[STAT 4400] HW-6

Michael Ghattas

4/6/2022

Problem 1

```
library(lme4)
library(lmerTest)

##
## Attaching package: 'lmerTest'

## The following object is masked from 'package:lme4':
##
##     lmer

## The following object is masked from 'package:stats':
##
##     step

library(extraoperators)
library(JWileymisc)

##
## Attaching package: 'JWileymisc'

## The following object is masked from 'package:rstanarm':
##
##     R2

library(multilevelTools)


df <- read.csv(file = "/Users/Home/Documents/Michael_Ghattas/School/
CU_Boulder/2022/Spring 2022/STAT - 4400/Data/ProfEvaltnsBeautyPublic.csv")
head(df)

##   tenured profnumber minority age beautyf2upper beautyflowerdiv
beautyfupperdiv
```

```
## 1       0          1        1  36             6               5
7
## 2       1          2        0  59             2               4
4
## 3       1          3        0  51             5               5
2
## 4       1          4        0  40             4               2
5
## 5       0          5        0  31             9               7
9
## 6       1          6        0  62             5               6
6
##   beautym2upper beautymlowerdiv beautymupperdiv  btystdave  btystdf2u
## 1             6               2               4  0.2015666  0.2893519
## 2             3               2               3 -0.8260813 -1.6193560
## 3             3               2               3 -0.6603327 -0.1878249
## 4             2               3               3 -0.7663125 -0.6650018
## 5             6               7               6  1.4214450  1.7208830
## 6             6               5               5  0.5002196 -0.1878249
##     btystdfl   btystdfu  btystdm2u   btystdml   btystdmu class1 class2
class3
## 1  0.4580018  0.8758139  0.6817153 -0.9000649 -0.1954181      0      0
1
## 2 -0.0735065 -0.5770065 -1.1319040 -0.9000649 -0.6546507      0      0
0
## 3  0.4580018 -1.5455530 -1.1319040 -0.9000649 -0.6546507      0      0
0
## 4 -1.1365230 -0.0927330 -1.7364440 -0.3125226 -0.6546507      0      1
0
## 5  1.5210190  1.8443610  0.6817153  2.0376470  0.7230470      0      0
0
## 6  0.9895102  0.3915404  0.6817153  0.8625621  0.2638144      0      0
0
##   class4 class5 class6 class7 class8 class9 class10 class11 class12
class13
## 1      0      0      0      0      0      0       0       0       0
0
## 2      0      0      0      0      0      0       0       0       0
```

```
0
## 3      1      0      0      0      0      0       0       0       0
0
## 4      0      0      0      0      0      0       0       0       0
0
## 5      0      0      0      0      0      0       0       0       0
0
## 6      0      0      0      0      0      0       0       0       0
0
##   class14 class15 class16 class17 class18 class19 class20 class21 class22
## 1       0       0       0       0       0       0       0       0       0
## 2       0       0       0       0       0       0       0       0       0
## 3       0       0       0       0       0       0       0       0       0
## 4       0       0       0       0       0       0       0       0       0
## 5       0       0       0       0       0       0       0       0       0
## 6       0       0       0       0       0       0       0       0       0
##   class23 class24 class25 class26 class27 class28 class29 class30
## 1       0       0       0       0       0       0       0       0
## 2       0       0       0       0       0       0       0       0
## 3       0       0       0       0       0       0       0       0
## 4       0       0       0       0       0       0       0       0
## 5       0       0       0       0       0       0       0       0
## 6       0       0       0       0       0       0       0       0
##   courseevaluation didevaluation female formal fulldept lower
multipleclass
## 1              4.3            24      1      0        1     0
1
## 2              4.5            17      0      0        1     0
0
## 3              3.7            55      0      0        1     0
1
## 4              4.3            40      1      0        1     0
1
## 5              4.4            42      1      0        1     0
0
## 6              4.2           182      0      1        1     0
0
```

```
##   nonenglish onecredit percentevaluating profevaluation students
tenuretrack
## 1          0         0          55.81395            4.7       43
1
## 2          0         0          85.00000            4.6       20
1
## 3          0         0         100.00000            4.1       55
1
## 4          0         0          86.95652            4.5       46
1
## 5          0         0          87.50000            4.8       48
1
## 6          0         0          64.53901            4.4      282
1
##   blkandwhite btystdvariance btystdavepos btystdaveneg
## 1           0      2.1298060     0.201567     0.000000
## 2           0      1.3860810     0.000000    -0.826081
## 3           0      2.5374350     0.000000    -0.660333
## 4           0      1.7605770     0.000000    -0.766312
## 5           0      1.6931000     1.421450     0.000000
## 6           0      0.9447419     0.500220     0.000000
```

```
courses <- data.frame(df[,19:48])
n <- nrow (df)
J <- ncol (courses) + 1
course.id <- rep (0, n)
for (i in 1:n){
  for (j in 1:30){
    if (courses[i,j]==1) course.id[i] <- j
  }
}
```

```
head(df)
```

```
##   tenured profnumber minority age beautyf2upper beautyflowerdiv
beautyfupperdiv
## 1       0          1        1  36             6               5
7
```

```
## 2       1          2        0  59             2              4
4
## 3       1          3        0  51             5              5
2
## 4       1          4        0  40             4              2
5
## 5       0          5        0  31             9              7
9
## 6       1          6        0  62             5              6
6
##   beautym2upper beautymlowerdiv beautymupperdiv  btystdave  btystdf2u
## 1             6               2               4  0.2015666  0.2893519
## 2             3               2               3 -0.8260813 -1.6193560
## 3             3               2               3 -0.6603327 -0.1878249
## 4             2               3               3 -0.7663125 -0.6650018
## 5             6               7               6  1.4214450  1.7208830
## 6             6               5               5  0.5002196 -0.1878249
##     btystdfl   btystdfu  btystdm2u   btystdml   btystdmu class1 class2
class3
## 1  0.4580018  0.8758139  0.6817153 -0.9000649 -0.1954181      0      0
1
## 2 -0.0735065 -0.5770065 -1.1319040 -0.9000649 -0.6546507      0      0
0
## 3  0.4580018 -1.5455530 -1.1319040 -0.9000649 -0.6546507      0      0
0
## 4 -1.1365230 -0.0927330 -1.7364440 -0.3125226 -0.6546507      0      1
0
## 5  1.5210190  1.8443610  0.6817153  2.0376470  0.7230470      0      0
0
## 6  0.9895102  0.3915404  0.6817153  0.8625621  0.2638144      0      0
0
##   class4 class5 class6 class7 class8 class9 class10 class11 class12
class13
## 1      0      0      0      0      0      0       0       0       0
0
## 2      0      0      0      0      0      0       0       0       0
0
## 3      1      0      0      0      0      0       0       0       0
```

```
0
## 4      0      0      0      0      0      0       0       0       0
0
## 5      0      0      0      0      0      0       0       0       0
0
## 6      0      0      0      0      0      0       0       0       0
0
##   class14 class15 class16 class17 class18 class19 class20 class21 class22
## 1       0       0       0       0       0       0       0       0       0
## 2       0       0       0       0       0       0       0       0       0
## 3       0       0       0       0       0       0       0       0       0
## 4       0       0       0       0       0       0       0       0       0
## 5       0       0       0       0       0       0       0       0       0
## 6       0       0       0       0       0       0       0       0       0
##   class23 class24 class25 class26 class27 class28 class29 class30
## 1       0       0       0       0       0       0       0       0
## 2       0       0       0       0       0       0       0       0
## 3       0       0       0       0       0       0       0       0
## 4       0       0       0       0       0       0       0       0
## 5       0       0       0       0       0       0       0       0
## 6       0       0       0       0       0       0       0       0
##   courseevaluation didevaluation female formal fulldept lower
multipleclass
## 1              4.3            24      1      0        1     0
1
## 2              4.5            17      0      0        1     0
0
## 3              3.7            55      0      0        1     0
1
## 4              4.3            40      1      0        1     0
1
## 5              4.4            42      1      0        1     0
0
## 6              4.2           182      0      1        1     0
0
##   nonenglish onecredit percentevaluating profevaluation students
tenuretrack
```

```
## 1           0         0          55.81395            4.7        43
1
## 2           0         0          85.00000            4.6        20
1
## 3           0         0         100.00000            4.1        55
1
## 4           0         0          86.95652            4.5        46
1
## 5           0         0          87.50000            4.8        48
1
## 6           0         0          64.53901            4.4       282
1
##   blkandwhite btystdvariance btystdavepos btystdaveneg
## 1           0      2.1298060     0.201567     0.000000
## 2           0      1.3860810     0.000000    -0.826081
## 3           0      2.5374350     0.000000    -0.660333
## 4           0      1.7605770     0.000000    -0.766312
## 5           0      1.6931000     1.421450     0.000000
## 6           0      0.9447419     0.500220     0.000000
```

(a)

$$y_i \sim N(\alpha_{j[i]} + \beta_{j[i]} x_i, \sigma_y^2), \text{ for } i = 1, \ldots, n$$

(b)

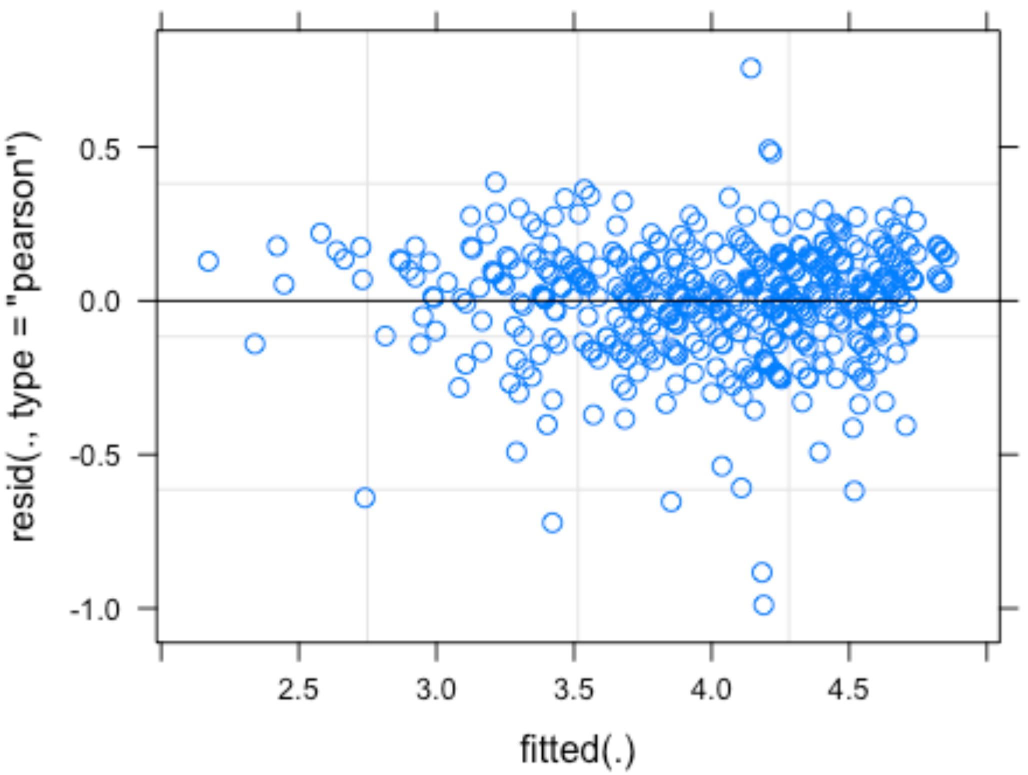
```
M1 <- lmer (courseevaluation ~ profevaluation + (1 + profevaluation |
course.id) + students + (1 + students | course.id) + tenuretrack + (1 +
tenuretrack | course.id) + tenured + (1 + tenured | course.id) +
percentevaluating + (1 + percentevaluating | course.id), data = df)

## boundary (singular) fit: see help('isSingular')

summary(M1)

## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## courseevaluation ~ profevaluation + (1 + profevaluation | course.id) +
##     students + (1 + students | course.id) + tenuretrack + (1 +
##     tenuretrack | course.id) + tenured + (1 + tenured | course.id) +
```

```
##     percentevaluating + (1 + percentevaluating | course.id)
##    Data: df
##
## REML criterion at convergence: -158.4
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.1976 -0.5388  0.1175  0.6546  3.9758
##
## Random effects:
##  Groups      Name              Variance  Std.Dev.  Corr
##  course.id   (Intercept)       5.199e-02 2.280e-01
##              profevaluation    2.345e-03 4.843e-02 -1.00
##  course.id.1 (Intercept)       8.580e-07 9.263e-04
##              students          9.228e-12 3.038e-06 -1.00
##  course.id.2 (Intercept)       2.396e-05 4.895e-03
##              tenuretrack       3.947e-05 6.282e-03 -1.00
##  course.id.3 (Intercept)       2.429e-03 4.929e-02
##              tenured           1.614e-03 4.017e-02 -1.00
##  course.id.4 (Intercept)       1.889e-02 1.374e-01
##              percentevaluating 2.221e-06 1.490e-03 -1.00
##  Residual                      3.622e-02 1.903e-01
## Number of obs: 463, groups:  course.id, 30
##
## Fixed effects:
##                     Estimate Std. Error        df t value Pr(>|t|)
## (Intercept)       -9.450e-02  1.297e-01  4.402e+00  -0.729   0.5031
## profevaluation     9.485e-01  2.736e-02  3.192e+00  34.673 3.19e-05 ***
## students          -6.214e-05  1.302e-04  1.684e+02  -0.477   0.6339
## tenuretrack       -6.614e-02  2.765e-02  7.418e+00  -2.392   0.0461 *
## tenured            5.425e-02  2.939e-02  5.508e+00   1.845   0.1189
## percentevaluating  1.981e-03  8.925e-04  3.473e+00   2.220   0.1009
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Correlation of Fixed Effects:
##             (Intr) prfvlt stdnts tnrtrc tenurd
## profevalutn -0.801
## students    -0.058 -0.082
## tenuretrack -0.188  0.113 -0.057
## tenured      0.069 -0.169 -0.079 -0.442
## percntvltng -0.385 -0.198  0.194 -0.043 -0.010
## optimizer (nloptwrap) convergence code: 0 (OK)
## boundary (singular) fit: see help('isSingular')
```

