[STAT 4400] HW-2 / Michael Ghattas

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1/31/2022

Question 1

(a)

```
library(ggplot2)
library(haven)
data <- read_dta("/Users/Home/Documents/Michael_Ghattas/School/CU_Boulder/
2022/Spring 2022/STAT - 4400/Data/heights.dta")
head(data)

## # A tibble: 6 × 9
##    earn height1 height2   sex  race  hisp    ed yearbn height
##   <dbl>   <dbl>   <dbl> <dbl> <dbl> <dbl> <dbl>  <dbl>  <dbl>
## 1    NA       5       6     2     1     2    12     53     66
## 2    NA       5       4     1     2     2    12     50     64
## 3 50000       6       2     1     1     2    16     45     74
## 4 60000       5       6     2     1     2    16     32     66
## 5 30000       5       4     2     1     2    16     61     64
## 6    NA       5       5     2     1     2    17     33     65

lmod = lm(earn ~ ., data = data)
summary(lmod)

##
## Call:
## lm(formula = earn ~ ., data = data)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -38659 -10081  -1953   6692 159119
##
## Coefficients: (1 not defined because of singularities)
```

```
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -19661.83   13559.54  -1.450   0.1473
## height1       4456.30    2122.47   2.100   0.0359 *
## height2        478.03     213.34   2.241   0.0252 *
## sex         -11651.40    1351.86  -8.619  < 2e-16 ***
## race          -427.21     718.08  -0.595   0.5520
## hisp          2718.34    1999.46   1.360   0.1742
## ed            2749.89     191.90  14.330  < 2e-16 ***
## yearbn        -167.41      29.81  -5.616 2.36e-08 ***
## height             NA         NA      NA       NA
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 17130 on 1371 degrees of freedom
##   (650 observations deleted due to missingness)
## Multiple R-squared:  0.2528, Adjusted R-squared:  0.249
## F-statistic: 66.27 on 7 and 1371 DF,  p-value: < 2.2e-16
```

We can transform the data by using different methods of indexing and/or linear transformation.

(b)

```
df = na.omit(data) # removing NA values

lmod = lm(earn ~ height, data = df)
summary(lmod)

##
## Call:
## lm(formula = earn ~ height, data = df)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -30031 -12497  -3215   7474 174659
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -84078.3     8901.1  -9.446   <2e-16 ***
```

```
## height        1563.1      133.4  11.713   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 18850 on 1377 degrees of freedom
## Multiple R-squared:  0.09061,    Adjusted R-squared:  0.08995
## F-statistic: 137.2 on 1 and 1377 DF,  p-value: < 2.2e-16
```

```
df$male <- 2 - df$sex
df$female <- (1 - df$sex) * -1

lmodM = lm(earn ~ height + ed + male, data = df)
summary(lmodM)
```

```
##
## Call:
## lm(formula = earn ~ height + ed + male, data = df)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -40589 -10563  -1563   6459 159369
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -40825.8    11285.6  -3.618 0.000308 ***
## height         319.4      174.1   1.835 0.066763 .
## ed            2632.3      192.7  13.661  < 2e-16 ***
## male         11718.6     1360.5   8.614  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 17330 on 1375 degrees of freedom
## Multiple R-squared:  0.2329, Adjusted R-squared:  0.2312
## F-statistic: 139.2 on 3 and 1375 DF,  p-value: < 2.2e-16
```

```
lmodF = lm(earn ~ height + ed + female, data = df)
summary(lmodF)
```

```
##
## Call:
## lm(formula = earn ~ height + ed + female, data = df)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -40589 -10563  -1563   6459 159369
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -29107.1    12242.8  -2.377   0.0176 *
## height         319.4      174.1   1.835   0.0668 .
## ed            2632.3      192.7  13.661   <2e-16 ***
## female      -11718.6     1360.5  -8.614   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 17330 on 1375 degrees of freedom
## Multiple R-squared:  0.2329, Adjusted R-squared:  0.2312
## F-statistic: 139.2 on 3 and 1375 DF,  p-value: < 2.2e-16
```

```
anova(lmodM, lmodF)
```

```
## Analysis of Variance Table
##
## Model 1: earn ~ height + ed + male
## Model 2: earn ~ height + ed + female
##   Res.Df        RSS Df  Sum of Sq F Pr(>F)
## 1   1375 4.1289e+11
## 2   1375 4.1289e+11  0 6.1035e-05
```

```
lmod = lm(earn ~ height + ed + male + female, data = df)
summary(lmod)
```

```
##
## Call:
## lm(formula = earn ~ height + ed + male + female, data = df)
```

```
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -40589 -10563  -1563   6459 159369
##
## Coefficients: (1 not defined because of singularities)
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -40825.8    11285.6  -3.618 0.000308 ***
## height         319.4      174.1   1.835 0.066763 .
## ed            2632.3      192.7  13.661  < 2e-16 ***
## male         11718.6     1360.5   8.614  < 2e-16 ***
## female            NA         NA      NA       NA
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 17330 on 1375 degrees of freedom
## Multiple R-squared:  0.2329, Adjusted R-squared:  0.2312
## F-statistic: 139.2 on 3 and 1375 DF,  p-value: < 2.2e-16
```

The preferred models are lmodM & lmodF, as they capture the significance of each of the three predictors (height, education, and sex) in realation to each sex. Each model explains about 23% of the data, meaning between both models we are able to explain approximately 40% of the data.

(c)

Based on the different models we tested in part (b), we can note from the lmodM & lmodF models that height increases the annual earnings by around $319 per inch for either sex. Additionally, we can see that education plays an important role as it contributes to an increase of about $2632 per academic year for either sex. From the ANOVA test we can hypothesize that there is little difference between the male and female models. Finally, from the lmod model and AIC we can confirm the significance of education and height on earnings, and further realize that being a male increases earnings by roughly $11719.

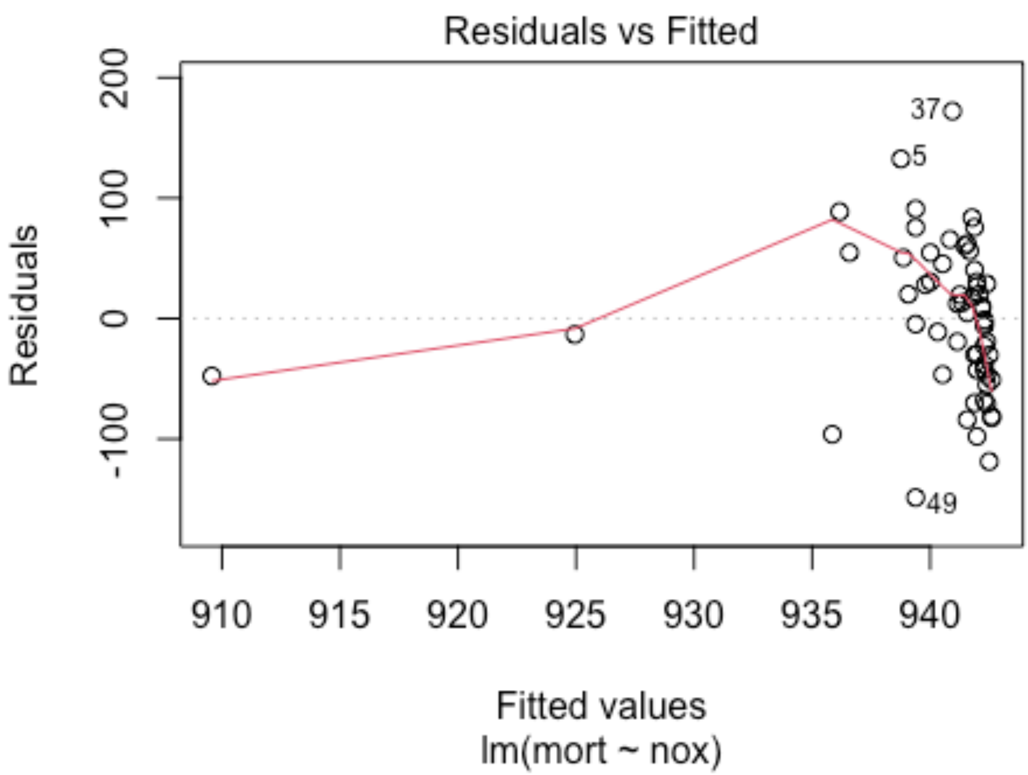
Question 2

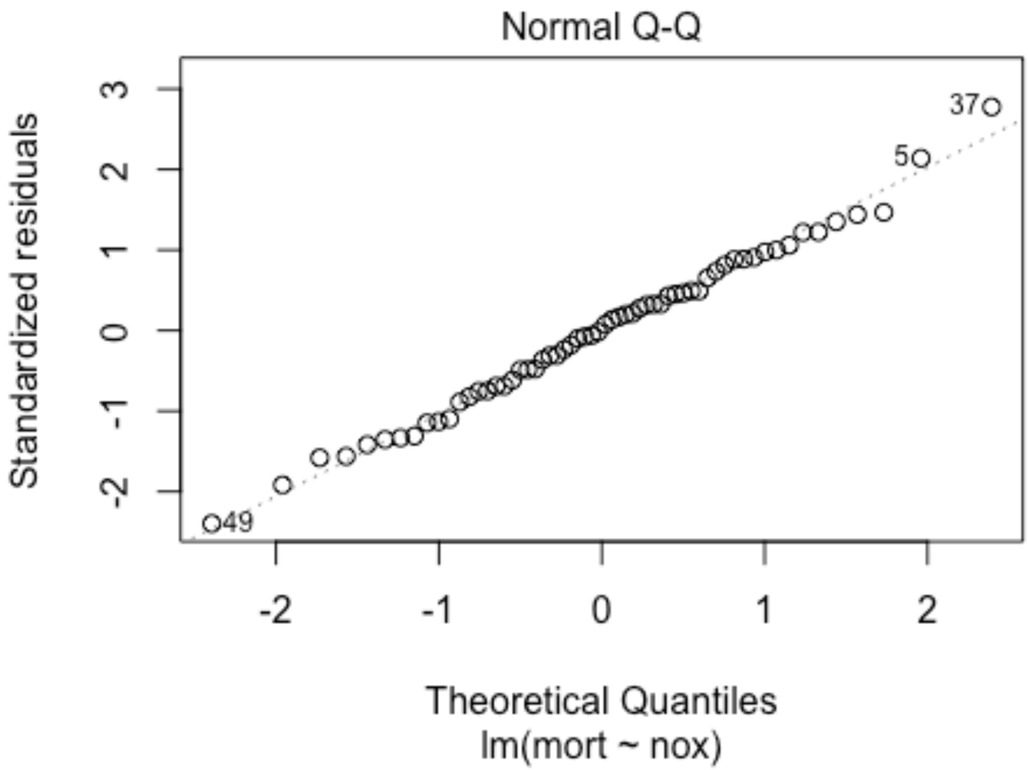
(a)

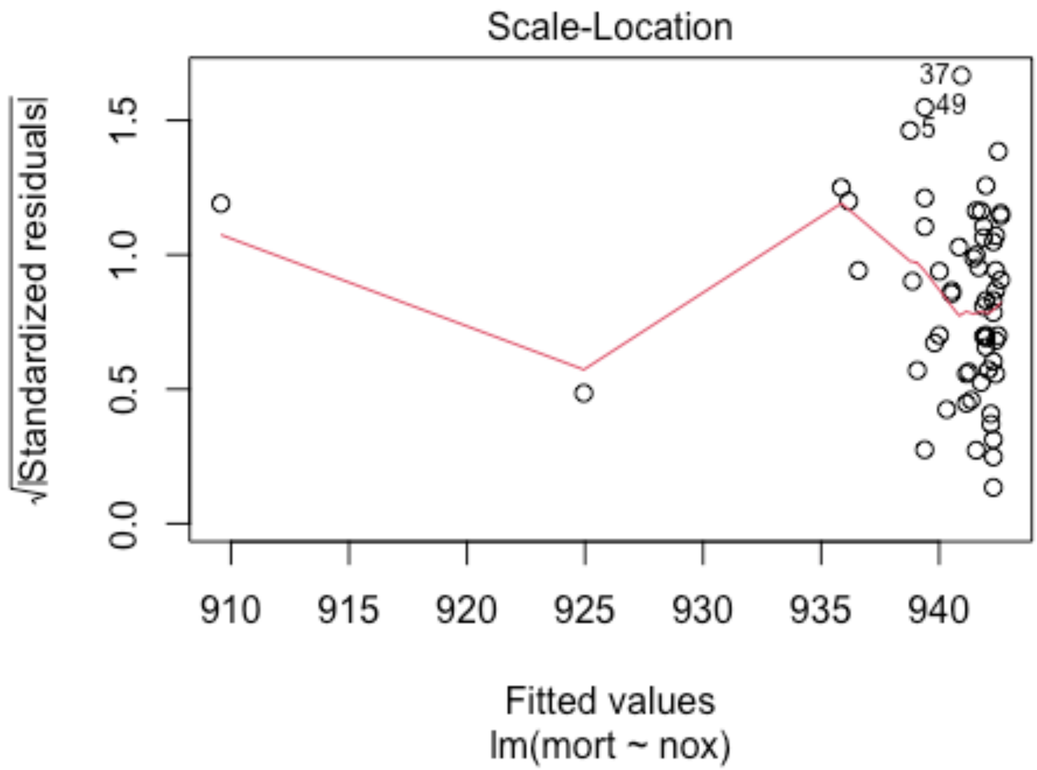
```
data <- read_dta("/Users/Home/Documents/Michael_Ghattas/School/CU_Boulder/
2022/Spring 2022/STAT - 4400/Data/pollution.dta")
head(data)

## # A tibble: 6 × 16
##    prec  jant  jult ovr65  popn  educ  hous  dens  nonw wwdrk  poor    hc
nox
##   <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
<dbl>
## 1    36    27    71   8.1  3.34  11.4  81.5  3243   8.8  42.6  11.7    21
15
## 2    35    23    72  11.1  3.14  11    78.8  4281   3.5  50.7  14.4     8
10
## 3    44    29    74  10.4  3.21   9.8  81.6  4260   0.8  39.4  12.4     6
6
## 4    47    45    79   6.5  3.41  11.1  77.5  3125  27.1  50.2  20.6    18
8
## 5    43    35    77   7.6  3.44   9.6  84.6  6441  24.4  43.7  14.3    43
38
## 6    53    45    80   7.7  3.45  10.2  66.8  3325  38.5  43.1  25.5    30
32
## # … with 3 more variables: so2 <dbl>, humid <dbl>, mort <dbl>

df = na.omit(data) # removing NA values

lmod = lm(mort ~ nox, data = df) #Do not believe this will be a good fit, as
nitric oxides might not be a main contributor to death on its own!
summary(lmod)

##
## Call:
## lm(formula = mort ~ nox, data = df)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -148.654  -43.710    1.751   41.663  172.211
```

```
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 942.7115     9.0034 104.706   <2e-16 ***
## nox          -0.1039     0.1758  -0.591    0.557
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 62.55 on 58 degrees of freedom
## Multiple R-squared:  0.005987,   Adjusted R-squared:  -0.01115
## F-statistic: 0.3494 on 1 and 58 DF,  p-value: 0.5568
```

