Absenteeism at the workplace

A Frequentist and Bayesian framework comparison

[STAT 4630] Final Project - Presentation / Michael Ghattas

Introduction

Bayesian statistics:

- Reliability and accuracy *(Especially in noisy data and small samples)*
- The possibility of incorporating prior knowledge into the analysis
- Intuitive interpretation of results.

R programming language:

- Used for statistical computing and graphics

“bayestestR” package:

- Provides tools to apply Bayesian methods easily
- Describe effects and their posterior distributions *(Coequals to the frequentist methods)*
- Implements Bayesian hypothesis testing
- Provides access to the established and exploratory indices of effect existence and significance
- Comprehensive and consistent set of functions to analyze and describe posterior distributions

Background

Data:
- Purely observational
- Downloaded from kaggle.com
- Identifies pockets of absence in an organization
- Exercise set to predict absence using decision trees or linear models
- Centered around absenteeism at the workplace
- Observational time-related variables are based on a collection of multiple parts over some time

Predictors:
- Workload
- Age
- BMI
- Duration at work
- Absence from work
- and more…

Analysis:
- Experiment with the utilization of Bayesian inference to build better models relevant to the subject
- Looks for an association between absence, season, workload, distance from work, BMI, and the other factors in the data set including hours worked, hours absent, Etc.
- Utilizes Frequentist and Bayesian inference techniques to analyze the data
- Models built around absenteeism
- Uses Ordinary and Generalized Least Squared frequentist linear regression models.

Analysis (Frequentist)

	ID	Reason.for.absence	Month.of.absence	Day.of.the.week	Seasons	Transportation.expense	Distance.from.Residence.to.Work
	<dbl>	<dbl>	<dbl>	<dbl>	<int>	<int>	<int>
1	11	26	7	3	1	289	36
2	36	0	7	3	1	118	13
3	3	23	7	4	1	179	51
4	7	7	7	5	1	279	5
5	11	23	7	5	1	289	36

Description: df [639 × 21]

Service.time	Age	Work.load.Average.day	Hit.target	Disciplinary.failure	Education	Son	Social.drinker	Social.smoker	Pet
<int>	<int>	<int>	<int>	<int>	<int>	<int>	<int>	<int>	<int>
13	33	239554	97	0	1	2	1	0	1
18	50	239554	97	1	1	1	1	0	0
18	38	239554	97	0	1	0	1	0	0
14	39	239554	97	0	1	2	1	1	0
13	33	239554	97	0	1	2	1	0	1

Description: df [639 × 21]

Disciplinary.failure	Education	Son	Social.drinker	Social.smoker	Pet	Weight	Height	Body.mass.index	Absenteeism.time.in.hours
<int>	<int>	<int>	<int>	<int>	<int>	<int>	<int>	<int>	<int>
0	1	2	1	0	1	90	172	30	4
1	1	1	1	0	0	98	178	31	0
0	1	0	1	0	0	89	170	31	2
0	1	2	1	1	0	68	168	24	4
0	1	2	1	0	1	90	172	30	2
0	1	4	1	0	0	65	168	23	4
0	1	2	1	0	0	95	196	25	40
0	3	1	0	0	1	88	172	29	8
0	1	4	1	0	0	65	168	23	8
0	1	4	1	0	0	65	168	23	8

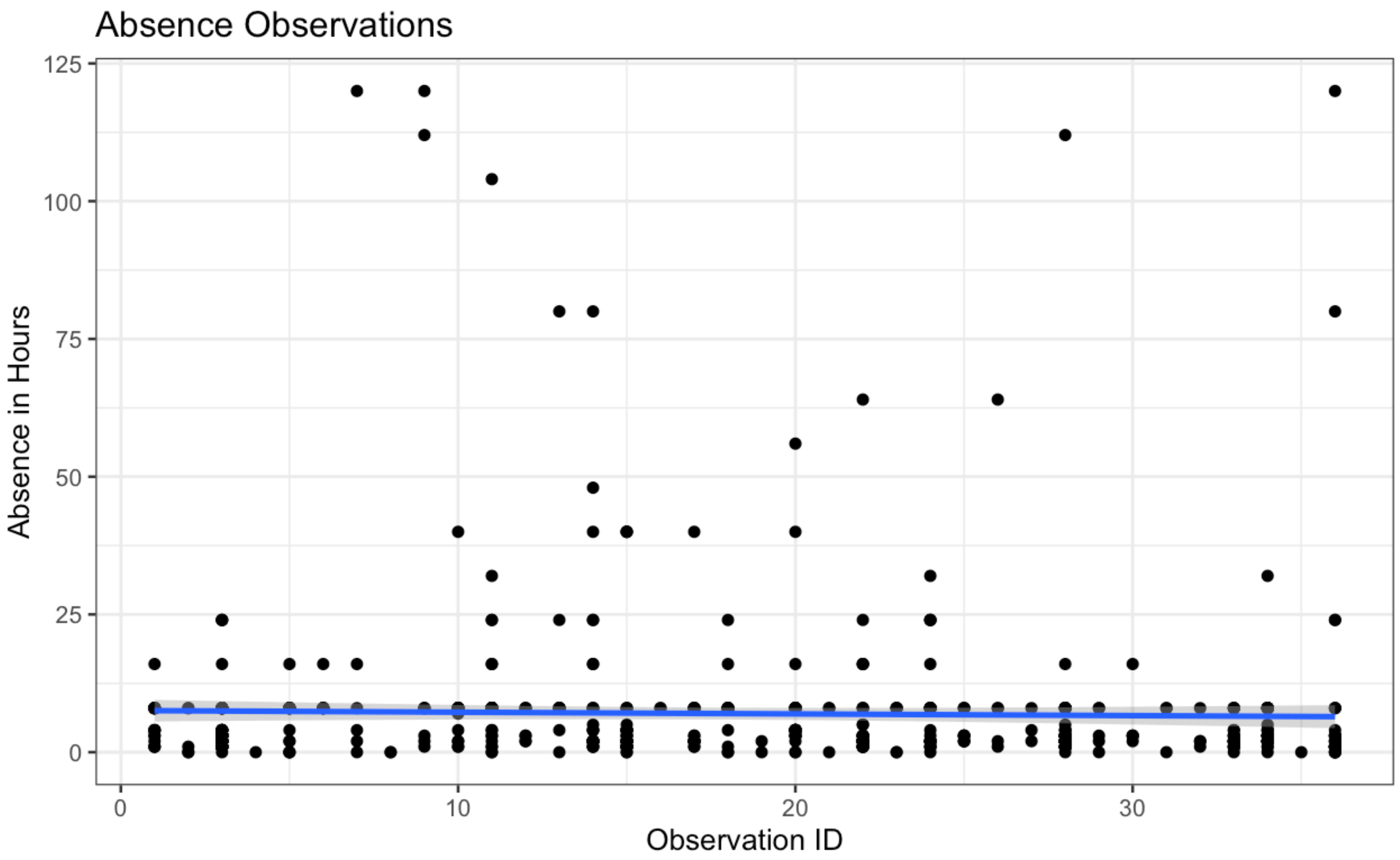
1-10 of 639 rows | 13-22 of 21 columns

Previous 1 2 3 4 5 6 ... 64 Next

Analysis (Frequentist)

```
lm(formula = Absenteeism.time.in.hours ~ ., data = rawDF)

Residuals:
    Min      1Q  Median      3Q     Max
-25.561  -4.943  -1.534   1.335 107.516

Coefficients:
                                  Estimate Std. Error t value Pr(>|t|)
(Intercept)                      1.302e+02  8.525e+01   1.528  0.12711
ID                              -1.862e-01  6.952e-02  -2.679  0.00759 **
Reason.for.absence              -4.623e-01  7.433e-02  -6.219 9.22e-10 ***
Month.of.absence                 3.730e-03  2.052e-01   0.018  0.98550
Day.of.the.week                 -7.737e-01  3.772e-01  -2.051  0.04067 *
Seasons                         -7.045e-02  5.542e-01  -0.127  0.89890
Transportation.expense           5.053e-03  1.066e-02   0.474  0.63569
Distance.from.Residence.to.Work -1.238e-01  5.738e-02  -2.157  0.03137 *
Service.time                    -1.229e-01  2.392e-01  -0.514  0.60776
Age                              2.511e-01  1.321e-01   1.901  0.05772 .
Work.load.Average.day           -4.777e-06  1.469e-05  -0.325  0.74510
Hit.target                       8.959e-02  1.657e-01   0.541  0.58889
Disciplinary.failure            -1.378e+01  2.879e+00  -4.787 2.12e-06 ***
Education                       -2.238e+00  1.009e+00  -2.218  0.02694 *
Son                              9.275e-01  5.644e-01   1.643  0.10083
Social.drinker                   1.968e+00  1.710e+00   1.151  0.25027
Social.smoker                   -6.293e-01  2.274e+00  -0.277  0.78206
Pet                             -4.785e-01  5.270e-01  -0.908  0.36426
Weight                           7.116e-01  5.312e-01   1.340  0.18082
Height                          -6.172e-01  4.801e-01  -1.286  0.19905
Body.mass.index                 -2.556e+00  1.533e+00  -1.668  0.09591 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 13.03 on 618 degrees of freedom
Multiple R-squared:  0.1375, Adjusted R-squared:  0.1096
F-statistic: 4.927 on 20 and 618 DF,  p-value: 2.447e-11
```

```r
modl <- lm(Absenteeism.time.in.hours ~ ., data = rawDF)
summary(modl)

get_parameters(modl)

ggplot(rawDF, aes(x = ID, y = Absenteeism.time.in.hours))  + geom_point() + theme_bw() + geom_smooth(method = "lm") +
  xlab("Observation ID") + ylab("Absence in Hours") + ggtitle("Absence Observations")
```



