[STAT 4400] Exam-2

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Problem 1

```
library (foreign)
library(arm)
library(cdlTools)
require(ggplot2)
require(GGally)
require(reshape2)
require(lme4)
require(compiler)
require(parallel)
require(boot)
require(lattice)
library(gridExtra)
library(grid)
library(dplyr)


frisk = read.table(file = "/Users/Home/Documents/Michael_Ghattas/School/
CU_Boulder/BA-BS/2022/Spring 2022/STAT - 4400/Data/frisk_with_noise.dat.txt",
header=TRUE, skip = 6)
head(frisk)

##   stops  pop past.arrests precinct eth crime
## 1    75 1720          191        1   1     1
## 2    36 1720           57        1   1     2
## 3    74 1720          599        1   1     3
## 4    17 1720          133        1   1     4
## 5    37 1368           62        1   2     1
## 6    39 1368           27        1   2     2

dim(frisk)   # 900 x 6
```

```
## [1] 900   6

names(frisk)[3] <- "arrests"
attach(frisk)
```

(a)

```
n.precinct <- max (precinct)
n.eth <- max (eth)
n.crime <- max(crime)
dcjs <- log(arrests*15/12)

##  first let's aggregate
friskagg <- aggregate(cbind(stops, arrests) ~ precinct + eth, data=frisk,
sum)

## These give me the percentages I want
fr2 <- friskagg  %>%
  group_by(precinct, eth) %>%
  summarise(n = sum(stops)) %>%
  mutate(percentage = n / sum(n))

table2 = aggregate(fr2$percentage, list(fr2$eth) ,FUN = mean)
colnames(table2) <- c("eth","percbyprec")

head(fr2)

## # A tibble: 6 × 4
## # Groups:   precinct [2]
##   precinct   eth     n percentage
##      <int> <int> <int>      <dbl>
## 1        1     1   202      0.525
## 2        1     2   102      0.265
## 3        1     3    81      0.210
## 4        2     1   132      0.380
## 5        2     2   144      0.415
## 6        2     3    71      0.205

dim(fr2)  #225 x 4
```

```
## [1] 225   4

##  Now I want to classify the precincts


precinct.category.vec = ifelse(fr2$eth==1 & fr2$percentage <.1, 1,
       ifelse(fr2$eth==1 & fr2$percentage < .4, 2,
              ifelse(fr2$eth==1 & fr2$percentage <= 1, 3,NA)))
fr3 = as.data.frame(na.omit(cbind(fr2$precinct,precinct.category.vec)))

# length 900, or 12 of each precinct
fr12 = cbind(frisk, dcjs,  rep(as.vector(fr3[,2]), each=12) )
colnames(fr12)[8] = "precinct.category"
head(fr12)

##   stops  pop arrests precinct eth crime     dcjs precinct.category
## 1    75 1720     191        1   1     1 5.475417                 3
## 2    36 1720      57        1   1     2 4.266195                 3
## 3    74 1720     599        1   1     3 6.618405                 3
## 4    17 1720     133        1   1     4 5.113493                 3
## 5    37 1368      62        1   2     1 4.350278                 3
## 6    39 1368      27        1   2     2 3.518980                 3

##  USE THIS as model 15.1log(arrests) is an offset
M1 <- as.list (rep (NA, 12))
index <- 0
for (j in 1:3){
  for (k in 1:4){
    index <- index + 1
    ok <- fr12$precinct.category==j & fr12$crime==k & fr12$arrests > 0
    M1[[index]] <- glmer (stops ~ 1 + (1 | eth) + (1|precinct) ,
        offset = log(arrests),
        family=poisson(link=log), data=fr12, subset=ok)
  }}
```

```r
allbeta = rep(0,12)
alltheta = matrix(rep(0,24), nrow= 12, ncol = 2)
alleth = matrix(rep(0,36), nrow= 12, ncol = 3)
for(i in 1:12){
  allbeta[i] = M1[[i]]@beta
  alltheta[i,] = M1[[i]]@theta
  alleth[i,] = as.data.frame(coef(M1[[i]])$eth)[,1]
}

##  USE THIS as model 15.5 # log(arrests) is a predictor not the dispersion
factor...
M2 <- as.list (rep (NA, 12))
index <- 0
for (j in 1:3){
  for (k in 1:4){
    index <- index + 1
    ok <- fr12$precinct.category==j & fr12$crime==k & fr12$arrests > 0
    M2[[index]] <- glmer (stops ~ 1 + log(arrests) + (1 | eth) + (1|precinct)
,
    family=poisson(link=log), data=fr12, subset=ok)
  }}

allbeta2 = matrix(rep(0,24), nrow= 12, ncol = 2)
alltheta2 = matrix(rep(0,24), nrow= 12, ncol = 2)
alleth2 = matrix(rep(0,36), nrow= 12, ncol = 3)
#allu = matrix(rep(0,84), nrow= 12, ncol = 7)
for(i in 1:12){
  allbeta2[i,] = M2[[i]]@beta
  alltheta2[i,] = M2[[i]]@theta
  alleth2[i,] = as.data.frame(coef(M2[[i]])$eth)[,1]
}

theta <- cbind(alltheta,alltheta2)
beta <- cbind(allbeta,allbeta2)
eths <- cbind(alleth,alleth2)
```

```
theta

##            [,1]       [,2]      [,3]       [,4]
##  [1,] 0.3892011 0.37654090 0.4209755 0.40424925
##  [2,] 0.2664679 0.34623141 0.2948581 0.40571387
##  [3,] 0.2712174 0.25946858 0.4564906 0.04537502
##  [4,] 0.5822364 0.13231499 0.7313298 0.33265600
##  [5,] 0.4289294 0.29870804 0.4131146 0.29297015
##  [6,] 0.3977210 0.18106453 0.4038553 0.19267373
##  [7,] 1.0844688 0.29421901 1.0593928 0.30389517
##  [8,] 0.9335737 0.28506005 0.8137612 0.30315303
##  [9,] 0.5902343 0.46840143 0.5604335 0.49240802
## [10,] 0.5009774 0.32484906 0.4998344 0.34206010
## [11,] 0.9813271 0.13573197 0.9422604 0.09272197
## [12,] 0.7582749 0.02935469 0.7616550 0.03032263

beta

##          allbeta
##  [1,] -0.3981067 -0.59031096 1.0397312
##  [2,]  0.4062756 -0.09508297 1.1380006
##  [3,]  0.1723844 -1.98836933 1.5071079
##  [4,] -0.9176366 -1.39692357 1.1285541
##  [5,] -0.5877452  0.18942226 0.8570694
##  [6,]  0.6419945  0.90380730 0.9402201
##  [7,] -0.1473768  0.24470419 0.9220117
##  [8,] -1.4625660 -0.59750342 0.8419582
##  [9,] -1.0464958 -0.65380304 0.9312586
## [10,]  0.5103797  0.63081152 0.9740098
## [11,] -0.8384846 -0.42146479 0.9186169
## [12,] -1.8688607 -1.89669751 1.0046404

eths

##             [,1]        [,2]        [,3]       [,4]       [,5]       [,6]
##  [1,] -0.2551779 -0.06464272 -0.86857358 -0.4015183 -0.2601472 -1.1033444
##  [2,]  0.4890077  0.75652331 -0.01628958  0.1813071  0.1735764 -0.6286888
```

```
##  [3,] -0.1520131  0.31058394  0.36388634 -1.9908851 -1.9598823 -2.0142507
##  [4,] -0.8109567 -0.92381955 -1.01626489 -1.0138804 -1.5226347 -1.6458958
##  [5,] -0.2750959 -0.51532798 -0.96886518  0.4635086  0.3069800 -0.1982845
##  [6,]  0.7759285  0.74768765  0.40520727  1.0289981  1.0353146  0.6500497
##  [7,] -0.5046673  0.11047986 -0.04557899 -0.1244835  0.5179323  0.3431861
##  [8,] -1.6256891 -1.10042829 -1.65643764 -0.7903844 -0.2036841 -0.7921550
##  [9,] -0.5640511 -0.90522627 -1.66596023 -0.1343643 -0.5210262 -1.3017503
## [10,]  0.7697108  0.69974341  0.06469146  0.9109909  0.8232484  0.1612331
## [11,] -1.0010210 -0.74173643 -0.77052895 -0.5248653 -0.3418552 -0.3964415
## [12,] -1.8860651 -1.84191328 -1.87833755 -1.9162100 -1.8689557 -1.9046459
```

```
M <- cbind(M1, M2)
M
```

```
##       M1
##  [1,] <S4 class 'glmerMod' [package "lme4"] with 13 slots>
##  [2,] <S4 class 'glmerMod' [package "lme4"] with 13 slots>
##  [3,] <S4 class 'glmerMod' [package "lme4"] with 13 slots>
##  [4,] <S4 class 'glmerMod' [package "lme4"] with 13 slots>
##  [5,] <S4 class 'glmerMod' [package "lme4"] with 13 slots>
##  [6,] <S4 class 'glmerMod' [package "lme4"] with 13 slots>
##  [7,] <S4 class 'glmerMod' [package "lme4"] with 13 slots>
##  [8,] <S4 class 'glmerMod' [package "lme4"] with 13 slots>
##  [9,] <S4 class 'glmerMod' [package "lme4"] with 13 slots>
## [10,] <S4 class 'glmerMod' [package "lme4"] with 13 slots>
## [11,] <S4 class 'glmerMod' [package "lme4"] with 13 slots>
## [12,] <S4 class 'glmerMod' [package "lme4"] with 13 slots>
##       M2
##  [1,] <S4 class 'glmerMod' [package "lme4"] with 13 slots>
##  [2,] <S4 class 'glmerMod' [package "lme4"] with 13 slots>
##  [3,] <S4 class 'glmerMod' [package "lme4"] with 13 slots>
##  [4,] <S4 class 'glmerMod' [package "lme4"] with 13 slots>
##  [5,] <S4 class 'glmerMod' [package "lme4"] with 13 slots>
##  [6,] <S4 class 'glmerMod' [package "lme4"] with 13 slots>
##  [7,] <S4 class 'glmerMod' [package "lme4"] with 13 slots>
##  [8,] <S4 class 'glmerMod' [package "lme4"] with 13 slots>
```

```
## [9,] <S4 class 'glmerMod' [package "lme4"] with 13 slots>
## [10,] <S4 class 'glmerMod' [package "lme4"] with 13 slots>
## [11,] <S4 class 'glmerMod' [package "lme4"] with 13 slots>
## [12,] <S4 class 'glmerMod' [package "lme4"] with 13 slots>

library(sjPlot) #for plotting lmer and glmer mods

## Registered S3 method overwritten by 'parameters':
##   method                           from
##   format.parameters_distribution datawizard

help(sjPlot)
tab_model(M1, show.re.var= TRUE, dv.labels= "OVERDISPERSED POISSON REGRESSION
OF POLICE STOPS")
```

