[STAT 4400] HW-1

Michael Ghattas

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Question-1

$$n = 3$$
$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i = \frac{1+2+3}{3} = \frac{6}{3} = 2$$
$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{2+0+4}{3} = \frac{6}{3} = 2$$

(a)

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$
$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} (y_i - \bar{y})(x_i - \bar{x})}{\sum_{i=1}^{n} (x_i - \bar{x})^2}$$

$$\hat{\sigma}^2 = \frac{1}{n-2}\sum_{i=1}^{n} (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2$$

(b)

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{3} (y_i - 2)(x_i - 2)}{\sum_{i=1}^{3} (x_i - 2)^2} = \frac{(-1+0+1)(0-2+2)}{(0)^2 + (-2)^2 + (2)^2} = \frac{(0)(0)}{4+4} = \frac{0}{8} = 0$$
$$\hat{\beta}_0 = 2 - (0)\bar{x} = 2 - 0 = 2$$

$$\hat{\sigma}^2 = \frac{1}{3-2}\sum_{i=1}^{3} (y_i - 2 - (0)x_i)^2 = \frac{1}{1}\sum_{i=1}^{3} (y_i - 2)^2 = 1[(-1)^2 + (0)^2 + (1)^2] = 1 + 0 + 1 = 2$$

(c)

$$\hat{\epsilon}_i = y_i - \hat{y}_i = y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i = y_i - 2 - 0 = y_i - 2$$

$$SSE = \sum_{i=1}^{3} \hat{\epsilon}_i^{\,2} = (1-2)^2 + (2-2)^2 + (3-2)^2 = (-1)^2 + (0)^2 + (1)^2 = 1 + 0 + 1 = 2$$

$$\hat{y}_i = \hat{\beta}_0 - \hat{\beta}_1 x_i + \hat{\epsilon}_i = 2 - 0 + \hat{\epsilon}_i = 2 + \hat{\epsilon}_i$$

$$SSR = \sum_{i=1}^{3} (\hat{y}_i - 2)^2 = ((2 + (1-2)) - 2)^2 + ((2 + (2-2)) - 2)^2 + ((2 + (3-2)) - 2)^2 = ((2-1) - 2)^2 + ((2-0) - 2)^2 + ((2+1) - 2)^2 = (1-2)^2 + (2-2)^2 + (3-2)^2 = (-1)^2 + (0)^2 + (1)^2 = 1 + 0 + 1 = 2$$

$$SST = \sum_{i=1}^{3} (y_i - 2)^2 = (1-2)^2 + (2-2)^2 + (3-2)^2 = (-1)^2 + (0)^2 + (1)^2 = 1 + 0 + 1 = 2$$

$$R^2 = 1 - \frac{SSE}{SST} = \frac{2}{2} = 1$$

(d)

$$H_0 : \beta_1 = 0$$

$$H_1 : \beta_1 \neq 0$$

$$\beta_1{}^* = 0$$

$$\hat{\beta}_1 \sim \mathrm{N}(0, \tau)$$

$$\hat{\tau} = \sqrt{\frac{\sigma^2}{\sum_{i=1}^{3} (x_i - \bar{x})^2}} = \sqrt{\frac{2}{\sum_{i=1}^{3} (x_i - 2)^2}} = \sqrt{\frac{2}{(1-2)^2 + (2-2)^2 + (3-2)^2}} = \sqrt{\frac{2}{(-1)^2 + (0)^2 + (1)^2}} = \sqrt{\frac{2}{1+0+1}} = \sqrt{\frac{2}{2}} = \sqrt{1} = 1$$