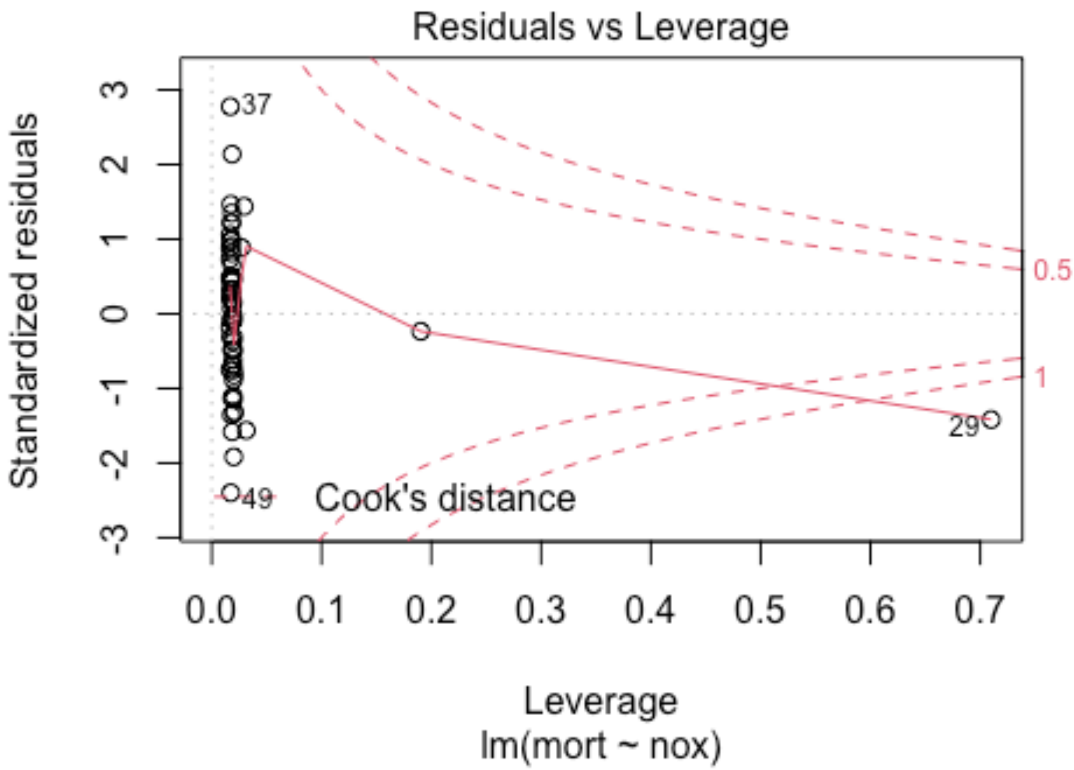
```
plot(lmod)
```


Residuals vs Fitted
Residuals
200
100
0
-100
37
5
49
910
915
920
925
930
935
940
Fitted values
lm(mort ~ nox)


Normal Q-Q
Standardized residuals
3
2
1
0
-1
-2
37
5
49
-2
-1
0
1
2
Theoretical Quantiles
lm(mort ~ nox)

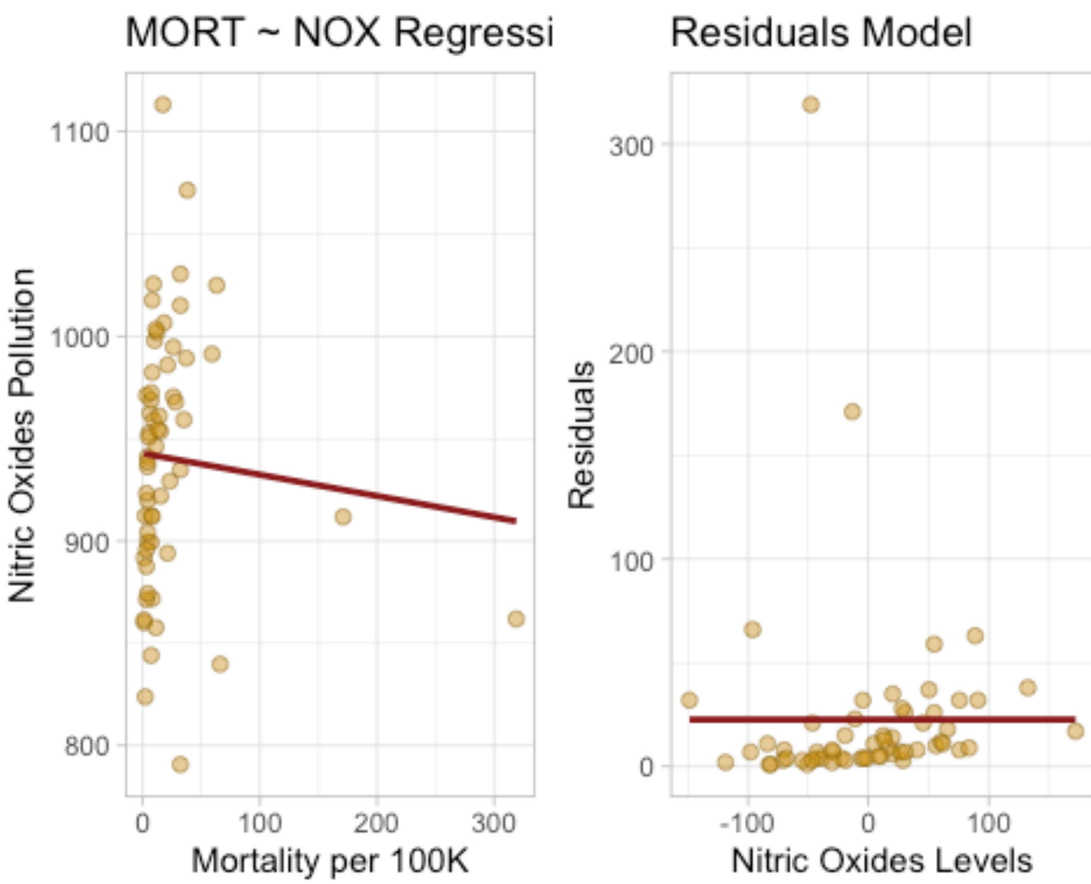


Scale-Location
37
49
5
0.0
0.5
1.0
1.5
√|Standardized residuals|
910
915
920
925
930
935
940
Fitted values
lm(mort ~ nox)




```
res = residuals(lmod)

plot1 = ggplot(df, aes(nox, mort)) +
  geom_point(shape = 21, color = "darkgoldenrod4", fill = "darkgoldenrod3",
size = 2,
             alpha = 0.5,show.legend = FALSE) +
  theme_light() + xlab("Mortality per 100K") +   ylab("Nitric Oxides
Pollution") +
  ggtitle("MORT ~ NOX Regression Model") +
  geom_smooth(method = lm, color = "firebrick4", se = FALSE)

plot2 = ggplot(lmod, aes(res, nox)) +
  geom_point(shape = 21, color = "darkgoldenrod4", fill = "darkgoldenrod3",
size = 2,
```

```
              alpha = 0.5,show.legend = FALSE) +
  theme_light() + xlab("Nitric Oxides Levels") +   ylab("Residuals") +
  ggtitle("Residuals Model") +
  geom_smooth(method = lm, color = "firebrick4", se = FALSE)

library(gridExtra)
grid.arrange(plot1, plot2, ncol = 2)

## `geom_smooth()` using formula 'y ~ x'
## `geom_smooth()` using formula 'y ~ x'
```



The assumption of a linearity and constant variance for the residual error appears to be in question. Ideally there should be symmetry in the scattering above and below the line.

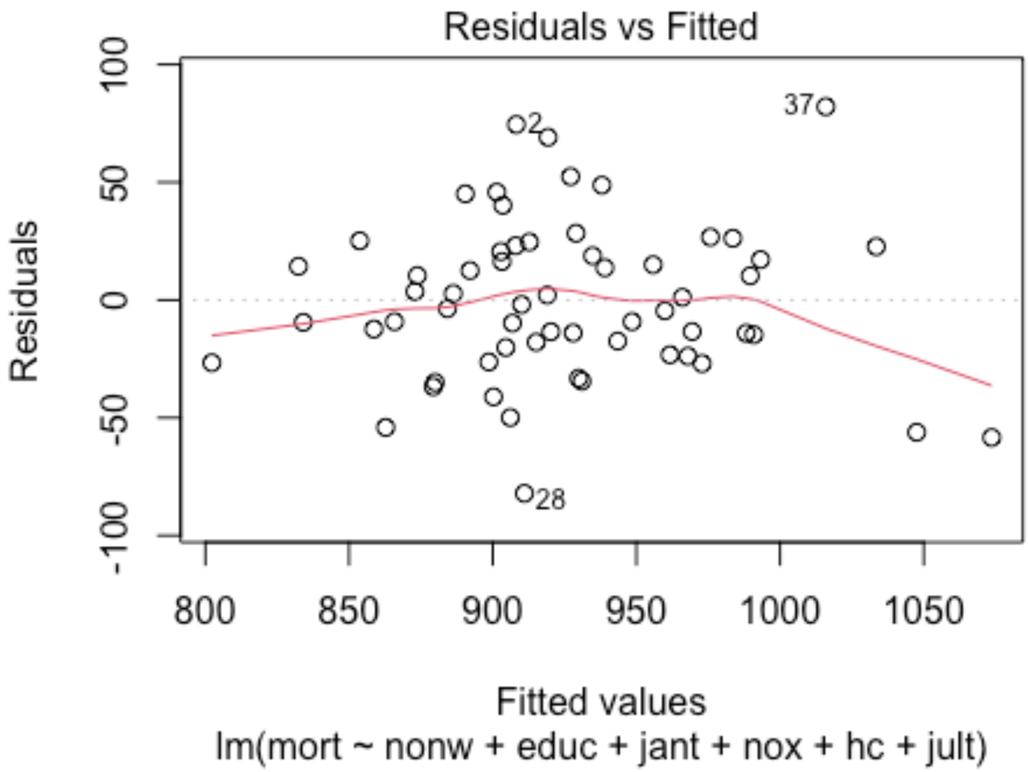
(b)

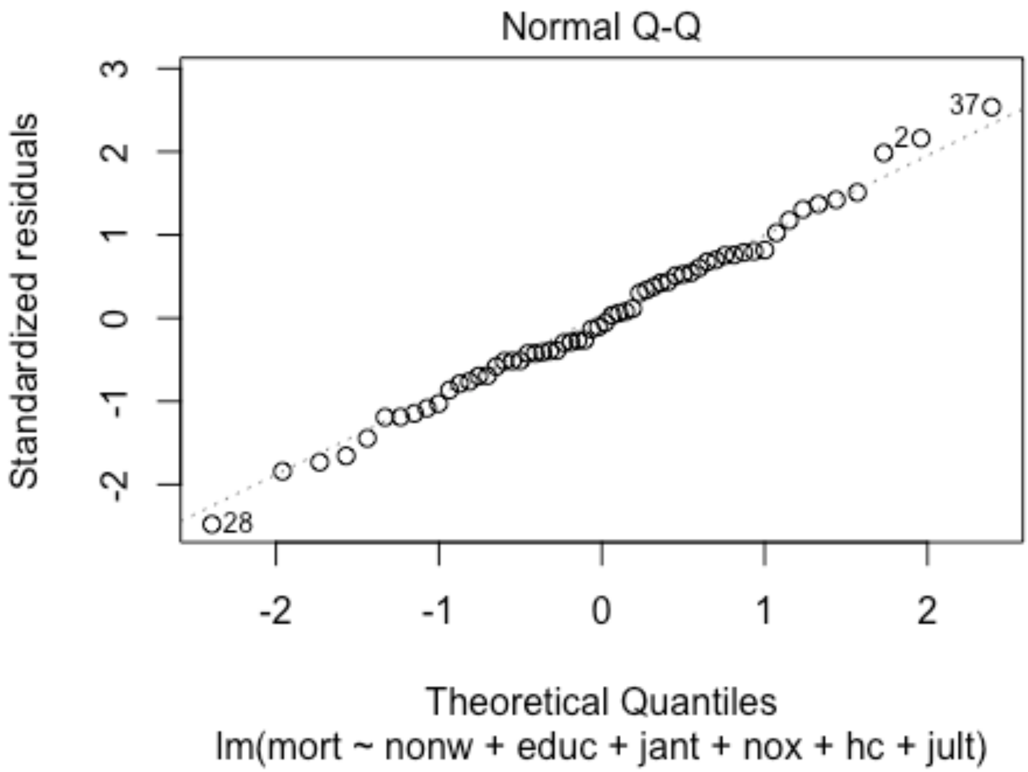
```
lmod = lm(mort ~ ., data = df)
summary(lmod)

##
## Call:
## lm(formula = mort ~ ., data = df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -68.066 -18.017   0.912  19.224  86.961
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.764e+03  4.373e+02   4.034 0.000215 ***
## prec         1.905e+00  9.237e-01   2.063 0.045071 *
## jant        -1.938e+00  1.108e+00  -1.748 0.087413 .
## jult        -3.100e+00  1.902e+00  -1.630 0.110159
## ovr65       -9.065e+00  8.486e+00  -1.068 0.291230
## popn        -1.068e+02  6.978e+01  -1.531 0.132952
## educ        -1.716e+01  1.186e+01  -1.447 0.155085
## hous        -6.511e-01  1.768e+00  -0.368 0.714393
## dens         3.600e-03  4.027e-03   0.894 0.376147
## nonw         4.460e+00  1.327e+00   3.360 0.001618 **
## wwdrk       -1.871e-01  1.662e+00  -0.113 0.910883
## poor        -1.676e-01  3.227e+00  -0.052 0.958807
## hc          -6.721e-01  4.910e-01  -1.369 0.177985
## nox          1.340e+00  1.006e+00   1.333 0.189506
## so2          8.625e-02  1.475e-01   0.585 0.561745
## humid        1.068e-01  1.169e+00   0.091 0.927644
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 34.93 on 44 degrees of freedom
## Multiple R-squared:  0.7649, Adjusted R-squared:  0.6847
## F-statistic: 9.542 on 15 and 44 DF,  p-value: 2.193e-09
```