(c)

```
plot(M1)
```



Problem 2

(a)

```r
I <- 100L
J <- 10L
W <- 3L
tau <- 2
sigma <- 1

assignment <- matrix(0L,I,J)
for (i in 1L:I) {
  workload <- colSums(assignment)
  available <- which (workload < W*I/J)
  if (i > 75L)
    cat("Round ",i,": available = ",
        paste(available,collapse=", "),"\n")
  while (length(available) < W) {
    slacker <- which.min(workload)
    pswaps <- which(!assignment[1L:(i-1L),slacker])
    swaprow <- sample(pswaps,1L)
    swapcol <- sample(which(as.logical(assignment[swaprow,])),1L)
    assignment[swaprow,swapcol] <- 0L
    assignment[swaprow,slacker] <- 1L
    workload <- colSums(assignment)
    available <- which(workload < W*I/J)
    cat("Round ",i,"x: availble=",paste(available,collapse=", "),
        "\n")
  }
  assignment[i,sample(available,W)] <- 1L
}
```

```
## Round  76 : available =  1, 2, 3, 4, 5, 6, 7, 8, 9, 10
## Round  77 : available =  1, 2, 3, 4, 5, 6, 7, 8, 9, 10
## Round  78 : available =  1, 2, 3, 4, 5, 6, 7, 8, 9, 10
## Round  79 : available =  1, 2, 3, 4, 5, 6, 7, 8, 9, 10
## Round  80 : available =  1, 2, 3, 4, 5, 6, 7, 8, 9, 10
```

```
## Round  81 : available =  1, 2, 3, 4, 5, 6, 7, 8, 9, 10
## Round  82 : available =  1, 2, 3, 4, 5, 6, 7, 8, 9, 10
## Round  83 : available =  1, 2, 3, 4, 5, 6, 7, 8, 10
## Round  84 : available =  1, 2, 3, 4, 5, 6, 7, 8, 10
## Round  85 : available =  1, 2, 3, 4, 5, 6, 7, 8, 10
## Round  86 : available =  1, 2, 3, 4, 5, 6, 7, 8, 10
## Round  87 : available =  1, 2, 3, 4, 5, 6, 7, 8, 10
## Round  88 : available =  1, 2, 3, 4, 5, 6, 7, 8, 10
## Round  89 : available =  1, 3, 4, 5, 6, 7, 8, 10
## Round  90 : available =  1, 3, 4, 5, 7, 8, 10
## Round  91 : available =  1, 4, 5, 7, 8, 10
## Round  92 : available =  1, 4, 7, 8, 10
## Round  93 : available =  1, 4, 7, 8, 10
## Round  94 : available =  4, 7, 8, 10
## Round  95 : available =  4, 7, 8, 10
## Round  96 : available =  4, 7, 8, 10
## Round  97 : available =  4, 7, 8, 10
## Round  98 : available =  4, 7, 8, 10
## Round  99 : available =  4, 7, 10
## Round  100 : available =  4, 7
## Round  100 x: availble= 4, 6, 7
```

```
colSums(assignment)
```

```
##  [1] 30 30 30 30 30 30 30 30 30 30
```

```
rowSums(assignment)
```

```
##   [1] 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
3 3 3
##  [38] 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
3 3 3
##  [75] 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
```

```
write.csv(assignment,"assignment.csv")

ability <- runif(I,1,10)
severity <- rnorm(J,0,tau)
```

```
applicant <- rep(1L:I,each=W)
rater <-
  sapply(1L:I,
            function (i)
              which(as.logical(assignment[i,])))
str(rater)
```

```
##  int [1:3, 1:100] 3 9 10 2 6 10 3 5 7 3 ...
```

```
rating <- ability[applicant] + severity[rater] + rnorm(I*W,0,sigma)
rating <- pmax(1,pmin(rating,10))
ratings.df <- data.frame(applicant=applicant, rater=as.vector(rater),
rating=rating)
ratings.df
```

```
##     applicant rater    rating
## 1           1     3  4.877936
## 2           1     9  4.071667
## 3           1    10  3.325303
## 4           2     2  4.722358
## 5           2     6  4.459272
## 6           2    10  4.966620
## 7           3     3  9.014091
## 8           3     5  6.626949
## 9           3     7  4.674131
## 10          4     3 10.000000
## 11          4     5  4.306151
## 12          4     9  9.498074
## 13          5     1  8.698292
## 14          5     2  8.464416
## 15          5     6  8.325377
## 16          6     1  1.000000
## 17          6     5  1.000000
## 18          6     9  4.064592
## 19          7     1  6.126157
## 20          7     2  6.782644
## 21          7    10  8.346936
```

```
## 22           8     3  5.744595
## 23           8     6  4.310485
## 24           8    10  5.341055
## 25           9     4  6.006205
## 26           9     6 10.000000
## 27           9     9  9.688982
## 28          10     2  3.490032
## 29          10     4  3.317153
## 30          10     8  8.055180
## 31          11     4  2.668839
## 32          11     9  6.275555
## 33          11    10  4.991605
## 34          12     2  4.947048
## 35          12     3  9.053500
## 36          12     5  3.011960
## 37          13     1  3.089924
## 38          13     8  6.741440
## 39          13     9  5.622164
## 40          14     1  1.000000
## 41          14     5  1.000000
## 42          14     6  3.149740
## 43          15     3  3.371135
## 44          15     5  1.000000
## 45          15     7  1.000000
## 46          16     7  6.901227
## 47          16     8 10.000000
## 48          16    10 10.000000
## 49          17     6  3.654946
## 50          17     8  9.787064
## 51          17     9  5.041957
## 52          18     3  6.313457
## 53          18     4  1.000000
## 54          18     9  5.100058
## 55          19     2  7.018081
## 56          19     3 10.000000
```

```
## 57          19     5  4.925819
## 58          20     5  2.895494
## 59          20     7  2.756140
## 60          20     8 10.000000
## 61          21     1  1.160127
## 62          21     4  1.000000
## 63          21     5  1.000000
## 64          22     3  5.088728
## 65          22     7  2.631927
## 66          22    10  6.151714
## 67          23     2  3.827981
## 68          23     3  7.779316
## 69          23     8  9.574754
## 70          24     1  1.000000
## 71          24     2  1.000000
## 72          24     7  1.000000
## 73          25     1  1.000000
## 74          25     7  1.000000
## 75          25     9  3.949869
## 76          26     4  1.000000
## 77          26     9  4.455132
## 78          26    10  4.983554
## 79          27     1  3.166579
## 80          27     8  8.654203
## 81          27     9  7.707843
## 82          28     5  4.412355
## 83          28     8 10.000000
## 84          28     9  8.301644
## 85          29     4  5.663889
## 86          29     6  6.901654
## 87          29     9  7.896750
## 88          30     3  3.566951
## 89          30     4  1.000000
## 90          30     8  6.506288
## 91          31     2  4.627617
```

```
## 92          31     9  5.050037
## 93          31    10  8.219304
## 94          32     4  8.003881
## 95          32     6  8.763371
## 96          32     7  5.793486
## 97          33     1  5.225754
## 98          33     5  2.938811
## 99          33    10  8.795114
## 100         34     2  2.252691
## 101         34     6  1.000000
## 102         34     8  6.319328
## 103         35     2  5.692678
## 104         35     4  2.766224
## 105         35    10  7.264668
## 106         36     1  6.285386
## 107         36     4  4.395731
## 108         36     8  8.163796
## 109         37     4  2.645935
## 110         37     7  1.000000
## 111         37     9  5.048189
## 112         38     1  6.346422
## 113         38     7  2.829043
## 114         38    10  8.132605
## 115         39     2  5.992719
## 116         39     5  3.875253
## 117         39     6  6.775163
## 118         40     1  4.483837
## 119         40     6  5.429065
## 120         40    10  6.767504
## 121         41     1  7.422351
## 122         41     3 10.000000
## 123         41     5  6.269211
## 124         42     2  7.717812
## 125         42     8 10.000000
## 126         42    10  9.975251
```

```
## 127        43     1  2.602405
## 128        43     4  1.396017
## 129        43     6  3.847542
## 130        44     2  8.585591
## 131        44     3  9.860183
## 132        44    10 10.000000
## 133        45     4  1.161482
## 134        45     9  4.608198
## 135        45    10  3.828891
## 136        46     2  6.205867
## 137        46     4  5.305351
## 138        46     7  4.121158
## 139        47     6 10.000000
## 140        47     8 10.000000
## 141        47     9 10.000000
## 142        48     1  3.305042
## 143        48     5  2.048494
## 144        48    10  5.544536
## 145        49     1  3.227400
## 146        49     4  1.700788
## 147        49    10  5.589945
## 148        50     1  3.750554
## 149        50     6  4.337544
## 150        50    10  4.569291
## 151        51     2  5.501920
## 152        51     6  7.727567
## 153        51     7  1.890945
## 154        52     3  9.860161
## 155        52     8  9.290609
## 156        52    10  7.840926
## 157        53     2  7.039270
## 158        53     3 10.000000
## 159        53     5  6.169399
## 160        54     3  8.821588
## 161        54     5  5.205619
```

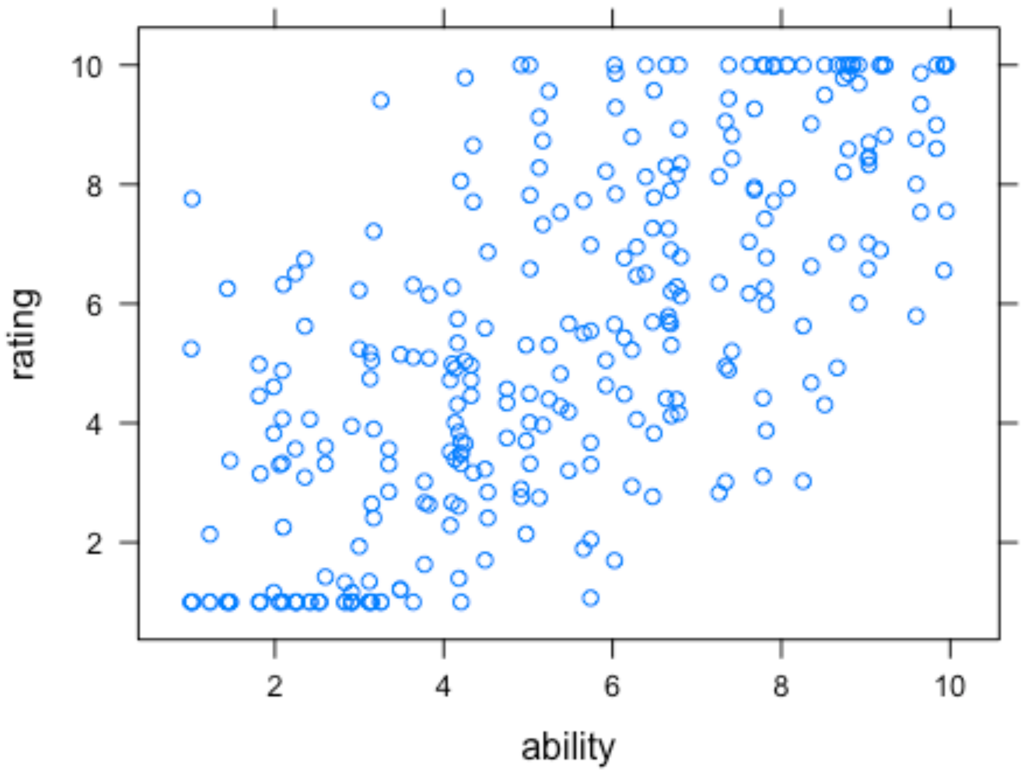
```
## 162         54     9  8.432689
## 163         55     1  9.864400
## 164         55     2  7.533943
## 165         55     6  9.342552
## 166         56     1  4.009182
## 167         56     6  4.492851
## 168         56     8 10.000000
## 169         57     2  1.421900
## 170         57     3  3.603976
## 171         57     6  3.316520
## 172         58     5  1.000000
## 173         58     7  1.000000
## 174         58     8  7.757806
## 175         59     3 10.000000
## 176         59     6  8.815973
## 177         59     9 10.000000
## 178         60     1  4.278819
## 179         60     2  4.825492
## 180         60    10  7.530537
## 181         61     3 10.000000
## 182         61     4 10.000000
## 183         61     7  7.554916
## 184         62     1  7.905044
## 185         62     3  7.949131
## 186         62     9  9.265588
## 187         63     1  8.996774
## 188         63     2  8.600547
## 189         63     3 10.000000
## 190         64     4  1.000000
## 191         64     5  1.000000
## 192         64     7  1.344268
## 193         65     2  1.932005
## 194         65     3  6.228581
## 195         65     9  5.244281
## 196         66     2  7.932318
```

```
## 197         66     3 10.000000
## 198         66     8 10.000000
## 199         67     1  3.395659
## 200         67     3  4.909747
## 201         67     9  4.009184
## 202         68     7  6.556078
## 203         68     8 10.000000
## 204         68     9 10.000000
## 205         69     2  5.697878
## 206         69     5  5.790073
## 207         69     6  7.253999
## 208         70     1  1.630763
## 209         70     2  2.666728
## 210         70     6  3.018002
## 211         71     2  2.849599
## 212         71     6  3.309543
## 213         71     9  3.563578
## 214         72     1  2.408196
## 215         72     3  3.899733
## 216         72     8  7.215165
## 217         73     1  1.000000
## 218         73     2  1.000000
## 219         73     5  1.000000
## 220         74     3  1.000000
## 221         74     6  2.135339
## 222         74     7  1.000000
## 223         75     4  4.190676
## 224         75     5  3.204214
## 225         75     6  5.663527
## 226         76     5  4.164806
## 227         76     8 10.000000
## 228         76     9  8.927404
## 229         77     2  2.842228
## 230         77     5  2.409205
## 231         77     9  6.866060
```

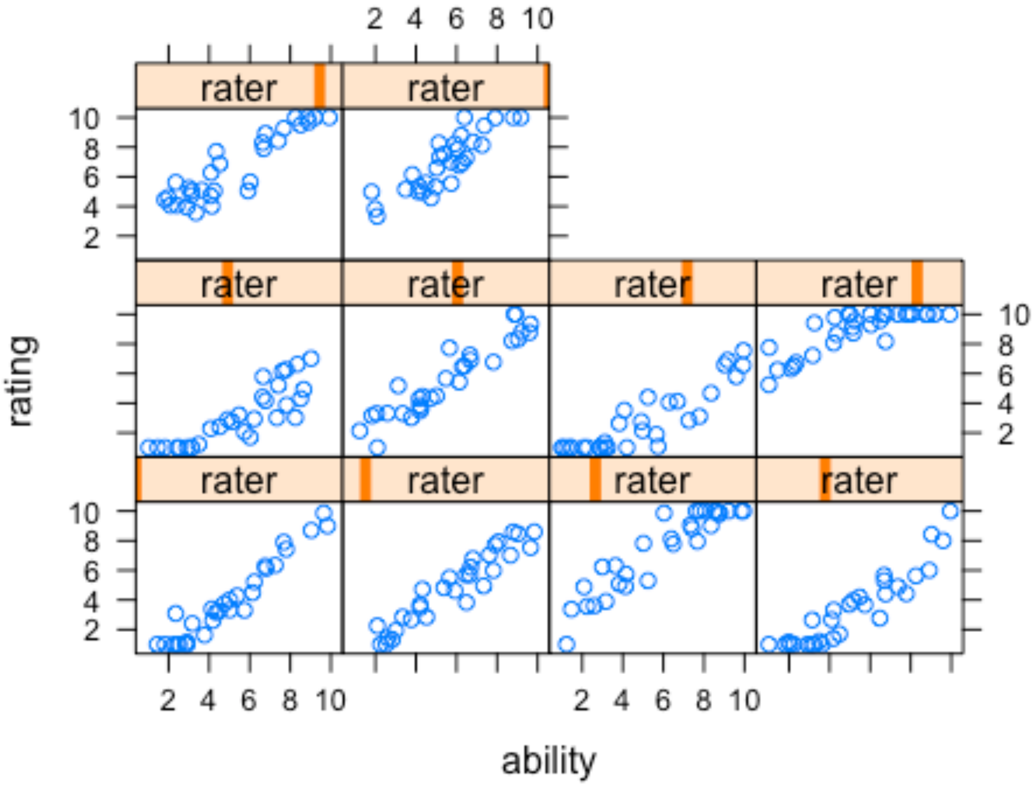
```
## 232         78     6  6.465609
## 233         78     7  4.058816
## 234         78    10  6.949901
## 235         79     5  1.696127
## 236         79     8 10.000000
## 237         79     9  5.652292
## 238         80     4  5.627440
## 239         80     5  3.022432
## 240         80     9 10.000000
## 241         81     5  1.000000
## 242         81     6  5.175261
## 243         81     9  4.744418
## 244         82     5  2.284002
## 245         82     7  3.522581
## 246         82     9  4.719841
## 247         83     3  5.304573
## 248         83     7  4.401000
## 249         83     8  9.562456
## 250         84     3  8.125913
## 251         84     6  6.507556
## 252         84    10 10.000000
## 253         85     4  1.204182
## 254         85     5  1.217691
## 255         85    10  5.148105
## 256         86     2  3.689308
## 257         86     6  3.486715
## 258         86     7  1.000000
## 259         87     5  2.748991
## 260         87     8  9.128233
## 261         87    10  8.278367
## 262         88     1  1.000000
## 263         88     2  1.326866
## 264         88     7  1.000000
## 265         89     3  9.778086
## 266         89     6  8.205507
```

```
## 267          89     8 10.000000
## 268          90     1  3.319124
## 269          90     3  7.821311
## 270          90    10  6.582771
## 271          91     4  8.430592
## 272          91     5  7.014987
## 273          91     7  6.582258
## 274          92     4  1.000000
## 275          92     7  1.000000
## 276          92     8  9.409213
## 277          93     1  1.000000
## 278          93     7  1.000000
## 279          93     8  6.253491
## 280          94     4  3.963601
## 281          94     8  8.730513
## 282          94    10  7.326352
## 283          95     4  4.890290
## 284          95     8 10.000000
## 285          95    10  9.437996
## 286          96     4  1.000000
## 287          96     7  1.000000
## 288          96     8  5.244696
## 289          97     4  3.700859
## 290          97     7  2.141669
## 291          97    10  5.305491
## 292          98     4  4.418442
## 293          98     7  3.106260
## 294          98     8 10.000000
## 295          99     4  3.670573
## 296          99     7  1.067972
## 297          99    10  6.983470
## 298         100     4  1.000000
## 299         100     6  3.300107
## 300         100     7  1.000000
```

