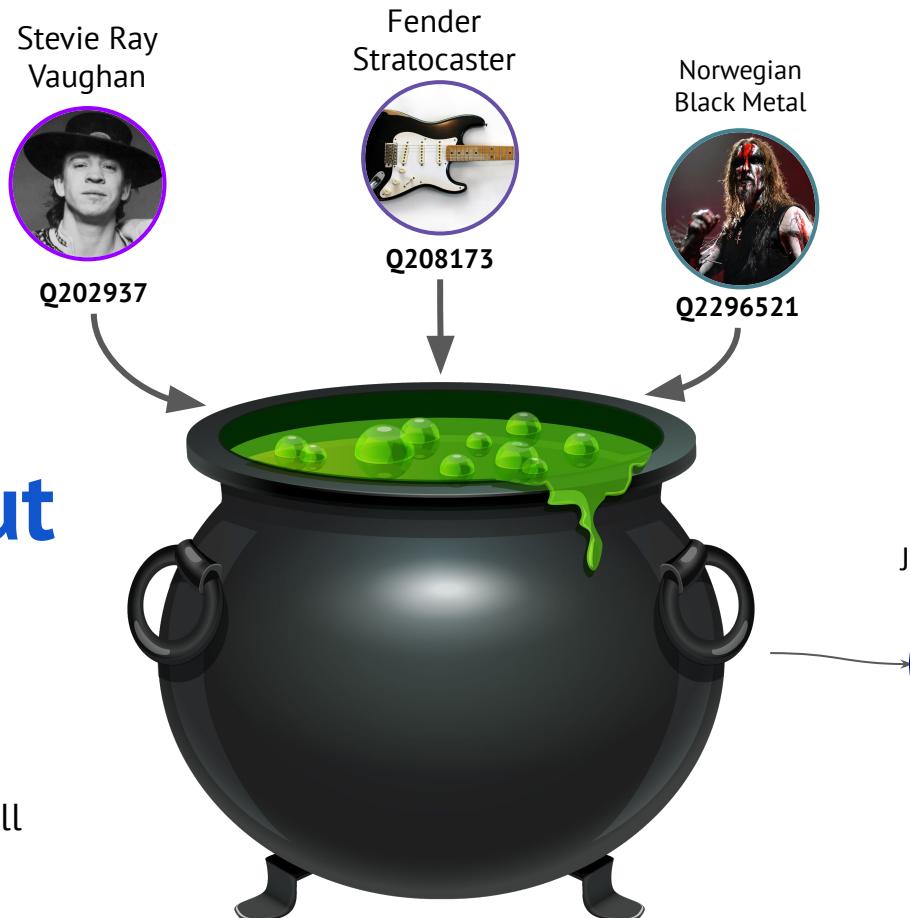


# Inductive Graph Reasoning without Node Features

Michael Galkin  
Postdoctoral Fellow @ Mila & McGill

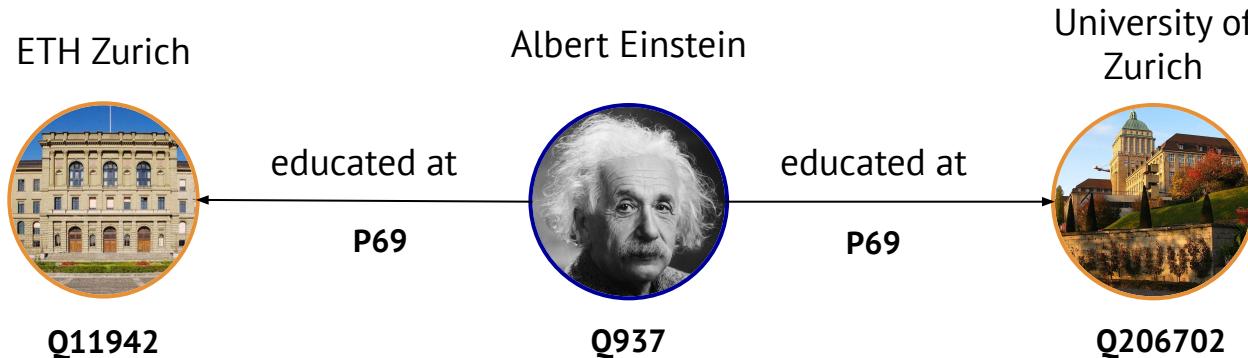


# Plan

- **Graph Reasoning Tasks**
- Featurization via Tokenization: NodePiece
- Featurization via Labeling Trick:  
    Neural Bellman-Ford and GNN-QE
- Past, Today, Future



# Triple-based Knowledge Graphs



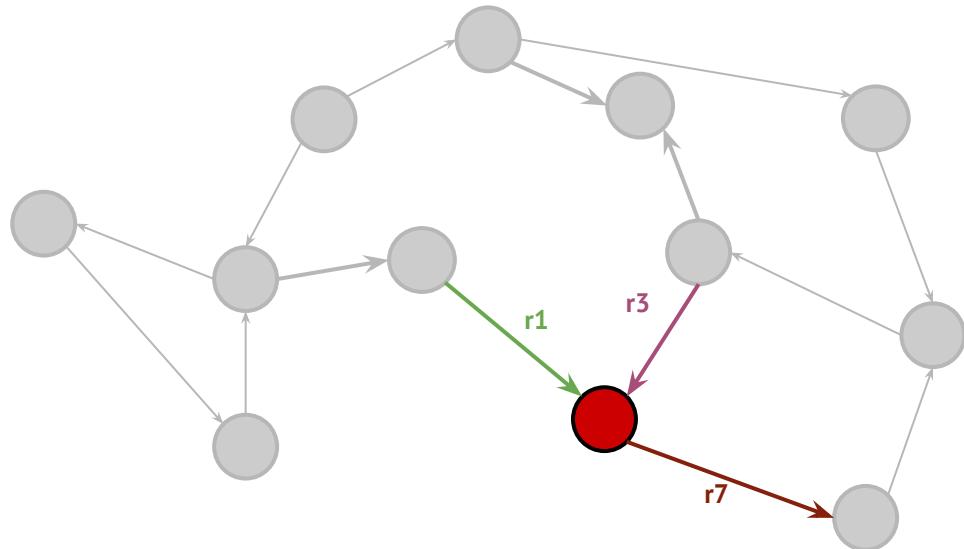
RDF

Albert Einstein  
Albert Einstein

educatedAt  
educatedAt

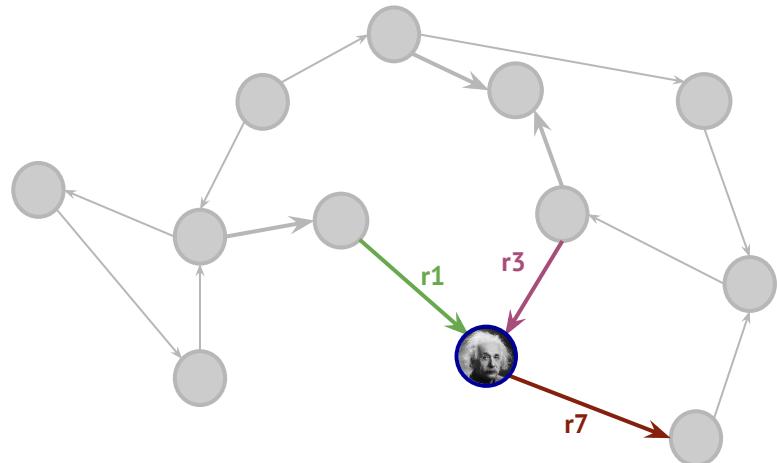
University of Zurich .  
ETH Zurich .

# Knowledge Graphs: Setup



- Directed graphs
- Explicit relation types (learnable edge features)
- Input node features are **not** given

# Graph Reasoning Tasks

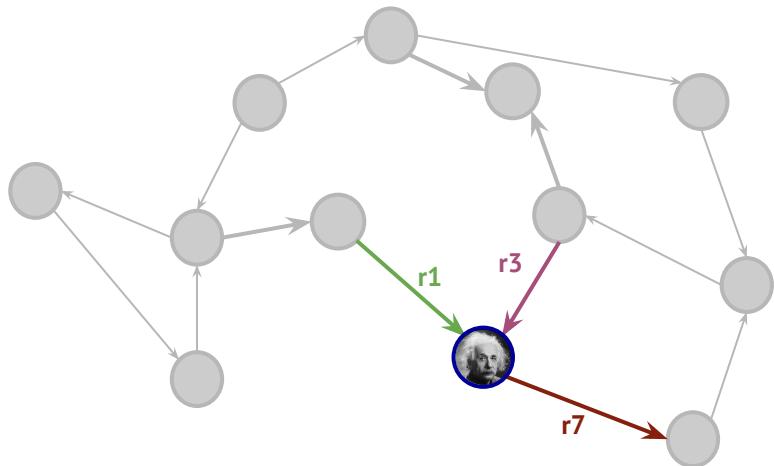


- Node Classification

$p(\text{ type(s) } | \text{ })$



# Graph Reasoning Tasks



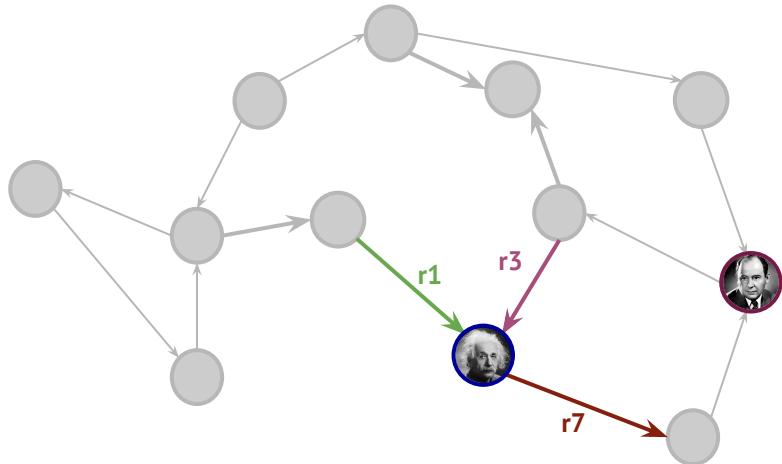
- **Node Classification**

$p(\text{ type(s)} | \text{Albert Einstein})$

- **Simple Link Prediction**

 educated at ?  $p(\text{tail} | \text{head, relation})$

# Graph Reasoning Tasks



- **Node Classification**

$p(\text{ type(s)} | \text{ })$



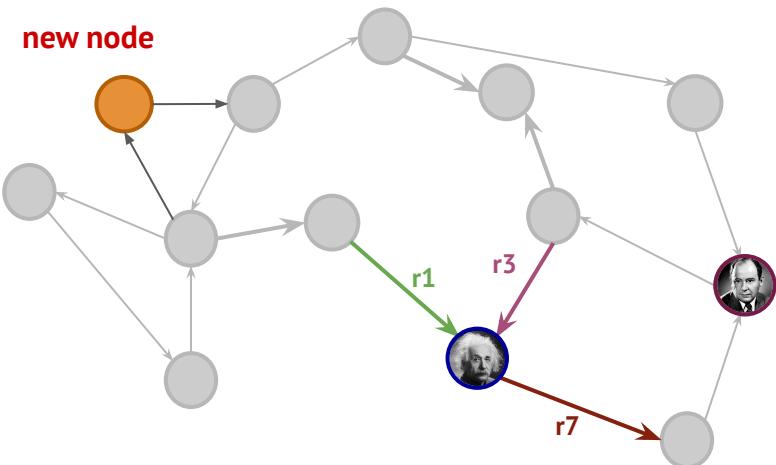
- **Simple Link Prediction**

 educated at → ?     $p(\text{ tail} | \text{ head, relation})$

- **Complex Query Answering**

 educated at → ?var     educated at → ?var    location → ?

# Inductive Graph Reasoning Tasks



Extend the same tasks to **new, unseen** nodes arriving at inference time

- **Node Classification**

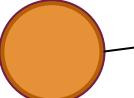
$$p(\text{ type(s) } | \text{new node})$$

- **Simple Link Prediction**



educated at  $\rightarrow ?$   $p(\text{tail} | \text{head, relation})$

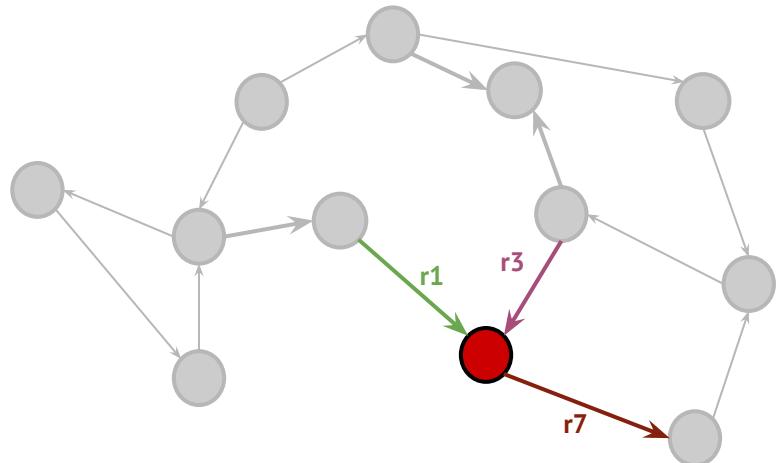
- **Complex Query Answering**

educated at  $\rightarrow ?var$  location  $\rightarrow ?$

educated at  $\rightarrow ?var$

# Knowledge Graphs: Setup

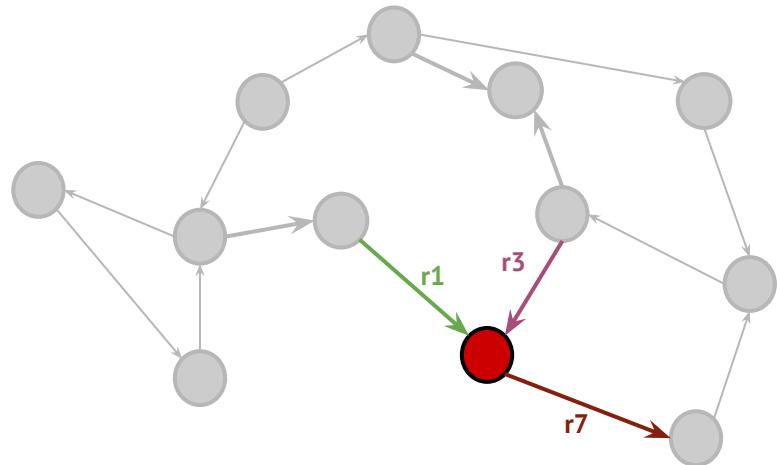


Any GNN-based pipeline needs features:

$$X' = \text{GNN}(X, A, W)$$

- **Input node features are not given**

# Knowledge Graphs: Setup



- Input node features are not given
- How do we get inductive features?

