HW8 solutions

Homework 8

Chapter 7, Exercise 9

This question uses the variables `dis` (the weighted mean of distances to five Boston employment centers) and `nox` (nitrogen oxides concentration in parts per 10 million) from the `Boston` data. We will treat `dis` as the predictor and `nox` as the response.

We load the Boston dataset

```
set.seed(1)
library(MASS)
```

```
attach(Boston)
```

a

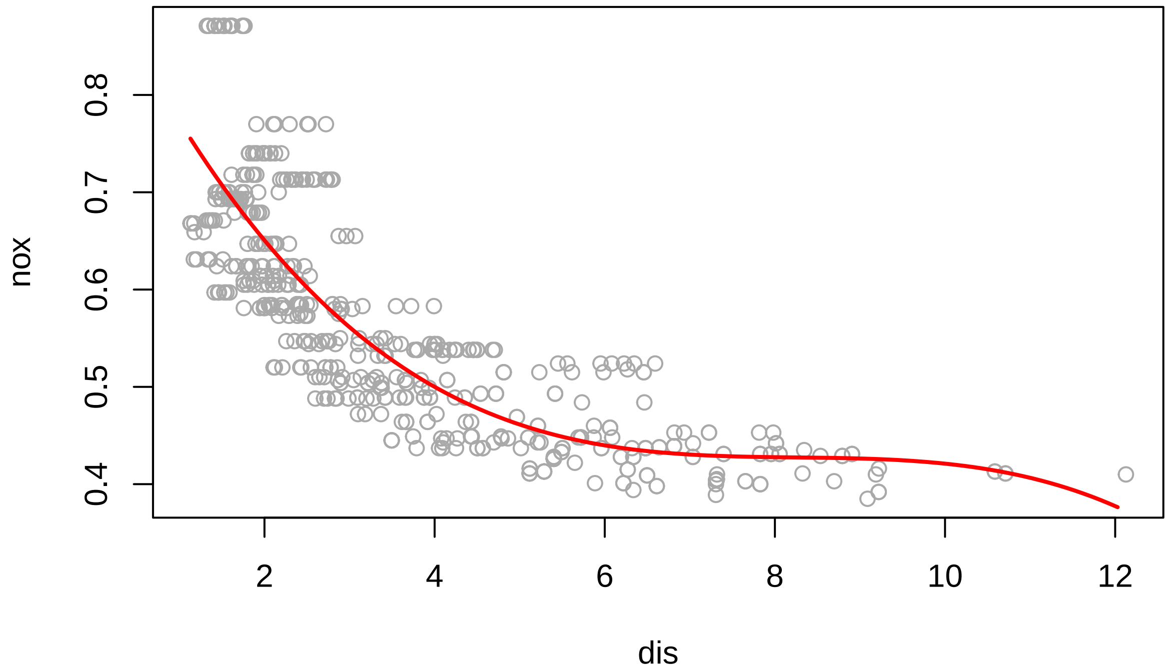
Use the `poly()` function to fit a cubic polynomial regression to predict `nox` using `dis`. Report the regression output, and plot the resulting data and polynomial fits.

```
lm.fit = lm(nox~poly(dis, 3), data=Boston)
summary(lm.fit)
```

```
##
## Call:
## lm(formula = nox ~ poly(dis, 3), data = Boston)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.121130 -0.040619 -0.009738  0.023385  0.194904
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)    0.554695   0.002759 201.021  < 2e-16 ***
## poly(dis, 3)1 -2.003096   0.062071 -32.271  < 2e-16 ***
## poly(dis, 3)2  0.856330   0.062071  13.796  < 2e-16 ***
## poly(dis, 3)3 -0.318049   0.062071  -5.124 4.27e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.06207 on 502 degrees of freedom
## Multiple R-squared:  0.7148, Adjusted R-squared:  0.7131
## F-statistic: 419.3 on 3 and 502 DF,  p-value: < 2.2e-16
```

```
dislim = range(dis)
dis.grid = seq(from=dislim[1], to=dislim[2], by=0.1)
lm.pred = predict(lm.fit, list(dis=dis.grid))
plot(nox~dis, data=Boston, col="darkgrey")
```

```r
lines(dis.grid, lm.pred, col="red", lwd=2)
```



Summary shows that all polynomial terms are significant while predicting nox using dis. Plot shows a smooth curve fitting the data fairly well.

b

Plot the polynomial fits for a range of different polynomial degrees (say, from 1 to 10), and report the associated residual sum of squares.

