Project Report: Conformal Prediction

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2025-03-14

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1 Introduction

Statistical inference methods play a critical role in data science. Traditional approaches, such as frequentist and Bayesian inference, offer probabilistic reasoning frameworks but come with certain limitations. This project explores **Conformal Prediction (CP)** as a scalable alternative to Bayesian uncertainty quantification. CP constructs **valid prediction intervals** with finite-sample guarantees without requiring prior distributions.

2 Frequentist vs. Bayesian Approaches

2.1 Frequentist Approach

- Parameters are fixed but unknown.
- Statistical inference is based on repeated sampling.
- Confidence intervals estimate the range of parameter values.

2.2 Bayesian Approach

- Uses **Bayes' Theorem** to update prior beliefs into a posterior distribution.
- Requires subjective prior selection.
- Computationally expensive, especially in high dimensions (e.g., MCMC sampling).

2.3 Limitations of Bayesian Inference

1. **Subjectivity in prior selection**
2. **Computational complexity** (costly MCMC calculations)
3. **Scalability issues** in large datasets

3 Conformal Prediction: An Alternative

Conformal Prediction (CP) provides a **distribution-free** method to construct valid prediction intervals.

3.1 How CP Works

- CP does not estimate a full probability distribution.
- Uses **nonconformity scores** to measure deviation from past observations.
- Constructs **prediction sets** based on empirical quantiles.

3.1.1 Comparison of Bayesian and Conformal Prediction

Feature	Bayesian Inference	Conformal Prediction
Prior Knowledge	Required	Not Needed
Uncertainty Estimation	Posterior Distributions	Prediction Sets
Computational Cost	High (MCMC)	Efficient (Quantile-based)
Scalability	Limited for Big Data	Highly Scalable

4 Applications of Conformal Prediction

CP is widely applicable in multiple domains:

- **Healthcare**: Uncertainty-aware diagnostics
- **Finance**: Risk assessment in stock markets
- **Cybersecurity**: Anomaly detection in network traffic
- **News Forecasting**: This project applies CP to forecast news article counts across four categories: *Entertainment, Politics, Sports, Technology.*

5 Implementation Details

5.1 Algorithmic Steps

1. Train a **base model** (linear regression).
2. Compute **residuals** (nonconformity scores).
3. Estimate **thresholds** using empirical quantiles.
4. Construct **prediction intervals** ensuring coverage probability.

Execution Time Summary:

```r
# Summary from execution log
training_time <- 0.1021  # seconds
prediction_time <- 0.0058  # seconds
total_run_time <- 0.1079  # seconds
```

6 Computational Complexity

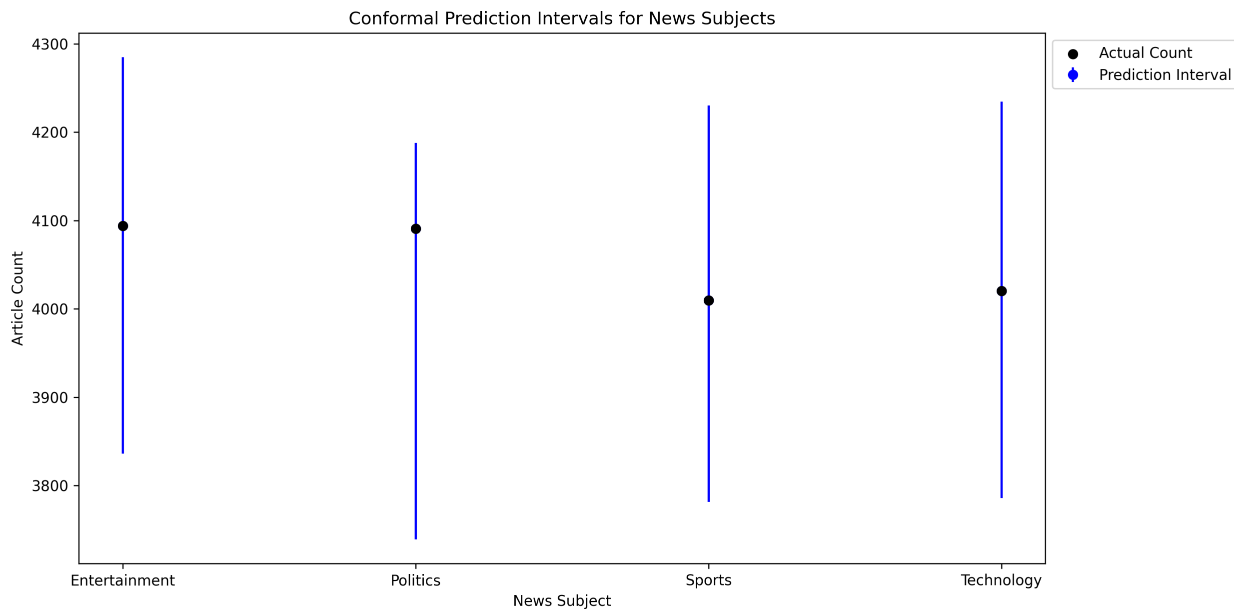
CP Method	Complexity	Pros	Cons
Inductive CP (ICP)	O(n)	Fast, scalable	Intervals may be wide
Transductive CP (TCP)	O(nk)	Tighter intervals	Computationally expensive
Mondrian CP	O(n log n)	Efficient and adaptive	Assumes conditional independence

ICP was selected due to its **efficiency and scalability**.

7 Results and Visualization

Below is the visual representation of Conformal Prediction intervals applied to news article forecasting:

```r
if (file.exists("COMP_4581_Project_Plot.png")) {
  knitr::include_graphics("COMP_4581_Project_Plot.png")
} else {
  cat("Warning: Image file 'COMP_4581_Project_Plot.png' not found. Skipping visualization.")
}
```



8 Future Considerations

To enhance Conformal Prediction, future research should explore:

1. **Adaptive nonconformity scores** to refine prediction intervals.
2. **Comparative evaluation with Bayesian inference** for deeper insights.
3. **Parallel processing techniques** for large-scale CP applications.

9 Conclusion

Conformal Prediction provides **valid prediction intervals** without relying on Bayesian priors. It is computationally efficient and scalable for **real-world applications** such as **finance, healthcare, and cybersecurity**. Future research should focus on improving interval tightness and extending CP methods to deep learning frameworks.