OVERDISPERSED POISSON REGRESSION OF POLICE STOPS

Predictors

Incidence Rate Ratios

CI

p

Incidence Rate Ratios

CI

p

Incidence Rate Ratios

CI

p

Incidence Rate Ratios

CI

p

Incidence Rate Ratios

CI

p

Incidence Rate Ratios

CI

p

Incidence Rate Ratios

CI

p

Incidence Rate Ratios

CI

p

Incidence Rate Ratios

CI

p

Incidence Rate Ratios

CI

p

Incidence Rate Ratios

CI

p

Incidence Rate Ratios

CI

p

(Intercept)

0.67

0.38 – 1.20

0.177

1.50

0.93 – 2.43

0.098

1.19

0.79 – 1.78

0.403

0.40

0.21 – 0.74

0.004

0.56

0.38 – 0.81

0.002

1.90

1.46 – 2.47

<0.001

0.86

0.50 – 1.49

0.598

0.23

0.14 – 0.38

<0.001

0.35

0.20 – 0.61

<0.001

1.67

1.12 – 2.47

0.011

0.43

0.31 – 0.60

<0.001

0.15

0.12 – 0.19

<0.001

Random Effects

σ^2

2.01

2.01

2.01

2.01

2.01

2.01

2.01

2.01

2.01

2.01

2.01

2.01

τ_{00}

0.15 precinct

0.07 precinct

0.07 precinct

0.34 precinct

0.18 precinct

0.16 precinct

1.18 precinct

0.87 precinct

0.35 precinct

0.25 precinct

0.96 precinct

0.57 precinct

0.14 eth

0.12 eth

0.07 eth

0.02 eth

0.09 eth

0.03 eth

0.09 eth

0.08 eth

0.22 eth

0.11 eth

0.02 eth

0.00 eth

ICC

0.13

0.09

0.07

0.15

0.12

0.09

0.39

0.32

0.22

0.15

0.33

0.22

N

3 eth

3 eth

3 eth

3 eth

3 eth

3 eth

3 eth

3 eth

3 eth

3 eth

3 eth

3 eth

4 precinct

4 precinct

4 precinct

4 precinct

24 precinct

24 precinct

24 precinct

24 precinct

47 precinct

47 precinct

47 precinct

47 precinct

Observations

12

12

12

12

72

72

72

72

141

141

140

141

Marginal R2 / Conditional R2

0.000 / 0.127

0.000 / 0.087

0.000 / 0.065

0.000 / 0.150

0.000 / 0.120

0.000 / 0.087

0.000 / 0.386

0.000 / 0.321

0.000 / 0.220

0.000 / 0.150

0.000 / 0.328

0.000 / 0.222

```
tab_model(M2, show.re.var= TRUE, dv.labels= "OVERDISPERSED POISSON REGRESSION
OF POLICE STOPS")
```

OVERDISPERSED POISSON REGRESSION OF POLICE STOPS

Predictors

Incidence Rate Ratios

CI

p

Incidence Rate Ratios

CI

p

Incidence Rate Ratios

CI

p

Incidence Rate Ratios

CI

p

Incidence Rate Ratios

CI

p

Incidence Rate Ratios

CI

p

Incidence Rate Ratios

CI

p

Incidence Rate Ratios

CI

p

Incidence Rate Ratios

CI

p

Incidence Rate Ratios

CI

p

Incidence Rate Ratios

CI

p

Incidence Rate Ratios

CI

p

(Intercept)

0.55

0.24 – 1.30

0.176

0.91

0.45 – 1.82

0.789

0.14

0.06 – 0.31

<0.001

0.25

0.07 – 0.91

0.036

1.21

0.77 – 1.89

0.409

2.47

1.77 – 3.44

<0.001

1.28

0.70 – 2.32

0.423

0.55

0.30 – 1.02

0.060

0.52

0.28 – 0.96

0.037

1.88

1.23 – 2.88

0.004

0.66

0.46 – 0.94

0.020

0.15

0.11 – 0.21

<0.001

arrests [log]

