EFFICIENT DISTRIBUTED RENDEZVOUS SCHEMES AND SPECTRUM MANAGEMENT FOR COGNITIVE RADIO NETWORKS

by

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ABSTRACT

JI LI. Efficient Distributed Rendezvous Schemes and Spectrum Management for Cognitive Radio Networks. (Under the direction of DR. JIANG (LINDA) XIE)

Cognitive radio emerges as a technology to realize the dynamic spectrum access by dynamically configuring its transmission parameters. With cognitive radios, unlicensed users can opportunistically access their available channels, which can improve the spectrum utilization. In a cognitive radio network (CRN), there are two types of users: primary users (PUs) and secondary users (SUs). PUs are the licensed users or the traditional wireless users who can access a specific licensed spectrum band. SUs are the unlicensed users equipped with cognitive radios which can opportunistically use currently unoccupied channels to transmit, but have to vacate channels for the returning PUs and then switch to other available channels for continuous transmissions. When two SUs want to establish a link, they have to meet on the same channel which must be available for both of them simultaneously. This process is called *rendezvous*.

Past research works on rendezvous only focused on designing the channel hopping sequence for the rendezvous process while ignoring some practical problems like rendezvous in wide-band CRNs, rendezvous without a predetermined sender and receiver, rendezvous considering directional antennas, and how to maximize the number of common available channels. In this research, we propose five schemes to realize efficient rendezvous and spectrum management considering these practical problems under different scenarios. We first propose a rendezvous and communication framework for wide-band CRNs. Furthermore, we propose two efficient rendezvous schemes without predetermined sender and receiver. Moreover, we propose a rendezvous scheme specifically for SUs equipped with directional antennas. Last, we propose a power control protocol to maximize the number of common available channels. All of the proposed schemes can realize efficient rendezvous and or spectrum management with

practical assumptions under different scenarios.

CHAPTER 1: INTRODUCTION

1.1 Background of Cognitive Radio Networks

According to the Federal Communications Commission (FCC), up to 85% of the spectrum allocated to existing wireless services is under utilized [1]. Cognitive radio is a promising technology which can realize the dynamic spectrum access to improve the utilization of the allocated spectrum [2]. In a cognitive radio network (CRN), unlicensed users, called secondary users (SUs), coexist with licensed users, called primary users (PUs). SUs are equipped with one or multiple cognitive radios which can opportunistically access the currently available channels, but have to vacate the channels for the returning PUs [3] [4]. In order to avoid generating interference to PUs, an SU should perform *spectrum sensing* to get its available channels before accessing any channel.

There are mainly three spectrum sensing methods: matched filter detection, energy detection, and cyclostationary feature detection [3]. Among these three, energy detection is the easiest one to implement. For energy detection, an SU calculates the energy of the signal received from a channel. If the energy is higher than a sensing threshold, the SU thinks that this channel is busy or unavailable [5, 6]. In addition, the sensing range of an SU is the range within which the SU can sense the channels occupied by PUs. Thus, the sensing range determines the available channels of a SU. The larger the sensing range, the more unavailable channels that could be sensed by a SU.

In a CRN, two SUs should only communicate through a channel which is available to both simultaneously to avoid the interference to PUs. We call this kind of channel the common available channel. A communicating SU pair could have several common available channels at a specific time. If the current occupied common available channel suddenly becomes unavailable for one or both of the two SUs, in order to finish the current transmission, the two SUs should hop to the next common available channel. The process of vacating the currently using channel and then switching to a new available channel is called *spectrum handoff* [3].

The available channel sets of a communicating SU pair are the important resources for establishing a physical link [7]. According to [8], there are two models based on the available channel sets of an SU pair, the *symmetrical model*: the available channel sets of any SU pair are the same, and the *asymmetrical model*: the available channel sets of any SU pair are different. In order to establish a link between two SUs, they should first meet on a common available channel. This process is called *rendezvous*.

Two SUs in a CRN should distributed perform a blind rendezvous process to establish a link on a common available channel. During a blind rendezvous process, an SU hops to a different channel at the beginning of each time slot by following a channel hopping sequence and sends out a Request-to-Send (RTS) message on each channel. If the SU receives a Clear-to-Send (CTS) message from another SU during the same time slot, it indicates that the two SUs meet on the same channel and the rendezvous succeeds. Since no information exchange is needed initially, a blind rendezvous is practical for CRNs. However, it is very challenging to design an efficient rendezvous algorithm that can guarantee rendezvous without knowing any information of other SUs. In order to evaluate a rendezvous scheme, three performance metrics are defined [9]: TTR, $MTTR$, and $ETTR$, where TTR is the time to rendezvous from the moment the channel hopping starts until the moment two SUs meet on a common available channel, $MTTR$ is the maximum possible time required to have a successful rendezvous, and $ETTR$ is the expectation of the time to a successful rendezvous.

1.2 Research Motivation

1.2.1 Framework for Wide-band Cognitive Radio Networks

The original purpose of cognitive radio is to improve the utilization of the spectrum which has been allocated. However, according to the FCC, the allocated spectrum, ranging from 3KHz to 300GHz, is very wide. Though we cannot expect cognitive radio to work in that wide spectrum, the number of channels a cognitive radio can access may be hundreds or thousands. For instance, according to [10], the TV band which can be utilized by SUs is from 54MHz to 862MHz and the bandwidth of each channel is 6MHz. This corresponds to a total of 134 channels. With the increase of the number of channels in a CRN, the time for an SU to sense the whole band and the energy consumed in the sensing process will increase. In [11], the importance of considering the wide-band scenario is explored and a scheme for mobile access of wide-band networks is proposed. However, to the best of our knowledge, there are no existing papers which can efficiently solve the communication problems in CRNs coming from the wide-band spectrum without a (common control channel) CCC. Therefore, addressing the communication problems stemming from the wide-band spectrum is necessary.

In [8, 12–20], several rendezvous schemes for CRNs are proposed. However, these schemes only focus on the rendezvous problem itself while ignoring the following practical communication scenarios in wide-band CRNs. First, they do not consider the wide-band scenario. All of them just consider that there are only a small number of channels (at most tens of channels) in the network and SUs will utilize all the channels to design the channel hopping sequence and perform channel hopping, while there may be hundreds of channels for a CRN. As the number of channels increases, the time to sense all the channels to get available channels and to rendezvous will increase significantly, according to the results shown in [9]. Second, they do not consider the scenario that time slots of two SUs are not perfectly aligned which may make a rendezvous unsuccessful, and the optimal time slot length to guarantee a successful rendezvous. Thus, a practical communication framework for wide-band CRNs without a CCC that can address the above problems is needed.

1.2.2 Rendezvous Schemes Without Predetermined Sender and Receiver

The past works in rendezvous only consider how to make two or multiple SUs rendezvous on a common available channel within bounded time slots, which is defined as the *channel rendezvous* problem in this research. However, the process of sending or receiving RTS or CTS packets is not considered. Past rendezvous schemes implicitly require a sender-and-receiver relation between the two rendezvous SUs, i.e., an SU sender always sends an RTS packet on each of its hopped channels during each time slot, while the other SU always listens on each of its hopped channels. If the other SU receives an RTS packet, it replies a CTS packet to set up a link with the SU sender. However, during the initialization phase of a CRN, every SU tries to rendezvous with other SUs to exchange their control information. There is no explicit sender or receiver role such that an SU is always a sender or receiver, since it cannot know if other SUs are sending or waiting for RTS packets. Therefore, for a SU, whether tuning its half-duplex radio to the sending or receiving mode during a time slot is a problem, which is defined as the *send-or-receive problem* in this research.

This *send-or-receive problem* is a practical problem when each SU is equipped with multiple cognitive radios or a single radio. For an SU equipped with a single radio, it can only be a sender or receiver during a time slot. When considering multiple radios, an SU can be a sender and receiver simultaneously by assigning sending or receiving tasks to different radios. Past rendezvous schemes considering multiple cognitive radios only focus on the *channel rendezvous* between two radios of two SUs [21]. They define a successful rendezvous between two SUs when any two radios of the two SUs hop to a common available channel at the same time slot. However, there also exists the *send-or-receive problem* for the two multiple-radio SUs. Therefore, when considering multiple-radio rendezvous, how to assign a sending or receiving task to each radio is a problem which has never been addressed in previous rendezvous schemes.

1.2.3 Rendezvous Process Considering Directional Antennas

Previous works in rendezvous only consider how to let two or multiple SUs rendezvous on a common available channel within a bounded time, when SUs are equipped with omni-directional antennas. However, omni-directional antennas may cause interference to all the PUs within the entire transmission area of an SU simultaneously, since omni-directional antennas transmit towards all directions. Especially, if the distance between two SUs increases, in order to keep them connected, the SUs increase their transmission power. As a result, the transmission range of an SU increases, which will cause interference to more PUs. Even though previous rendezvous schemes assume that during each time slot, an SU should first sense a channel before accessing it, PUs can still return to that channel during the transmission period of an SU as PUs' traffic is unknown to SUs. Therefore, in a CRN, we should try to decrease the number of PUs who could potentially be interfered by a SU's rendezvous process.

One way to overcome the above problem is to equip each SU a directional antenna. With a directional antenna, an SU can transmit toward a specific direction with a certain angle. The area covered by the signals from a directional antenna is called a *transmission sector*. Therefore, an SU with a directional antenna can only cause interference to PUs within its transmission sector rather than the whole transmission area during each time slot. Another benefit of the directional antenna is that after a successful rendezvous, two SUs can know each other's location. For the data transmission or the next rendezvous process, two SUs can just tune their directional antennas to the transmission sectors which can cover each other.

However, the blind rendezvous problem with directional antennas has not been considered in the literature yet. In this research, we propose fully distributed rendezvous schemes for SUs equipped with directional antennas. We first propose a new rendezvous problem called the *sector rendezvous problem* between two SUs. For a sector rendezvous process, there exist the *indexing problem* and *sector number* problem that make the sector rendezvous problem different from the existing channel rendezvous problem. We cannot directly apply existing channel hopping sequences in [8, 14, 15, 17, 22–26] to the sector rendezvous problem since they do not consider the indexing problem and assume that each SU can access the same number of channels. In order to tackle the sector rendezvous problem, we design fully distributed sector rendezvous schemes for an SU only based on the SU's own information. Our proposed schemes can guarantee successful sector rendezvous and channel rendezvous simultaneously within a bounded time. The proposed sector rendezvous schemes can work on top of any existing channel hopping scheme which guarantees a successful channel rendezvous within a bounded time.

1.2.4 Power Control to Maximize the Number of Common Available Channels

The available channels of an SU are critical for its transmission because with more available channels, the SU can have more channel sources to choose. Moreover, in a CRN, it is necessary for two SUs to rendezvous on the same available channel to communicate with each other. More common available channels also mean more opportunities for the two SUs to meet on the same channel. Therefore, the most important factor for an SU communicating pair is their common available channels rather than their own available channels. In CRNs, the number of the common available channels plays an important role in many network operations such as channel rendezvous, spectrum handoff, and broadcast.

In a cognitive radio network, in order to know which channels are available currently, an SU has to sense the spectrum. The sensing range of an SU is the range within which the SU can sense the channels occupied by PUs. Thus, the sensing range determines the available channels of a SU. The larger the sensing range, the more unavailable channels that could be sensed by a SU. Now, the issue is how to determine the sensing ranges of an SU sender and SU receiver in order to get a maximum number of common available channels? However, none of the existing papers in rendezvous or spectrum handoff consider improving the performance by increasing the number of available channels, especially the common available channels between two SUs in CRNs. In this research, we first illustrate the relationship among the number of common available channels, SU transmission power, sensing threshold, and sensing range of an SU sender and receiver. Then, we propose a power control protocol to maximize the number of common available channels between two SUs in CRNs. Our proposed protocol is practical and easy to implement.

1.3 Overview of Proposed Research

Fig. 1.1 shows the overview of our proposed Research. In order to achieve efficient rendezvous and spectrum management in CRNs, we first propose a rendezvous framework for wide-band cognitive radio networks considering hundreds of channels. In this framework, we first propose a spectrum split scheme to split the wide-band spectrum into several spectrum segments and map each SU to a specific spectrum segment. Next, we propose an efficient rendezvous scheme specific for this framework considering spectrum splitting which enables two SUs to reach a fast and successful rendezvous. In the designed framework, we assume that a CCC is not required, there is not a central controller or base station for information exchange, and each SU does not know any information about the other SU before they achieve a successful rendezvous. All these assumptions are very practical concerning the characteristics and limitations of cognitive radio networks.

Next, we propose two efficient rendezvous schemes under the scenarios that there is not a predetermined sender or receiver for a rendezvous process during the initialization phase of cognitive radio networks, which is very practical. Under this scenario,

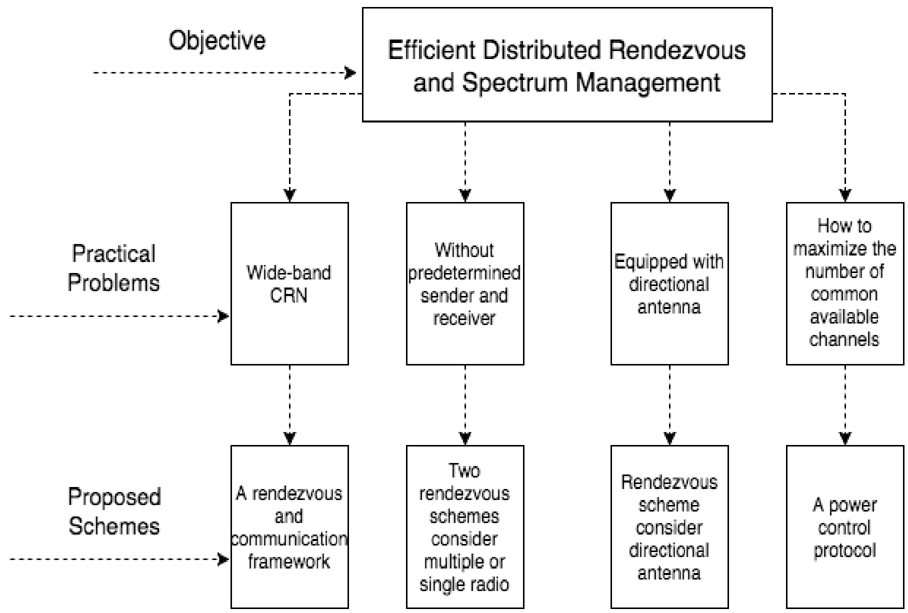

Figure 1.1: Overview of the proposed research

we define a new type of rendezvous called *link rendezvous*. In order to design the rendezvous schemes to achieve successful link rendezvous, we designed two innovative spectrum management schemes called *channel group* and *virtual channels*. In the proposed rendezvous schemes, we also assume each SU do not know any control information about the other SUs, which is very practical.

