CSCI 3104 Fall 2021
Instructors: Profs. Grochow and Waggoner

Quiz- Standard 11

Due Date .... October / $11th$ / 2021
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Contents

1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
- You **must** virtually sign the Honor Code (see Section 2). Failure to do so will result in your assignment not being graded.

2 Honor Code (Make Sure to Virtually Sign)

Problem 1.

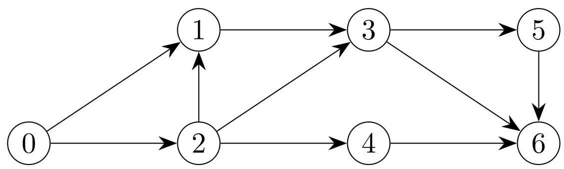
- My submission is in my own words and reflects my understanding of the material.
- I have not collaborated with any other person.
- I have not posted to external services including, but not limited to Chegg, Discord, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

I agree to the above, Michael Ghattas. $\square$

3 Standard 11- Network Flows: Reductions

Problem 2. We say that two $i \rightsquigarrow j$ paths are *edge-disjoint* if they do not share any common edges. Note however, these paths can (and in fact, often do) share common vertices (aside from i and j). As an example, consider the following graph.

- Observe that $0 \to 1 \to 3 \to 5 \to 6$ and $0 \to 2 \to 3 \to 6$ are edge-disjoint paths, as they do not share any directed edges. It is fine that they share the common vertex 3.
- Note however that $0 \to 2 \to 4 \to 6$ and $0 \to 2 \to 3 \to 6$ are **not** edge-disjoint paths, as they both share the $(0, 2)$ edge.



Consider the following problem.

- Input: A directed graph $G(V, E)$, as well as a start node i and an end node j.
- Solution: We seek to find a set $\mathcal{P}$ of $i \to j$ paths such that any two distinct paths $P_1, P_2 \in \mathcal{P}$ are edge-disjoint, and $|\mathcal{P}|$ is maximum. That is, we seek to find a maximum set of edge-disjoint $i \to j$ paths.

(a) Describe how to reduce the above problem to the (one-source, one-sink) max-flow problem from class. Your description should be **general**, and not tied to a specific example.

Answer:

We start by noting that the maximum number of edge-disjoint $i \to j$ is equal to the maximum flow value. Thus our approach can be considered as follows:

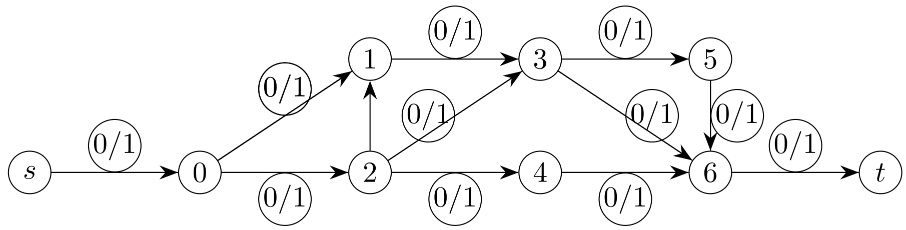
- Initially, assume that there are (n) edge-disjoint paths $P_1 \to P_n$.
- Set $w(e) = 1$ if the edge is traversed in any P_i, where $i = \{1, ..., n\}$
- Set $w(e) = 0$ if the edge is not traversed in any P_i, where $i = \{1, ..., n\}$
- The maximum flow formulation assigns a unit capacity to every edge
- Given that our paths are edge-disjoint, our flow-augmenting path f has a cardinality $val(f) = n$
- The maximum number of edge-disjoint $i \to j$ paths equals maximum flow value.
- Assume that the maximum flow value is (n), then there exists $0 - 1$ flow f of value (n)
- Consider the *source* node s, and the edge (s, u) with $f(s, u) = 1$, then there should exist an edge (u, v) with $f(u, v) = 1$.
- Consider the *sink* node t, proceed while continuously choosing a new edge along the chosen path P_i until reaching t
- Finally, repeating this process until there are no more edge-disjoint paths should produces (n) edge-disjoint paths

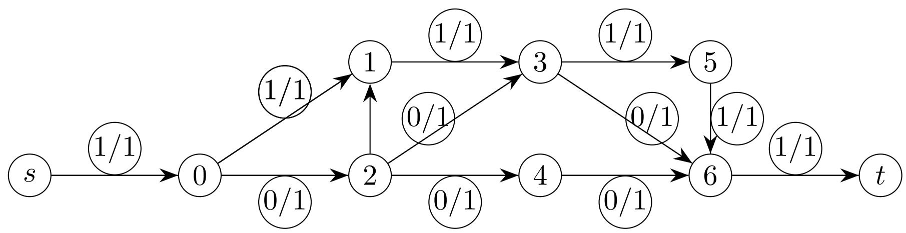
□

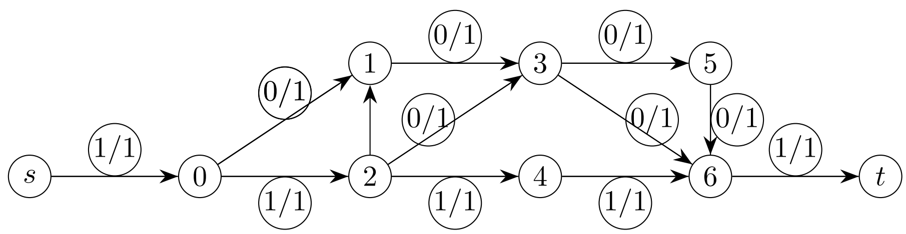
(b) Using your reduction, find a maximum set of edge-disjoint paths from $0 \rightsquigarrow 6$ in the graph above. Show your work, as well as your final answer. Note that there may be multiple maximum-size sets $\mathcal{P}$ in the graph above; you need only find one such set $\mathcal{P}$ of edge-disjoint paths, as long as it has the largest number of paths possible.

Answer:

We will utilize the approach from part (a) to avoid repetition of information. Thus our chosen paths that make up $\mathcal{P}$ are:







- $P_1 = s \to 0 \to 1 \to 3 \to 5 \to 6 \to t$
- $P_2 = s \to 0 \to 2 \to 4 \to 6 \to t$

Since there are no more edge-disjoint possible paths available for us to choose from after choosing P_1 and P_2, $\mathcal{P} = \{P_1, P_2\}$, and $|\mathcal{P}| = 2$, which is the maximum flow value.

$\square$