Any GNN-based pipeline needs features:

$$X' = \text{GNN}(X, A, W)$$



# Brief History of Transductive Learning: 2011 -

**RESCAL**

[Nickel et al, ICML 2011]

**TransE**

[Bordes et al, NeurIPS 2013]

100+ KG embedding models since then 😱



# Brief History of Transductive Learning: 2011 -

Transductive

Triples

Supervised

**RESCAL**

[Nickel et al, ICML 2011]

**TransE**

[Bordes et al, NeurIPS 2013]

100+ KG embedding models since then 😱



Link Prediction on FB15k-237

Leaderboard   Dataset



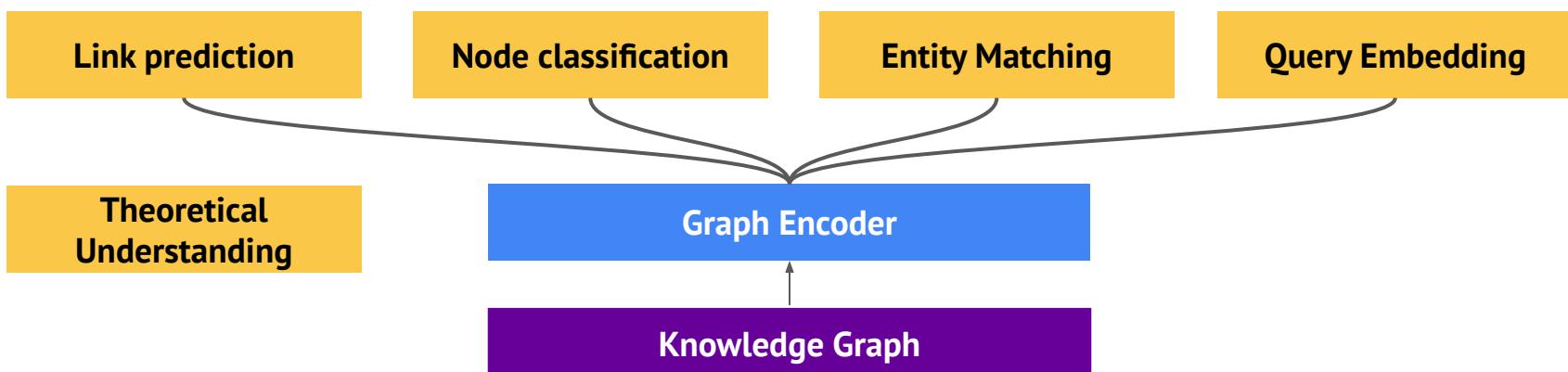
No substantial progress since 2018

# Big Picture in $\mathbb{R}^5$

Transductive	Triples	Supervised	Unimodal	Small
Inductive	Hyper-relational	Unsupervised	Multimodal	Large (sampling)

**SETTING**

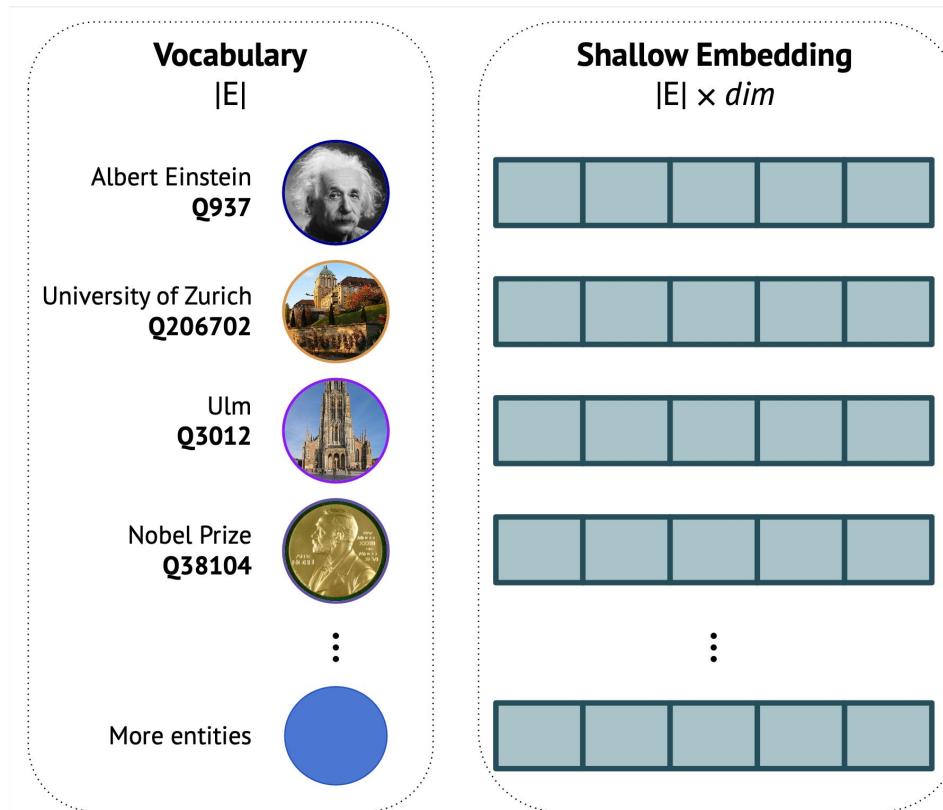
**TASK**



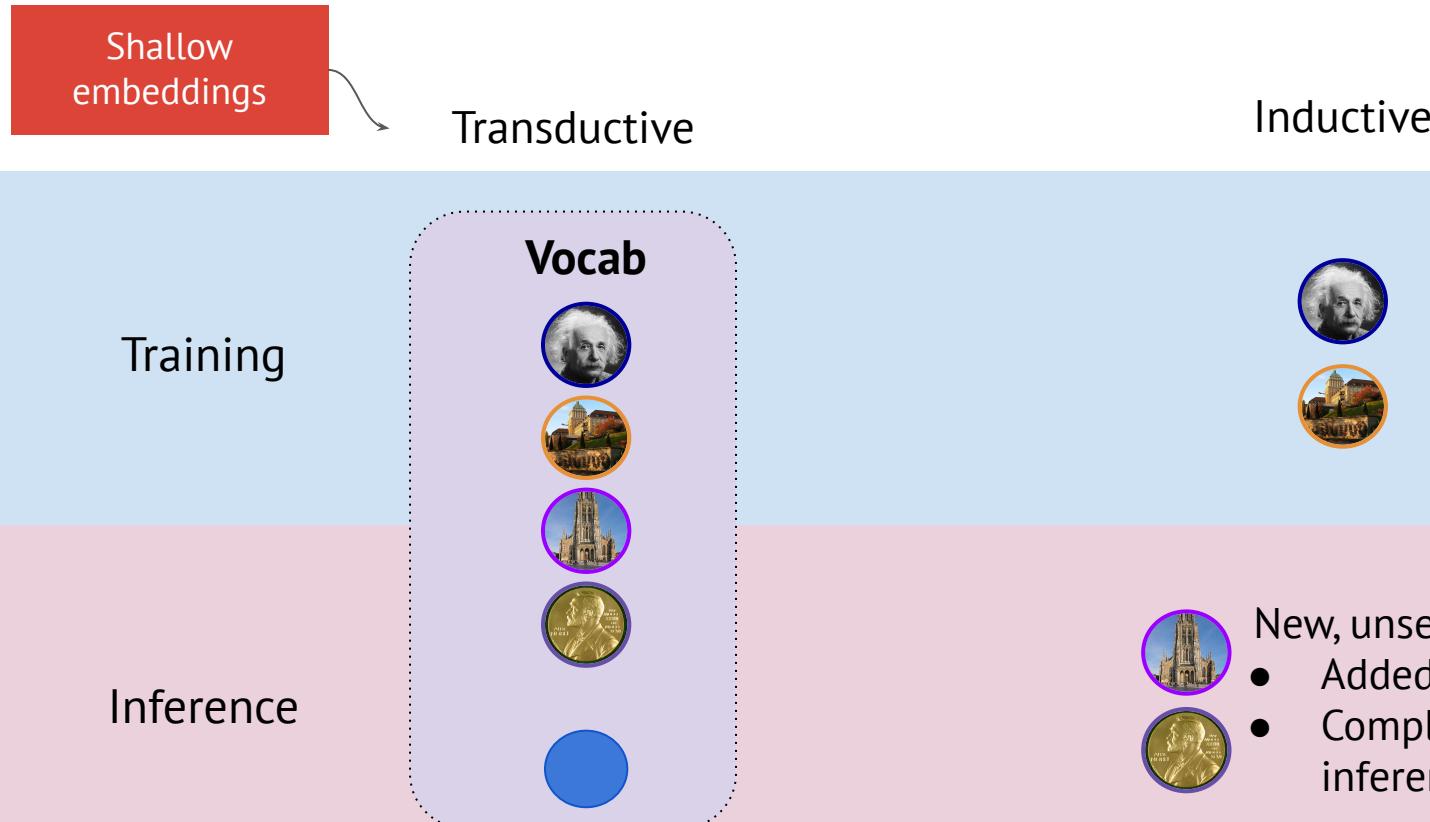
# Shallow Embedding

Looks like a  
Representation  
Learning challenge 🤔

Can we do better?



# Transductive vs Inductive



# OGB WikiKG: Just 2.5M nodes (June'21)

## Leaderboard for [ogbl-wikikg2](#)

The MRR score on the test and validation sets. The higher, the better.

Package:  $\geq 1.2.4$

Deprecated [ogbl-wikikg](#) leaderboard can be found [here](#).

BERT-Large is  $\sim 340M$  params

Rank	Method	Test MRR	Validation MRR	Contact	References	#Params	Hardware	Date
1	PairRE (200dim)	0.5208 ± 0.0027	0.5423 ± 0.0020	Linlin Chao	<a href="#">Paper</a> , <a href="#">Code</a>	500,334,800	Tesla P100 (16GB GPU)	Jan 28, 2021
2	RotatE (250dim)	0.4332 ± 0.0025	0.4353 ± 0.0028	Hongyu Ren – OGB team	<a href="#">Paper</a> , <a href="#">Code</a>	1,250,435,750	Quadro RTX 8000 (45GB GPU)	Jan 23, 2021
3	TransE (500dim)	0.4256 ± 0.0030	0.4272 ± 0.0030	Hongyu Ren – OGB team	<a href="#">Paper</a> , <a href="#">Code</a>	1,250,569,500	Quadro RTX 8000 (45GB GPU)	Jan 23, 2021
4	ComplEx (250dim)	0.4027 ± 0.0027	0.3759 ± 0.0016	Hongyu Ren – OGB team	<a href="#">Paper</a> , <a href="#">Code</a>	1,250,569,500	Quadro RTX 8000 (45GB GPU)	Jan 23, 2021

BERT (340M params) - disruption in NLP   
KG embs (>1B params) - 😬

## Life beyond shallow embedding?

Do we really need to learn & store the  
whole **shallow** embedding matrix  $|E| \times \text{dim}$  ?

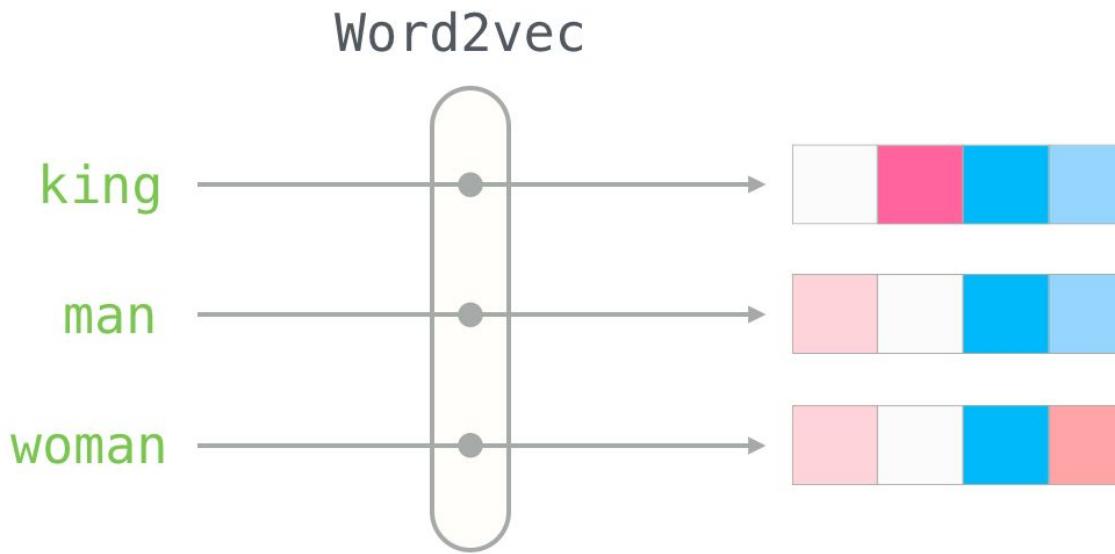
Trying to fit a  $100M \times 200$  tensor on a Tesla V100 ->



# Plan

- Graph Reasoning Tasks
- **Featurization via Tokenization: NodePiece**
- Featurization via Labeling Trick:  
Neural Bellman-Ford and GNN-QE
- Past, Today, Future

# Back to 2014



Unseen words = [OOV] (out-of-vocabulary)

# Byte-Pair Encoding / WordPiece

*Dictionary*

5 low  
2 lower  
6 newest  
3 widest

*Vocabulary*

l, o, w, e, r, n, w, s, t, i, d, es, est

Add a pair (es, t) with freq 9

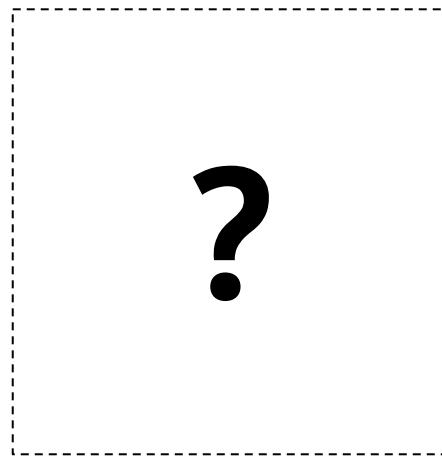
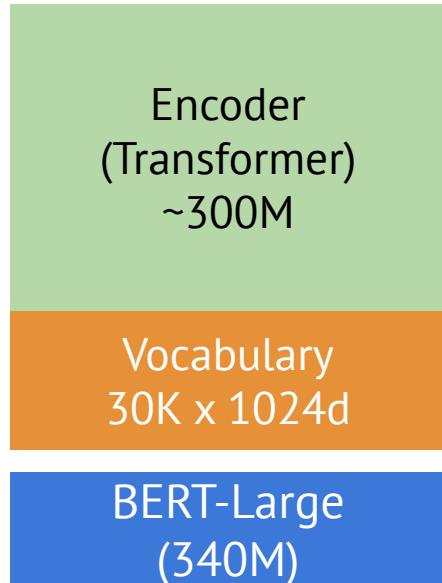
# Byte-Pair Encoding / WordPiece

