

# VAEM: a Deep Generative Model for Heterogeneous **Mixed Type Data**

VAEM is an extension of variational autoencoders (VAEs) in order to handle such heterogeneous data. It is a deep generative model that is trained in a two stage manner. In the first stage we fit a different VAE independently to each data dimension  $x_{nd}$ . We call the resulting D models marginal VAEs. Then, in the second stage, in order to capture the inter-variable dependencies, a new multi-dimensional VAE, called the dependency network, is build on top of the latent representations provided by the first-stage encoders. Finally, if the model is used in down stream tasks such as sequential active information acquisition, we often introduce a third stage, which is to add a new discriminator (preditor) model on top of the VAEM outputs.

Different stages are referred as 1,2, and 3 in list\_stages in the .json files.

#### **Usage**

То Bank Marketing UCI dataset run the demo, you need to first download the (https://archive.ics.uci.edu/ml/datasets/Bank+Marketing), and put the csv file under data/bank. You will need to preprocess the data into the format according to our example .csv file (which does not contain any real data). This can done by splitting the text into columns using; as delimiters. Then, simply run Main\_Notebook.ipynb. This notebook train/or load a VAEM model on Bank dataset, and demonstrates how to perform sequential active information acquisition (SAIA) and imputation. By default, it trains a new model on Bank dataset. If you would like to load a pre-trained model, by default it will load a pre-trained tensorflow model from saved\_weights/bank/. Note that in order to perform active information acquisition, an additional third stage training is required. This will add a discriminator (predictor) to the model, which is required for SAIA. The configurations for VAEM can be found in .json files in hyperparameters/bank, which include:

- "list stage": list of stages that you would like the model to be trained. stage 1 = training marginal VAEs, stage 2 = training dependency network, stage 3 = add predictor and improve predictive performance. The default is [1,2].
- "epochs": number of epochs for training VAEM. If you would like to load a pretrained model rather than training a new one, you can simply set this to zero.
- "latent\_dim": size of latent dimensions of dependency network,
- "p": upper bound for artificial missingness probability. For example, if set to 0.9, then during each training epoch, the algorithm will randomly choose a probability smaller than 0.9, and randomly drops observations according to this probability. Our suggestion is that if original dataset already contains missing data, you can just set p to 0.
- "iteration": iterations (number of mini batches) used per epoch. set to -1 to run the full epoch. If your dataset is large, please set to other values such as 10.
- "batch\_size": iterations (number of mini batches) used per epoch. set to -1 to run the full epoch. If your dataset is large, please set to other values such as 10.
- "K": the dimension of the feature map (h) dimension of PNP encoder.
- "M": Number of MC samples when perform imputing.
- "repeat": number of repeats.
- "data name": name of the dataset being used. Our default is "bank".
- "output dir": Directory where the model is stored. Our default is "./saved weights/bank/",
- "data\_dir": Directory where the data is stored. Our default is "./data/bank/",
- "list strategy": list of strategies for active learning, 0 = random, 1 = single ordering. Default: [1]

#### Load modules

T. [4].

```
TH [T]:
        Tillbort numpy as np
        import tensorflow as tf
        print(tf.__version__)
        from scipy.stats import bernoulli
        import os
        import random
        from random import sample
        from sklearn.feature_selection import mutual_info_regression
        import matplotlib.pyplot as plt
        from sklearn.model_selection import train_test_split
        import pandas as pd
        import sklearn.preprocessing as preprocessing
        from sklearn.metrics import mean_squared_error
        from sklearn.feature_selection import mutual_info_regression, mutual_info_clas
        sif
        plt.switch_backend('agg')
        tfd = tf.contrib.distributions
        import utils.process as process
        import json
        import utils.params as params
        import seaborn as sns; sns.set(style="ticks", color_codes=True)
        /home/paperspace/anaconda3/lib/python3.6/site-packages/h5py/__init__.py:36: Fu
```

```
/home/paperspace/anaconda3/lib/python3.6/site-packages/h5py/__init__.py:36: Fu
tureWarning: Conversion of the second argument of issubdtype from `float` to `
np.floating` is deprecated. In future, it will be treated as `np.float64 == n
p.dtype(float).type`.
    from ._conv import register_converters as _register_converters
1.4.1
```