2.83

2.51 – 3.19

<0.001

3.12

2.78 – 3.51

<0.001

4.51

3.87 – 5.26

<0.001

3.09

2.38 – 4.02

<0.001

2.36

2.25 – 2.47

<0.001

2.56

2.45 – 2.67

<0.001

2.51

2.40 – 2.64

<0.001

2.32

2.16 – 2.50

<0.001

2.54

2.45 – 2.63

<0.001

2.65

2.59 – 2.71

<0.001

2.51

2.41 – 2.61

<0.001

2.73

2.62 – 2.84

<0.001

Random Effects

σ2

0.02

0.02

0.02

0.02

0.02

0.02

0.02

0.02

0.02

0.02

0.02

0.02

τ00

0.18 precinct

0.09 precinct

0.21 precinct

0.53 precinct

0.17 precinct

0.16 precinct

1.12 precinct

0.66 precinct

0.31 precinct

0.25 precinct

0.89 precinct

0.58 precinct

0.16 eth

0.16 eth

0.00 eth

0.11 eth

0.09 eth

0.04 eth

0.09 eth

0.09 eth

0.24 eth

0.12 eth

0.01 eth

0.00 eth

ICC

0.94

0.92

0.91

0.97

0.92

0.90

0.98

0.97

0.96

0.94

0.98

0.96

N

3 eth

3 eth

3 eth

3 eth

3 eth

3 eth

3 eth

3 eth

3 eth

3 eth

3 eth

3 eth

4 precinct

4 precinct

4 precinct

4 precinct

24 precinct

24 precinct

24 precinct

24 precinct

47 precinct

47 precinct

47 precinct

47 precinct

Observations

12

12

12

12

72

72

72

72

141

141

140

141

Marginal R2 / Conditional R2

0.849 / 0.991

0.912 / 0.993

0.914 / 0.992

0.862 / 0.995

0.573 / 0.966

0.729 / 0.973

0.278 / 0.987

0.550 / 0.987

0.689 / 0.988

0.789 / 0.988

0.610 / 0.991

0.764 / 0.991

(b)

The advantage of using the level of past arrests as an offset rather than a linear predictor is the reduction of bias in terms of our model and arrests. Since past arrests are taken into consideration as an offset for the model instead of a predictor of outcome.

Problem 2

```
library(arm)
library(ggplot2)
library(RColorBrewer)
library(reshape)
library(wesanderson)
library(gridExtra)
library(grid)
hiv.dataf <- read.csv ("/Users/Home/Documents/Michael_Ghattas/School/
CU_Boulder/BA-BS/2022/Spring 2022/STAT - 4400/Data/allvar.csv")
head(hiv.dataf)

##   VISIT newpid      VDATE CD4PCT arv   visage treatmnt CD4CNT baseage
## 1     1      1  6/29/1988     18   0 3.910000        1    323    3.91
## 2     4      1  1/19/1989     37   0 4.468333        1    610    3.91
## 3     7      1  4/13/1989     13   0 4.698333        1    324    3.91
## 4    10      1                NA   0 5.005000        1     NA    3.91
## 5    13      1 11/30/1989     13   0 5.330833        1    626    3.91
## 6    16      1                NA  NA       NA        1    220    3.91

dim(hiv.dataf)  # 1254 x 9

## [1] 1254    9
```

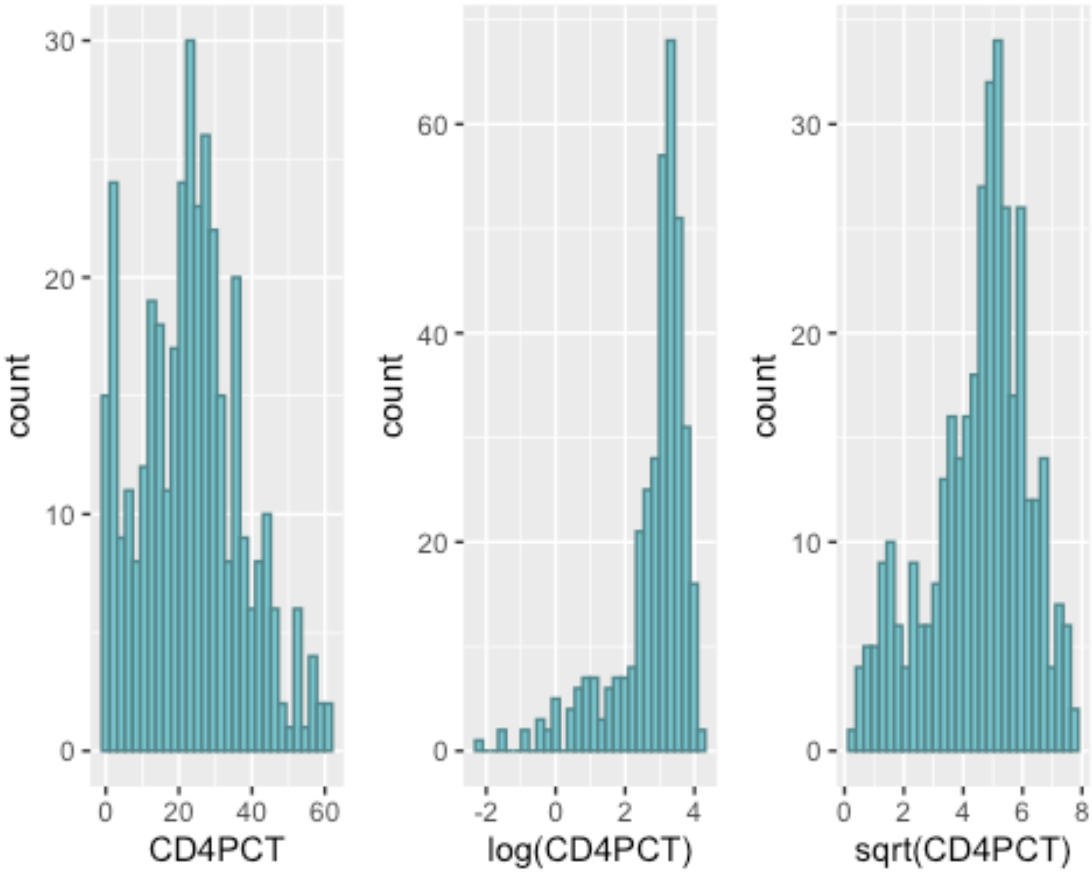
```
table(hiv.dataf$treatmnt)

##
##   1   2
## 675 579

summary(hiv.dataf$treatmnt)

##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   1.000   1.000   1.000   1.462   2.000   2.000
```

(a)

```
attach(hiv.dataf)
ok <- treatmnt==1 & !is.na(CD4PCT) & (baseage>1 & baseage<5)& !is.na(baseage)
table(ok)  # 369 meet the criteria

## ok
## FALSE  TRUE
##   885   369

hiv.data = (hiv.dataf[ok,])
head(hiv.data)

##    VISIT newpid      VDATE CD4PCT arv   visage treatmnt CD4CNT baseage
## 1      1      1  6/29/1988     18   0 3.910000        1    323  3.9100
## 2      4      1  1/19/1989     37   0 4.468333        1    610  3.9100
## 3      7      1  4/13/1989     13   0 4.698333        1    324  3.9100
## 5     13      1 11/30/1989     13   0 5.330833        1    626  3.9100
## 7     19      1   6/7/1990     12   1 5.848333        1    220  3.9100
## 17     1      4  6/23/1988     30   0 2.302500        1   1021  2.3025

dim(hiv.data)  # 369 x 9

## [1] 369   9

attach(hiv.data)

## The following objects are masked from hiv.dataf:
##
##     arv, baseage, CD4CNT, CD4PCT, newpid, treatmnt, VDATE, visage,
##     VISIT
```

```
p1 = ggplot(hiv.data, aes(x=CD4PCT))+
  geom_histogram(color="cadetblue4", fill="cadetblue3")

p2 = ggplot(hiv.data, aes(x=log(CD4PCT)))+
  geom_histogram(color="cadetblue4", fill="cadetblue3")

p3 = ggplot(hiv.data, aes(x=sqrt(CD4PCT)))+
  geom_histogram(color="cadetblue4", fill="cadetblue3")

grid.arrange(p1,p2,p3, ncol=3)
```