We plot polynomials of degrees 1 to 10 and save train RSS.

```r
all.rss = rep(NA, 10)
for (i in 1:10) {
  lm.fit = lm(nox~poly(dis, i), data=Boston)
  all.rss[i] = sum(lm.fit$residuals^2)
}
all.rss
```

```
##  [1] 2.768563 2.035262 1.934107 1.932981 1.915290 1.878257 1.849484 1.835630
##  [9] 1.833331 1.832171
```

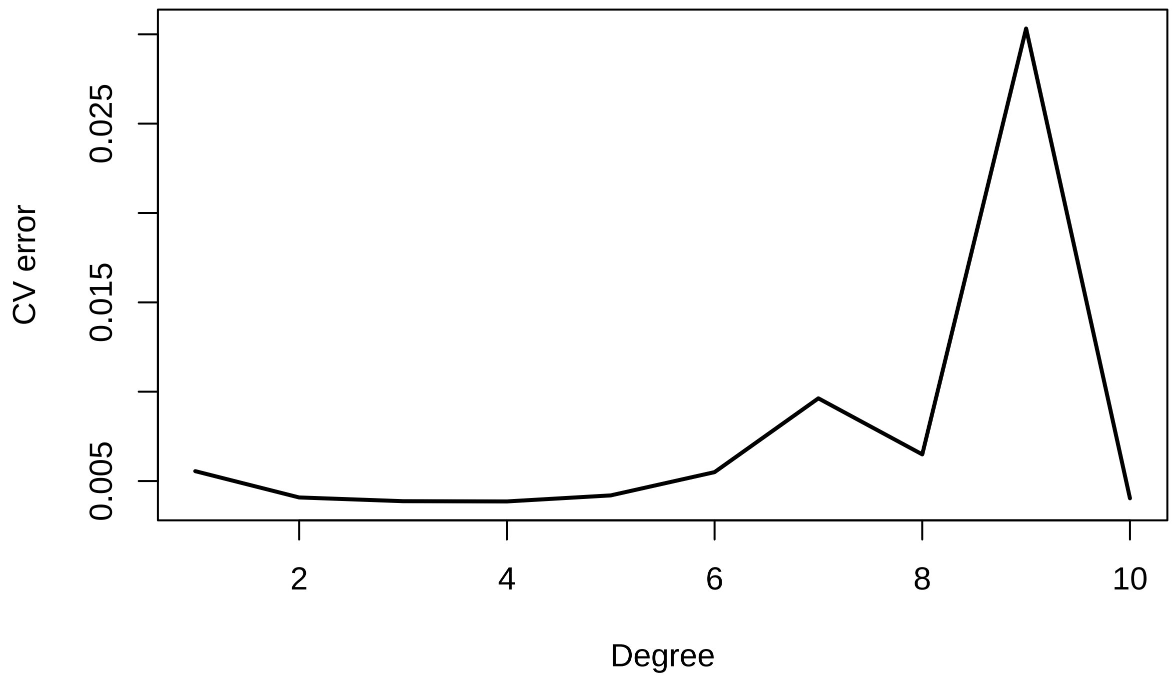
As expected, train RSS monotonically decreases with degree of polynomial.

c

Perform cross-validation or another approach to select the optimal degree for the polynomial, and explain your results.

We use a 10-fold cross validation to pick the best polynomial degree.

```
library(boot)
all.deltas = rep(NA, 10)
for (i in 1:10) {
  glm.fit = glm(nox~poly(dis, i), data=Boston)
  all.deltas[i] = cv.glm(Boston, glm.fit, K=10)$delta[2]
}
plot(1:10, all.deltas, xlab="Degree", ylab="CV error", type="l", pch=20, lwd=2)
```



A 10-fold CV shows that the CV error reduces as we increase degree from 1 to 3, stay almost constant till degree 5, and the starts increasing for higher degrees. We pick 4 as the best polynomial degree.

d

Use the `bs()` function to fit a regression spline to predict `nox` using `dis`. Report the output for the fit using four degrees of freedom. How did you choose the knots? Plot the resulting fit.

