1 Undulator and Electron Beam Parameters

Electron Energy	17.5	GeV
Bunch Charge (Q)	1.0	nC
Bunch Length ($l_{\rm lab}$)	25.0	μm
Beta Function (β_x)	32.0	m
Emittance (ϵ_x)	1.4	mm-$mrad$
Undulator Period ($\lambda_{\rm lab}$)	35.6	mm
Maximum Magnetic Field (B_0)	1.0	T

$$\gamma m_e \frac{d^2 x_{\rm lab}}{dt^2} = -Q v_z B_y = -Q B_0 v_z \cos\left(k_{\rm lab} z_{\rm lab}\right). \tag{1}$$

$$\frac{d^2 x_{\rm lab}}{dt^2} = -\frac{Q B_0}{\gamma m_e} \cos\left(k_{\rm lab} z_{\rm lab}\right), \tag{2}$$

where $z_{\rm lab} = v_{\rm lab} t = \beta c$.

$$x_{\rm lab} = \frac{Q B_0 v_z}{\gamma m_e} \frac{1}{k_{\rm lab}^2 v_z^2} \cos\left(k_{\rm lab} v_{\rm lab} t\right) = \frac{Q B_0}{\gamma m_e k_{\rm lab}^2 v_z} \cos\left(k_{\rm lab} v_{\rm lab} t\right). \tag{3}$$

Since $k_{\rm lab} = \gamma k_{\rm rest}$,

$$k_{\rm rest} = \frac{2\pi}{\lambda_{\rm rest}} = \frac{2\pi}{\lambda_{\rm lab}/\gamma} = \frac{2\pi\gamma}{\lambda_{\rm lab}}, \tag{4}$$

$$\omega_{\rm rest} = \frac{2\pi\gamma v_z}{\lambda_{\rm lab}} = \frac{2\pi(\beta\gamma)c}{\lambda_{\rm lab}}, \tag{5}$$

and

$$x_0 = \frac{Q B_0}{\gamma m_e k_{\rm lab}^2 v_z} = \frac{Q B_0}{\gamma^3 m_e k_{\rm rest}^2 v_z} = \frac{Q B_0}{\gamma^2 m_e k_{\rm rest}^2 (\beta\gamma) c}. \tag{6}$$

2 Beam Distribution

$$\rho(x,y) = \rho_0 \exp\left[-\frac{(x-a/2)^2}{2\sigma_x^2} - \frac{(y-b/2)^2}{2\sigma_y^2} \right]. \tag{7}$$

$$\begin{aligned}
Q_\lambda =& \frac{Q}{l_{\text{rest}}} = \frac{Q}{(\gamma l_{\text{lab}})} \\
=& \int d^2\vec{r}_\perp \rho(x,y) \\
=& \int d^2\vec{r}_\perp \rho_0 \exp\left[-\frac{(x-a/2)^2}{2\sigma_x^2} - \frac{(y-b/2)^2}{2\sigma_y^2} \right] \\
=& \rho_0 \left\{ \int_0^a dx \exp\left[-\frac{(x-a/2)^2}{2\sigma_x^2} \right] \right\} \left\{ \int_0^b dy \exp\left[-\frac{(y-b/2)^2}{2\sigma_y^2} \right] \right\} \\
=& 2\pi\sigma_x\sigma_y\rho_0 \left\{ \int_{-\infty}^a dx \rho_x(x) + \int_0^\infty dx \rho_x(x) - \int_{-\infty}^\infty dx \rho_x(x) \right\} \\
& \times \left\{ \int_{-\infty}^b dy \rho_y(y) + \int_0^\infty dy \rho_y(y) - \int_{-\infty}^\infty dy \rho_y(y) \right\} \\
=& 2\pi\sigma_x\sigma_y\rho_0 \left\{ \frac{1}{2}\left[1 + \text{erf}\left(\frac{a-a/2}{\sqrt{2\pi\sigma_x^2}} \right) \right] + \frac{1}{2}\left[1 + \text{erf}\left(\frac{0+a/2}{\sqrt{2\pi\sigma_x^2}} \right) \right] - 1 \right\} \\
& \times \left\{ \frac{1}{2}\left[1 + \text{erf}\left(\frac{b-b/2}{\sqrt{2\pi\sigma_y^2}} \right) \right] + \frac{1}{2}\left[1 + \text{erf}\left(\frac{0+b/2}{\sqrt{2\pi\sigma_y^2}} \right) \right] - 1 \right\} \\
=& 2\pi\sigma_x\sigma_y \left[\text{erf}\left(\frac{a/2}{\sqrt{2\pi\sigma_x^2}} \right) \text{erf}\left(\frac{b/2}{\sqrt{2\pi\sigma_y^2}} \right) \right] \rho_0,
\end{aligned} \tag{8}$$