$$\hat{t} = \frac{\hat{\beta}_1}{\hat{\tau}}$$

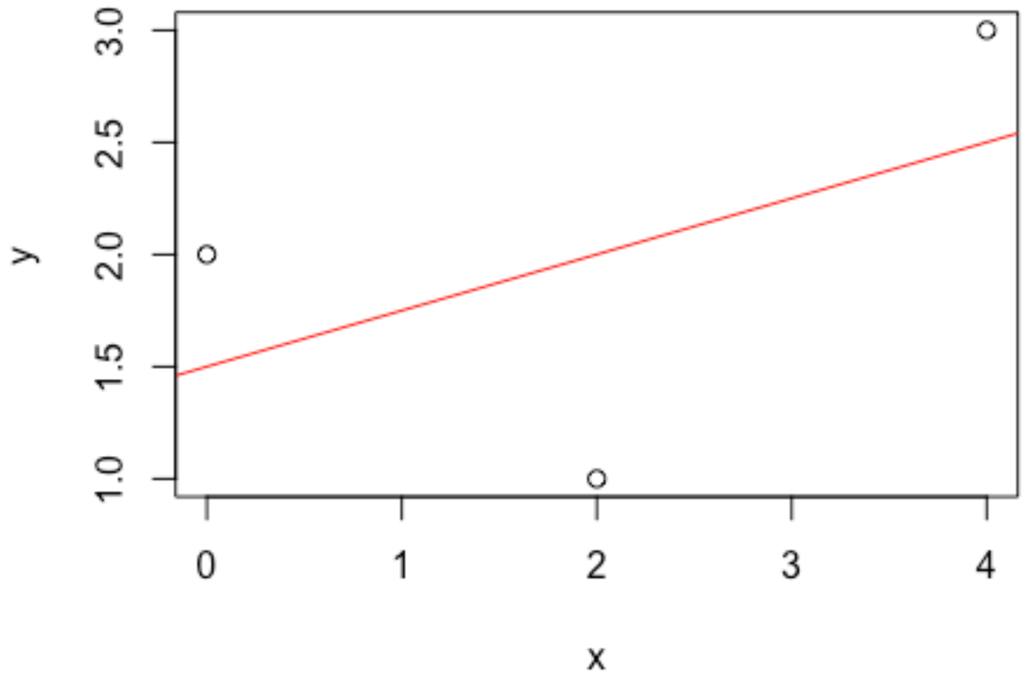
$$p[\,|t| \geq t^*] = \alpha$$

```
y = c(1 , 2 , 3)
x = c(2 , 0 , 4)
alpha = 0.5
n = length(x)

beta.0.hat = 2
beta.1.hat = 0
```

```
sigma2.hat = 2
std.t = dt(x , (n - 2))
t.star = qt((1 - alpha / 2) , df = (n - 2))

y.hat = beta.0.hat + (beta.1.hat * x)
eps.hat = y - y.hat
SSE = sum(eps.hat ** 2)
sigma2.hat = SSE / (n - 2)
tau.hat = sqrt(sigma2.hat / sum((x - mean(x)) ** 2))
t.hat = beta.1.hat / tau.hat

t.star

## [1] 1

t.hat

## [1] 0

print("(|t| < t*) Thus we fail to reject the the null-hypothesis")

## [1] "(|t| < t*) Thus we fail to reject the the null-hypothesis"
```

(e)

```
lmod = lm(y ~ x)
plot(x, y)
abline(lmod, col = "red")
```



Question-2

$0 \le x \le 50\ \overline{x} = 35\ \sigma_x = 10$

$y = a + bx\ \overline{y} = a + b\overline{x} = a + 35b$

$\sigma_y = 15\ \overline{y} = 100$

$\sigma_y = |b|\sigma_x = 10|b|\ 15 = 10|b| \to |b| = \dfrac{15}{10} = 1.5 \to b = \pm 1.5$

$a + 35b = 100 \to a + 35(\pm 1.5) = 100$
$a + 35(-1.5) = 100 \to a - 52.5 = 100 \to a = 100 + 52.5 = 152.5$
$a + 35(1.5) = 100 \to a + 52.5 = 100 \to a = 100 - 52.5 = 47.5$

(a)

$$y = 152.5 - 1.5x$$

(b)

$$y = 47.5 + 1.5x$$

(c)

I would recommend the solution from part (a), as it has an increasing function that exhibits a positive correlation.

Question-3

(a)

```
set.seed(123)

var1 = rnorm(1000, mean = 0, sd = 1)
var2 = rnorm(1000, mean = 0, sd = 1)

lmod1 = lm(var1 ~ var2)
summary(lmod1)

##
## Call:
## lm(formula = var1 ~ var2)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.7168 -0.6290 -0.0060  0.6451  3.2383
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.01252    0.03129   0.400  0.68909
## var2         0.08494    0.03097   2.742  0.00621 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## Residual standard error: 0.9885 on 998 degrees of freedom
## Multiple R-squared:  0.007479,   Adjusted R-squared:  0.006484
## F-statistic:  7.52 on 1 and 998 DF,  p-value: 0.006211

print("Yes, based on the p-value generated we can conclude that the slope coefficient var2 is statistically significant.")

## [1] "Yes, based on the p-value generated we can conclude that the slope coefficient var2 is statistically significant."
```