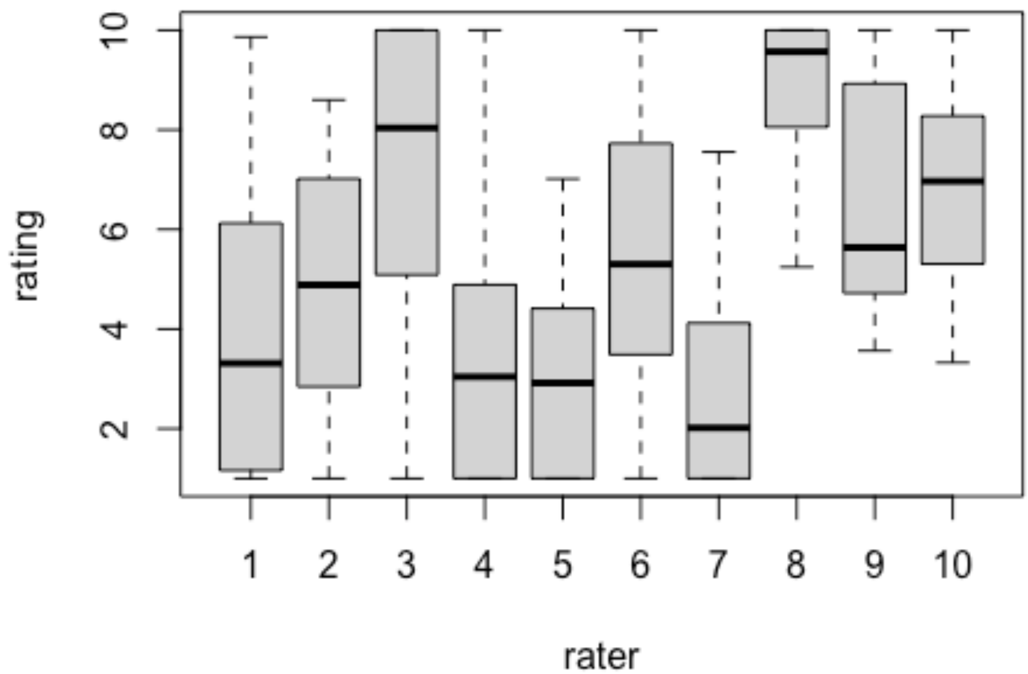
```
write.csv(ratings.df,"ratings.csv")

library(lattice)
ratings.df1 <- data.frame(ratings.df, ability=ability[applicant],
severity=severity[rater])

xyplot(rating~ability,data=ratings.df1)
```




```
xyplot(rating~ability|rater,data=ratings.df1)
```




```
boxplot(rating~rater,data=ratings.df1)
```



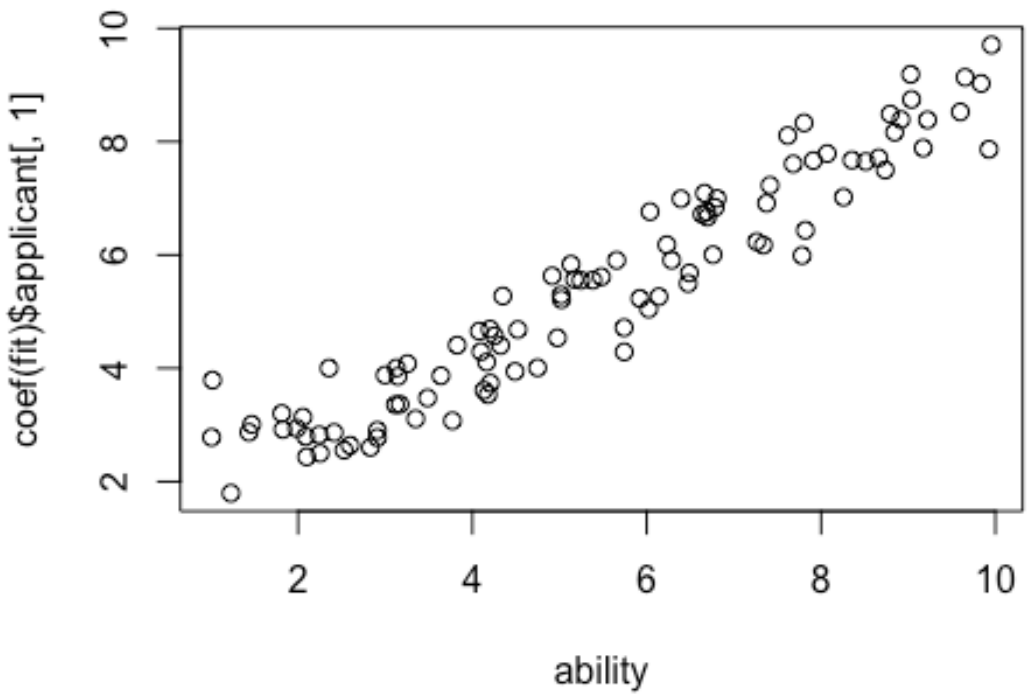

```
library(arm)
fit <- lmer(rating ~ (1|applicant) + (1|rater), data=ratings.df)

display(fit)

## lmer(formula = rating ~ (1 | applicant) + (1 | rater), data = ratings.df)
## coef.est  coef.se
##     5.36     0.64
##
## Error terms:
##  Groups    Name        Std.Dev.
##  applicant (Intercept) 2.08
##  rater     (Intercept) 1.92
##  Residual              0.94
```

```
## ---
## number of obs: 300, groups: applicant, 100; rater, 10
## AIC = 1137.8, DIC = 1131.7
## deviance = 1130.8
```

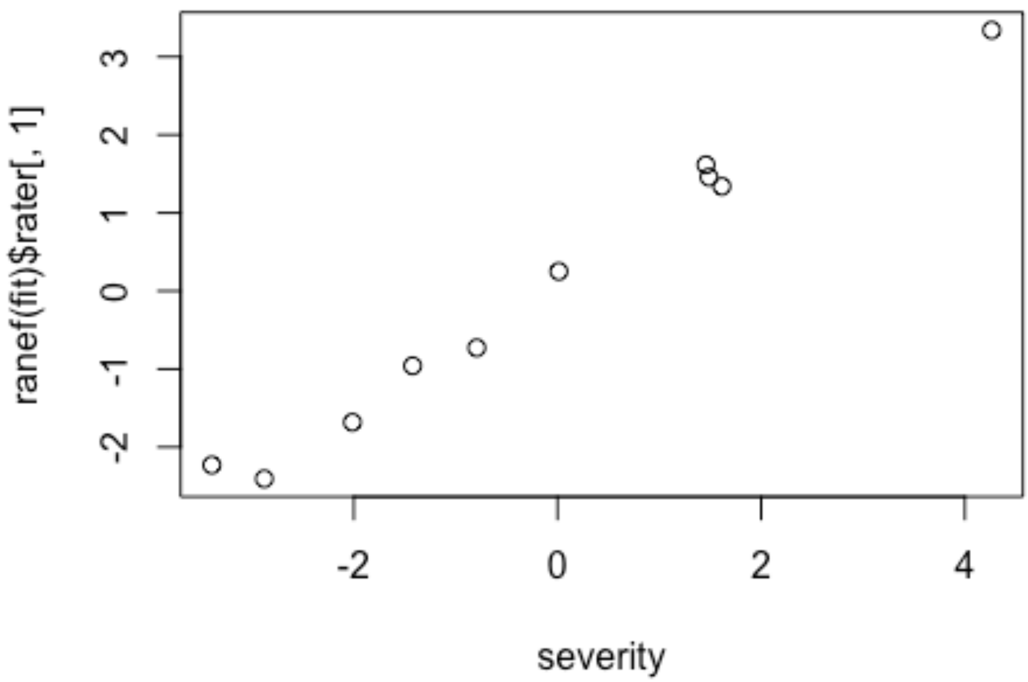
```r
sqrt(9^2/12)
```

```
## [1] 2.598076
```

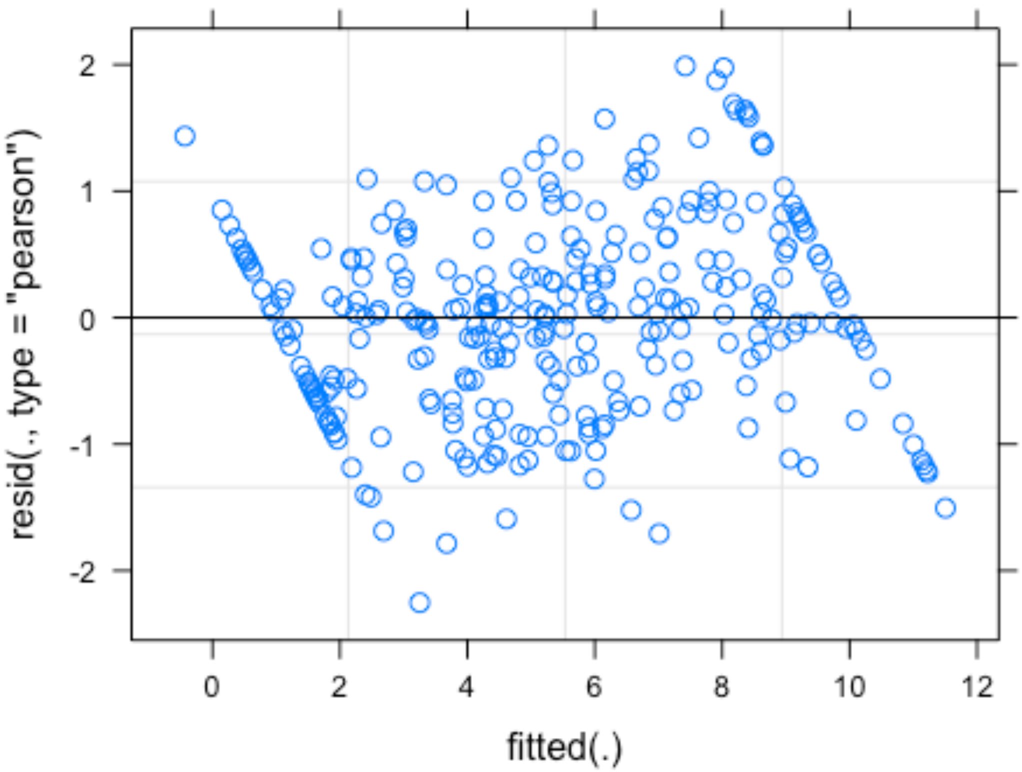
```r
plot(ability,coef(fit)$applicant[,1])
```



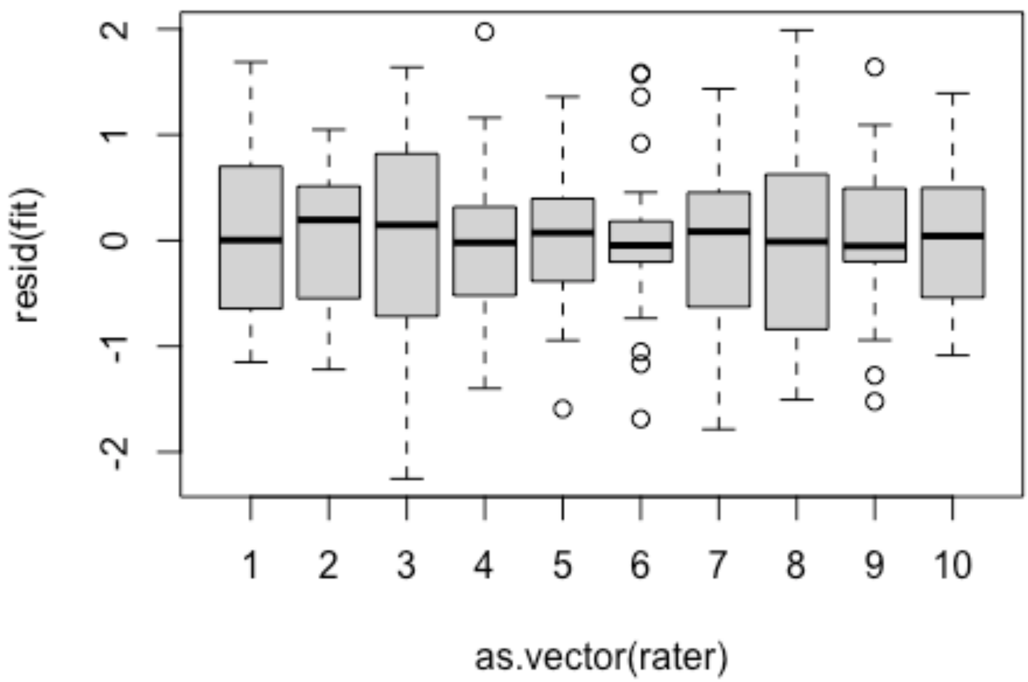

```r
plot(severity,ranef(fit)$rater[,1])
```




```
plot(fit)
```

