

### Homework #3

Due: Monday, September 22

- 1) In a forced vibration test under harmonic excitation it was noted that the amplitude of motion at  $\omega = \omega_n$  was exactly three times the amplitude at an excitation frequency 20% higher than  $\omega_n$ . Determine the damping ratio of the system.
- 2) A machine is supported on four steel springs for which damping can be neglected. The natural frequency of vertical vibration of the machine–spring system is 200 cycles per minute. The machine generates a vertical force  $p(t) = p_0 \sin \omega t$ . The amplitude of the resulting steady state vertical displacement of the machine is  $u_0 = 0.2$  inches when the machine is running at 20 revolutions per minute (rpm), 1.042 in. at 180 rpm, and 0.0248 in. at 600 rpm. Calculate the amplitude of vertical motion of the machine if the steel springs are replaced by four rubber isolators that provide the same stiffness but introduce damping equivalent to  $\zeta = 30\%$  for the system. Comment on the effectiveness of the isolators at various machine speeds.
- 3) Consider an industrial machine of mass  $m$  supported on spring-type isolators of total stiffness  $k$ . The machine operates at a frequency of  $f$  Hertz with a force unbalance of  $p_0$ .
  - (a) Determine an expression giving the fraction of force transmitted to the foundation as a function of the forcing frequency  $f$  and the static deflection  $\delta_{st} = mg/k$ . Consider only the steady-state response. Note: your expressions should include  $\delta_{st}$  and  $f$ .
  - (b) Determine the static deflection  $\delta_{st}$  for the force transmitted to be 20% of  $p_0$  if  $f = 10$  Hz.
- 4) An automobile is traveling along a multispan elevated roadway supported every 100 ft. Long-term creep has resulted in a 4-in. deflection at the middle of each span (see Figure a). The roadway profile can be approximated as sinusoidal with an amplitude of 2 in. and a period of 100 ft. The SDF system shown in Figure (b) is a simple idealization of an automobile, appropriate for a “first approximation” study of the ride quality of the vehicle. When fully loaded, the weight of the automobile is 3600 lbs. The stiffness of the automobile suspension system is 700 lb/in., and its viscous damping coefficient is such that the damping ratio of the system is 50%. Assuming the automobile does not lift off the road surface, determine the maximum and minimum contact force between the road and the automobile when the automobile is traveling at 65 mph. Would the automobile actually lift off the road?

\*Note: this problem uses the scenario of Example 3.4 in the textbook.

