Мухрасов Роман

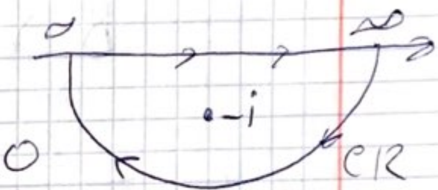
Домашняя работа №5

А Задача №1

$$\int_{-\infty}^{\infty} \frac{e^{-kix}}{(x+i)^3}dx \quad ; \quad \underbrace{\frac{1}{(x+i)^3}}_{g(z)} \text{ при } x\to\infty \ \frac{1}{(x+i)^3}\to 0$$

По лемме Жордана при $k>0$ $\quad -k = k'$

$$\oint = \int_{-\infty}^{\infty} + \int_{CR} = -2\pi i \sum \underset{z=z_0}{res}\, e^{ik'z} g(z)$$



$$\int_{CR} \to 0 \text{ при } z\to\infty \ g(z)\to 0$$

$$\underset{z=-i}{res}\, e^{ik'z} g(z) = \frac{1}{(3-1)!}\frac{d^2}{dz^2} e^{ik'z}\frac{1}{(z+i)^3}(z+i)^3\Big|_{z\to -i} =$$

полюс 3 порядка

$$= \frac{1}{2} i^2 k'^2 e^{ik'(-i)} = -\frac{k'^2}{2}e^{k'} \Rightarrow$$

$$\int_{-\infty}^{\infty}\frac{e^{-kiz}}{(z+i)^3}dz \text{ при } k<0 = +\pi i k^2 e^{-k}$$

при $k \leq 0$

$$\oint = \int_{-\infty}^{\infty} + \int_{CR}^{\to 0} = -2\pi i \sum \underset{z=z_0}{res}\, e^{ik'z} g(z)$$

Мигдасов Роман

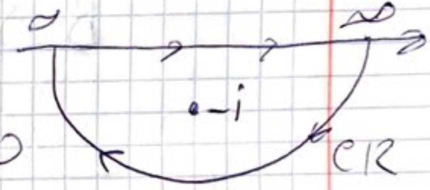
Домашняя работа №5

Задача №1

$$\int_{-\infty}^{\infty}\frac{e^{-kix}}{(x+i)^3}\,dx \; ; \; \underbrace{\frac{1}{(x+i)^3}}_{g(z)} \text{ при } x\to\infty \; \frac{1}{(x+i)^3}\to 0$$

По лемме Жордана при $k>0$ $\quad -k=k'$

$$\oint = \int_{-\infty}^{\infty} + \int_{C_R} = -2\pi i \sum \underset{z=z_0}{\mathrm{res}}\, e^{ik'z} g(z)$$



$\int_{C_R}\to 0$ при $z\to\infty$ $g(z)\to 0$

$$\underset{z\to -i}{\mathrm{res}}\, e^{ik'z}g(z) = \frac{1}{(3-1)!}\frac{d^2}{dz^2}\, e^{ik'z}\frac{1}{(z+i)^3}(z+i)^3\Big|_{z\to -i} =$$

полюс 3 порядка

$$= \frac{1}{2}\, i^2 k'^2 e^{ik'(-i)} = -\frac{k'^2}{2}e^{k'} \Rightarrow$$

$$\int_{-\infty}^{\infty}\frac{e^{-kiz}}{(z+i)^3}\,dz \text{ ; при } k>0 = \pi i k^2 e^{-k}$$

при $k\le 0$

$$\oint = \int_{-\infty}^{\infty} + \int_{C_R}^{\to 0} = -2\pi i\sum \underset{z=z_0}{\mathrm{res}}\, e^{ik'z}g(z)$$

Т.К. область ~~интегрирования является~~ односвязной

Т.К. функция $g(z)e^{ikz}$ аналитична в некоторой замкнутой ~~области~~ односвязной области без особых точек =>

$\mathrm{res}\, g(z)e^{ikz} = 0 \Rightarrow \int\limits_{-\infty}^{\infty} \frac{e^{-kiz}}{(z+i)^3}\,dz = 0$ при $k<0$

В Задание 12

$a(x) = \frac{1}{x^4+1}$ ($a(x)$ - четная)