"I love tacos, apples, and tea!"

i	love	tacos	,	app	##les	,	and	t	##e	##a	!
6	7	8	5	10	11	5	9	30	41	37	3

- Fixed-size vocab of subword units (30-50K)
- We can tokenize any unseen word

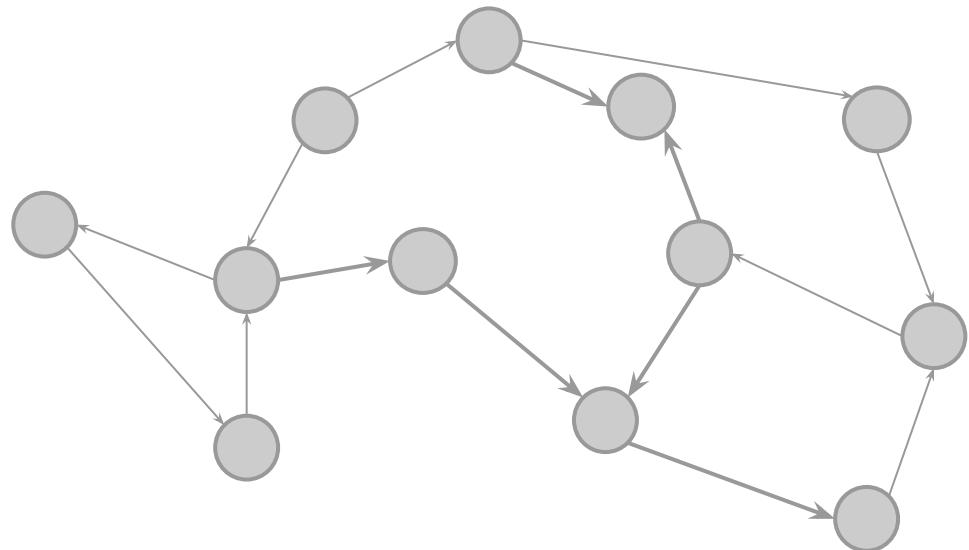
# Tokenizing KGs



Vocabulary  
2.5M x 500d

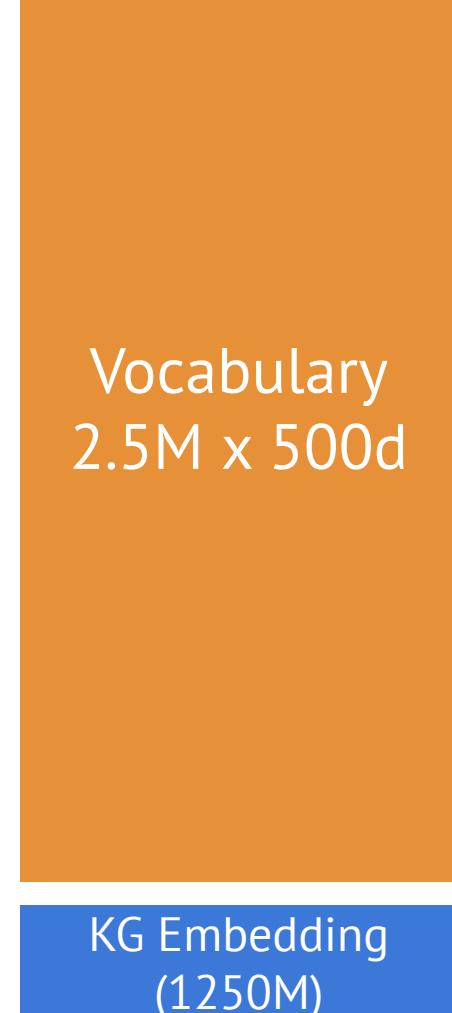
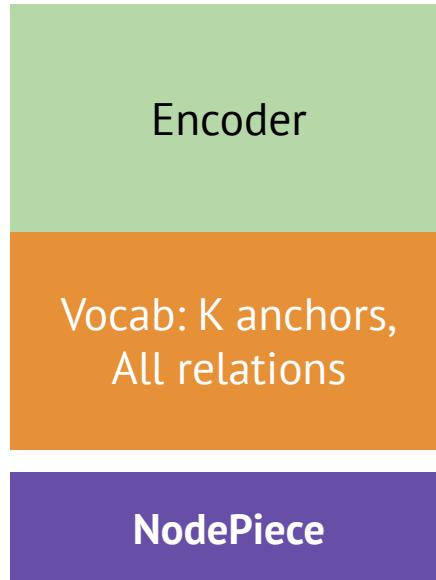
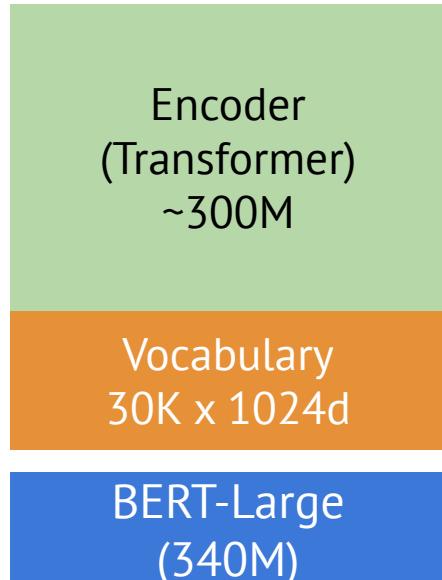
KG Embedding  
(1250M)

# Tokenization + Graphs?



If nodes in a graph are  
**"words"**,  
can we design a  
fixed-size vocab of  
**"sub-word"** units?

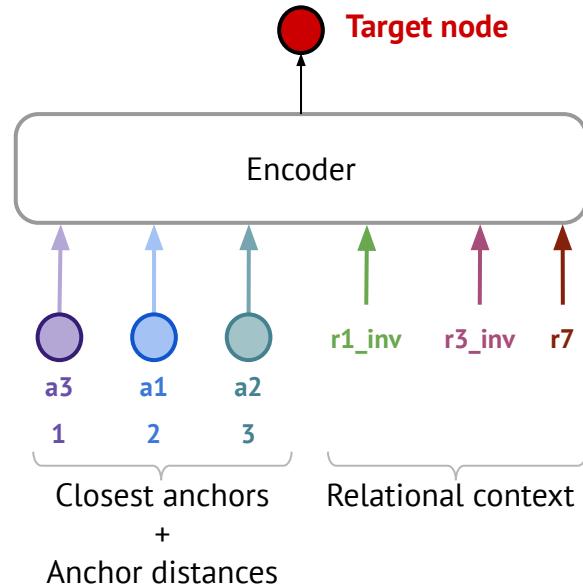
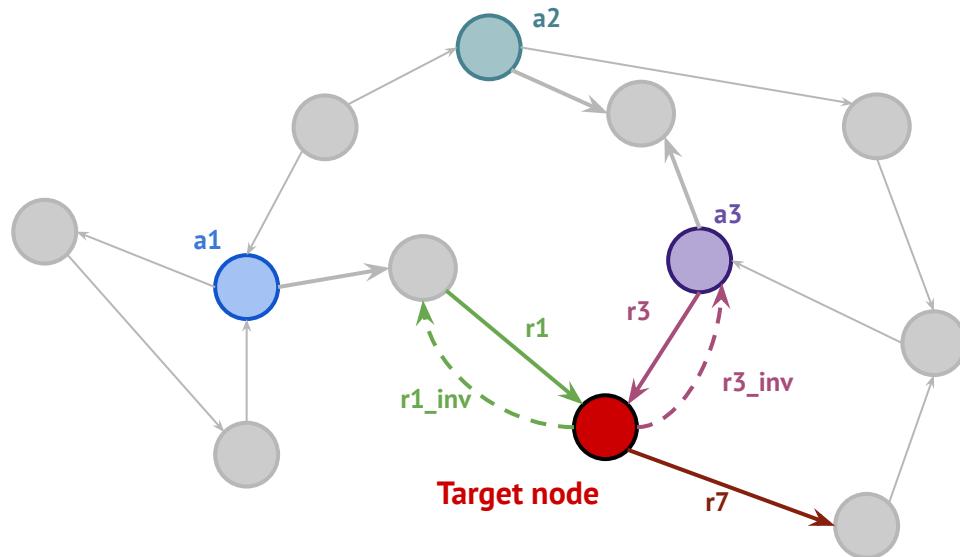
# Tokenizing KGs



# Tokenizing KGs

	Shallow embedding, only known words, otherwise OOV	Compositional representations, subword units
Language	Word2vec, GloVe	Byte-Pair Encoding, WordPiece
Graphs	All KG embedding algorithms (TransE, etc)	<b>NodePiece</b>

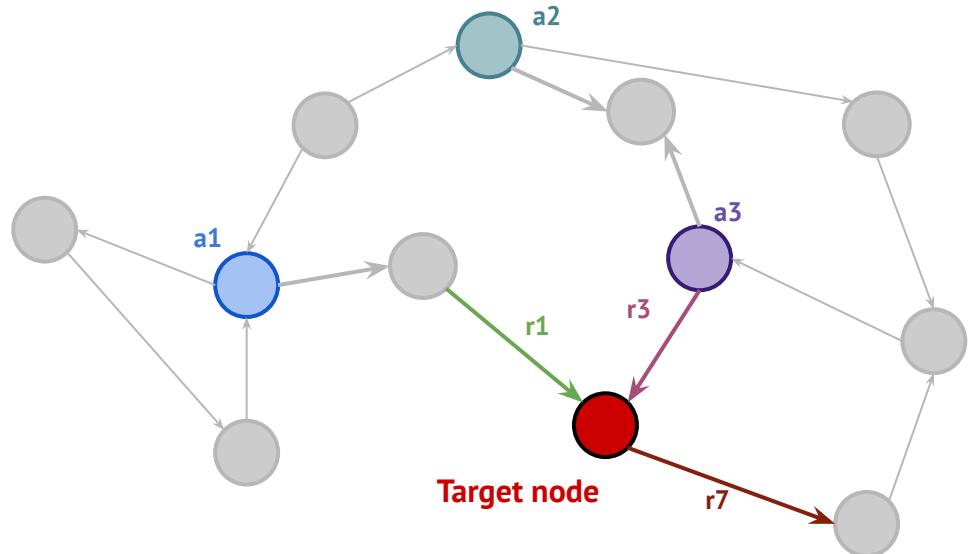
# NodePiece - “*subword units*” for KGs



Vocabulary = Anchors + Relation types

Inductive out-of-the-box: unseen nodes are “tokenized” with the same Vocab

# Tokenization Idea

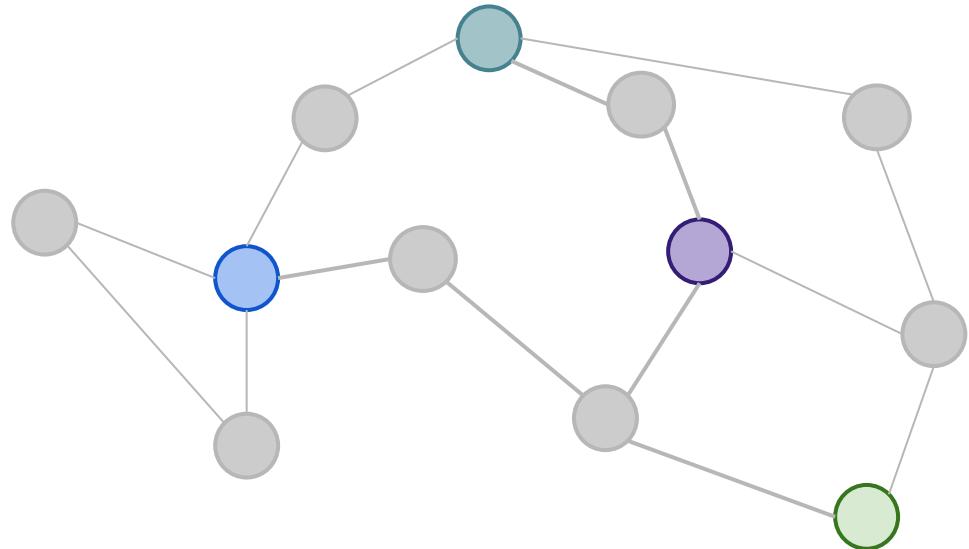


Represent an entity  $e$  as  
a set of  **$k$  most similar** tokens  $t$

$$\max \text{sim}(e, \{t_i\}_{i \in k})$$

- Basic case: similarity as shortest path distance
- Can be generalized to non-Euclidean spaces

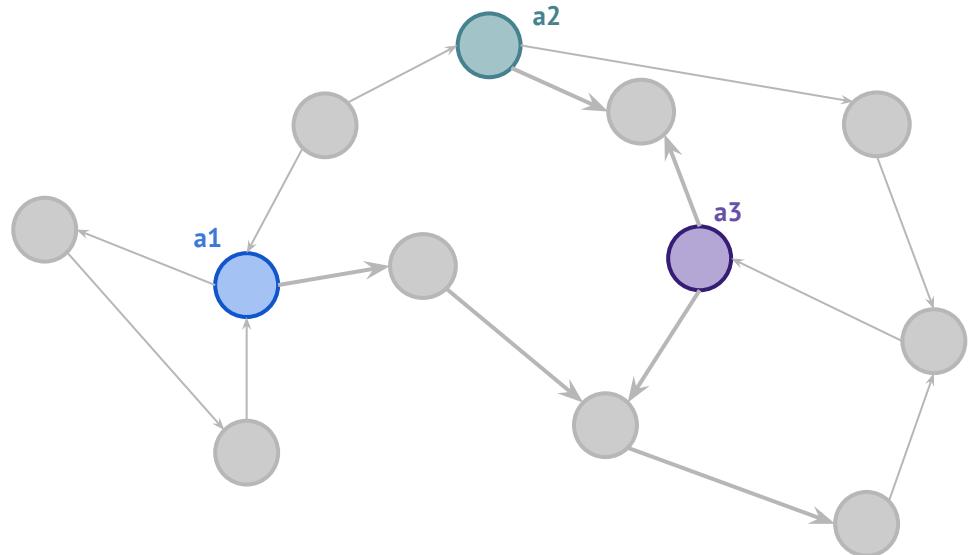
# Anchor Node Selection



**Ideal: Anchors = Dominating Set**

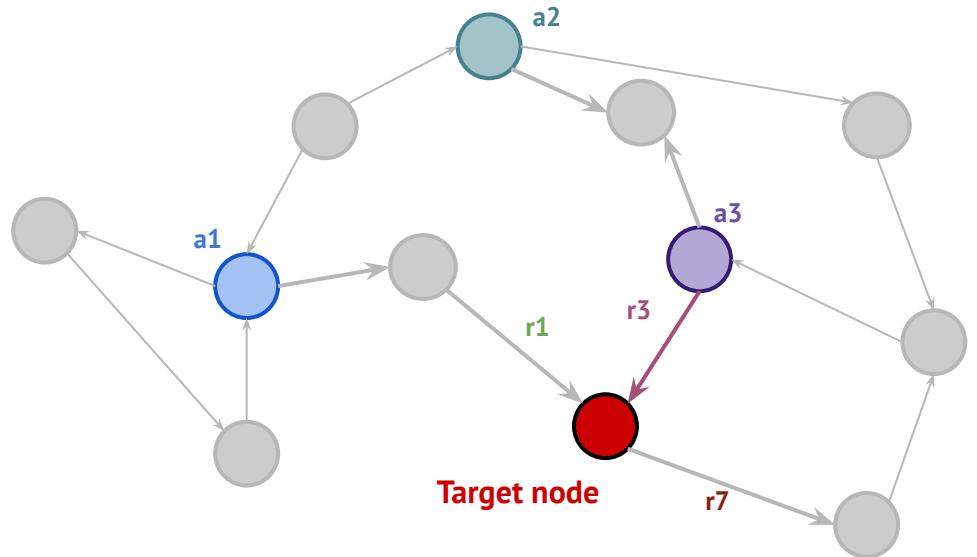
- ✓ Minimized distances
- 😢 NP complete
- 😢 Even k-hop Dominating Set is NP complete