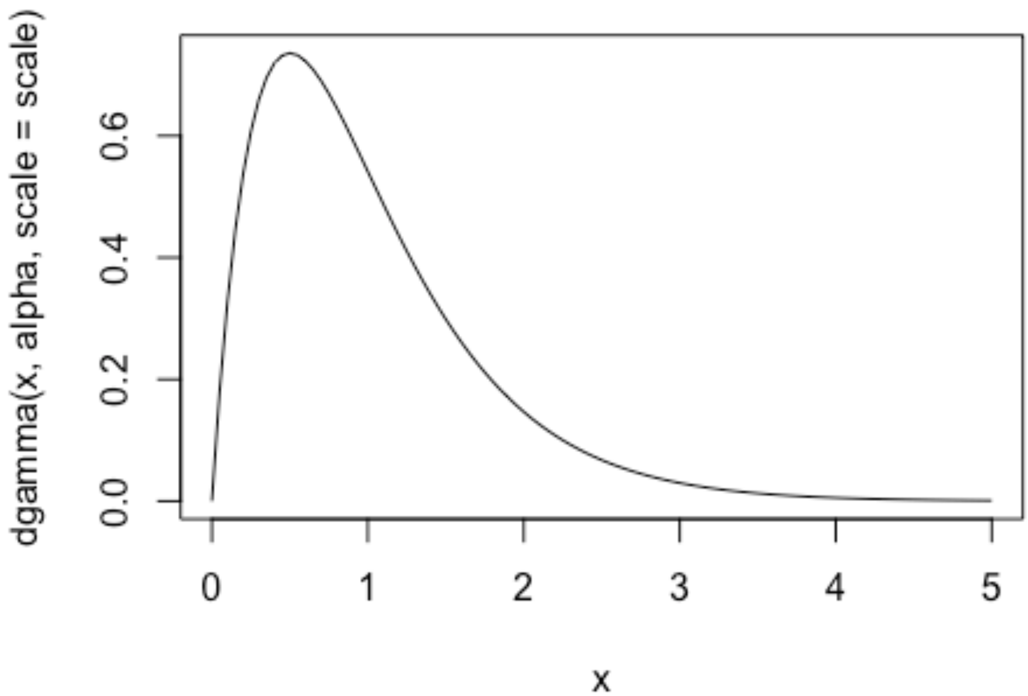



```
boxplot(resid(fit)~as.vector(rater))
```



(b)

```
alpha <- 2
scale <- .5
curve(dgamma(x,alpha,scale=scale),xlim=c(0,5))
```




```
sigma2 <- rgamma(J,alpha,scale=scale)
rating2 <- ability[applicant] + severity[rater] + rnorm(I*W,0,sigma2[rater])
rating2 <- pmax(1,pmin(rating2,10))
ratings2.df <- data.frame(applicant=applicant, rater=as.vector(rater),
rating=rating2, severity=severity[rater], ability=ability[applicant],
sigma2=sigma2[rater])
ratings2.df

##     applicant rater    rating   severity  ability    sigma2
## 1           1     3  2.265984  1.4877767 2.088463 0.7904247
## 2           1     9  2.722904  1.6139248 2.088463 1.1312162
## 3           1    10  2.551737  1.4578290 2.088463 1.2604318
## 4           2     2  3.978599 -0.7929399 4.323272 0.6828268
## 5           2     6  3.973206  0.0132853 4.323272 1.0132648
```

```
## 6          2    10  4.496550  1.4578290 4.323272 1.2604318
## 7          3     3  8.599447  1.4877767 8.353930 0.7904247
## 8          3     5  4.782393 -2.8789604 8.353930 0.3411132
## 9          3     7  6.352362 -3.3938644 8.353930 1.5997597
## 10         4     3  9.198944  1.4877767 8.513316 0.7904247
## 11         4     5  5.419206 -2.8789604 8.513316 0.3411132
## 12         4     9 10.000000  1.6139248 8.513316 1.1312162
## 13         5     1  8.965668 -1.4225642 9.037567 1.0591885
## 14         5     2  9.009166 -0.7929399 9.037567 0.6828268
## 15         5     6  8.889410  0.0132853 9.037567 1.0132648
## 16         6     1  1.162306 -1.4225642 2.414470 1.0591885
## 17         6     5  1.000000 -2.8789604 2.414470 0.3411132
## 18         6     9  2.881571  1.6139248 2.414470 1.1312162
## 19         7     1  5.086573 -1.4225642 6.807349 1.0591885
## 20         7     2  6.454767 -0.7929399 6.807349 0.6828268
## 21         7    10  8.461029  1.4578290 6.807349 1.2604318
## 22         8     3  5.764736  1.4877767 4.163829 0.7904247
## 23         8     6  5.049613  0.0132853 4.163829 1.0132648
## 24         8    10  5.486119  1.4578290 4.163829 1.2604318
## 25         9     4  5.409174 -2.0135056 8.916556 0.9744233
## 26         9     6  9.697904  0.0132853 8.916556 1.0132648
## 27         9     9  9.594964  1.6139248 8.916556 1.1312162
## 28        10     2  3.296038 -0.7929399 4.202830 0.6828268
## 29        10     4  2.205034 -2.0135056 4.202830 0.9744233
## 30        10     8  9.835656  4.2636140 4.202830 1.4674738
## 31        11     4  1.896995 -2.0135056 4.098462 0.9744233
## 32        11     9  6.182575  1.6139248 4.098462 1.1312162
## 33        11    10  2.865148  1.4578290 4.098462 1.2604318
## 34        12     2  5.689450 -0.7929399 7.338838 0.6828268
## 35        12     3  7.641267  1.4877767 7.338838 0.7904247
## 36        12     5  4.346300 -2.8789604 7.338838 0.3411132
## 37        13     1  1.000000 -1.4225642 2.355169 1.0591885
## 38        13     8  4.931244  4.2636140 2.355169 1.4674738
## 39        13     9  1.584704  1.6139248 2.355169 1.1312162
## 40        14     1  1.000000 -1.4225642 1.826703 1.0591885
```

```
## 41          14      5  1.000000 -2.8789604 1.826703 0.3411132
## 42          14      6  1.000000  0.0132853 1.826703 1.0132648
## 43          15      3  2.668110  1.4877767 1.468557 0.7904247
## 44          15      5  1.000000 -2.8789604 1.468557 0.3411132
## 45          15      7  1.000000 -3.3938644 1.468557 1.5997597
## 46          16      7  6.404400 -3.3938644 9.171770 1.5997597
## 47          16      8 10.000000  4.2636140 9.171770 1.4674738
## 48          16     10  9.202186  1.4578290 9.171770 1.2604318
## 49          17      6  4.689566  0.0132853 4.253096 1.0132648
## 50          17      8  5.930066  4.2636140 4.253096 1.4674738
## 51          17      9  6.399998  1.6139248 4.253096 1.1312162
## 52          18      3  4.291185  1.4877767 3.639056 0.7904247
## 53          18      4  1.000000 -2.0135056 3.639056 0.9744233
## 54          18      9  6.483971  1.6139248 3.639056 1.1312162
## 55          19      2  8.002506 -0.7929399 8.661620 0.6828268
## 56          19      3  8.752655  1.4877767 8.661620 0.7904247
## 57          19      5  5.836227 -2.8789604 8.661620 0.3411132
## 58          20      5  2.500173 -2.8789604 4.915200 0.3411132
## 59          20      7  1.981867 -3.3938644 4.915200 1.5997597
## 60          20      8  8.878019  4.2636140 4.915200 1.4674738
## 61          21      1  3.132281 -1.4225642 2.910086 1.0591885
## 62          21      4  1.000000 -2.0135056 2.910086 0.9744233
## 63          21      5  1.000000 -2.8789604 2.910086 0.3411132
## 64          22      3  5.960817  1.4877767 3.825461 0.7904247
## 65          22      7  1.000000 -3.3938644 3.825461 1.5997597
## 66          22     10  4.658806  1.4578290 3.825461 1.2604318
## 67          23      2  5.570780 -0.7929399 6.490266 0.6828268
## 68          23      3  7.764521  1.4877767 6.490266 0.7904247
## 69          23      8 10.000000  4.2636140 6.490266 1.4674738
## 70          24      1  1.140621 -1.4225642 2.253875 1.0591885
## 71          24      2  2.701198 -0.7929399 2.253875 0.6828268
## 72          24      7  1.000000 -3.3938644 2.253875 1.5997597
## 73          25      1  2.298491 -1.4225642 2.910328 1.0591885
## 74          25      7  1.000000 -3.3938644 2.910328 1.5997597
## 75          25      9  5.163721  1.6139248 2.910328 1.1312162
```

```
## 76          26     4  1.000000 -2.0135056 1.813251 0.9744233
## 77          26     9  1.878704  1.6139248 1.813251 1.1312162
## 78          26    10  1.531699  1.4578290 1.813251 1.2604318
## 79          27     1  3.337821 -1.4225642 4.349065 1.0591885
## 80          27     8  6.493254  4.2636140 4.349065 1.4674738
## 81          27     9  5.471387  1.6139248 4.349065 1.1312162
## 82          28     5  4.417050 -2.8789604 6.634164 0.3411132
## 83          28     8 10.000000  4.2636140 6.634164 1.4674738
## 84          28     9  8.713403  1.6139248 6.634164 1.1312162
## 85          29     4  3.741923 -2.0135056 6.688691 0.9744233
## 86          29     6  7.827849  0.0132853 6.688691 1.0132648
## 87          29     9  6.777405  1.6139248 6.688691 1.1312162
## 88          30     3  3.853280  1.4877767 2.243513 0.7904247
## 89          30     4  1.819021 -2.0135056 2.243513 0.9744233
## 90          30     8  5.017635  4.2636140 2.243513 1.4674738
## 91          31     2  5.730237 -0.7929399 5.921472 0.6828268
## 92          31     9  9.598133  1.6139248 5.921472 1.1312162
## 93          31    10  7.687680  1.4578290 5.921472 1.2604318
## 94          32     4  6.425930 -2.0135056 9.597532 0.9744233
## 95          32     6  9.953044  0.0132853 9.597532 1.0132648
## 96          32     7  6.751721 -3.3938644 9.597532 1.5997597
## 97          33     1  3.173758 -1.4225642 6.229940 1.0591885
## 98          33     5  2.967219 -2.8789604 6.229940 0.3411132
## 99          33    10  8.368607  1.4578290 6.229940 1.2604318
## 100         34     2  1.272435 -0.7929399 2.098488 0.6828268
## 101         34     6  3.708905  0.0132853 2.098488 1.0132648
## 102         34     8  6.134369  4.2636140 2.098488 1.4674738
## 103         35     2  5.466916 -0.7929399 6.475130 0.6828268
## 104         35     4  4.115230 -2.0135056 6.475130 0.9744233
## 105         35    10  7.915441  1.4578290 6.475130 1.2604318
## 106         36     1  6.378768 -1.4225642 6.760145 1.0591885
## 107         36     4  4.748522 -2.0135056 6.760145 0.9744233
## 108         36     8 10.000000  4.2636140 6.760145 1.4674738
## 109         37     4  2.568151 -2.0135056 3.146508 0.9744233
## 110         37     7  1.000000 -3.3938644 3.146508 1.5997597
```

```
## 111         37     9  3.932255  1.6139248 3.146508 1.1312162
## 112         38     1  4.545484 -1.4225642 7.262176 1.0591885
## 113         38     7  3.878847 -3.3938644 7.262176 1.5997597
## 114         38    10 10.000000  1.4578290 7.262176 1.2604318
## 115         39     2  5.373374 -0.7929399 7.820055 0.6828268
## 116         39     5  5.320612 -2.8789604 7.820055 0.3411132
## 117         39     6  6.208908  0.0132853 7.820055 1.0132648
## 118         40     1  3.274473 -1.4225642 6.139561 1.0591885
## 119         40     6  7.043255  0.0132853 6.139561 1.0132648
## 120         40    10  5.159053  1.4578290 6.139561 1.2604318
## 121         41     1  5.841576 -1.4225642 7.804232 1.0591885
## 122         41     3  8.310068  1.4877767 7.804232 0.7904247
## 123         41     5  4.889318 -2.8789604 7.804232 0.3411132
## 124         42     2  6.661505 -0.7929399 7.910548 0.6828268
## 125         42     8  9.040901  4.2636140 7.910548 1.4674738
## 126         42    10  7.844730  1.4578290 7.910548 1.2604318
## 127         43     1  1.815095 -1.4225642 4.179907 1.0591885
## 128         43     4  2.795057 -2.0135056 4.179907 0.9744233
## 129         43     6  4.208141  0.0132853 4.179907 1.0132648
## 130         44     2  5.848825 -0.7929399 8.792783 0.6828268
## 131         44     3 10.000000  1.4877767 8.792783 0.7904247
## 132         44    10  9.034837  1.4578290 8.792783 1.2604318
## 133         45     4  1.000000 -2.0135056 1.986082 0.9744233
## 134         45     9  3.051181  1.6139248 1.986082 1.1312162
## 135         45    10  1.666043  1.4578290 1.986082 1.2604318
## 136         46     2  4.612726 -0.7929399 6.696083 0.6828268
## 137         46     4  4.577345 -2.0135056 6.696083 0.9744233
## 138         46     7  4.318521 -3.3938644 6.696083 1.5997597
## 139         47     6  7.308289  0.0132853 8.844184 1.0132648
## 140         47     8 10.000000  4.2636140 8.844184 1.4674738
## 141         47     9 10.000000  1.6139248 8.844184 1.1312162
## 142         48     1  5.016681 -1.4225642 5.741620 1.0591885
## 143         48     5  2.655368 -2.8789604 5.741620 0.3411132
## 144         48    10  3.935296  1.4578290 5.741620 1.2604318
## 145         49     1  3.976630 -1.4225642 4.489395 1.0591885
```

```
## 146         49    4  1.000000 -2.0135056 4.489395 0.9744233
## 147         49   10  4.709164  1.4578290 4.489395 1.2604318
## 148         50    1  3.052025 -1.4225642 4.747852 1.0591885
## 149         50    6  5.648262  0.0132853 4.747852 1.0132648
## 150         50   10  7.649129  1.4578290 4.747852 1.2604318
## 151         51    2  4.201468 -0.7929399 5.654936 0.6828268
## 152         51    6  4.048520  0.0132853 5.654936 1.0132648
## 153         51    7  1.014419 -3.3938644 5.654936 1.5997597
## 154         52    3  6.287783  1.4877767 6.037426 0.7904247
## 155         52    8  9.156966  4.2636140 6.037426 1.4674738
## 156         52   10  6.438349  1.4578290 6.037426 1.2604318
## 157         53    2  7.531082 -0.7929399 7.617902 0.6828268
## 158         53    3  9.815168  1.4877767 7.617902 0.7904247
## 159         53    5  4.348453 -2.8789604 7.617902 0.3411132
## 160         54    3  7.765720  1.4877767 7.414140 0.7904247
## 161         54    5  3.994895 -2.8789604 7.414140 0.3411132
## 162         54    9  8.525242  1.6139248 7.414140 1.1312162
## 163         55    1  7.627407 -1.4225642 9.652258 1.0591885
## 164         55    2  8.509741 -0.7929399 9.652258 0.6828268
## 165         55    6 10.000000  0.0132853 9.652258 1.0132648
## 166         56    1  3.798667 -1.4225642 5.018150 1.0591885
## 167         56    6  4.117701  0.0132853 5.018150 1.0132648
## 168         56    8 10.000000  4.2636140 5.018150 1.4674738
## 169         57    2  1.900784 -0.7929399 2.597991 0.6828268
## 170         57    3  4.793938  1.4877767 2.597991 0.7904247
## 171         57    6  2.989017  0.0132853 2.597991 1.0132648
## 172         58    5  1.000000 -2.8789604 1.019432 0.3411132
## 173         58    7  1.000000 -3.3938644 1.019432 1.5997597
## 174         58    8  3.106449  4.2636140 1.019432 1.4674738
## 175         59    3 10.000000  1.4877767 9.221392 0.7904247
## 176         59    6  8.347065  0.0132853 9.221392 1.0132648
## 177         59    9 10.000000  1.6139248 9.221392 1.1312162
## 178         60    1  4.385438 -1.4225642 5.381858 1.0591885
## 179         60    2  4.395961 -0.7929399 5.381858 0.6828268
## 180         60   10  6.922029  1.4578290 5.381858 1.2604318
```