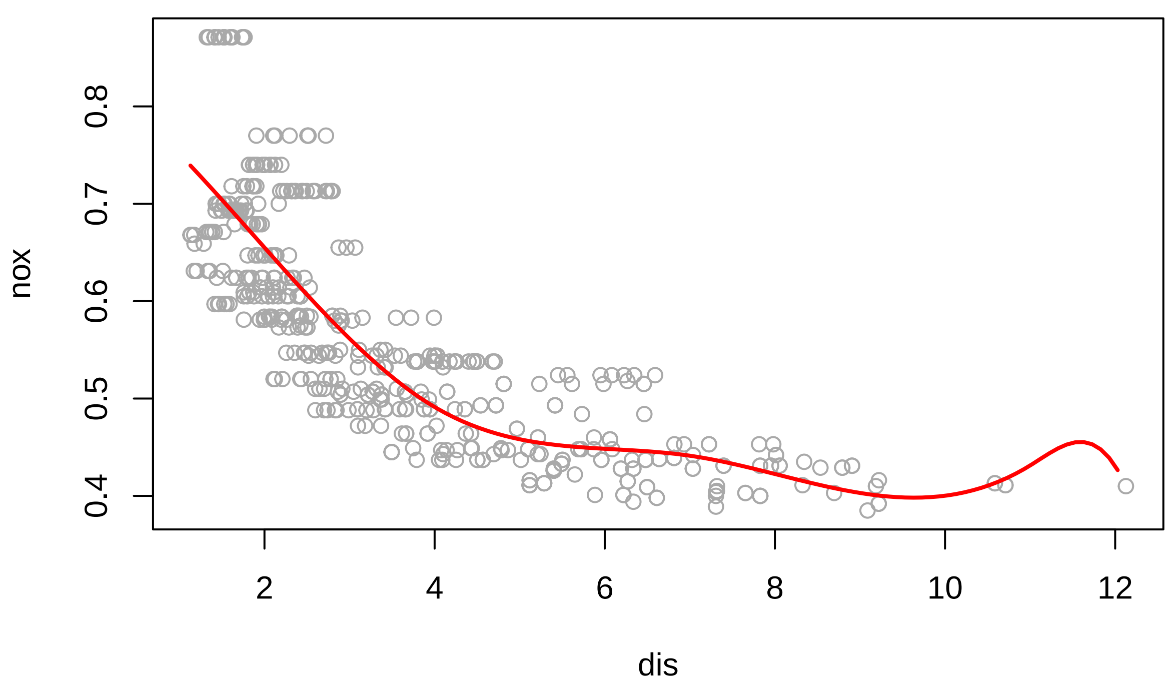
We see that dis has limits of about 1 and 13 respectively. We split this range in roughly equal 4 intervals and establish knots at $[4, 7, 11]$. Note: `bs` function in R expects either `df` or `knots` argument. If both are specified, knots are ignored.

```
library(splines)
sp.fit = lm(nox~bs(dis, df=4, knots=c(4, 7, 11)), data=Boston)
summary(sp.fit)
```

```
##
## Call:
## lm(formula = nox ~ bs(dis, df = 4, knots = c(4, 7, 11)), data = Boston)
```

```
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.124567 -0.040355 -0.008702  0.024740  0.192920
##
## Coefficients:
##                                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)                            0.73926    0.01331  55.537  < 2e-16 ***
## bs(dis, df = 4, knots = c(4, 7, 11))1 -0.08861    0.02504  -3.539  0.00044 ***
## bs(dis, df = 4, knots = c(4, 7, 11))2 -0.31341    0.01680 -18.658  < 2e-16 ***
## bs(dis, df = 4, knots = c(4, 7, 11))3 -0.26618    0.03147  -8.459 3.00e-16 ***
## bs(dis, df = 4, knots = c(4, 7, 11))4 -0.39802    0.04647  -8.565  < 2e-16 ***
## bs(dis, df = 4, knots = c(4, 7, 11))5 -0.25681    0.09001  -2.853  0.00451 **
## bs(dis, df = 4, knots = c(4, 7, 11))6 -0.32926    0.06327  -5.204 2.85e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.06185 on 499 degrees of freedom
## Multiple R-squared:  0.7185, Adjusted R-squared:  0.7151
## F-statistic: 212.3 on 6 and 499 DF,  p-value: < 2.2e-16
```

```r
sp.pred = predict(sp.fit, list(dis=dis.grid))
plot(nox~dis, data=Boston, col="darkgrey")
lines(dis.grid, sp.pred, col="red", lwd=2)
```



The summary shows that all terms in spline fit are significant. Plot shows that the spline fits data well except at the extreme values of dis, (especially $dis > 10$).

e

Now fit a regression spline for a range of degrees of freedom, and plot the resulting fits and report the resulting RSS. Describe the results obtained.

