

# 1 review

Boltzmann probability:

$$S = -k \sum p_i \ln p_i$$

Entropy of mixing:

$$S = R \sum x_i \ln x_i$$

Standard Entropy:

$$S^\circ = \int_0^{298} \frac{C_p}{T} dT$$

## 2 Section 4A - Gibbs and Auxillary functions

Gibbs:

$$dG = dH - Tds$$

### 2.1 Gibbs derivation

All systems tend toward a lower "Energy state". This process acquires entropy (maximizes configuration). Consider the three laws applied to a reversible process:

- 1st law:

$$dU = \delta q - \delta w$$

$$dU = \delta q_{rev} - PdV \text{ (I)}$$

- 2nd law:

$$dS \geq \frac{\delta q}{T}$$

$$dS = \frac{\delta q_{rev}}{T}$$

$$\delta q_{rev} = Tds \text{ (II)}$$

- 3rd law:

$$S = 0 \text{ at } 0\text{K}$$

- Enthalpy:

$$H \equiv U + PV$$

$$dH = dU + PdV + vdP \text{ (III)}$$

### 2.2 cont.

$$(II) \circ (I): dU = Tds - PdV \text{ (IV)}$$

$$(IV) \circ (III): dH = (Tds - PdV) + PdV + VdP$$

$$dH = TdS + VdP \text{ (IV)}$$

### 2.3 For a process at constant T and P

$$(IV) \Big|_{P,T}: dH = Tds$$

- This equation is a criteria for spontaneity
- It displays an equilibrium which can be used to predict the direction of a reaction
- This equation balances both:
  - enthalpy (naturally lower energy state is preferred)
  - entropy (naturally tends towards a statistically favored state)

## 2.4 Gibbs Free Energy

$$\begin{aligned}
G &\equiv H - TS \\
\Delta G &= \Delta H - T\Delta S \\
dG &= dU + VdP + PdV - SdT - TdS \\
dG &= \delta q - \delta w + VdP + PdV - SdT - TdS \\
dG &= TdS - PdV + VdP + PdV - SdT - TdS \\
dG &= VdP - SdT
\end{aligned}$$

The final line indicates that the equilibrium criteria varies with temperature and pressure

$$\left[ \frac{\delta G}{\delta T} \Big|_P = -S \quad \frac{\delta G}{\delta P} \Big|_T = V \quad \frac{\delta H}{\delta P} \Big|_S = V \quad \frac{\delta H}{\delta S} \Big|_P = T \right]$$

## 2.5 Gibbs: Standard State

$$\Delta G^\circ = \Delta H^\circ - \Delta S^\circ T$$

## 3 First order Homogenous functions

$$F(x_1, \dots, x_i) = \frac{dF}{dx_1}x_1 + \frac{dF}{dx_2}x_2 + \dots + \frac{dF}{dx_i}x_i$$

Applying this theorem to Thermodynamic equations:

$$G(P, T, n) = \frac{dG}{dP}P + \frac{dG}{dT}T + \frac{dG}{dn}n$$

## 4 Legendre Transform