Moreover, we propose an efficient rendezvous scheme specific for SUs which are equipped directional antennas. Under this scenario, the rendezvous process could be more completed since each SU could be equipped with heterogeneous directional antenna and it does not know any information about the other SUs. We define a new type of rendezvous called *sector rendezvous* when considering directional antennas. Our propose rendezvous scheme can efficiently solve the sector rendezvous problem under practical scenarios.

Last, we propose a power control protocol which can maximize the number of common available channels between two SUs. Common available channels between two SUs are critical in achieving a successful rendezvous and spectrum handoff. In the proposed scheme, after two SUs successfully achieve a rendezvous, they first exchange some control information, based on which they could perform distributed power control to maximize the number of common available channels, which could improve the performance of their next rendezvous or spectrum handoff process.

1.4 Proposal Organization

The rest of this proposal is organized as follows. In Chapter 2, the related works in CRNs are introduced. In Chapter 3, five schemes are proposed regarding practical problems for the rendezvous process in CRNs. We conclude our research, address our future works, and list publications in Chapter 4.

CHAPTER 2: RELATED WORK

The initialization of a CRN is critical, because each SU may not know any information about other SUs in the same network before information exchange. However, due to the dynamic characteristics of CRNs, obtaining the basic but time-varying control information, such as the available channel sets and locations, of SUs is very challenging. For simplicity, many existing papers address this challenge by assuming the existence of a common control channel (CCC) in the network and using the CCC for all control information exchange [10, 27, 28]. However, a CCC may suffer the following problems. First, a CCC may not be available simultaneously to all the SUs in a network. Second, even though a CCC exists, due to the dynamics of CRNs, its availability over the time is subject to PUs' traffic. Third, a single CCC in a network may suffer the congestion problem and is fragile under attacks. In order to overcome these drawbacks, some schemes are proposed to form several SU clusters in a network and choose a common available channel as the CCC in each cluster [29, 30]. However, in order to form a cluster, each SU has to know certain information about its neighboring SUs, which is also not practical for a CRN without any control information exchange before the initialization.

In [8, 12, 13, 15–18, 20, 22, 24–26, 31–33], several rendezvous schemes for CRNs are proposed. However, these schemes only focus on the rendezvous problem itself or designing efficient channel hopping sequences, while ignoring the following practical communication scenarios in wide-band CRNs. First, they do not consider the wide-band scenario. All of them just consider that there are only a small number of channels (at most tens of channels) in the network and SUs will utilize all the channels to design the channel hopping sequence and perform channel hopping, while there may be hundreds of channels for a CRN. As the number of channels increases, the time to sense all the channels to get available channels and to rendezvous will increase significantly, according to the results shown in [9, 14, 19]. Second, the process of sending or receiving RTS or CTS packets is not considered. Past rendezvous schemes implicitly require a sender-and-receiver relation between the two rendezvous SUs, i.e., an SU sender always sends an RTS packet on each of its hopped channels during each time slot, while the other SU always listens on each of its hopped channels. If the other SU receives an RTS packet, it replies a CTS packet to set up a link with the SU sender. However, during the initialization phase of a CRN, every SU tries to rendezvous with other SUs to exchange their control information. There is no explicit sender or receiver role such that an SU is always a sender or receiver, since it cannot know if other SUs are sending or waiting for RTS packets. Third, they cannot directly apply to the rendezvous problem considering directional antennas since they do not consider the indexing problem and assume that each SU can access the same number of channels.

Many existing papers in CRNs have proposed different algorithms for rendezvous, broadcast, and spectrum handoff by directly using the common available channels of SUs. A random channel selection scheme is proposed in [13] to make sure two users can rendezvous on the same available channel. Rendezvous among multiple SUs in [34] is realized by generating the same channel hopping sequence. Blind rendezvous schemes based on common available channels and channel hopping are proposed in [8, 12, 13, 15, 21, 35–38]. In addition, distributed algorithms are proposed in [34, 39] to avoid collisions during a spectrum handoff based on the common available channels between two SUs. An optimal channel selection sequence for the two SUs after each spectrum handoff process is designed in [40, 41]. Moreover, multiple distributed broadcast schemes based on the common available channels between two users are proposed in [16, 24, 26, 42]. However, all these papers just directly utilize the common available channels while ignoring how to maximize the number of common available channels.

CHAPTER 3: PROPOSED RESEARCH

In this chapter, we propose five schemes considering rendezvous in wide-band CRNs, rendezvous without a predetermined sender and receiver, rendezvous considering directional antennas, and how to maximize the number of common available channels.

3.1 Communication Framework for Wide-band Cognitive Radio Networks

In this section, we first propose a rendezvous framework for wide-band cognitive radio networks considering hundreds of channels. In this framework, we first propose a spectrum split scheme to split the wide-band spectrum into several spectrum segments and map each SU to a specific spectrum segment. Next, we propose an efficient rendezvous scheme specific for this framework considering spectrum splitting which enables two SUs to reach a fast and successful rendezvous.

3.1.1 System Model

We consider that SUs in a CRN can dynamically access a wide-band spectrum which has a total of M channels indexed from 1 to M, where M is a large number which could be hundreds. There are totally N SUs. Each SU is equipped with only one cognitive radio that cannot perform spectrum sensing and data transmission at the same time. Each SU has a unique ID which can be a positive integer. Furthermore, we assume that each SU can obtain its available channel set using a spectrum sensing algorithm [3]. Initially, an SU does not know any information about other SUs, even their unique IDs. A time-slotted system is adopted in this CRN. In each time slot, an SU either hops onto a channel according to its hopping sequence or stays on its current channel to continue the current communication with another SU. Time slots of different SUs are not necessarily synchronized, which is practical and easy to implement. SUs in the CRN attempt to get information about other SUs within its transmission range, send packets to other SUs, or receive packets from other SUs.

In the rest of the section, we consider the following two scenarios concerning the available channel sets of an SU pair.

Symmetrical model: Any SU pair has exactly the same available channel set. According to [24, 26, 42], the similarity of the available channel sets between two SUs within one hop is over 85%. Thus, this model is reasonable. Moreover, studying this scenario can help us to get solutions for more complicated scenarios.

Asymmetrical model: Any SU pair may have different but overlapping available channel sets. This scenario is more practical because of the dynamics caused by PUs' and SUs' traffic and locations.

3.1.2 The Proposed Framework

The goal of our proposed framework is to design efficient distributed schemes for each SU to guarantee that it can communicate with other SUs successfully in a wide-band CRN. The framework mainly contains three parts: the initialization process, the process to get control information from neighboring SUs, and the process to send packets to a specific SU. Moreover, the framework does not require a common control channel (CCC) for information exchange. The flow charts of these three parts are shown in Fig. 3.1.

During the initialization process, each SU first divides the whole wide-band spectrum into several virtual spectrum segments (SS). Based on **Algorithm 5** (which is explained in Section 3.1.7), each SU can get a unique list containing the number of channels in each SS. We call it the spectrum splitting list (SSL). Using the unique SSL, an SU can obtain the channel index of the start and end channel in each SS (shown in **Algorithm 1**). Then, each SU uses its ID (based on **Algorithm 1**) to determine its home spectrum segment (HS). **Algorithm 1** is based on mapping all the SUs evenly to the whole spectrum, where we assume that the IDs of SUs are evenly distributed within a certain range. An SU hops from a channel to another according to the designed channel hopping sequence (based on **Algorithm 9** which

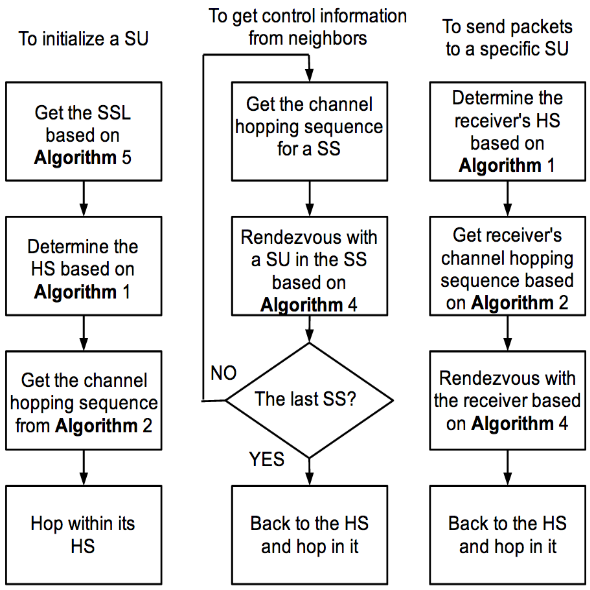

Figure 3.1: The flow charts of our proposed framework.

is explained in Section 3.1.3) within its HS. Since SUs are not synchronized, they may be at different positions of the same hopping sequence even they locate in the same HS. This is the initialization process of an SU who just starts in a CRN, as shown in the left of Fig. 3.1.

Algorithm 1 The algorithm to determine a SU's home spectrum's index

Input:
    The SU's ID x.
Output:
    The SU's home spectrum's index;
Use **Algorithm 5** to get the SSL l;
$y = (x-1)\%M + 1$; //The channel index starts from 1
$sum = 0$;
for $i = 0$ to $l.length - 1$ **do**
    if $y > sum$ and $y \leq sum + l[i]$ **then**
        return i;
    end if
    $sum += l[i]$;
end for

After the initialization process, an SU tries to get the control information of all its neighboring SUs which are hopping in the same or different spectrum segments

(SSs). We assume that when an SU is not transmitting data packets, it first performs **Algorithm 3** or **Algorithm 4** (which is explained in Section 3.1.3) to rendezvous with an SU in its home spectrum segment (HS) and exchange all their known control information about themselves and others. Through this process, each SU can get all the other SUs' information within the same SS and exchange all of their known information. Therefore, after rendezvous and exchange control information with one existing SU in a SS, a SU can get the control information of all the other SUs in that SS, and all the existing SUs in the SS can also get the information of this SU and all its known information about other SUs. Then, the SU can sequentially hop to each SS, rendezvous with one SU in that SS, and exchange all the known control information about other SUs by executing **Algorithm 3** or **Algorithm 4**. The SU will hop to a next SS only when it has rendezvoused with an SU in the current SS or the time spent in the current SS has already been more than the MTTR of the applied rendezvous scheme (there are no SUs in current SS) . Finally, the SU will obtain and update the control information of all other SUs in the network. An SU can perform the above process periodically to update the control information about other SUs and inform its own change due to the dynamic characteristics of CRNs. The process of getting the control information of all the neighboring SUs is shown in the middle of Fig. 3.1.

After the above process of control information exchange, an SU will get some information about other SUs such as their IDs. When an SU wants to send packets to a specific SU, it first uses the receiver's ID to determine the receiver's home spectrum segment (HS) (based on **Algorithm 1**). Then, the SU sender executes **Algorithm 3** or **Algorithm 4** to realize rendezvous with the SU receiver in the SU receiver's HS for communications. The flow charts of communicating with a specific SU is shown in the right of Fig. 3.1. Because of the spectrum splitting design, the time to obtain control information and to set up communications can be significantly reduced since an SU pair only needs to rendezvous within a specific spectrum segment (SS). In addition, the probability that multiple pairs of SUs rendezvous on the same channel is also reduced since different pairs of SUs may rendezvous in different spectrum segments (SSs). In the following sections, we will give the details of the proposed algorithms in our framework.

3.1.3 Proposed Rendezvous Schemes Under the Symmetrical Model

An SU sender and receiver have exactly the same available channel set under the symmetrical model. This may not be very practical. However, the algorithms developed under this model are the basis for the asymmetrical model.

Under this scenario, each SU has the same available channels in each SS. Each SU hops within its HS according to a channel hopping sequence generated based on its available channel set. Assume that the number of the available channels of both the SU sender and receiver in the SU receiver's HS is n. Since an SU sender and receiver have the same available channel set, we use the indexes of all the available channels in a SS, assuming to be from 1 to n, to design the channel hopping sequence. We desire to get a target sequence $s = s_1 s_2 \ldots s_{2n}$ whose length is $2n$ and each channel index from 1 to n appears exactly twice in s. When an SU sender wants to rendezvous with an SU receiver, it first hops to the HS of the SU receiver, generates the same channel hopping sequence as the SU receiver's, and then starts to hop on channel s_1. Both the SU sender and receiver hop according to the sequence sequentially and circularly, which means that when an SU reaches the end of the hopping sequence, it will continue to hop onto the first channel of the sequence in the next time slot. Assume that the SU receiver is initially on channel $s_c, 1 \leq c \leq 2n$. We denote ε as the distance between the SU sender and SU receiver on the channel hopping sequence due to the asynchronization. Therefore, $\varepsilon = c-1$ or $\varepsilon = 2n-(c-1)$ when considering the circulation of the channel hopping sequence. We define the rendezvous sequence (RS) as follows:

Rendezvous Sequence (RS): The sequence s is a RS when no matter what ε is, within $2n-1$ time slots, the SU sender and receiver will hop on a same channel if both of them hop according to the same hopping sequence sequentially and circularly.

We use s to represent our desired rendezvous sequence for the rest of the section. We induce the following lemma to help us design our rendezvous sequence.