Analysis (Frequentist)

```
dataFrame <- data.frame(cbind(absenceHrs = cleanDF$Absenteeism.time.in.hours, weekDay = cleanDF$Day.of.the.week,
 commuteDist = cleanDF$Distance.from.Residence.to.Work, workLoad = cleanDF$Work.load.Average.day, achieve = cleanDF$Hit.target,
 repeats = cleanDF$Disciplinary.failure, education = cleanDF$Education, BMI = cleanDF$Body.mass.index)); dataFrame

mod <- lm(absenceHrs ~ ., data = dataFrame)
summary(mod)

get_parameters(mod)
mean(dataFrame$absenceHrs)
```

Description: df [639 × 8]

absenceHrs <dbl>	weekDay <dbl>	commuteDist <dbl>	workLoad <dbl>	achieve <dbl>	repeats <dbl>	education <dbl>	BMI <dbl>
4	3	36	239554	97	0	1	30
0	3	13	239554	97	1	1	31
2	4	51	239554	97	0	1	31
4	5	5	239554	97	0	1	24
2	5	36	239554	97	0	1	30
4	6	50	239554	97	0	1	23
40	2	12	239554	97	0	1	25
8	2	11	239554	97	0	3	29
8	2	50	239554	97	0	1	23
8	3	50	239554	97	0	1	23

1-10 of 639 rows   Previous 1 2 3 4 5 6 ... 64 Next

```
Call:
lm(formula = absenceHrs ~ ., data = dataFrame)

Residuals:
    Min      1Q  Median      3Q     Max 
-11.612  -5.385  -2.371   0.547 111.705 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)   
(Intercept)  1.557e+01  1.577e+01   0.987  0.32387   
weekDay     -9.938e-01  3.817e-01  -2.603  0.00945 **
commuteDist -1.018e-01  3.858e-02  -2.640  0.00851 **
workLoad     5.473e-06  1.408e-05   0.389  0.69763   
achieve      8.327e-02  1.458e-01   0.571  0.56810   
repeats     -3.689e+00  2.543e+00  -1.451  0.14740   
education   -2.200e+00  8.865e-01  -2.481  0.01334 * 
BMI         -2.983e-01  1.370e-01  -2.177  0.02984 * 
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 13.63 on 631 degrees of freedom
Multiple R-squared:  0.0368,	Adjusted R-squared:  0.02611 
F-statistic: 3.444 on 7 and 631 DF,  p-value: 0.00126
```

Analysis (Frequentist)

Response:

- Absence duration (abs)

Predictors:

- Distance from work (dis)
- Day of observation (day)
- Body Mass Index (bmi)
- Education (edu)
  - 1 = None to High-school/GED
  - 2 = Undergraduate/Associates
  - 3 = Graduate/Professional
  - 4 = Ph.D/Doctorate

```
df <- data.frame(cbind(abs = dataFrame$absenceHrs, dis = dataFrame$commuteDist, day = dataFrame$weekDay, edu = dataFrame$education, bmi = dataFrame$BMI)); df

freqMod <- glm(abs ~ ., data = df)
summary(freqMod)
get_parameters(freqMod)
```

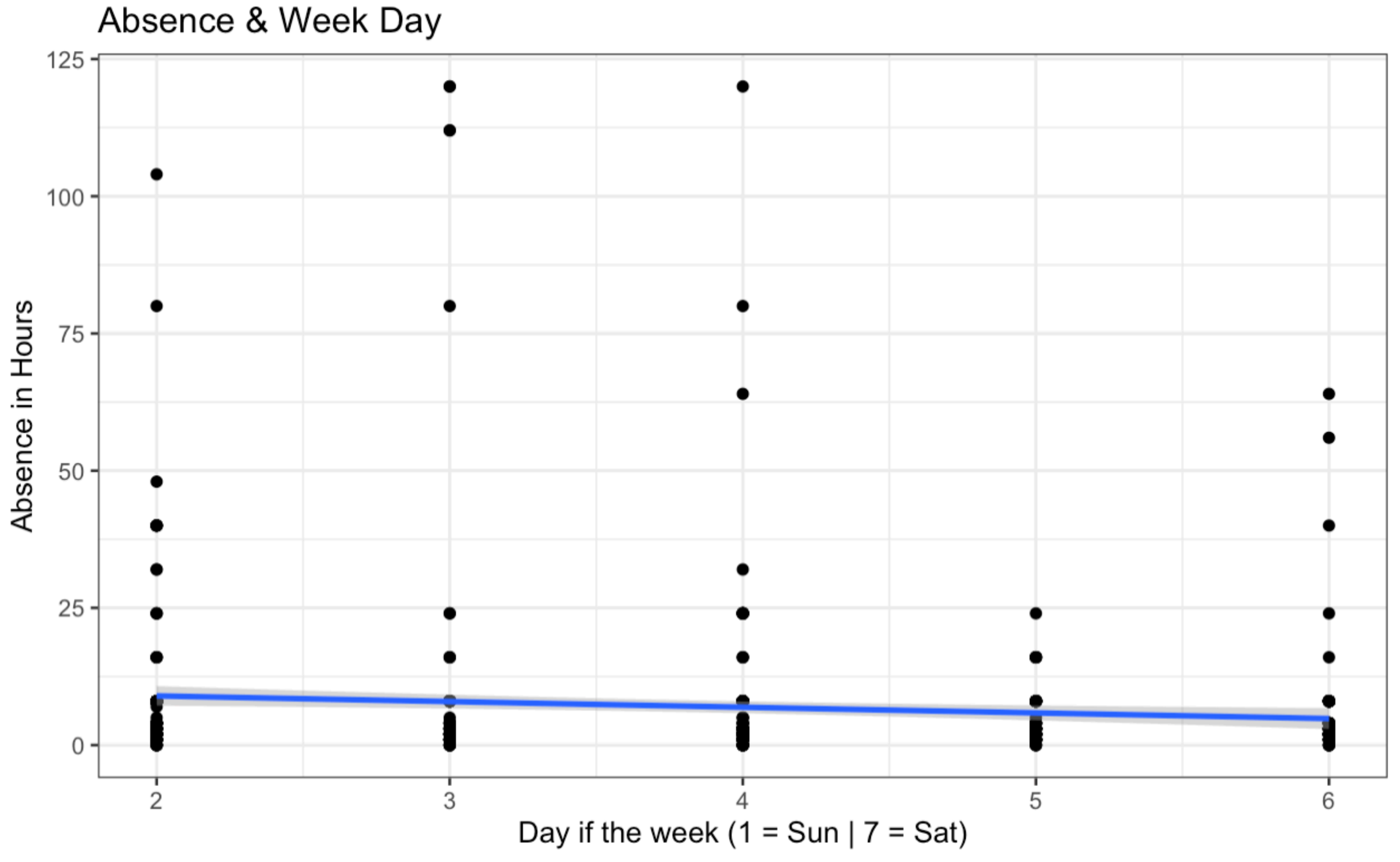
Description: df [639 × 5]