# Anchor Node Selection



**Current strategy:**  
40% top degrees  
40% top PPR  
20% random

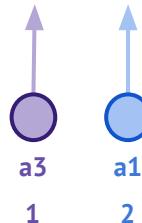
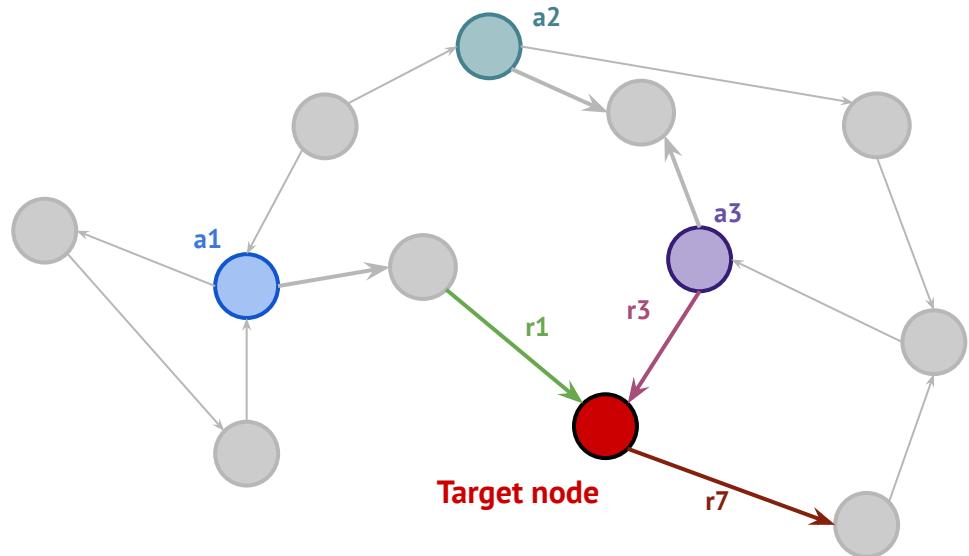
# Tokenization



BFS from the target node until we reach  $|K|$  anchors

- Can be done in forward pass
- Can be pre-processed and saved

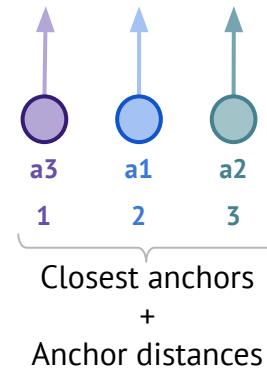
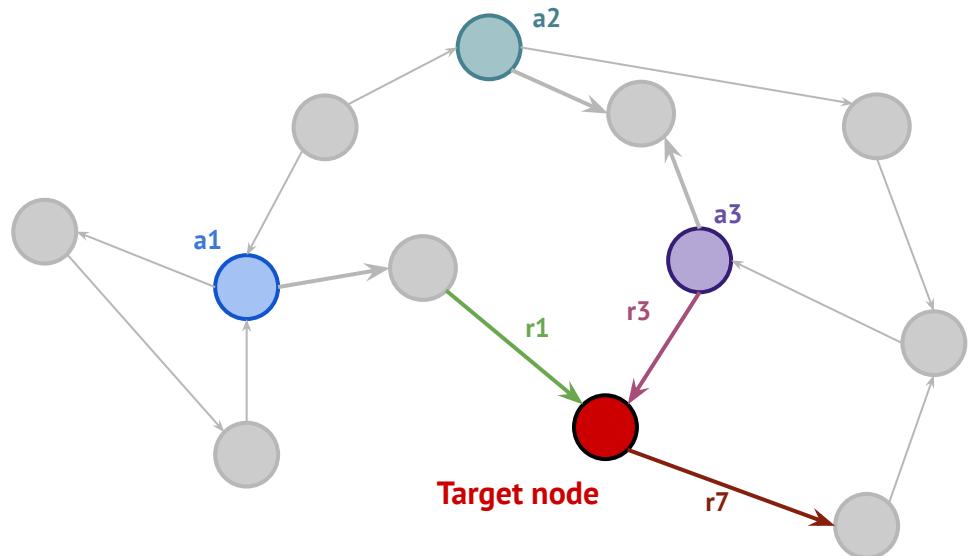
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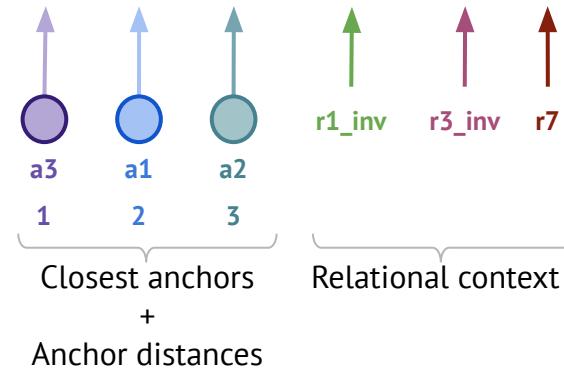
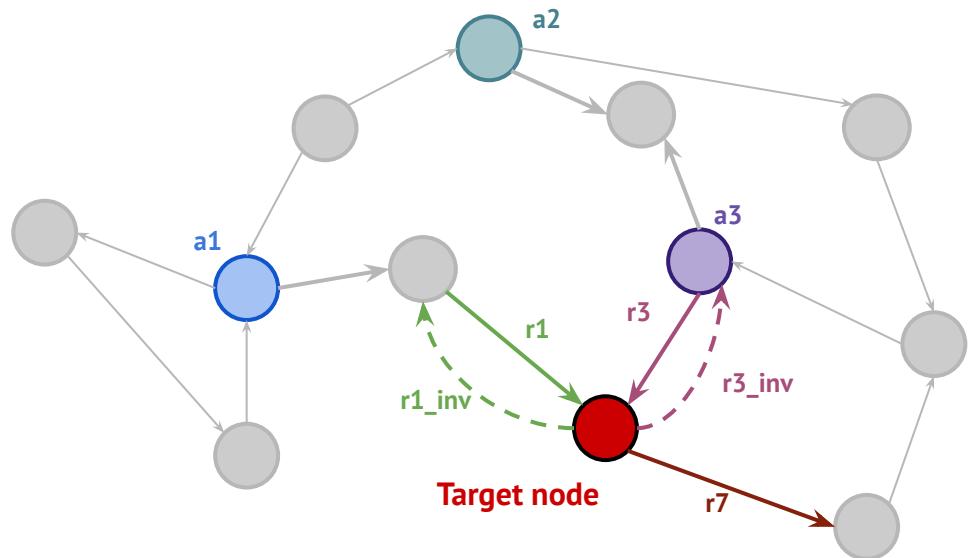
# Tokenization



BFS from the target node until we reach  $|K|$  anchors

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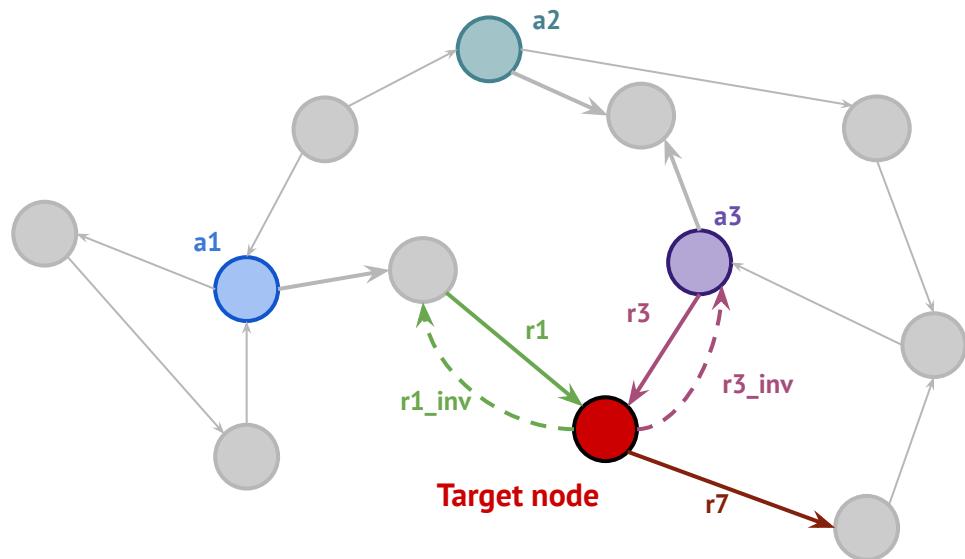
# Tokenization



BFS from the target node until we reach  $|K|$  anchors

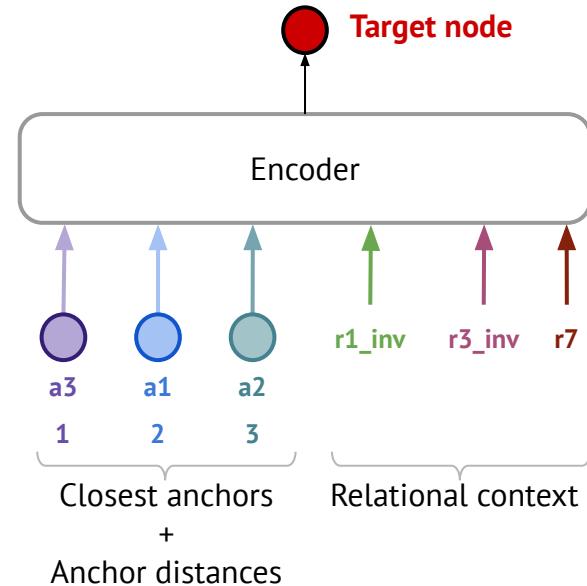
- Can be done in forward pass
- Can be pre-processed and saved

# Tokenization

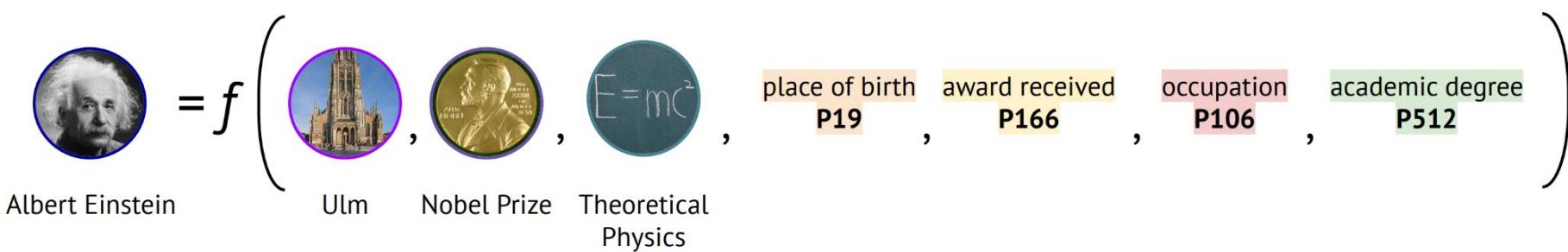


BFS from the target node until we reach  $|K|$  anchors

- Can be done in forward pass
- Can be pre-processed and saved



# Tokenizing Einstein



3 nearest anchors

4 unique outgoing relations in the context



# Tokenizing John Mayer



John Mayer

$$= f \left( \begin{array}{l} \text{Stevie Ray Vaughan}, \quad \text{Fender Stratocaster}, \quad \text{NORWEGIAN BLACK METAL}, \\ \text{instrument P1303}, \quad \text{award received P166}, \quad \text{occupation P106}, \quad \text{genre P136} \end{array} \right)$$

Stevie Ray  
Vaughan



Fender  
Stratocaster



NORWEGIAN  
BLACK METAL

instrument  
**P1303**

award received  
**P166**

occupation  
**P106**

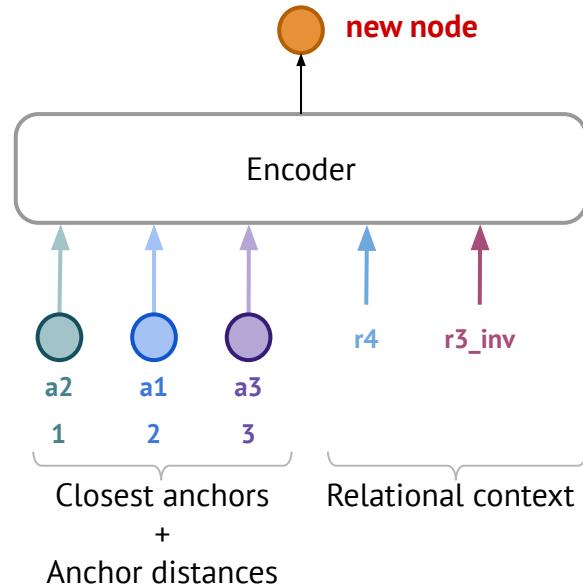
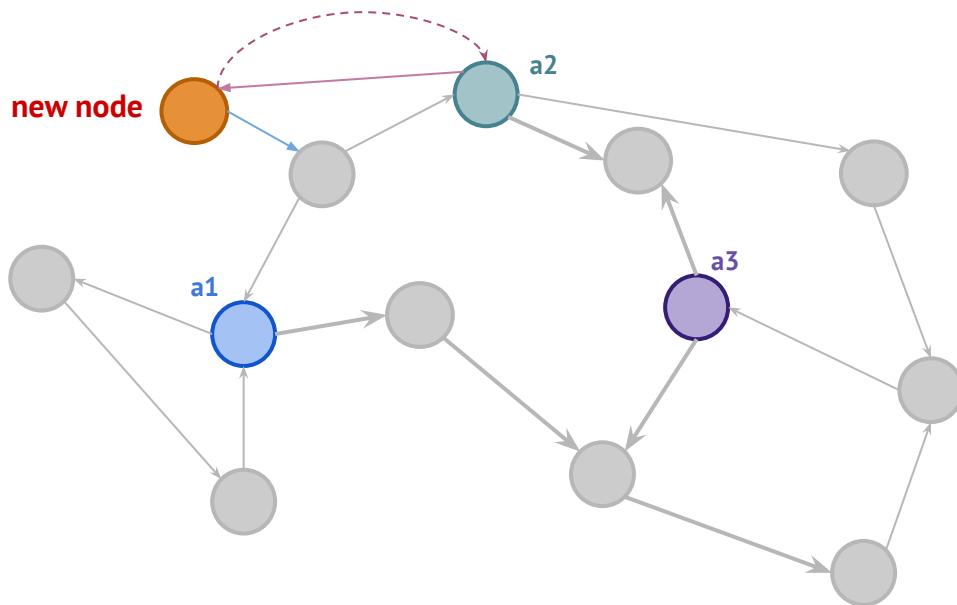
genre  
**P136**