Lemma 1. *Assume that $s_i = s_j = k, i < j$, $1 \leq k \leq n$. If $j - i = k$, s could be a RS.*

Proof. We define the *Sequential Distance* of the element k in the sequence s as $d_k = j - i = k$, and the *Circular Distance* of the element k in the sequence s as $D_k = 2n + i - j = 2n - k$, which is the number of time slots taken to hop from s_j to s_i circularly in s. We define the distance pair of the channel index k as $p_k = (d_k, D_k)$. Since each d_k is different and bounded in $[1, n]$ and each $D_k = 2n - d_k$ is different and bounded in $[n, 2n-1]$, each p_k is different. Assume that initially the SU sender is on channel s_1 and the SU receiver is on channel s_c Therefore, $\varepsilon = c-1$ or $\varepsilon = 2n-(c-1)$, where $0 \leq \varepsilon \leq 2n-1$. We ignore the case when $\varepsilon = 0$, because under this case, the two SUs are already on the same channel. Thus, we only need to prove our lemma when $1 \leq \varepsilon \leq 2n-1$. Since each p_i is different, we can definitely find a p_t such that $\varepsilon = d_t$ or D_t. Suppose $s_{i1} = s_{j1} = t, i1 < j1$. Thus, when the SU sender hops on channel s_{i1}, it will rendezvous with the SU receiver on channel s_{j1}, or when the SU sender hops on channel s_{j1}, it will rendezvous with the SU receiver on channel s_{i1}. Therefore, sequence s can guarantee a successful rendezvous within $2n-1$ time slots no matter which channel the SU receiver dwells on at the beginning of the rendezvous process. □

In order to efficiently generate s, we notice that s has the following properties.

Lemma 2. *When $n = 4t+2$ or $4t+3$, where t is an integer and $t \geq 1$, the rendezvous sequence does not exist.*

Proof. Designing a rendezvous sequence s as described in **Lemma 1** is equivalent to dividing $1, 2, \ldots, 2n$, these $2n$ numbers into two lists l_l and l_r, each of which has exactly n different numbers, such that for $k = 1, 2, \ldots, n$, $l_r[k] - l_l[k] = k$. Then, we have

$$\sum_{k=1}^{n} l_r[k] - \sum_{k=1}^{n} l_l[k] = \sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \tag{3.1}$$

$$\sum_{k=1}^{n} l_r[k] + \sum_{k=1}^{n} l_l[k] = \sum_{k=1}^{2n} k = n(2n+1). \tag{3.2}$$

Combining (3.7) and (3.8), we can get

$$\sum_{k=1}^{n} l_r[k] = \frac{5n^2 + 3n}{4}. \tag{3.3}$$

When $n = 4t+2$ or $4t+3$,where t is an integer and $t \geq 1$, the right-hand side of (3.9) is not an integer which contradicts the fact that it is a sum of n integers. □

Lemma 3. *When $n = 4t$ or $4t+1$, where t is an integer and $t \geq 1$, the rendezvous sequence always exists.*

Proof. When $n = 4$, sequence "$1, 1, 4, 2, 3, 2, 4, 3$" satisfies the requirements of our desired rendezvous sequence. When $n = 5$, sequence "$1, 1, 5, 2, 4, 2, 3, 5, 4, 3$" satisfies the requirements of our desired rendezvous sequence. When $n > 5$, the proof can be found in the second page of [43]. □

Based on the proof of **Lemma 2** and **Lemma 18**, we propose **Algorithm 9** to generate our desired rendezvous sequence. Using the channel hopping sequence generated based on **Algorithm 9** and **Lemma 1**, we design the rendezvous algorithm under the symmetrical model as shown in **Algorithm 3**.

We define a round of channel hopping as the length of the $2n$ time slots an SU hops sequentially and circularly according to the channel hopping sequence. An example of rendezvous in one channel hopping round under the symmetrical model is shown

Algorithm 2 The algorithm to generate a rendezvous sequence

Input:

The number of channels to generate the channel hopping sequence: n

Output:

The desired rendezvous sequence: s

if $n < 4$ or $n\%4 == 2$ or $n\%4 == 3$ **then**
    $s = \phi$;
    return s;
end if
if $n == 4$ **then**
    $s = [1, 1, 4, 2, 3, 2, 4, 3]$;
    return s;
end if
if $n == 5$ **then**
    $s = [1, 1, 5, 2, 4, 2, 3, 5, 4, 3]$;
    return s;
end if
Let $pl = \phi$ be the list of all the position pairs;
$m = \lfloor n/4 \rfloor$;
if $n > 4$ and $n\%4 == 0$ **then**
    Add all pairs $(4m + r, 8m - r)$, for $r = 0, 1, \ldots, 2m - 1$, to pl;
    Add pairs $(2m + 1, 6m)$ and $(2m, 4m - 1)$ to pl;
    Add all pairs $(r, 4m - 1 - r)$, for $r = 1, 3, \ldots, m - 1$, to pl;
    Add pair $(m, m + 1)$ to pl;
    Add all pairs $(m + 2 + r, 3m - 1 - r)$, for $r = 0, 1, \ldots, m - 3$, to pl;
end if
if $n > 4$ and $n\%4 == 1$ **then**
    Add all pairs $(4m + 2 + r, 8m + 2 - r)$, for $r = 0, 1, \ldots, 2m - 1$, to pl;
    Add pairs $(2m + 1, 6m + 2)$ and $(2m + 2, 4m + 1)$ to pl;
    Add all pairs $(r, 4m + 1 - r)$, for $r = 1, 3, \ldots, m$, to pl;
    Add pair $(m + 1, m + 2)$ to pl;
    Add all pairs $(m + 2 + r, 3m + 1 - r)$, for $r = 1, 2, \ldots, m - 2$, to pl;
end if
for $i = 1$ to n **do**
    $s[pl[i].first] = s[pl[i].second] = pl[i].second - pl[i].first$;
end for
return s;

Algorithm 3 The rendezvous algorithm for an SU sender under the symmetrical model

Input:
The available channel set C of the SU sender within the SU sender's HS.

n equals the size of C;
if $n\%4 == 2$ or $n\%4 == 3$ **then**
    $m = 4(\lfloor n/4 \rfloor + 1)$; /* To guarantee the existence of the RS */
else
    $m = n$;
end if
Generate the channel hopping sequence s based on m and **Algorithm 9**;
for $k = 1$ to $2m$ **do**
    if $s_k > n$ **then**
        $s_k = s_k\%n$; /* To guarantee s_k to be within n */
    end if
end for
$i = 1$;
for $j = 1$ to $2m - 1$ **do**
    if The sender receives a CTS packet on channel C_{s_i} of the receiver's HS **then**
        Perform communications;
    else
        $i++$;
    end if
end for

in Fig. 3.2. We assume that the number of available channels of an SU pair is 4. We label each available channel using the index from 1 to 4. Based on **Algorithm 9**, their channel hopping sequence is "$1, 1, 4, 2, 3, 2, 4, 3$". Assume that the SU sender starts hopping from the channel whose index is $s_1 = 1$ and the SU receiver currently stays on the channel whose index is $s_3 = 4$. Then, the SU sender and receiver will rendezvous on the channel whose index is $s_4 = 2$ after 3 time slots.

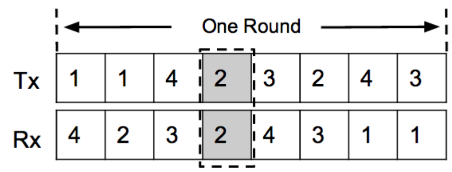

Figure 3.2: An example of rendezvous under the symmetrical model.

From **Lemma 1** and **Algorithm 3**, we can induce **Theorem 1**.

Theorem 1. *Under the symmetrical model,* ***Algorithm 3*** *can guarantee rendezvous in one round of the channel hopping. The MTTR of our proposed rendezvous scheme is* $2m - 1$ *and the ETTR is* $m - 1$*, where* n *is the number of channels in current SS,* $m = n$ *if* $n\%4 = 0$ *or* $n\%4 = 1$*, and* $m = 4(\lfloor n/4 \rfloor + 1)$ *if* $n\%4 = 2$ *or* $n\%4 = 3$.

Proof. From **Lemma 1** and **Algorithm 3**, the length of our channel hopping sequence is $2m$ and our rendezvous algorithm can guarantee rendezvous. Thus, according to **Algorithm 3**, the *MTTR* is $2m - 1$. When the SU sender is on channel s_1, assume that the SU receiver has equal probability, which is $1/2m$, to be on any channel from s_1 to s_{2m}. According to the proof of **Lemma 1**, the total number of time slots needed to rendezvous of all possible situations is $\sum_{i=0}^{2m-1} i = m(2m - 1)$. Thus, the *ETTR* is $\lfloor m(2m - 1)/2m \rfloor = m - 1$. □

3.1.4 Proposed Rendezvous Schemes Under the Asymmetrical Model

Under the asymmetrical model, the SU sender and receiver have different available channel sets. The challenge here is that before rendezvous happens, the SU sender does not know the available channel set of the SU receiver. Here, we assume that they have at least one common available channel. Otherwise, they can never achieve a successful rendezvous.

As an SU pair may have different available channel sets, each SU's channel hopping sequence should contain all the channels within the SU receiver's HS. We assume that there are totally n channels in the SU receiver's HS which are indexed from 1 to n. The spectrum splitting scheme in Section 3.1.7 can guarantee the existence of the channel hopping sequences. We still use **Algorithm 9** to design the basic channel hopping sequence based on these n channels for both the SU sender and receiver. Additionally, we will replace the unavailable channels in the base channel hopping sequence with the ones randomly chosen from the SU sender's or receiver's own available channels to get their own channel hopping sequences. According to **Lemma 1**, the replacements will not change the property of our channel hopping sequence. Our goal is to design a rendezvous algorithm which can guarantee rendezvous on different channels during several rounds of the channel hopping until rendezvous on the channel which is commonly available for an SU pair. According to the property of the rendezvous sequence, we can simply change the starting position of an SU sender's channel hopping during each channel hopping round. During a new channel hopping round, after changing the distance between the SU pair's positions in the channel hopping sequence, the SU sender and receiver can rendezvous on a new channel. The details of the rendezvous algorithm under the asymmetrical model are shown in **Algorithm 4**.

An example of rendezvous under the asymmetric model is shown in Fig. 3.3. Assume that the available channels of the SU sender are $\{1, 2\}$ and the available channels of the SU receiver are $\{1, 3\}$. Assume that $n = 4$, so the base channel hopping sequence is "$1, 1, 4, 2, 3, 2, 4, 3$". The SU sender starts from channel $s_1 = 1$ and we

Algorithm 4 The rendezvous algorithm for an SU sender under the asymmetrical model

Input:

The SU receiver's ID x;

The available channel set C of the SU sender in the SU sender's HS.

```
Use Algorithm 5 to get the SSL of the whole spectrum band: l;
Use Algorithm 1 to get the index of the HS of the SU receiver: r;
n = l[r];  /* The index starts from 0 */
Generate the channel hopping sequence s based on n and Algorithm 9;
for i = 1 to 2n do
    j = i;  /* The starting hopping position of the current round */
    for k = 0 to 2n − 1 do
        ν = s_j;
        if ν ∉ C then
            Let ν be a randomly chosen channel in C;
        end if
        if The rendezvous on ν succeeds then
            Perform communications;
        else
            j++;
            if j > 2n then
                j = 1; /* Circularly hopping */
            end if
        end if
    end for
end for
```

assume that the SU receiver is on channel $s_3 = 4$. During the first round of channel hopping, after the random replacement process, the channel hopping sequences for the SU sender and SU receiver are "$1, 1, 2, 2, 1, 2, 2, 2$" and "$3, 3, 3, 1, 3, 3, 1, 1$", respectively. According to the channel hopping sequences, the SU sender and receiver do not rendezvous on the same channel for the first round. Then, the SU sender continues the second round of rendezvous by starting from channel $s_2 = 1$ and the base channel hopping sequence will be "$1, 4, 2, 3, 2, 4, 3, 1$". However, the SU receiver will keep using the base rendezvous sequence "$1, 1, 4, 2, 3, 2, 4, 3$" during the second round. Then, during the second round, after the random replacement process, the channel hopping sequences for the SU sender and SU receiver are "$1, 1, 2, 2, 2, 1, 2, 1$" and "$3, 3, 3, 3, 1, 3, 1, 1$", respectively. Finally, the SU sender and receiver rendezvous on channel 1 which is available for both of them at the end of the second round after 15 time slots.

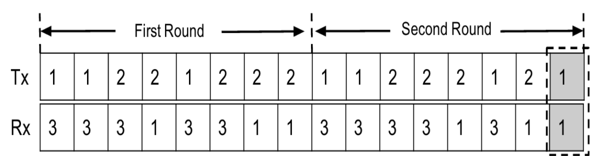

Figure 3.3: An example of rendezvous under the asymmetrical model.

According to the conclusions under the symmetrical model and **Algorithm 4**, we can induce **Theorem 2**.

Theorem 2. *Under the asymmetrical model,* ***Algorithm 4*** *can guarantee rendezvous. The MTTR of the guaranteed rendezvous is $2n(n-G+1)$, and the upper bound of the ETTR is approximately $(2n-1)/p$, where n is the number of channels in current SS, G is the number of common available channels in current SS, and p is the probability that a channel is available for both the SU sender and receiver.*

Proof. The length of the designed sequence s is $2n$. According to **Algorithm 4** and the proof of **Lemma 1**, during each channel hopping round, two SUs can meet on a different available channel. Since there are total G common available channels in current SS, there are at most $n - G + 1$ channel hopping rounds before two SUs hop a common available channel. Therefore, the maximum time slots to a successful rendezvous ($MTTR$) is $2n(n - G + 1)$.

Since the probability that a channel is available for both the SU sender and receiver is p, the probability that a successful rendezvous does not happen until the kth round is $(1-p)^{k-1}p$. Therefore,

$$
\begin{aligned}
ETTR &\leq \sum_{k=1}^{2n} (1-p)^{k-1} pk(2n-1) \\
&= \left(\frac{1-(1-p)^{2n}}{p} - 2np^{2n} \right) (2n-1) \\
&\approx \frac{2n-1}{p}.
\end{aligned}
$$

□

Note that the $ETTR$ of our proposed rendezvous algorithms linearly increases with n. This is much better than other existing rendezvous algorithms under the asymmetrical model whose $ETTR$ is at least $O(n^2)$ such as the ones in [8, 17, 37].

