CSCI 3104 Fall 2021
Instructors: Profs. Grochow and Waggoner

Quiz- Standard 14

Due Date .... October / 22^{nd} / 2021
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Contents

1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
- You **must** virtually sign the Honor Code (see Section 2). Failure to do so will result in your assignment not being graded.

2 Honor Code (Make Sure to Virtually Sign)

Problem 1.

- My submission is in my own words and reflects my understanding of the material.
- I have not collaborated with any other person.
- I have not posted to external services including, but not limited to Chegg, Discord, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

I agree to the above, Michael Ghattas. □

3 Standard 14- Analyzing Code I: Nested Independent Loops

Problem 2. Analyze the *worst-case* runtime of the following algorithm. Clearly derive the runtime complexity function $T(n)$ for this algorithm, and then find a tight asymptotic bound for $T(n)$ (that is, find a function $f(n)$ such that $T(n) \in \Theta(f(n))$). Avoid heuristic arguments from 2270/2824 such as multiplying the complexities of nested loops. [**Note:** $A[1, \ldots, n][1, \ldots, m]$ is a two-dimensional array with row indices in $\{1, \ldots, n\}$ and column indices in $\{1, \ldots, m\}$.]

Assume that $A[i][j]$ takes 2 steps, one for accessing $A[i]$ and a second for accessing the jth element of $A[i]$.

Algorithm 1 Nested Independent Loops

procedure FOO1($A[1, \ldots, n][1, \ldots, n]$)
    for $i \leftarrow 1; i \leq n; i \leftarrow i + 1$ **do**
        for $j \leftarrow 1; j \leq n; j \leftarrow j + 1$ **do**
            for $k \leftarrow 1; k \leq n; k \leftarrow k * 2$ **do**
                if $A[i][k] + A[k][j] \leq A[i][j]$ **then**
                    print $A[i][j]$

Answer:

Starting with the inner most loop (k):

- +1 step for $k \leftarrow 1$
- The boundaries of the loop summation = $\sum_{k=1}^{\lceil(\log_2 n)+1\rceil}$
- The loop has +1 step for $k \leq n$, +1 step for $k * 2$, +1 step for the $k \leftarrow k * 2$, +1 step for $A[i][k] + A[k][j]$, +1 step for $A[i][k] + A[k][j] \leq A[i][j]$, and +1 step for **print**$A[i][j]$ for a total of +6 steps
- So the k-loop = 1+ $\sum_{k=1}^{\lceil(\log_2 n)+1\rceil} 6 = 1 + 6[\lceil(\log_2 n) + 1\rceil]$

Next we look at the following loop (j):

- +1 step for $j \leftarrow 1$
- The boundaries of the loop summation = $\sum_{j=1}^{n}$
- The loop has +1 step for $j \leq n$, +1 step for $j + 1$, +1 step for the $j \leftarrow j + 1$, + (k-loop)
- So the j-loop = 1+ $\sum_{j=1}^{n} 3 + 1 + 6[\lceil(\log_2 n) + 1\rceil] = 1 + 4n + 6n[\lceil(\log_2 n) + 1\rceil]$

Next we look at the last loop (i):

- +1 step for $i \leftarrow 1$
- The boundaries of the loop summation = $\sum_{i=1}^{n}$
- The loop has +1 step for $i \leq n$, +1 step for $i + 1$, +1 step for the $i \leftarrow i + 1$, + (j-loop)
- So the i-loop = 1+ $\sum_{i=1}^{n} 3 + 1 + 4n + 6n[\lceil(\log_2 n) + 1\rceil] = 1 + 4n + 4n^2 + 6n^2[\lceil(\log_2 n) + 1\rceil]$

$T(n) = 1 + 4n + 4n^2 + 6n^2[\lceil(\log_2 n) + 1\rceil]$

Thus, $T(n) \in \Theta(n^2[\log_2 n])$ □