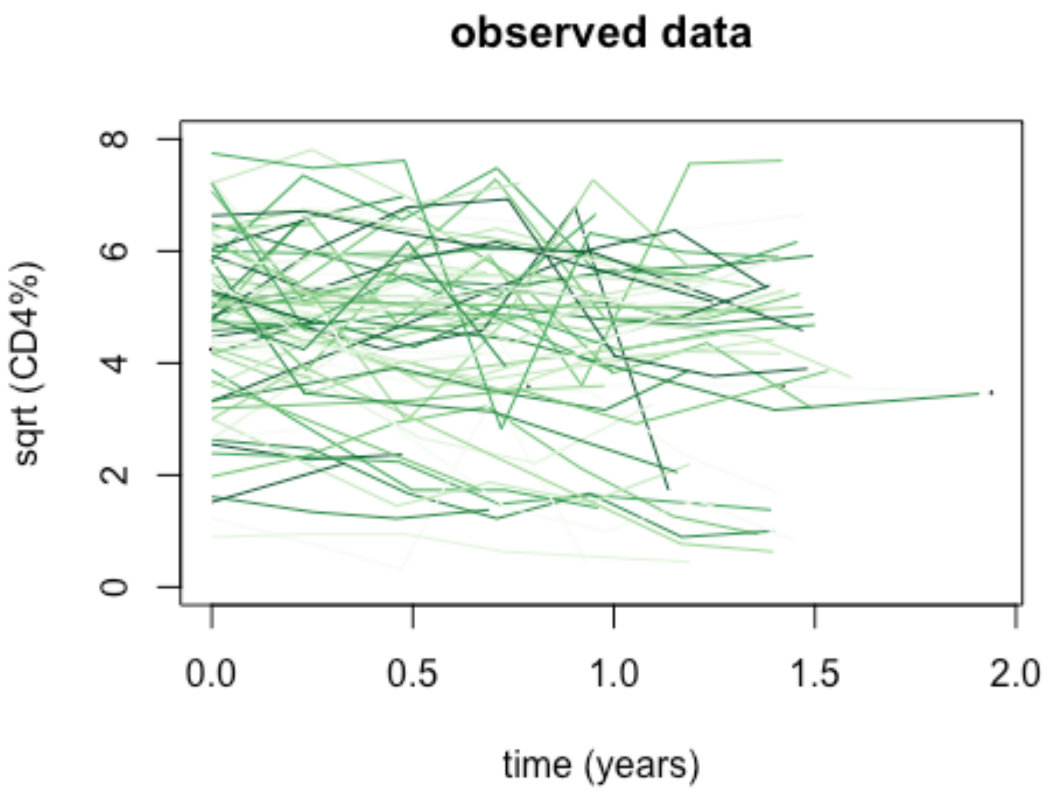
```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.

## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```



```r
## Redefining variables
y <- sqrt (CD4PCT)              # we are using the square root of the
percentage
age.baseline <- baseage         # kid's age (yrs) at the beginning of the
study
age.measurement <- visage       # kids age (yrs) at the time of measurement
treatment <- treatmnt
time <- visage - baseage

length(unique (hiv.data$newpid))   # there are 83 patients in the dataset of
length 369
```

```
## [1] 83
```

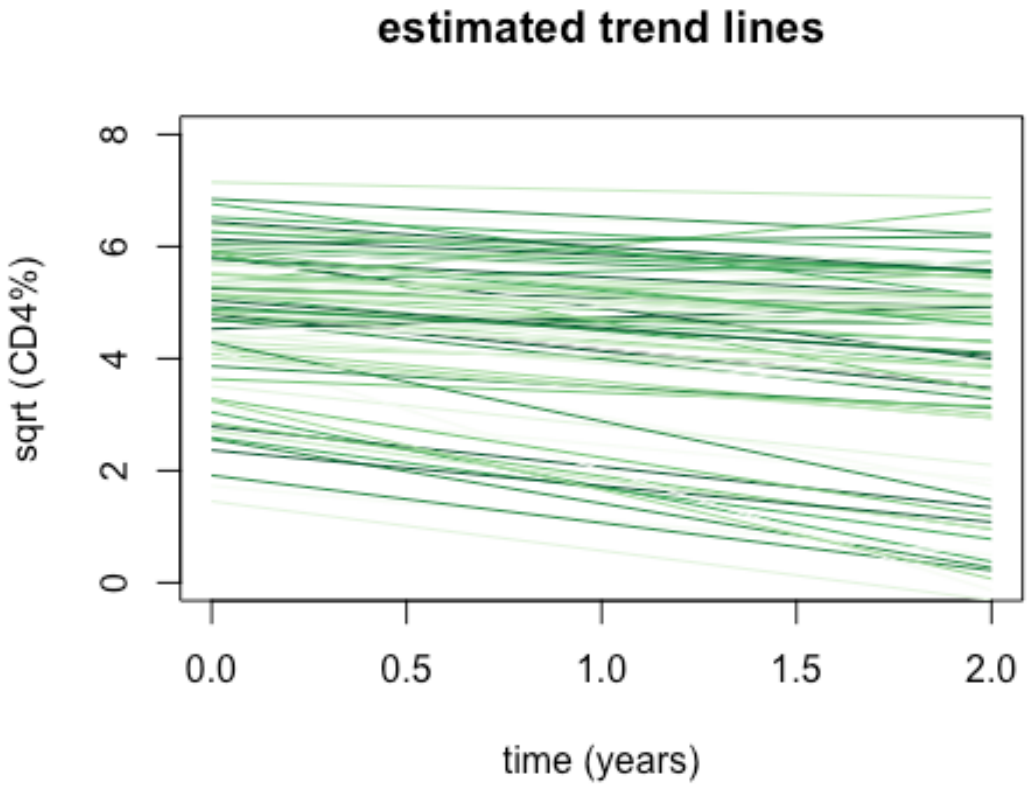
```r
## Set up new patient id numbers from 1 to J
unique.pid <- unique (newpid)
n <- length (y)
J <- length (unique.pid)
person <- rep (NA, n)
for (j in 1:J){
person[newpid==unique.pid[j]] <- j
}

cols <- rep(brewer.pal(8,'Greens'),20)
for (j in 1:J){
if(j==1){
plot(time[newpid==unique.pid[j]], y[newpid==unique.pid[j]], xlab="time
(years)", ylab="sqrt (CD4%)",
     main="observed data", cex = .1, ylim=c(0,8))
}
points(time[newpid==unique.pid[j]], y[newpid==unique.pid[j]], col = cols[j],
type="l", ylim=c(0,8))
}
```




```
M1 <- lmer (y ~ time + (1 + time | person))
display (M1)

## lmer(formula = y ~ time + (1 + time | person))
##              coef.est coef.se
## (Intercept)   4.85     0.16
## time         -0.47     0.13
##
## Error terms:
##  Groups   Name        Std.Dev. Corr
##  person   (Intercept) 1.33
##           time        0.68     0.15
##  Residual             0.75
## ---
```