where $\rho_x = \frac{1}{\sqrt{2\pi\sigma_x^2}} \exp\left[-\frac{(x-a/2)^2}{2\sigma_x^2} \right]$ and $\rho_y = \frac{1}{\sqrt{2\pi\sigma_y^2}} \exp\left[-\frac{(y-b/2)^2}{2\sigma_y^2} \right]$. And therefore,

$$\rho_0 = \frac{Q_\lambda}{2\pi\sigma_x\sigma_y} \left[\text{erf}\left(\frac{a/2}{\sqrt{2\pi\sigma_x^2}} \right) \text{erf}\left(\frac{b/2}{\sqrt{2\pi\sigma_y^2}} \right) \right]^{-1}, \tag{9}$$

and

$$\rho(x,y) = \frac{Q_\lambda}{2\pi\sigma_x\sigma_y} \left[\text{erf}\left(\frac{a/2}{\sqrt{2\pi\sigma_x^2}} \right) \text{erf}\left(\frac{b/2}{\sqrt{2\pi\sigma_y^2}} \right) \right]^{-1} \exp\left[-\frac{(x-a/2)^2}{2\sigma_x^2} - \frac{(y-b/2)^2}{2\sigma_y^2} \right]. \tag{10}$$

3 Test Particle Generation

$$
\begin{aligned}
N_\lambda(x,y) =& \frac{1}{Q_\lambda}\int d^2\vec{r}'_\perp \rho\left(x',y'\right) \\
=& \frac{\rho_0}{Q_\lambda}\left\{\int_0^x dx' \exp\left[-\frac{\left(x'-a/2\right)^2}{2\sigma_x^2}\right]\right\}\left\{\int_0^y dy' \exp\left[-\frac{\left(y'-b/2\right)^2}{2\sigma_y^2}\right]\right\} \\
=& \frac{\rho_0}{Q_\lambda}2\pi\sigma_x\sigma_y\left\{\int_{-\infty}^x dx'\rho_x(x') + \int_0^\infty dx'\rho_x(x') - \int_{-\infty}^\infty dx'\rho_x(x')\right\} \\
& \qquad \times \left\{\int_{-\infty}^y dy'\rho_y(y') + \int_0^\infty dy'\rho_y(y') - \int_{-\infty}^\infty dy'\rho_y(y')\right\} \\
=& \frac{\rho_0}{Q_\lambda}2\pi\sigma_x\sigma_y\left\{\frac{1}{2}\left[1+\operatorname{erf}\left(\frac{x-a/2}{\sqrt{2\pi\sigma_x^2}}\right)\right]+\frac{1}{2}\left[1+\operatorname{erf}\left(\frac{0+a/2}{\sqrt{2\pi\sigma_x^2}}\right)\right]-1\right\} \\
& \qquad \times \left\{\frac{1}{2}\left[1+\operatorname{erf}\left(\frac{y-b/2}{\sqrt{2\pi\sigma_y^2}}\right)\right]+\frac{1}{2}\left[1+\operatorname{erf}\left(\frac{0+b/2}{\sqrt{2\pi\sigma_y^2}}\right)\right]-1\right\} \\
=& \frac{\rho_0}{4Q_\lambda}2\pi\sigma_x\sigma_y\left[\operatorname{erf}\left(\frac{x-a/2}{\sqrt{2\pi\sigma_x^2}}\right)+\operatorname{erf}\left(\frac{a/2}{\sqrt{2\pi\sigma_x^2}}\right)\right]\left[\operatorname{erf}\left(\frac{y-b/2}{\sqrt{2\pi\sigma_y^2}}\right)+\operatorname{erf}\left(\frac{b/2}{\sqrt{2\pi\sigma_y^2}}\right)\right] \\
=& \frac{1}{4}\frac{\left[\operatorname{erf}\left(\frac{x-a/2}{\sqrt{2\pi\sigma_x^2}}\right)+\operatorname{erf}\left(\frac{a/2}{\sqrt{2\pi\sigma_x^2}}\right)\right]\left[\operatorname{erf}\left(\frac{y-b/2}{\sqrt{2\pi\sigma_y^2}}\right)+\operatorname{erf}\left(\frac{b/2}{\sqrt{2\pi\sigma_y^2}}\right)\right]}{\operatorname{erf}\left(\frac{a/2}{\sqrt{2\pi\sigma_x^2}}\right)\operatorname{erf}\left(\frac{b/2}{\sqrt{2\pi\sigma_y^2}}\right)}.
\end{aligned}
\tag{11}
$$

