CSCI 3104 Fall 2021
Instructor: Profs. Grochow and Waggoner

Requizzing Period 1- Standard 5

Due Date .... October / 11^{th} / 2021
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Contents

1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
- You **must** virtually sign the Honor Code (see Section 2). Failure to do so will result in your assignment not being graded.

2 Honor Code (Make Sure to Virtually Sign)

Problem 1.

- My submission is in my own words and reflects my understanding of the material.
- I have not collaborated with any other person.
- I have not posted to external services including, but not limited to Chegg, Discord, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

I agree to the above, Michael Ghattas. □

3 Standard 5- Exchange Arguments

3.1 Problem 2

Problem 2. Consider the interval scheduling problem from class. You are given a set of intervals $\mathcal{I}$, where each interval has a start and finish time $[s_i, f_i]$. Your goal is to select a subset S of the given intervals such that (i) no two intervals in S overlap, and (ii) S contains as many intervals as possible subject to condition (i).

Suppose we have two intervals with the same start time but different finish times. That is, let $I_1 = [s, f_1]$ and $I_2 = [s, f_2]$ with $f_2 > f_1$.

(a) Let overlap($[s, f]$) denote the number of intervals of $\mathcal{I}$ (excluding $[s, f]$) with which $[s, f]$ overlaps. Explain carefully why overlap(I_1) $\leq$ overlap(I_2).

Answer:

This is due to the fact that any additional interval(s) will either overlap both I_1 and I_2, only I_2, or neither. Our reasoning is as follows:

Given that $I_1 = [s, f_1]$ and $I_2 = [s, f_2]$, with $f_2 > f_1$, means that I_1 is the first interval with the earliest end-time. Any additional interval(s) will either start at the same time as I_1 and I_2 (s), before the end-time of I_1 (f_1) and therefore before the end-time of I_2 (f_2) as well, after the end-time of I_1 (f_1) but before the end-time of I_2 (f_2), or after the end-time of I_2 (f_2) and therefore after the end-time of I_1 (f_1) too. If the additional interval(s) starts at the same time as I_1 and I_2 (s), or before the end-time of I_1 (f_1) and therefore before the end-time of I_2 (f_2) as well, then it would overlap with I_1 and I_2. If the additional interval(s) starts after the end-time of I_1 (f_1) yet before the end-time of I_2 (f_2), then it would only overlap with I_2. Though if the additional interval(s) starts after the end-time of I_1 (f_1) and the end-time of I_2 (f_2), then it would not overlap with I_1 or I_2. Thus, overlap(I_1) will always be either less or equal to overlap(I_2). $\square$

(b) Suppose that overlap(I_1) $<$ overlap(I_2). Explain carefully why I_2 can safely be exchanged for I_1 (that is, in any non-overlapping set of intervals containing I_2, replacing I_2 by I_1 always results in another non-overlapping set of intervals, no smaller than the one we started with).

Answer:

Following our reasoning in part (a) we proceed. Accordingly, while I_2 overlaps I_1, excluding I_1 and I_2, there is an additional equal number of intervals that do not overlap with I_1, as there are intervals that do not overlap with I_2. Yet, each interval in the set of intervals that do not overlap I_1, overlaps with one interval in the set of intervals that do not overlap with I_2. Our reasoning is as follows:

Excluding I_1 and I_2, let A be the set of intervals $1, ..., n$ that do not overlap with I_1, and let B be the set of intervals $1, ..., n$ that do not overlap with I_2. The first interval (A_1) in A will overlap with the first interval in B (B_1), and all the way through, the n^{th} interval (A_n) in A will overlap with the n^{th} interval in B (B_n). Therefore, replacing I_2 by I_1, will always result in a set with the same number of non-overlapping intervals respectively to I_1 and I_2. Thus, while I_2 overlaps I_1 causing overlap(I_1) $<$ overlap(I_2), $|A|$ will always be equal to $|B|$, so replacing I_2 by I_1 always results in another non-overlapping set of intervals, no smaller than the one we started with. $\square$