```
## number of obs: 369, groups: person, 83
## AIC = 1108.1, DIC = 1087.8
## deviance = 1091.9
```

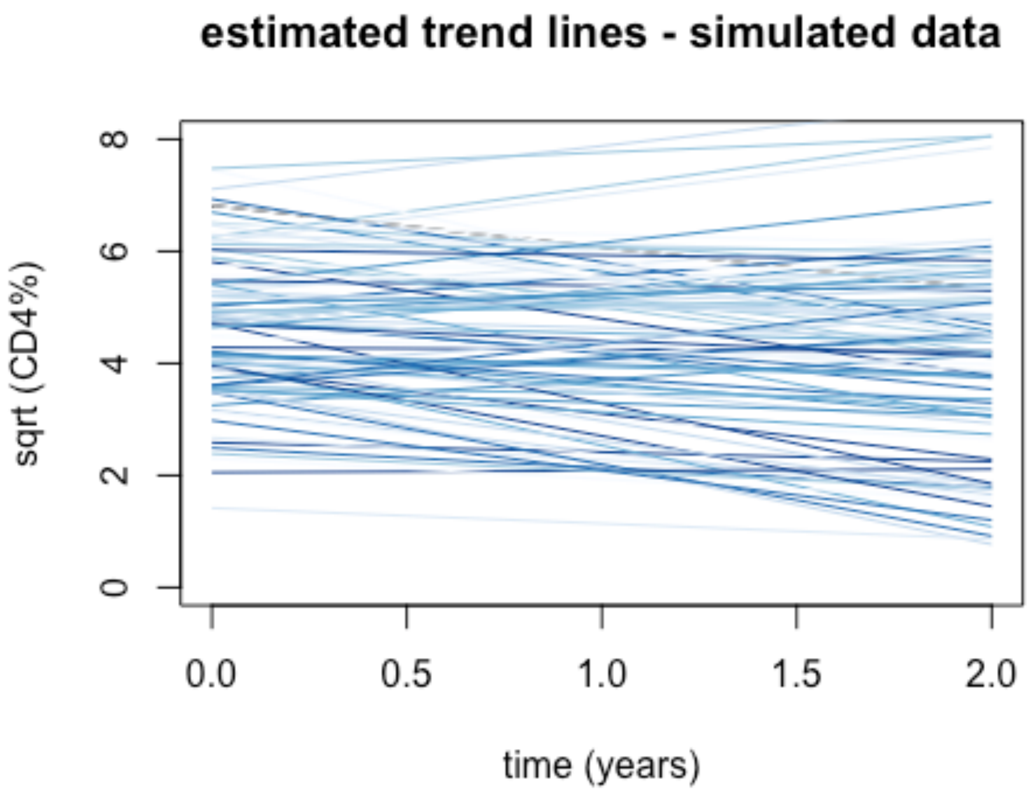
```
coef.1 <- matrix(0, J, 1)
coef.2 <- matrix(0, J, 1)
coef.1 <- coef(M1)$person[1]
coef.2 <- coef(M1)$person[2]
t = time[newpid==unique.pid[1]]
for (j in 1:J){
    if(j==1){
    plot(t , y=coef.1[j,1] + coef.2[j,1]*t, type="l", xlab="time (years)",
ylab="sqrt (CD4%)",
              main="estimated trend lines", xlim=c(0,2), ylim=c(0,8))
    }
  curve(coef.1[j,1] + coef.2[j,1]*x,col=cols[j], add=T)
}
```



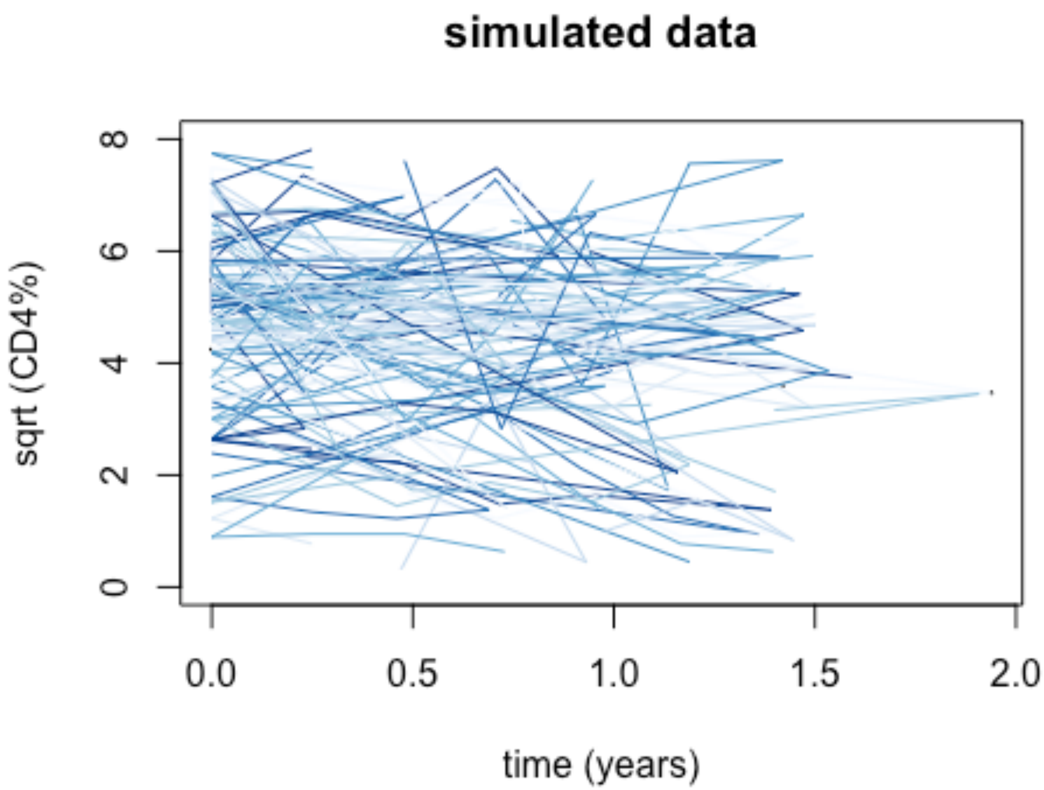

```
CD4.fake <- function(J, K){
  time <- rep (seq(0,1,length=K), J)  # K measurements during the year
  person <- rep (1:J, each=K)         # person ID's
  treatment <- sample (rep(0:1, J/2))
  treatment1 <- treatment[person]
#                                     # hyperparameters
  mu.a.true <- 4.8                    # more generally, these could
  g.0.true <- -.5                     # be specified as additional
  g.1.true <- .5                      # arguments to the function
  sigma.y.true <- .7
  sigma.a.true <- 1.3
  sigma.b.true <- .7
#                                     # personal-level parameters
```

```
  a.true <- rnorm (J, mu.a.true, sigma.a.true)
  b.true <- rnorm (J, g.0.true + g.1.true*treatment, sigma.b.true)
#                                     # data
  y <- rnorm (J*K, a.true[person] + b.true[person]*time, sigma.y.true)
  return (data.frame (y, time, person, treatment1))
}

fake.83.7 = CD4.fake (83,7)
head(fake.83.7)

##          y      time person treatment1
## 1 7.567169 0.0000000      1          0
## 2 6.485642 0.1666667      1          0
## 3 6.203211 0.3333333      1          0
## 4 7.384666 0.5000000      1          0
## 5 6.281348 0.6666667      1          0
## 6 6.220887 0.8333333      1          0

dim(fake.83.7)   # 581 x 4    83*7 = 581

## [1] 581   4

unique.pidf <- unique (fake.83.7$person)
nf <- length (y)
Jf <- length (unique.pidf)
personf <- rep (NA, n)
for (j in 1:Jf){
  personf[fake.83.7$person==unique.pidf[j]] <- j
}

## Fit the model
M1f <- lmer (y ~ time + (1 + time | person), data=fake.83.7)
display (M1f)

## lmer(formula = y ~ time + (1 + time | person), data = fake.83.7)
##             coef.est coef.se
## (Intercept)  4.68     0.16
## time        -0.27     0.13
```

```
##
## Error terms:
##  Groups   Name        Std.Dev. Corr
##  person   (Intercept) 1.36
##           time        0.82     -0.12
##  Residual             0.70
## ---
## number of obs: 581, groups: person, 83
## AIC = 1584.1, DIC = 1563.6
## deviance = 1567.9