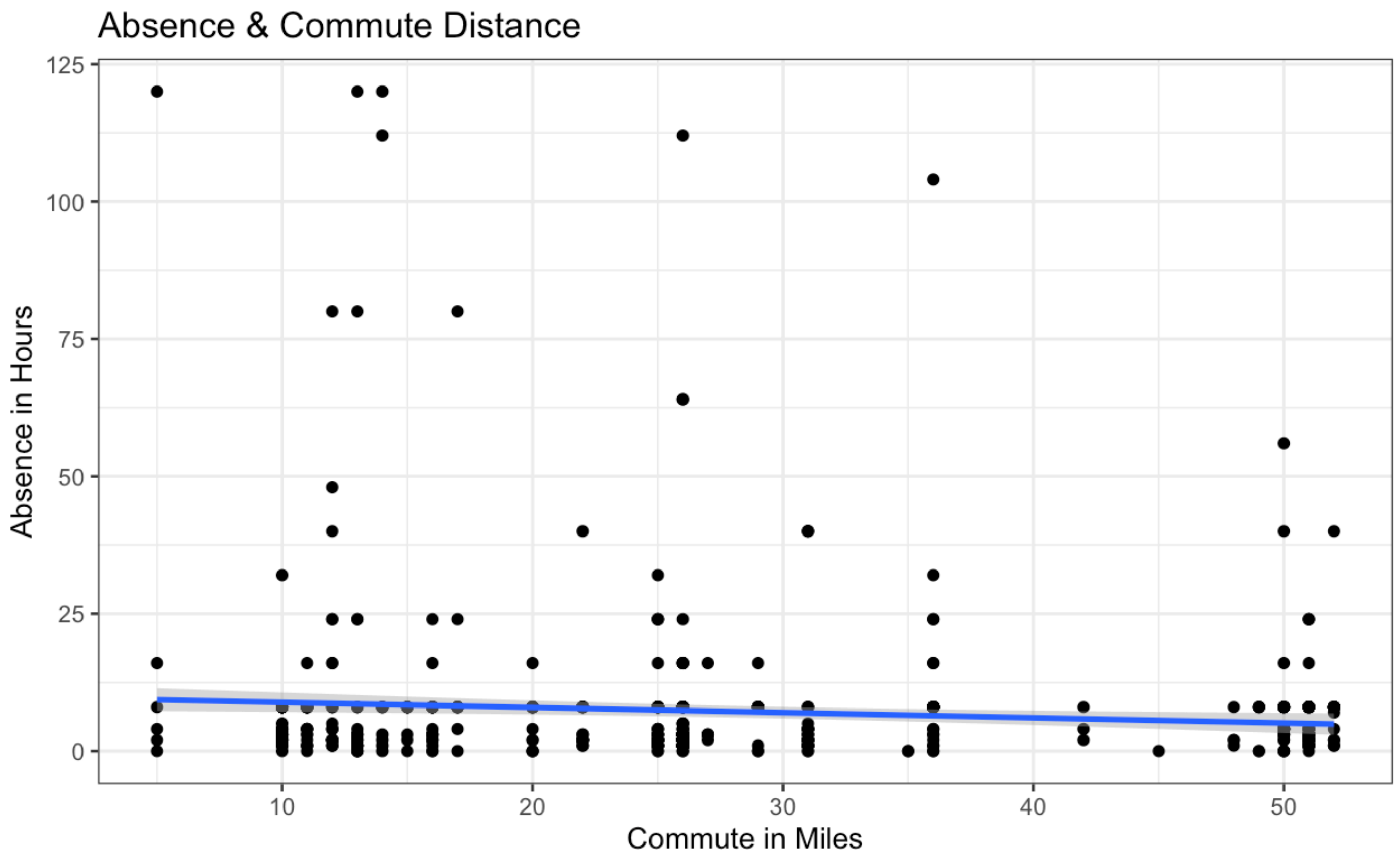
abs <dbl>	dis <dbl>	day <dbl>	edu <dbl>	bmi <dbl>
4	36	3	1	30
0	13	3	1	31
2	51	4	1	31
4	5	5	1	24
2	36	5	1	30
4	50	6	1	23
40	12	2	1	25
8	11	2	3	29
8	50	2	1	23
8	50	3	1	23

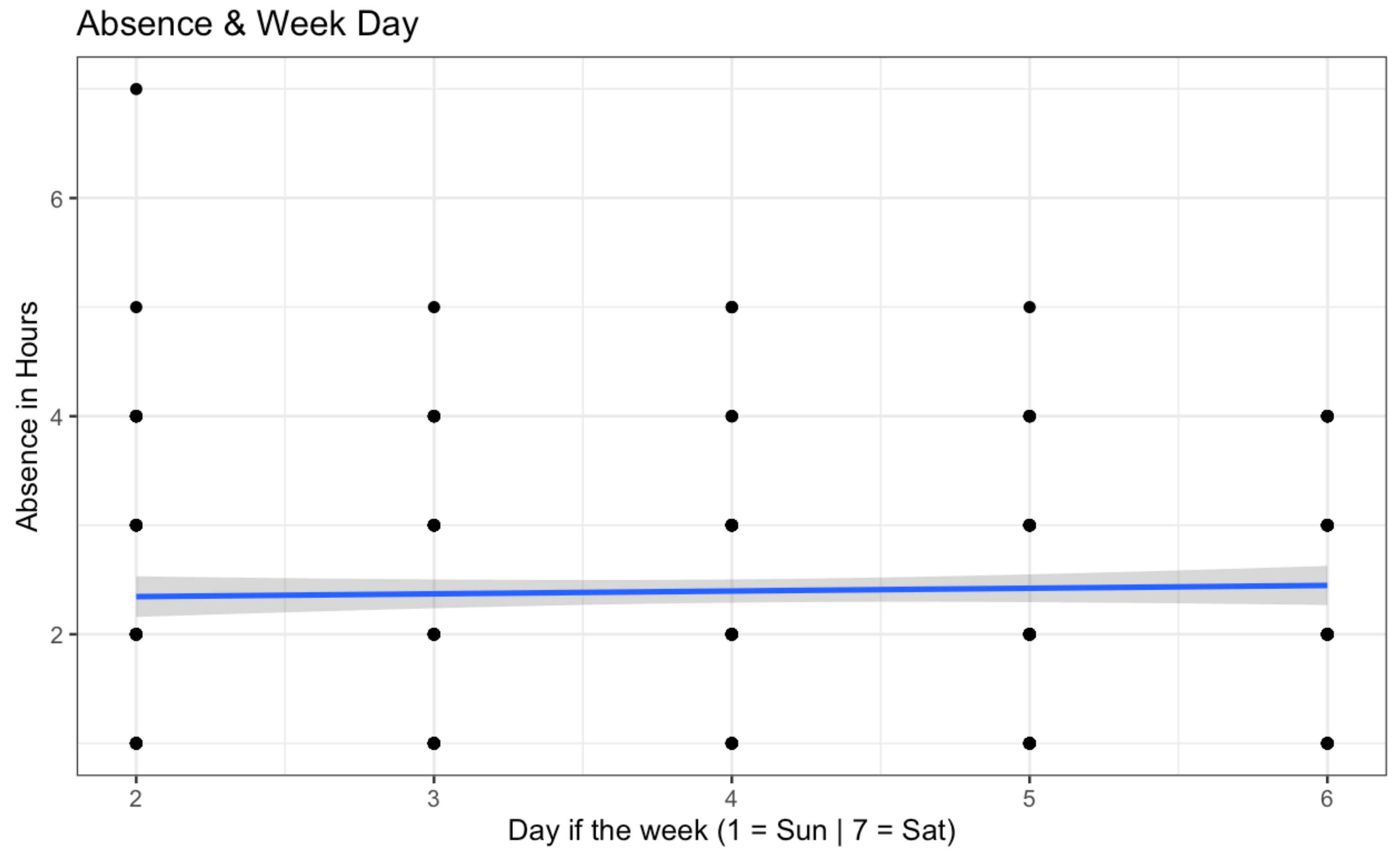
1-10 of 639 rows Previous 1 2 3 4 5 6 ... 64 Next