N_λ can be separated as $N_\lambda = N_x(x)N_y(y)$, where

$$
\begin{aligned}
N_x(x) =& \frac{1}{2}\frac{\operatorname{erf}\left(\frac{x-a/2}{\sqrt{2\pi\sigma_x^2}}\right)+\operatorname{erf}\left(\frac{a/2}{\sqrt{2\pi\sigma_x^2}}\right)}{\operatorname{erf}\left(\frac{a/2}{\sqrt{2\pi\sigma_x^2}}\right)}, \\
N_y(y) =& \frac{1}{2}\frac{\operatorname{erf}\left(\frac{y-b/2}{\sqrt{2\pi\sigma_y^2}}\right)+\operatorname{erf}\left(\frac{b/2}{\sqrt{2\pi\sigma_y^2}}\right)}{\operatorname{erf}\left(\frac{b/2}{\sqrt{2\pi\sigma_y^2}}\right)}.
\end{aligned}
\tag{12}
$$

4 Electromagnetic Potentials

$$\begin{aligned}\phi\left(\vec{r},t\right)=&\frac{1}{\epsilon_0}\sum_{mn}\sum_{l=0}^{\infty}\sum_{j=0}^{l}C_j^l\frac{(-x_0)^l}{l!}\frac{1}{2^l}\int d^2\vec{r}'_\perp\psi_{mn}\left(\vec{r}_\perp\right)\psi^*_{mn}\left(\vec{r}'_\perp\right)\frac{\partial^l\rho\left(x',y'\right)}{\partial x'^l}\\&\times\frac{\exp\left[i(2j-l)(kz+\omega t)\right]}{(2j-l)^2\left(1-\beta^2\right)k^2+k^2_{\perp mn}}.\end{aligned}\tag{13}$$

$$\begin{aligned}A_x\left(\vec{r},t\right)=&-\mu_0x_0\omega\sum_{mn}\sum_{l=0}^{\infty}\sum_{j=0}^{l}C_j^l\frac{(-x_0)^l}{l!}\frac{1}{2^l2i}\int d^2\vec{r}'_\perp\psi_{mn}\left(\vec{r}_\perp\right)\psi^*_{mn}\left(\vec{r}'_\perp\right)\frac{\partial^l\rho\left(x',y'\right)}{\partial x'^l}\\&\times\left[\frac{\exp\left[i(2j-l+1)(kz+\omega t)\right]}{(2j-l+1)^2\left(1-\beta^2\right)k^2+k^2_{\perp mn}}-\frac{\exp\left[i(2j-l-1)(kz+\omega t)\right]}{(2j-l-1)^2\left(1-\beta^2\right)k^2+k^2_{\perp mn}}\right]\end{aligned}\tag{14}$$