3.1.5 Unaligned Time Slots

Considering a more practical scenario where each SU pair's time-slot system cannot be synchronized perfectly before information exchange due to hardware constrains, which causes the time slots between a SU pair not aligned and the time they can meet on a common available channel not the length of a whole time slot. A successful rendezvous requires both meeting on a common available channel and a successful exchange of RTS and CTS packets. When the time slots of a SU pair are not aligned, the time they meet on a common available channel may not be long enough for them to exchange RTS and CTS packets, which will lead to an unsuccessful rendezvous even though they have met on a common available channel. In this section, we justify that our proposed rendezvous schemes in **Algorithm 3** and **Algorithm 4** can be applied to the asynchronous time-slot scenario without any modification.

We denote λ as the time required to complete a RTS and CTS packet exchange, Δ as the offset of the time slots between a SU pair, δ as the length of a time slot, and ε as the distance between the positions of the channels the SU pair are on in the hopping sequence when the SU sender starts a rendezvous process. We assume that $\lambda \leq \delta/2$. Otherwise, two SUs may never rendezvous under certain scenarios because of insufficient time to exchange RTS-CTS packets. For example, in Fig. 3.4, if $\Delta < \lambda$ and $\delta - \Delta < \lambda$, the overlap of two time slots is not long enough to complete a RTS-CTS exchange. We will address this problem in the next subsection by designing an optimal length of a time slot which can guarantee a successful rendezvous.

We can induce the following four lemmas.

Lemma 4. *When $\delta - \Delta \geq \lambda$ and the system time of the SU sender is Δ seconds ahead of the SU receiver's, ε does not change.*

Proof. Under this scenario, $\delta - \Delta \geq \lambda$ indicates that the overlapping part of the time slots of a SU pair is long enough for a RTS and CTS packet exchange. Therefore, this scenario is equivalent to the original rendezvous discussed previously. $\square$

An example of Lemma 4 is shown in Fig. 3.4.

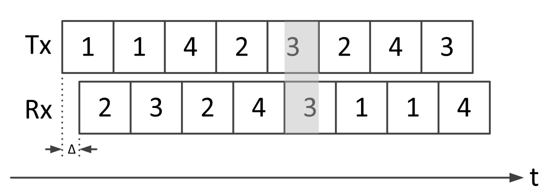

Figure 3.4: The first scenario regarding a rendezvous considering unaligned time slots.

Lemma 5. *When $\delta - \Delta < \lambda$ and the system time of the SU sender is Δ seconds ahead of the SU receiver's, ε changes to $(\varepsilon - 1 + 2n)$ mod $2n$, where n is the total number of channels in a SS.*

Proof. Under this scenario, $\delta - \Delta < \lambda$ indicates that the overlapping part of the time slots of a SU pair is not long enough for a RTS and CTS packet exchange. However, the overlapping part between the current time slot of the SU sender and the previous time slot of the SU receiver is Δ, where $\Delta > \delta/2 \geq \lambda$. Therefore, ε is decreased by one under this scenario. Considering the circular property of ε, ε should be $(\varepsilon - 1 + 2n)$ mod $2n$. □

An example of Lemma 5 is shown in Fig. 3.5.

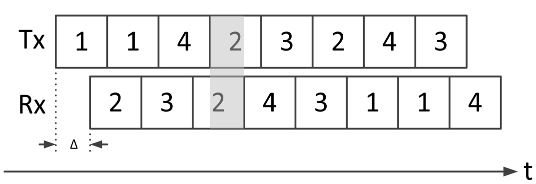

Figure 3.5: The second scenario regarding a rendezvous considering unaligned time slots.

Lemma 6. *When $\delta - \Delta \geq \lambda$ and the system time of the SU receiver is Δ seconds ahead of the SU sender's, ε does not change.*

Proof. Under this scenario, $\delta - \Delta \geq \lambda$ indicates that the overlapping part of the time slots of a SU pair is long enough for a RTS and CTS packet exchange. Therefore, this scenario is equivalent to the scenario in Lemma 4. □

An example of Lemma 6 is shown in Fig. 3.6.

Lemma 7. *When $\delta - \Delta < \lambda$ and the system time of the SU receiver is Δ seconds ahead of the SU sender's, ε changes to $(\varepsilon + 1)$ mod $2n$, where n is the total number of channels in a SS.*

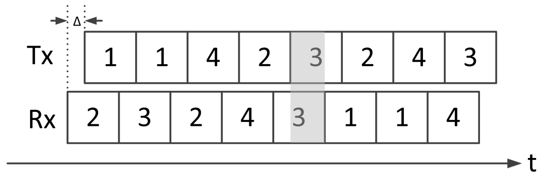

Figure 3.6: The third scenario regarding a rendezvous considering unaligned time slots.

Proof. Under this scenario, $\delta - \Delta < \lambda$ indicates that the overlapping part of the time slots of a SU pair is not long enough for a RTS and CTS packet exchange. However, the overlapping part between the current time slot of the SU sender and the next time slot of the SU receiver is Δ, where $\Delta > \delta/2 \geq \lambda$. Therefore ε is increased by one under this scenario. Considering the circular property of ε, ε should be $(\varepsilon + 1)$ mod $2n$. □

An example of Lemma 7 is shown in Fig. 3.7.

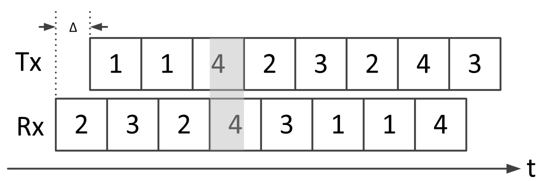

Figure 3.7: The fourth scenario regarding a rendezvous considering unaligned time slots.

According to the above examples we can get the following theorem:

Theorem 3. *Considering unaligned time slots, our proposed rendezvous schemes under both the symmetrical and asymmetrical models can still work without any modification.*

Proof. According to the four scenarios above, under the asynchronous scenario, ε has three different values: ε, $(\varepsilon - 1 + 2n)$ mod $2n$, and $(\varepsilon + 1)$ mod $2n$, where $2n$ is the length of the channel hopping sequence. All of the three values are still within the range $[0, 2n-1]$. Therefore, according to the proof of Lemma 1, considering the asynchronous time-slot scenario has no difference comparing to without considering it when implementing our proposed channel hopping sequence. Therefore, under this scenario, we do not need to modify our proposed rendezvous schemes. □

3.1.6 The Optimal Time Slot Length

A successful rendezvous requires a RTS-CTS packet exchange. Therefore, when considering unaligned time slots, a successful rendezvous not only requires two SUs on a common available channel, but also enough overlapping time in that channel to finish exchanging RTS-CTS packets. In this subsection, we address how to find the optimal length of a time slot which can guarantee a successful RTS-CTS packet exchange under our rendezvous schemes.

During a rendezvous process, a time slot with a length δ can be divided into two parts: one is the sensing part with a length t_s and the other is the part for exchanging RTS and CTS packets with a length t_t. Our goal is to minimize t_t, since more exchanges of RTS and CTS mean more energy consumption and are not necessary. Assume that a successful RTS-CTS packet exchange requires at least λ seconds. Considering the duality, there are only two scenarios regarding the overlapping time.

For the first scenario shown in Fig. 3.8, the shadow area represents an overlap of the RTS-CTS exchanging period. We have

$$\begin{aligned} Minimize \quad & t_t; \\ s.t. \quad & \delta = t_s + t_t; \\ & \lambda \leq \delta/2; \\ & \lambda \leq \delta - \Delta - t_s; \\ & 0 \leq \Delta \leq t_s; \end{aligned}$$

The answer for the first scenario $t_{t\_min}^1 = max\{2\lambda - t_s, \lambda + t_s\}$, with the known λ and t_s.

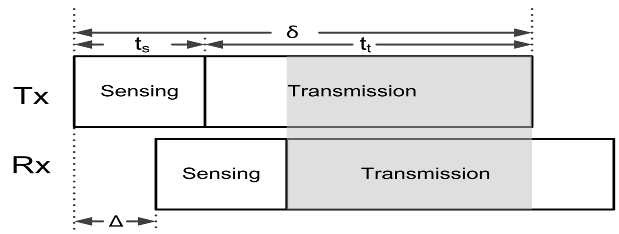

Figure 3.8: The first scenario considering unaligned time slots

For the second scenario shown in Fig. 3.9, we have

$$
\begin{aligned}
Minimize \quad & t_t; \\
s.t. \quad & \delta = t_s + t_t; \\
& \lambda \leq \delta/2; \\
& \lambda \leq \delta - \Delta - t_s \ or \ \lambda \leq \Delta - t_s; \\
& t_s < \Delta < \delta.
\end{aligned}
$$

The answer for the second scenario $t_{t\_min}^2 = max\{2\lambda - t_s, 2\lambda + t_s\} = 2\lambda + t_s$, with the known λ and t_s.

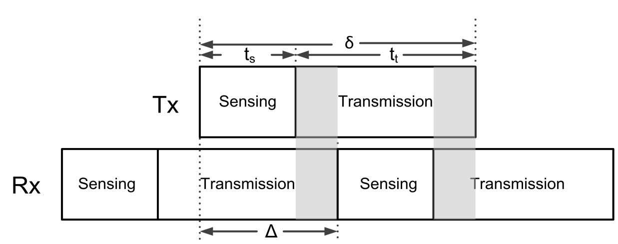

Figure 3.9: The second scenario considering unaligned time slots

Therefore, the optimal t_t that can guarantee a successful rendezvous for the two scenarios is $t_{t\_min} = max\{t^1_{t\_min}, t^2_{t\_min}\}\} = 2\lambda + t_s$. Based on this and **Theorem 3**, we can get the following theorem:

Theorem 4. *When considering unaligned time slots, the minimum length of a time slot to guarantee a successful rendezvous is* $2(\lambda + t_s)$*, where* λ *is the time for a RTS-CTS exchange and* t_s *is the time for spectrum sensing.*

3.1.7 Proposed Spectrum Splitting Scheme

First, our proposed spectrum splitting algorithm is based on the following mathematical truth.

Lemma 8. *For any integer* $a = 4t + 2, t > 2$*,* a *can be represented as* $a = x + y$*, where* $x = 4u + 1, u \geq 1$ *and* $y = 4v + 1, v \geq 1$*,* $a, t, u,$ *and* v *are integers.*

Proof. When $a = 4t + 2, t > 2$, if $t = 2q$, q is an integer and $q \geq 1$, then $a = 8q + 2$, thus $a = (4q + 1) + (4q + 1)$. If $t = 2q + 1, q \geq 1$, then $a = 8q + 6$, thus $a = [4(q + 1) + 1] + (4q + 1)$. □

Lemma 9. *For any integer* $a = 4t + 3, t > 2$*,* a *can be represented as* $a = x + y + z$*, where* $x = 4u + 1, u \geq 1$*,* $y = 4v + 1, v \geq 1$*, and* $z = 4w + 1$*,* $a, t, u, v,$ *and* w *are integers.*

Proof. When $a = 4t + 3$, if $t = 3q$, q is an integer and $q \geq 1$, then $a = 12q + 3$, thus $a = (4q + 1) + (4q + 1) + (4q + 1)$. If $t = 3q + 1, q \geq 1$, then $a = 12q + 7$, thus $a = [4(q + 1) + 1] + (4q + 1) + (4q + 1)$. If $t = 3q + 2, q \geq 1$, then $a = 12q + 11$, thus $a = [4(q + 1) + 1] + [4(q + 1) + 1] + (4q + 1)$. □

Lemma 10. *For any integer* $a = 4t, t >= 2$*,* a *can be represented as* $a = x + y$*, where* $x = 4u, u \geq 1$ *and* $y = 4v, u \geq 1$*,* $a, t, u,$ *and* v *are integers.*

Proof. When $a = 4t, t >= 2$, if $t = 2q$, q is an integer and $q \geq 1$, then $a = 8q$, thus $a = 4q + 4q$. If $t = 2q + 1, q \geq 1$, then $a = 8q + 4$, thus $a = 4q + 4(q + 1)$. □

Lemma 11. *For any integer $a = 4t + 1, t >= 2$, a can be represented as $a = x + y$, where $x = 4u, u \geq 1$ and $y = 4v + 1, v \geq 1$, a, t, u, and v are integers.*

Proof. When $a = 4t + 1, t >= 2$, if $t = 2q$, q is an integer and $q \geq 1$, then $a = 8q + 1$, thus $a = 4q + (4q + 1)$. If $t = 2q + 1, q \geq 1$, then $a = 8q + 5$, thus $a = 4(q + 1) + (4q + 1)$. □

Let θ be the minimum number of channels in a SS. We will discuss how to choose the value of θ in Section 3.1.8 through simulation results. Given θ, SUs can execute **Algorithm 5** to determine the SSL.

Algorithm 5 The algorithm to determine the number of channels in each spectrum segment

---

Input:

The total number of channels in the whole spectrum band: M; the threshold of the minimum number of channels in each spectrum segment: θ

Output:

The SSL: l

Initialize l as an empty list;
Initialize Q as an empty queue;
$Q.push(M)$;
while Q is not empty **do**
  Let ω be the element in Q's head;
  Pop ω out of Q;
  Use **Lemma 8** to **Lemma 11** to represent ω using a set of numbers so that ω is the sum of these numbers; Let the set be A;
  if for each element $\beta \in A$, $\beta \geq \theta$ **then**
    Push all the elements in A into Q;
  else
    Add ω to l;
  end if
end while
return l;

Based on **Lemma 8** to **Lemma 11** and **Algorithm 5**, each SU can use the same scheme to split the whole spectrum which guarantees the existence of our designed channel hopping sequence in each spectrum segment. An example of our spectrum splitting scheme is that when $M = 100$ and $\theta = 20$, the SSL is $[28, 24, 24, 24]$ and each element in this SSL can guarantee the existence of our rendezvous sequence according to **Lemma 18**.