3 nearest anchors

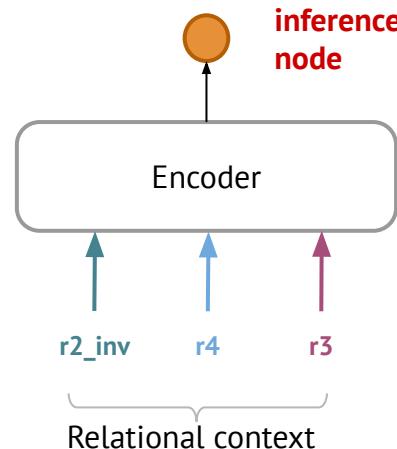
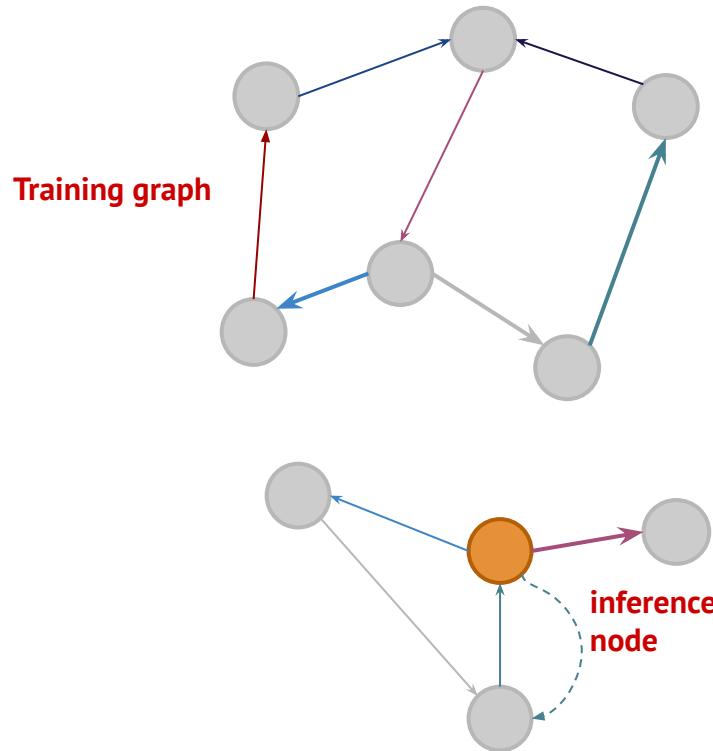
4 unique outgoing relations in the context



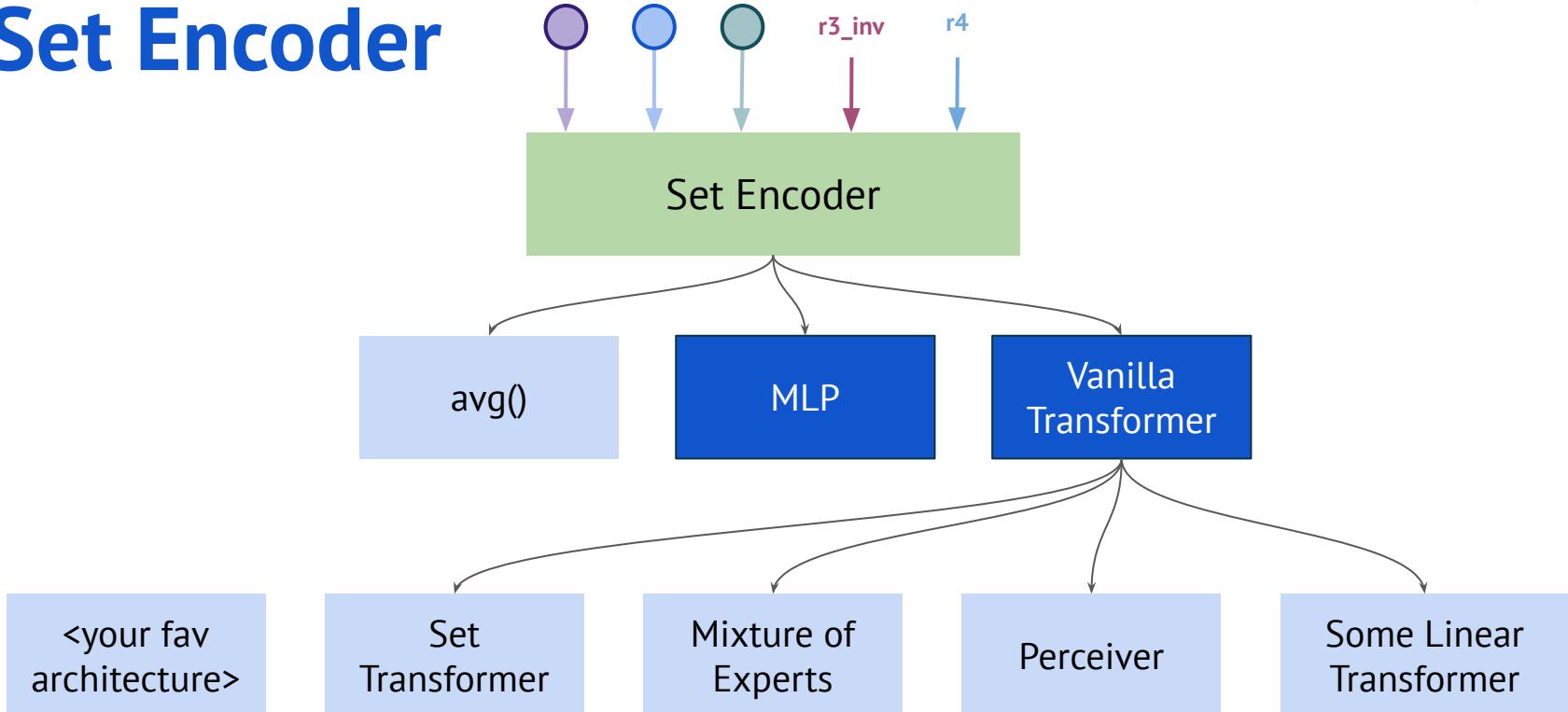
# Unseen Node Tokenization



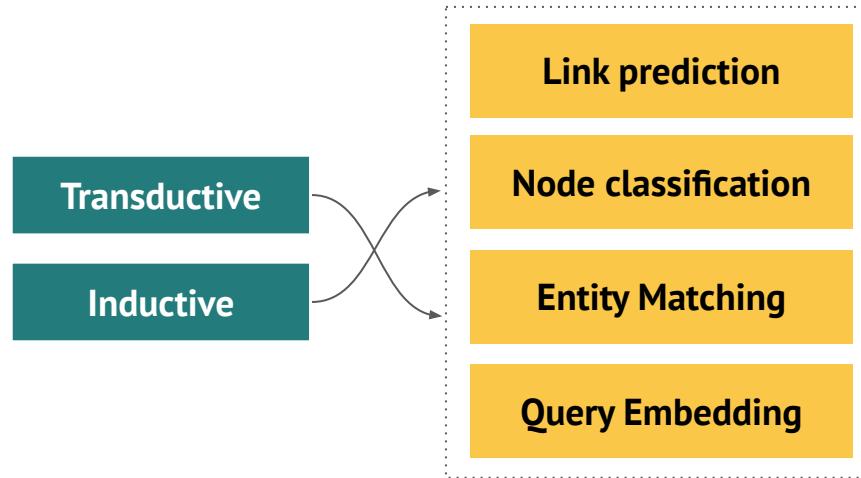
# Inductive Node Tokenization



# Set Encoder



# New Downstream Tasks



# Transductive Link Prediction

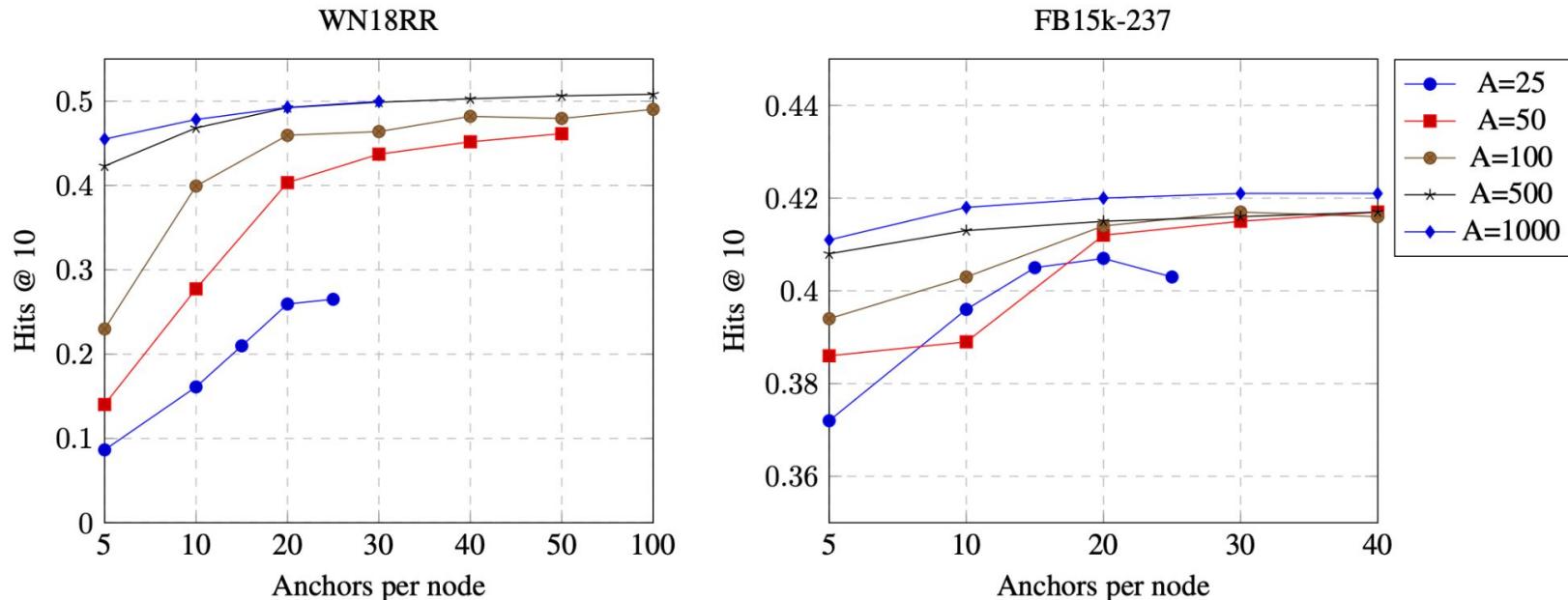


Figure 2: Combinations of total anchors  $A$  and anchors per node. Denser FB15k-237 saturates faster on smaller  $A$  while sparse WN18RR saturates at around 500 anchors.

# NodePiece Experiments: Summary



**10x** fewer parameters while retaining **90%** of transductive LP



**2x** better compared to shallow models of similar #params



Relation Prediction and Node Classification: no anchors is better!



**Inductive out-of-the-box** and very competitive

Table 6: Node classification results.  $|V|$  denotes vocabulary size (anchors + relations), #P is a total parameter count (millions).

	$ V $	#P (M)	WD50K (5% labeled)			WD50K (10% labeled)		
			ROC-AUC	PRC-AUC	Hard Acc	ROC-AUC	PRC-AUC	Hard Acc
MLP	46k + 1k	4.1	0.503	0.016	0.001	0.510	0.017	0.002
CompGCN	46k + 1k	4.4	0.836	0.280	0.176	0.834	0.265	0.161
NodePiece + GNN	50 + 1k	0.75	0.981	0.443	0.513	0.981	0.450	0.516
- no rel. context	50 + 1k	0.64	0.982	0.446	0.534	0.982	0.449	0.530
- no distances	50 + 1k	0.74	0.981	0.448	0.516	0.981	0.448	0.513
- no anchors, rels only	0 + 1k	0.54	0.984	0.453	0.532	0.984	0.456	0.533



**Yesterday this slide  
had a UMAP  
visualization**

# OGB WikiKG 2 : NodePiece is New SOTA

## Leaderboard for [ogbl-wikikg2](#)

The MRR score on the test and validation sets. The higher, the better.

Package: >=1.2.4

Deprecated [ogbl-wikikg](#) leaderboard can be found [here](#).