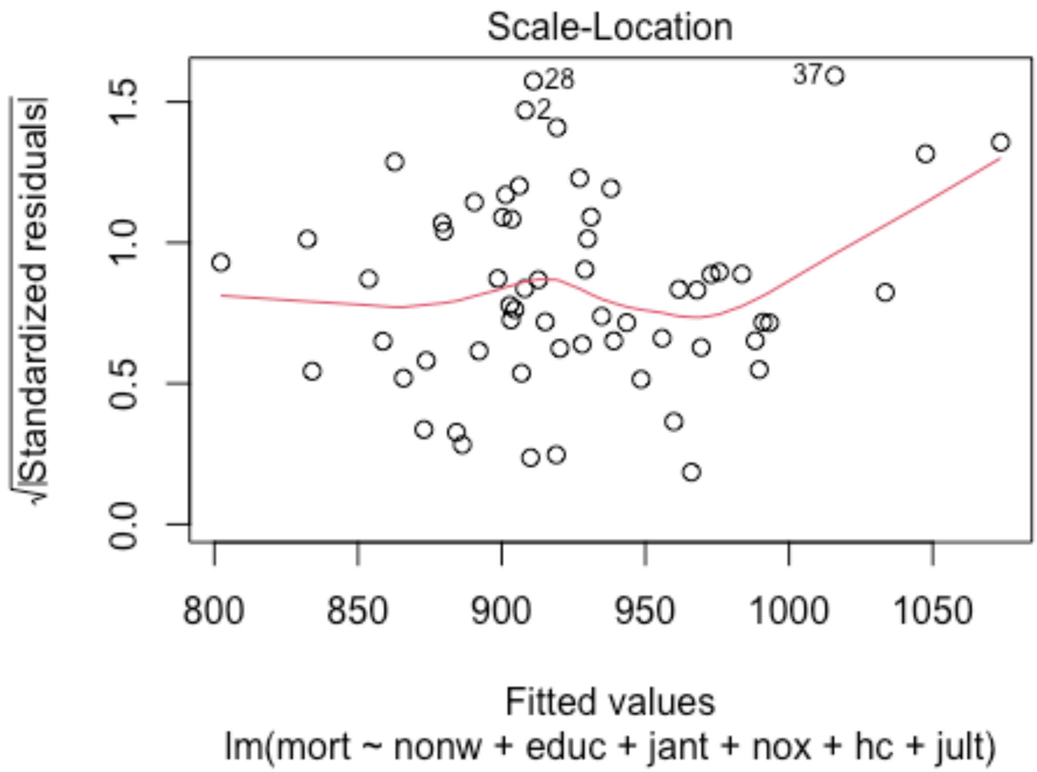
```r
df$mort <- (df$mort - mean(df$mort) / sd(df$mort))
df$nonw <- (df$nonw - mean(df$nonw) / sd(df$nonw))
df$educ <- (df$educ - mean(df$educ) / sd(df$educ))
df$jant <- (df$jant - mean(df$jant) / sd(df$jant))
df$nox <- (df$nox - mean(df$nox) / sd(df$nox))
df$hc <- (df$hc - mean(df$hc) / sd(df$hc))
df$jult <- (df$jult - mean(df$jult) / sd(df$jult))

lmod = lm(mort ~ nonw + educ + jant + nox + hc + jult, data = df)
summary(lmod)
```

```
##
## Call:
## lm(formula = mort ~ nonw + educ + jant + nox + hc + jult, data = df)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -82.10 -20.93  -2.80  21.10  82.11
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 997.2196    74.9580  13.304  < 2e-16 ***
## nonw          4.9460     0.7093   6.973 4.99e-09 ***
## educ        -20.2172     6.1102  -3.309  0.00169 **
## jant         -1.2197     0.6219  -1.961  0.05512 .
## nox           1.9879     0.6247   3.182  0.00245 **
## hc           -1.0336     0.3273  -3.158  0.00262 **
## jult         -2.2518     1.3871  -1.623  0.11044
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 35.25 on 53 degrees of freedom
## Multiple R-squared:  0.7115, Adjusted R-squared:  0.6789
## F-statistic: 21.79 on 6 and 53 DF,  p-value: 1.007e-12
```

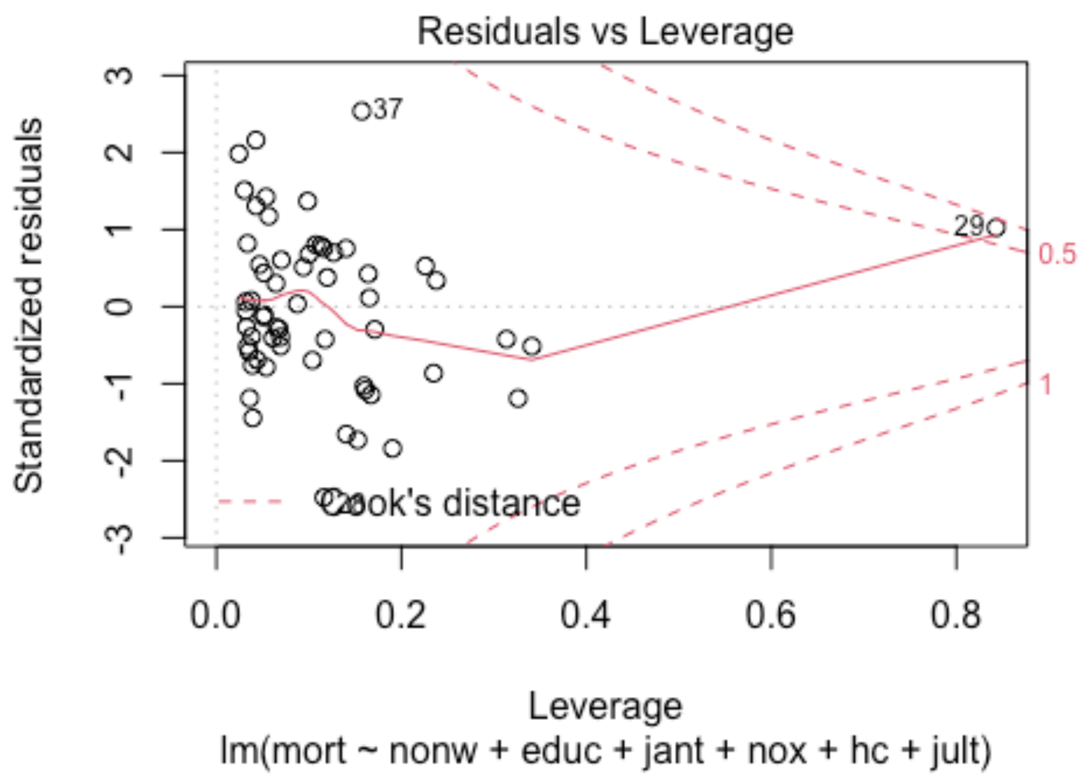
```r
plot(lmod)
```


Residuals vs Fitted
Residuals
100
50
0
-50
-100
37
2
28
800
850
900
950
1000
1050
Fitted values
lm(mort ~ nonw + educ + jant + nox + hc + jult)


Normal Q-Q
Standardized residuals
Theoretical Quantiles
lm(mort ~ nonw + educ + jant + nox + hc + jult)
37
2
28
3
2
1
0
-1
-2
-2
-1
0
1
2

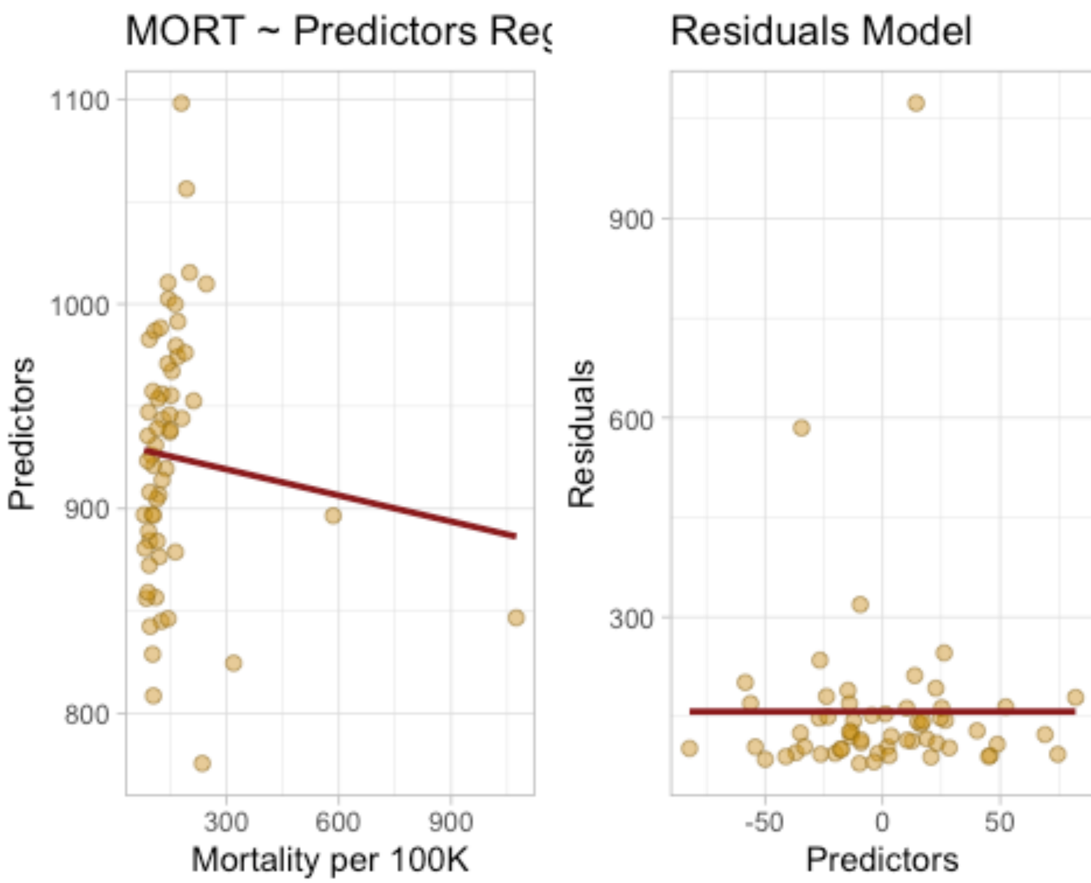


Scale-Location
√|Standardized residuals|
0.0
0.5
1.0
1.5
800
850
900
950
1000
1050
28
2
37
Fitted values
lm(mort ~ nonw + educ + jant + nox + hc + jult)




```
res = residuals(lmod)

plot1 = ggplot(df, aes(nonw + educ + jant + nox + hc + jult, mort)) +
  geom_point(shape = 21, color = "darkgoldenrod4", fill = "darkgoldenrod3",
size = 2,
             alpha = 0.5,show.legend = FALSE) +
  theme_light() + xlab("Mortality per 100K") +   ylab("Predictors") +
  ggtitle("MORT ~ Predictors Regression Model") +
  geom_smooth(method = lm, color = "firebrick4", se = FALSE)

plot2 = ggplot(lmod, aes(res, nonw + educ + jant + nox + hc + jult)) +
  geom_point(shape = 21, color = "darkgoldenrod4", fill = "darkgoldenrod3",
size = 2,
             alpha = 0.5,show.legend = FALSE) +
```

```
  theme_light() + xlab("Predictors") +   ylab("Residuals") +
  ggtitle("Residuals Model") +
  geom_smooth(method = lm, color = "firebrick4", se = FALSE)

library(gridExtra)
grid.arrange(plot1, plot2, ncol = 2)

## `geom_smooth()` using formula 'y ~ x'
## `geom_smooth()` using formula 'y ~ x'
```



The assumption of a linearity and constant variance for the residual error appears to be better. There seems to be symmetry in the scattering above and below the line.

(c)

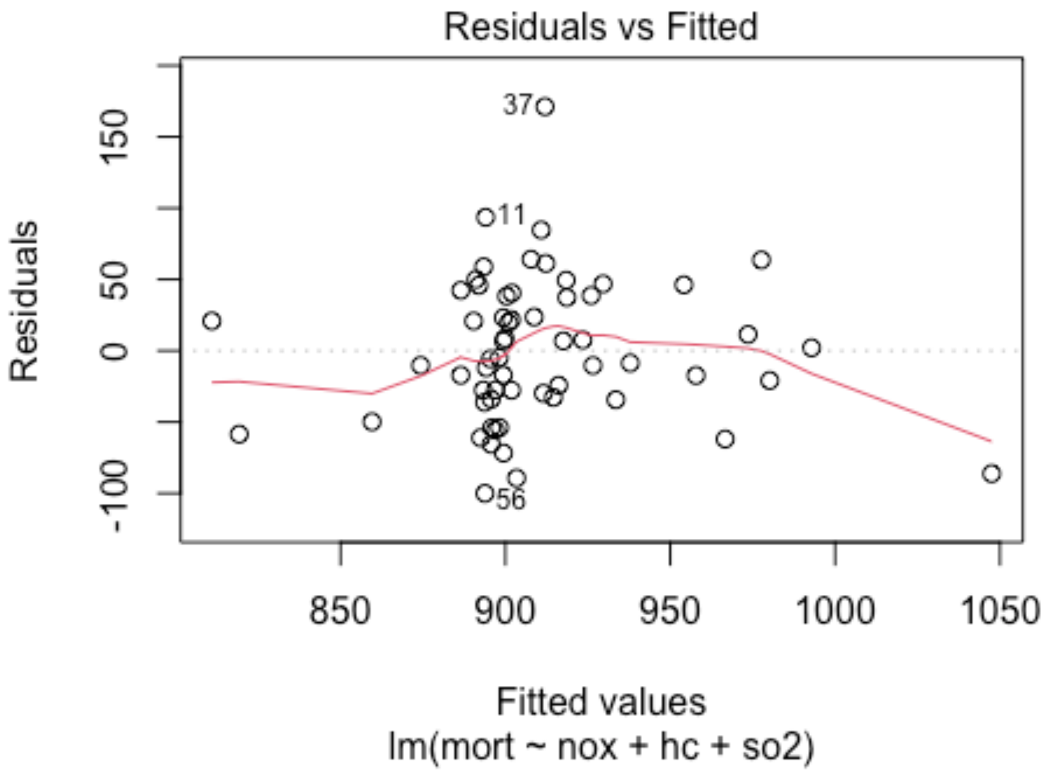
The slope coefficients suggest a high positive correlation between the non-white population in urbanized areas and relative Nitric-Oxides pollution potential and mortality. Additionally, there is a moderate negative correlation between the average January temperature, relative hydrocarbon pollution potential, and average July temperature and mortality. Finally, there is a strong correlation between the median school years completed by those over 22 and mortality. While the reasons behind most of the correlations requires more investigation, it is clear that higher education leads to lower mortality, most likely driven by better decision making and standard of living.