### load hyperparameters

```
In [2]: args = params.Params('./hyperparameters/bank_plot.json')

if not os.path.exists(args.output_dir):
    os.makedirs(args.output_dir)
rs = 42 # random seed
fast_plot = 0
```

#### **Load Bank Data**

```
In [3]: seed = 3000
        bank raw = pd.read csv("./data/bank/bankmarketing train.csv")
        print(bank raw.info())
        label column="y"
        matrix1 = bank_raw.copy()
        process.encode_catrtogrial_column(matrix1, ["job"])
        process.encode catrtogrial column(matrix1, ["marital"])
        process.encode_catrtogrial_column(matrix1, ["education"])
        process.encode_catrtogrial_column(matrix1, ["default"])
        process.encode_catrtogrial_column(matrix1, ["housing"])
        process.encode_catrtogrial_column(matrix1, ["loan"])
        process.encode_catrtogrial_column(matrix1, ["contact"])
        process.encode_catrtogrial_column(matrix1, ["month"])
        process.encode_catrtogrial_column(matrix1, ["day_of_week"])
        process.encode_catrtogrial_column(matrix1, ["poutcome"])
        process.encode_catrtogrial_column(matrix1, ["y"])
        Data = ((matrix1.values).astype(float))[0:,:]
        <class 'pandas.core.frame.DataFrame'>
```

Range Index: 32950 entries 0 to 32919

```
tangernack. Debbe chicked, o co
 Data columns (total 21 columns):
                            32950 non-null int64
 job
                                                  32950 non-null object
 marital
                                               32950 non-null object
32950 non-null object
 education
default

        default
        32950 non-null object

        housing
        32950 non-null object

        loan
        32950 non-null object

        contact
        32950 non-null object

        month
        32950 non-null object

        day_of_week
        32950 non-null int64

        campaign
        32950 non-null int64

        pdays
        32950 non-null int64

        previous
        32950 non-null int64

        poutcome
        32950 non-null float64

        cons.price.idx
        32950 non-null float64

        cons.conf.idx
        32950 non-null float64

        euribor3m
        32950 non-null float64

                                                 32950 non-null object
                                                     32950 non-null float64
 euribor3m
 nr.employed 32950 non-null float64
y 32950 non-null object
 dtypes: float64(5), int64(5), object(11)
 memory usage: 5.3+ MB
 None
```

## Specify parameters for data preprocessing

```
In []: # the data will be mapped to interval [min_Data,max_Data]. Usually this will b
        e [0,1] but you can also specify other values.
        max_Data = 0.7
        min_Data = 0.3
        # list of categorical variables
        list_cat = np.array([0,1,2,3,4,5,6,7])
        # list of numerical variables
        list_flt = np.array([8,9,10,11,12,13,14,15,16,17,18,19,20])
        # among numerical variables, which ones are discrete. This is referred as cont inuous-discrete variables in Appendix C.1.3 in our paper.
        # Examples include variables that take integer values, for example month, day of week, number of custumors etc. Other examples include numerical variables that are recorded on a discrete grid (for example salary).
        list_discrete = np.array([8,9])
```