(b)

```
set.seed(321)

z.scores <- rep (NA, 100)

for (k in 1:100)
{
  var1 <- rnorm (1000 ,0 ,1)
  var2 <- rnorm (1000 ,0 ,1)
  fit <- lm (var2  ~ var1)
  z.scores[k] <- coef(fit )[2] / summary(fit)$coef[2,"Std. Error"]
}

alpha = .05
cutoffn = qnorm((1 - alpha) / 2, lower.tail=TRUE)
sum(abs(z.scores) > cutoffn)

## [1] 100

for (k in 1:100)
{
  result = sum(abs(z.scores) < 1.96)
}

result

## [1] 95
```

```
print("95 estimated slope coefficients are statistically significant at the α = .05 level of significance.")
```

```
## [1] "95 estimated slope coefficients are statistically significant at the α = .05 level of significance."
```

Question-4

(a)

```
library(haven)
data <- read_dta("/Users/Home/Documents/Michael_Ghattas/School/CU_Boulder/2022/Spring 2022/STAT - 4400/HW/1/child.iq.dta")
head(data)
```

```
## # A tibble: 6 × 3
##    ppvt educ_cat momage
##   <dbl>    <dbl>  <dbl>
## 1   120        2     21
## 2    89        1     17
## 3    78        2     19
## 4    42        1     20
## 5   115        4     26
## 6    97        1     20
```

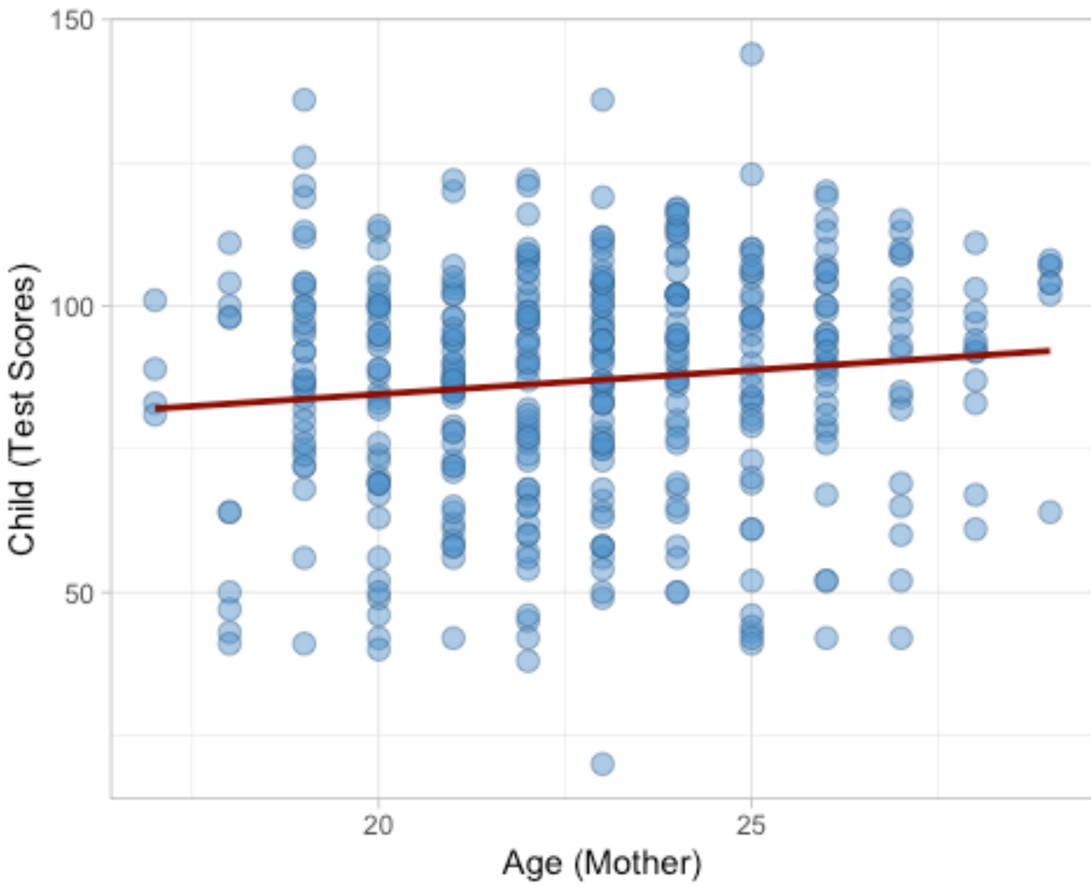
```
lmod = lm(ppvt ~ momage, data = data)
summary(lmod)
```

```
##
## Call:
## lm(formula = ppvt ~ momage, data = data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -67.109 -11.798   2.971  14.860  55.210
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  67.7827     8.6880   7.802 5.42e-14 ***
```

```
## momage        0.8403     0.3786   2.219    0.027 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 20.34 on 398 degrees of freedom
## Multiple R-squared:  0.01223,    Adjusted R-squared:  0.009743
## F-statistic: 4.926 on 1 and 398 DF,  p-value: 0.02702
```

```
library(ggplot2)
ggplot(data, aes(momage, ppvt)) +
geom_point(shape = 21, color="steelblue4", fill="steelblue3", size = 3,
alpha=0.5,show.legend = FALSE) +
theme_light() + xlab("Age (Mother)") + ylab("Child (Test Scores)") +
geom_smooth(method = lm, color="darkred", se=FALSE)
```

```
## `geom_smooth()` using formula 'y ~ x'
```




```
print("Based on our summary and plots, it seems that the mother's age is a
significant predictor, though not enough on its own to find a direct
correlation. That being said, the plot clearly shows that the majority of the
observations with a higher test score belong to the mothers in their late
20's. Thus mothers should give birth in their late 20s.")
```

```
## [1] "Based on our summary and plots, it seems that the mother's age is a
significant predictor, though not enough on its own to find a direct
correlation. That being said, the plot clearly shows that the majority of the
observations with a higher test score belong to the mothers in their late
20's. Thus mothers should give birth in their late 20s."
```