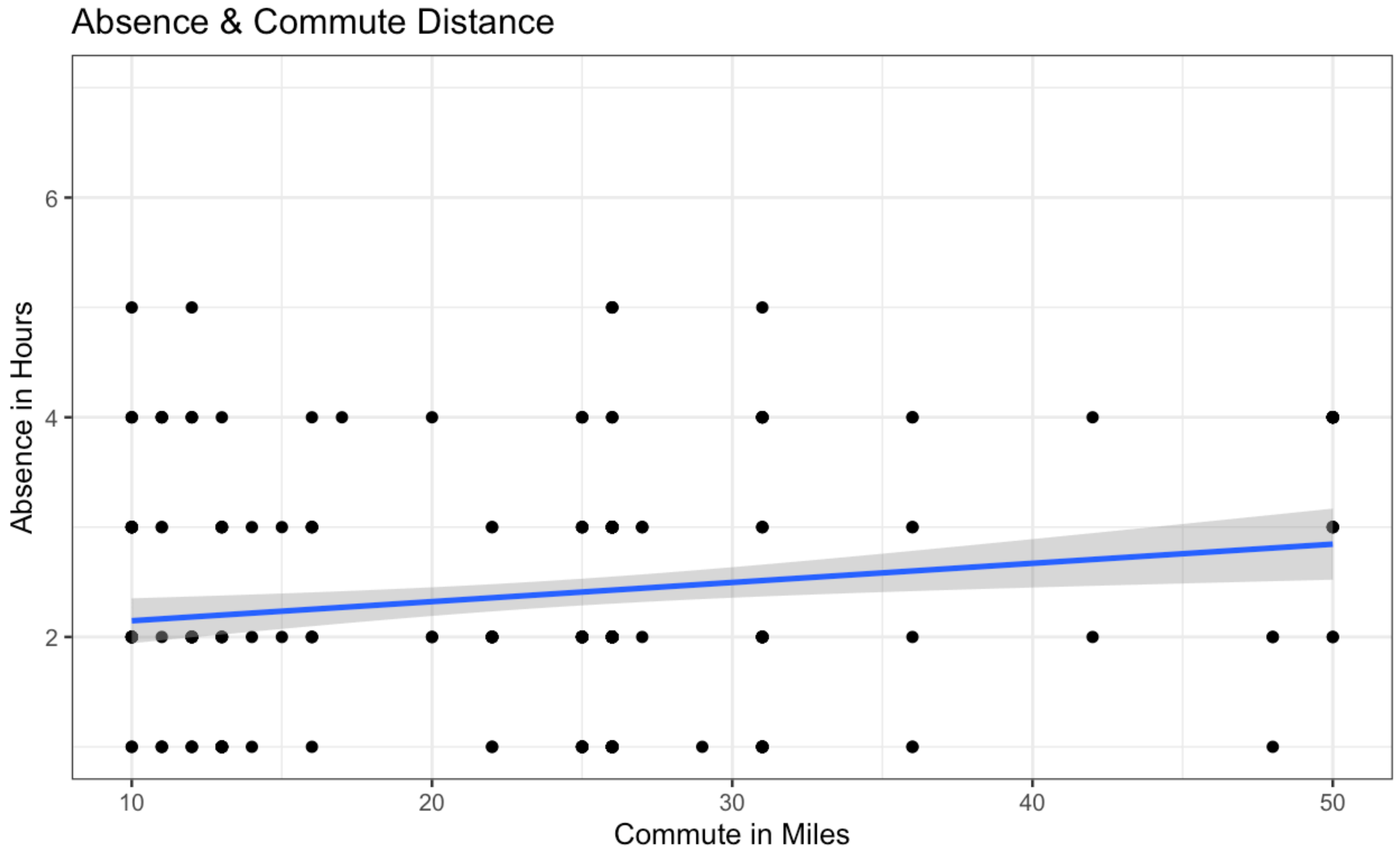
```
Call:
glm(formula = abs ~ ., data = df)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-11.098   -5.799   -2.153    0.801  112.128

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 25.05775    4.69432   5.338 1.31e-07 ***
dis         -0.09918    0.03833  -2.587  0.00989 **
day         -0.98927    0.38150  -2.593  0.00973 **
edu         -2.12504    0.87606  -2.426  0.01556 *
bmi         -0.31660    0.13581  -2.331  0.02006 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

(Dispersion parameter for gaussian family taken to be 185.6514)

    Null deviance: 121671  on 638  degrees of freedom
Residual deviance: 117703  on 634  degrees of freedom
AIC: 5158.4

Number of Fisher Scoring iterations: 2
```

Analysis (Frequentist)