## Figure 20.5 (c) (using fake data)
cols <- rep(brewer.pal(8,'Blues'),20)
coef.1 <- matrix(0, J, 1)
coef.2 <- matrix(0, J, 1)
coef.1 <- coef(M1f)$person[1]
coef.2 <- coef(M1f)$person[2]
t = time[fake.83.7$person==unique.pidf[1]]
for (j in 1:J){
  if(j==1){
    plot(t , y=coef.1[j,1] + coef.2[j,1]*t, type="l", xlab="time (years)",
ylab="sqrt (CD4%)",
         main="estimated trend lines - simulated data", xlim=c(0,2),
ylim=c(0,8))
  }
  curve(coef.1[j,1] + coef.2[j,1]*x, col = cols[j], add=T)
}
```




```
for (j in 1:J){
  if(j==1){
    plot(time[fake.83.7$person==unique.pidf[j]],
y[fake.83.7$person==unique.pidf[j]], xlab="time (years)", ylab="sqrt (CD4%)",
         main="simulated data", cex = .1, ylim=c(0,8))
  }
  points(time[fake.83.7$person==unique.pidf[j]],
y[fake.83.7$person==unique.pidf[j]], type="l", col=cols[j], ylim=c(0,8))
}
```



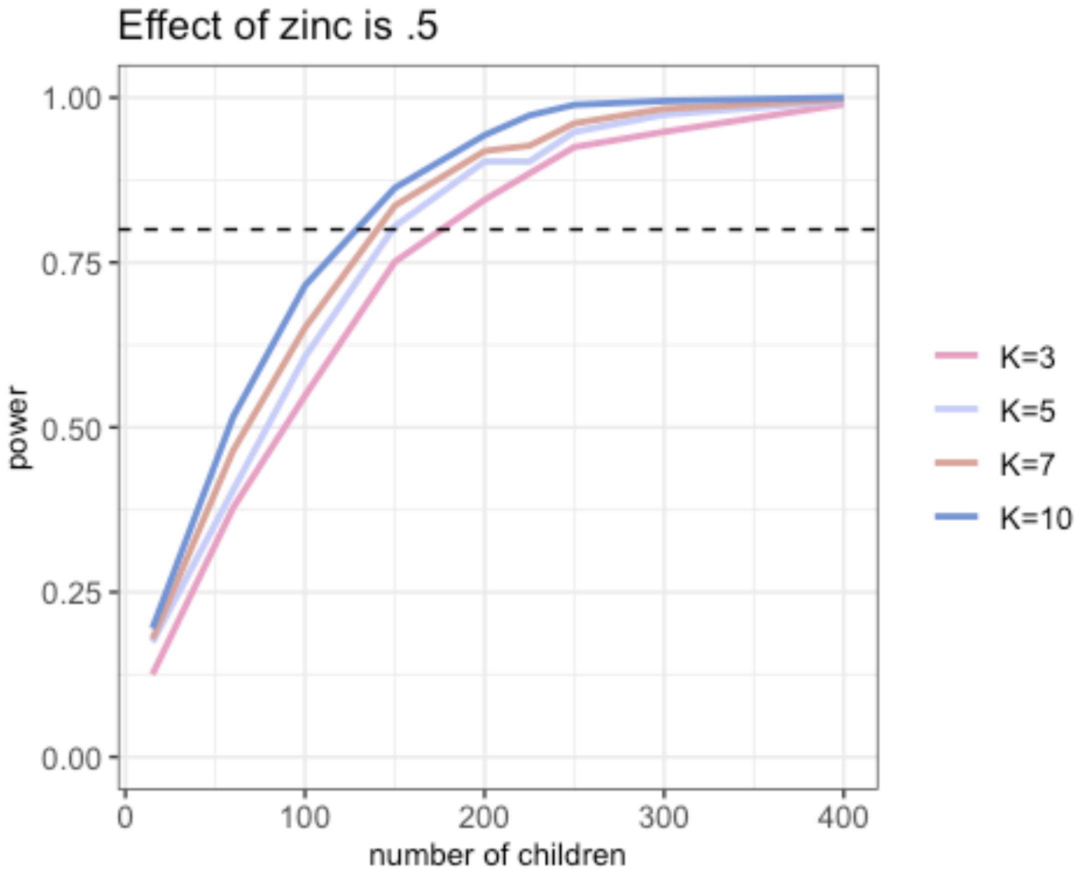

```
CD4.power <- function (J, K, n.sims=1000){
  signif <- rep (NA, n.sims)
  for (s in 1:n.sims){
    fake <- CD4.fake (J,K)
    lme.power <- lmer (y ~ time + time:treatment1 + (1 + time | person),
         data=fake)
    theta.hat <- fixef(lme.power)["time:treatment1"]
    theta.se <- se.fixef(lme.power)["time:treatment1"]
    signif[s] <- (theta.hat - 2*theta.se) > 0    # return TRUE or FALSE
  }
  power <- mean (signif)                         # proportion of TRUE
  return (power)
}
```

```r
##  these really vary wildly from run to run if nsims is only 100
CD4.power (J=150, K=7, n.sims=100)

## [1] 0.8

CD4.power (J=110, K=7, n.sims=100)

## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
control$checkConv, :
## Model failed to converge with max|grad| = 0.0169073 (tol = 0.002,
component 1)

## [1] 0.71

CD4.power (J=80, K=7, n.sims=100)

## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
control$checkConv, :
## Model failed to converge with max|grad| = 0.00349175 (tol = 0.002,
component 1)

## [1] 0.56

CD4.power (J=50, K=7, n.sims=100)

## boundary (singular) fit: see help('isSingular')

## [1] 0.44

J.values <- c(15, 60, 100, 150, 200, 225, 250, 300, 400)
n.sims.values <- rep(1000,9)
K.values <- c(3,5,7,10)
#power.values <- array (NA, c(length(J.values),length(K.values)))
#  for (i1 in 1:length(J.values)){
#  for (i2 in 1:length(K.values)){
#    #cat ("computing power calculation for J =", J.values[i1], ", K =",
K.values[i2], "\n")
#    power.values[i1,i2] <- CD4.power (J=J.values[i1], K=K.values[i2],
n.sims=n.sims.values[i1])
#    #cat ("power =", power.values[i1,i2], "\n")
#  }
#}
```

```r
#save(power.values, J.values, n.sims.values, K.values, file =
'powervalues3.RData')
load('powervalues3.RData')

dfp = as.data.frame(cbind(seq(1:length(J.values)), J.values, power.values))
colnames(dfp) = c("ID", "J.values", "K=3", "K=5","K=7", "K=10")
dfpmelt = melt(dfp,id = c("ID", "J.values"))
```

(b)

```r
p <- ggplot(dfpmelt, aes(x = J.values, y = value, color = variable)) +
  geom_line(size=1) + ylim(0, 1) +
  scale_color_manual(values = wes_palette("GrandBudapest2", n = 4)) +
  theme_bw() +
  theme(axis.text=element_text(size=10),
        axis.title=element_text(size=10),
        legend.text=element_text(size=10)) +
  geom_hline(yintercept = 0.80, linetype = 2) +
  xlab("number of children") + ylab("power") +
  ggtitle("Effect of zinc is .5") +
  theme(legend.title = element_blank())
p
```



Problem 3

```
load("/Users/Home/Documents/Michael_Ghattas/School/CU_Boulder/BA-BS/2022/
Spring 2022/STAT - 4400/Data/schooldata.Rdata")
head(schooldata)

##   id    extro     open    agree    social class school
## 1  1 63.69356 43.43306 38.02668  75.05811     d     IV
## 2  2 69.48244 46.86979 31.48957  98.12560     a     VI
## 3  3 79.74006 32.27013 40.20866 116.33897     d     VI
## 4  4 62.96674 44.40790 30.50866  90.46888     c     IV
## 5  5 64.24582 36.86337 37.43949  98.51873     d     IV
## 6  6 50.97107 46.25627 38.83196  75.21992     d      I
```

(a)