## Data pre-processing (decompressing)

Here we basically do two things. First, the raw data matrix is sorted and normalized (squashed). Then, we will "decompress" the categorical variables in the raw data matrix into one-hot encoding features. In our implementations, we always use the suffix \_decompressed to indicate any variables that are based on one-hot representations for categorical variables. For example, Mask\_decompressed is the missingness indicator of the data matrix after decompressed into one-hot encodings.

```
In [4]: # sort the variables in the data matrix, so that categorical variables appears
    first. The resulting data matrix is Data_sub
    list_discrete_in_flt = (np.in1d(list_flt, list_discrete).nonzero()[0])
    list_discrete_compressed = list_discrete_in_flt + len(list_cat)

if len(list_flt)>0 and len(list_cat)>0:
    list_var = np.concatenate((list_cat,list_flt))
    elif len(list_flt)>0:
        list_var = list_flt
```

```
else:
    list var = list cat
Data_sub = Data[:,list_var]
dic_var_type = np.zeros(Data_sub.shape[1])
dic_var_type[0:len(list_cat)] = 1
# In this notebook we assume the raw data matrix is fully observed
Mask = np.ones(Data_sub.shape)
# Normalize/squash the data matrix
Data_std = (Data_sub - Data_sub.min(axis=0)) / (Data_sub.max(axis=0) - Data_su
b.min(axis=0))
scaling_factor = (Data_sub.max(axis=0) - Data_sub.min(axis=0))/(max_Data - min
Data_sub = Data_std * (max_Data - min_Data) + min_Data
# decompress categorical data into one hot representation
Data_cat = Data[:,list_cat].copy()
Data_flt = Data[:,list_flt].copy()
Data_compressed = np.concatenate((Data_cat,Data_flt),axis = 1)
Data decompressed, Mask decompressed, cat dims, DIM FLT = process.data preproc
ess(Data_sub,Mask,dic_var_type)
Data train decompressed, Data test decompressed, mask train decompressed, mask
_test_decompressed,mask_train_compressed, mask_test_compressed,Data_train_comp
ressed, Data_test_compressed = train_test_split(
        Data decompressed, Mask decompressed, Mask, Data compressed, test size=0.
1, random state=rs)
list_discrete = list_discrete_in_flt + (cat_dims.sum()).astype(int)
Data_decompressed = np.concatenate((Data_train_decompressed, Data_test_decompr
essed), axis=0)
Data_train_orig = Data_train_decompressed.copy()
Data test orig = Data test decompressed.copy()
# Note that here we have added some noise to continuous-discrete variables to
help training. Alternatively, you can also disable this by changing the noise
ratio to 0.
Data noisy decompressed, records d, intervals d = process.noisy transform(Data
decompressed, list_discrete, noise_ratio = 0.99)
noise_record = Data_noisy_decompressed - Data_decompressed
Data_train_noisy_decompressed = Data_noisy_decompressed[0:Data_train_decompres
sed.shape[0],:]
Data test noisy decompressed = Data noisy decompressed[Data train decompressed
.shape[0]:,:]
```

(32950, 21)

#### Load or Train a VAEM model

```
In [5]: import utils.active_learning as active_learning
    vae = active_learning.p_vae_active_learning(Data_train_compressed, Data_train_
    noisy_decompressed,mask_train_decompressed,Data_test_decompressed,mask_test_compressed,mask_test_decompressed,cat_dims,DIM_FLT,dic_var_type,args)
```

```
Tensor("is/generator/mul_5:0", shape=(?, 124), dtype=float32)
Tensor("is/generator/mul_43:0", shape=(?, 124), dtype=float32)
```

/home/paperspace/anaconda3/lib/python3.6/site-packages/tensorflow/python/ops/g radients\_impl.py:96: UserWarning: Converting sparse IndexedSlices to a dense T ensor of unknown shape. This may consume a large amount of memory.

"Converting sparse IndexedSlices to a dense Tensor of unknown shape. "

INFO:tensorflow:Restoring parameters from ./saved\_weights/bank3\_gen/encoder.te
nsorflow

INFO:tensorflow:Restoring parameters from ./saved\_weights/bank3\_gen/generator.
tensorflow

```
WARNING:tensorflow:From /home/paperspace/Desktop/VAEM NIPS/models/model.py:55 4: all_variables (from tensorflow.python.ops.variables) is deprecated and will be removed after 2017-03-02. Instructions for updating: Please use tf.global_variables instead.
```