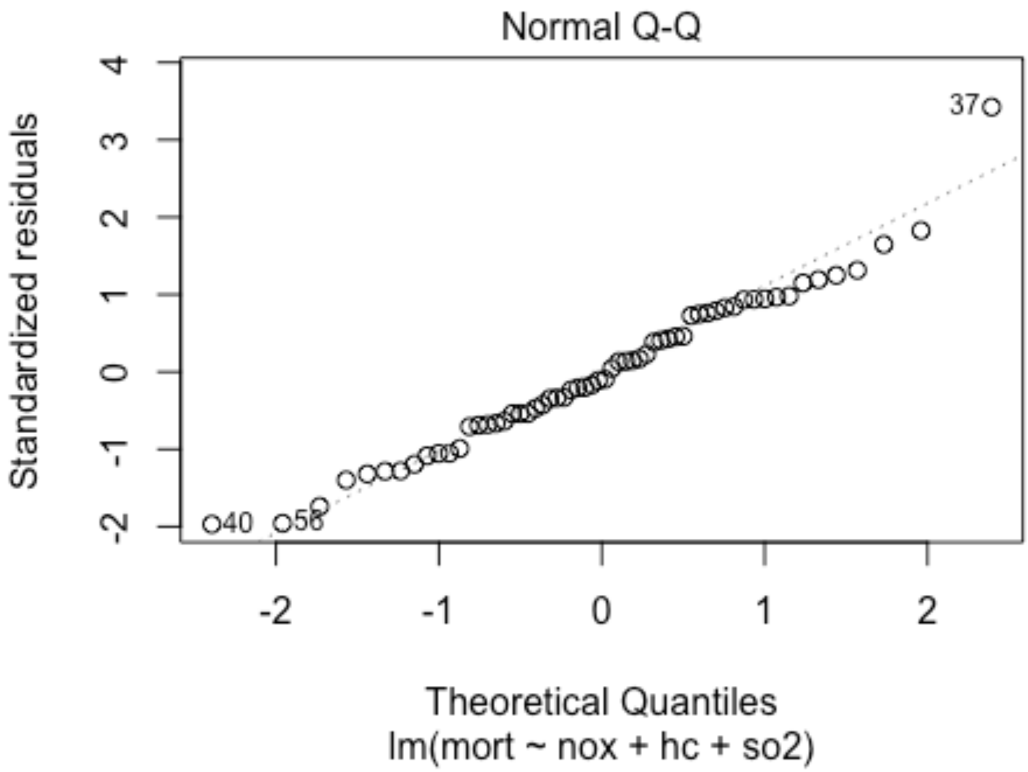
(d)

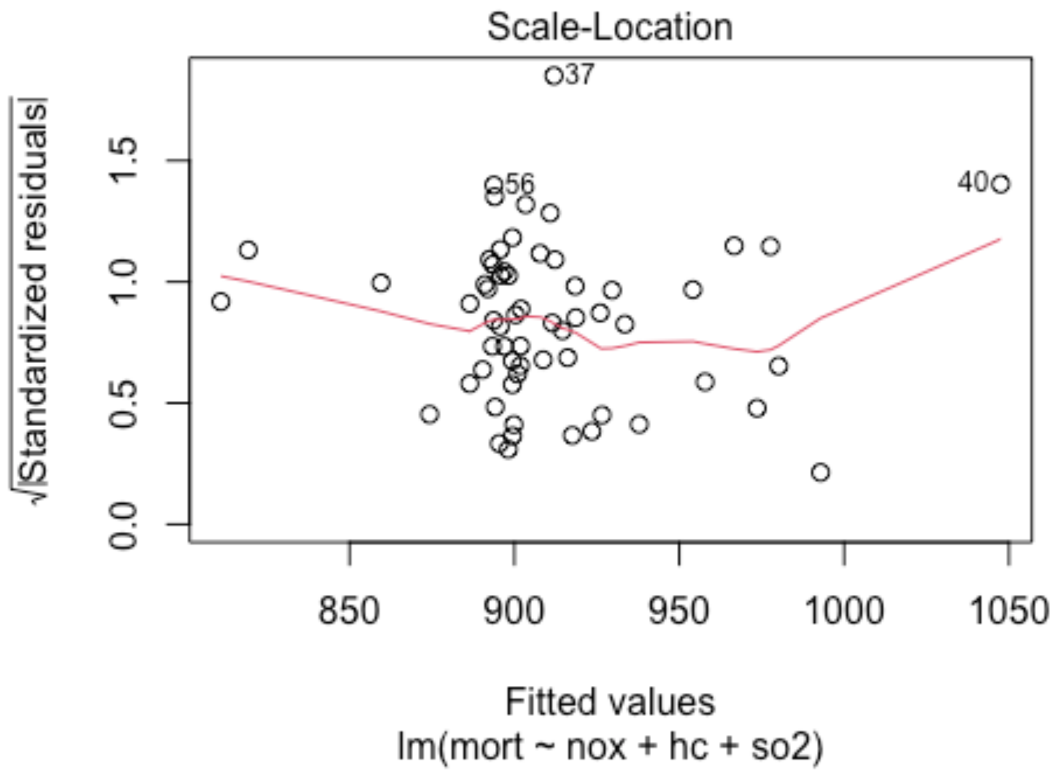
```r
df$mort <- (df$mort - mean(df$mort) / sd(df$mort))
df$nox <- (df$nox - mean(df$nox) / sd(df$nox))
df$hc <- (df$hc - mean(df$hc) / sd(df$hc))
df$so2 <- (df$so2 - mean(df$so2) / sd(df$so2))

lmod = lm(mort ~ nox + hc + so2, data = df)
summary(lmod)

##
## Call:
## lm(formula = mort ~ nox + hc + so2, data = df)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -100.020  -33.058   -5.287   38.398  171.163
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 895.8644     9.0166  99.357   <2e-16 ***
## nox           2.9350     1.2668   2.317   0.0242 *
## hc           -1.6135     0.6069  -2.659   0.0102 *
## so2           0.2006     0.1728   1.161   0.2507
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 51.84 on 56 degrees of freedom
```

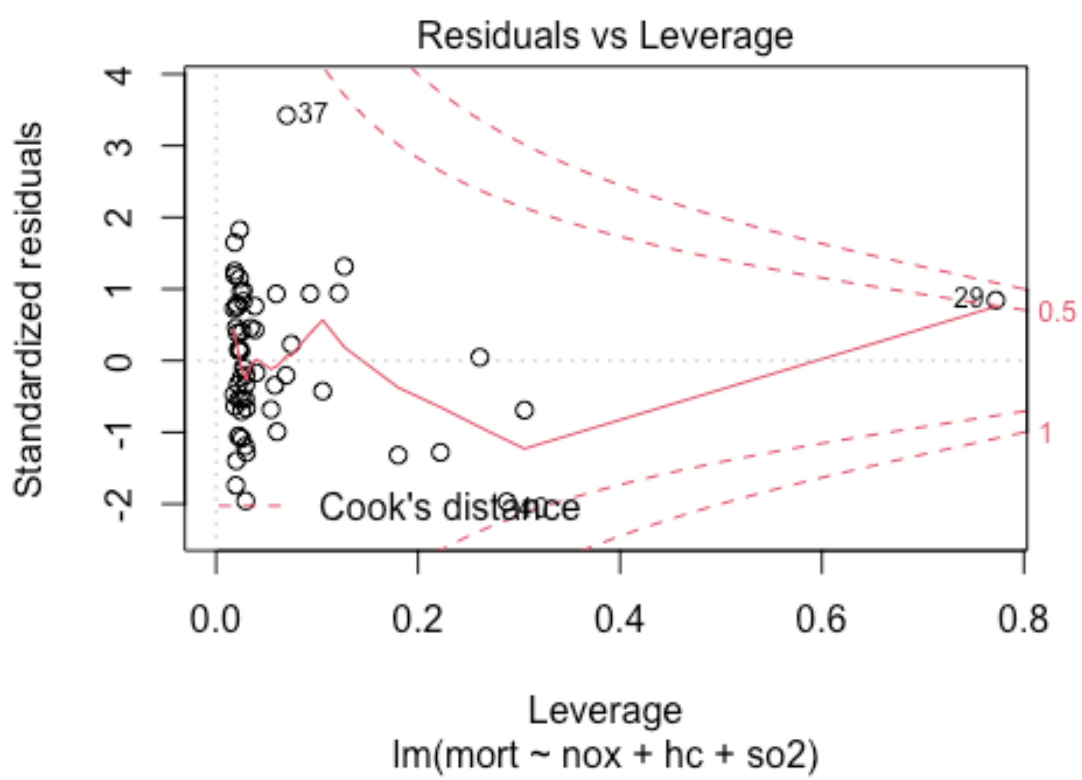
```
## Multiple R-squared:  0.3407, Adjusted R-squared:  0.3054
## F-statistic: 9.647 on 3 and 56 DF,  p-value: 3.131e-05
```

```
plot(lmod)
```

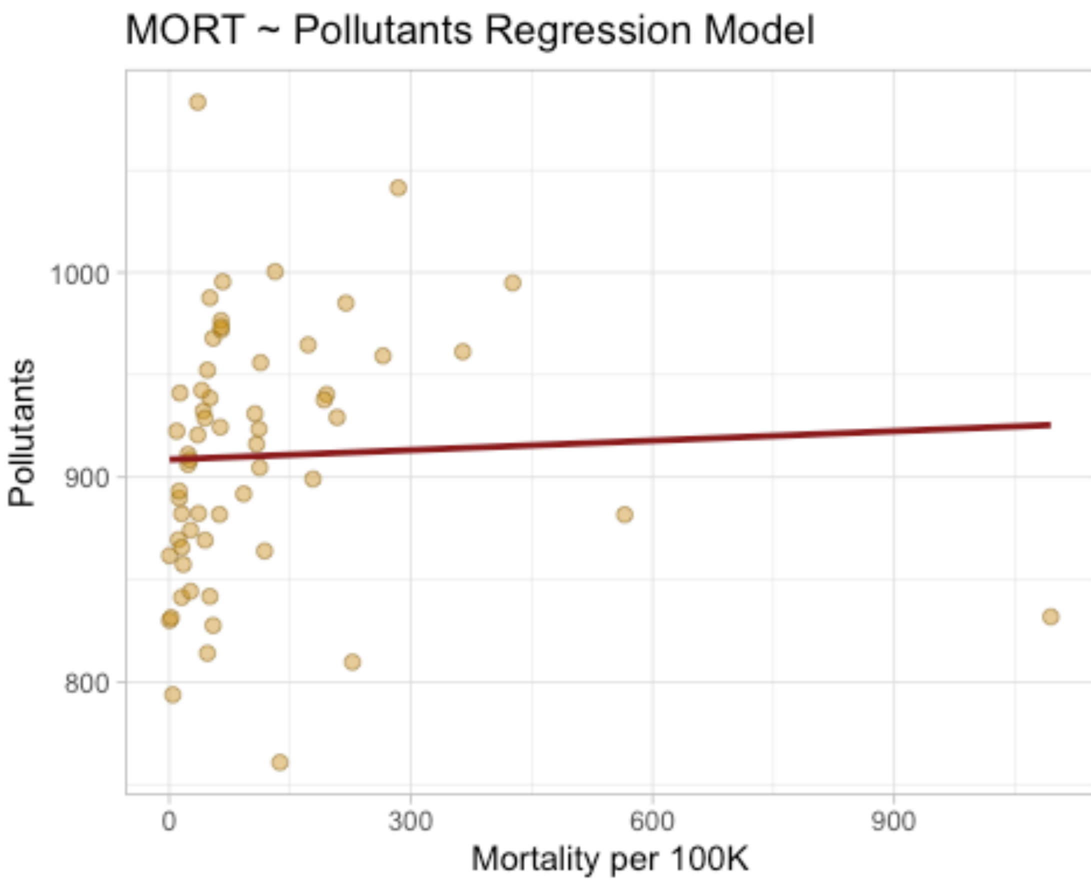

Residuals vs Fitted
Residuals
Fitted values
lm(mort ~ nox + hc + so2)
37
11
56
150
50
0
-100
850
900
950
1000
1050


Normal Q-Q
Standardized residuals
Theoretical Quantiles
lm(mort ~ nox + hc + so2)
37
40
56
-2
-1
0
1
2
3
4


Scale-Location
√|Standardized residuals|
0.0
0.5
1.0
1.5
37
56
40
850
900
950
1000
1050
Fitted values
lm(mort ~ nox + hc + so2)




```
res = residuals(lmod)

ggplot(df, aes(nox + hc + so2, mort)) +
  geom_point(shape = 21, color="darkgoldenrod4", fill = "darkgoldenrod3",
size = 2,
             alpha = 0.5,show.legend = FALSE) +
  theme_light() + xlab("Mortality per 100K") +   ylab("Pollutants") +
  ggtitle("MORT ~ Pollutants Regression Model") +
  geom_smooth(method = lm, color = "firebrick4", se = FALSE)

## `geom_smooth()` using formula 'y ~ x'
```



We can note that Nitric-oxides pollutants have a moderate positive correlation on the rate of mortality, while Sulfur-dioxides seem to have a slight positive correlation. However, Hydrocarbon pollutants seem ti have a moderate negative correlation with the rate of mortality. The findings need further investigation with an understanding of the physical and chemical mechanisms in effect.

(e)

```
df$mort <- (df$mort - mean(df$mort) / sd(df$mort))
df$nox <- (df$nox - mean(df$nox) / sd(df$nox))
df$hc <- (df$hc - mean(df$hc) / sd(df$hc))
df$so2 <- (df$so2 - mean(df$so2) / sd(df$so2))

# split dataset into training and test sets
train <- df[1:(nrow(df) / 2), ]
test <- df[((nrow(df) / 2) + 1):nrow(df), ]
```

```r
# fit linear model
lmodT <- lm(log(mort) ~ nox + so2 + hc, data = train)
summary(lmodT)
```

```
##
## Call:
## lm(formula = log(mort) ~ nox + so2 + hc, data = train)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.110207 -0.030891 -0.005169  0.038273  0.092587
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  6.7814638  0.0123673 548.338   <2e-16 ***
## nox          0.0010814  0.0024328   0.445   0.6603
## so2          0.0004547  0.0002654   1.713   0.0986 .
## hc          -0.0007324  0.0011548  -0.634   0.5315
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.0529 on 26 degrees of freedom
## Multiple R-squared:  0.3753, Adjusted R-squared:  0.3033
## F-statistic: 5.208 on 3 and 26 DF,  p-value: 0.005971
```

```
## lm(formula = log(mort) ~ z.nox + z.so2 + z.hc, data = train)
##             coef.est coef.se
## (Intercept) -4.66     0.01
## z.nox        0.10     0.21
## z.so2        0.05     0.03
## z.hc        -0.13     0.20
## ---
## n = 30, k = 4
## residual sd = 0.05, R-Squared = 0.38
```

```r
# predict
predictions <- predict(lmodT, test)
cbind(predictions = exp(predictions), observed = test$mort)
```

```
##    predictions  observed
## 1     901.4267  961.8648
## 2     879.5232  816.8138
## 3     932.2266  884.5248
## 4     888.1223  812.9968
## 5     914.3313  916.3838
## 6     883.5898  878.6088
## 7     883.8008 1068.5308
## 8     922.6332  950.0228
## 9     959.6859  970.3978
## 10   1022.9651  946.6648
## 11    881.6019  849.3658
## 12    887.2760  893.8748
## 13    919.9731  901.5598
## 14    900.3723  980.8768
## 15    886.6263  829.6558
## 16    901.8733  908.9348
## 17    858.3745  795.0838
## 18    876.8046  867.0758
## 19    844.8060  746.1078
## 20    881.8798  854.6388
## 21    888.7338  859.5298
## 22    889.7631  906.0468
## 23    889.7325  927.8388
## 24    883.1924  867.5768
## 25    906.0448  923.1778
## 26    879.8302  779.1388
## 27    898.5465  958.8768
## 28    881.6504  851.0708
## 29    902.4481  867.1918
## 30    899.2646  909.8168
```

We can not that this is not really cross-validation, but rather providing a sense of how the steps of cross-validation can be implemented.