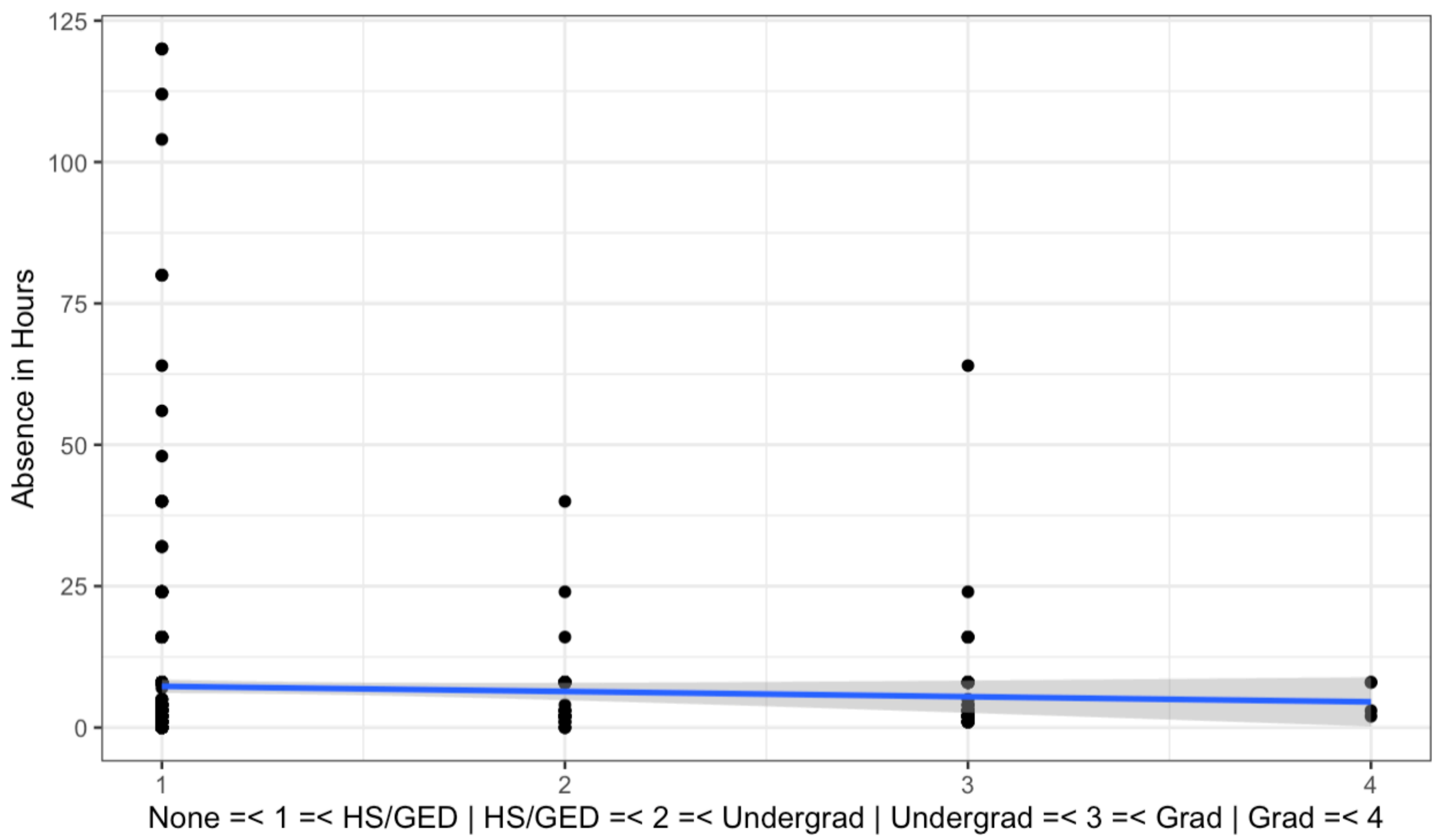




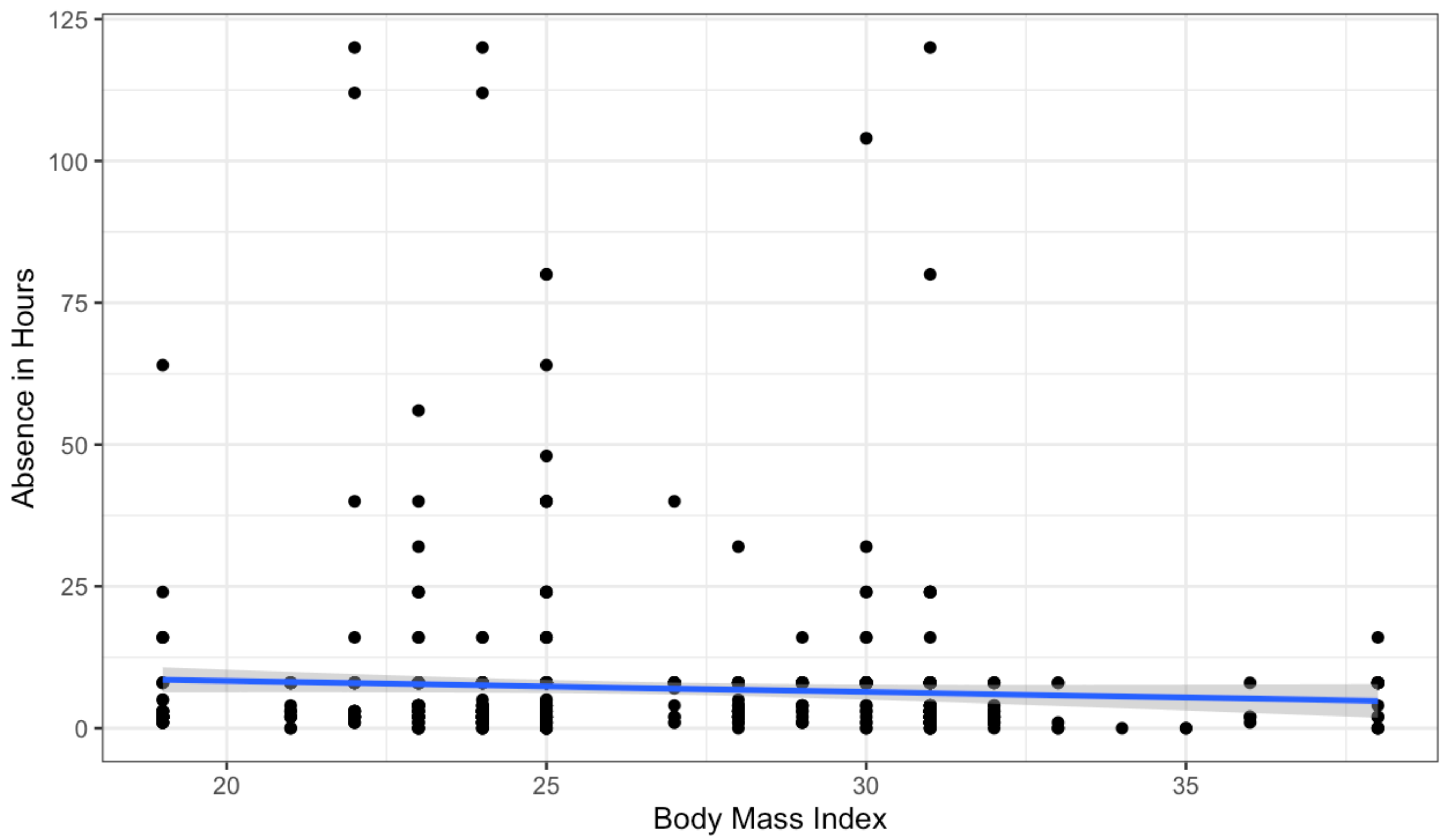


Analysis (Frequentist)