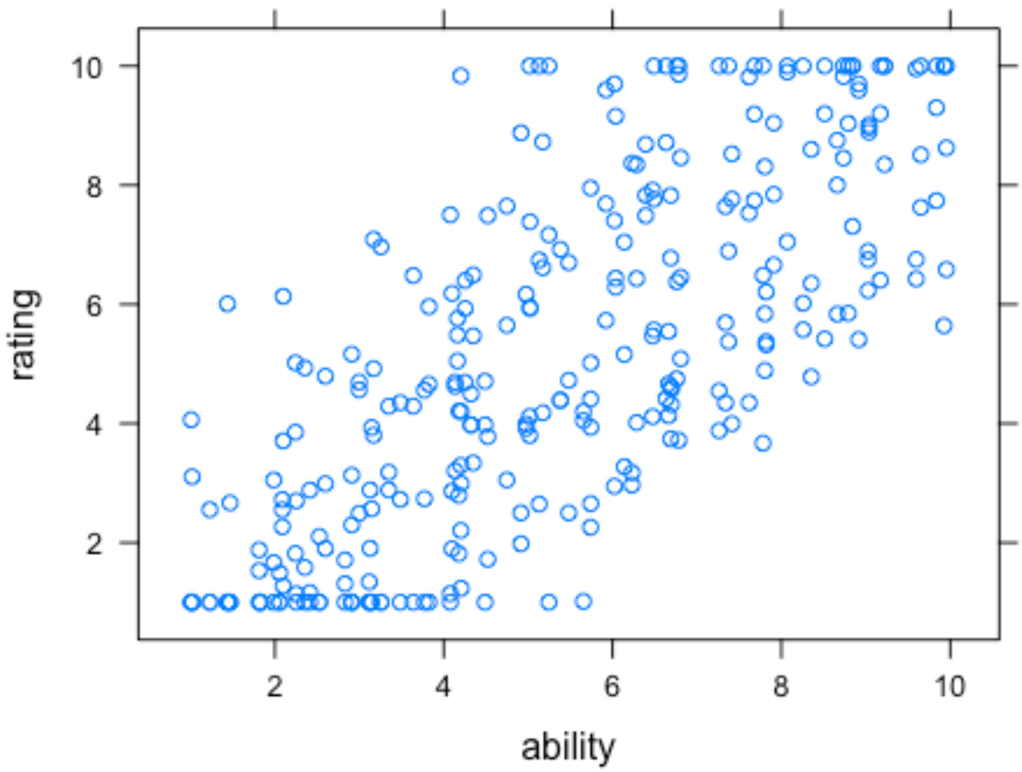
```
## 181         61     3 10.000000  1.4877767 9.954520 0.7904247
## 182         61     4  8.625937 -2.0135056 9.954520 0.9744233
## 183         61     7  6.583096 -3.3938644 9.954520 1.5997597
## 184         62     1  7.744351 -1.4225642 7.680190 1.0591885
## 185         62     3  9.194236  1.4877767 7.680190 0.7904247
## 186         62     9 10.000000  1.6139248 7.680190 1.1312162
## 187         63     1  7.738936 -1.4225642 9.836159 1.0591885
## 188         63     2  9.299800 -0.7929399 9.836159 0.6828268
## 189         63     3 10.000000  1.4877767 9.836159 0.7904247
## 190         64     4  1.000000 -2.0135056 3.118112 0.9744233
## 191         64     5  1.000000 -2.8789604 3.118112 0.3411132
## 192         64     7  1.343082 -3.3938644 3.118112 1.5997597
## 193         65     2  2.491874 -0.7929399 2.998104 0.6828268
## 194         65     3  4.695982  1.4877767 2.998104 0.7904247
## 195         65     9  4.564155  1.6139248 2.998104 1.1312162
## 196         66     2  7.048009 -0.7929399 8.070390 0.6828268
## 197         66     3  9.899033  1.4877767 8.070390 0.7904247
## 198         66     8 10.000000  4.2636140 8.070390 1.4674738
## 199         67     1  3.207271 -1.4225642 4.133904 1.0591885
## 200         67     3  4.625902  1.4877767 4.133904 0.7904247
## 201         67     9  4.692481  1.6139248 4.133904 1.1312162
## 202         68     7  5.640096 -3.3938644 9.926123 1.5997597
## 203         68     8 10.000000  4.2636140 9.926123 1.4674738
## 204         68     9 10.000000  1.6139248 9.926123 1.1312162
## 205         69     2  4.664207 -0.7929399 6.663364 0.6828268
## 206         69     5  4.131881 -2.8789604 6.663364 0.3411132
## 207         69     6  5.545681  0.0132853 6.663364 1.0132648
## 208         70     1  1.000000 -1.4225642 3.771571 1.0591885
## 209         70     2  2.737629 -0.7929399 3.771571 0.6828268
## 210         70     6  4.559160  0.0132853 3.771571 1.0132648
## 211         71     2  3.181820 -0.7929399 3.349516 0.6828268
## 212         71     6  2.883867  0.0132853 3.349516 1.0132648
## 213         71     9  4.289663  1.6139248 3.349516 1.1312162
## 214         72     1  3.802950 -1.4225642 3.169514 1.0591885
## 215         72     3  4.920800  1.4877767 3.169514 0.7904247
```

```
## 216         72     8  7.090122  4.2636140 3.169514 1.4674738
## 217         73     1  1.000000 -1.4225642 2.527244 1.0591885
## 218         73     2  2.103419 -0.7929399 2.527244 0.6828268
## 219         73     5  1.000000 -2.8789604 2.527244 0.3411132
## 220         74     3  2.554078  1.4877767 1.229988 0.7904247
## 221         74     6  1.000000  0.0132853 1.229988 1.0132648
## 222         74     7  1.000000 -3.3938644 1.229988 1.5997597
## 223         75     4  4.724665 -2.0135056 5.481126 0.9744233
## 224         75     5  2.500596 -2.8789604 5.481126 0.3411132
## 225         75     6  6.700413  0.0132853 5.481126 1.0132648
## 226         76     5  3.719256 -2.8789604 6.784910 0.3411132
## 227         76     8 10.000000  4.2636140 6.784910 1.4674738
## 228         76     9  9.858765  1.6139248 6.784910 1.1312162
## 229         77     2  3.780474 -0.7929399 4.521682 0.6828268
## 230         77     5  1.721056 -2.8789604 4.521682 0.3411132
## 231         77     9  7.489306  1.6139248 4.521682 1.1312162
## 232         78     6  8.344831  0.0132853 6.285877 1.0132648
## 233         78     7  4.014822 -3.3938644 6.285877 1.5997597
## 234         78    10  6.432031  1.4578290 6.285877 1.2604318
## 235         79     5  2.946874 -2.8789604 6.022823 0.3411132
## 236         79     8  9.699309  4.2636140 6.022823 1.4674738
## 237         79     9  7.405026  1.6139248 6.022823 1.1312162
## 238         80     4  6.018396 -2.0135056 8.257412 0.9744233
## 239         80     5  5.573616 -2.8789604 8.257412 0.3411132
## 240         80     9 10.000000  1.6139248 8.257412 1.1312162
## 241         81     5  1.000000 -2.8789604 3.127866 0.3411132
## 242         81     6  1.903936  0.0132853 3.127866 1.0132648
## 243         81     9  2.883881  1.6139248 3.127866 1.1312162
## 244         82     5  1.144139 -2.8789604 4.078103 0.3411132
## 245         82     7  1.000000 -3.3938644 4.078103 1.5997597
## 246         82     9  7.502053  1.6139248 4.078103 1.1312162
## 247         83     3  7.166909  1.4877767 5.245415 0.7904247
## 248         83     7  1.000000 -3.3938644 5.245415 1.5997597
## 249         83     8 10.000000  4.2636140 5.245415 1.4674738
## 250         84     3  7.829607  1.4877767 6.392900 0.7904247
```

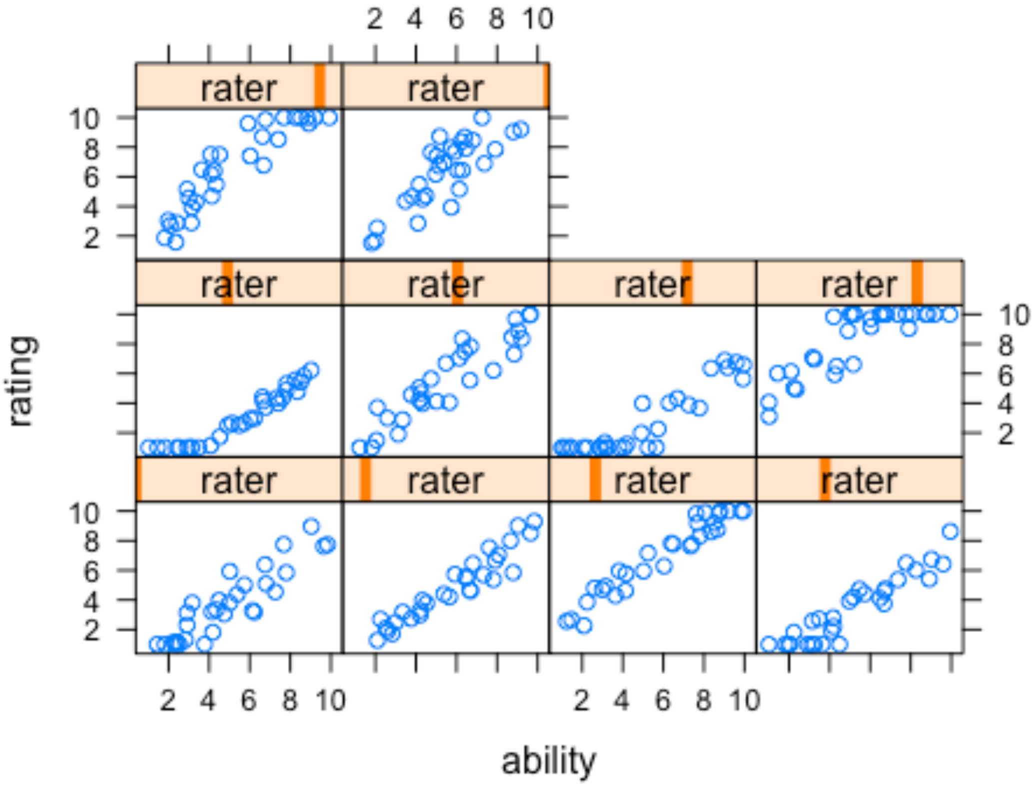
```
## 251         84     6  7.488603  0.0132853 6.392900 1.0132648
## 252         84    10  8.685398  1.4578290 6.392900 1.2604318
## 253         85     4  2.725155 -2.0135056 3.485888 0.9744233
## 254         85     5  1.000000 -2.8789604 3.485888 0.3411132
## 255         85    10  4.349165  1.4578290 3.485888 1.2604318
## 256         86     2  2.991369 -0.7929399 4.205899 0.6828268
## 257         86     6  4.202519  0.0132853 4.205899 1.0132648
## 258         86     7  1.234071 -3.3938644 4.205899 1.5997597
## 259         87     5  2.648121 -2.8789604 5.131233 0.3411132
## 260         87     8 10.000000  4.2636140 5.131233 1.4674738
## 261         87    10  6.741864  1.4578290 5.131233 1.2604318
## 262         88     1  1.310699 -1.4225642 2.829816 1.0591885
## 263         88     2  1.708580 -0.7929399 2.829816 0.6828268
## 264         88     7  1.000000 -3.3938644 2.829816 1.5997597
## 265         89     3  9.822772  1.4877767 8.735545 0.7904247
## 266         89     6  8.454078  0.0132853 8.735545 1.0132648
## 267         89     8 10.000000  4.2636140 8.735545 1.4674738
## 268         90     1  5.929252 -1.4225642 5.022025 1.0591885
## 269         90     3  5.950469  1.4877767 5.022025 0.7904247
## 270         90    10  7.385487  1.4578290 5.022025 1.2604318
## 271         91     4  6.748669 -2.0135056 9.027522 0.9744233
## 272         91     5  6.232131 -2.8789604 9.027522 0.3411132
## 273         91     7  6.890102 -3.3938644 9.027522 1.5997597
## 274         92     4  1.000000 -2.0135056 3.255177 0.9744233
## 275         92     7  1.000000 -3.3938644 3.255177 1.5997597
## 276         92     8  6.965140  4.2636140 3.255177 1.4674738
## 277         93     1  1.000000 -1.4225642 1.437316 1.0591885
## 278         93     7  1.000000 -3.3938644 1.437316 1.5997597
## 279         93     8  6.007842  4.2636140 1.437316 1.4674738
## 280         94     4  4.180819 -2.0135056 5.173128 0.9744233
## 281         94     8  6.613593  4.2636140 5.173128 1.4674738
## 282         94    10  8.724699  1.4578290 5.173128 1.2604318
## 283         95     4  5.373512 -2.0135056 7.374701 0.9744233
## 284         95     8 10.000000  4.2636140 7.374701 1.4674738
## 285         95    10  6.891867  1.4578290 7.374701 1.2604318
```

```
## 286        96     4  1.000000 -2.0135056 1.008387 0.9744233
## 287        96     7  1.000000 -3.3938644 1.008387 1.5997597
## 288        96     8  4.063008  4.2636140 1.008387 1.4674738
## 289        97     4  3.918358 -2.0135056 4.975807 0.9744233
## 290        97     7  3.987732 -3.3938644 4.975807 1.5997597
## 291        97    10  6.169025  1.4578290 4.975807 1.2604318
## 292        98     4  6.487007 -2.0135056 7.782192 0.9744233
## 293        98     7  3.669940 -3.3938644 7.782192 1.5997597
## 294        98     8 10.000000  4.2636140 7.782192 1.4674738
## 295        99     4  4.410284 -2.0135056 5.739992 0.9744233
## 296        99     7  2.258524 -3.3938644 5.739992 1.5997597
## 297        99    10  7.951786  1.4578290 5.739992 1.2604318
## 298       100     4  1.000000 -2.0135056 2.056819 0.9744233
## 299       100     6  1.490828  0.0132853 2.056819 1.0132648
## 300       100     7  1.000000 -3.3938644 2.056819 1.5997597
```

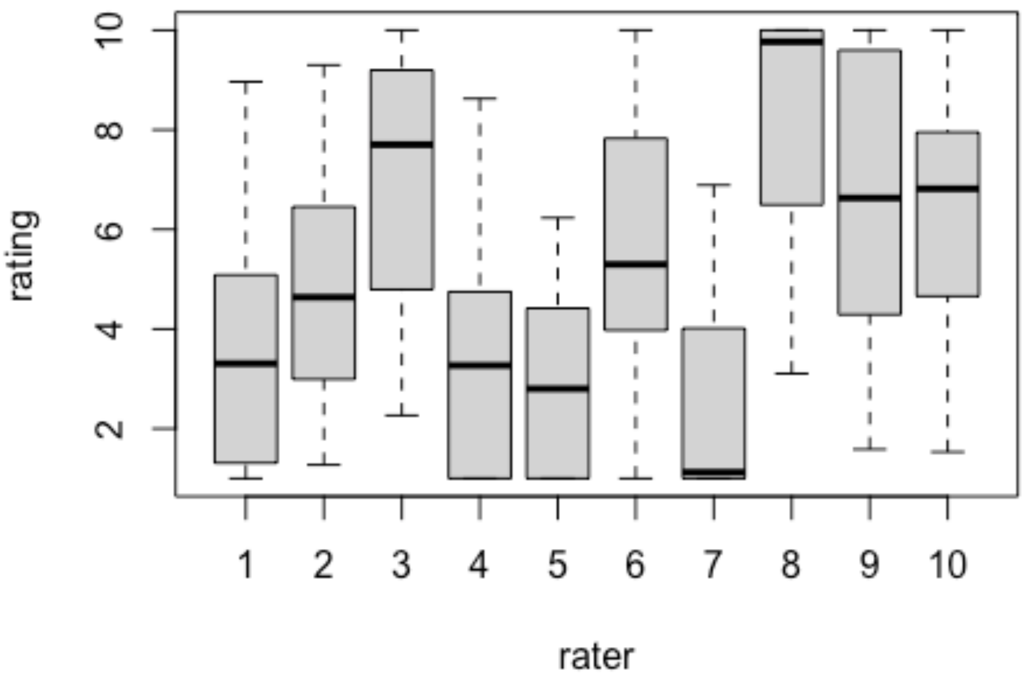
```
xyplot(rating~ability,data=ratings2.df)
```




```
xyplot(rating~ability|rater,data=ratings2.df)
```