интеграл Фурье имеет вид

$$f(x) = \frac{1}{2\pi}\int\limits_{-\infty}^{\infty} d\lambda \int\limits_{-\infty}^{\infty} f(t)\, e^{-i\lambda(t-x)}\, dt$$

$$\frac{1}{2\pi}\int\limits_{-\infty}^{\infty} d\lambda \int\limits_{-\infty}^{\infty} \frac{1}{t^4+1}\, e^{-\lambda i(t-x)}\, dt =$$

$$= \frac{1}{2\pi}\int\limits_{-\infty}^{\infty} e^{i\lambda x}\, d\lambda \int\limits_{-\infty}^{\infty} \frac{e^{-i\lambda t}}{t^4+1}\, dt = I(t)$$

$-\lambda = \lambda'$

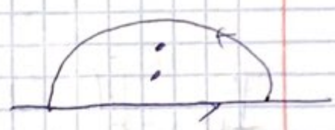
по лемме Жордана (для $\lambda' < 0$) ($\lambda > 0$)

$$\int\limits_{-\infty}^{\infty} \frac{e^{i\lambda' t}}{t^4+1}\, dt = -2\pi i \sum \mathrm{res}\, \frac{e^{i\lambda' t}}{t^4+1}$$

$$= -2\pi i \left(\frac{e^{i\lambda'(-\sqrt{-i})}}{(x+\sqrt{i})(x^2-i)}\Big|_{\sqrt{i}} + \frac{e^{i\lambda'(\sqrt{-i})}}{(x^2+i)(x+\sqrt{-i})}\Big|_{\sqrt{-i}} \right) =$$

$$= -2\pi i \left(\frac{e^{i\lambda'(-\sqrt{i})}}{(x-\sqrt{i})(x-\sqrt{-i})(x+\sqrt{-i})}\Big|_{-\sqrt{i}} + \frac{e^{i\lambda'(\sqrt{-i})}}{(x-\sqrt{i})(x+\sqrt{i})(x+\sqrt{-i})}\Big|_{\sqrt{-i}} \right)$$

для $(\lambda<0) = \lambda'>0$



$$\int_{-\infty}^{\infty} \frac{e^{i\lambda' t}}{t^4+1}\,dt = 2\pi i \sum \operatorname{res} \frac{e^{i\lambda' t}}{t^4+1}\Big|_{\substack{t_0=\sqrt{i}\\ t_0=-\sqrt{-i}}} =$$

$$= 2\pi i \left(\frac{e^{i\lambda'(\sqrt{i})}}{(x+\sqrt{i})(x-\sqrt{-i})(x+\sqrt{-i})}\Big|_{\sqrt{i}} + \frac{e^{i\lambda'(-\sqrt{-i})}}{(x+\sqrt{i})(x-\sqrt{i})(x-\sqrt{-i})}\Big|_{-\sqrt{-i}} \right) =$$

Преобразуем все

~~Сгруппируем [illegible]~~

$\lambda>0$

$$\int_{-\infty}^{\infty} = \frac{-2\pi i\, e^{-i\lambda'\sqrt{i}}}{(-\sqrt{i}-\sqrt{i})(-\sqrt{i}-\sqrt{-i})(-\sqrt{i}+\sqrt{-i})} - \frac{2\pi i\, e^{i\lambda'\sqrt{-i}}}{(\sqrt{-i}-\sqrt{i})(\sqrt{-i}+\sqrt{i})(\sqrt{-i}+\sqrt{-i})} =$$

$$= \frac{\cancel{1}\pi i\, e^{-i\lambda'\sqrt{i}}}{2\sqrt{2}\, i\sqrt{i}} + \frac{\pi\, e^{i\lambda'\sqrt{-i}}}{2\sqrt{2}\sqrt{-i}}$$

2)

$\lambda<0$

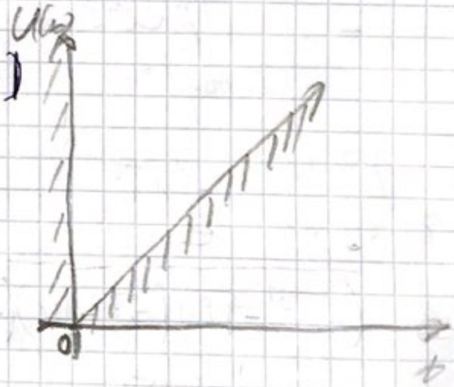
$$\int_{-\infty}^{\infty} = \frac{\pi e^{i\lambda'(-\sqrt{i})}}{\sqrt{2}\sqrt{i}\cdot 2} + \frac{\pi e^{-i\lambda'\sqrt{-i}}}{2\sqrt{2}\sqrt{-i}}$$