July 2022

Rank	Method	Ext.		Validation			References	#Params	Hardware	Date
		data	Test MRR	MRR	Contact					
1	<b>StarGraph + TripleRE</b>	No	0.7201 ± 0.0011	0.7288 ± 0.0008	Hongzhu Li (360AI)	Paper, <a href="#">Code</a>	86,762,146	Tesla A100(40GB)	May 30, 2022	
					Xuanyu Zhang (DXM AI)	Paper, <a href="#">Code</a>	38,430,804	Tesla V100 (16GB)	Apr 19, 2022	
2	<b>TranS</b>	No	0.6939 ± 0.0011	0.7058 ± 0.0018	Xuanyu Zhang (DXM AI)	Paper, <a href="#">Code</a>	19,215,402	Tesla V100 (16GB)	Apr 28, 2022	
					Xuanyu Zhang (DXM AI)	Paper, <a href="#">Code</a>	36,421,802	Tesla A100(40GB)	Feb 24, 2022	
4	<b>TripleRE + NodePiece</b>	No	0.6866 ± 0.0014	0.6955 ± 0.0008	Long Yu (360AI)	Paper, <a href="#">Code</a>	19,215,402	Tesla V100 (32GB)	Feb 10, 2022	
					Baoxin Wang (HFL)	Paper, <a href="#">Code</a>	7,289,002	Tesla A100(40GB)	Dec 25, 2021	
6	<b>TripleRE + NodePiece</b>	No	0.6582 ± 0.0020	0.6616 ± 0.0018	Long Yu (360AI)	Paper, <a href="#">Code</a>	250,167,400	Tesla V100 (32GB)	Nov 23, 2021	
					Yihong Chen (UCL NLP & FAIR London)	Paper, <a href="#">Code</a>				
7	<b>ComplEx-RP (50dim)</b>	No	0.6392 ± 0.0045	0.6561 ± 0.0070						

NodePiece-enabled models

# OGB WikiKG 2

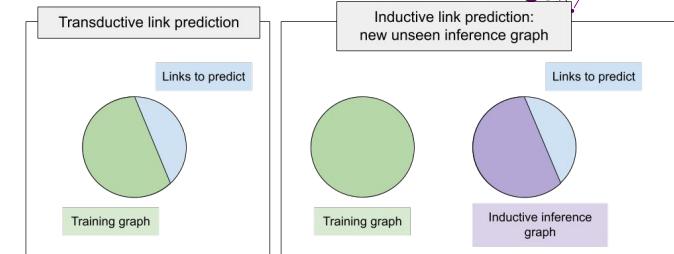
Input graph: 2.5M nodes, 16M edges, ~1K edge types

- **20K anchors (< 1% total nodes)** -> 4M params
- **0 anchors / 0 node embeddings** -> **0.476 MRR**
- No relations in node hashes -> also OK
- “Word length” - 32 tokens
  - 20 anchors per node
  - 12 relations in context

**Table 4: Test MRR and parameter budget on OGB WikiKG 2.**

Model	#Params	MRR
NP + AutoSF	6.9M	$0.570 \pm 0.003$
- rel. context	5.9M	$0.592 \pm 0.003$
- anc. dists	6.9M	$0.570 \pm 0.004$
- no anchors	1.3M	$0.476 \pm 0.001$
AutoSF	500M	$0.546 \pm 0.005$
PairRE	500M	$0.521 \pm 0.003$
RotatE	1250M	$0.433 \pm 0.002$
TransE	1250M	$0.426 \pm 0.003$

# Inductive Link Prediction



Inference graphs are disjoint with training (new nodes)

NodePiece + CompGCN encoder = SOTA on many tasks on relation-rich graphs

Table 5: Inductive link prediction results, Hits@10. Best results are in **bold**, second best are underlined. † results taken from Teru et al. (2020). NBFNet results taken from Zhu et al. (2021).

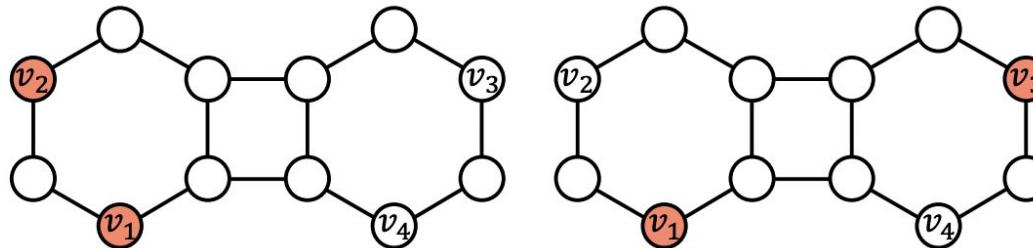
Class	Method	FB15k-237				WN18RR				NELL-995			
		V1	V2	V3	V4	V1	V2	V3	V4	V1	V2	V3	V4
Path	Neural LP †	0.529	0.589	0.529	0.559	0.744	0.689	0.462	0.671	0.408	0.787	0.827	<u>0.806</u>
	DRUM †	0.529	0.587	0.529	0.559	0.744	0.689	0.462	0.671	0.194	0.786	0.827	<u>0.806</u>
	RuleN †	0.498	0.778	0.877	0.856	0.809	0.782	0.534	0.716	0.535	0.818	0.773	0.614
GNN	GraIL †	0.642	0.818	0.828	0.893	0.825	0.787	0.584	0.734	<u>0.595</u>	<b>0.933</b>	<u>0.914</u>	0.732
	NBFNet	<u>0.834</u>	<b>0.949</b>	<b>0.951</b>	<b>0.960</b>	<b>0.948</b>	<b>0.905</b>	<b>0.893</b>	<b>0.890</b>	-	-	-	-
	NP + CompGCN	<b>0.873</b>	<b>0.939</b>	<b>0.944</b>	<b>0.949</b>	<u>0.830</u>	<b>0.886</b>	<u>0.785</u>	<u>0.807</u>	<b>0.890</b>	<u>0.901</u>	<b>0.936</b>	<b>0.893</b>

# Plan

- Graph Reasoning Tasks
- Featurization via Tokenization: NodePiece
- **Featurization via Labeling Trick:  
Neural Bellman-Ford and GNN-QE**
- Past, Today, Future

# The Labeling Trick

Idea: for **each** link we predict, **instantiate** a graph with **unique** initial node labels (**features**)

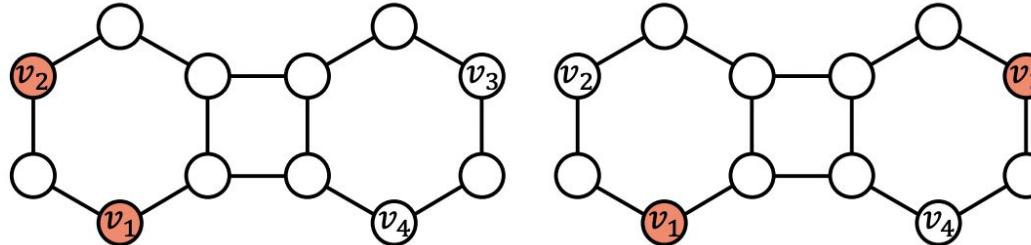


**Figure 2:** When we predict  $(v_1, v_2)$ , we will label these two nodes differently from the rest, so that a GNN is aware of the target link when learning  $v_1$  and  $v_2$ 's representations. Similarly, when predicting  $(v_1, v_3)$ , nodes  $v_1$  and  $v_3$  will be labeled differently. This way, the representation of  $v_2$  in the left graph will be different from that of  $v_3$  in the right graph, enabling GNNs to distinguish the non-isomorphic links  $(v_1, v_2)$  and  $(v_1, v_3)$ .

# The Labeling Trick

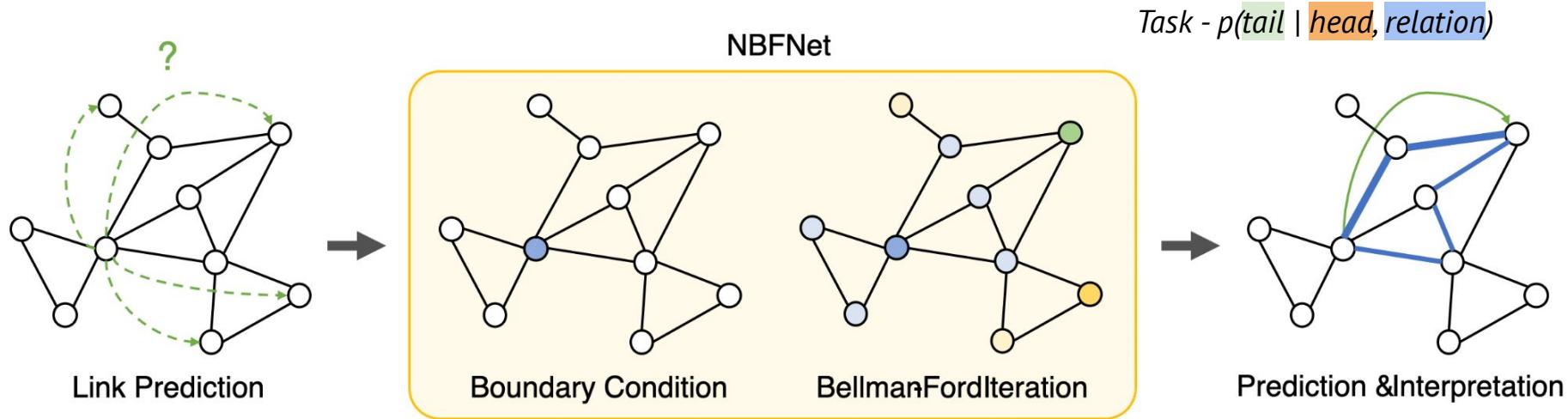
Idea: for **each** link we predict, **instantiate** a graph with **unique** initial node labels (**features**)

- SEAL (homogeneous link prediction, still SOTA on OGB)
- GraIL (KG link prediction, first inductive method)
- Neural Bellman-Ford (homogeneous + KG link prediction, current SOTA)



**Figure 2:** When we predict  $(v_1, v_2)$ , we will label these two nodes differently from the rest, so that a GNN is aware of the target link when learning  $v_1$  and  $v_2$ 's representations. Similarly, when predicting  $(v_1, v_3)$ , nodes  $v_1$  and  $v_3$  will be labeled differently. This way, the representation of  $v_2$  in the left graph will be different from that of  $v_3$  in the right graph, enabling GNNs to distinguish the non-isomorphic links  $(v_1, v_2)$  and  $(v_1, v_3)$ .

# Neural Bellman-Ford



Idea:

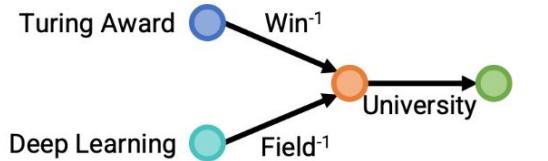
1. Relations do not change at inference -> we can learn relation (edge type) embeddings
2. Initialize **head node feature** with the learnable **relation vector (query)**
3. Propage for L layers, take final representations as final node features

# Neural Bellman-Ford

Table 4: Homogeneous graph link prediction results. Results of VGAE and S-VGAE are taken from their original papers [32, 12].

Class	Method	Cora		Citeseer		PubMed	
		AUROC	AP	AUROC	AP	AUROC	AP
<b>Path-based</b>	Katz Index [30]	0.834	0.889	0.768	0.810	0.757	0.856
	Personalized PageRank [42]	0.845	0.899	0.762	0.814	0.763	0.860
	SimRank [28]	0.838	0.888	0.755	0.805	0.743	0.829
<b>Embeddings</b>	DeepWalk [43]	0.831	0.850	0.805	0.836	0.844	0.841
	LINE [53]	0.844	0.876	0.791	0.826	0.849	0.888
	node2vec [17]	0.872	0.879	0.838	0.868	0.891	0.914
<b>GNNs</b>	VGAE [32]	0.914	0.926	0.908	0.920	0.944	0.947
	S-VGAE [12]	0.941	0.941	<b>0.947</b>	<b>0.952</b>	0.960	0.960
	SEAL [73]	0.933	0.942	0.905	0.924	0.978	0.979
	TLC-GNN [67]	0.934	0.931	0.909	0.916	0.970	0.968
	NBFNet	<b>0.956</b>	<b>0.962</b>	0.923	0.936	<b>0.983</b>	<b>0.982</b>

# GNN-QE: NBFNet + Multi-hop Reasoning



Symbolic Query  
Decomposition

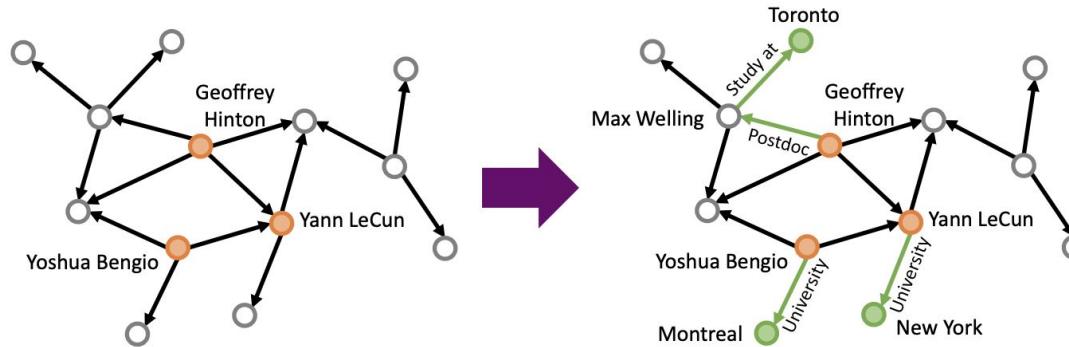
$$\begin{aligned}
 \textcolor{blue}{\bullet} &= \{ \text{Turing Award: 1.0} \} & \textcolor{cyan}{\bullet} &= \{ \text{Deep Learning: 1.0} \} \\
 \textcolor{red}{\bullet} &= \text{Win}^{-1}(\textcolor{blue}{\bullet}) & \textcolor{yellow}{\bullet} &= \text{Field}^{-1}(\textcolor{cyan}{\bullet}) \\
 \textcolor{orange}{\bullet} &= \textcolor{red}{\bullet} \wedge \textcolor{yellow}{\bullet} \\
 \textcolor{green}{\bullet} &= \text{University}(\textcolor{orange}{\bullet})
 \end{aligned}$$

Each variable is a fuzzy set of entities, where each element in the set has a probability.

- Each relation projection (simple link prediction) step is modelled by a L-layer NBFNet
- NBFNet returns a probability distribution (scalars) over all entities (fuzzy set)

# GNN-QE: NBFNet + Multi-hop Reasoning

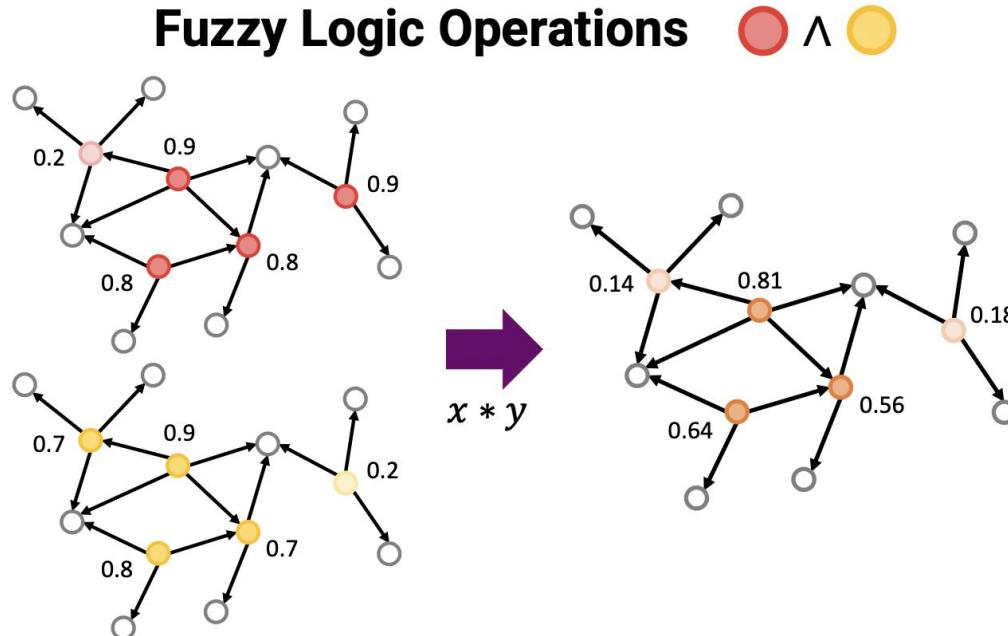
**Neural Relation Projection** *University*(



Use a **GNN** to propagate the input **fuzzy set** and get the **output fuzzy set**.