For a rectangular pipe,

$$\begin{aligned}\psi^{\phi}_{mn}\left(\vec{r}_\perp\right)=&\frac{2}{\sqrt{ab}}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right),\\\psi^{A_x}_{mn}\left(\vec{r}_\perp\right)=&\frac{2}{\sqrt{ab}}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right),\end{aligned}\tag{15}$$

and including only $l=0$ and $l=1$ for ϕ and $l=0$ for A_x, then

$$\begin{aligned}\phi\left(\vec{r},t\right)=&\frac{4}{\epsilon_0ab}\sum_{mn}\frac{\sin\left(m\pi x/a\right)\sin\left(n\pi y/b\right)}{\left(m\pi/a\right)^2+\left(n\pi/b\right)^2}\int d^2\vec{r}'_\perp\sin\left(\frac{m\pi x'}{a}\right)\sin\left(\frac{n\pi y'}{b}\right)\rho\left(x',y'\right)\\&+\frac{4}{\epsilon_0ab}\left[x_0\cos(kz+\omega t)\right]\sum_{mn}\left(\frac{m\pi}{a}\right)\frac{\sin\left(m\pi x/a\right)\sin\left(n\pi y/b\right)}{\left(1-\beta^2\right)k^2+\left[\left(m\pi/a\right)^2+\left(n\pi/b\right)^2\right]}\\&\times\int d^2\vec{r}'_\perp\cos\left(\frac{m\pi x'}{a}\right)\sin\left(\frac{n\pi y'}{b}\right)\rho\left(x',y'\right)\end{aligned}\tag{16}$$

and

$$\begin{aligned}A_x\left(\vec{r},t\right)=&-\frac{4\mu_0}{ab}\left[x_0\omega\sin(kz+\omega t)\right]\sum_{mn}\left(\frac{m\pi}{a}\right)\frac{\cos\left(m\pi x/a\right)\sin\left(n\pi y/b\right)}{\left(1-\beta^2\right)k^2+\left[\left(m\pi/a\right)^2+\left(n\pi/b\right)^2\right]}\\&\times\int d^2\vec{r}'_\perp\cos\left(\frac{m\pi x'}{a}\right)\sin\left(\frac{n\pi y'}{b}\right)\rho\left(x',y'\right)\end{aligned}\tag{17}$$

5 Electromagnetic Fields

$$\begin{aligned}
\mathbf{E}(x,y,z) &= \Big\{E_{0x}(x,y) + E_{1x}(x,y)\left[x_0\cos(kz+\omega t)\right]\Big\}\hat{e}_x \\
&+ \Big\{E_{0y}(x,y) + E_{1y}(x,y)\left[x_0\cos(kz+\omega t)\right]\Big\}\hat{e}_y \\
&+ E_{1z}(x,y)\left[x_0 k\sin(kz+\omega t)\right]\hat{e}_z \\
\mathbf{B}(x,y,z) &= B_{1y}(x,y)\left[x_0\omega k\cos(kz+\omega t)\right]\hat{e}_y \\
&+ B_{1z}(x,y)\left[x_0\omega\sin(kz+\omega t)\right]\hat{e}_z
\end{aligned} \tag{18}$$

Define

$$E_0 = \frac{2Q_\lambda}{\pi^3\epsilon_0\sigma_x\sigma_y}\left[\mathrm{erf}\left(\frac{a/2}{\sqrt{2\pi\sigma_x}}\right)\mathrm{erf}\left(\frac{b/2}{\sqrt{2\pi\sigma_y}}\right)\right]^{-1} \tag{19}$$