$$I(\lambda) = \begin{cases} \dfrac{\pi e^{-i\lambda'\sqrt{i}}}{2\sqrt{2}\sqrt{i}} + \dfrac{\pi e^{i\lambda'\sqrt{-i}}}{2\sqrt{2}\sqrt{-i}} & \lambda \geq 0 \\ \dfrac{\pi e^{i\lambda'\sqrt{i}}}{2\sqrt{2}\sqrt{i}} + \dfrac{\pi e^{-i\lambda'\sqrt{-i}}}{2\sqrt{2}\sqrt{i}} & \lambda < 0 \end{cases}$$

$$f(x) = \begin{cases} \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty} e^{i\lambda x}\dfrac{\pi}{\sqrt{2i}}\left(\dfrac{e^{i\lambda\sqrt{i}}}{2} + \dfrac{e^{-i\lambda\sqrt{-i}}}{2i}\right) & \lambda > 0 \\ \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty} e^{i\lambda x}\left(\dfrac{\pi}{\sqrt{2i}}\right)\left(\dfrac{e^{-i\lambda\sqrt{i}}}{2} + \dfrac{e^{+i\lambda\sqrt{-i}}}{2i}\right) & \lambda < 0 \end{cases}$$

C. Задача 3

$U(x) = \alpha x \qquad E(p) = c|p|$



Заметим

$\varphi(0) = 0$

ψ_p на собств. знач энергии

$\hat{H}|\psi\rangle = E|\psi\rangle$

$\hat{H} = \dfrac{\hat{p}^2}{2m} + \alpha\hat{x}$ перейдем в импульсное пред

$\hat{p} \Rightarrow p \ ; \ \hat{x} \Rightarrow i\hbar\dfrac{\partial}{\partial p}$

Перепишем ур.

$$\frac{p^2}{2m}a(p) + \alpha i\hbar \frac{da(p)}{dp} = E\,a(p)$$

Запишем

$$\psi(x) = \frac{1}{\sqrt{L}}\int_{-\infty}^{\infty} e^{ipx/\hbar}\, a(p)\, \frac{dp\, L}{2\pi\hbar}$$

используем граничное условие

$x=0 \quad \psi(0)=0$

$$\psi(0) = \frac{1}{\sqrt{L}}\int_{-\infty}^{\infty} a(p)\, dp\, \frac{L}{2\pi\hbar} = 0$$

$\Downarrow$

$$\int_{-\infty}^{\infty} a(p)\, dp = 0$$

далее из ур Шредингера выразим $a(p)$

$$\frac{d(a)}{dp} = \frac{1}{\alpha i\hbar}\left(E - \frac{p^2}{2m}\right) a(p)$$

$$\frac{d(a)}{a(p)} = \frac{1}{\alpha i\hbar}\left(E - \frac{p^2}{2m}\right) dp$$

возьмем т.к. $\boxed{E = \frac{c k^2(p)}{2m}}$

$$\frac{d(a)}{a(p)} = \frac{-i}{\alpha\hbar}\left(E - \frac{p^2}{2m}\right) dp$$

$$\int \frac{d(a)}{a(p)} = \int \frac{-i}{\alpha\hbar\, 2m}\left(c k^2(p) - p^2\right) dp$$

$$\ln a_1 = -\frac{i}{2\hbar 2m}\left(+\frac{P_c^2 k^2}{2} - \frac{p^3}{3}\right) + const_1 \quad \Big| \text{для } p>0$$

$$\ln a_2 = -\frac{i}{2\hbar 2m}\left(-\frac{P_c^2 k^2}{2} - \frac{p^3}{3}\right) + const_2 \quad \Big| \text{для } p<0$$

$$\underset{p>0}{a(p)_1} = C_1 e^{-\frac{i}{2m2\hbar}\left(\frac{P_c^2 k^2}{2} - \frac{p^3}{3}\right)} \Rightarrow$$

$$\underset{p<0}{a(p)_2} = C_2 e^{+\frac{i}{2m2\hbar}\left(\frac{p_c^2 k^2}{2} + \frac{p^3}{3}\right)}$$

$$\int_{-\infty}^{\infty} a(p)\,dp = 0 \Rightarrow \int_0^{\infty} a_1(p)\,dp + \int_{-\infty}^{0} a_2(p)\,dp = 0$$

$$\int_0^{\infty} C_1 e^{-\frac{i}{2m2\hbar}\left(\frac{P_c^2 k^2}{2} - \frac{p^3}{3}\right)} dp = -\int_{-\infty}^{0} C_2 e^{\frac{i}{2m2\hbar}\left(\frac{P_c^2 k^2}{2} + \frac{p^3}{3}\right)} dp$$

