[STAT 4540] HW-1 / Michael Ghattas

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Problem 1

a)

$X_t = \{X_1, X_2, X_3\}$

$\{X_1, X_2, X_3\} \to independent$

$X_1 \sim Poisson(1, 2)$

$X_2 \sim Expo(1, 0.5)$

$X_3 \sim \Gamma(1, 0.1)$

b)

$X_t \sim^{iid} Normal(0, 1)\ \forall t$

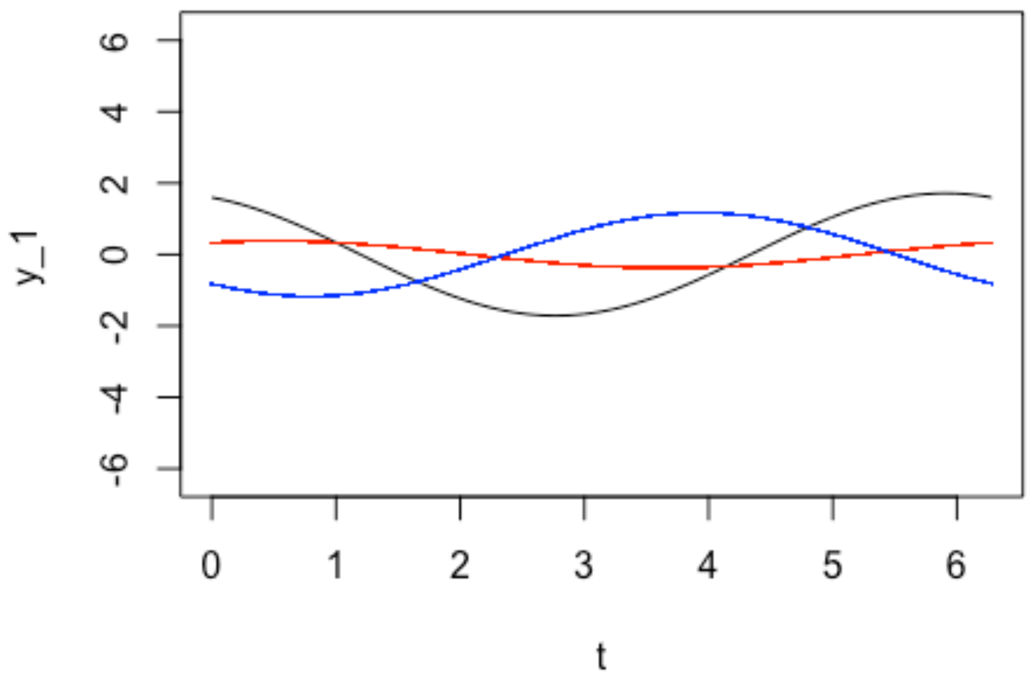
$t = \{0, \infty\}$

Problem 2

a)

```
set.seed(1)
```

```r
U_1 <- rnorm(1,0,1)
U_2 <- rnorm(1,0,1)
U_3 <- rnorm(1,0,1)
U <- c(U_1, U_2, U_3)


V_1 <- rnorm(1,0,1)
V_2 <- rnorm(1,0,1)
V_3 <- rnorm(1,0,1)
V <- c(V_1, V_2, V_3)


t <- seq(0, (2 * pi), by = 0.01)


y_1 <- ((U_1 * sin(t)) + (V_1 * cos(t)))
y_2 <- ((U_2 * sin(t)) + (V_2 * cos(t)))
y_3 <- ((U_3 * sin(t)) + (V_3 * cos(t)))
Y <- c(y_1, y_2, y_3)



plot(t, y_1, ylim = c(-(2 * pi), (2 * pi)), type = 'l')
points(t , y_2, col = "red", pch = ".")
points(t, y_3, col = "blue", pch = ".")
```



b)

Using the given information:

$$E[X_t] = E[U]*\sin(t) + E[V]*\cos(t) = 0$$

$$E[X_s] = E[U]*\sin(s) + E[V]*\cos(s) = 0$$

We will test if $E[X_t, X_s] = E[X_{t+h} \cdot X_{s+h}]\ \forall(h)\ \&\ \forall(t, s)$:

$E[X_t \cdot X_s] = (E[U]*\sin(t) + E[V]*\cos(t))*(E[U]*\sin(s) + E[V]*\cos(s))$

$= (U*\sin(t)*U\sin(s)) + (U*\sin(t)*V*\cos(s)) + (V*\cos(t)*U*\sin(s)) + (V*\cos(t)*V*\cos(s))$

$= U^2 \cdot \sin(t)\sin(s)) + UV \cdot \sin(t)\cos(s) + UV \cdot \cos(t)\sin(s)) + V^2 \cdot \cos(t)\cos(s))$

$= 0.5U^2\sin 0.5(t+s)\sin 0.5(t-s) + 0.5V^2\cos 0.5(t+s)\cos 0.5(t-s)) + UV\sin(t+s)$

Since the ACVF will not be only dependent on $(t - s)$, $\{X_t\}$ is NOT weakly stationary!

Problem 3

a

Since $Y_0 = 0$ is a provided condition, this is not a standard condition of a n AR(1) model. This means we set a limitation on the starting point of the series.

b

$$y_t = \phi \cdot y_{t-1} + z_t$$

$$E[y_t] = E[\phi \cdot y_{t-1} + z_t] = \phi \cdot E[y_{t-1}] + E[z_t] = \phi \cdot \mu + 0 = \phi \cdot \mu$$

$$\mu = \phi\mu$$

$$\mu - \phi\mu = 0$$

$$\mu \cdot (1 - \phi) = 0$$

$$\mu = 0$$

$$E[y_t] = \mu = 0$$

c

$$Var[y_t] = E[y_{t-1}^2] - E[y_t]^2 = E[y_{t-1}^2] - 0^2 = E[(\phi \cdot y_{t-1} + z_t)^2] - 0$$
$$= E[(\phi y_{t-1})^2 + 2 \cdot \phi y_{t-1} \cdot z_1 + z^2] = E[(\phi)^2] \cdot E[(y_{t-1})^2] + 2\phi(0)(0) + 0^2]$$
$$= \phi^2 E[(y_{t-1})^2] + 0 + 0 = \phi^2 E[(0)^2] = \phi^2(0) = 0$$

d

Using the information we have so far:

1. $E[y_t] = \mu = 0$

2. $Var[y_t] = 0 \rightarrow \sigma = 0$

3. $E[y_t, y_s] = E[y_{t+h} \cdot y_{s+h}] = E[y_0 y_{s-t}$, where $(h = -t)$

   $E[y_t \cdot y_s] = 0$, given $(y_0 = 0)$

Thus, $\{y_t\}$ is weakly stationary!

END.