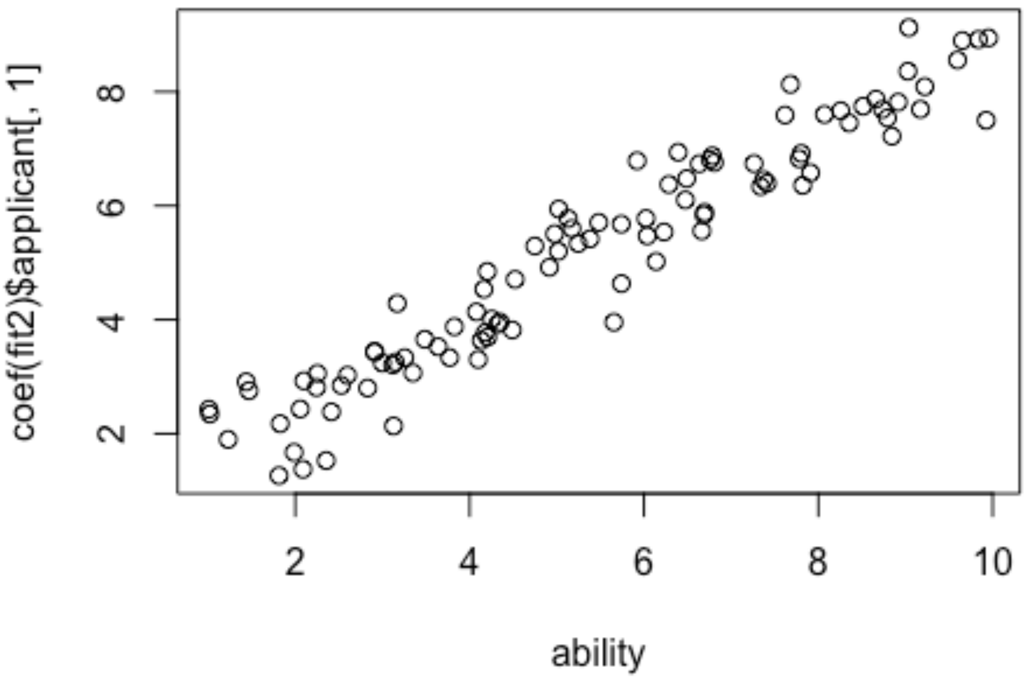



```
boxplot(rating~rater,data=ratings2.df)
```

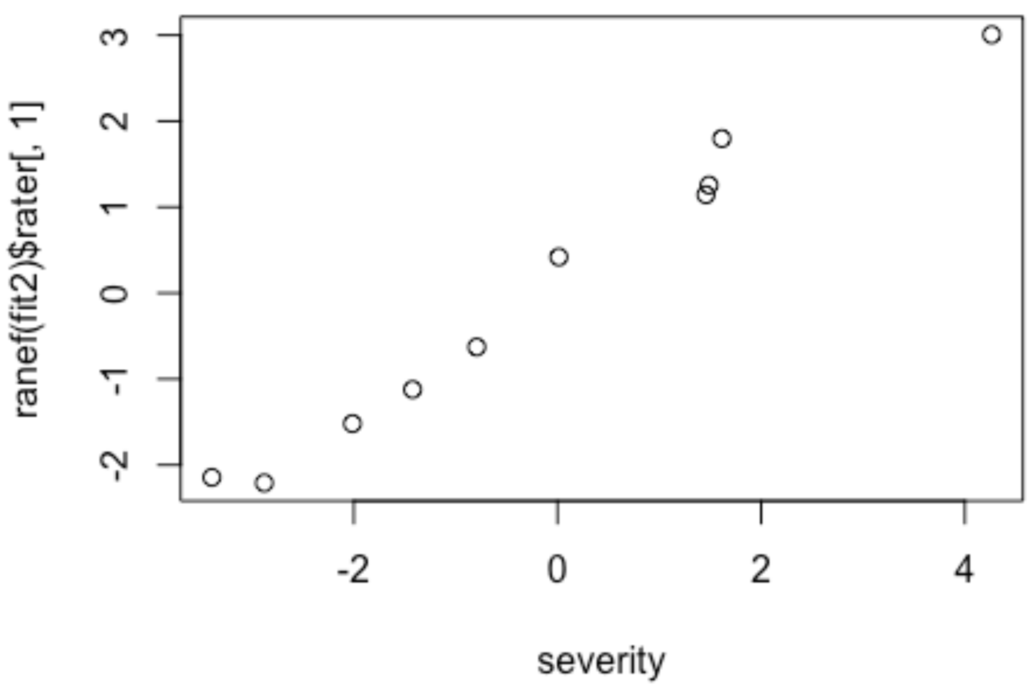



```
fit2 <- lmer(rating ~ (1|applicant) + (1|rater), data=ratings2.df)
display(fit2)

## lmer(formula = rating ~ (1 | applicant) + (1 | rater), data = ratings2.df)
## coef.est  coef.se
##     5.16     0.61
##
## Error terms:
##  Groups    Name        Std.Dev.
##  applicant (Intercept) 2.13
##  rater     (Intercept) 1.80
##  Residual              0.97
## ---
## number of obs: 300, groups: applicant, 100; rater, 10
```

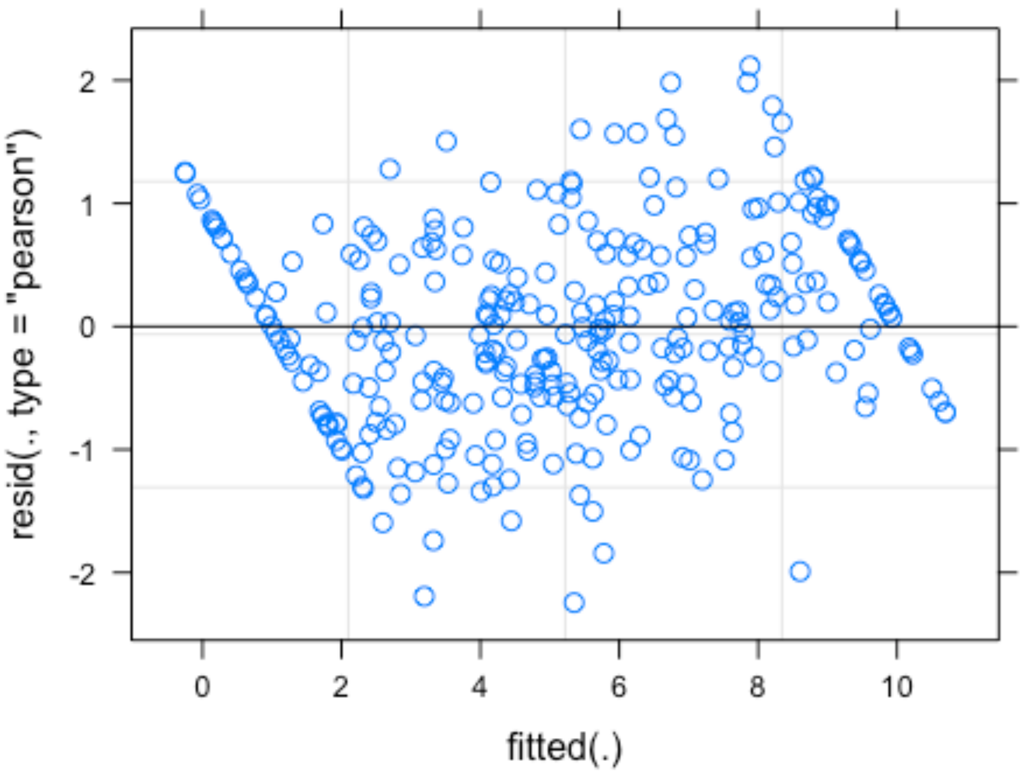
```
## AIC = 1154.9, DIC = 1148.5
## deviance = 1147.7

plot(ability,coef(fit2)$applicant[,1])
```



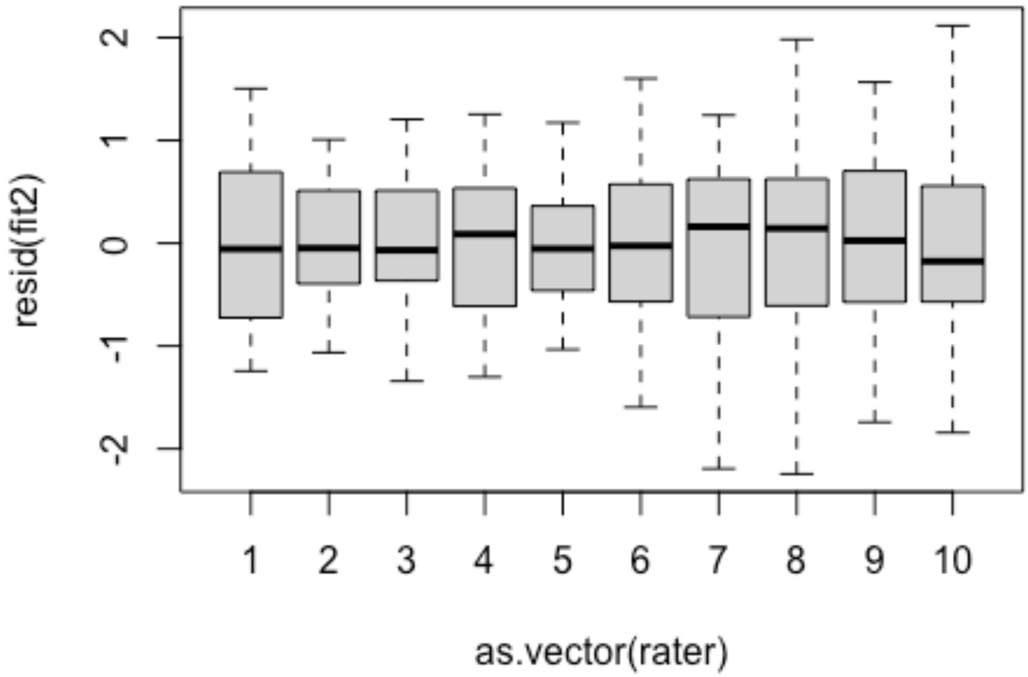

```
plot(severity,ranef(fit2)$rater[,1])
```




```
plot(fit2)
```




```
boxplot(resid(fit2)~as.vector(rater))
```



Problem 3

```
library(reshape)

##
## Attaching package: 'reshape'

## The following object is masked from 'package:data.table':
##
##     melt

## The following object is masked from 'package:Matrix':
##
##     expand
```

```
filename<- "http://www.stat.columbia.edu/~gelman/arm/examples/olympics/olympics1932.txt"
olympics1932_na<-
read.fwf(filename,widths=c(2,14,9,9,9,9,9,9,9),skip=21,header = FALSE)
colnames(olympics1932_na)<- c("pair", "criterion", "judge_1",  "judge_2",
"judge_3",
                              "judge_4",  "judge_5" , "judge_6",  "judge_7")
olympics1932<-na.locf(olympics1932_na)
olympics1932$criterion<-str_trim(olympics1932_na$criterion)

arr_olym<-melt(data = olympics1932,id.vars=c("pair","criterion"),
               measure.vars=c(colnames(olympics1932)[3:9]))

olym_984 <- rename(arr_olym, c("pair"="skater_ID", "variable"="judge_ID"))
olym_984 <- olym_984[order(olym_984$judge_ID),]
olym_984 <- olym_984[c("criterion", "value", "skater_ID", "judge_ID")]

olym_984$SameCountry <-ifelse(olym_984[,3] == " 1"&olym_984[,4] ==
"judge_5",1,
  ifelse(olym_984[,3] == " 2"&olym_984[,4] == "judge_7",1,
  ifelse(olym_984[,3] == " 3"&olym_984[,4] == "judge_1",1,
  ifelse(olym_984[,3] == " 4"&olym_984[,4] == "judge_1",1,
  ifelse(olym_984[,3] == " 7"&olym_984[,4] == "judge_7",1,0
  )))))

olym_984

##      criterion value skater_ID judge_ID SameCountry
## 1      Program   5.6         1  judge_1           0
## 2  Performance   5.6         1  judge_1           0
## 3      Program   5.5         2  judge_1           0
## 4  Performance   5.5         2  judge_1           0
## 5      Program   6.0         3  judge_1           0
## 6  Performance   6.0         3  judge_1           0
## 7      Program   5.6         4  judge_1           0
## 8  Performance   5.6         4  judge_1           0
## 9      Program   5.4         5  judge_1           0
```

```
## 10 Performance   4.8         5  judge_1          0
## 11     Program   5.2         6  judge_1          0
## 12 Performance   4.8         6  judge_1          0
## 13     Program   4.8         7  judge_1          0
## 14 Performance   4.3         7  judge_1          0
## 15     Program   5.5         1  judge_2          0
## 16 Performance   5.5         1  judge_2          0
## 17     Program   5.2         2  judge_2          0
## 18 Performance   5.7         2  judge_2          0
## 19     Program   5.3         3  judge_2          0
## 20 Performance   5.5         3  judge_2          0
## 21     Program   5.3         4  judge_2          0
## 22 Performance   5.3         4  judge_2          0
## 23     Program   4.5         5  judge_2          0
## 24 Performance   4.8         5  judge_2          0
## 25     Program   5.1         6  judge_2          0
## 26 Performance   5.6         6  judge_2          0
## 27     Program   4.0         7  judge_2          0
## 28 Performance   4.6         7  judge_2          0
## 29     Program   5.8         1  judge_3          0
## 30 Performance   5.8         1  judge_3          0
## 31     Program   5.8         2  judge_3          0
## 32 Performance   5.6         2  judge_3          0
## 33     Program   5.8         3  judge_3          0
## 34 Performance   5.7         3  judge_3          0
## 35     Program   5.8         4  judge_3          0
## 36 Performance   5.8         4  judge_3          0
## 37     Program   5.8         5  judge_3          0
## 38 Performance   5.5         5  judge_3          0
## 39     Program   5.3         6  judge_3          0
## 40 Performance   5.0         6  judge_3          0
## 41     Program   4.7         7  judge_3          0
## 42 Performance   4.5         7  judge_3          0
## 43     Program   5.3         1  judge_4          0
## 44 Performance   4.7         1  judge_4          0
```

```
## 45     Program   5.8          2  judge_4           0
## 46 Performance   5.4          2  judge_4           0
## 47     Program   5.0          3  judge_4           0
## 48 Performance   4.9          3  judge_4           0
## 49     Program   4.4          4  judge_4           0
## 50 Performance   4.8          4  judge_4           0
## 51     Program   4.0          5  judge_4           0
## 52 Performance   4.4          5  judge_4           0
## 53     Program   5.4          6  judge_4           0
## 54 Performance   4.7          6  judge_4           0
## 55     Program   4.0          7  judge_4           0
## 56 Performance   4.0          7  judge_4           0
## 57     Program   5.6          1  judge_5           0
## 58 Performance   5.7          1  judge_5           0
## 59     Program   5.6          2  judge_5           0
## 60 Performance   5.5          2  judge_5           0
## 61     Program   5.4          3  judge_5           0
## 62 Performance   5.5          3  judge_5           0
## 63     Program   4.5          4  judge_5           0
## 64 Performance   4.5          4  judge_5           0
## 65     Program   5.5          5  judge_5           0
## 66 Performance   4.6          5  judge_5           0
## 67     Program   4.5          6  judge_5           0
## 68 Performance   4.0          6  judge_5           0
## 69     Program   3.7          7  judge_5           0
## 70 Performance   3.6          7  judge_5           0
## 71     Program   5.2          1  judge_6           0
## 72 Performance   5.3          1  judge_6           0
## 73     Program   5.1          2  judge_6           0
## 74 Performance   5.3          2  judge_6           0
## 75     Program   5.1          3  judge_6           0
## 76 Performance   5.2          3  judge_6           0
## 77     Program   5.0          4  judge_6           0
## 78 Performance   5.0          4  judge_6           0
## 79     Program   4.8          5  judge_6           0
```

```
## 80 Performance   4.8         5  judge_6          0
## 81     Program   4.5         6  judge_6          0
## 82 Performance   4.6         6  judge_6          0
## 83     Program   4.0         7  judge_6          0
## 84 Performance   4.0         7  judge_6          0
## 85     Program   5.7         1  judge_7          0
## 86 Performance   5.4         1  judge_7          0
## 87     Program   5.8         2  judge_7          0
## 88 Performance   5.7         2  judge_7          0
## 89     Program   5.3         3  judge_7          0
## 90 Performance   5.7         3  judge_7          0
## 91     Program   5.1         4  judge_7          0
## 92 Performance   5.5         4  judge_7          0
## 93     Program   5.5         5  judge_7          0
## 94 Performance   5.2         5  judge_7          0
## 95     Program   5.0         6  judge_7          0
## 96 Performance   5.2         6  judge_7          0
## 97     Program   4.8         7  judge_7          0
## 98 Performance   4.8         7  judge_7          0
```