```r
mod1 <- lm(extro ~ open + agree + social, data = schooldata)
summary(mod1)
```

```
##
## Call:
## lm(formula = extro ~ open + agree + social, data = schooldata)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -30.3151  -6.0743  -0.1586   6.2851  30.0167
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 57.839518   3.148056  18.373   <2e-16 ***
## open         0.024749   0.046471   0.533    0.594
## agree        0.026538   0.053347   0.497    0.619
## social       0.005082   0.017303   0.294    0.769
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 9.342 on 1196 degrees of freedom
## Multiple R-squared:  0.0005222,  Adjusted R-squared:  -0.001985
## F-statistic: 0.2083 on 3 and 1196 DF,  p-value: 0.8907
```

```r
display(mod1)
```

```
## lm(formula = extro ~ open + agree + social, data = schooldata)
##             coef.est coef.se
## (Intercept) 57.84     3.15
## open         0.02     0.05
## agree        0.03     0.05
## social       0.01     0.02
## ---
## n = 1200, k = 4
## residual sd = 9.34, R-Squared = 0.00
```

The model is unhelpful, we are unable to make any inference with certinty, and the response looks independent from the predictors for the most part.

(b)

```
require(lme4)

mod2 <- lmer (extro ~ open + agree + social + (open | school) + (agree |
school) + (social | school), data = schooldata)

## boundary (singular) fit: see help('isSingular')

summary(mod2)

## Linear mixed model fit by REML ['lmerMod']
## Formula: extro ~ open + agree + social + (open | school) + (agree |
school) +
##     (social | school)
##    Data: schooldata
##
## REML criterion at convergence: 5821
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.8769 -0.5140  0.0006  0.5174  6.0296
##
## Random effects:
##  Groups   Name        Variance  Std.Dev. Corr
##  school   (Intercept) 2.469e+01 4.96860
##           open        2.504e-04 0.01582  1.00
##  school.1 (Intercept) 2.474e+01 4.97437
##           agree       1.414e-05 0.00376  -1.00
##  school.2 (Intercept) 2.455e+01 4.95520
##           social      1.123e-04 0.01059  1.00
##  Residual             7.102e+00 2.66487
## Number of obs: 1200, groups:  school, 6
##
## Fixed effects:
##                Estimate Std. Error t value
```

```
## (Intercept) 59.121156   3.625935  16.305
## open         0.009540   0.014814   0.644
## agree        0.027026   0.015359   1.760
## social      -0.001843   0.006576  -0.280
##
## Correlation of Fixed Effects:
##        (Intr) open   agree
## open    0.112
## agree  -0.201 -0.008
## social  0.265  0.002 -0.003
## optimizer (nloptwrap) convergence code: 0 (OK)
## boundary (singular) fit: see help('isSingular')
```

```
display(mod2)
```

```
## lmer(formula = extro ~ open + agree + social + (open | school) +
##     (agree | school) + (social | school), data = schooldata)
##             coef.est coef.se
## (Intercept) 59.12     3.63
## open         0.01     0.01
## agree        0.03     0.02
## social       0.00     0.01
##
## Error terms:
##  Groups   Name        Std.Dev. Corr
##  school   (Intercept) 4.97
##           open        0.02     1.00
##  school.1 (Intercept) 4.97
##           agree       0.00     -1.00
##  school.2 (Intercept) 4.96
##           social      0.01     1.00
##  Residual             2.66
```

```
## Warning in optwrap(optimizer, devfun, x@theta, lower = x@lower,
calc.derivs =
## TRUE, : convergence code 1 from bobyqa: bobyqa -- maximum number of
```

```
function
## evaluations exceeded

## ---
## number of obs: 1200, groups: school, 6
## AIC = 5849, DIC = 5785.7
## deviance = 5803.4
```

With this model, the parameters are estimated as follows:

Unexplained within-school variation $\hat{\sigma}_y$ = 2.66 \

School-Open intercepts variation $\hat{\sigma}_\alpha$ = 4.97 \ School-agree intercepts variation $\hat{\sigma}_\alpha$ = 4.97 \ School-Social intercepts variation $\hat{\sigma}_\alpha$ = 4.96 \

School-Open slopes variation $\hat{\sigma}_\beta$ = 0.02 \ School-agree slopes variation $\hat{\sigma}_\beta$ = 0.00 \ School-Social slopes variation $\hat{\sigma}_\beta$ = 0.01 \

Correlation between intercepts and slopes (School-Open) $ $ = 1 \ Correlation between intercepts and slopes (School-agree) $ $ = -1 \ Correlation between intercepts and slopes (School-Social) $ $ = 1 \

Fixed effect, school mean intercept $\hat{\mu}_\alpha$ = 59.12 \ Fixed effect, School-Open mean slope & $\hat{\mu}_\beta$ = 0.01 \ Fixed effect, School-Open mean slope & $\hat{\mu}_\beta$ = 0.03 \ Fixed effect, School-Open mean slope & $\hat{\mu}_\beta$ = 0.00 \

(c)

```
require(lme4)

mod3 <- lmer (extro ~ open + agree + social + school:class + (1 + open |
school) + (1 + open | school:class) + (1 + agree | school:class) + (1 +
social | school:class), data = schooldata)

## fixed-effect model matrix is rank deficient so dropping 1 column /
coefficient

## boundary (singular) fit: see help('isSingular')

summary(mod3)

## Linear mixed model fit by REML ['lmerMod']
## Formula: extro ~ open + agree + social + school:class + (1 + open |
```

```
school) +
##     (1 + open | school:class) + (1 + agree | school:class) +
##     (1 + social | school:class)
##    Data: schooldata
##
## REML criterion at convergence: 3418.4
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -9.9923 -0.3343  0.0042  0.3386 10.6753
##
## Random effects:
##  Groups         Name        Variance  Std.Dev.  Corr
##  school.class   (Intercept) 9.690e-01 0.9843771
##                 social      2.401e-11 0.0000049 -1.00
##  school.class.1 (Intercept) 9.652e-01 0.9824428
##                 agree       4.023e-05 0.0063426 -0.95
##  school.class.2 (Intercept) 9.693e-01 0.9845173
##                 open        7.326e-06 0.0027066 -1.00
##  school         (Intercept) 9.638e-01 0.9817460
##                 open        5.878e-06 0.0024245 0.13
##  Residual                   9.670e-01 0.9833553
## Number of obs: 1200, groups:  school:class, 24; school, 6
##
## Fixed effects:
##                      Estimate Std. Error t value
## (Intercept)         7.999e+01  1.854e+00  43.137
## open                6.150e-03  5.093e-03   1.208
## agree              -7.650e-03  5.848e-03  -1.308
## social              5.361e-04  1.852e-03   0.289
## schoolI:classa     -3.973e+01  2.574e+00 -15.434
## schoolII:classa    -2.775e+01  2.569e+00 -10.799
## schoolIII:classa   -2.309e+01  2.573e+00  -8.972
## schoolIV:classa    -1.946e+01  2.575e+00  -7.558
## schoolV:classa     -1.517e+01  2.572e+00  -5.900
```