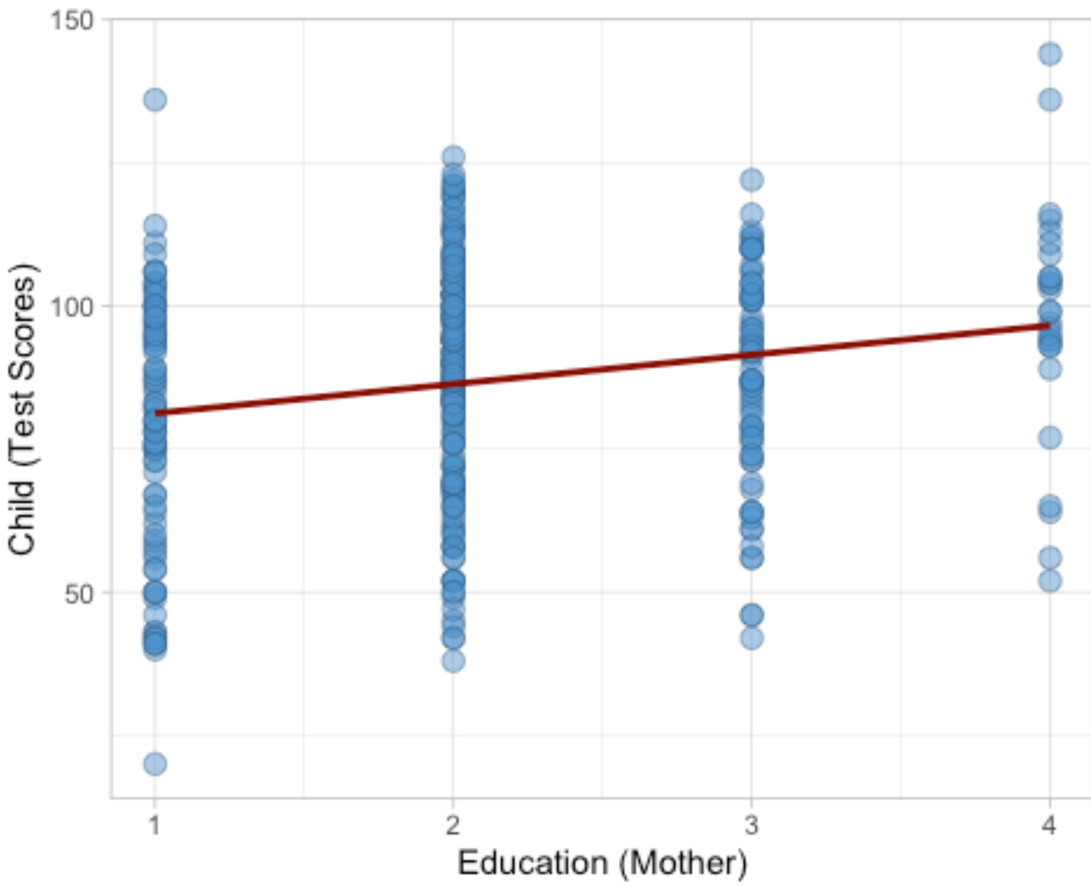
(b)

```
lmod = lm(ppvt ~ momage + educ_cat, data = data)
summary(lmod)
```

```
##
## Call:
## lm(formula = ppvt ~ momage + educ_cat, data = data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -61.763 -13.130   2.495  14.620  55.610
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  69.1554     8.5706   8.069 8.51e-15 ***
## momage        0.3433     0.3981   0.862 0.389003
## educ_cat      4.7114     1.3165   3.579 0.000388 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 20.05 on 397 degrees of freedom
## Multiple R-squared:  0.04309,    Adjusted R-squared:  0.03827
## F-statistic: 8.939 on 2 and 397 DF,  p-value: 0.0001594
```

```
library(ggplot2)
ggplot(data, aes(educ_cat, ppvt)) +
geom_point(shape = 21, color="steelblue4", fill="steelblue3", size = 3,
alpha=0.5,show.legend = FALSE) +
theme_light() + xlab("Education (Mother)") + ylab("Child (Test Scores)") +
geom_smooth(method = lm, color="darkred", se=FALSE)
```

```
## `geom_smooth()` using formula 'y ~ x'
```