Question 3

(a)

```
require(arm)

## Loading required package: arm

## Loading required package: MASS

## Loading required package: Matrix

## Loading required package: lme4

##
## arm (Version 1.12-2, built: 2021-10-15)

## Working directory is /Users/Home/Documents/Michael_Ghattas/School/
CU_Boulder/2022/Spring 2022/STAT - 4400/HW/2

require(ggplot2)
require(foreign)

## Loading required package: foreign

data <- read.csv("/Users/Home/Documents/Michael_Ghattas/School/CU_Boulder/
2022/Spring 2022/STAT - 4400/Data/ProfEvaltnsBeautyPublic.csv")

df = na.omit(data) # removing NA values
df$profnumber <- as.factor(df$profnumber)
df$female <- as.factor(df$female)

dummies <- df[, 18:47]
df$class <- factor(apply(dummies, FUN = function(r) r %*% 1:30, MARGIN = 1))
df <- df[-c(18:47)]

lmod1 <- lm(courseevaluation ~ female + profnumber + class, data = df)
summary(lmod1)
```

```
##
## Call:
## lm(formula = courseevaluation ~ female + profnumber + class,
##     data = df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.5286 -0.2062  0.0000  0.2000  0.9667
##
## Coefficients: (1 not defined because of singularities)
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)   4.150e+00  1.595e-01  26.019  < 2e-16 ***
## female1      -2.539e-02  3.557e-01  -0.071 0.943130
## profnumber2  -6.167e-01  2.763e-01  -2.232 0.026253 *
## profnumber3  -4.477e-01  3.515e-01  -1.274 0.203696
## profnumber4  -1.664e-01  3.520e-01  -0.473 0.636665
## profnumber5   2.042e-01  3.576e-01   0.571 0.568428
## profnumber6   3.650e-01  2.214e-01   1.648 0.100220
## profnumber7  -5.782e-02  3.377e-01  -0.171 0.864156
## profnumber8  -9.604e-02  3.505e-01  -0.274 0.784277
## profnumber9   1.068e-01  3.529e-01   0.303 0.762334
## profnumber10  5.109e-01  2.102e-01   2.431 0.015595 *
## profnumber11 -6.310e-01  3.132e-01  -2.014 0.044754 *
## profnumber12 -4.354e-02  2.443e-01  -0.178 0.858651
## profnumber13 -3.395e-01  2.425e-01  -1.400 0.162463
## profnumber14 -4.488e-01  2.628e-01  -1.708 0.088539 .
## profnumber15 -1.062e+00  4.164e-01  -2.550 0.011218 *
## profnumber16  9.558e-02  3.372e-01   0.283 0.777001
## profnumber17 -2.674e-01  4.048e-01  -0.661 0.509321
## profnumber18 -1.000e-01  2.110e-01  -0.474 0.635844
## profnumber19  2.198e-01  3.436e-01   0.640 0.522683
## profnumber20 -6.446e-01  3.411e-01  -1.890 0.059626 .
## profnumber21 -5.189e-01  3.651e-01  -1.421 0.156094
## profnumber22 -1.050e+00  7.871e-01  -1.333 0.183281
## profnumber23 -3.141e-01  3.640e-01  -0.863 0.388866
```

```
## profnumber24  3.214e-01  2.174e-01   1.479 0.140123
## profnumber25  1.254e-01  4.212e-01   0.298 0.766101
## profnumber26 -7.500e-02  4.885e-01  -0.154 0.878083
## profnumber27 -1.926e-01  2.802e-01  -0.687 0.492273
## profnumber28 -2.628e-01  3.029e-01  -0.868 0.386150
## profnumber29  3.611e-01  4.098e-01   0.881 0.378819
## profnumber30 -3.246e-01  6.947e-01  -0.467 0.640631
## profnumber31 -4.850e-01  2.230e-01  -2.175 0.030319 *
## profnumber32  1.500e-01  3.190e-01   0.470 0.638495
## profnumber33 -2.738e-02  2.404e-01  -0.114 0.909386
## profnumber34 -2.642e-01  3.628e-01  -0.728 0.467015
## profnumber35 -1.327e-01  3.604e-01  -0.368 0.712983
## profnumber36 -2.281e-01  3.818e-01  -0.597 0.550629
## profnumber37 -6.435e-01  2.278e-01  -2.825 0.005012 **
## profnumber38 -1.115e-01  2.836e-01  -0.393 0.694316
## profnumber39  2.303e-01  2.349e-01   0.980 0.327547
## profnumber40 -2.926e-01  5.545e-01  -0.528 0.598049
## profnumber41  5.100e-01  2.366e-01   2.156 0.031804 *
## profnumber42  5.500e-01  3.190e-01   1.724 0.085589 .
## profnumber43  1.316e-01  3.834e-01   0.343 0.731664
## profnumber44  2.833e-01  4.785e-01   0.592 0.554153
## profnumber45  4.812e-01  2.672e-01   1.801 0.072573 .
## profnumber46  1.500e-01  4.785e-01   0.313 0.754105
## profnumber47 -1.350e+00  5.557e-01  -2.429 0.015678 *
## profnumber48 -1.579e-01  5.519e-01  -0.286 0.774916
## profnumber49 -3.695e-01  3.582e-01  -1.031 0.303056
## profnumber50 -3.500e-01  2.059e-01  -1.700 0.090091 .
## profnumber51  4.754e-01  3.557e-01   1.337 0.182266
## profnumber52  2.536e-02  2.597e-01   0.098 0.922275
## profnumber53  2.334e-01  3.783e-01   0.617 0.537722
## profnumber54 -4.079e-01  3.557e-01  -1.147 0.252227
## profnumber55 -1.064e+00  2.964e-01  -3.588 0.000382 ***
## profnumber56 -3.182e-01  2.452e-01  -1.298 0.195316
## profnumber57 -2.246e-01  4.212e-01  -0.533 0.594187
## profnumber58 -2.146e-01  3.411e-01  -0.629 0.529644
```

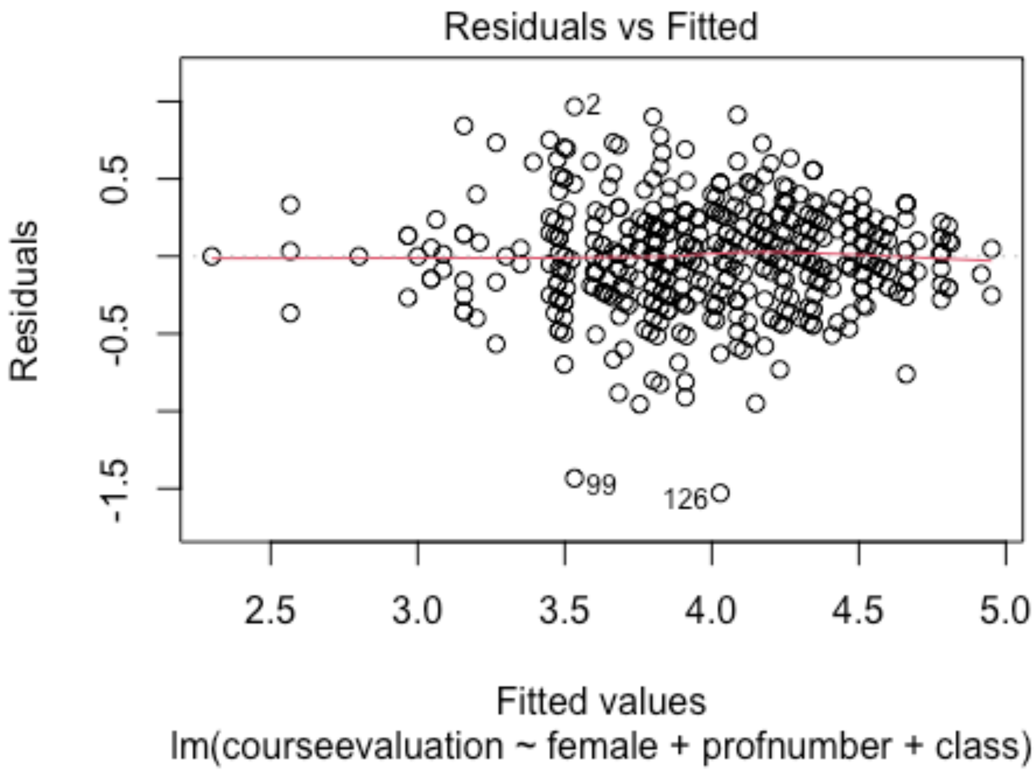
```
## profnumber59 -8.000e-01  3.888e-01  -2.058 0.040375 *
## profnumber60 -1.158e+00  3.898e-01  -2.971 0.003185 **
## profnumber61 -1.500e-01  4.220e-01  -0.355 0.722467
## profnumber62  1.500e-01  4.220e-01   0.355 0.722467
## profnumber63 -3.246e-01  4.212e-01  -0.771 0.441414
## profnumber64 -4.913e-01  3.898e-01  -1.260 0.208432
## profnumber65 -1.103e-01  3.505e-01  -0.315 0.753168
## profnumber66 -4.667e-01  2.256e-01  -2.069 0.039313 *
## profnumber67 -2.000e-01  3.190e-01  -0.627 0.531101
## profnumber68 -1.583e+00  2.763e-01  -5.731 2.20e-08 ***
## profnumber69 -1.125e+00  5.037e-01  -2.233 0.026221 *
## profnumber70  3.611e-01  2.059e-01   1.754 0.080380 .
## profnumber71  6.300e-01  2.017e-01   3.123 0.001946 **
## profnumber72 -3.000e-01  2.256e-01  -1.330 0.184411
## profnumber73  4.300e-01  2.366e-01   1.818 0.070004 .
## profnumber74 -3.000e-01  2.522e-01  -1.190 0.235039
## profnumber75 -7.461e-02  5.298e-01  -0.141 0.888085
## profnumber76 -9.246e-01  4.212e-01  -2.195 0.028821 *
## profnumber77  1.254e-01  3.860e-01   0.325 0.745464
## profnumber78  2.570e-02  2.790e-01   0.092 0.926647
## profnumber79 -3.500e-01  2.763e-01  -1.267 0.206048
## profnumber80 -2.996e-01  3.731e-01  -0.803 0.422573
## profnumber81  2.780e-01  2.577e-01   1.078 0.281596
## profnumber82 -6.270e-02  2.034e-01  -0.308 0.758140
## profnumber83  5.539e-02  3.628e-01   0.153 0.878731
## profnumber84  8.613e-02  4.583e-01   0.188 0.851032
## profnumber85  6.643e-01  3.102e-01   2.141 0.032972 *
## profnumber86  1.167e-01  2.763e-01   0.422 0.673065
## profnumber87  5.000e-02  3.190e-01   0.157 0.875541
## profnumber88 -9.929e-01  2.174e-01  -4.568 6.91e-06 ***
## profnumber89 -5.246e-01  3.898e-01  -1.346 0.179266
## profnumber90 -4.000e-01  3.190e-01  -1.254 0.210728
## profnumber91  3.421e-01  3.898e-01   0.878 0.380836
## profnumber92 -2.103e-01  3.505e-01  -0.600 0.548912
## profnumber93         NA         NA      NA       NA
```

```
## profnumber94 -6.496e-01  3.731e-01  -1.741 0.082604 .
## class1        1.986e-01  2.489e-01   0.798 0.425551
## class2        3.052e-01  3.021e-01   1.010 0.313120
## class3       -1.246e-01  2.508e-01  -0.497 0.619662
## class4       -2.523e-01  1.476e-01  -1.709 0.088355 .
## class5        1.869e-01  2.144e-01   0.872 0.383960
## class6       -2.074e-01  3.220e-01  -0.644 0.519901
## class7       -6.000e-01  2.996e-01  -2.002 0.046040 *
## class8        2.250e-01  6.049e-01   0.372 0.710143
## class9       -1.706e-02  2.465e-01  -0.069 0.944860
## class10       4.713e-01  2.992e-01   1.575 0.116151
## class11       5.409e-01  3.226e-01   1.677 0.094519 .
## class12      -1.419e-01  2.612e-01  -0.543 0.587402
## class13      -1.167e-01  3.041e-01  -0.384 0.701456
## class14      -3.574e-01  3.597e-01  -0.994 0.321115
## class15      -1.500e+00  4.785e-01  -3.135 0.001870 **
## class16       2.881e-01  2.821e-01   1.021 0.307973
## class17       2.846e-01  1.920e-01   1.482 0.139175
## class18       1.719e-01  2.629e-01   0.654 0.513643
## class19      -6.861e-01  3.230e-01  -2.124 0.034375 *
## class20       4.882e-01  2.127e-01   2.296 0.022307 *
## class21      -3.624e-01  1.764e-01  -2.054 0.040698 *
## class22      -5.000e-01  3.907e-01  -1.280 0.201491
## class23       7.250e-01  2.348e-01   3.088 0.002181 **
## class24      -3.000e-01  4.642e-01  -0.646 0.518544
## class25       3.858e-15  4.444e-01   0.000 1.000000
## class26       1.272e-01  2.476e-01   0.514 0.607819
## class27       6.769e-02  3.044e-01   0.222 0.824151
## class28       1.375e-01  3.383e-01   0.406 0.684712
## class29      -4.508e-01  3.202e-01  -1.408 0.160153
## class30      -9.570e-03  2.792e-01  -0.034 0.972677
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3907 on 339 degrees of freedom
```

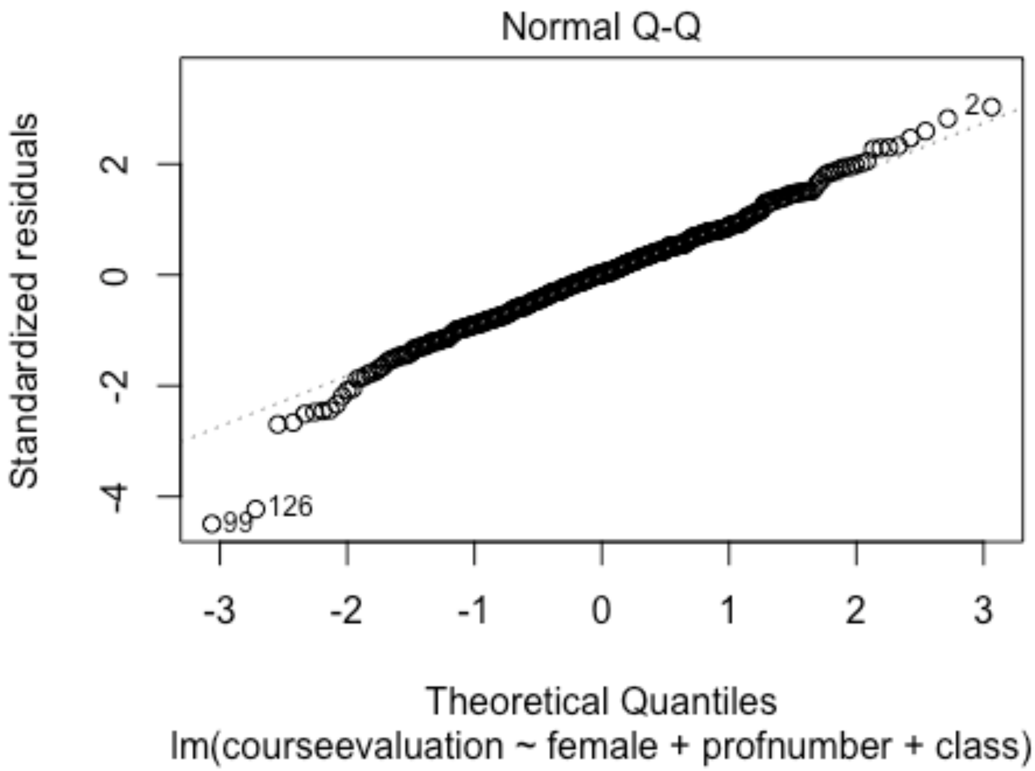
```
## Multiple R-squared:  0.6362, Adjusted R-squared:  0.5042
## F-statistic:  4.82 on 123 and 339 DF,  p-value: < 2.2e-16
```