(a)

```
library(dplyr)

## Warning: package 'dplyr' was built under R version 4.1.2

##
## Attaching package: 'dplyr'

## The following object is masked from 'package:reshape':
##
##     rename

## The following object is masked from 'package:car':
##
##     recode

## The following object is masked from 'package:gridExtra':
##
##     combine
```

```
## The following objects are masked from 'package:data.table':
##
##     between, first, last

## The following object is masked from 'package:MASS':
##
##     select

## The following objects are masked from 'package:stats':
##
##     filter, lag

## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union

data_tech <- olym_984 %>% filter(criterion == "Program")
data_art <- olym_984 %>% filter(criterion == "Performance")
reg_tech <- lmer(value ~ 1 + (1 | skater_ID) + (1 | judge_ID), data =
data_tech)
summary(reg_tech)

## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: value ~ 1 + (1 | skater_ID) + (1 | judge_ID)
##    Data: data_tech
##
## REML criterion at convergence: 60
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -2.51025 -0.45646 -0.05459  0.63866  1.89709
##
## Random effects:
##  Groups    Name        Variance Std.Dev.
##  skater_ID (Intercept) 0.17488  0.4182
##  judge_ID  (Intercept) 0.07664  0.2768
##  Residual              0.11057  0.3325
```

```
## Number of obs: 49, groups:  skater_ID, 7; judge_ID, 7
##
## Fixed effects:
##             Estimate Std. Error     df t value Pr(>|t|)
## (Intercept)   5.1347     0.1954 9.5399   26.28  3.2e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

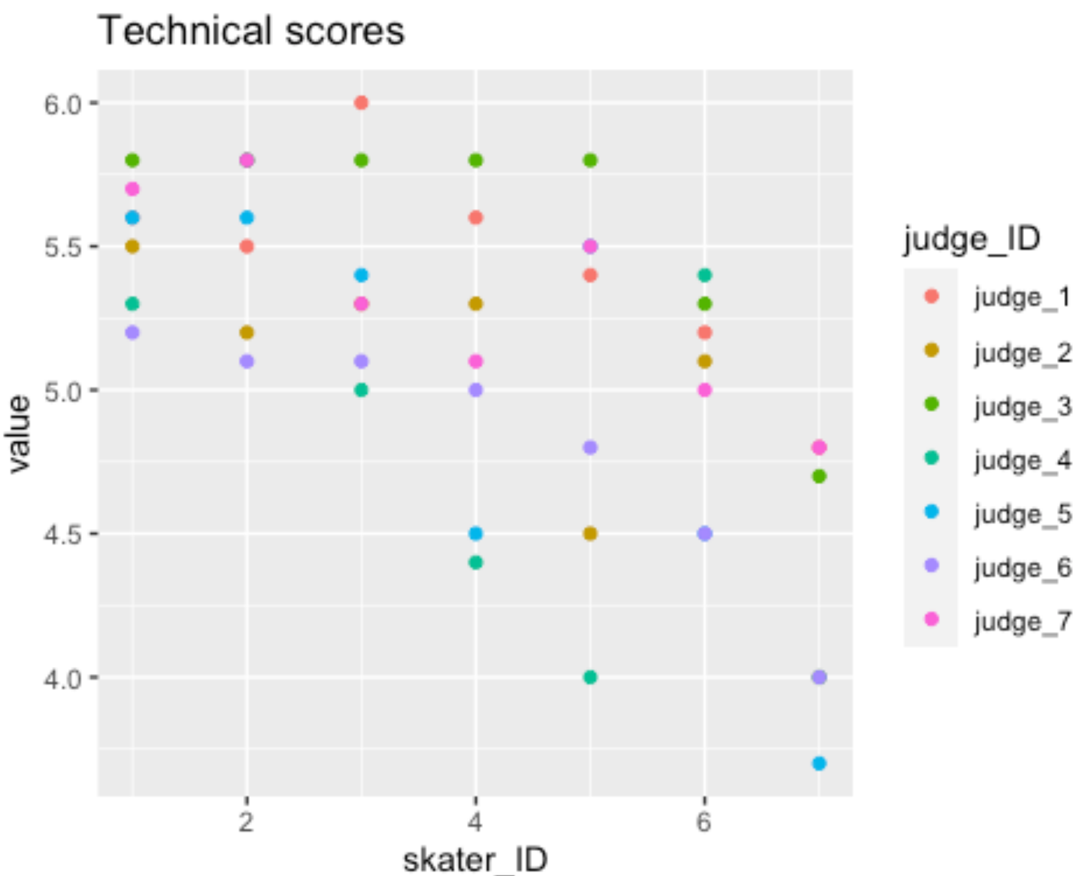
(b)

```
reg_art <- lmer(value ~ 1 + (1|skater_ID) + (1|judge_ID),data=data_art)
summary(reg_tech)
```

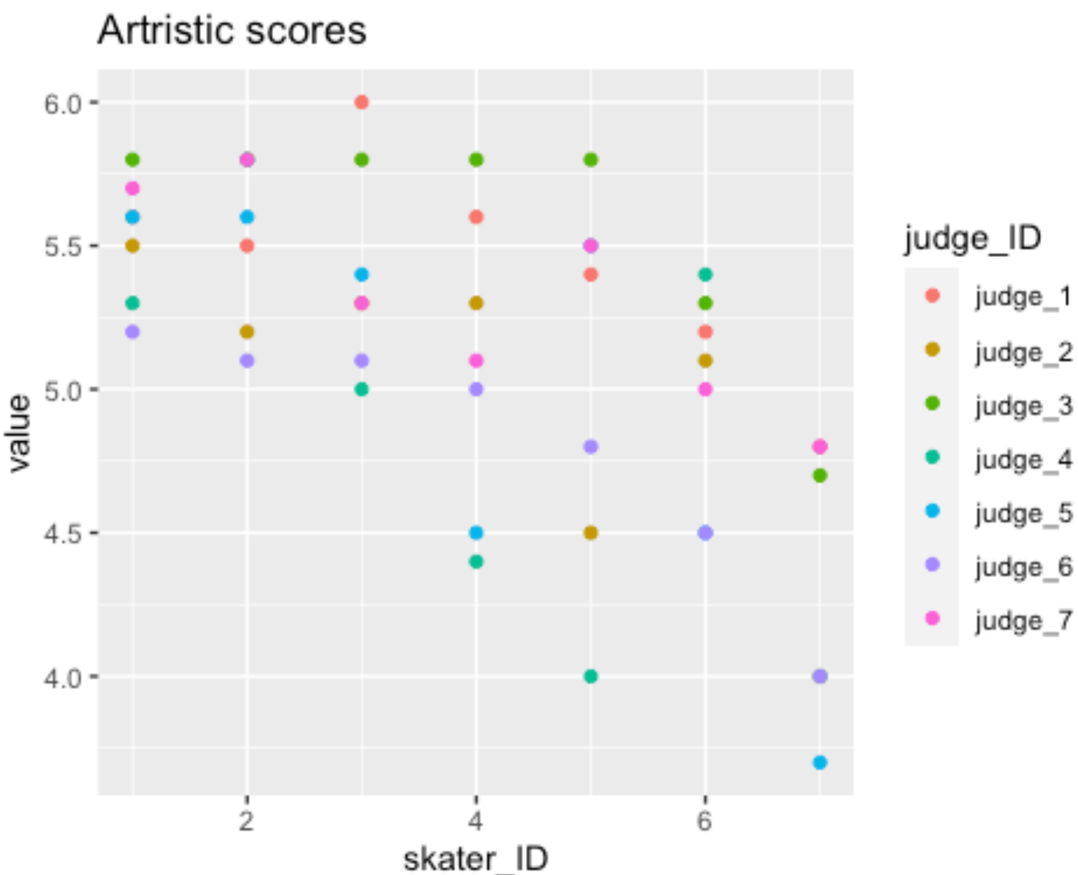
```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: value ~ 1 + (1 | skater_ID) + (1 | judge_ID)
##    Data: data_tech
##
## REML criterion at convergence: 60
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -2.51025 -0.45646 -0.05459  0.63866  1.89709
##
## Random effects:
##  Groups    Name        Variance Std.Dev.
##  skater_ID (Intercept) 0.17488  0.4182
##  judge_ID  (Intercept) 0.07664  0.2768
##  Residual              0.11057  0.3325
## Number of obs: 49, groups:  skater_ID, 7; judge_ID, 7
##
## Fixed effects:
##             Estimate Std. Error     df t value Pr(>|t|)
## (Intercept)   5.1347     0.1954 9.5399   26.28  3.2e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

(c)

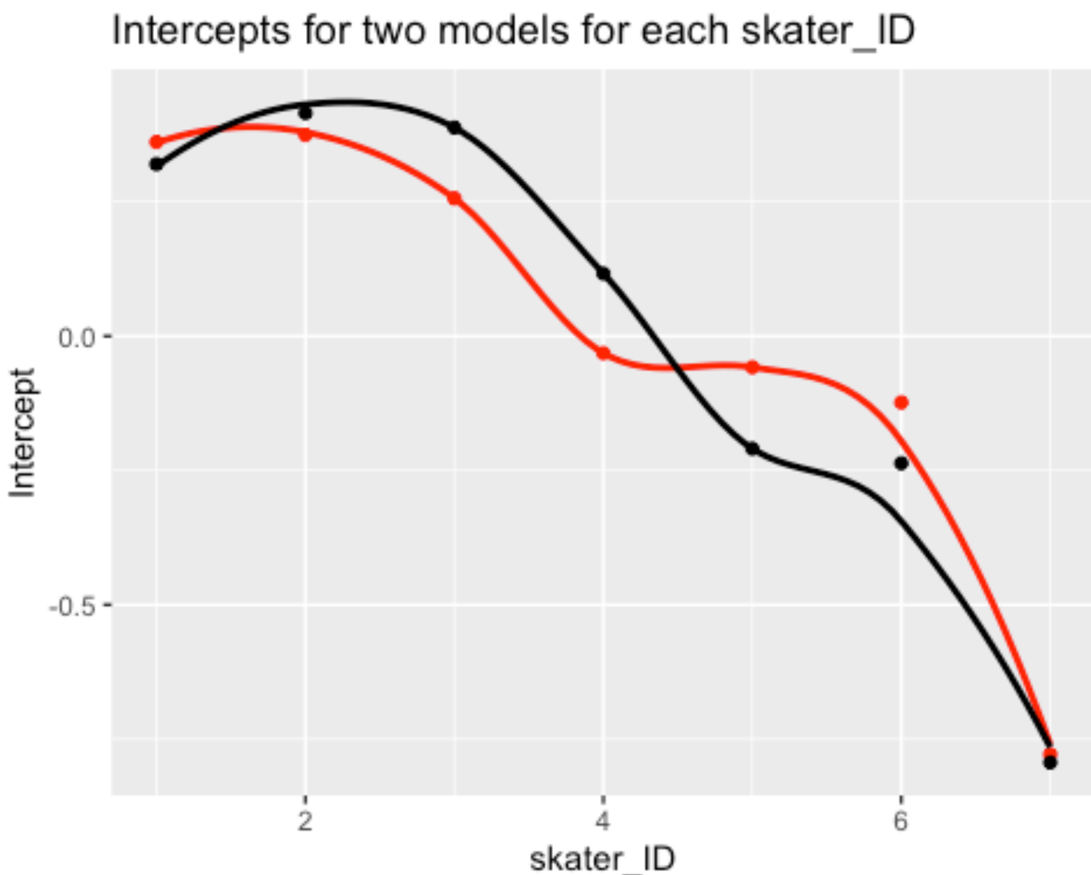
```
ggplot(data_tech,aes(x=skater_ID,y=value,color=judge_ID))+geom_point()+
  ggtitle("Technical scores")
```




```
ggplot(data_tech,aes(x=skater_ID,y=value,color=judge_ID))+geom_point()+
  ggtitle("Artristic scores")
```




```r
inter_skate <- as.data.frame(cbind(unlist(ranef(reg_tech))
[1:7],unlist(ranef(reg_art))[1:7]))
inter_skate$skater_ID <-c(1:7)
ggplot(data=inter_skate)+
  geom_point(col="red",aes(x=skater_ID,y=V1))
+geom_smooth(col="red",aes(x=skater_ID,y=V1),se=FALSE)+
  geom_point(col="black",aes(x=skater_ID,y=V2))
+geom_smooth(col="black",aes(x=skater_ID,y=V2),se=FALSE)+
  ggtitle("Intercepts for two models for each skater_ID")+
  ylab("Intercept")

## `geom_smooth()` using method = 'loess' and formula 'y ~ x'
## `geom_smooth()` using method = 'loess' and formula 'y ~ x'
```




```
inter_judge <- as.data.frame(cbind(unlist(ranef(reg_tech))
[1:7],unlist(ranef(reg_art))[1:7]))
inter_judge$judge_ID <-c(1:7)
ggplot(data=inter_judge)+
  geom_point(col="red",aes(x=judge_ID,y=V1))
+geom_smooth(col="red",aes(x=judge_ID,y=V1),se=FALSE)+
  geom_point(col="black",aes(x=judge_ID,y=V2))
+geom_smooth(col="black",aes(x=judge_ID,y=V2),se=FALSE)+
  ggtitle("Intercepts for two models for each judge_ID")+
  ylab("Intercept")

## `geom_smooth()` using method = 'loess' and formula 'y ~ x'
## `geom_smooth()` using method = 'loess' and formula 'y ~ x'
```

(d)

Please see graphs above.