3.1.8 Performance Evaluation

In this section, we evaluate the performance of our proposed framework in terms of $ETTR$ of rendezvous, the average normalized throughput, the time to finish information exchange, and the effect of θ under the symmetrical and asymmetrical models. Our simulation programs are based the Python language. Since our simulations are under different scenarios, which require different simulation parameters, we introduce the simulation parameters before showing results of each scenario rather than giving a table containing all parameters at the beginning of this section.

3.1.8.1 The $ETTR$ of Rendezvous under the Symmetrical Model

In order to evaluate our proposed rendezvous algorithm under the symmetrical model, we compare our proposed schemes with an existing CRN rendezvous algorithm called jump-stay [8] with guaranteed rendezvous. We consider a scenario that there are two SUs: one is the SU sender and the other is the SU receiver. The SU sender tries to rendezvous with the SU receiver by using our proposed scheme and the jump-stay scheme.

Fig. 3.10 shows the $ETTR$ of our proposed scheme and the jump-stay scheme when the SU sender tries to rendezvous with a SU receiver whose HS has 10 to 50 channels. In this simulation, we assume that the ratio of the number of available channels for the SU pair to the total number of channels in the SU receiver's HS is 0.8. From the figure we can see that the average time to rendezvous under both schemes increases linearly with the total number of channels. This is consistent with our conclusion in **Theorem 1** and the results in [8]. The figure shows that the $ETTR$ of our proposed rendezvous algorithm is always lower than that of the jump-stay scheme when a single

SS of a wide-band spectrum is considered.

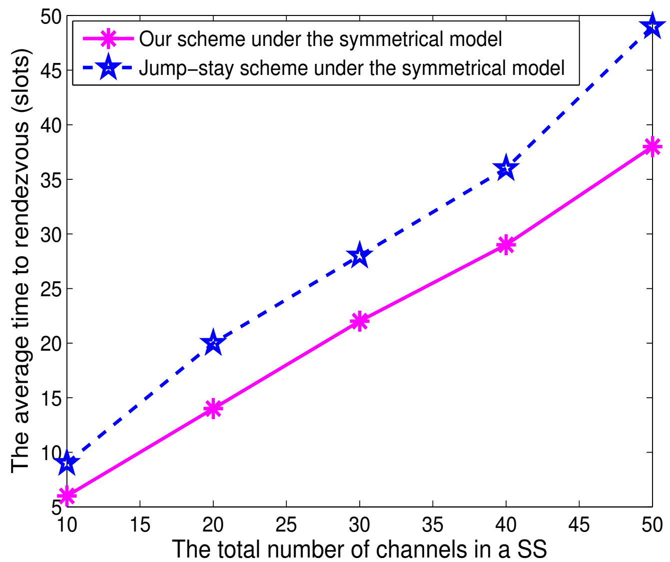

Figure 3.10: The $ETTR$ within a SS under the symmetrical model

Fig. 3.11 shows the $ETTR$ of our proposed scheme and the jump-stay scheme in a wide-band scenario with the total number of channels changing from 50 to 300. In this simulation, we assume that the ratio of the number of available channels for the SU pair to the total number of channels in the wide-band spectrum is 0.8 and $\theta = 20$ in the spectrum splitting scheme. The figure shows that our proposed rendezvous algorithm can always achieve a much lower $ETTR$ as compared to the jump-stay scheme in a wide-band scenario. This is because that the channel hopping of the jump-stay scheme includes all the channels of the whole spectrum, while our scheme can perform rendezvous in a smaller SS because of the proposed spectrum splitting scheme. Furthermore, due to the spectrum splitting without changing θ, the performance of our scheme does not change much even when the total number of channels increases.

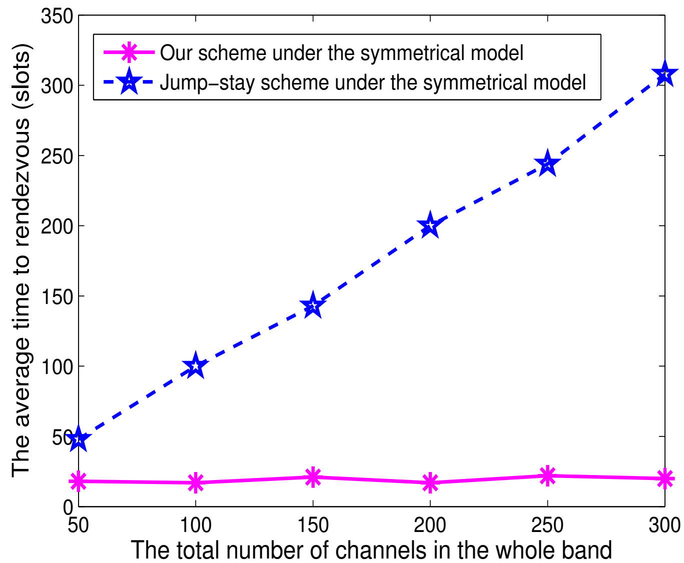

Figure 3.11: The $ETTR$ of the whole spectrum under the symmetrical model

3.1.8.2 The $ETTR$ of Rendezvous under the Asymmetrical Model

We compare our proposed rendezvous algorithm with the jump-stay scheme [8] and enhanced jump-stay scheme [37] under the asymmetrical model which is more practical (the enhanced jump-stay scheme only considers the asymmetrical model).

Fig. 3.12 shows the $ETTR$ of our proposed scheme, the jump-stay scheme, and the enhanced jump-stay scheme, when the SU sender tries to rendezvous with the SU receiver whose HS has 10 to 50 channels. In this simulation, we assume that the ratio of the number of available channels for the SU pair to the total number of channels in the SU receiver's HS is 0.5, the SU pair has the same number of available channels, and 50% of their available channels are the same. The figure shows that the performance of our proposed rendezvous algorithm is better than the jump-stay scheme and enhanced jump-stay scheme under the asymmetrical model within a single SS.

Fig. 3.13 shows the $ETTR$ of our proposed scheme, the jump-stay scheme, and the enhanced jump-stay scheme in a wide-band scenario with the total number of channels

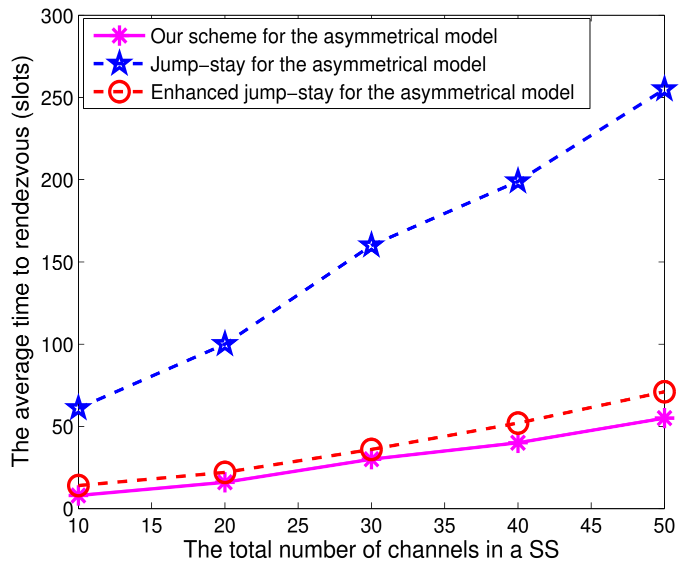

Figure 3.12: The $ETTR$ within a SS under the asymmetrical model.

changing from 50 to 300. In this simulation, we assume that $\theta = 20$ in the spectrum splitting scheme, the ratio of the number of available channels for the SU pair to the total number of channels in the whole wide-band spectrum is 0.5, the SU pair has the same number of available channels, and 80% of their available channels are the same. This figure shows that the performance of our proposed rendezvous algorithm is always much better than the jump-stay scheme and the enhanced jump-stay scheme in a wide-band scenario.

Fig. 3.14 shows the $ETTR$ of our proposed scheme, the jump-stay scheme, and the enhanced jump-stay scheme within a SS with a total of 40 channels when the ratio (defined as the common available ratio) of the number of common available channels between two SUs to the number of their individual total available channels (assume each SU has the same number of available channels) changes from 0.1 to 0.9. In this simulation, we assume that the ratio of the number of available channels for the SU pair to the total number of channels in the SS is 0.5. This figure shows that the performance of our proposed rendezvous algorithm is always much better than the

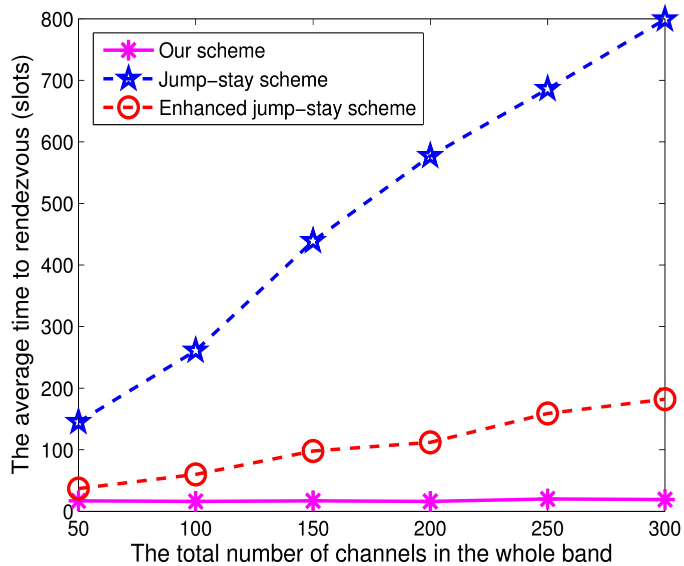

Figure 3.13: The $ETTR$ of the whole spectrum under the asymmetrical model.

jump-stay scheme. Our scheme performs better than the enhanced jump-stay scheme when the common available ratio changes from 0.1 to 0.5 and performs almost the same as the enhanced jump-stay scheme when the common available ratio changes from 0.6 to 0.9, which means that our scheme has an advantage when the available channel sets of the two SUs are more distinct.

3.1.8.3 The Time for Information Exchange

As we discussed in Section 3.1.2, in order to get the control information of other SUs, a SU has to periodically hop to each spectrum segment (SS) to rendezvous with a SU currently in that SS and exchanges their own and known control information. In this subsection, we show simulation results on the time of this process. In the following simulations, the same spectrum splitting scheme is applied for all the three rendezvous schemes and each SU's neighbors are randomly generated to form a practical multiple-hop network. We assume that a SU can only rendezvous with its one-hop neighbors.

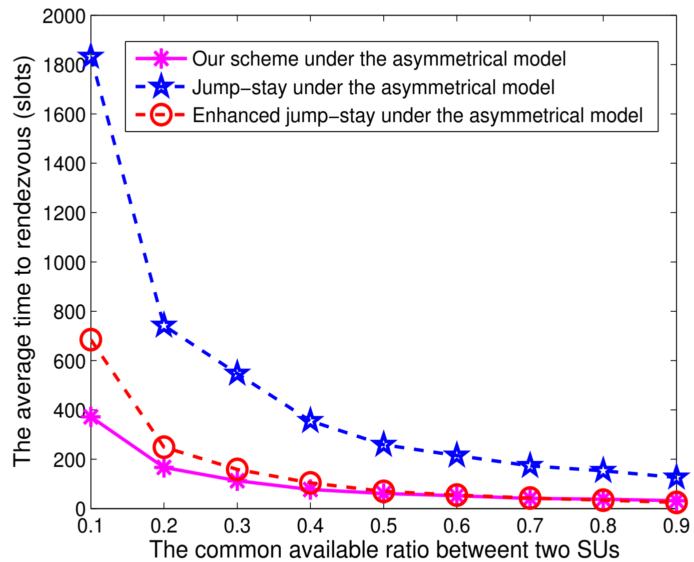

Figure 3.14: The $ETTR$ under different common available ratio.

First, we consider how long it takes for a SU to finish hopping and rendezvousing within all the SSs, assuming that during the process, other SUs only hop within their home SS. This simulates the scenario that a newly joined SU tries to get other SUs' control information. As discussed in Section 3.1.2, a SU will hop to the next SS only after it has rendezvoused with a SU in the current SS or it has stayed in that SS for more than MTTR time slots, where MTTR is the maximum time to guarantee a rendezvous for the applied rendezvous algorithm.

Fig. 3.15 shows the time for a SU to finish hopping within all the SSs under the three rendezvous schemes, when the total number of channels M changes from 50 to 150 and the number of SUs N is 20. Our proposed scheme has the same good performance as the enhanced jump-stay scheme and outperforms the original jump-stay scheme when M is below 150, while outperforms the other two jump-stay schemes when M changes from 200 to 300. The reason is that our rendezvous scheme can achieve the fastest rendezvous and has the smallest MTTR among all the three schemes. The time increases drastically when M changes from 150 to 300, due to the increase of the number of SSs and the decrease of the number of SUs in a SS.

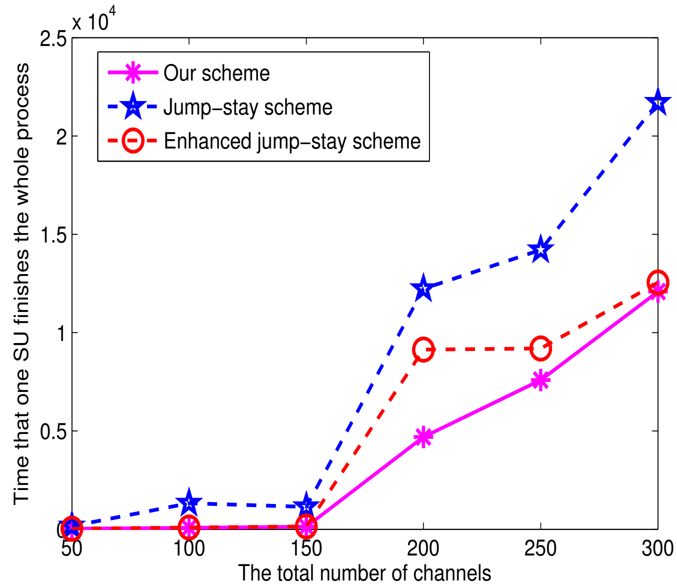

Figure 3.15: The time for a SU to finish the whole information exchange process as M increases.