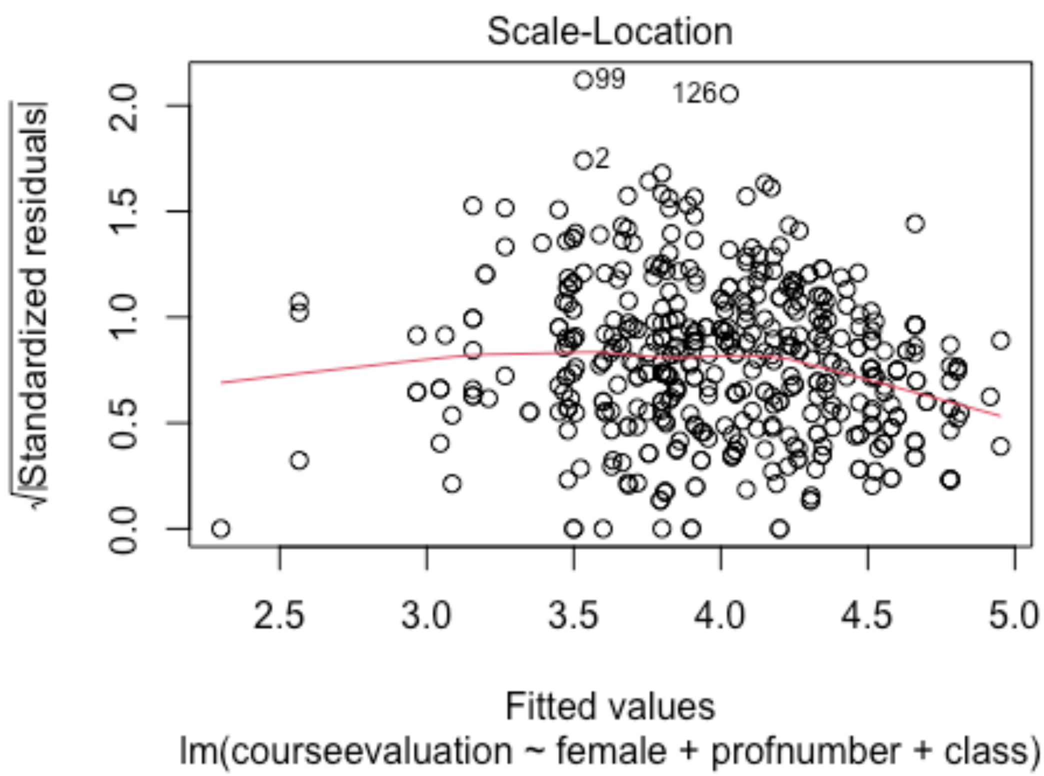
```
plot(lmod1)
```

```
## Warning: not plotting observations with leverage one:
##   22, 61, 62, 69, 211, 234
```

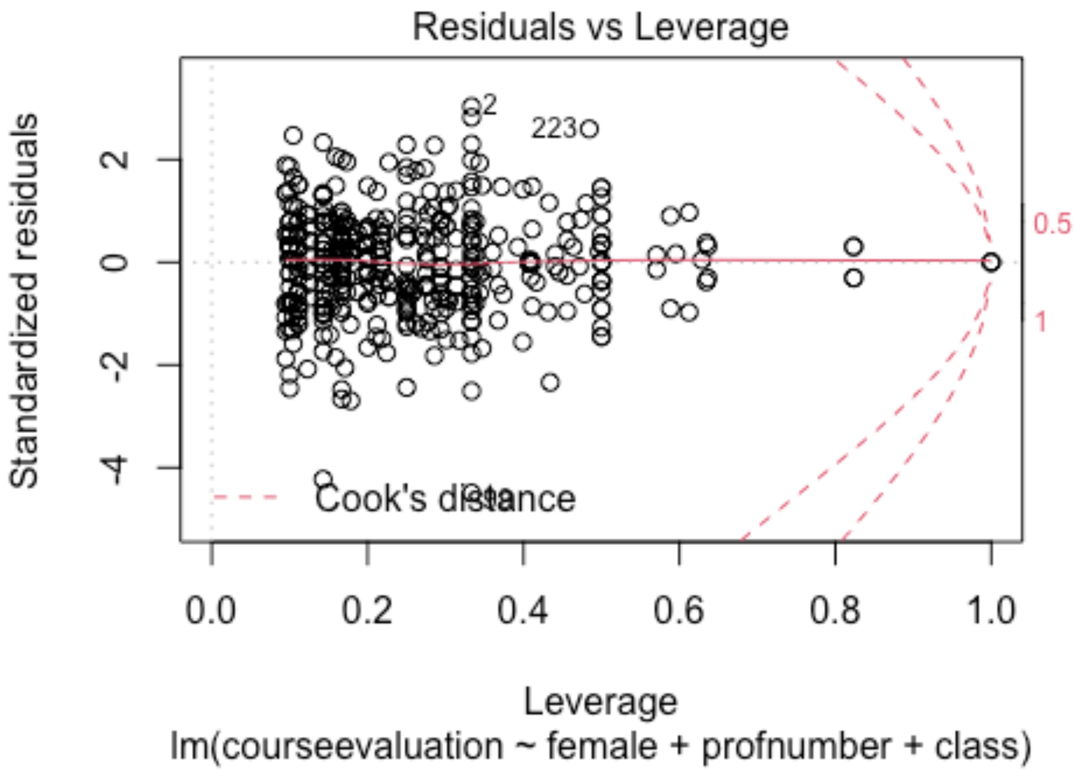


Residuals vs Fitted
Residuals
0.5
-0.5
-1.5
2
99
126
2.5
3.0
3.5
4.0
4.5
5.0
Fitted values
lm(courseevaluation ~ female + profnumber + class)


Normal Q-Q
Standardized residuals
Theoretical Quantiles
lm(courseevaluation ~ female + profnumber + class)
2
0
-2
-4
-3
-2
-1
0
1
2
3
99
126
2



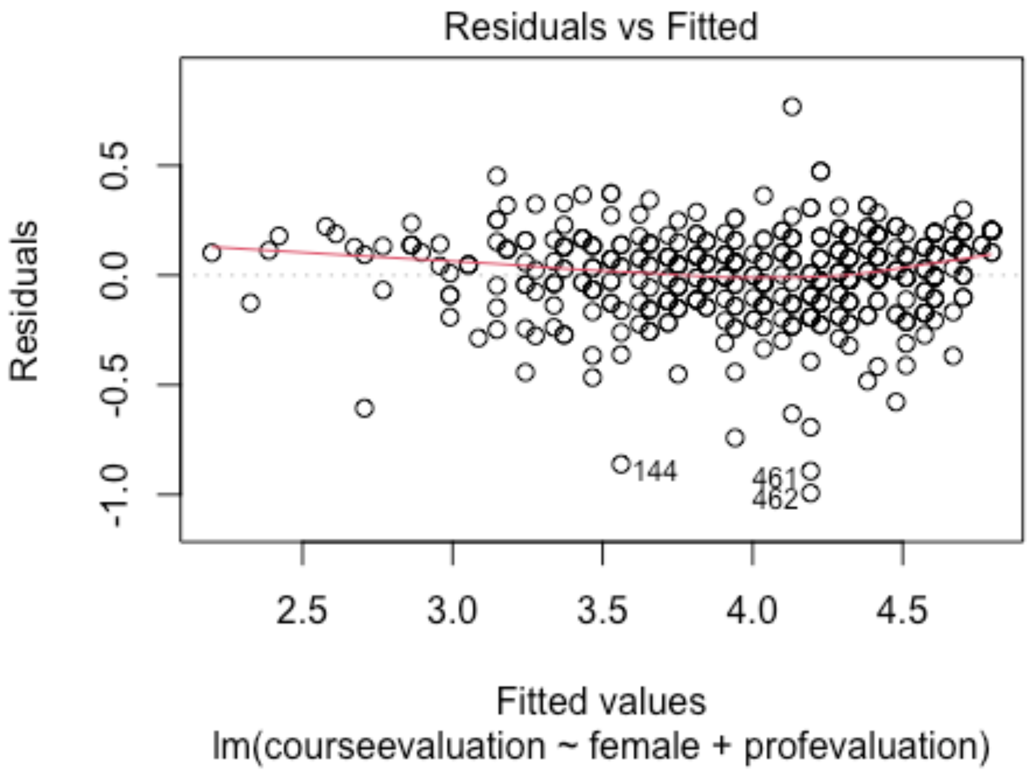

```
## Warning in sqrt(crit * p * (1 - hh)/hh): NaNs produced

## Warning in sqrt(crit * p * (1 - hh)/hh): NaNs produced
```

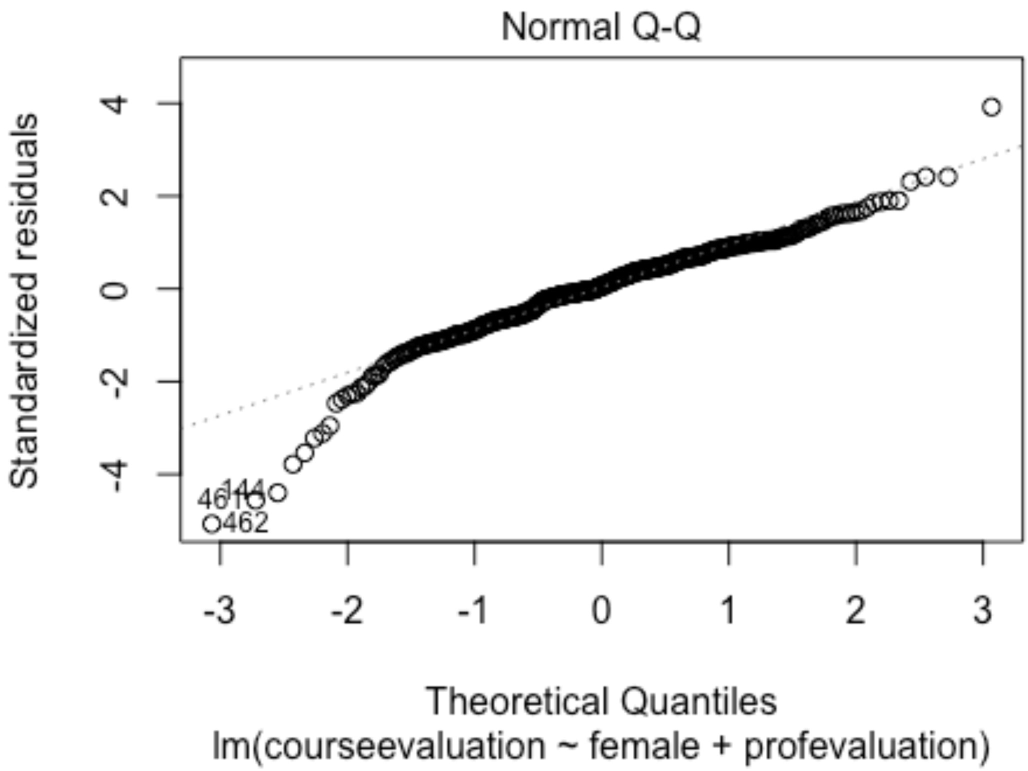


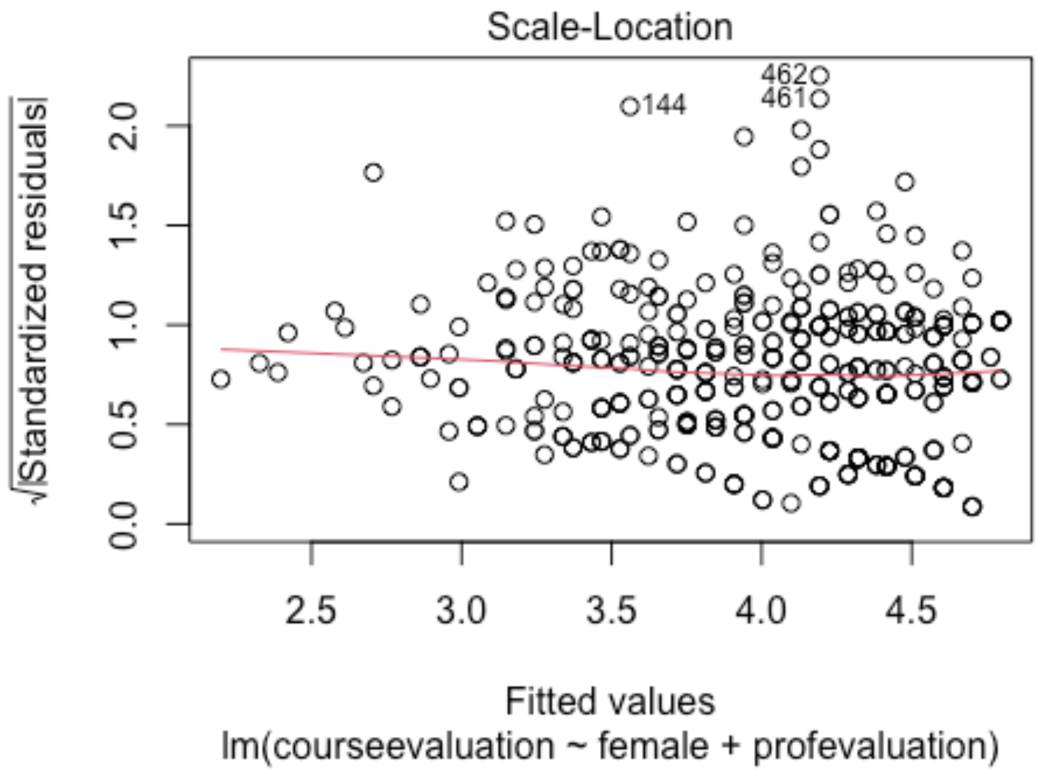

```
lmod2 <- lm(courseevaluation ~ female + profevaluation, data = df)
summary(lmod2)

##
## Call:
## lm(formula = courseevaluation ~ female + profevaluation, data = df)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.99287 -0.11464  0.01212  0.12865  0.76858
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)      0.04604    0.07272   0.633   0.5270
```