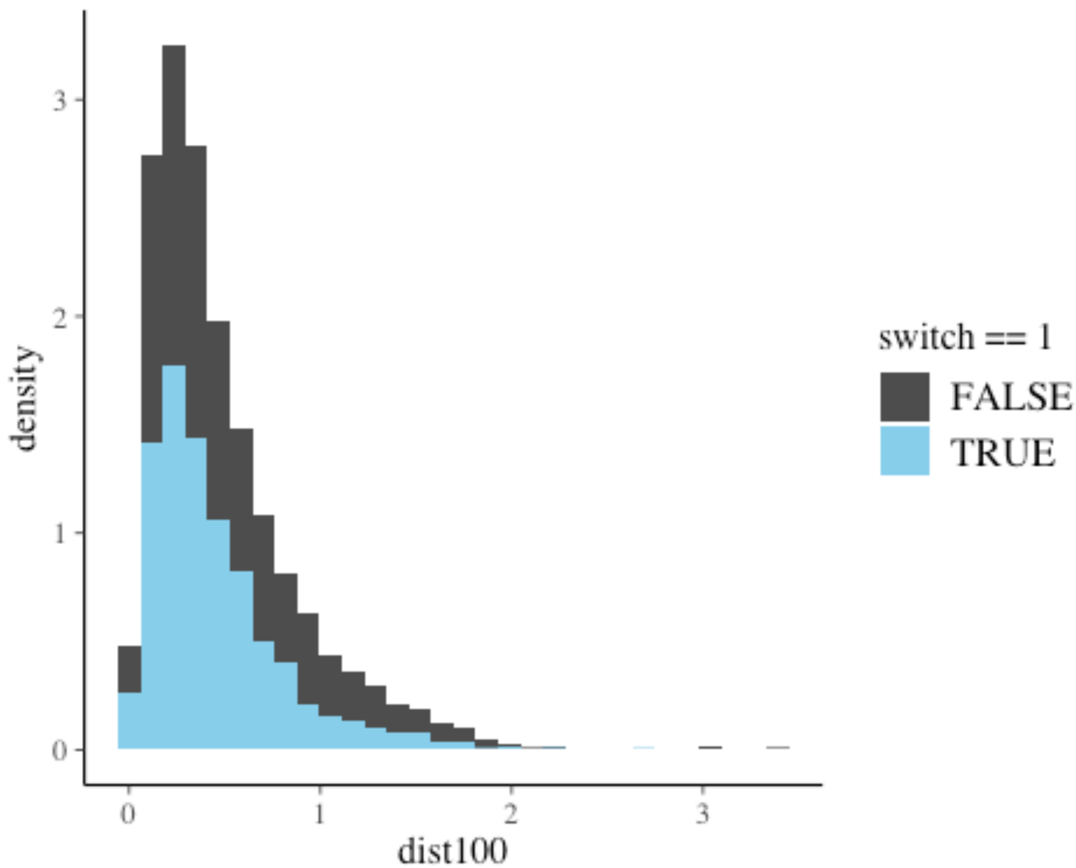
Problem 4

```
library(ggplot2)
library(bayesplot)

## Warning: package 'bayesplot' was built under R version 4.1.2

## This is bayesplot version 1.9.0

## - Online documentation and vignettes at mc-stan.org/bayesplot

## - bayesplot theme set to bayesplot::theme_default()
```

```
##     * Does _not_ affect other ggplot2 plots
##     * See ?bayesplot_theme_set for details on theme setting
```

```
library(rstanarm)
theme_set(bayesplot::theme_default())

data(wells)
wells$dist100 <- wells$dist / 100

ggplot(wells, aes(x = dist100, y = ..density.., fill = switch == 1)) +
  geom_histogram() +
  scale_fill_manual(values = c("gray30", "skyblue"))
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```



(a)

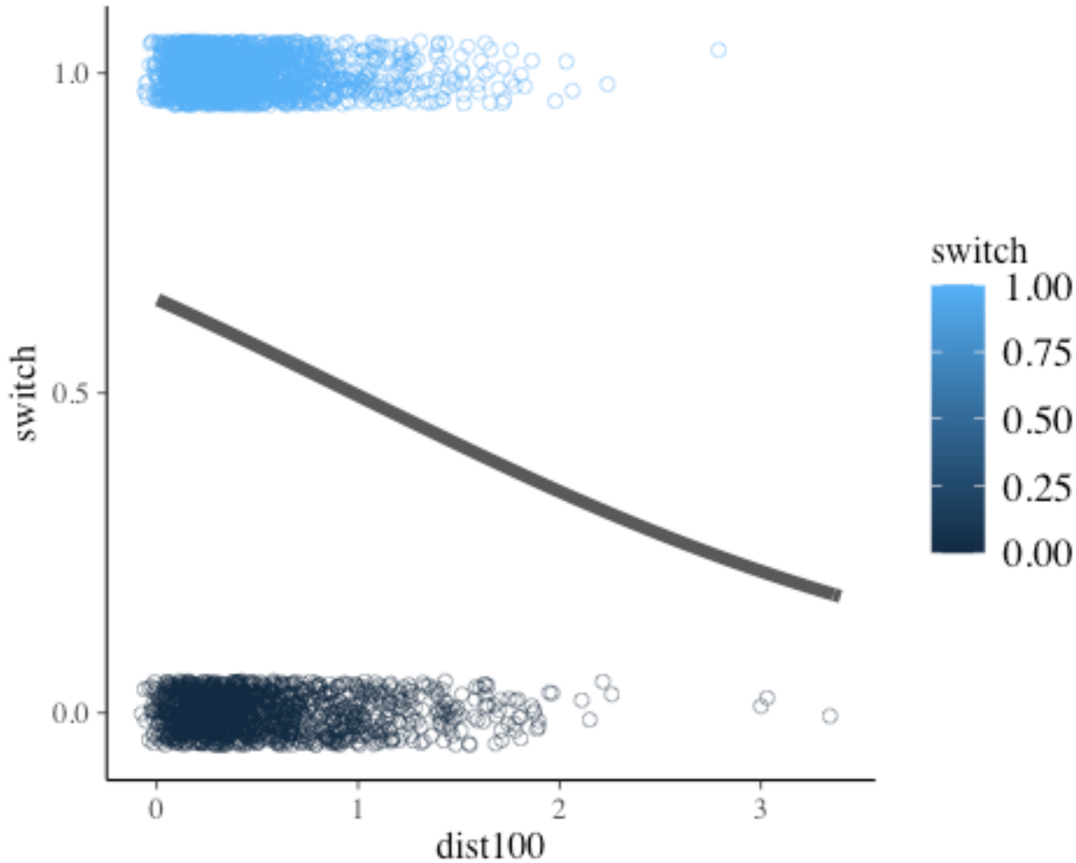
```
t_prior <- student_t(df = 7, location = 0, scale = 2.5)
fit1 <- stan_glm(switch ~ dist100, data = wells,
                 family = binomial(link = "logit"),
                 prior = t_prior, prior_intercept = t_prior,
                 cores = 2, seed = 12345)
```

(b)

```
round(posterior_interval(fit1, prob = 0.5), 2)

##                25%   75%
## (Intercept)  0.57  0.65
## dist100     -0.69 -0.56
```

(c)

```
# Predicted probability as a function of x
pr_switch <- function(x, ests) plogis(ests[1] + ests[2] * x)
# A function to slightly jitter the binary data
jitt <- function(...) {
  geom_point(aes_string(...), position = position_jitter(height = 0.05, width
= 0.1),
             size = 2, shape = 21, stroke = 0.2)
}
ggplot(wells, aes(x = dist100, y = switch, color = switch)) +
  scale_y_continuous(breaks = c(0, 0.5, 1)) +
  jitt(x="dist100") +
  stat_function(fun = pr_switch, args = list(ests = coef(fit1)),
                size = 2, color = "gray35")
```



Problem 5

```
df <- read.table("/Users/Home/Documents/Michael_Ghattas/School/CU_Boulder/
2022/Spring 2022/STAT - 4400/Data/rodents.dat")
df$race <- factor(df$race, labels=c("White (non-hispanic)", "Black (non-
hispanic)", "Puerto Rican", "Other Hispanic", "Asian/Pacific Islander",
"Amer-Indian/Native Alaskan", "Two or more races"))
df$unitflr2 <- as.factor(df$unitflr2)
df$numunits <- as.factor(df$numunits)
df$stories <- as.factor(df$stories)
df$extwin4_2 <- as.factor(df$extwin4_2)
df$extflr5_2 <- as.factor(df$extflr5_2)
df$borough <- factor(df$borough, labels=c("Bronx", "Brooklyn", "Manhattan",
"Queens", "Staten Island"))
```

```r
df$cd <- as.factor(df$cd)
df$intcrack2 <- as.factor(df$intcrack2)
df$inthole2 <- as.factor(df$inthole2)
df$intleak2 <- as.factor(df$intleak2)
df$intpeel_cat <- as.factor(df$intpeel_cat)
df$help <- as.factor(df$help)
df$old <- as.factor(df$old)
df$dilap <- as.factor(df$dilap)
df$regext <- as.factor(df$regext)
df$poverty <- as.factor(df$poverty)
df$povertyx2 <- as.factor(df$povertyx2)
df$housing <- factor(df$housing, labels=c("public", "rent controlled/
stabilized", "owned", "other rentals"))
df$board2 <- as.factor(df$board2)
df$subsidy <- as.factor(df$subsidy)
df$under6 <- as.factor(df$under6)
# Missing values
missingNA <- sapply(df, function(x) sum(is.na(x)))
df <- na.omit(df)
```

(a)

```r
model.14.3A <- glmer(rodent2 ~ 1+race+personrm +intcrack2 + inthole2 +
intleak2 +
                            struct +regext+extflr5_2 +
                           # old+dilap+intpeel_cat+extwin4_2+housing +
                            (1|bldg),
                            data=df,
                            family=binomial(link="logit"),
                            control=glmerControl(
                                                  optimizer="bobyqa",
                                                 optCtrl=list(maxfun=200000))
                            )
summary(model.14.3A)
```

```
## Generalized linear mixed model fit by maximum likelihood (Laplace
##   Approximation) [glmerMod]
##  Family: binomial  ( logit )
```

```
## Formula: rodent2 ~ 1 + race + personrm + intcrack2 + inthole2 + intleak2 +
##     struct + regext + extflr5_2 + (1 | bldg)
##    Data: df
## Control: glmerControl(optimizer = "bobyqa", optCtrl = list(maxfun =
2e+05))
##
##      AIC      BIC   logLik deviance df.resid
##    757.5    826.6   -363.7    727.5      729
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -2.2234 -0.4474 -0.2733  0.4820  3.4320
##
## Random effects:
##  Groups Name        Variance Std.Dev.
##  bldg   (Intercept) 1.065    1.032
## Number of obs: 744, groups:  bldg, 491
##
## Fixed effects:
##                                 Estimate Std. Error z value Pr(>|z|)
## (Intercept)                     -1.89276    0.41929  -4.514 6.36e-06 ***
## raceBlack (non-hispanic)         1.05538    0.32064   3.291 0.000997 ***
## racePuerto Rican                 0.93419    0.37042   2.522 0.011670 *
## raceOther Hispanic               1.14651    0.33310   3.442 0.000578 ***
## raceAsian/Pacific Islander       0.08936    0.53164   0.168 0.866517
## raceAmer-Indian/Native Alaskan  1.42349    1.28831   1.105 0.269190
## raceTwo or more races            1.04408    1.08211   0.965 0.334618
## personrm                         0.80507    0.28081   2.867 0.004144 **
## intcrack21                       1.13764    0.31495   3.612 0.000304 ***
## inthole21                        0.92155    0.39585   2.328 0.019909 *
## intleak21                        0.50604    0.25998   1.947 0.051594 .
## struct                          -1.18173    0.24889  -4.748 2.05e-06 ***
## regext1                         -0.33257    0.22167  -1.500 0.133544
## extflr5_21                       1.11165    0.57035   1.949 0.051288 .
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

##
## Correlation matrix not shown by default, as p = 14 > 12.
## Use print(x, correlation=TRUE)  or
##     vcov(x)        if you need it
```

(b)

```
model.14.3B <- glmer(rodent2 ~ 1+race+personrm +intcrack2 + inthole2 +
intleak2 +
                            struct +regext+extflr5_2 +
                           # old+dilap+intpeel_cat+extwin4_2+housing +
                            (1|bldg)+
                            (1|cd),
                            data=df,
                            family=binomial(link="logit"),
                            # increase convergence iterations
                            control=glmerControl(
                                                  optimizer="bobyqa",
                                                 optCtrl=list(maxfun=200000))
                            )
summary(model.14.3B)

## Generalized linear mixed model fit by maximum likelihood (Laplace
##   Approximation) [glmerMod]
##  Family: binomial  ( logit )
## Formula: rodent2 ~ 1 + race + personrm + intcrack2 + inthole2 + intleak2 +
##     struct + regext + extflr5_2 + (1 | bldg) + (1 | cd)
##    Data: df
## Control: glmerControl(optimizer = "bobyqa", optCtrl = list(maxfun =
2e+05))
##
##      AIC      BIC   logLik deviance df.resid
##    758.7    832.5   -363.3    726.7      728
##
## Scaled residuals:
```

```
##     Min      1Q  Median      3Q     Max
## -2.1093 -0.4523 -0.2703  0.4710  3.4232
##
## Random effects:
##  Groups Name        Variance Std.Dev.
##  bldg   (Intercept) 0.9167   0.9574
##  cd     (Intercept) 0.1313   0.3624
## Number of obs: 744, groups:  bldg, 491; cd, 55
##
## Fixed effects:
##                                  Estimate Std. Error z value Pr(>|z|)
## (Intercept)                       -1.8858     0.4214  -4.475 7.64e-06 ***
## raceBlack (non-hispanic)           1.0288     0.3282   3.135 0.001719 **
## racePuerto Rican                   0.8610     0.3824   2.252 0.024334 *
## raceOther Hispanic                 1.0946     0.3416   3.204 0.001353 **
## raceAsian/Pacific Islander         0.1302     0.5331   0.244 0.807097
## raceAmer-Indian/Native Alaskan     1.4623     1.2740   1.148 0.251035
## raceTwo or more races              0.9959     1.0878   0.916 0.359907
## personrm                           0.8326     0.2815   2.957 0.003104 **
## intcrack21                         1.1008     0.3157   3.487 0.000488 ***
## inthole21                          0.9186     0.3934   2.335 0.019548 *
## intleak21                          0.4901     0.2606   1.880 0.060079 .
## struct                            -1.1613     0.2492  -4.659 3.18e-06 ***
## regext1                           -0.3549     0.2230  -1.592 0.111479
## extflr5_21                         1.0929     0.5673   1.926 0.054043 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

##
## Correlation matrix not shown by default, as p = 14 > 12.
## Use print(x, correlation=TRUE)  or
##     vcov(x)        if you need it
```

(c)

```
anova_logit.14 <- anova(model.14.3B,model.14.3A); anova_logit.14
```

```
## Data: df
## Models:
## model.14.3A: rodent2 ~ 1 + race + personrm + intcrack2 + inthole2 +
intleak2 + struct + regext + extflr5_2 + (1 | bldg)
## model.14.3B: rodent2 ~ 1 + race + personrm + intcrack2 + inthole2 +
intleak2 + struct + regext + extflr5_2 + (1 | bldg) + (1 | cd)
##              npar    AIC    BIC  logLik deviance Chisq Df Pr(>Chisq)
## model.14.3A   15 757.46 826.64 -363.73   727.46
## model.14.3B   16 758.67 832.46 -363.33   726.67 0.786  1     0.3753
```