Fig. 3.16 shows the time to finish the whole information exchange process when the number of SUs N changes from 10 to 50 and the total number of channels M is 100. Our proposed scheme has the same good performance as the enhanced jump-stay scheme when $N = 10, 40, 50$ and always outperforms the original jump-stay scheme. However, our scheme performs much better than the other two schemes when $N = 20, 30$. The vibration of the results comes from the trade-off that when N increases, the chance to achieve a rendezvous in one SS increases, but the probability that a collision happens also increases.

Second, we consider how long it takes all SUs to finish hopping and rendezvousing within all the SSs, assuming that all the SUs start this process simultaneously. This simulates the scenario that several SUs start to form a CRN and try to get other SUs' control information.

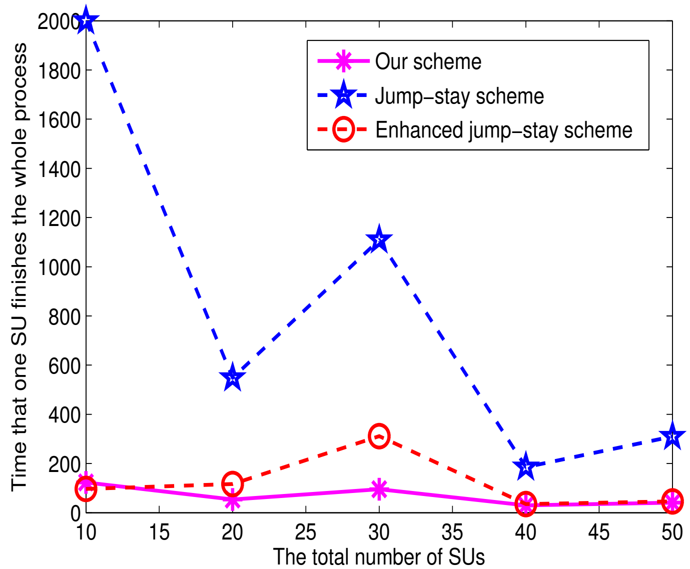

Figure 3.16: The time for a SU to finish the whole information exchange process as N increases.

Fig. 3.17 shows the time for all SUs to finish the whole process , when $N = 20$ and M changes from 50 to 300. In this figure, our proposed scheme outperforms both jump-stay schemes because of faster rendezvous and smaller MTTR. The time increases with M since more channels increase the time to rendezvous.

Fig. 3.18 shows the time for all SUs to finish the whole process, when $M = 100$ and N changes from 10 to 50. Our scheme still outperforms the other two jump-stay schemes. The time to finish the whole process decreases as N increases. This is because that more SUs in the whole network results in more SUs in a SS, which increases the chance of one SU to rendezvous with each other.

Therefore, our proposed scheme outperforms the two jump-stay schemes when considering a practical CRN where the control information exchange and update between SUs are necessary.

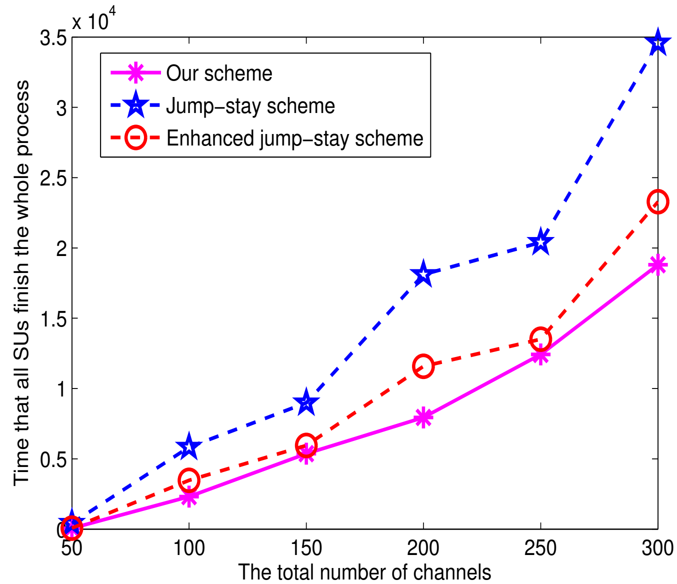

Figure 3.17: The time for all SUs to finish the whole information exchange process as M increases.

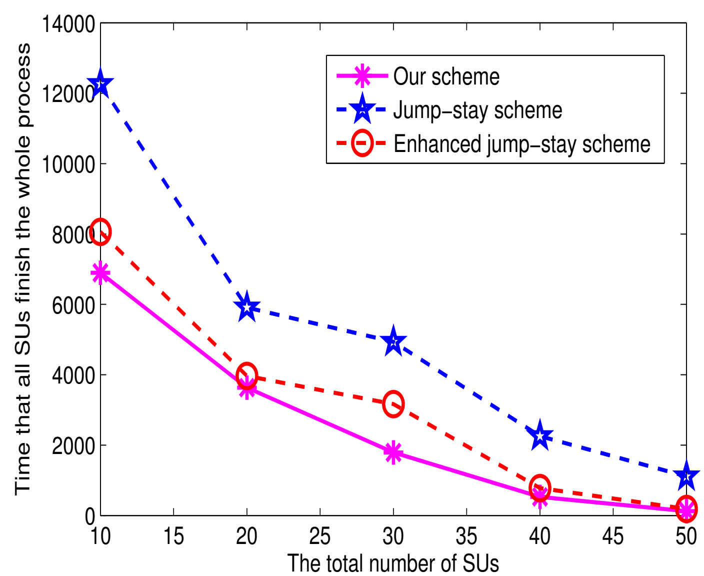

Figure 3.18: The time for all SUs to finish the whole information exchange process as N increases.

3.1.8.4 The Average Normalized Throughput

We consider practical communication scenarios in a CRN with multiple SU pairs and traffic flows. We evaluate the average normalized throughput for each user in a CRN which is the average ratio of the number of data packets successfully transmitted out by a SU sender to the total number of data packets generated in a period of time. Since there are no other existing similar rendezvous schemes deigned for wide-band CRNs considering the practical communication scenario we consider here, we apply both jump-stay and enhanced jump-stay rendezvous schemes to a wide-band CRN under both the symmetrical and asymmetrical models. We assume that there are 30 SUs in the network: 15 are the SU senders and 15 are the SU receivers. The average packet arrival rate for each SU sender is 5 $packets/sec$. The length of each packet in the unit of time slots is a random number in $[1, 20]$. We assume that $\theta = 20$, the total number of channels in the wide-band CRN changes from 50 to 300, and the ratio of the number of available channels for the SU pair to the total number of channels in the whole wide-band spectrum is 0.8.

Fig. 3.19 shows the average normalized throughput of our proposed scheme and the jump-stay scheme under the symmetrical model when the total number of channels within a wide-band spectrum changes from 50 to 300. The result shows that our proposed scheme can maintain a very high average normalized throughput which is much better than the jump-stay scheme when considering the whole wide-band spectrum. This is because that after splitting the spectrum, the time to rendezvous of each transmission pair is much shorter. Thus, the overall throughput is benefited.

Fig. 3.20 shows the average normalized throughput of our proposed scheme and the jump-stay scheme under the symmetrical model when the SU packet arrival rate changes from 1 to 20. We consider a wide-band spectrum with 200 channels here. The result shows that our proposed scheme can maintain a very high average normalized

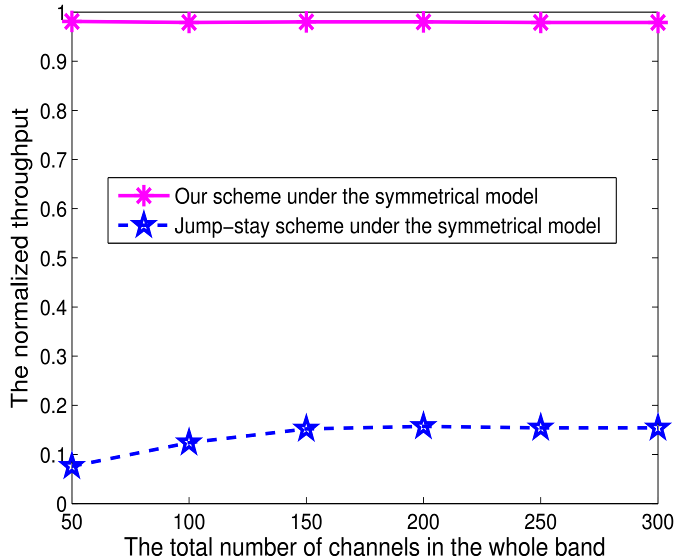

Figure 3.19: The average normalized throughput under the symmetrical model as the number of total channels increases.

throughput which is much better than the jump-stay scheme under different SU packet arrival rates. We can also notice that the average normalized throughput of the jump-stay scheme decreases significantly when the SU packet arrival rate increases, while our scheme can still keep a high throughput.

Fig. 3.21 shows the average normalized throughput of our proposed scheme, the jump-stay scheme, and the enhanced jump-stay scheme under the asymmetrical model when the total number of channels varies from 50 to 300. The result shows that the average normalized throughput of our proposed scheme is much higher than that of the jump-stay scheme and enhanced jump-stay scheme when the whole wide-band spectrum is considered. We can also notice that the performance of the jump-stay scheme and the enhanced jump-stay scheme fluctuates with the increment of the total number of channels, while our scheme can still keep a high normalized throughput because of the proposed framework and rendezvous schemes.

Fig. 3.22 shows the average normalized throughput of our proposed scheme, the jump-stay scheme, and the enhanced jump-stay scheme under the asymmetrical model

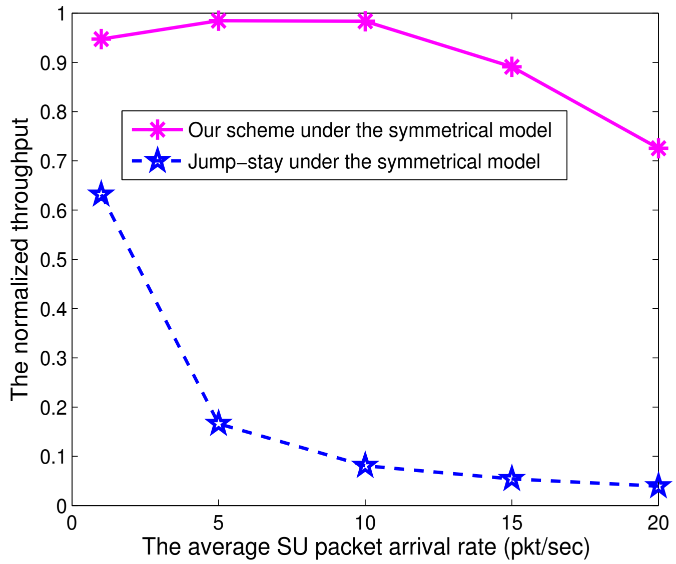

Figure 3.20: The average normalized throughput under the symmetrical model as the SU packet arrival rate increases.

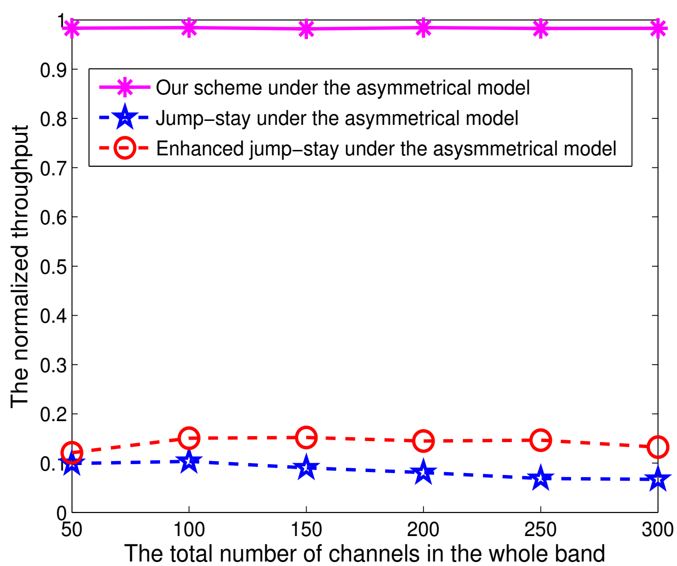

Figure 3.21: The average normalized throughput under the asymmetrical model as the number of total channels increases.

when the SU packet arrival rate changes from 1 to 20. We consider a wide-band spectrum with 200 channels. The result shows that our proposed scheme performs much better than both the jump-stay scheme and enhanced jump-stay scheme when the SU packet arrival rate changes. Therefore, our proposed scheme can adapt to different SU traffic under the asymmetrical model.

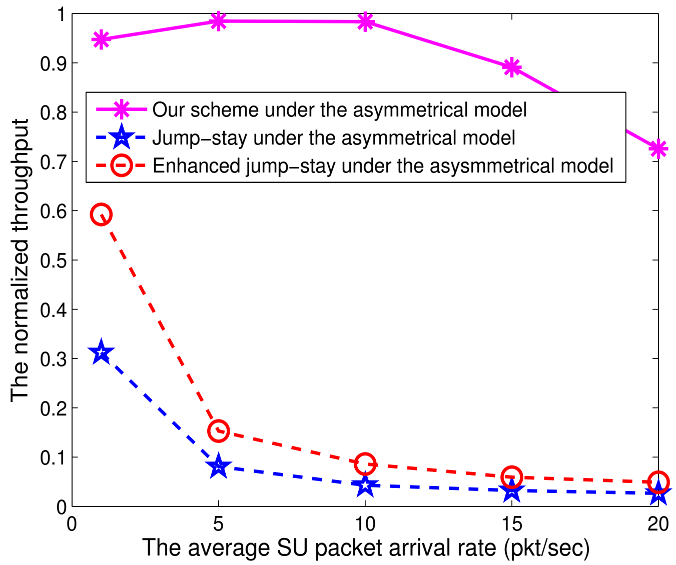

Figure 3.22: The average normalized throughput under the asymmetrical model as the SU packet arrival rate increases.