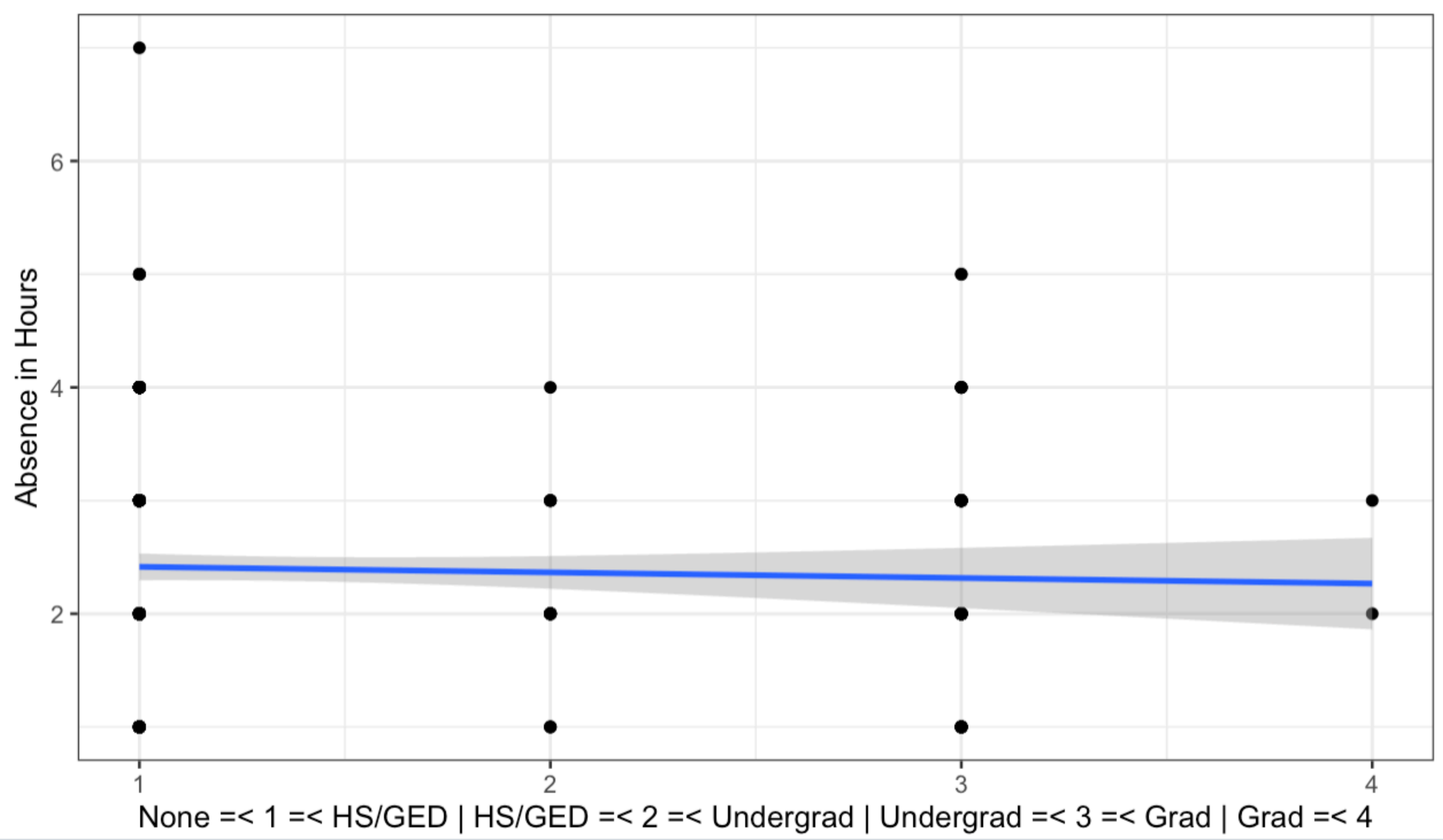
Absence & Education Level



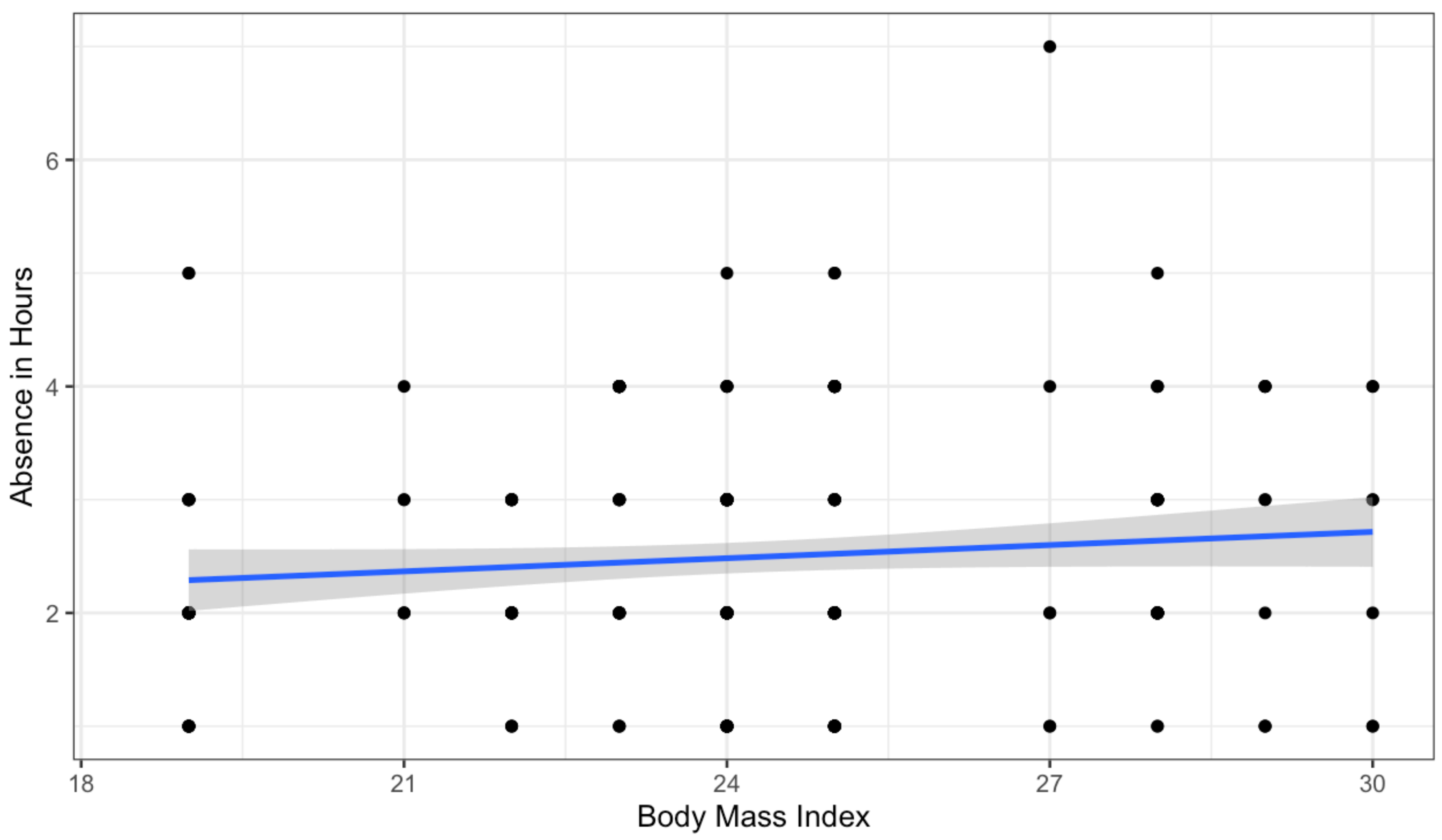
Absence &BMI



Absence & Education Level



Absence &BMI



Analysis (Bayesian)

```
bayMod <- stan_glm(abs ~ ., data = df, chains = 4, iter = 4000, warmup = 2000)
posteriors <- get_parameters(bayMod)
posteriors

ggplot(posteriors, aes(x = dis)) + geom_density(fill = "orange") +  theme_bw() + xlab("Coefficient Estimate") + ylab("Density") + ggtitle("Distance")
ggplot(posteriors, aes(x = day)) + geom_density(fill = "orange") +  theme_bw() + xlab("Coefficient Estimate") + ylab("Density") + ggtitle("Day of the week")
ggplot(posteriors, aes(x = edu)) + geom_density(fill = "orange") +  theme_bw() + xlab("Coefficient Estimate") + ylab("Density") + ggtitle("Education Level")
ggplot(posteriors, aes(x = bmi)) + geom_density(fill = "orange") +  theme_bw() + xlab("Coefficient Estimate") + ylab("Density") + ggtitle("Body Mass Index")

ggplot(posteriors, aes(x = dis)) + geom_density(fill = "orange") +  theme_bw() + xlab("Coefficient Estimate") + ylab("Density") + ggtitle("Distance") +
  geom_vline(xintercept = mean(posteriors$dis), color = "green", size = 1) + geom_vline(xintercept = median(posteriors$dis), color = "red", size = 1) +
  geom_vline(xintercept = map_estimate(posteriors$dis), color = "purple", size = 1)

ggplot(posteriors, aes(x = day)) + geom_density(fill = "orange") +  theme_bw() + xlab("Coefficient Estimate") + ylab("Density") + ggtitle("Day of the week") +
  geom_vline(xintercept = mean(posteriors$day), color = "green", size = 1) + geom_vline(xintercept = median(posteriors$day), color = "red", size = 1) +
  geom_vline(xintercept = map_estimate(posteriors$day), color = "purple", size = 1)

ggplot(posteriors, aes(x = edu)) + geom_density(fill = "orange") +  theme_bw() + xlab("Coefficient Estimate") + ylab("Density") + ggtitle("Education Level") +
  geom_vline(xintercept = mean(posteriors$edu), color = "green", size = 1) + geom_vline(xintercept = median(posteriors$edu), color = "red", size = 1) +
  geom_vline(xintercept = map_estimate(posteriors$edu), color = "purple", size = 1)

ggplot(posteriors, aes(x = bmi)) + geom_density(fill = "orange") +  theme_bw() + xlab("Coefficient Estimate") + ylab("Density") + ggtitle("Body Mass Index") +
  geom_vline(xintercept = mean(posteriors$bmi), color = "green", size = 1) + geom_vline(xintercept = median(posteriors$bmi), color = "red", size = 1) +
  geom_vline(xintercept = map_estimate(posteriors$bmi), color = "purple", size = 1)
```

Description: df [8,000 × 5]

(Intercept) <dbl>	dis <dbl>	day <dbl>	edu <dbl>	bmi <dbl>
27.61617	-0.137068024	-1.129772411	-2.41648668	-0.354133067
29.57639	-0.131556189	-1.213011751	-1.75551546	-0.433741791
26.05764	-0.105284714	-0.594892006	-3.09663877	-0.367683614
16.78638	-0.109454666	-1.153906469	-0.53295495	-0.033360272
23.80253	-0.103237760	-0.201277636	-2.77046336	-0.348215152
24.84257	-0.110220273	-0.433692755	-3.31955664	-0.310789020
29.69123	-0.168798173	-1.264370872	-2.23843588	-0.358353367
27.19844	-0.143794276	-0.646018490	-2.55999190	-0.374668054
25.80101	-0.063727117	-1.281233530	-3.25694944	-0.291835348
29.55205	-0.058820149	-1.580765860	-2.64914142	-0.440715555