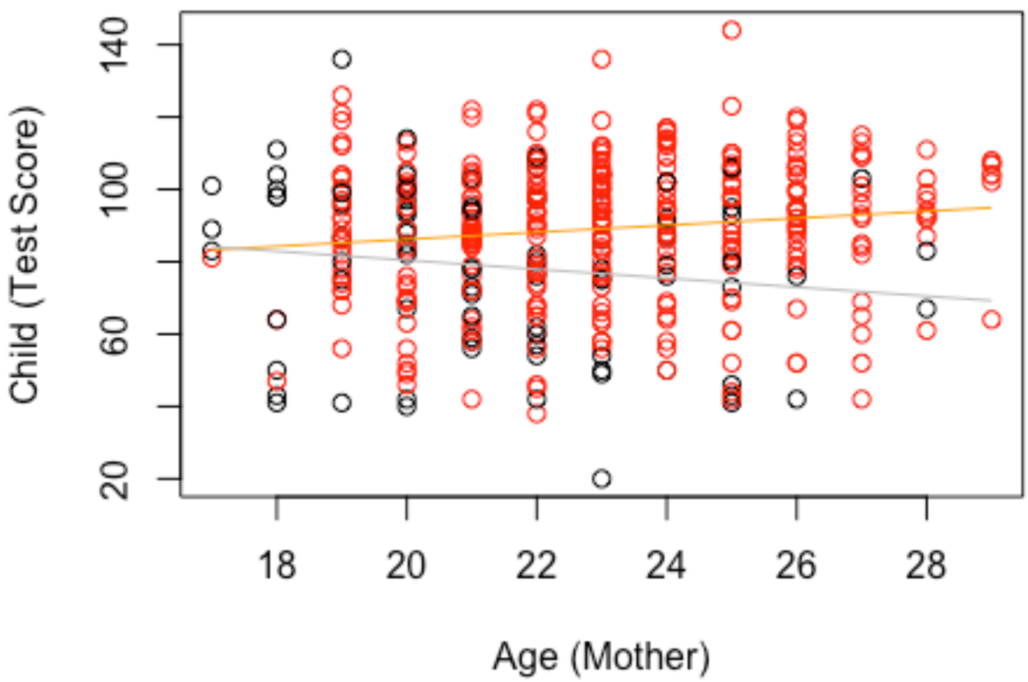



```
print("Based on our summary and plots, it seems that the mother's education is a strong and significant predictor. That being said, the plot clearly shows that the majority of the observations with a higher test score belong to the mothers having completed a high-school education.")

## [1] "Based on our summary and plots, it seems that the mother's education is a strong and significant predictor. That being said, the plot clearly shows that the majority of the observations with a higher test score belong to the mothers having completed a high-school education."
```

