured to be 0.02g.

Determine the damping ratio of the structure (ζ).





Transmissibility

A vibration isolation block is to be installed in a laboratory so that the vibration from adjacent factory operations will not disturb certain experiments. If the isolation block weighs 2000 lb and the surrounding floor and foundation vibrate at 1500 cycles per minute, determine the stiffness of the isolation system such that the absolute motion of the isolation block is limited to 10% of the floor vibration; neglect damping