We fit regression splines with dfs between 3 and 16.

```
all.cv = rep(NA, 16)
for (i in 3:16) {
  lm.fit = lm(nox~bs(dis, df=i), data=Boston)
  all.cv[i] = sum(lm.fit$residuals^2)
}
all.cv[-c(1, 2)]
```

```
##  [1] 1.934107 1.922775 1.840173 1.833966 1.829884 1.816995 1.825653 1.792535
##  [9] 1.796992 1.788999 1.782350 1.781838 1.782798 1.783546
```

Train RSS monotonically decreases till df=14 and then slightly increases for df=15 and df=16.

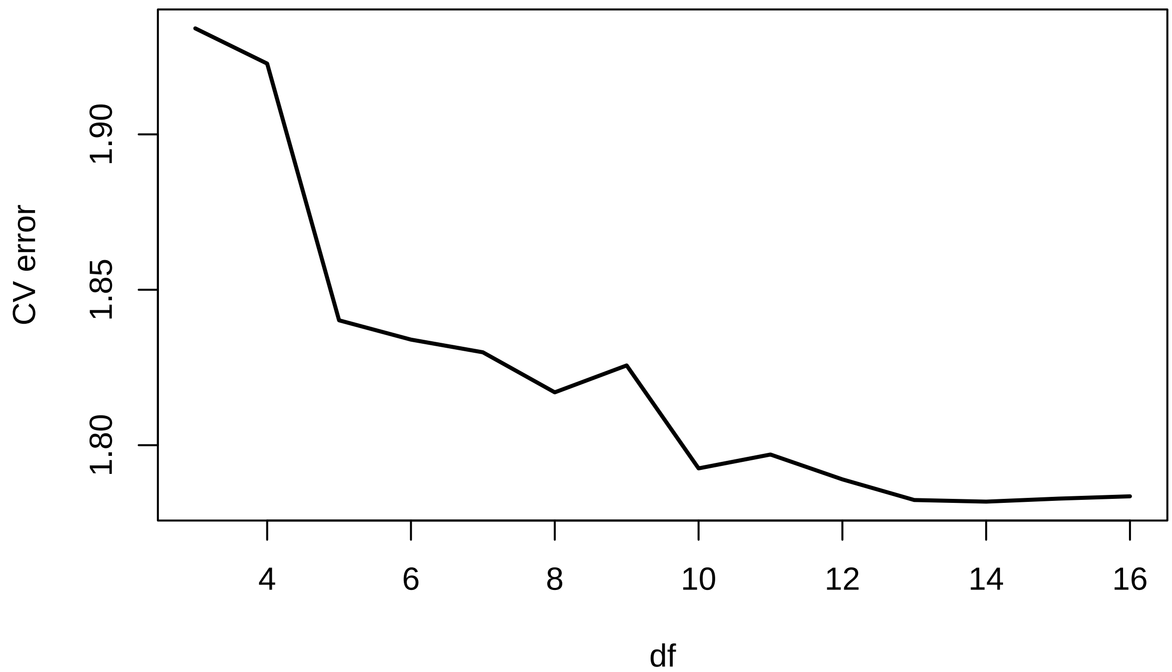
f

Perform cross-validation or another approach in order to select the best degrees of freedom for a regression spline on this data. Describe your results.

Finally, we use a 10-fold cross validation to find best df. We try all integer values of df between 3 and 16.

```
all.cv = rep(NA, 16)
for (i in 3:16) {
  lm.fit = glm(nox~bs(dis, df=i), data=Boston)
  cvres=cv.glm(Boston, lm.fit, K=10)
  all.cv[i] = cvres$delta[2]
}
```

```
plot(3:16, all.cv[-c(1, 2)], lwd=2, type="l", xlab="df", ylab="CV error")
```



CV error is more jumpy in this case, but attains minimum, and we can pick the optimal degrees of freedom based on it.