$$\begin{aligned}
E_x &= -\frac{\partial\phi}{\partial x} - \frac{\partial A_x}{\partial t} \\
&= -\frac{\pi^2}{ab}E_0\sum_{mn}\left(\frac{m\pi}{a}\right)\frac{\cos\left(m\pi x/a\right)\sin\left(n\pi y/b\right)}{\left(m\pi/a\right)^2+\left(n\pi/b\right)^2} \\
&\quad\times\int d^2\vec{r}'_\perp \sin\left(\frac{m\pi x'}{a}\right)\sin\left(\frac{n\pi y'}{b}\right)\exp\left[-\frac{\left(x'-a/2\right)^2}{2\sigma_x^2}-\frac{\left(y'-b/2\right)^2}{2\sigma_y^2}\right] \\
&\quad-\frac{\pi^2}{ab}E_0\left[x_0\cos(kz+\omega t)\right]\sum_{mn}\left[\left(\frac{m\pi}{a}\right)^2-\left(\frac{\omega}{c}\right)^2\right]\frac{\cos\left(m\pi x/a\right)\sin\left(n\pi y/b\right)}{\left(1-\beta^2\right)k^2+\left[\left(m\pi/a\right)^2+\left(n\pi/b\right)^2\right]} \\
&\quad\times\int d^2\vec{r}'_\perp \cos\left(\frac{m\pi x'}{a}\right)\sin\left(\frac{n\pi y'}{b}\right)\exp\left[-\frac{\left(x'-a/2\right)^2}{2\sigma_x^2}-\frac{\left(y'-b/2\right)^2}{2\sigma_y^2}\right] \\
&= -E_0\sum_{mn}\left(\frac{m}{\tilde{a}}\right)\frac{\cos\left(m\tilde{x}\right)\sin\left(n\tilde{y}\right)}{\left(m/\tilde{a}\right)^2+\left(n/\tilde{b}\right)^2} \\
&\quad\times\int d^2\vec{\tilde{r}}'_\perp \sin\left(m\tilde{x}'\right)\sin\left(n\tilde{y}'\right)\exp\left[-\frac{\tilde{a}^2}{2\sigma_x^2}\left(\tilde{x}'-\frac{\pi}{2}\right)^2-\frac{\tilde{b}^2}{2\sigma_y^2}\left(\tilde{y}'-\frac{\pi}{2}\right)^2\right] \\
&\quad-E_0\left[x_0\cos(kz+\omega t)\right]\sum_{mn}\left[\left(\frac{m}{\tilde{a}}\right)^2-\left(\frac{\omega}{c}\right)^2\right]\frac{\cos\left(m\tilde{x}\right)\sin\left(n\tilde{y}\right)}{\left(1-\beta^2\right)k^2+\left[\left(m/\tilde{a}\right)^2+\left(n/\tilde{b}\right)^2\right]} \\
&\quad\times\int d^2\vec{\tilde{r}}'_\perp \cos\left(m\tilde{x}'\right)\sin\left(n\tilde{y}'\right)\exp\left[-\frac{\tilde{a}^2}{2\sigma_x^2}\left(\tilde{x}'-\frac{\pi}{2}\right)^2-\frac{\tilde{b}^2}{2\sigma_y^2}\left(\tilde{y}'-\frac{\pi}{2}\right)^2\right].
\end{aligned} \tag{20}$$