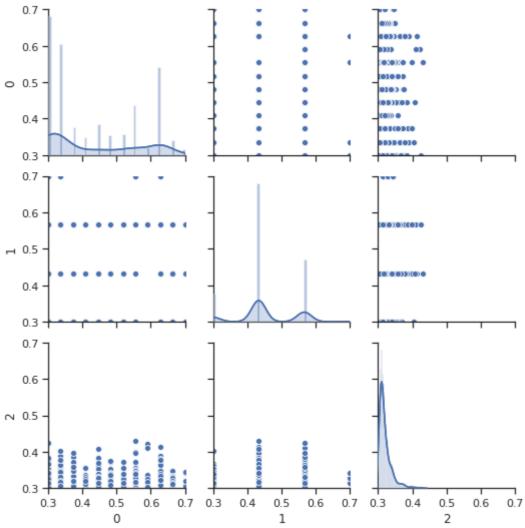
# Use the model to impute data and generate pairplots

```
In [6]: tf.reset_default_graph()
        ### Impute missing data. Fthe mask to be zeros
        x_recon,z_posterior,x_recon_cat_p = vae.get_imputation( Data_train_noisy_decom
        pressed, mask_train_decompressed*0,cat_dims,dic_var_type) ## one hot already c
        pverted to integer
        x_real = process.compress_data(Data_train_decompressed,cat_dims, dic_var_type)
        ## x_real still needs conversion
        x real cat p = Data train decompressed[:,0:(cat dims.sum()).astype(int)]
        \# max Data = 0.7
        # min Data = 0.3
        Data_std = (x_real - x_real.min(axis=0)) / (x_real.max(axis=0) - x_real.min(ax
        is=0))
        scaling factor = (x real.max(axis=0) - x real.min(axis=0))/(max Data - min Dat
        Data_real = Data_std * (max_Data - min_Data) + min_Data
        fast plot = 1
        sub id = [1,2,10]
        if fast_plot ==1:
            Data_real = pd.DataFrame(Data_real[:,sub_id])
            g = sns.pairplot(Data_real.sample(min(1000,x_real.shape[0])),diag_kind =
            g = g.map_diag(sns.distplot, bins = 50,norm_hist = True)
            g.set(xlim=(min_Data,max_Data), ylim = (min_Data,max_Data))
        else:
            Data real = pd.DataFrame(Data real[:,sub id])
            g = sns.pairplot(Data_real.sample(min(10000,x_real.shape[0])),diag_kind =
        'kde')
            g = g.map_diag(sns.distplot, bins = 50,norm_hist = True)
            g = g.map_upper(plt.scatter,marker='+')
            g = g.map_lower(sns.kdeplot, cmap="hot", shade=True, bw=.1)
            g.set(xlim=(min_Data,max_Data), ylim = (min_Data,max_Data))
        76.0
        /home/paperspace/Desktop/VAEM NIPS/models/model.py:1087: RuntimeWarning: inval
        id value encountered in true divide
          decoded_cat_int_p = decoded_cat_int_p/np.sum(decoded_cat_int_p,1,keepdims=Tr
        /home/paperspace/Desktop/VAEM NIPS/models/model.py:1099: RuntimeWarning: inval
        id value encountered in less
          decoded_cat_int[n,d] = np.random.choice(len(decoded_cat_int_p[n,:]), 1 , p=d
        ecoded_cat_int_p[n,:])
        11.0
        2.0
        7.0
```