(c)

```
data$mom.hs <- ifelse(data$educ_cat >= 2, 1, 0)

lmod <- lm(ppvt ~ (mom.hs * momage), data = data)
summary(lmod)
```

```
##
## Call:
## lm(formula = ppvt ~ (mom.hs * momage), data = data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -56.696 -12.407   2.022  14.804  54.343
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)   105.2202    17.6454   5.963 5.49e-09 ***
## mom.hs        -38.4088    20.2815  -1.894   0.0590 .
## momage         -1.2402     0.8113  -1.529   0.1271
## mom.hs:momage   2.2097     0.9181   2.407   0.0165 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 19.85 on 396 degrees of freedom
## Multiple R-squared:  0.06417,    Adjusted R-squared:  0.05708
## F-statistic: 9.051 on 3 and 396 DF,  p-value: 8.276e-06

spread <- ifelse(data$mom.hs == 1, "red", "black")
plot(data$momage, data$ppvt, xlab = "Age (Mother)", ylab = "Child (Test
Score)", col = spread, pch = 1)
curve(cbind(1, 1, x, 1 * x) %*% coef(lmod), add = TRUE, col = "orange")
# Mother finished hs
curve(cbind(1, 0, x, 0 * x) %*% coef(lmod), add = TRUE, col = "grey")
# Mother did not finish hs
```