$$
\begin{aligned}
E_y &= -\frac{\partial \phi}{\partial y} \\
&= -\frac{\pi^2}{ab} E_0 \sum_{mn} \left(\frac{n\pi}{b}\right) \frac{\sin\left(m\pi x/a\right)\cos\left(n\pi y/b\right)}{\left(m\pi/a\right)^2 + \left(n\pi/b\right)^2} \\
&\quad\times \int d^2\vec{r}'_\perp \sin\left(\frac{m\pi x'}{a}\right) \sin\left(\frac{n\pi y'}{b}\right) \exp\left[-\frac{\left(x'-a/2\right)^2}{2\sigma_x^2} - \frac{\left(y'-b/2\right)^2}{2\sigma_y^2}\right] \\
&\quad -\frac{\pi^2}{ab} E_0 \left[x_0 \cos(kz+\omega t)\right] \sum_{mn} \left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right) \frac{\sin\left(m\pi x/a\right)\cos\left(n\pi y/b\right)}{\left(1-\beta^2\right)k^2 + \left[\left(m\pi/a\right)^2 + \left(n\pi/b\right)^2\right]} \\
&\quad\times \int d^2\vec{r}'_\perp \cos\left(\frac{m\pi x'}{a}\right) \sin\left(\frac{n\pi y'}{b}\right) \exp\left[-\frac{\left(x'-a/2\right)^2}{2\sigma_x^2} - \frac{\left(y'-b/2\right)^2}{2\sigma_y^2}\right] \qquad (21) \\
&= -E_0 \sum_{mn} \left(\frac{n}{\tilde{b}}\right) \frac{\sin\left(m\tilde{x}\right)\cos\left(n\tilde{y}\right)}{\left(m/\tilde{a}\right)^2 + \left(n/\tilde{b}\right)^2} \\
&\quad\times \int d^2\vec{\tilde{r}}'_\perp \sin\left(m\tilde{x}'\right) \sin\left(n\tilde{y}'\right) \exp\left[-\frac{\tilde{a}^2}{2\sigma_x^2}\left(\tilde{x}'-\frac{\pi}{2}\right)^2 - \frac{\tilde{b}^2}{2\sigma_y^2}\left(\tilde{y}'-\frac{\pi}{2}\right)^2\right] \\
&\quad -E_0 \left[x_0 \cos(kz+\omega t)\right] \sum_{mn} \left(\frac{m}{\tilde{a}}\right)\left(\frac{n}{\tilde{b}}\right) \frac{\sin\left(m\tilde{x}\right)\cos\left(n\tilde{y}\right)}{\left(1-\beta^2\right)k^2 + \left[\left(m/\tilde{a}\right)^2 + \left(n/\tilde{b}\right)^2\right]} \\
&\quad\times \int d^2\vec{\tilde{r}}'_\perp \cos\left(m\tilde{x}'\right) \sin\left(n\tilde{y}'\right) \exp\left[-\frac{\tilde{a}^2}{2\sigma_x^2}\left(\tilde{x}'-\frac{\pi}{2}\right)^2 - \frac{\tilde{b}^2}{2\sigma_y^2}\left(\tilde{y}'-\frac{\pi}{2}\right)^2\right].
\end{aligned}
$$

$$
\begin{aligned}
E_z &= -\frac{\partial \phi}{\partial z} \\
&= \frac{\pi^2}{ab} E_0 \left[x_0 k \sin(kz+\omega t)\right] \sum_{mn} \left(\frac{m\pi}{a}\right) \frac{\sin\left(m\pi x/a\right)\sin\left(n\pi y/b\right)}{\left(1-\beta^2\right)k^2 + \left[\left(m\pi/a\right)^2 + \left(n\pi/b\right)^2\right]} \\
&\quad\times \int d^2\vec{r}'_\perp \cos\left(\frac{m\pi x'}{a}\right) \sin\left(\frac{n\pi y'}{b}\right) \exp\left[-\frac{\left(x'-a/2\right)^2}{2\sigma_x^2} - \frac{\left(y'-b/2\right)^2}{2\sigma_y^2}\right] \qquad (22) \\
&= E_0 \left[x_0 k \sin(kz+\omega t)\right] \sum_{mn} \left(\frac{m}{\tilde{a}}\right) \frac{\sin\left(m\tilde{x}\right)\sin\left(n\tilde{y}\right)}{\left(1-\beta^2\right)k^2 + \left[\left(m/\tilde{a}\right)^2 + \left(n/\tilde{b}\right)^2\right]} \\
&\quad\times \int d^2\vec{\tilde{r}}'_\perp \cos\left(m\tilde{x}'\right) \sin\left(n\tilde{y}'\right) \exp\left[-\frac{\tilde{a}^2}{2\sigma_x^2}\left(\tilde{x}'-\frac{\pi}{2}\right)^2 - \frac{\tilde{b}^2}{2\sigma_y^2}\left(\tilde{y}'-\frac{\pi}{2}\right)^2\right].
\end{aligned}
$$