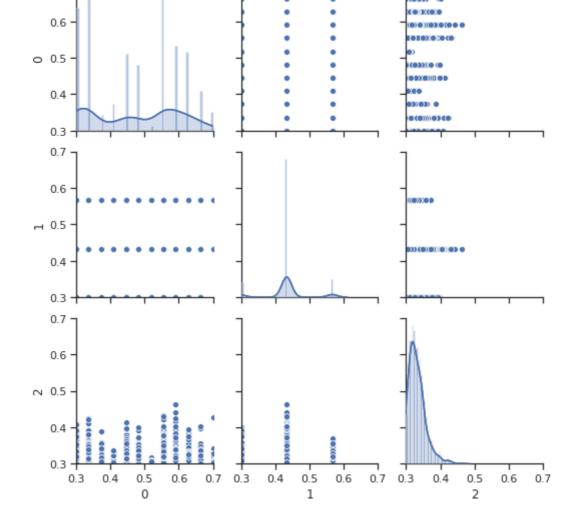
1.0 2.0 0.0 1.0

```
76.0
11.0
3.0
7.0
2.0
2.0
2.0
```

0.7



```
In [7]:
        Data_fake_noisy= x_recon
        Data_fake = process.invert_noise(Data_fake_noisy,list_discrete_compressed,reco
        rds_d)
        Data_std = (Data_fake - x_real.min(axis=0)) / (x_real.max(axis=0) - x_real.min
        (axis=0))
        Data_fake = Data_std * (max_Data - min_Data) + min_Data
        sub_id = [1,2,10]
        if fast_plot ==1:
            g = sns.pairplot(pd.DataFrame(Data_fake[:,sub_id]).sample(min(1000,x_real.
        shape[0])),diag_kind = 'kde')
            g = g.map_diag(sns.distplot, bins = 50,norm_hist = True)
            g.set(xlim=(min_Data,max_Data), ylim = (min_Data,max_Data))
        else:
            g = sns.pairplot(pd.DataFrame(Data_fake[:,sub_id]).sample(min(1000,x_real.
        shape[0])),diag_kind = 'kde')
            g = g.map_diag(sns.distplot, bins = 50,norm_hist = True)
            g = g.map_upper(plt.scatter,marker='+')
            g = g.map_lower(sns.kdeplot, cmap="hot",shade=True,bw=.1)
            g.set(xlim=(min_Data,max_Data), ylim = (min_Data,max_Data))
```



### Train a discriminator on top of the model and perform SAIA

```
In [8]: args = params.Params('./hyperparameters/bank_SAIA.json')
import utils.active_learning as active_learning
vae = active_learning.p_vae_active_learning(Data_train_compressed, Data_train_
noisy_decompressed,mask_train_decompressed,Data_test_decompressed,mask_test_compressed,mask_test_decompressed,cat_dims,DIM_FLT,dic_var_type,args)
```

Tensor("is/generator/mul\_5:0", shape=(?, 124), dtype=float32)
Tensor("is/generator/mul\_43:0", shape=(?, 124), dtype=float32)

/home/paperspace/anaconda3/lib/python3.6/site-packages/tensorflow/python/ops/g radients\_impl.py:96: UserWarning: Converting sparse IndexedSlices to a dense T ensor of unknown shape. This may consume a large amount of memory.

"Converting sparse IndexedSlices to a dense Tensor of unknown shape. "

INFO:tensorflow:Restoring parameters from ./saved\_weights/bank3\_gen/encoder.te
nsorflow

INFO:tensorflow:Restoring parameters from ./saved\_weights/bank3\_gen/generator.
tensorflow

Epoch: 0 negative training ELBO per observed feature: 5081.59, Cat\_ter m: 0.09, Flt\_term: -0.57,z\_term: 62.83

Epoch: 1 negative training ELBO per observed feature: 2226.58, Cat\_ter

m: 0.09, Flt\_term: -0.56,z\_term: 26.07

Epoch: 2 negative training ELBO per observed feature: 1549.46, Cat\_ter

m: 0.09, Flt\_term: -0.56,z\_term: 16.15

Epoch: 3 negative training ELBO per observed feature: 1267.07, Cat\_ter

m: 0.09, Flt\_term: -0.59,z\_term: 10.53

Epoch: 4 negative training ELBO per observed feature: 1094.83, Cat\_ter

m: 0.09, Flt\_term: -0.58,z\_term: 9.83

Epoch: 5 negative training ELBO per observed feature: 972.37, Cat\_term:

0.08, Flt\_term: -0.59,z\_term: 7.71

Epoch: 6 negative training ELBO per observed feature: 891.73, Cat\_term:

0.09, Flt\_term: -0.59,z\_term: 7.22

Epoch: 7 negative training ELBO per observed feature: 811.60, Cat\_term:

```
0.09, Flt_term: -0.58,z_term: 6.13
              negative training ELBO per observed feature: 771.66, Cat_term:
0.09, Flt_term: -0.59,z_term: 5.25
Epoch: 9
               negative training ELBO per observed feature: 718.78, Cat_term:
0.09, Flt_term: -0.61,z_term: 4.92
Epoch: 10
               negative training ELBO per observed feature: 676.23, Cat_term:
0.09, Flt_term: -0.60,z_term: 3.82
               negative training ELBO per observed feature: 627.54, Cat_term:
Epoch: 11
0.08, Flt_term: -0.59,z_term: 3.43
Epoch: 12
              negative training ELBO per observed feature: 590.13, Cat_term:
0.09, Flt_term: -0.60,z_term: 2.60
Epoch: 13
              negative training ELBO per observed feature: 554.96, Cat_term:
0.08, Flt term: -0.59, z term: 2.53
Epoch: 14
               negative training ELBO per observed feature: 529.37, Cat_term:
0.08, Flt_term: -0.60,z_term: 1.83
Epoch: 15
               negative training ELBO per observed feature: 496.02, Cat_term:
0.09, Flt_term: -0.58,z_term: 1.85
               negative training ELBO per observed feature: 467.30, Cat_term:
Epoch: 16
0.08, Flt_term: -0.60,z_term: 1.76
              negative training ELBO per observed feature: 442.08, Cat term:
Epoch: 17
0.08, Flt_term: -0.60,z_term: 1.39
Epoch: 18
               negative training ELBO per observed feature: 413.42, Cat_term:
0.09, Flt_term: -0.60,z_term: 1.00
Epoch: 19
               negative training ELBO per observed feature: 382.13, Cat_term:
0.09, Flt term: -0.60, z term: 0.44
Epoch: 20
               negative training ELBO per observed feature: 359.26, Cat term:
0.08, Flt_term: -0.61,z_term: -0.41
               negative training ELBO per observed feature: 328.52, Cat_term:
Epoch: 21
0.08, Flt_term: -0.60,z_term: -0.48
Epoch: 22
               negative training ELBO per observed feature: 301.39, Cat_term:
0.08, Flt term: -0.59, z term: -0.56
Epoch: 23
               negative training ELBO per observed feature: 278.36, Cat_term:
0.08, Flt_term: -0.62,z_term: -1.18
Epoch: 24
              negative training ELBO per observed feature: 252.63, Cat_term:
0.08, Flt term: -0.60, z term: -1.29
Epoch: 25
               negative training ELBO per observed feature: 229.46, Cat_term:
0.08, Flt term: -0.60, z term: -2.14
               negative training ELBO per observed feature: 207.38, Cat term:
Epoch: 26
0.08, Flt_term: -0.60,z_term: -2.08
Epoch: 27
               negative training ELBO per observed feature: 192.34, Cat_term:
0.08, Flt_term: -0.60,z_term: -2.66
               negative training ELBO per observed feature: 169.82, Cat_term:
Epoch: 28
0.08, Flt_term: -0.62,z_term: -2.85
               negative training ELBO per observed feature: 157.50, Cat term:
Epoch: 29
0.08, Flt_term: -0.60,z_term: -2.86
               negative training ELBO per observed feature: 145.15, Cat term:
0.08, Flt_term: -0.61,z_term: -3.18
Epoch: 31
               negative training ELBO per observed feature: 134.72, Cat_term:
0.08, Flt term: -0.62, z term: -3.41
Epoch: 32
               negative training ELBO per observed feature: 123.38, Cat term:
0.08, Flt_term: -0.58,z_term: -3.62
               negative training ELBO per observed feature: 115.80, Cat_term:
Epoch: 33
0.08, Flt_term: -0.59,z_term: -3.77
Epoch: 34
               negative training ELBO per observed feature: 106.00, Cat_term:
0.08, Flt term: -0.61,z term: -3.62
Epoch: 35
               negative training ELBO per observed feature: 99.92, Cat_term:
0.08, Flt term: -0.59, z term: -4.08
Epoch: 36
               negative training ELBO per observed feature: 91.88, Cat_term:
0.08, Flt_term: -0.59,z_term: -3.97
            negative training ELBO per observed feature: 86.50, Cat_term:
Epoch: 37
0.08, Flt_term: -0.59,z_term: -4.16
Epoch: 38
               negative training ELBO per observed feature: 81.59, Cat_term:
0.08, Flt_term: -0.61,z_term: -4.35
Epoch: 39
            negative training ELBO per observed feature: 77.57, Cat_term:
0.08, Flt_term: -0.60,z_term: -4.35
Epoch: 40
               negative training ELBO per observed feature: 71.84, Cat_term:
```

```
0.08, Flt_term: -0.59,z_term: -4.44
Epoch: 41
              negative training ELBO per observed feature: 68.15, Cat_term:
0.08, Flt_term: -0.59,z_term: -4.48
Epoch: 42
               negative training ELBO per observed feature: 63.78, Cat_term:
0.08, Flt_term: -0.59,z_term: -4.52
Epoch: 43
               negative training ELBO per observed feature: 60.68, Cat_term:
0.08, Flt_term: -0.58,z_term: -4.65
               negative training ELBO per observed feature: 56.97, Cat_term:
Epoch: 44
0.08, Flt_term: -0.60,z_term: -4.69
Epoch: 45
               negative training ELBO per observed feature: 53.56, Cat_term:
0.08, Flt_term: -0.60,z_term: -4.82
Epoch: 46
              negative training ELBO per observed feature: 51.72, Cat_term:
0.08, Flt term: -0.58, z term: -4.91
Epoch: 47
               negative training ELBO per observed feature: 49.12, Cat_term:
0.08, Flt_term: -0.61,z_term: -4.92
Epoch: 48
               negative training ELBO per observed feature: 47.77, Cat_term:
0.08, Flt_term: -0.58,z_term: -4.92
Epoch: 49
               negative training ELBO per observed feature: 44.49, Cat_term:
0.08, Flt_term: -0.59,z_term: -5.01
Epoch: 50
               negative training ELBO per observed feature: 42.90, Cat term:
0.08, Flt_term: -0.60,z_term: -4.98
Epoch: 51
               negative training ELBO per observed feature: 40.53, Cat term:
0.08, Flt_term: -0.60,z_term: -4.97
Epoch: 52
               negative training ELBO per observed feature: 39.43, Cat_term:
0.08, Flt term: -0.60, z term: -5.05
Epoch: 53
               negative training ELBO per observed feature: 37.83, Cat term:
0.08, Flt_term: -0.60,z_term: -5.07
               negative training ELBO per observed feature: 35.81, Cat_term:
Epoch: 54
0.08, Flt_term: -0.59,z_term: -5.13
Epoch: 55
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Epoch: 56
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0.08, Flt_term: -0.58,z_term: -5.17
Epoch: 57
              negative training ELBO per observed feature: 32.19, Cat_term:
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Epoch: 58
               negative training ELBO per observed feature: 30.92, Cat_term:
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Epoch: 59
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Epoch: 60
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Epoch: 61
               negative training ELBO per observed feature: 28.27, Cat_term:
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               negative training ELBO per observed feature: 26.76, Cat term:
Epoch: 62
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Epoch: 63
               negative training ELBO per observed feature: 25.90, Cat term:
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Epoch: 64
               negative training ELBO per observed feature: 24.86, Cat_term:
0.08, Flt term: -0.59, z term: -5.28
Epoch: 65
               negative training ELBO per observed feature: 23.85, Cat term:
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               negative training ELBO per observed feature: 23.22, Cat_term:
Epoch: 66
0.08, Flt_term: -0.57,z_term: -5.28
Epoch: 67
               negative training ELBO per observed feature: 21.70, Cat_term:
0.08, Flt term: -0.58, z term: -5.37
Epoch: 68
               negative training ELBO per observed feature: 21.81, Cat_term:
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Epoch: 69
               negative training ELBO per observed feature: 21.22, Cat_term:
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              negative training ELBO per observed feature: 20.35, Cat_term:
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Epoch: 71
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              negative training ELBO per observed feature: 19.11, Cat_term:
Epoch: 72
0.08, Flt_term: -0.58,z_term: -5.45
Epoch: 73
               negative training ELBO per observed feature: 18.69, Cat_term:
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Epoch: 75
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Epoch: 76
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               negative training ELBO per observed feature: 16.15, Cat_term:
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Epoch: 78
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0.08, Flt_term: -0.56,z_term: -5.48
Epoch: 79
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0.08, Flt term: -0.58, z term: -5.48
Epoch: 80
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               negative training ELBO per observed feature: 14.06, Cat_term:
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Epoch: 83
0.08, Flt_term: -0.55,z_term: -5.47
Epoch: 84
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Epoch: 85
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Epoch: 86
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               negative training ELBO per observed feature: 12.31, Cat_term:
Epoch: 87
0.08, Flt_term: -0.56,z_term: -5.54
Epoch: 88
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Epoch: 89
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Epoch: 91
               negative training ELBO per observed feature: 11.51, Cat_term:
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Epoch: 92
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Epoch: 93
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Epoch: 94
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               negative training ELBO per observed feature: 10.16, Cat term:
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Epoch: 96
               negative training ELBO per observed feature: 10.36, Cat term:
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Epoch: 97
               negative training ELBO per observed feature: 10.10, Cat_term:
0.09, Flt term: -0.56, z term: -5.56
Epoch: 98
               negative training ELBO per observed feature: 9.51, Cat term:
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               negative training ELBO per observed feature: 9.36, Cat_term:
Epoch: 99
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Epoch: 100
               negative training ELBO per observed feature: 8.92, Cat_term:
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Epoch: 101
               negative training ELBO per observed feature: 9.13, Cat_term:
0.08, Flt_term: -0.55,z_term: -5.56
Epoch: 102
               negative training ELBO per observed feature: 8.73, Cat_term:
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             negative training ELBO per observed feature: 8.58, Cat_term:
Epoch: 103
0.08, Flt_term: -0.56,z_term: -5.59
Epoch: 104
               negative training ELBO per observed feature: 8.76, Cat_term:
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              negative training ELBO per observed feature: 8.69, Cat_term:
Epoch: 105
0.08, Flt_term: -0.55,z_term: -5.60
Epoch: 106
               negative training ELBO per observed feature: 8.00, Cat_term:
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0.08, Flt\_term: -0.57,z\_term: -5.61 negative training ELBO per observed feature: 8.36, Cat\_term: Epoch: 107 0.08, Flt\_term: -0.56,z\_term: -5.63 Epoch: 108 negative training ELBO per observed feature: 8.21, Cat\_term: 0.08, Flt\_term: -0.55,z\_term: -5.63 Epoch: 109 negative training ELBO per observed feature: 7.67, Cat\_term: 0.08, Flt\_term: -0.57,z\_term: -5.64 Epoch: 110 negative training ELBO per observed feature: 7.50, Cat\_term: 0.09, Flt\_term: -0.57,z\_term: -5.60 Epoch: 111 negative training ELBO per observed feature: 7.12, Cat\_term: 0.08, Flt\_term: -0.56,z\_term: -5.60 Epoch: 112 negative training ELBO per observed feature: 6.98, Cat\_term:

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