3.1.8.5 The Effect of the Value of θ

Our spectrum splitting scheme is based on a predetermined value θ. Therefore, how to determine an appropriate θ is crucial for our proposed communication framework. In this subsection, we show the effect of θ on the average normalized throughput and the average collision ratio which can help us to determine θ under both the symmetrical and asymmetrical models. The average collision ratio is defined as the ratio of the number of successful rendezvous with collisions to the total number of rendezvous. The parameters in the following simulation are the same as the ones in the last subsection, except the total number of channels and θ. We set the total number of channels to be 100 and change the value of θ from 10 to 50.

Fig. 3.23 shows the effect of θ under the symmetrical model on the average normalized throughput. When θ increases, the number of channels in each SS increases. This may result in a longer time to rendezvous in our proposed scheme. Thus, the average normalized throughput decreases.

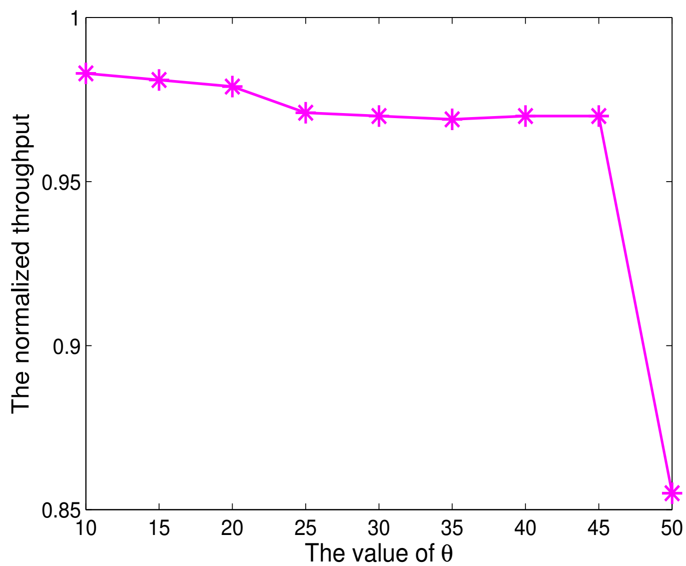

Figure 3.23: The effect of θ to the throughput under the symmetrical model.

Fig. 3.24 shows the effect of θ on the average collision ratio under the symmetrical model. When θ increases, the average collision ratio decreases. This is reasonable since when θ increases, a SU pair can rendezvous on more channels in a SS. Thus, the probability that multiple SUs rendezvous on the same channel is reduced. By observing Fig. 3.23 and Fig. 3.24 together, we can notice that when the collision ratio increases, the average normalized throughput increases also. This indicates that the rendezvous collisions under the symmetrical model may not affect the throughput.

Fig. 3.25 shows the effect of θ under the asymmetrical model on the average normalized throughput. When θ increases, the average normalized throughput first increases

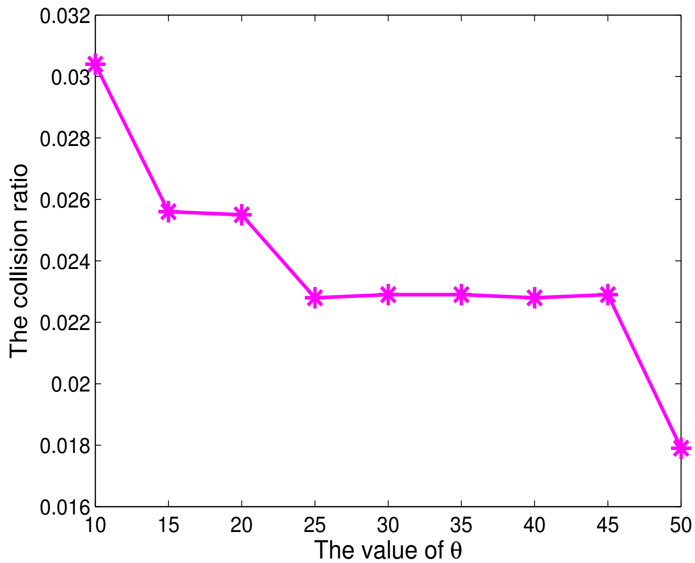

Figure 3.24: The effect of θ to the collision ratio under the symmetrical model.

and then decreases. The reason is that under the asymmetrical model, each SU will use all the channels in a SS to design a channel hopping sequence. When the number of channels in a SS is small, the $ETTR$ decreases, but the probability of a collision between the communicating pairs in the same SS increases, which may decease the average normalized throughput. On the other hand, when the number of channels in a SS increases, the $ETTR$ increases, which can also decrease the average normalized throughput. Therefore, there exists an optimal θ for the asymmetrical model to maximize the average normalized throughput. Under this particular scenario, we can set θ to be 35 or 40 to get the maximum average normalized throughput.

Fig. 3.26 shows the effect of θ on the average collision ratio under the asymmetrical model. When θ increases, the collision ratio decreases. By observing Fig. 3.25 and Fig. 3.26 together, we can notice that when the collision ratio decreases, the throughput fluctuates. This indicates that the rendezvous collision under the asymmetrical model is a factor which can affect the throughput.

Based on the above analysis, when the system performance requirements are given,

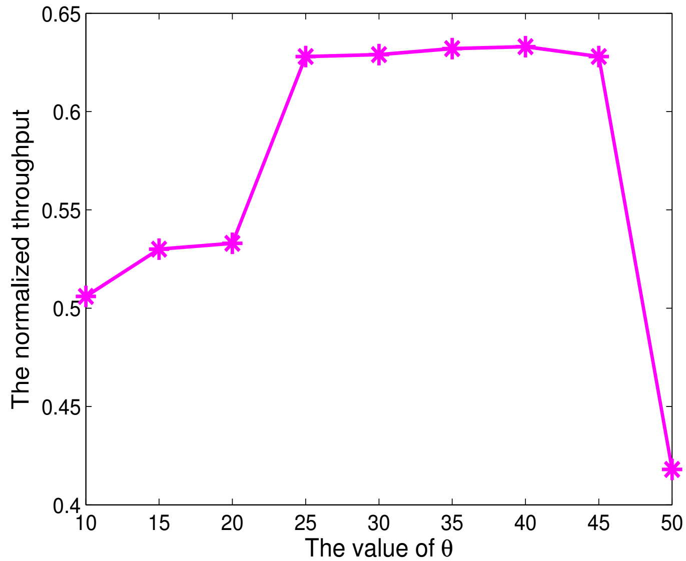

Figure 3.25: The effect of θ to the throughput under the asymmetrical model.

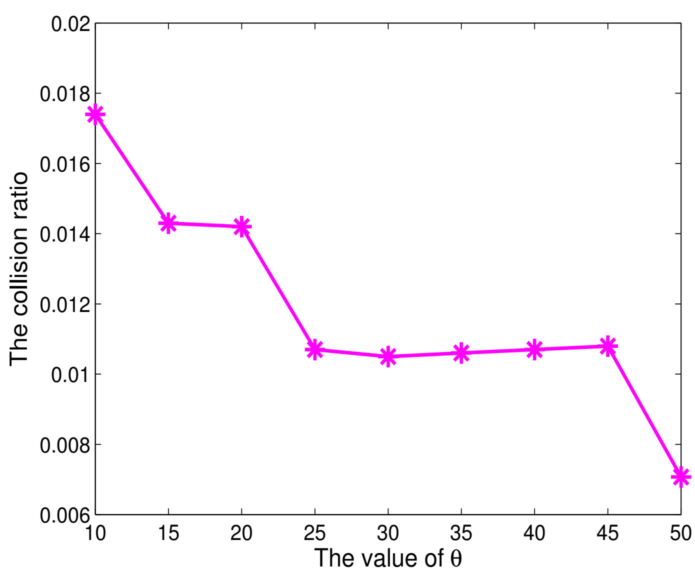

Figure 3.26: The effect of θ to the collision ratio under the asymmetrical model.

an appropriate θ can be predetermined.

3.2 Rendezvous Schemes Without Predetermined Sender and Receiver Considering Multiple Cognitive Radios

In this section, we propose two efficient rendezvous schemes under the scenarios that there is not a predetermined sender or receiver for a rendezvous process during the initialization phase of cognitive radio networks when SUs are equipped with multiple radios.

3.2.1 System Model and Problem Description

In this research, we assume that there are totally N SUs and N_p PUs coexisting. There are totally M channels that all the SUs and PUs can access. Each SU is equipped with a GPS which can get its location coordinates. Furthermore, each SU is equipped with exactly R homogeneous half-duplex cognitive radios. Therefore, a cognitive radio cannot send and receive signals simultaneously. We assume that for a SU, all its cognitive radios use the same transmission power and have the same interference range.

There is no CCC in the network through which all SUs can exchange their control information. Before the initialization process of a CRN, each SU does not know any information about other SUs, which is a very practical assumption. A time-slotted system is deployed among all SUs and they do no need to be strictly synchronized.

A channel hopping scheme is adopted for the rendezvous process. During a rendezvous process, each cognitive radio of an SU hops to a channel chosen from its available channel set, senses the current availability of the channel, and sends an RTS packet or waits for an RTS packet from another SU. Therefore, each time slot includes the sensing phase and the sending or receiving phase during the rendezvous process. According to existing rendezvous schemes, a successful rendezvous occurs when two

SUs hop to a common available channel at the same time slot, but the process of sending an RTS packet and receiving a CTS reply is not considered. In this research, we define this kind of rendezvous as *channel rendezvous*. This may not be a problem when one SU is always the sender while the other is the receiver, which means that there is an explicit send-or-receive relationship between the two SUs. Under this scenario, after one SU sends an RTS packet, if the other SU receives the RTS packet and replies a CTS packet, the rendezvous succeeds.

However, during the initialization phase of a CRN, each SU may want to rendezvous with other SUs to exchange control information. There is no explicit role for a SU, since it cannot know if other SUs are senders or receivers currently. If two SUs hop to a common available channel and they both have their transceivers on, none of them can receive the RTS packets, which can fail a successful rendezvous. Furthermore, each SU does not know if its target SU is on the current common available channel and is sending or receiving in the current time slot. Practically, only when the two SUs hop to a common available channel and when one SU is sending while the other is trying to receive on that channel, a successful rendezvous can finally happen. In this research, we denote this kind of rendezvous as *link rendezvous*. Therefore, when an SU is equipped with multiple cognitive radios, during a rendezvous process, the SU should not only consider which available channel each cognitive radio should tune to but also determine that each radio should be in the sending or receiving mode. We call this problem as the *send-or-receive problem* in a rendezvous scheme.

For the ith SU with R cognitive radios, during time slot t of a rendezvous process, it generates a *task allocation list* $\mathbb{A}_i^t = (c_{i1}^t, s_{i1}^t), (c_{i2}^t, s_{i2}^t), \ldots, (c_{iR}^t, s_{iR}^t)$, where $1 \leq c_{iu}^t \leq M$, $s_{iu}^t = 1$ (send) or $s_{iu}^t = 0$ (receive), and (c_{iu}^t, s_{iu}^t) is called a *task pair* for the uth cognitive radio of the ith SU during time slot t. We also denote C_i^t as the available channel set of the ith SU during time slot t. We define *link rendezvous* as follows.

Link Rendezvous: A successful link rendezvous between the ith SU and the jth SU during time slot t requires that there exists a task pair (c^t_{iu}, s^t_{iu}) for the uth cognitive radio of the ith SU and a task pair (c^t_{jv}, s^t_{jv}) for the vth cognitive radio of the jth SU such that $c^t_{iu} \in C^t_i$, $c^t_{jv} \in C^t_j, c^t_{iu} = c^t_{jv}$, and s^t_{iu} XOR $s^t_{jv} = 1$.

Moreover, we can notice that $\mathbb{A}^t_i$ should satisfy the following constraints:

1. No two task pairs (c^t_{iu}, s^t_{iu}) and (c^t_{iv}, s^t_{iv}) satisfy that $c^t_{iu} = c^t_{iv}$ and $s^t_{iu} = s^t_{iv} = 1$, which means that two cognitive radios of the same SU cannot send RTS packets simultaneously on the same channel.

2. No two task pairs (c^t_{iu}, s^t_{iu}) and (c^t_{iv}, s^t_{iv}) satisfy that $c^t_{iu} = c^t_{iv}$ and s^t_{iu} XOR $s^t_{iv} = 1$, which means that when one cognitive radio of an SU is sending an RTS packet, there should be no other cognitive radio of the same SU that is trying to receive RTS packets on the same channel.

Therefore, the problem is how to generate a *task allocation list* $\mathbb{A}$ for each SU to realize a fast and guaranteed rendezvous considering the *send-or-receive problem*.

3.2.2 The Basic Rendezvous Scheme

In this section, we discuss the blind rendezvous problem between two SUs when considering multiple cognitive radios and the *send-or-receive problem*. At the beginning of the initialization phase of a CRN, each SU does not know any information about other SUs. Therefore, each SU can only perform a blind rendezvous process based on its local information to set up links with other SUs. However, previous research works only focus on designing the channel-hopping sequence so that the two SUs can hop to a common available channel at the same time, while ignore the *send-or-receive problem*, especially when each SU is equipped with multiple cognitive radios. Our goal is to generate a *task allocation list* for each SU during each time slot based on an existing channel hopping sequence so that two SUs can achieve a successful *link rendezvous* within bounded time slots.