$$
\begin{aligned}
B_y =& \frac{\partial A_x}{\partial z} \\
=& -\frac{\pi^2}{ab}\frac{E_0}{c^2}\left[x_0\omega k\cos(kz+\omega t)\right]\sum_{mn}\frac{\cos\left(m\pi x/a\right)\sin\left(n\pi y/b\right)}{\left(1-\beta^2\right)k^2+\left[\left(m\pi/a\right)^2+\left(n\pi/b\right)^2\right]} \\
&\times\int d^2\vec{r}'_\perp\cos\left(\frac{m\pi x'}{a}\right)\sin\left(\frac{n\pi y'}{b}\right)\exp\left[-\frac{\left(x'-a/2\right)^2}{2\sigma_x^2}-\frac{\left(y'-b/2\right)^2}{2\sigma_y^2}\right] \\
=& -\frac{E_0}{c^2}\left[x_0\omega k\cos(kz+\omega t)\right]\sum_{mn}\frac{\cos\left(m\tilde{x}\right)\sin\left(n\tilde{y}\right)}{\left(1-\beta^2\right)k^2+\left[\left(m/\tilde{a}\right)^2+\left(n/\tilde{b}\right)^2\right]} \\
&\times\int d^2\vec{\tilde{r}}'_\perp\cos\left(m\tilde{x}'\right)\sin\left(n\tilde{y}'\right)\exp\left[-\frac{\tilde{a}^2}{2\sigma_x^2}\left(\tilde{x}'-\frac{\pi}{2}\right)^2-\frac{\tilde{b}^2}{2\sigma_y^2}\left(\tilde{y}'-\frac{\pi}{2}\right)^2\right].
\end{aligned}
\tag{23}
$$

$$
\begin{aligned}
B_z =& -\frac{\partial A_x}{\partial y} \\
=& \frac{\pi^2}{ab}\frac{E_0}{c^2}\left[x_0\omega\sin(kz+\omega t)\right]\sum_{mn}\left(\frac{n\pi}{b}\right)\frac{\cos\left(m\pi x/a\right)\cos\left(n\pi y/b\right)}{\left(1-\beta^2\right)k^2+\left[\left(m\pi/a\right)^2+\left(n\pi/b\right)^2\right]} \\
&\times\int d^2\vec{r}'_\perp\cos\left(\frac{m\pi x'}{a}\right)\sin\left(\frac{n\pi y'}{b}\right)\exp\left[-\frac{\left(x'-a/2\right)^2}{2\sigma_x^2}-\frac{\left(y'-b/2\right)^2}{2\sigma_y^2}\right] \\
=& \frac{E_0}{c^2}\left[x_0\omega\sin(kz+\omega t)\right]\sum_{mn}\left(\frac{n}{\tilde{b}}\right)\frac{\cos\left(m\tilde{x}\right)\cos\left(n\tilde{y}\right)}{\left(1-\beta^2\right)k^2+\left[\left(m/\tilde{a}\right)^2+\left(n/\tilde{b}\right)^2\right]} \\
&\times\int d^2\vec{\tilde{r}}'_\perp\cos\left(m\tilde{x}'\right)\sin\left(n\tilde{y}'\right)\exp\left[-\frac{\tilde{a}^2}{2\sigma_x^2}\left(\tilde{x}'-\frac{\pi}{2}\right)^2-\frac{\tilde{b}^2}{2\sigma_y^2}\left(\tilde{y}'-\frac{\pi}{2}\right)^2\right].
\end{aligned}
\tag{24}
$$