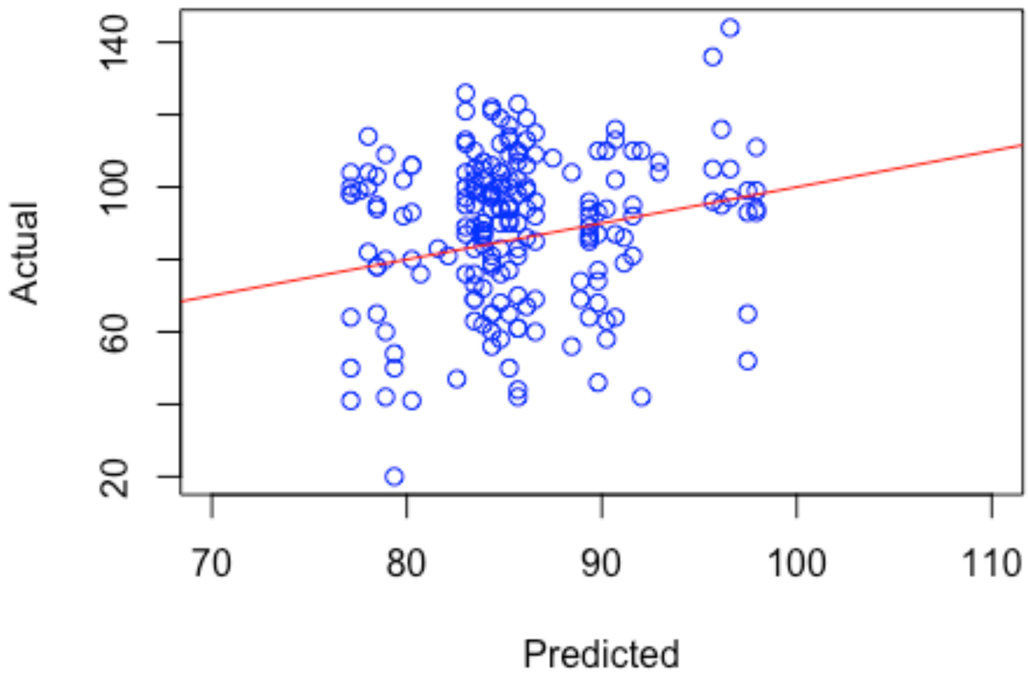


(d)

```
lmod <- lm(ppvt ~ momage + educ_cat, data = data[1:200, ])
summary(lmod)

##
## Call:
## lm(formula = ppvt ~ momage + educ_cat, data = data[1:200, ])
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -46.358 -12.967   2.866  14.435  58.428
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
```

```
## (Intercept)  63.6295    11.8202   5.383 2.07e-07 ***
## momage         0.4473     0.5516   0.811  0.41836
## educ_cat       5.4434     1.8228   2.986  0.00318 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 19.58 on 197 degrees of freedom
## Multiple R-squared:  0.06199,    Adjusted R-squared:  0.05246
## F-statistic: 6.509 on 2 and 197 DF,  p-value: 0.001831
```

```r
pmod <- predict(lmod, data[201:400, ])
plot(pmod, data$ppvt[201:400], xlim = c(70, 110), xlab = "Predicted",
    ylab = "Actual", col = "blue")
abline(a = 0, b = 1, col = "red")
```



Question-5

(a)

```
library(faraway)
data(prostate)
head(prostate)

##       lcavol lweight age      lbph svi      lcp gleason pgg45     lpsa
## 1 -0.5798185  2.7695  50 -1.386294   0 -1.38629       6     0 -0.43078
## 2 -0.9942523  3.3196  58 -1.386294   0 -1.38629       6     0 -0.16252
## 3 -0.5108256  2.6912  74 -1.386294   0 -1.38629       7    20 -0.16252
## 4 -1.2039728  3.2828  58 -1.386294   0 -1.38629       6     0 -0.16252
## 5  0.7514161  3.4324  62 -1.386294   0 -1.38629       6     0  0.37156
## 6 -1.0498221  3.2288  50 -1.386294   0 -1.38629       6     0  0.76547

lmod = lm(log(lpsa) ~ log(lcavol) + ., data = prostate)

## Warning in log(lpsa): NaNs produced

## Warning in log(lcavol): NaNs produced

summary(lmod)

##
## Call:
## lm(formula = log(lpsa) ~ log(lcavol) + ., data = prostate)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.42557 -0.12542  0.02419  0.19314  0.51973
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.228166   0.665962  -0.343  0.73286
## log(lcavol) -0.073779   0.082178  -0.898  0.37221
## lcavol       0.298653   0.093642   3.189  0.00209 **
## lweight      0.132774   0.086027   1.543  0.12700
## age         -0.008582   0.005988  -1.433  0.15596
## lbph         0.068535   0.030029   2.282  0.02535 *
```

```
## svi          0.247254   0.116662   2.119  0.03741 *
## lcp         -0.049417   0.044109  -1.120  0.26619
## gleason      0.086846   0.075419   1.152  0.25323
## pgg45        0.001981   0.002088   0.948  0.34605
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3323 on 74 degrees of freedom
##   (13 observations deleted due to missingness)
## Multiple R-squared:  0.4948, Adjusted R-squared:  0.4334
## F-statistic: 8.053 on 9 and 74 DF,  p-value: 3.254e-08
```

```
confint(lmod, 'log(lcavol)', level = 0.95)
```

```
##                  2.5 %     97.5 %
## log(lcavol) -0.2375227 0.08996445
```

(b)

```
lmod = lm(log(lpsa) ~ log(lcavol) + lcavol + lbph + svi, data = prostate)
```

```
## Warning in log(lpsa): NaNs produced
```

```
## Warning in log(lcavol): NaNs produced
```

```
summary(lmod)
```

```
##
## Call:
## lm(formula = log(lpsa) ~ log(lcavol) + lcavol + lbph + svi, data =
prostate)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.48196 -0.12939  0.05611  0.22173  0.48638
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.35496    0.12566   2.825 0.005987 **
## log(lcavol) -0.07497    0.08041  -0.932 0.354038
```

```
## lcavol       0.29829    0.08713   3.423 0.000983 ***
## lbph         0.07834    0.02645   2.961 0.004043 **
## svi          0.24564    0.10428   2.356 0.020976 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3384 on 79 degrees of freedom
##   (13 observations deleted due to missingness)
## Multiple R-squared:  0.4407, Adjusted R-squared:  0.4124
## F-statistic: 15.56 on 4 and 79 DF,  p-value: 1.98e-09
```

```
confint(lmod, 'log(lcavol)', level = 0.95)
```

```
##                   2.5 %     97.5 %
## log(lcavol) -0.2350253 0.08509177
```

(c)

The model from part (a) has a slightly better fit, as the R^2 value is slightly higher, indicating a better fitted model.