CSCI 3104 Fall 2021
Instructors: Profs. Grochow and Waggoner

Quiz- Standard 15

Due Date ..... October / 22^{nd} / 2021
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Contents

1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
- You **must** virtually sign the Honor Code (see Section 2). Failure to do so will result in your assignment not being graded.

2 Honor Code (Make Sure to Virtually Sign)

Problem 1.

- My submission is in my own words and reflects my understanding of the material.
- I have not collaborated with any other person.
- I have not posted to external services including, but not limited to Chegg, Discord, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

I agree to the above, Michael Ghattas. □

3 Standard 15- Analyzing Code II: Nested Dependent Loops

Problem 2. Analyze the *worst-case* runtime of the following algorithm. Clearly derive the runtime complexity function $T(n)$ for this algorithm, and then find a tight asymptotic bound for $T(n)$ (that is, find a function $f(n)$ such that $T(n) \in \Theta(f(n))$). Avoid heuristic arguments from 2270/2824 such as multiplying the complexities of nested loops.

Algorithm 1 Nested Dependent Loops

```
1: procedure FOO1(Integer n)
2:     for i ← 1; i ≤ 2 * n; i ← i + 1 do
3:         for j ← 1; j ≤ i; j ← j + 1 do
4:             print (i + j)
```

Answer:

We begin with analyzing the inner $j-loop$:

The initialization of the loop takes 1 step.

Observe that the $j-loop$ takes i iterations. At each iteration, the loop does the following;

The comparison $j \leq i$ takes 1 step.

The update $j \leftarrow j+1$ takes 2 steps: 1 step to evaluate $j+1$ and 1 step for the assignment.

The body of the loop consists of a single *print* statement, which takes 1 step, and 1 step to evaluate $(i+j)$.

So the runtime complexity of the j-loop is,

$$1 + \sum_{j=1}^{i} (1+2+2) = 1 + \sum_{j=1}^{i} 5 = 1 + 5i$$

We now analyze the outer $i-loop$:

Initializing the outer loop takes 1 step.

The outer $i-loop$ takes $2n$ iterations. At each iteration, the loop does the following;

The comparison $i \leq 2n$ takes 1 step.

The update $i \leftarrow i+1$ takes 2 steps: 1 step to evaluate $i+1$ and 1 step for the assignment.

The body the $i-loop$ consists solely of the $j-loop$.

So the runtime complexity function $T(n)$ is,

$$\begin{aligned} T(n) &= 1 + \sum_{i=1}^{2n} [1+2+(1+5i)] \\ &= 1 + \sum_{i=1}^{2n} (4+5i) \\ &= 1 + \sum_{i=1}^{2n} 4 + 5 \cdot \sum_{i=1}^{2n} i \\ &= 1 + 8n + 5 \cdot [\frac{2n(2n+1)}{2}] \end{aligned}$$

Thus, $T(n) \in \Theta(n^2)$.

□