6 Electromagnetic Field Components

$$
\begin{aligned}
E_{0x}(x,y) =& -\frac{\pi^2}{ab}E_0\sum_{mn}\left(\frac{m\pi}{a}\right)\frac{\cos\left(m\pi x/a\right)\sin\left(n\pi y/b\right)}{\left(m\pi/a\right)^2+\left(n\pi/b\right)^2} \\
&\times\int d^2\vec{r}'_\perp \sin\left(\frac{m\pi x'}{a}\right)\sin\left(\frac{n\pi y'}{b}\right)\exp\left[-\frac{\left(x'-a/2\right)^2}{2\sigma_x^2}-\frac{\left(y'-b/2\right)^2}{2\sigma_y^2}\right] \\
E_{1x}(x,y) =& -\frac{\pi^2}{ab}E_0\sum_{mn}\left[\left(\frac{m\pi}{a}\right)^2-\left(\frac{\omega}{c}\right)^2\right]\frac{\cos\left(m\pi x/a\right)\sin\left(n\pi y/b\right)}{\left(1-\beta^2\right)k^2+\left[\left(m\pi/a\right)^2+\left(n\pi/b\right)^2\right]} \\
&\times\int d^2\vec{r}'_\perp \cos\left(\frac{m\pi x'}{a}\right)\sin\left(\frac{n\pi y'}{b}\right)\exp\left[-\frac{\left(x'-a/2\right)^2}{2\sigma_x^2}-\frac{\left(y'-b/2\right)^2}{2\sigma_y^2}\right] \\
E_{0y}(x,y) =& -\frac{\pi^2}{ab}E_0\sum_{mn}\left(\frac{n\pi}{b}\right)\frac{\sin\left(m\pi x/a\right)\cos\left(n\pi y/b\right)}{\left(m\pi/a\right)^2+\left(n\pi/b\right)^2} \\
&\times\int d^2\vec{r}'_\perp \sin\left(\frac{m\pi x'}{a}\right)\sin\left(\frac{n\pi y'}{b}\right)\exp\left[-\frac{\left(x'-a/2\right)^2}{2\sigma_x^2}-\frac{\left(y'-b/2\right)^2}{2\sigma_y^2}\right] \\
E_{1y}(x,y) =& -\frac{\pi^2}{ab}E_0\sum_{mn}\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right)\frac{\sin\left(m\pi x/a\right)\cos\left(n\pi y/b\right)}{\left(1-\beta^2\right)k^2+\left[\left(m\pi/a\right)^2+\left(n\pi/b\right)^2\right]} \\
&\times\int d^2\vec{r}'_\perp \cos\left(\frac{m\pi x'}{a}\right)\sin\left(\frac{n\pi y'}{b}\right)\exp\left[-\frac{\left(x'-a/2\right)^2}{2\sigma_x^2}-\frac{\left(y'-b/2\right)^2}{2\sigma_y^2}\right] \\
E_{1z}(x,y) =& \frac{\pi^2}{ab}E_0\sum_{mn}\left(\frac{m\pi}{a}\right)\frac{\sin\left(m\pi x/a\right)\sin\left(n\pi y/b\right)}{\left(1-\beta^2\right)k^2+\left[\left(m\pi/a\right)^2+\left(n\pi/b\right)^2\right]} \\
&\times\int d^2\vec{r}'_\perp \cos\left(\frac{m\pi x'}{a}\right)\sin\left(\frac{n\pi y'}{b}\right)\exp\left[-\frac{\left(x'-a/2\right)^2}{2\sigma_x^2}-\frac{\left(y'-b/2\right)^2}{2\sigma_y^2}\right] \\
B_{1y}(x,y) =& -\frac{\pi^2}{ab}\frac{E_0}{c^2}\pi ab\sigma_x\sigma_y\sum_{mn}\frac{\cos\left(m\pi x/a\right)\sin\left(n\pi y/b\right)}{\left(1-\beta^2\right)k^2+\left[\left(m\pi/a\right)^2+\left(n\pi/b\right)^2\right]} \\
&\times\int d^2\vec{r}'_\perp \cos\left(\frac{m\pi x'}{a}\right)\sin\left(\frac{n\pi y'}{b}\right)\exp\left[-\frac{\left(x'-a/2\right)^2}{2\sigma_x^2}-\frac{\left(y'-b/2\right)^2}{2\sigma_y^2}\right] \\
B_{1z}(x,y) =& \frac{\pi^2}{ab}\frac{E_0}{c^2}\pi ab\sigma_x\sigma_y\sum_{mn}\left(\frac{n\pi}{b}\right)\frac{\cos\left(m\pi x/a\right)\cos\left(n\pi y/b\right)}{\left(1-\beta^2\right)k^2+\left[\left(m\pi/a\right)^2+\left(n\pi/b\right)^2\right]} \\
&\times\int d^2\vec{r}'_\perp \cos\left(\frac{m\pi x'}{a}\right)\sin\left(\frac{n\pi y'}{b}\right)\exp\left[-\frac{\left(x'-a/2\right)^2}{2\sigma_x^2}-\frac{\left(y'-b/2\right)^2}{2\sigma_y^2}\right]
\end{aligned}
\tag{25}
$$