The relation *University* is used to guide the propagation towards **paths** that can predict the relation *University*.

# GNN-QE: NBFNet + Multi-hop Reasoning



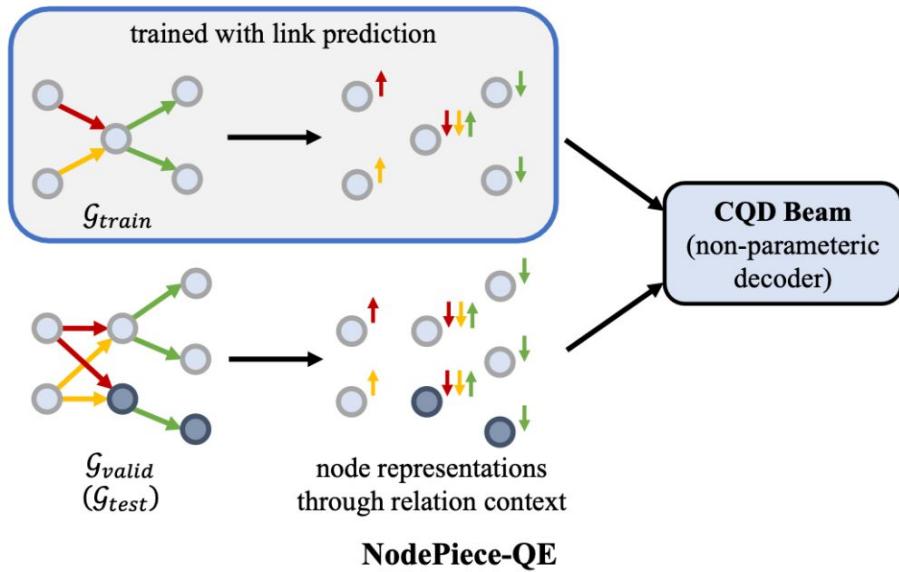
Use **product fuzzy logic** to model logic operations (e.g.  $\neg$ ,  $\wedge$ ,  $\vee$ ) over fuzzy sets.

1. Logical operators as algebraic operations
2. Resulting probability **distribution** is used as **scalar weights** for the next hop graph initialization

# The Essence of Inductiveness

What is **invariant** in inductive reasoning setups? **Relation types**

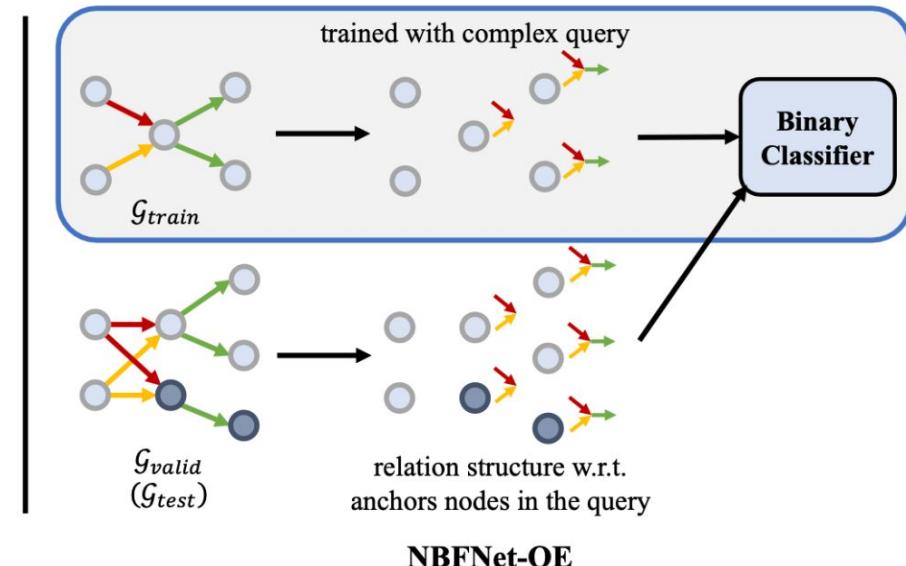
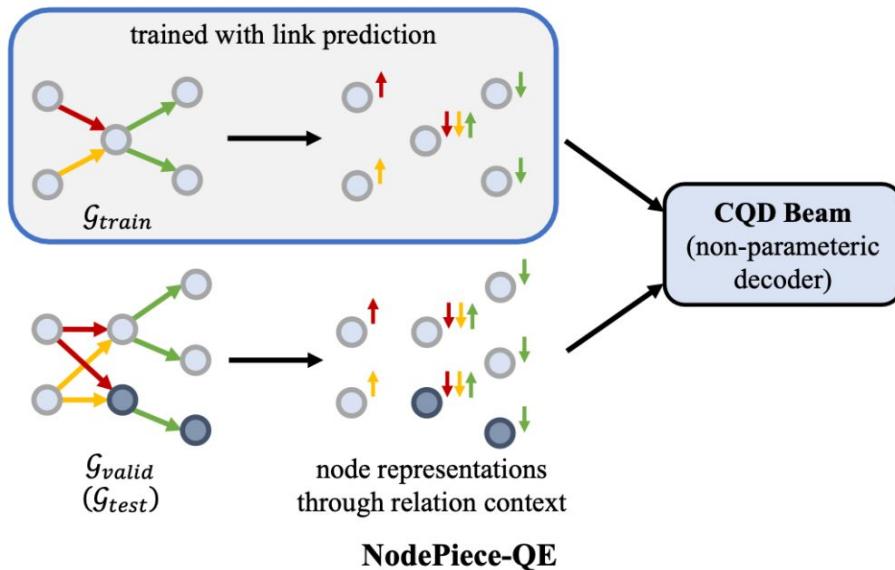
- NodePiece - parameterization through relational context



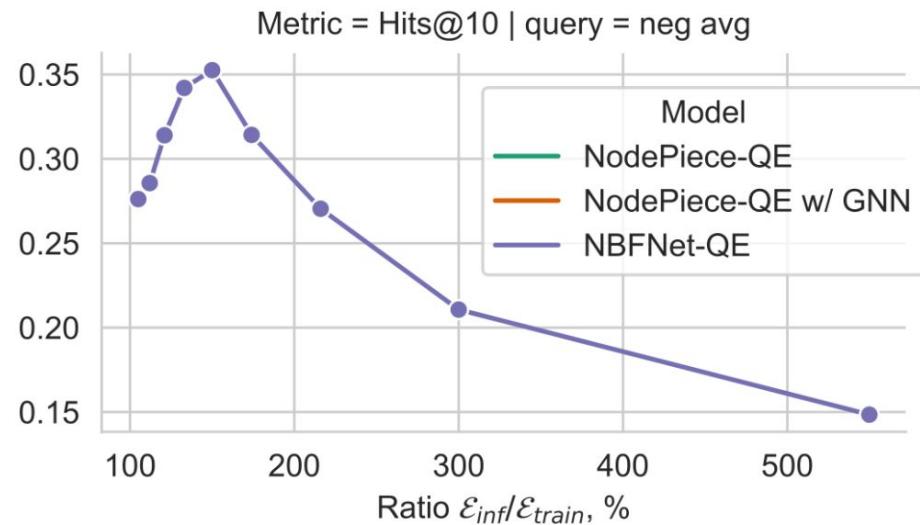
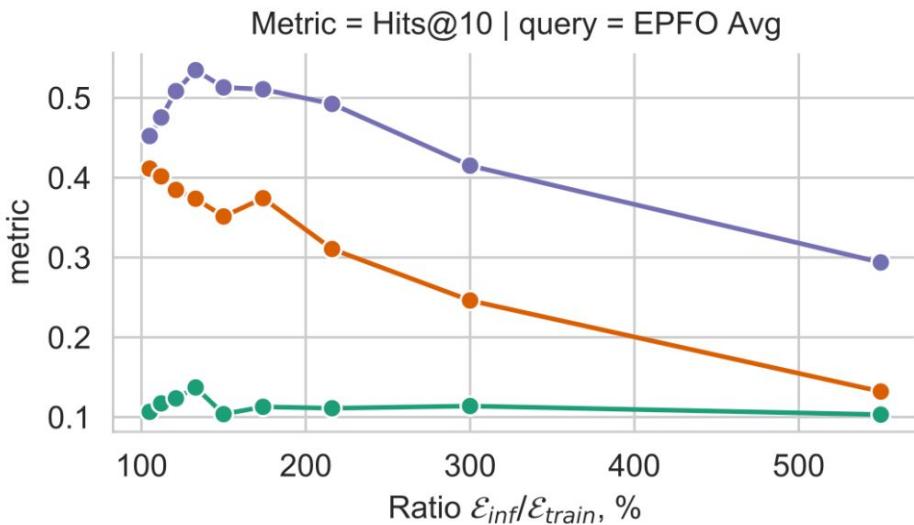
# The Essence of Inductiveness

What is **invariant** in inductive reasoning setups? **Relation types**

- NodePiece - parameterization through relational context
- GNN-QE - parameterization through relational structure



# Inductive Generalization to Larger Test Graphs is Still a Problem



# Plan

- Graph Reasoning Tasks
- Featurization via Tokenization: NodePiece
- Featurization via Labeling Trick:  
Neural Bellman-Ford and GNN-QE
- **Past, Today, Future**

# Space of KG Tasks in 2019

Transductive

Triples

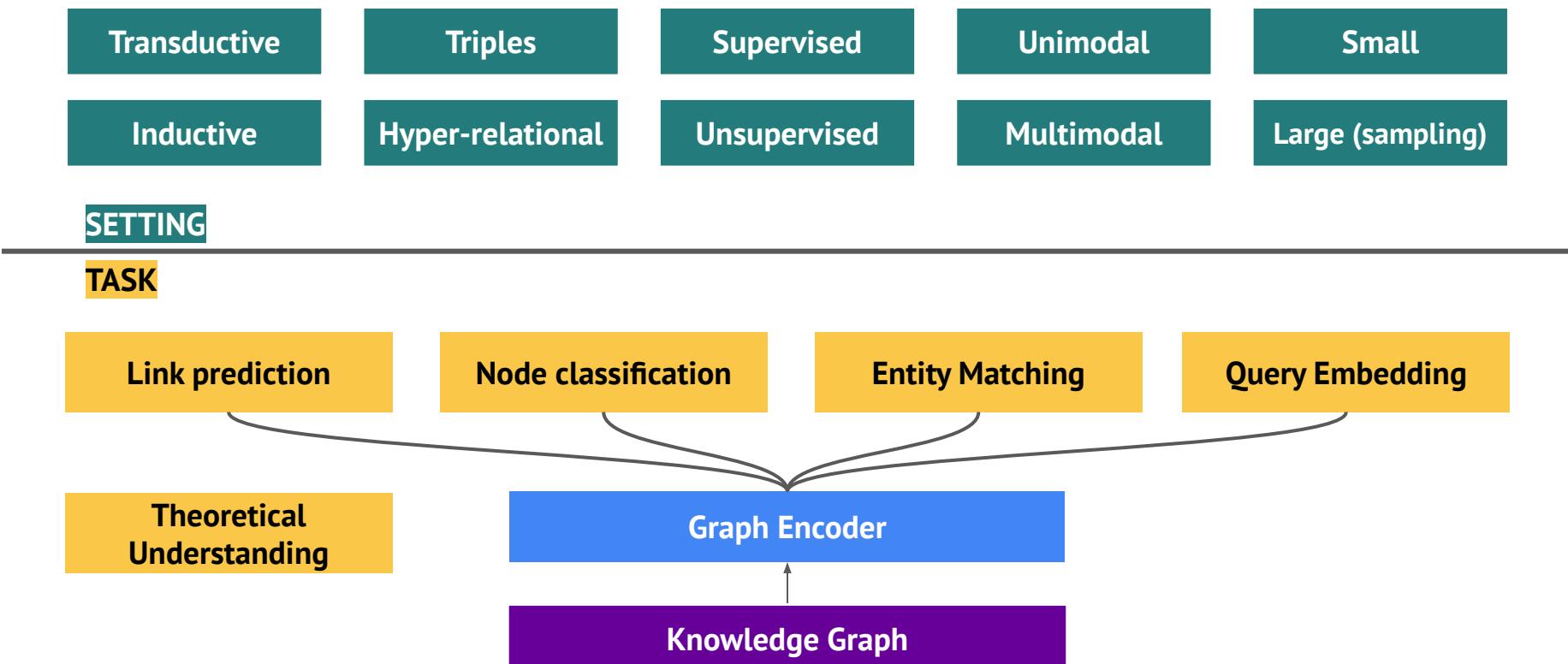
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SETTING

TASK

Link prediction

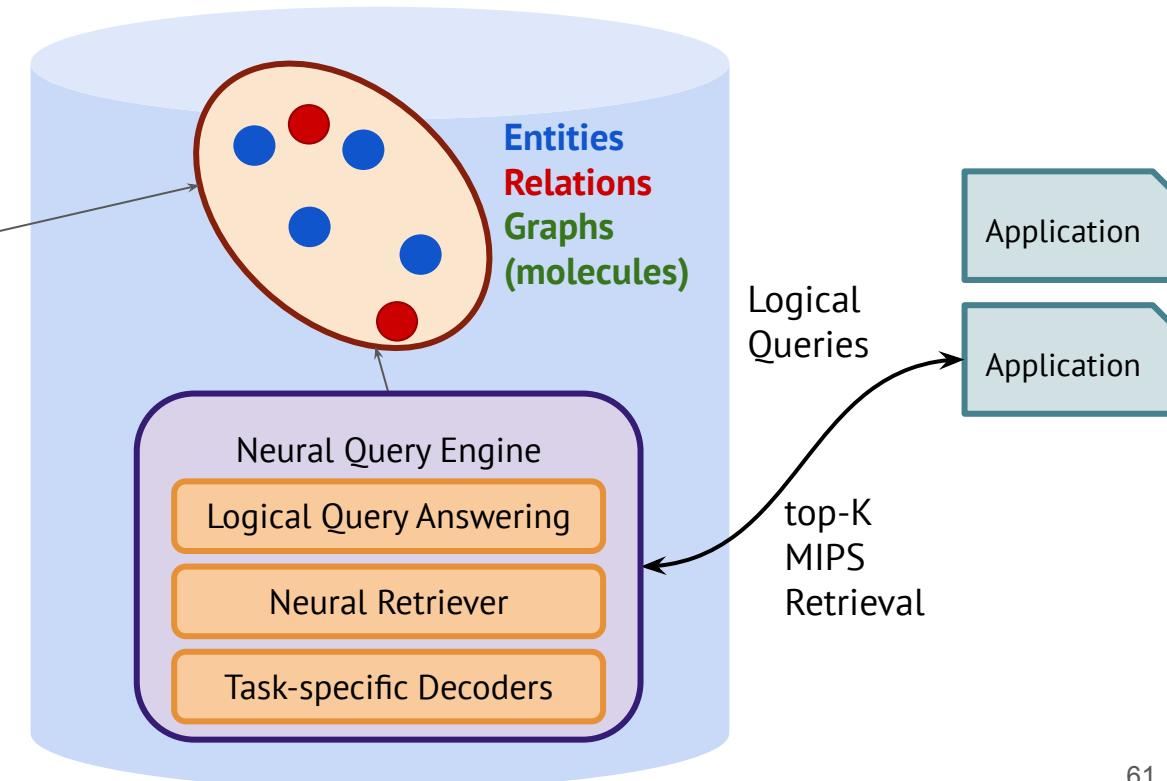
# Space of KG Tasks Today



# Future: Neural Graph Databases

(Montreal, location, Quebec)  
 (Quebec, location, Canada)  
 (Canada, bordersWith, USA)