1-10 of 8,000 rows

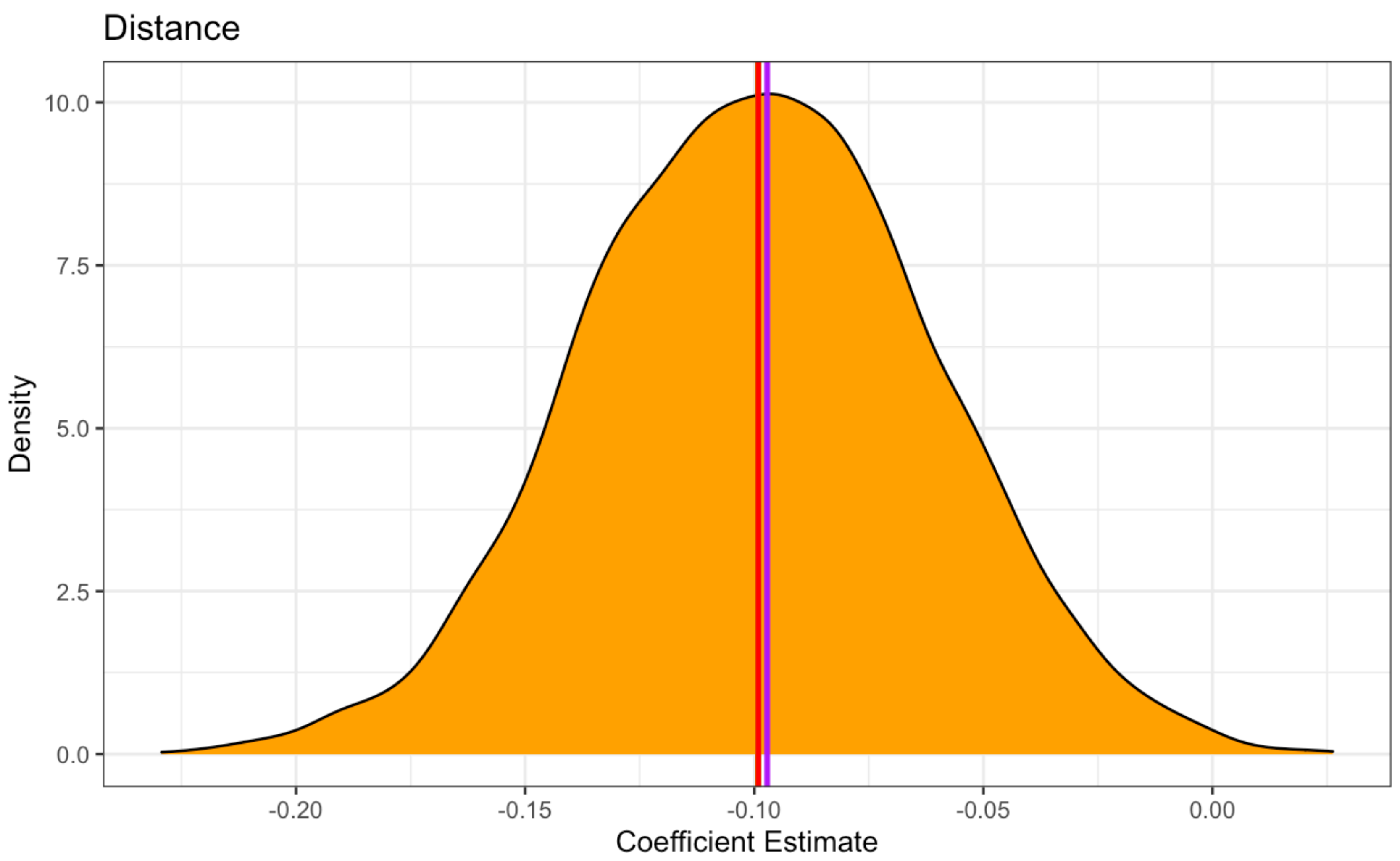
Previous 1 2 3 4 5 6 ... 100 Next

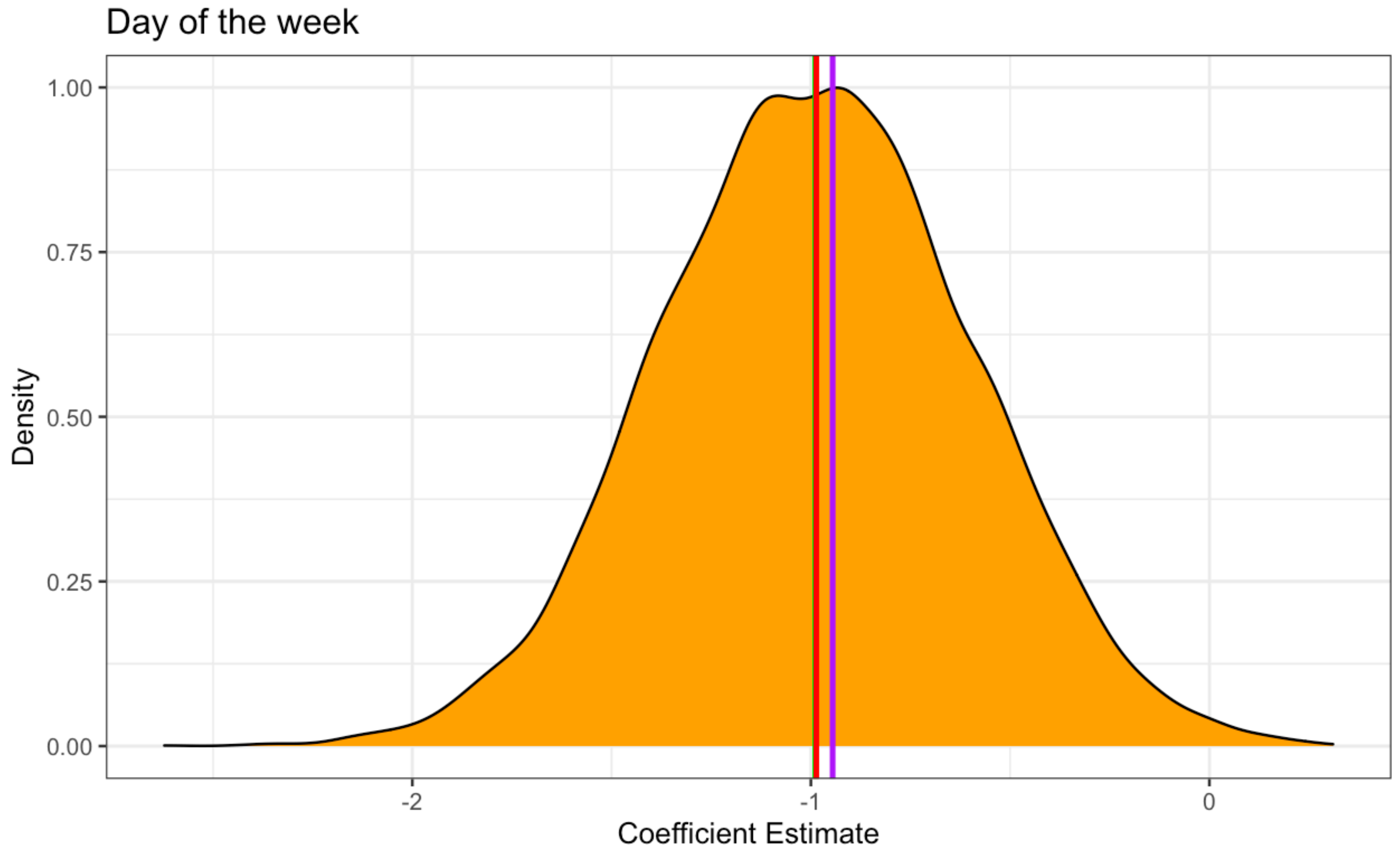
Description: df [3 × 4]

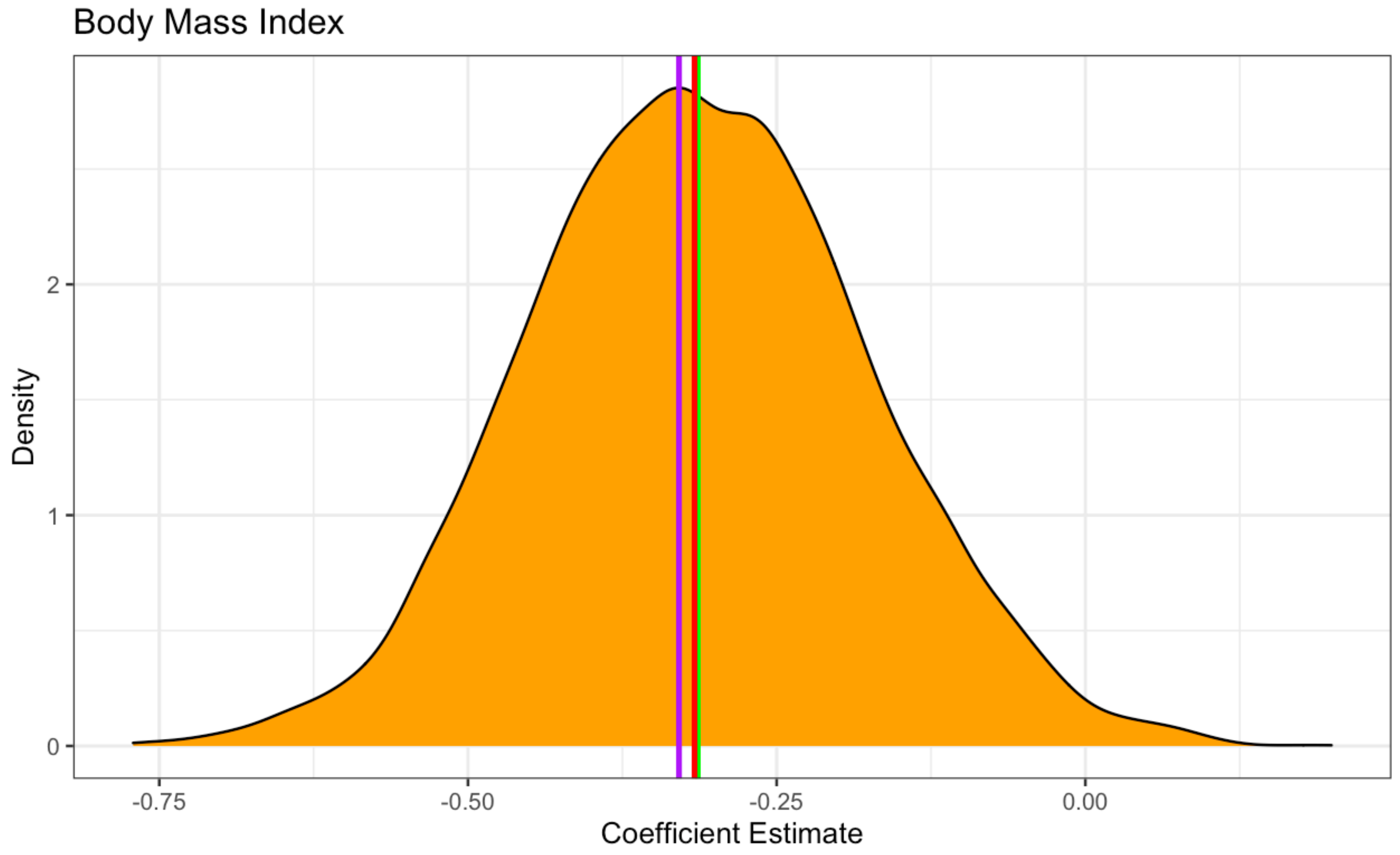
	dis <dbl>	day <dbl>	edu <dbl>	bmi <dbl>
Mean	-0.09915903	-0.9884541	-2.122505	-0.3138754
Median	-0.09914091	-0.9863515	-2.128213	-0.3164964
MAP	-0.09717782	-0.9453622	-2.277893	-0.3290778