сделаем замену $p = k^2 t^2$ | $p^2 k^2 \pm p^3$

$dp = 2k^2 t\,dt$ | $k^4 t^4 k^2 \pm k^6 t^6$

$k^6(t^4 \pm t^6)$

$$\int_0^{\infty} C_1 e^{-\frac{i k^6}{2m2\hbar}\left(\frac{t^4}{2} - \frac{t^6}{3}\right)} 2k^2 t\,dt = -\int_{-\infty}^{0} C_2 e^{+\frac{i k^6}{2m2\hbar}\left(\frac{t^4}{2} + \frac{t^6}{3}\right)} 2k^2 t\,dt$$

$$\int_0^\infty C_1 e^{-i\lambda\left(\frac{t_c^4}{2}-\frac{t^6}{3}\right)} t\,dt = \int_{-\infty}^{0} C_2 e^{i\lambda\left(\frac{t_c^4}{2}+\frac{t^6}{3}\right)} t\,dt \quad \frac{t K^6}{2m2\hbar^2}=\lambda$$

Примем $\lambda \gg 1$

$$C_1\int_0^\infty e^{i\lambda\left(\frac{t^6}{3}-\frac{t_c^4}{2}\right)} t\,dt = C_2\int_{-\infty}^{0} e^{i\lambda\left(\frac{t_c^4}{2}+\frac{t^6}{3}\right)} t\,dt$$

$I_1(\lambda) \qquad I(\lambda)_2$

$I(\lambda)_1$ при $\lambda \gg 1$ $\quad \int_a^b e^{i\lambda S(x)} g(x)\,dx$

$S(x) \approx S(x_0) + \frac{S''(x_0)}{2}(x-x_0)^2$

x_0 – экстремум

$g(x) \approx g(x_0)$

$S'(x_0) = 0$

$$\int_0^\infty \underbrace{e^{i\lambda S(x_0)}\, g(x_0)}_{const}\, e^{i\lambda \frac{S''(x_0)}{2}(x-x_0)^2} dx$$

$S(t) = \left(\frac{t^6}{3} - \frac{t_c^4}{2}\right) \qquad S'(t_0) = \frac{6t^5}{3} - \frac{4t_c^3}{2} = 0$

$S'(t_0) = 2t^3(t^2 - t_c) = 0$

$t_0 = \sqrt{t_c} \Rightarrow \quad S''(t_0) = 10t^4 - 6t_c^2$

$t_0 = -\sqrt{t_c}$ – не подх. от 0 до ∞ $\quad S''(t_0) = 10c^2 - 6c^2 = 4c^2$

$$\int_0^{\infty} e^{-\frac{\lambda}{2} S''(t_0)\rho^2} d\rho\, e^{i\pi/4}$$

$t - t_0 = x$

$x^2 = \rho^2 e^{i\pi/2} = \rho^2 i$

$$\int_0^{\infty} e^{-\frac{\lambda}{2} S''(t_0)\rho^2} d\rho\, e^{i\pi/4} = \sqrt{\frac{\pi}{\lambda S''(t_0)}}\, e^{i\pi/4}$$

$S''(t_0) > 0$ т.к. от 0 до ∞

$$I(x_1) = \sqrt{\frac{\pi}{\lambda S''(t_0)}}\, g(t_0)\, e^{i\lambda S(t_0) + i\pi/4} =$$

$$= \sqrt{\frac{\pi}{\lambda x}}\sqrt{c}\, e^{i\lambda S(t_0) + i\pi/4} = \sqrt{\frac{\pi}{\lambda 4c^2}}\sqrt{c}\, e^{i\lambda(-\frac{c^3}{6}) + i\pi/4}$$

для $I(x_2)$: $S'(t_0) = 2t^3 c + 2t^5 = 0$

$S(t) = \frac{t^4 c}{2} + \frac{t^6}{3}$ $\quad t_0 = \pm i\sqrt{c} \to$ берем $-i\sqrt{c}$

$$I(x_2) = \sqrt{\frac{\pi}{\lambda |S''(t_0)|}}\,(-i\sqrt{c})\, e^{i\lambda S(x_0) - i\pi/4} =$$