- No symbolic storage
- Embedding-based storage
- Inferring Missing Links
- Complex Query Answering
- Updatable





# Q&A Time!



@michael\_galkin



@mgalkin



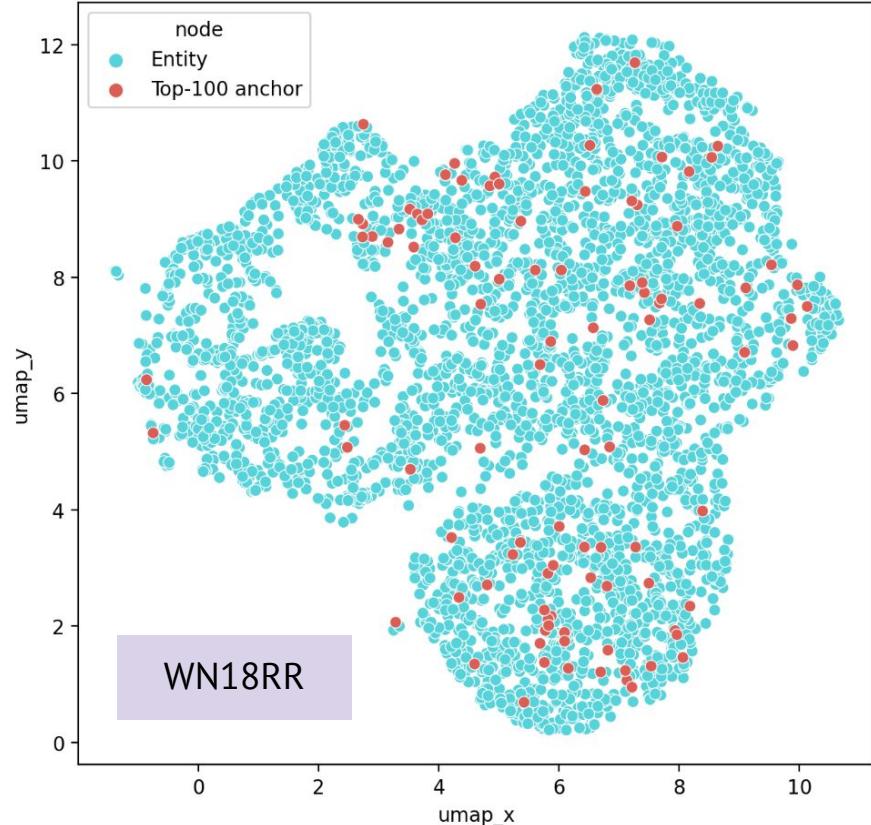
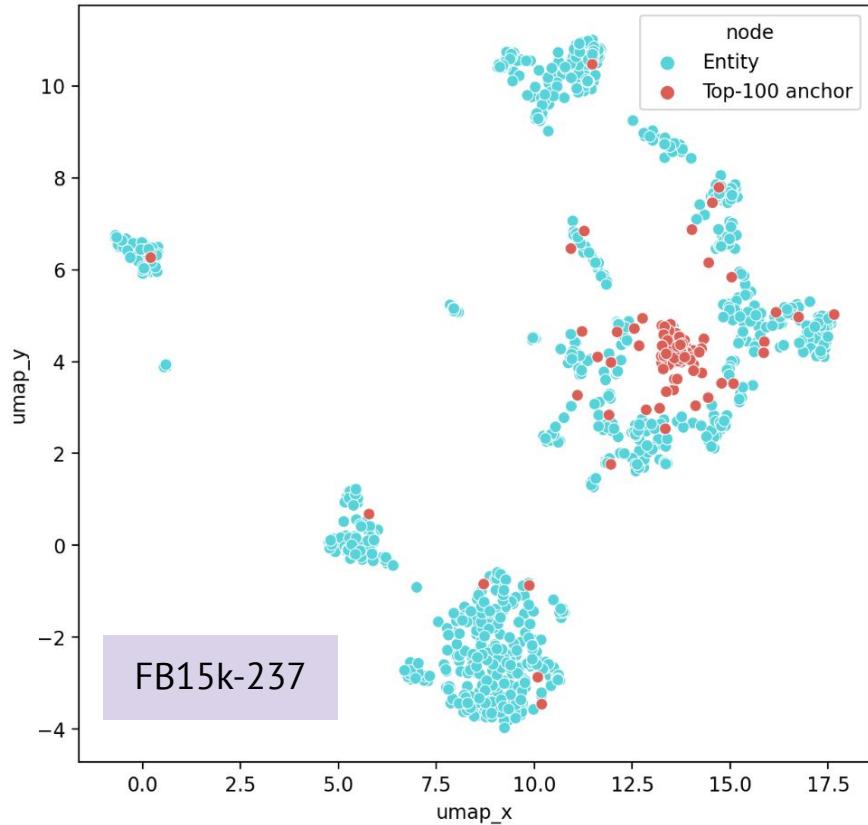
mikhail.galkin@mila.quebec



migalkin.github.io

# Backup

# Visualizations: Anchors + Entities

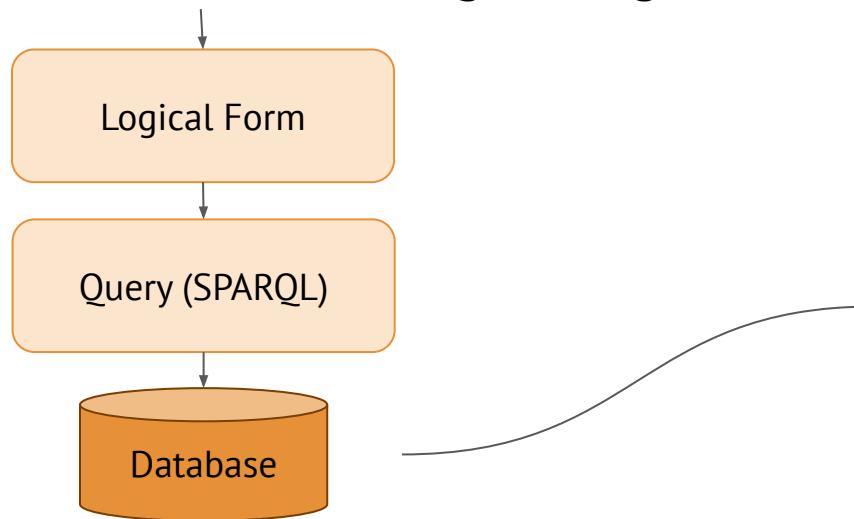


# Complex Logical Query Answering: Why?



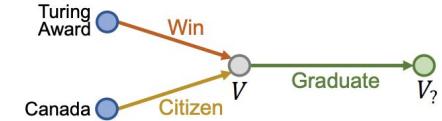
Where did Canadian citizens with Turing Award graduate?

Typical KGQA pipeline

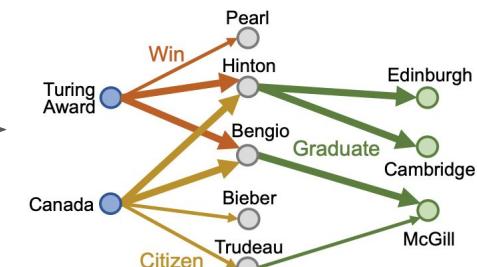


(A) Query  $q$  and Its Dependency Graph

$$q = V_? . \exists V : Win(TuringAward, V) \wedge Citizen(Canada, V) \wedge Graduate(V, V_?)$$



(C) Knowledge Graph Space

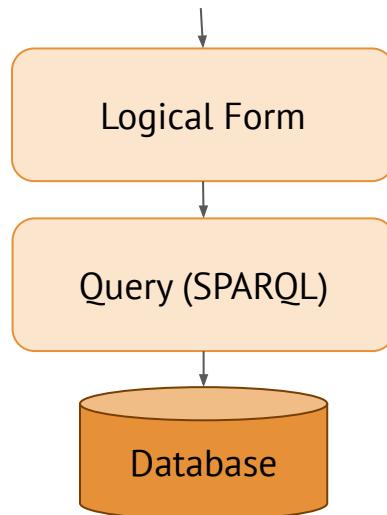


# Complex Logical Query Answering: Why?



Where did Canadian citizens with Turing Award graduate?

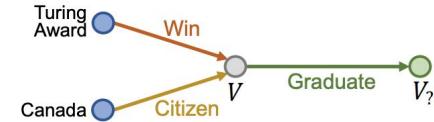
Typical KGQA pipeline



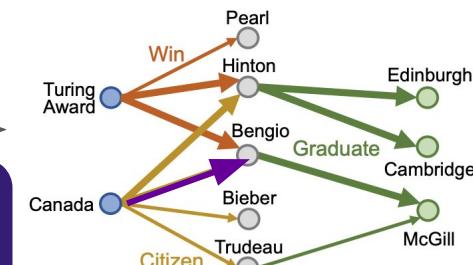
What if this edge is missing?

(A) Query  $q$  and Its Dependency Graph

$$q = V_? . \exists V : Win(TuringAward, V) \wedge Citizen(Canada, V) \wedge Graduate(V, V_?)$$



(C) Knowledge Graph Space



# Complex Logical Query Answering

Transductive

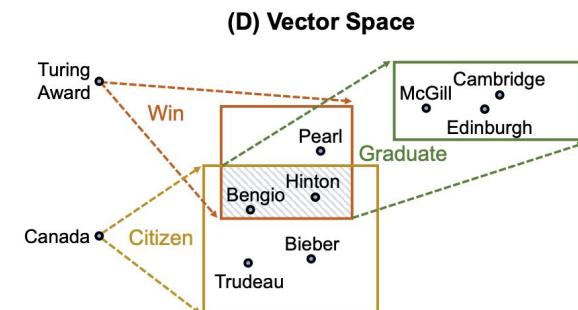
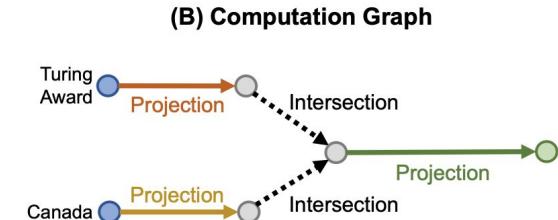
Triples

Inductive

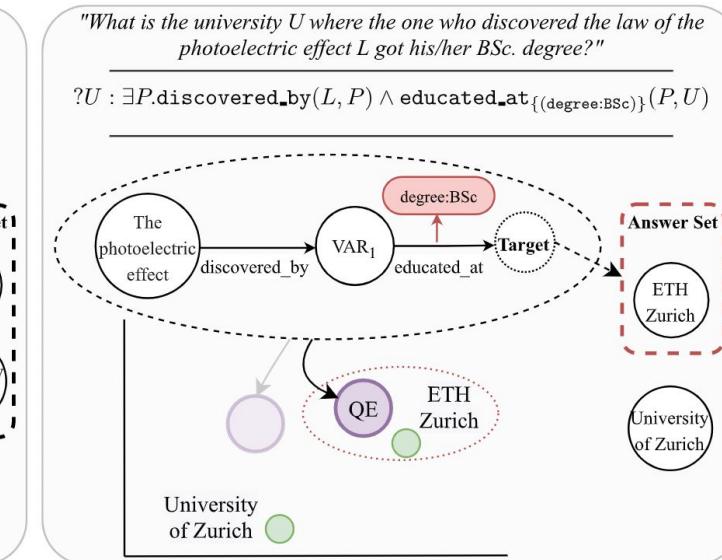
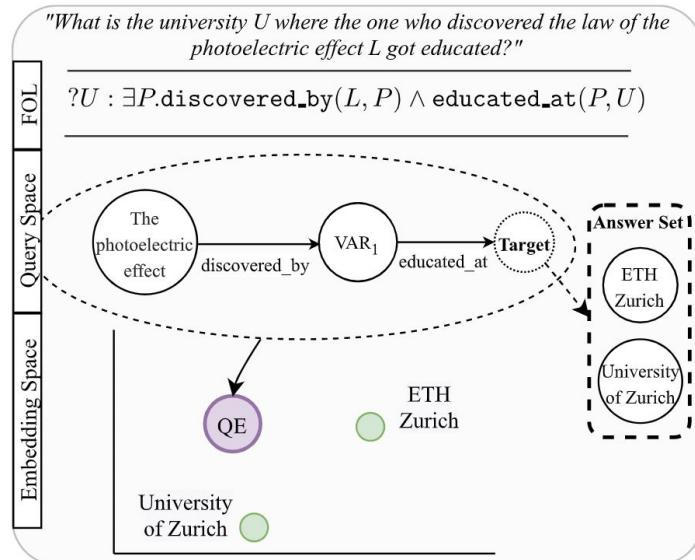
Hyper-relational

Query Embedding

- Databases assume KGs are **complete**
  - In reality - they are not
- We want to answer FOL queries over **incomplete** graphs with **neural** operators
- Embed a query in a latent space, MIPS decoder for kNN answers



# StarQE: Complex Logical Query Answering on HR KGs



1. Extending FOL to HR KGs
2. Qualifiers help A LOT
3. New query types are enabled

# ★ StarQE for Logical Queries: Summary

- Extend FOL to hyper-relational graphs with qualifiers
- Enabling new query types (eg, joins over qualifier entities)
- Robust to inner representation: RDF\* vs reified RDF
- Qualifiers help A LOT in answering complex queries