3 rows

Analysis (Bayesian)







Analysis (Bayesian)

Description: df [4 × 3]

	Credible Interval <dbl>	Low <dbl>	High <dbl>
dis	0.89	-0.1583286	-0.03833184
day	0.89	-1.5939230	-0.36997340
edu	0.89	-3.5450719	-0.78052170
bmi	0.89	-0.5362896	-0.10512166

4 rows

Description: df [4 × 1]

	Uncertainty <dbl>
dis	-0.2293638
day	-2.6222454
edu	-5.4068510
bmi	-0.7711571

```r
post <- as.data.frame(matrix(nrow = 3, ncol = 4))
mean <- cbind(mean(posteriors$dis), mean(posteriors$day), mean(posteriors$edu), mean(posteriors$bmi))
median <- cbind(median(posteriors$dis), median(posteriors$day), median(posteriors$edu), median(posteriors$bmi))
map <- cbind(map_estimate(posteriors$dis), map_estimate(posteriors$day), map_estimate(posteriors$edu), map_estimate(posteriors$bmi))
post[1, ] <- mean
post[2, ] <- median
post[3, ] <- map
colnames(post) <- c("dis", "day", "edu", "bmi")
rownames(post) <- c("Mean", "Median", "MAP")
post <- as.data.frame(post)
post

range <- as.data.frame(matrix(nrow = 4, ncol = 1))
range[1, ] <- t(range(posteriors$dis))
range[2, ] <- t(range(posteriors$day))
range[3, ] <- t(range(posteriors$edu))
range[4, ] <- t(range(posteriors$bmi))
colnames(range) <- c("Uncertainty")
rownames(range) <- c("dis", "day", "edu", "bmi")
range

CI <- as.data.frame(matrix(nrow = 4, ncol = 3))
CI[1, ] <- t(hdi(posteriors$dis, ci = 0.89))
CI[2, ] <- t(hdi(posteriors$day, ci = 0.89))
CI[3, ] <- t(hdi(posteriors$edu, ci = 0.89))
CI[4, ] <- t(hdi(posteriors$bmi, ci = 0.89))
colnames(CI) <- c("Credible Interval", "Low", "High")
rownames(CI) <- c("dis", "day", "edu", "bmi")
CI

rope_range <- rope_range(bayMod)
print("dis")
rope(posteriors$dis, range = rope_range, ci = 0.89)
print("day")
rope(posteriors$day, range = rope_range, ci = 0.89)
print("edu")
rope(posteriors$edu, range = rope_range, ci = 0.89)
print("bmi")
rope(posteriors$bmi, range = rope_range, ci = 0.89)

print("Probability of Direction: dis")
n_positive <- posteriors %>% filter(dis > 0) %>% nrow()
nDis <- (n_positive / nrow(posteriors)) * 100
nDis
print("Probability of Direction: day")
n_positive <- posteriors %>% filter(day > 0) %>% nrow()
nDay <- (n_positive / nrow(posteriors)) * 100
nDay
print("Probability of Direction: edu")
n_positive <- posteriors %>% filter(edu > 0) %>% nrow()
nEdu <- (n_positive / nrow(posteriors)) * 100
nEdu
print("Probability of Direction: bmi")
n_positive <- posteriors %>% filter(bmi > 0) %>% nrow()
nBMI <- (n_positive / nrow(posteriors)) * 100
nBMI

print("Frequentist p-Value: dis")
onesided_p_dis <- (1 - nDis) / 100
twosided_p_dis <- onesided_p_dis * 2
twosided_p_dis
print("Frequentist p-Value: day")
onesided_p_day <- (1 - nDay) / 100
twosided_p_day <- onesided_p_day * 2
twosided_p_day
print("Frequentist p-Value: edu")
onesided_p_edu <- (1 - nEdu) / 100
twosided_p_edu <- onesided_p_edu * 2
twosided_p_edu
print("Frequentist p-Value: bmi")
onesided_p_bmi <- (1 - nBMI) / 100
twosided_p_bmi <- onesided_p_bmi * 2
twosided_p_bmi
```

Analysis (Bayesian)

```
[1] "dis"
# Proportion of samples inside the ROPE [-1.38, 1.38]:

inside ROPE
-----------
100.00 %

[1] "day"
# Proportion of samples inside the ROPE [-1.38, 1.38]:

inside ROPE
-----------
88.71 %

[1] "edu"
# Proportion of samples inside the ROPE [-1.38, 1.38]:

inside ROPE
-----------
15.11 %

[1] "bmi"
# Proportion of samples inside the ROPE [-1.38, 1.38]:

inside ROPE
-----------
100.00 %
```

```
[1] "Probability of Direction: dis"
[1] 0.3375
[1] "Probability of Direction: day"
[1] 0.5375
[1] "Probability of Direction: edu"
[1] 0.875
[1] "Probability of Direction: bmi"
[1] 1.1
```

```
[1] "Frequentist p-Value: dis"
[1] 0.01325
[1] "Frequentist p-Value: day"
[1] 0.00925
[1] "Frequentist p-Value: edu"
[1] 0.0025
[1] "Frequentist p-Value: bmi"
[1] -0.002
```

Analysis (Bayesian)

```
discribePosteriors <- describe_posterior(bayMod, test = c("p_direction", "rope", "bayesfactor"))
discribePosteriors
print_md(discribePosteriors, digits = 2)
```

```
Parameter   | Median |          95% CI |     pd |          ROPE | % in ROPE |  Rhat |     ESS |
BF
-----------------------------------------------------------------------------------------------
----
(Intercept) |  25.04 | [15.88, 34.26] |   100% | [-1.38, 1.38] |        0% | 1.000 | 7747.00 | >
1000
dis         |  -0.10 | [-0.17, -0.02] | 99.66% | [-1.38, 1.38] |      100% | 1.000 | 9560.00 |
0.550
day         |  -0.99 | [-1.73, -0.23] | 99.46% | [-1.38, 1.38] |    86.38% | 1.000 | 8888.00 |
0.489
edu         |  -2.13 | [-3.87, -0.43] | 99.12% | [-1.38, 1.38] |    17.64% | 1.000 | 7937.00 |
0.332
bmi         |  -0.32 | [-0.57, -0.04] | 98.90% | [-1.38, 1.38] |      100% | 1.000 | 8582.00 |
```

Table 1: Summary of Posterior Distribution

Parameter	Median	95% CI	pd	ROPE	% in ROPE	Rhat	ESS	BF
(Intercept)	25.06	[15.99, 34.46]	100%	[-1.38, 1.38]	0%	1.000	8546.00	> 1000
dis	-0.10	[-0.18, -0.02]	99.42%	[-1.38, 1.38]	100%	1.000	10462.00	0.454
day	-0.98	[-1.73, -0.23]	99.58%	[-1.38, 1.38]	87.03%	1.000	11640.00	0.471
edu	-2.14	[-3.79, -0.31]	99.16%	[-1.38, 1.38]	18.47%	1.000	9242.00	0.348
bmi	-0.32	[-0.58, -0.05]	98.95%	[-1.38, 1.38]	100%	1.000	9496.00	0.258

Conclusion

The frequentist approach tries to estimate the actual effect; the models return a point- estimate, a single value, not the distribution of the correlation estimated under several assumptions. While the Bayesian method, based on the observed data and a prior belief about the result, the Bayesian sampling algorithm, MCMC sampling, returns a probability distribution called the posterior of the effect compatible with the observed data. Furthermore, to illustrate the statistical significance of effects, we do not use p- values. Instead, we describe the posterior distribution of the effect, reporting the median, the 89% Credible Interval, and other indices. We conclude that Bayesian methods provide enhanced reliability (Etz & Vandekerckhove, 2016), accuracy (Kruschke, Aguinis, & Joo, 2012), the possibility of introducing prior knowledge into the analysis (Andrews & Baguley, 2013; Kruschke et al., 2012), and provides intuitive results with clear interpretation (Kruschke, 2010; Wagenmakers et al., 2018).

Thank You!

Happy to answer any further questions.