Assignment 3: Individual Report

Assignment 3 is assessed on an INDIVIDUAL basis. Your solutions should be uploaded to Moodle as a single written document, which may contain graphics and mathematics (but which does not just list your code), and which makes clear links to well-labelled MATLAB files which should be uploaded to Moodle (as .m files) at the same time. Your solutions may be in PDF (e.g. generated via LaTeX), Word, or any other appropriate format, but they must NOT depend on the marker having access to any additional software beyond a PDF reader, Microsoft Word, and a copy of MATLAB. Credit will be given for providing working code, and for providing suitable comments within the code to allow it to be used accurately.
Simply submitting a collection of MATLAB files is not enough.

There are 3 questions in this assignment, worth 15, 50 and 35 marks.

Your answers need to be submitted to Moodle by **noon on Monday 15th January**.
Late submission.
Late submissions without penalty are only allowed for participants who have been granted an extension. To request an extension please see the relevant form on the moodle. Otherwise, the project is subject to the standard University policy: *"Work which is up to one hour late will have five percent of marks deducted. After one hour, ten percent of the available marks will be deducted for each day (or part of each day) that the work is late, up to a total of five days, including weekends and bank holidays e.g. if work is awarded a mark of 30 out of 50, and the work is up to one day late, the final mark is 25. After five days, the work is marked at zero."* For more details, see [Guide to Assessment, Standards, Marking and Feedback.]
Academic misconduct.
Any collaboration with your fellow students should be avoided. The work submitted for assessment must be yours and yours alone. Remember that there are severe penalties for academic misconduct offences such as plagiarism and collusion. For more details, see [Guide to Assessment, Standards, Marking and Feedback].

1. Deterministic SIR model of epidemic.
Consider a version of the SIR model of an epidemic, given by

$$\frac{dS}{dt} = -\beta\frac{IS}{N},$$
$$\frac{dI}{dt} = \beta\frac{IS}{N} - \gamma I - \mu I,$$
$$\frac{dR}{dt} = \gamma I,$$
$$N = S + I + R,$$

where $S(t)$, $I(t)$ and $R(t)$ are the numbers of susceptible, infected and recovered individuals; β is a coefficient characterising the transmission of infection from an infected individual to a susceptible one; γ is the recovery rate and μ is the mortality rate due to infection. Let the parameter values be as follows:

$$\beta = 2, \gamma = 0.1, \mu = 0.01.$$

a) Solve the system with these parameters using MATLAB's ode45 with initial conditions

$S(0) = 100$, $I(0) = 1$ and $R(0) = 0$ and plot the result.

b) Compute and plot the total number of deaths due to the epidemic for a range of values of β. Keep the other parameters and the initial conditions fixed.

c) Describe how the equations could be altered to include the effects of vaccination or reinfection?

[15 marks]

2. Stochastic SIR model of epidemic.

a) Create MATLAB code which simulates an epidemic as a sum of three independent Poisson processes:

Poisson Process	Rate
Infection of susceptible individual	$\beta SI/N$
Death of infected individual	μI
Recovery of infected individual	γI

Simulate this process with parameter values and initial conditions from question 1. Provide graphical output showing two independent examples of the process and compare it with the deterministic model of question 1.

b) For a suitably large sample size,

i) calculate the mean duration of the epidemic (i.e. the time needed for the number of infected individuals to drop to zero) and plot the histogram;

ii) calculate the mean of the total number of deaths due to the epidemic and compare it with the prediction of the deterministic model of question 1.

iii) calculate the mean time needed for the epidemic to reach its peak (i.e. the maximum number of infected individuals);

iv) approximate the probability that the number of infected individuals at the peak will be more than 10;

v) If there is a vaccine which is 100% effective, what level of vaccination (a percentage of vaccinated population) is needed to reduce the probability computed in part (iv) by a factor of 2?

[50 marks]

3. Modified Stochastic Model.

a) The model of question 2 is based on the assumption that recovered individuals are immune to infection. Assume that each recovered individual, with equal probability, is immune or susceptible and modify your code for question 2 to take this assumption into account.
Simulate this modified process with the same parameter values and initial conditions as in question 2 and produce graphical output showing two independent examples of the process.

b) Answer questions (i)-(v) of part (b) of question 2 using the modified model.

c) Compare your results for the two stochastic models. In your opinion, which model is better? What other modifications can you propose?

[35 marks]