$S''(x_0) < 0$

$$= \sqrt{\frac{\pi}{\lambda|-16c^2|}}\,(-i\sqrt{c})\, e^{i\lambda(\frac{c^3}{2} - \frac{c^3}{3}) - i\pi/4}$$

В итоге

$$C_1 \sqrt{\frac{\pi}{\lambda 4c^2}}\sqrt{c}\, e^{i\lambda(-\frac{c^3}{6}) + i\pi/4} = -C_2 \sqrt{\frac{\pi}{\lambda 16c^4}}\, i\sqrt{c}\, e^{i\lambda\frac{c^3}{6} - i\pi/4}$$

т.к. $\lambda \gg 1$ $\quad C_1\sqrt{c}\, e^{i\lambda(-\frac{c^3}{6}) + i\frac{\pi}{4}} = -C_2\, i\sqrt{c}\, e^{i\lambda\frac{c^3}{6} - i\pi/4}$

$$\left(C_1\sqrt{c}\, e^{-i\lambda\frac{c^3}{6} - i\frac{\pi}{4}} + C_2 i\sqrt{c}\, e^{(i\lambda\frac{c^3}{6} - i\frac{\pi}{4})}\right) = 0$$

$$2C_{1,2}\,\pi c\,\cos\left(\lambda\frac{c^3}{6}-\frac{\pi}{4}\right)=0$$

$$\lambda\frac{c^3}{6}-\frac{\pi}{4}=0$$

$$\lambda\frac{c^3}{6}-\frac{\pi}{4}=-\frac{\pi}{2}+\pi n$$

$$\left(\pi n-\frac{\pi}{2}\right)=\frac{k^6c^3}{2m\alpha\hbar 6}$$

$$\boxed{k=\left(\frac{\left(\pi n-\frac{\pi}{2}\right)}{c^3}\cdot 12m\alpha\hbar\right)^{1/6}}$$

при больших n

$$E=\frac{ck^2|p|}{2m}\sim n^{\frac{1}{3}}$$

Задача 8

$$I(p)=\int_0^\infty e^{-pt}t^{z-1}dt$$

Заметим $\Gamma(z)=\int_0^\infty e^{-t}t^{z-1}dt$

$$\int_0^\infty e^{-\lambda t}dt=\frac{1}{\lambda}\Rightarrow\int_0^\infty t^n e^{-\lambda t}dt=\frac{n!}{\lambda^{n+1}}$$

$$I(p)=\int_0^\infty e^{-pt}t^{z-1}dt=\frac{\Gamma(z)}{p^z}=\frac{(z-1)!}{p^z}$$

Сл. Задание № 7

при $\lambda \to +\infty$

$$I(\lambda) = \int_{-\infty}^{\infty} e^{i\lambda(x+\frac{x^4}{4})} \frac{1}{x^2+1}\, dx$$

Приближение стационарной фазы

$$\int_{-\infty}^{\infty} e^{i\lambda S(x)} g(x)\, dx \quad \text{при } \lambda \to \infty \Rightarrow$$

$$S''(x) > 0$$

$$\Rightarrow \int_{-\infty}^{\infty} e^{i\lambda S(x)} g(x)\, d(x) = g(x_0)\sqrt{\frac{2\pi}{\lambda S''(x_0)}}\, e^{i(\lambda S(x_0) + i\frac{\pi}{4})}$$

$S(x) = x + \frac{x^4}{4}$ $\quad S'(x) = 1 + \frac{4x^3}{4} = 1 + x^3 = 0 \Rightarrow$

$S''(x_0) = 3x^2 = 3 > 0 \quad \Rightarrow x_0 = -1$ (экстрем)

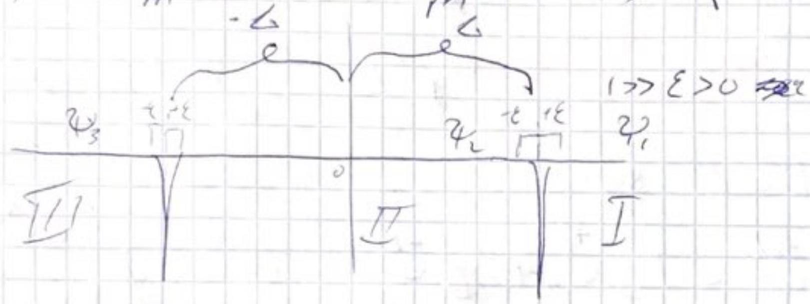
$S(x_0) = -1 + \frac{1}{4} = -\frac{3}{4}$