```
## schoolVI:classa  -1.015e+01  2.151e+00  -4.719
## schoolI:classb   -3.432e+01  2.573e+00 -13.339
## schoolII:classb  -2.639e+01  2.571e+00 -10.265
## schoolIII:classb -2.224e+01  2.572e+00  -8.646
## schoolIV:classb  -1.829e+01  2.570e+00  -7.115
## schoolV:classb   -1.390e+01  2.574e+00  -5.401
## schoolVI:classb  -8.153e+00  2.144e+00  -3.803
## schoolI:classc   -3.161e+01  2.572e+00 -12.289
## schoolII:classc  -2.523e+01  2.572e+00  -9.807
## schoolIII:classc -2.124e+01  2.575e+00  -8.246
## schoolIV:classc  -1.726e+01  2.571e+00  -6.713
## schoolV:classc   -1.287e+01  2.573e+00  -5.003
## schoolVI:classc  -5.309e+00  2.146e+00  -2.473
## schoolI:classd   -2.974e+01  2.574e+00 -11.554
## schoolII:classd  -2.423e+01  2.570e+00  -9.429
## schoolIII:classd -2.011e+01  2.571e+00  -7.825
## schoolIV:classd  -1.623e+01  2.572e+00  -6.312
## schoolV:classd   -1.161e+01  2.570e+00  -4.518

##
## Correlation matrix not shown by default, as p = 27 > 12.
## Use print(x, correlation=TRUE)  or
##     vcov(x)        if you need it

## fit warnings:
## fixed-effect model matrix is rank deficient so dropping 1 column /
coefficient
## optimizer (nloptwrap) convergence code: 0 (OK)
## boundary (singular) fit: see help('isSingular')
```

```
display(mod3)
```

```
## lmer(formula = extro ~ open + agree + social + school:class +
##     (1 + open | school) + (1 + open | school:class) + (1 + agree |
##     school:class) + (1 + social | school:class), data = schooldata)
##                  coef.est coef.se
## (Intercept)       79.99    1.85
```

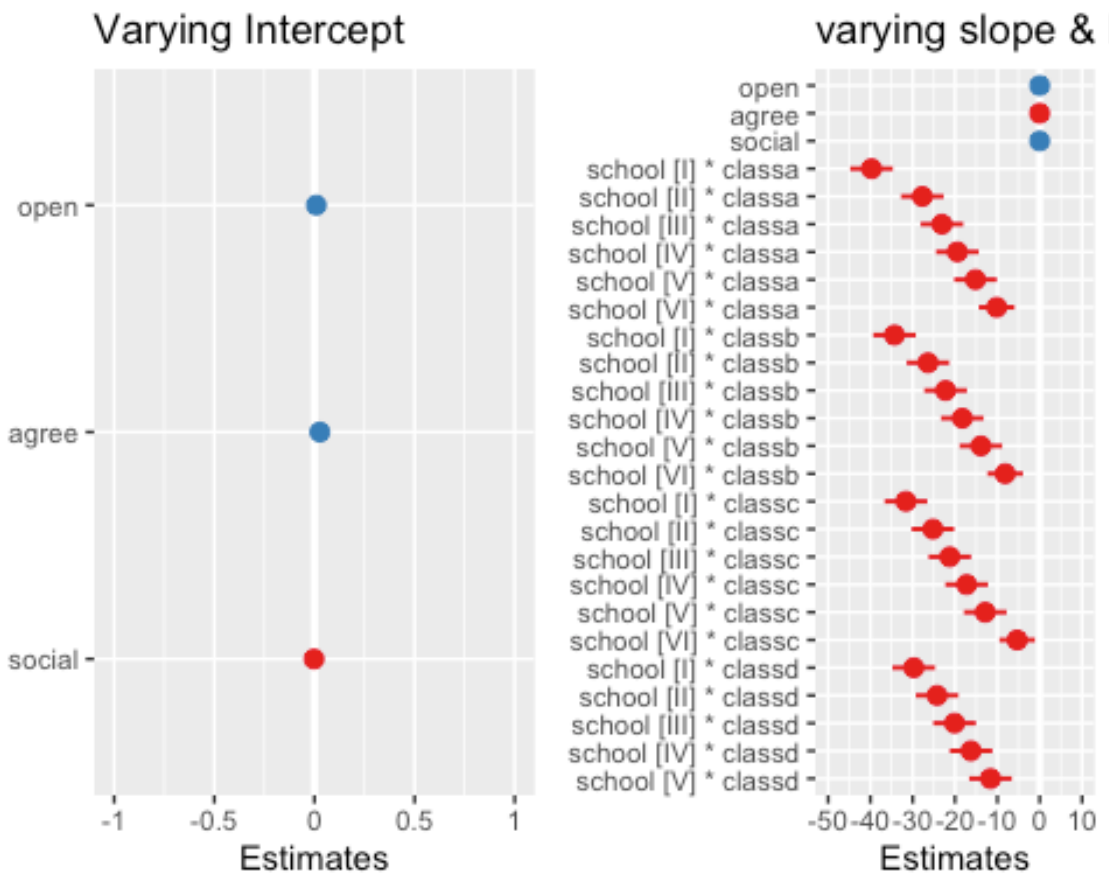
```
## open               0.01     0.01
## agree             -0.01     0.01
## social             0.00     0.00
## schoolI:classa   -39.73     2.57
## schoolII:classa  -27.75     2.57
## schoolIII:classa -23.09     2.57
## schoolIV:classa  -19.46     2.58
## schoolV:classa   -15.17     2.57
## schoolVI:classa  -10.15     2.15
## schoolI:classb   -34.32     2.57
## schoolII:classb  -26.39     2.57
## schoolIII:classb -22.24     2.57
## schoolIV:classb  -18.29     2.57
## schoolV:classb   -13.90     2.57
## schoolVI:classb   -8.15     2.14
## schoolI:classc   -31.61     2.57
## schoolII:classc  -25.23     2.57
## schoolIII:classc -21.24     2.58
## schoolIV:classc  -17.26     2.57
## schoolV:classc   -12.87     2.57
## schoolVI:classc   -5.31     2.15
## schoolI:classd   -29.74     2.57
## schoolII:classd  -24.23     2.57
## schoolIII:classd -20.11     2.57
## schoolIV:classd  -16.23     2.57
## schoolV:classd   -11.61     2.57
##
## Error terms:
##  Groups         Name        Std.Dev. Corr
##  school.class   (Intercept) 0.98
##                 social      0.00     -1.00
##  school.class.1 (Intercept) 0.98
##                 agree       0.01     -0.95
##  school.class.2 (Intercept) 0.98
##                 open        0.00     -1.00
```

```
##  school         (Intercept) 0.98
##                 open        0.00     0.13
##  Residual                   0.98
```

```
## Warning in optwrap(optimizer, devfun, x@theta, lower = x@lower,
calc.derivs =
## TRUE, : convergence code 1 from bobyqa: bobyqa -- maximum number of
function
## evaluations exceeded

## ---
## number of obs: 1200, groups: school:class, 24; school, 6
## AIC = 3498.4, DIC = 3281
## deviance = 3349.7
```

This model would be helpful if we are trying to predict extroversion based on openness, agreeableness, and social ability within a school and per class. It should be helpful as it takes into account multiple predictors and effects, allowing for a more accurate model and improved certainty. ### (d)

```
library(sjPlot) #for plotting lmer and glmer mods
library(gridExtra)

plot1 = plot_model(mod2, show.values=FALSE, show.p=TRUE, title="Varying
Intercept")
plot2 = plot_model(mod3, show.values=FALSE, show.p=TRUE, title="varying slope
& Intercept")

grid.arrange(plot1, plot2, ncol = 2)
```



Varying Intercept
open
agree
social
-1
-0.5
0
0.5
1
Estimates
varying slope &
open
agree
social
school [I] * classa
school [II] * classa
school [III] * classa
school [IV] * classa
school [V] * classa
school [VI] * classa
school [I] * classb
school [II] * classb
school [III] * classb
school [IV] * classb
school [V] * classb
school [VI] * classb
school [I] * classc
school [II] * classc
school [III] * classc
school [IV] * classc
school [V] * classc
school [VI] * classc
school [I] * classd
school [II] * classd
school [III] * classd
school [IV] * classd
school [V] * classd
-50
-40
-30
-20
-10
0
10
Estimates