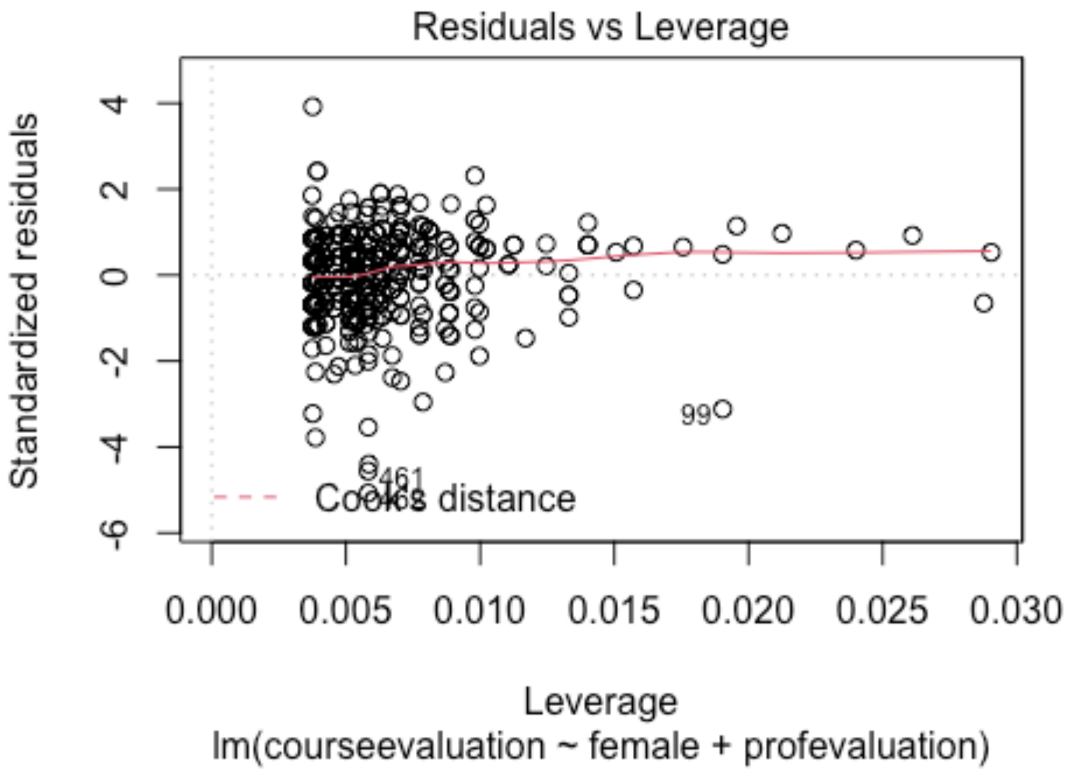
```
## female1        -0.03356    0.01864  -1.801   0.0724 .
## profevaluation  0.95009    0.01694  56.087   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1964 on 460 degrees of freedom
## Multiple R-squared:  0.8753, Adjusted R-squared:  0.8747
## F-statistic:  1614 on 2 and 460 DF,  p-value: < 2.2e-16
```

```
plot(lmod2)
```


Residuals vs Fitted
Residuals
0.5
0.0
-0.5
-1.0
144
461
462
2.5
3.0
3.5
4.0
4.5
Fitted values
lm(courseevaluation ~ female + profevaluation)


Normal Q-Q
Standardized residuals
4
2
0
-2
-4
461
144
462
-3
-2
-1
0
1
2
3
Theoretical Quantiles
lm(courseevaluation ~ female + profevaluation)

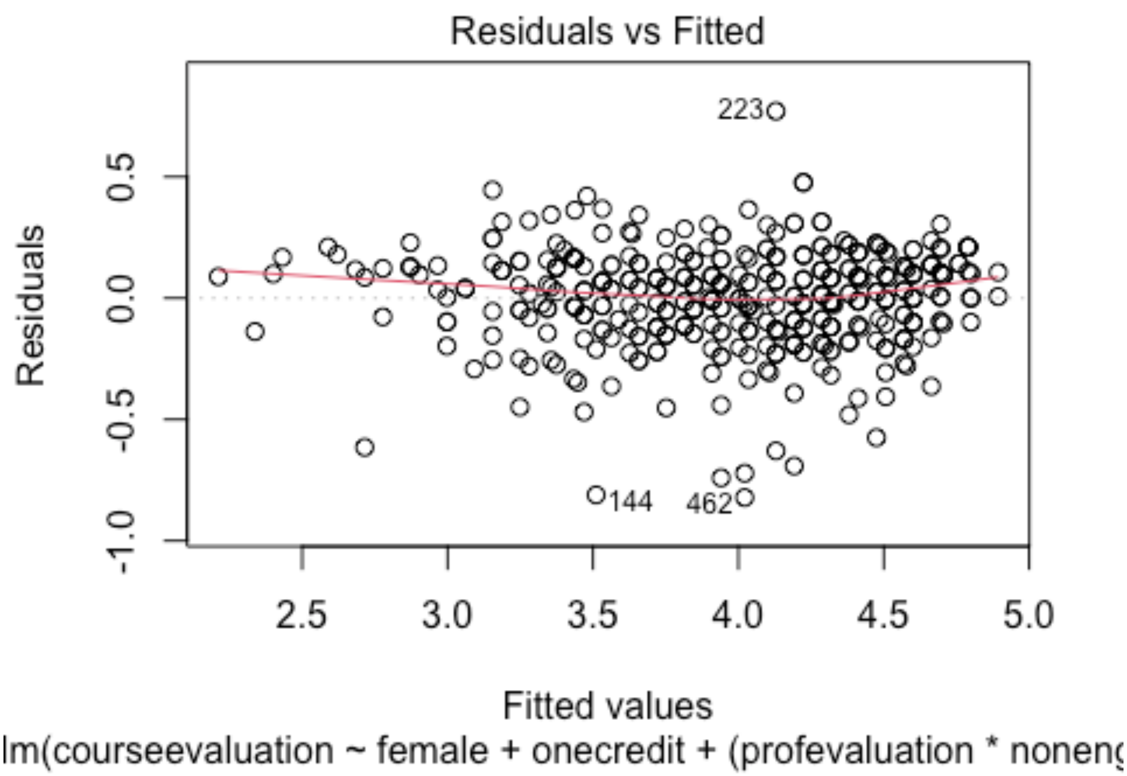

Scale-Location
√|Standardized residuals|
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
462
461
144
Fitted values
lm(courseevaluation ~ female + profevaluation)

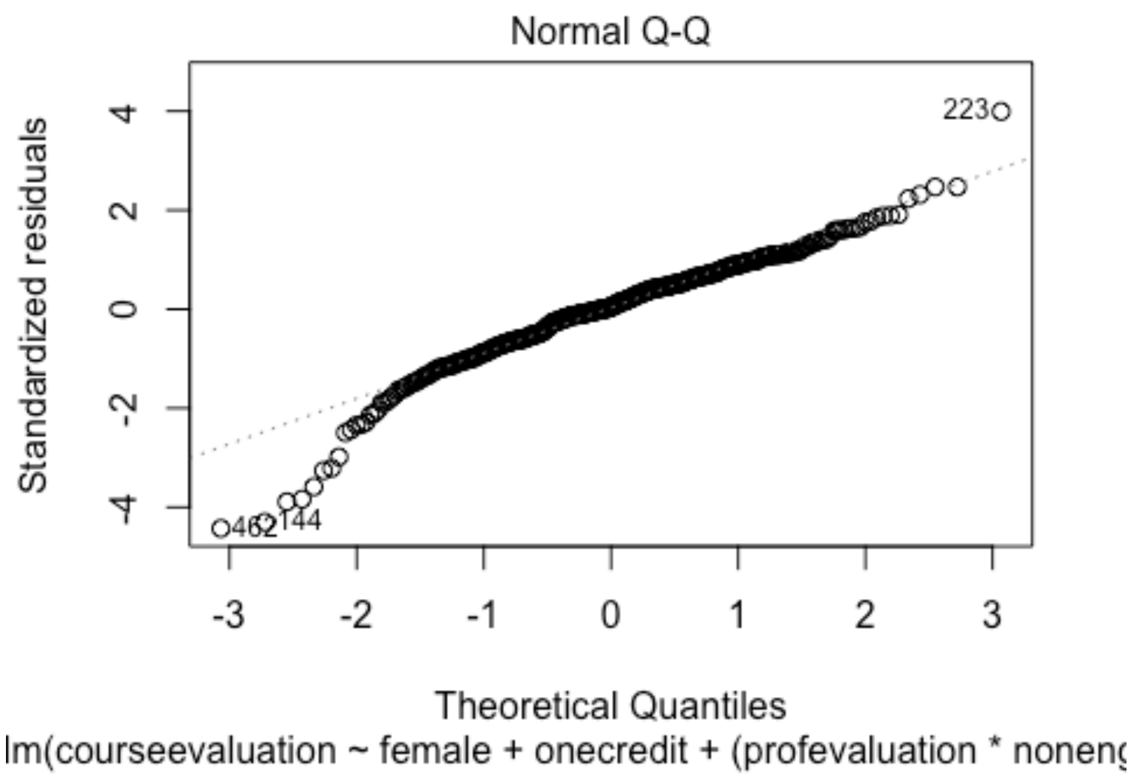


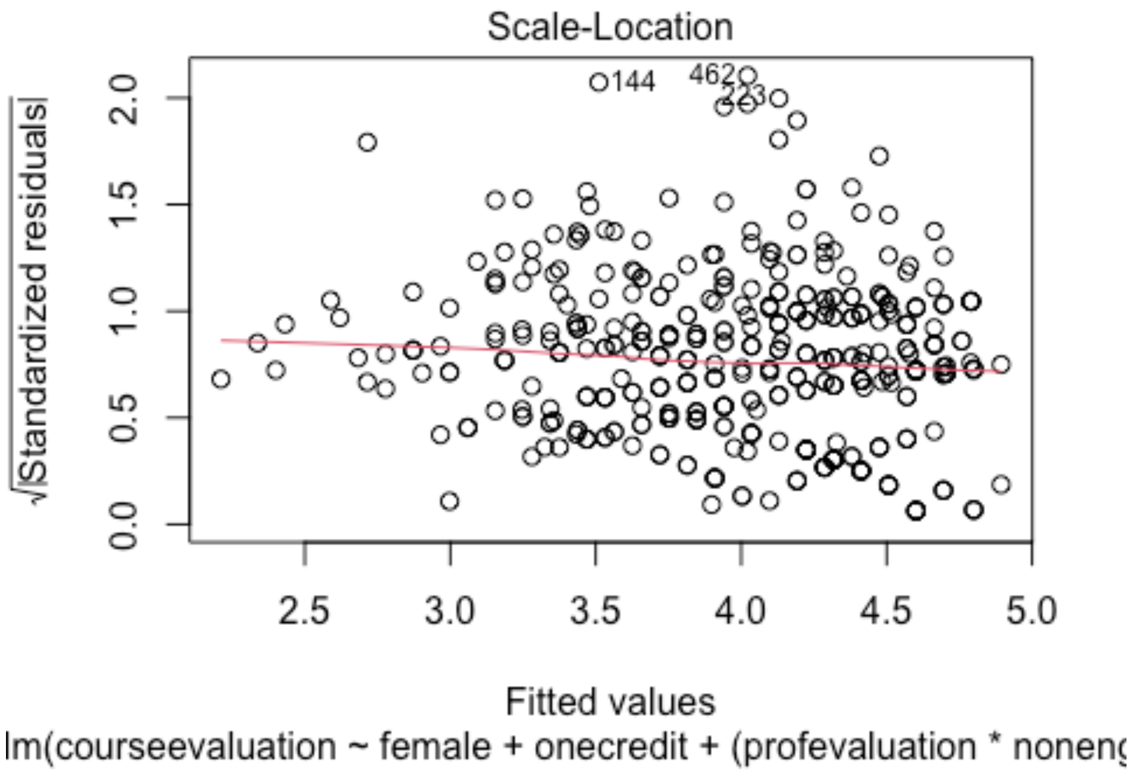

```
df$profevaluation <- (df$profevaluation - mean(df$profevaluation)) / (2 *
sd(df$profevaluation))

lmod3 <- lm(courseevaluation ~ female + onecredit + (profevaluation *
nonenglish), data = df)
summary(lmod3)

##
## Call:
## lm(formula = courseevaluation ~ female + onecredit + (profevaluation *
##     nonenglish), data = df)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
```

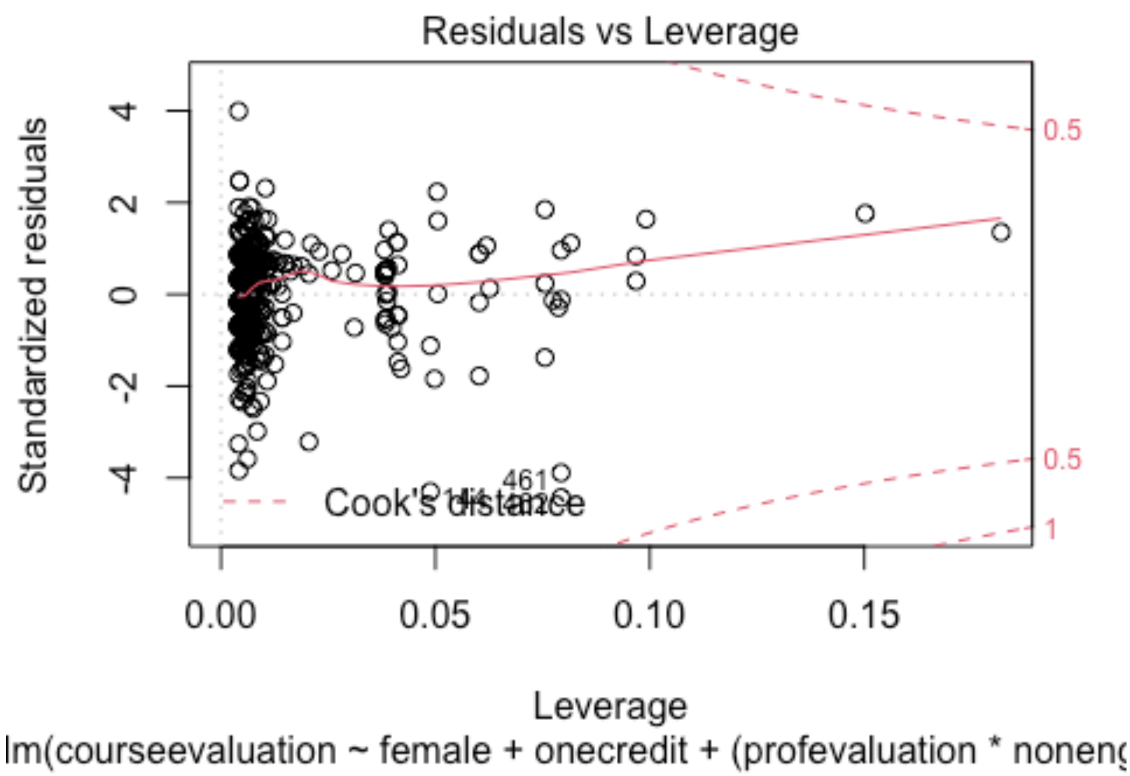
```
## -0.82174 -0.11348  0.00804  0.12507  0.77070
##
## Coefficients:
##                           Estimate Std. Error t value Pr(>|t|)
## (Intercept)                4.01118    0.01234 325.144  < 2e-16 ***
## female1                   -0.03164    0.01838  -1.722  0.08581 .
## onecredit                  0.10406    0.03920   2.654  0.00822 **
## profevaluation             1.02568    0.01901  53.954  < 2e-16 ***
## nonenglish                -0.13237    0.04261  -3.106  0.00201 **
## profevaluation:nonenglish -0.18278    0.09500  -1.924  0.05496 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1934 on 457 degrees of freedom
## Multiple R-squared:  0.8798, Adjusted R-squared:  0.8785
## F-statistic: 669.3 on 5 and 457 DF,  p-value: < 2.2e-16
```

```
plot(lmod3)
```


Residuals vs Fitted
223
144
462
Residuals
0.5
0.0
-0.5
-1.0
2.5
3.0
3.5
4.0
4.5
5.0
Fitted values
lm(courseevaluation ~ female + onecredit + (profevaluation * noneng


Normal Q-Q
Standardized residuals
Theoretical Quantiles
lm(courseevaluation ~ female + onecredit + (profevaluation * noneng
223
462
144
4
2
0
-2
-4
-3
-2
-1
0
1
2
3


Scale-Location
144
462
223
√|Standardized residuals|
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Fitted values
lm(courseevaluation ~ female + onecredit + (profevaluation * noneng



(b)

based on the above three models, we can note that lmod3 provided the best fit, while maintaining normality and constant variance. Additionally, the predictors seems to provide the most significance.