$g(x_0) = \frac{1}{2} \Rightarrow I(\lambda) = \frac{1}{2}\sqrt{\frac{2\pi}{\lambda 3}}\, e^{i\lambda \cdot -\frac{3}{4} + i\frac{\pi}{4}}$

Подробнее метод показан в 3 номере

Задача №4

$$U(x) = -\frac{\hbar^2 \varkappa}{m}\delta(x-L) - \frac{\hbar^2 \varkappa}{m}\delta(x+L) \quad (\varkappa L \gg 1)$$



$$-\frac{\hbar^2}{2m}\psi''(x) + U(x)\psi(x) = E\psi(x)$$

Проинтегрируем вблизи L $\qquad \int \delta(0) = 1$

$$\underbrace{E\int_{L-\varepsilon}^{L+\varepsilon}\psi(x)dx}_{0} = -\frac{\hbar^2}{2m}\int_{L-\varepsilon}^{L+\varepsilon}\psi''(x)dx - \frac{\hbar^2\varkappa}{m}\psi(L) \Rightarrow$$

т.к. кон. функц. дз. мало $\qquad \frac{\hbar^2}{2m}\left(\psi'(L+0) - \psi'(L-0)\right) = \frac{\hbar^2\varkappa}{m}\psi(L)$

$$\psi_1'(L+0) - \psi_2'(L-0) = -2\varkappa\psi(L) \qquad (\mathrm{II} \sim \mathrm{I})$$

аналогично получится для второго пика

$$\psi_2'(-L+0) - \psi_3'(-L-0) = -2\varkappa\psi(-L)$$

$-\frac{\hbar^2}{2m}\psi''(x) = E\psi(x)$ $\qquad$ допустим $-\frac{E2m}{\hbar^2} = \varkappa^2$

$$\psi'' = \varkappa^2\psi \Rightarrow \psi(x) = Ae^{-\varkappa x} + Be^{\varkappa x} \Rightarrow$$

$\Rightarrow \psi_I(x) = Ae^{-\varkappa x}$ т.к. волновая функ. на ∞ обр. в 0

$\psi_{III}(x) = Be^{\varkappa x}$

$\psi_{II} = A_{II} e^{-\varkappa x} + B_{II} e^{\varkappa x}$

Нех. сшит $\psi_{III}(-0-L) = \psi_{II}(0-L)$

$\psi_{I}(L+0) = \psi_{II}(L-0)$

$\psi_{III}(-0-L) = \psi_{II}(0-L)$

$B_{III} e^{-\varkappa L} = A_{II} e^{+\varkappa L} + B_{II} e^{-\varkappa L}$

$\psi(x) = \begin{cases} A_{II} e^{-\varkappa x} + B_{II} e^{\varkappa x} & -L < x < L \\ B_{III} e^{+\varkappa x} & x < -L \end{cases}$

$\cancel{(B_{III} - B_{II}) e^{-\varkappa L} = A_{II} e^{\varkappa L}}$

производне

$(x=-L)$ $-A_2 \varkappa e^{+\varkappa L} + B_2 \varkappa e^{-\varkappa L} + \varkappa B_3 e^{\varkappa L} = -2K B_3 e^{\varkappa L}$

$\varkappa e^{\varkappa L}(B_3 - A_2) + B_2 \varkappa e^{-\varkappa L} = -2K B_3 e^{\varkappa L}$

$\varkappa B_2 e^{-\varkappa L} + B_2 \varkappa e^{-\varkappa L} = -2K B_3 e^{\varkappa L}$

$2\varkappa B_2 e^{-\varkappa L} = -2K B_3 e^{\varkappa L}$; $-e^{2\varkappa L} \frac{K B_3}{B_2} = \varkappa$

$(x=L)$ $\psi_{I}(L+0) = \psi_{II}(L-0)$

$A_1 e^{-\varkappa L} = A_2 e^{-\varkappa L} + B_2 e^{\varkappa L}$

произв. $-\varkappa A_1 e^{-\varkappa L} + A_2 \varkappa e^{-\varkappa L} + \varkappa B_2 e^{\varkappa L} = -2K A_1 e^{-\varkappa L}$