Question 4

y-intercept:
$logit(0.27) = -0.9946$

Coefficient of earnings:
$logit(0.88) = -0.9946 + x_6\ 1.9924301646902063 = -0.9946 + x_6$
$$x = \frac{1.9924301646902063 + 0.9946}{6} = 0.4978$$

Equation:
$Pr(y = 1) = logit^{-1}(-0.9946 + (0.4978 * x_i))$

Question 5

(a)

```
require(arm)
require(foreign)
require(ggplot2)

data <- read.csv("/Users/Home/Documents/Michael_Ghattas/School/CU_Boulder/
2022/Spring 2022/STAT - 4400/Data/hvs02_sorted.csv")

df = na.omit(data) # removing NA values
df$race <- factor(df$race, labels = c("White (non-hispanic)", "Black (non-
hispanic)", "Puerto Rican", "Other Hispanic", "Asian/Pacific Islander",
"Native", "Mixed"))

df$unitflr2 <- as.factor(df$unitflr2)
df$numunits <- as.factor(df$numunits)
df$stories <- as.factor(df$stories)
df$extwin4_2 <- as.factor(df$extwin4_2)
df$extflr5_2 <- as.factor(df$extflr5_2)
df$borough <- factor(df$borough, labels = c("Bronx", "Brooklyn", "Manhattan",
"Queens", "Staten Island"))
df$cd <- as.factor(df$cd)
df$intcrack2 <- as.factor(df$intcrack2)
df$inthole2 <- as.factor(df$inthole2)
df$intleak2 <- as.factor(df$intleak2)
df$intpeel_cat <- as.factor(df$intpeel_cat)
df$help <- as.factor(df$help)
df$old <- as.factor(df$old)
```

```
df$dilap <- as.factor(df$dilap)
df$regext <- as.factor(df$regext)
df$poverty <- as.factor(df$poverty)
df$povertyx2 <- as.factor(df$povertyx2)
df$housing <- factor(df$housing, labels = c("public", "rent controlled",
"owned"))
df$board2 <- as.factor(df$board2)
df$subsidy <- as.factor(df$subsidy)
df$under6 <- as.factor(df$under6)

df$hispanic_Mean = (df$hispanic_Mean * 10)
df$black_Mean = (df$black_Mean * 10)

lmod1 <- glm(rodent2 ~ race + hispanic_Mean + black_Mean, data = df)
summary(lmod1)

##
## Call:
## glm(formula = rodent2 ~ race + hispanic_Mean + black_Mean, data = df)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -0.5462  -0.3731  -0.1487   0.5437   0.9001
##
## Coefficients:
##                              Estimate Std. Error t value Pr(>|t|)
## (Intercept)                  0.089800   0.011249   7.983 1.67e-15 ***
## raceBlack (non-hispanic)     0.168594   0.018363   9.181  < 2e-16 ***
## racePuerto Rican             0.169905   0.020204   8.410  < 2e-16 ***
## raceOther Hispanic           0.232621   0.018062  12.879  < 2e-16 ***
## raceAsian/Pacific Islander 0.133452   0.021525   6.200 5.99e-10 ***
## raceNative                   0.133106   0.124816   1.066   0.2863
## raceMixed                    0.152896   0.067555   2.263   0.0236 *
## hispanic_Mean                0.025349   0.003122   8.119 5.53e-16 ***
## black_Mean                   0.015715   0.002717   5.783 7.66e-09 ***
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 0.1994306)
##
##     Null deviance: 1478.8  on 6777  degrees of freedom
## Residual deviance: 1349.9  on 6769  degrees of freedom
## AIC: 8318
##
## Number of Fisher Scoring iterations: 2
```

Intercept:
An apartment where white (non-Hispanic) people live, situated in an area with average black and hispanic population, has probability 6.79% of having rodent infestation in the building

Race:
We can notice the coefficients for all level are positive and statistically significant, with the only exception of Natives in particular, if anything else is hold at the average point, apartments where Hispanic, 29.75% more likely, and Puerto-Rican, 25% more likely, live have a higher chance to be in building infested by rodents.

hispanic_Mean:
10% increase in Hispanic presence in the district is associated with a 4.75% increase in probability that the building is infested by rodents.

black_Mean:
A flat occupied by whites, with average Hispanic presence in the district, is 2.75% more likely to be infested if the ratio of black people living in the district is 10% higher.

(b)

```
lmod2 <- glm(rodent2 ~ race + hispanic_Mean + black_Mean + borough + old +
housing + personrm + struct + foreign, data = df)
summary(lmod2)
```

```
##
## Call:
## glm(formula = rodent2 ~ race + hispanic_Mean + black_Mean + borough +
##     old + housing + personrm + struct + foreign, data = df)
##
## Deviance Residuals:
```

```
##     Min       1Q   Median       3Q      Max
## -0.9566  -0.3188  -0.1341   0.3923   1.1825
##
## Coefficients:
##                              Estimate Std. Error t value Pr(>|t|)
## (Intercept)                 -0.029120   0.030554  -0.953   0.3406
## raceBlack (non-hispanic)     0.164515   0.017910   9.185  < 2e-16 ***
## racePuerto Rican             0.173350   0.020017   8.660  < 2e-16 ***
## raceOther Hispanic           0.156429   0.019080   8.199 2.88e-16 ***
## raceAsian/Pacific Islander   0.052994   0.022439   2.362   0.0182 *
## raceNative                   0.125151   0.118562   1.056   0.2912
## raceMixed                    0.112955   0.064179   1.760   0.0785 .
## hispanic_Mean                0.019574   0.003610   5.422 6.10e-08 ***
## black_Mean                   0.006378   0.002721   2.344   0.0191 *
## boroughBrooklyn              0.086505   0.018729   4.619 3.93e-06 ***
## boroughManhattan             0.070004   0.017552   3.988 6.73e-05 ***
## boroughQueens               -0.005357   0.019635  -0.273   0.7850
## boroughStaten Island         0.034708   0.035763   0.970   0.3318
## old1                         0.073131   0.012119   6.034 1.68e-09 ***
## housingrent controlled       0.172833   0.020278   8.523  < 2e-16 ***
## housingowned                 0.103665   0.020099   5.158 2.57e-07 ***
## personrm                     0.101100   0.012726   7.944 2.27e-15 ***
## struct                      -0.210232   0.011743 -17.903  < 2e-16 ***
## foreign                      0.050231   0.012435   4.039 5.42e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 0.1795936)
##
##     Null deviance: 1478.8  on 6777  degrees of freedom
## Residual deviance: 1213.9  on 6759  degrees of freedom
## AIC: 7617.9
##
## Number of Fisher Scoring iterations: 2
```

Intercept:
a public flat occupied by whites and owned by a non-foreign born individual, located in the Bronx borough in a district of average black and Hispanic presence, and an average number of persons per room, has a probability of 6.18% to be in a building infested by rodents.

race:
A non white race has a higher probability to be associated with a building infested by rodents.

Hispanic_Mean:
A 10% increase in Hispanic population in the district is associated with 3.25% more likelihood to live in a building infested by rodents.

black_Mean:
A 10% increase in black population in the district is associated with a 1.5% higher probability to live in a building infested by rodents.

borough:
Brooklyn and Manhattan have the highest probability to rats infestations, and Queens and Staten Island don't differ from Bronx.

old:
Buildings built before 1947 have 9% more likely to have rodent infestations.

housing:
Privately owned apartments are -6.50% more likely to have rodent infestations.

personrm:
Higher the number of people per room leads to higher the chances of rodent infestations.

struct:
Good or excellent building structure have less chance of having a rodent infestations.

foreign:
Foreign-born owners tend to possess apartments located in buildings 5% more likely to be infested by rodents.

Question 6

(a)

```
require(arm)
require(ggplot2)
```

```r
require(foreign)

data <- read.table("/Users/Home/Documents/Michael_Ghattas/School/CU_Boulder/
2022/Spring 2022/STAT - 4400/Data/wells.dat")
df = na.omit(data) # removing NA values
head(df)

##   switch arsenic   dist assoc educ
## 1      1    2.36 16.826     0    0
## 2      1    0.71 47.322     0    0
## 3      0    2.07 20.967     0   10
## 4      1    1.15 21.486     0   12
## 5      1    1.10 40.874     1   14
## 6      1    3.90 69.518     1    9

df$logArsenic <- log(df$arsenic)
lmod1 <- glm(switch ~ (dist * logArsenic), data = df)
summary(lmod1)

##
## Call:
## glm(formula = switch ~ (dist * logArsenic), data = df)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -1.0058  -0.4949   0.2456   0.4274   0.8136
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)      0.6138520  0.0155893  39.376  < 2e-16 ***
## dist            -0.0020308  0.0002994  -6.782 1.42e-11 ***
## logArsenic       0.2140817  0.0234125   9.144  < 2e-16 ***
## dist:logArsenic -0.0003792  0.0004054  -0.935     0.35
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 0.2275286)
```

```
##
##     Null deviance: 737.94  on 3019  degrees of freedom
## Residual deviance: 686.23  on 3016  degrees of freedom
## AIC: 4105.3
##
## Number of Fisher Scoring iterations: 2
```

Intercept:
a person with an average distance from a well with clean water and average logArsenic has a 62.01% probability to switch wells.

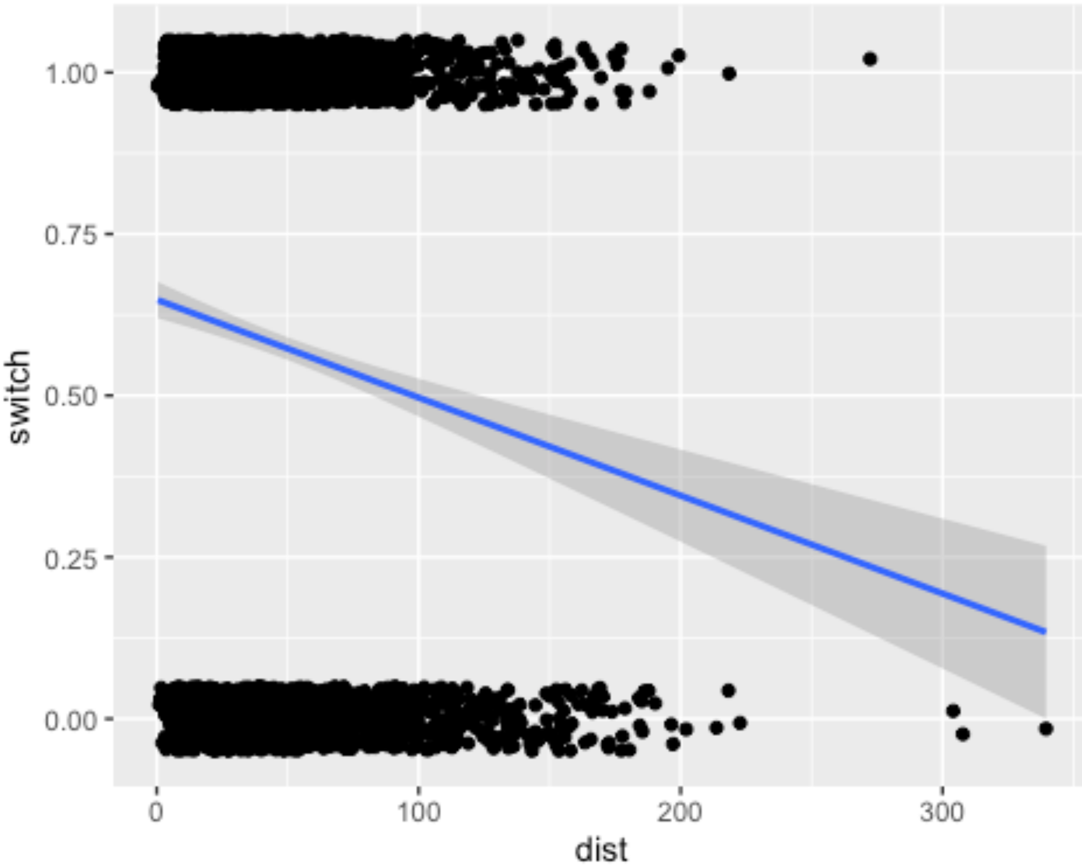
dist:
a one meter increase in distance from a well with safe water has a decreasing the probability of switching wells by -0.25%.

logArsenic:
A 10% increase in arsenic corresponds in a difference in the expected probability of switching well of 9.34%$.

dist:log.arsenic:
Insignificant, exclude it from next model.

(b)

```
ggplot(data = df, aes(x = dist, y = switch)) +
  geom_jitter(position = position_jitter(height = .05)) +
  geom_smooth(method = "glm")

## `geom_smooth()` using formula 'y ~ x'
```



(c)

```
b <- coef(lmod1)
hi <- 100
lo <- 0
delta <- invlogit(b[1] + (b[2] * hi) + (b[3] * df$logArsenic + (b[4] *
df$logArsenic * hi)) - invlogit(b[1] + (b[2] * lo) + (b[3] * df$logArsenic) +
(b[4] * df$logArsenic * lo)))
mean(delta)

## [1] 0.4509107
```

Households that are 100 meters from the nearest safe well are 45% more likely to switch.

II

```
b <- coef(lmod1)
hi <- 200
lo <- 100
delta <- invlogit(b[1] + (b[2] * hi) + (b[3] * df$logArsenic) + (b[4] *
df$logArsenic * hi)) - invlogit(b[1] + (b[2] * lo) + (b[3] * df$logArsenic) +
(b[4] * df$logArsenic * lo))
mean(delta)

## [1] -0.05180368
```

5% less likely to switch.

III

```
b <- coef(lmod1)
lo <- 0.5
delta <- invlogit(b[1] + (b[2] * df$dist) + (b[3] * hi) + (b[4] * df$dist *
hi)) - invlogit(b[1] + (b[2] * df$dist) + (b[3] * lo) + (b[4] * df$dist *
lo))
mean(delta)

## [1] 0.3514743
```

35% more likely to switch.

IIV

```
b <- coef(lmod1)
hi <- 2.0
lo <- 1.0
delta <- invlogit(b[1] + (b[2] * df$dist) + (b[3] * hi) + (b[4] * df$dist *
hi)) - invlogit(b[1] + (b[2] * df$dist) + (b[3] * lo) + (b[4] * df$dist *
lo))
mean(delta)

## [1] 0.04153164
```

4% more likely to switch.