$\varkappa =$

$\psi_1 = A_1 e^{-\varkappa x}$ $\qquad$ $\psi_2 = A_2 e^{-\varkappa x} + B_2 e^{+\varkappa x}$

$\psi_3 = B_3 e^{\varkappa x}$

$B_3 e^{-\varkappa L} = A_2 e^{+\varkappa L} + B_2 e^{-\varkappa L}$

$$\begin{cases} A_2 e^{+\varkappa L} = (B_3 - B_2) e^{-\varkappa L} \qquad (x = -L) \\ -\varkappa A_2 e^{\varkappa L} + \varkappa B_2 e^{-\varkappa L} - \varkappa B_3 e^{-\varkappa L} = -2k B_3 e^{-\varkappa L} \end{cases}$$

$-\varkappa A_2 e^{\varkappa L} + (-\varkappa)(B_3 - B_2) e^{-\varkappa L} = -2k B_3 e^{-\varkappa L}$

$-2\varkappa A_2 e^{\varkappa L} = -2k B_5 e^{-\varkappa L}$

$\varkappa e^{2\varkappa L} = \frac{k B_3}{A_2}$ $\qquad$ $k = \frac{\varkappa A_2 e^{\varkappa L}}{(A_2 e^{\varkappa L} + B_2 e^{-\varkappa L})}$

$$\begin{cases} A_1 e^{-\varkappa L} = A_2 e^{-\varkappa L} + B_2 e^{\varkappa L} \qquad (x = L) \\ -\varkappa A_1 e^{-\varkappa L} + \varkappa A_2 e^{-\varkappa L} - \varkappa B_2 e^{\varkappa L} = -2k A_1 e^{-\varkappa L} \end{cases}$$

$-\varkappa A_1 e^{-\varkappa L} + \varkappa A_2 e^{-\varkappa L} - \varkappa B_2 e^{\varkappa L} = -2k (A_2 e^{-\varkappa L} + B_2 e^{\varkappa L})$

$\cancel{\varkappa A_2 e^{-\varkappa L}} - \varkappa B_2 e^{\varkappa L} - \varkappa B_2 e^{\varkappa L} = -2k (A_2 e^{-\varkappa L} + B_2 e^{\varkappa L})$

$-2\varkappa B_2 e^{\varkappa L} = -2k (A_2 e^{-\varkappa L} + B_2 e^{\varkappa L})$

$k = \frac{\varkappa B_2 e^{\varkappa L}}{(A_2 e^{\varkappa L} + B_2 e^{\varkappa L})}$

$$2K = \frac{\varkappa B_2 e^{\varkappa L}}{A_2 e^{-\varkappa L} + B_2 e^{\varkappa L}} + \frac{\varkappa A_2 e^{\varkappa L}}{A_2 e^{\varkappa L} + B_2 e^{-\varkappa L}}$$

$$2K = \varkappa e^{\varkappa L} \left(\frac{A_2 e^{\varkappa L} B_2 + B_2^2 e^{-\varkappa L} + A_2^2 e^{-\varkappa L} + A_2 B_2 e^{\varkappa L}}{A_2^2 e^{0} + A_2 B_2 e^{-2\varkappa L} + B_2^2 e^{0} + B_2 A_2 e^{2\varkappa L}} \right)$$

$$2K = \varkappa \left(\frac{A_2 B_2 e^{2\varkappa L} + B_2^2 + A_2^2 + A_2 B_2 e^{2\varkappa L}}{A_2 B_2 e^{-2\varkappa L} + B_2^2 + A_2^2 + A_2 B_2 e^{2\varkappa L}} \right)$$

$$2K = \varkappa \frac{A_2 B_2 \left(e^{2\varkappa L} + e^{2\varkappa L} \right) + A_2^2 + B_2^2}{A_2 B_2 \left(e^{-2\varkappa L} + e^{2\varkappa L} \right) + A_2^2 + B_2^2}$$

$$2K = \varkappa \frac{A_2 B_2 \, 2 \, e^{2\varkappa L} + A_2^2 + B_2^2}{A_2 B_2 \, \mathrm{ch}\, 2\varkappa L + A_2^2 + B_2^2}$$

$$E = -\frac{\varkappa^2 \hbar^2}{2m}$$