Our proposed basic link rendezvous scheme is to generate a *task allocation list* $\mathbb{A}$ for each SU during each time slot based on an existing channel hopping sequence which can guarantee a successful *channel rendezvous* within bounded time slots. Given a specific channel hopping sequence (e.g., the jump-stay channel hopping sequence in [8]), we denote the channel-hopping sequence for the jth radio of the ith SU as $\mathbb{S}_{ij}$, and the element in $\mathbb{S}_{ij}$ at time slot t as $\mathbb{S}_{ij}^t$. After $\mathbb{S}_{ij}$ is generated, for the task allocation list $\mathbb{A}_i^t$, all the values of $c_{i1}^t, c_{i2}^t, \ldots, c_{iR}^t$ are determined as $c_{i1}^t = \mathbb{S}_{i1}^t, c_{i2}^t = \mathbb{S}_{i2}^t, \ldots, c_{iR}^t = \mathbb{S}_{iR}^t$. Therefore, the next step is to set the binary values of 0 or 1 to $s_{i1}^t, s_{i2}^t, \ldots, s_{iR}^t$ to guarantee that $\mathbb{A}_i^t$ satisfy the constraints explained in Section 3.5.7.

For any $c_{ij}^t, 1 \leq j \leq R$, if c_{ij}^t appears more than once among all the R radios, which means that more than one radio will hop on channel c_{ij}^t during time slot t, the radios allocated to channel c_{ij}^t should wait for RTS packets according to the constraints in Section 3.5.7 Other radios will randomly choose sending or receiving roles to themselves.

Here is an example of the $0-1$ allocation solution. Assume that $R = 5$ and $c_{i1}^t = 1, c_{i2}^t = 3, c_{i3}^t = 3, c_{i4}^t = 2, c_{i5}^t = 1$. Since channel 3 appears twice, $s_{i2}^t = 0$ and $s_{i3}^t = 0$, which means that the second and third radio should wait for RTS packets during the current time slot on channel 3. Since channel 1 also appears twice, $s_{i1}^t = 0$ and $s_{i5}^t = 0$, which means that the first and fifth radio should also wait for RTS packets during the current time slot on channel 1. For the fourth radio, we can randomly set $s_{i4}^t = 1$. Finally, the generated *task allocation list* for the ith SU during time slot t is $\mathbb{A}_i^t = \{(c_{i1}^t = 1, s_{i1}^t = 0), (c_{i2}^t = 3, s_{i2}^t = 0), (c_{i3}^t = 3, s_{i3}^t = 0), (c_{i4}^t = 2, s_{i4}^t = 1), (c_{i5}^t = 1, s_{i5}^t = 0)\}$.

The details of our proposed rendezvous scheme for the ith SU is shown in **Algo-**

rithm 6.

Algorithm 6 The basic rendezvous algorithm considering the send-or-receive problem

Input:

The number of cognitive radios: R

The current time slot: t

A pre-designed channel-hopping sequence $\mathbb{S}$

Assume that the jth radio starts the channel hopping at time t_j;
for $j = 1$ to R **do**
    $c_{ij}^t = \mathbb{S}_{t+t_j}$;
end for
for $j = 1$ to R **do**
    if c_{ij}^t appears more than once **then**
        $s_{ij}^t = 0$;
    else
        s_{ij}^t is randomly set to be either 0 or 1;
    end if
    Tune the jth radio to channel c_{ij}^t;
    if $c_{ij}^t \in C_i^t$ and $s_{ij}^t = 1$ **then**
        Send an RTS packet on channel c_{ij}^t;
    else
        Wait for an RTS packet on channel c_{ij}^t;
    end if
end for

Based on **Algorithm 6**, we can get **Theorem 12**.

Theorem 12. *Algorithm 6 can guarantee a successful* ***channel*** *rendezvous, and the generated task allocation list $\mathbb{A}_i^t$ can satisfy the constraints in Section 3.5.7*

Proof. Since each cognitive radio of the ith SU hops following a pre-defined channel hopping sequence which can guarantee a successful channel rendezvous, our proposed scheme can also guarantee a successful channel rendezvous between two SUs as long as two of their radios achieve a channel rendezvous. Furthermore, according to **Algorithm 6**, if multiple radios hop to a same channel, all of them will wait for RTS packets. Therefore, all the constraints in Section 3.5.7 can be satisfied. □

3.2.3 The Enhanced Rendezvous Scheme

One problem of the basic rendezvous scheme is that multiple radios may all use one same channel to receive RTS packets, if they all hop to the same channel simultaneously. Therefore, the cognitive radios of an SU are not well utilized. For example, when each SU is equipped with 4 cognitive radios, if 3 of them hop to channel 1 and the last radio hops to channel 2 at the same time, under the basic scheme it is equivalent to just using two radios during the current time slot. In this section, we propose an enhanced rendezvous scheme, considering multiple cognitive radios and the *send-or-receive problem*, which can well utilize all the radios of an SU during each time slot to achieve a fast *link rendezvous.*

The basic idea of our proposed enhanced rendezvous scheme is to split the total M channels into several continuous groups. Each group has exactly R consecutive channels, where R is the number of cognitive radios of a SU. If $M \bmod R \neq 0$, then sequentially add channel 1, 2, ..., $R-1$ to the last group until the number of channels is R. There is an example in Fig. 3.27 when $M = 10$, $R = 4$, the generated $\lceil \frac{10}{4} \rceil = 3$ channel groups are $\{1, 2, 3, 4\}$, $\{5, 6, 7, 8\}$, and $\{9, 10, 1, 2\}$.

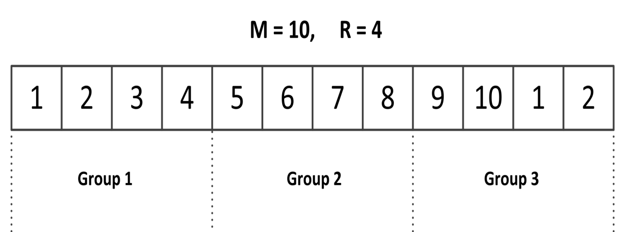

Figure 3.27: An example of generating all channel groups.

We denote $M_s = \lceil \frac{M}{R} \rceil$ as the total number of channel groups, $\mathcal{G} = \{G_1, G_2, \ldots, G_{M_s}\}$ as the set of all the channel groups, and $G_i = \{g_{i1}, g_{i2}, \ldots, g_{iR}\}$ as the channels in the ith channel group. As all SUs are equipped with exactly R cognitive radios and can access totally M channels, they can generate the same $\mathcal{G}$ independently. For a

SU, it applies M_s rather than the number of all channels M to generate a hopping sequence, based on an existing channel hopping sequence generating algorithm. We denote this sequence as the *channel-group hopping sequence* (CGHS).

Our basic idea is to assign each cognitive radio with a different current available channel during each time slot. During time slot t, if the ith SU hops to the jth channel group, it allocates current available channels in G_j sequentially to its R cognitive radios. If some channels become unavailable during the current slot which makes the total available channels in G_j are not enough for all R radios, the SU also allocates the available channels in the next channel groups until each of the R radios is allocated with a different current available channel. If the number of available channels among the total M channels is less than R, then allocate each of them to a radio and let the rest radios keep silent during the current time slot. Here is an example when $M = 10$, $R = 3$, and the available channels in each channel group are $\{2, 3\}$, $\{5\}$, $\{7, 9\}$, and $\{10, 2\}$. Assume that an SU hops to the second channel group during the current time slot according to the CGHS. The 3 allocated available channels are channel 5 from the second channel group and channel $7, 9$ from the third channel group.

The last step is to solve the *send-or-receive problem*. Note that this problem is equivalent to design a binary string of length R containing only 0 or 1. We define such a string of length R as β_R. Therefore, during time slot t, solving the *send-or-receive problem* for the ith SU is equivalent to designing a binary string β_{iR}^t. We derive the following lemmas regarding a successful *link rendezvous* between the ith and jth SU based on β_{iR}^t and β_{jR}^t.

Lemma 12. *A necessary condition of a successful **link** rendezvous between the ith and jth SU during time slot t is that β_{iR}^t XOR $\beta_{jR}^t > 0$, if we regard β_{iR}^t and β_{iR}^t as the binary representations of two integers.*

Proof. If β_{iR}^t XOR $\beta_{jR}^t = 0$, i.e., every bit of the two binary strings is the same, there does not exist a send-and-receive pair between the two SUs which can guarantee successfully sending and receiving RTS packets. However, if β_{iR}^t XOR $\beta_{jR}^t > 0$, there are at least two corresponding bits that are different and they can make one radio of an SU send and one radio of the other SU receive. □

The challenge of designing β_{iR}^t is that the two SUs do not know any information about each other before a successful rendezvous. Our goal is to find a good distributed scheme to generate β so that β_{iR}^t XOR $\beta_{jR}^t = 0$ happens as less as possible. The following lemma can help us design a well performed β_R.

Lemma 13. *Two different binary strings β_{iR} and β_{jR}, always satisfy β_{iR} XOR $\beta_{jR} > 0$, as long as they are the binary representations of two different integers from the range $[0, 2^R - 1]$.*

Proof. If β_{iR} XOR $\beta_{jR} = 0$, the decimal representations of the two binary strings should be the same, as each bit of the two strings is the same, which contradicts the fact that they represent two different numbers. □

The details of our proposed enhanced rendezvous scheme for the ith SU is shown in **Algorithm 7**.

Based on **Lemma 13** and **Algorithm 7**, we can get **Theorem 13**.

Theorem 13. *The enhanced rendezvous scheme in Algorithm 7 can guarantee a successful* ***channel*** *rendezvous. The probability that two SUs independently choose the same binary string for the send-or-receive problem, which may fail a link rendezvous, is $\frac{1}{2^{R\eta}}$, where R is the number of cognitive radios each SU is equipped with and η*

Algorithm 7 The enhanced blind rendezvous algorithm considering the send-or-receive problem

Input:

SU's unique ID i
The number of cognitive radios R
The current time slot t
An existing channel hopping sequence generating algorithm

Get the list of channel groups $\mathcal{G}$;
Based on $M_s = \lceil \frac{M}{R} \rceil$, generate a group hopping sequence $\mathbb{S}_i$;
Current channel group id $k = \mathbb{S}_i^t$;
for $j = 1$ to R **do**
    c_{ij}^t = an available channel in $G_k, G_{k+1}, \ldots$;
end for
Randomly choose an integer from the range $[0, 2^R - 1]$;
Get the binary representation β_R of length R of the chosen number;
for $j = 1$ to R **do**
    if $\beta_R[j] = 1$ **then**
        $s_{ij}^t = 1$;
    else
        $s_{ij}^t = 0$;
    end if
end for
for $j = 1$ to R **do**
    if $s_{ij}^t = 1$ **then**
        Turn on the transceiver of the jth radio and send an RTS packet on channel c_{ij}^t;
    else
        Turn on the receiver of the jth radio and stay on channel c_{ij}^t waiting for RTS packets;
    end if
end for

is the number of times that two radios of the two SUs hop to a common available channel during a blind rendezvous process.

Proof. We define a channel group between two SUs as a valid group if there is a common available channel in it. As long as two SUs have at least one common available channel, there is a valid group between them as they split all the channels into the sam channel groups. Therefore, the two SUs will hop to a valid group as long as they follow the hopping sequence which can guarantee a successful *channel rendezvous* on a common available channel. According to **Algorithm 7**, two SUs have the same send-or-receive allocations only when they choose the same random integer from the range $[0, 2^R - 1]$. The probability that two SUs independently choose the same integer from $[0, 2^R - 1]$ is $\frac{1}{2^R} \times \frac{1}{2^R} = \frac{1}{2^{2R}}$. There are totally 2^R different integers from $[0, 2^R - 1]$. Therefore, the final probability of a link rendezvous failure is $(\frac{1}{2^{2R}} \times 2^R)^\eta = \frac{1}{2^{R\eta}}$. □

3.3 Rendezvous Schemes Without Predetermined Send and Receiver Considering a Single Radio

In this section, we propose two efficient rendezvous schemes under the scenarios that there is not a predetermined sender or receiver for a rendezvous process during the initialization phase of cognitive radio networks when SUs are equipped with single radio.

3.3.1 System Model and Problem Description

In this research, we assume that there are totally N SUs and N_p PUs coexisting. There are totally M channels that all the SUs and PUs can access. Furthermore, each SU is equipped with a half-duplex cognitive radio. Therefore, a cognitive radio cannot send and receive signals simultaneously. There is no CCC in the network through which all SUs can exchange their control information. Before the initialization process of a CRN, each SU does not know any control information about other

SUs, which is a very practical assumption. A time-slotted system is deployed among all SUs and PUs and they do no need to be strictly synchronized.

A channel hopping scheme is adopted for the rendezvous process between two SUs. During a rendezvous process, each SU hops to the current channel based on a channel hopping sequence, senses the current availability of the channel, and sends an RTS packet or waits for an RTS packet from another SU. Therefore, each time slot includes the sensing phase and the sending or receiving phase during the rendezvous process. According to existing rendezvous schemes, a successful rendezvous occurs when two SUs hop to a common available channel at the same time slot, but the process of sending an RTS packet and receiving a CTS reply is not considered. In this research we define this kind of rendezvous as *channel rendezvous*. This may not be a problem when one SU is always the sender while the other is the receiver, which means that there is an explicit send-or-receive relationship between the two SUs. Under this scenario, after one SU sends an RTS packet, if the other SU receives the RTS packet and replies a CTS packet, the rendezvous succeeds.

However, during the initialization phase of a CRN, each SU may try to rendezvous with other SUs to exchange control information. There is no explicit role for an SU, since it cannot know if other SUs are senders or receivers currently. If two SUs hop to a common available channel and their transceivers are both tuning to the sending mode , none of them can receive the RTS packets, which can fail a successful rendezvous. Furthermore, each SU does not know if its target SU is on the current common available channel and is in the sending or receiving mode during the current time slot. Practically, only when the two SUs hop to a common available channel and when one SU is sending while the other is trying to receive on that channel, a successful rendezvous can finally happen. In this research, we denote this kind of rendezvous as *link rendezvous*. Therefore, in a *link rendezvous* process without the explicit sender or receiver , an SU should not only determine the channel it should hop to but also whether tuning its half-duplex radio to the sending or receiving mode during a time slot. We call this problem as the *send-or-receive problem* in a *link rendezvous* scheme.

For the ith SU, during time slot t of a link rendezvous process, it generates a *task allocation* $\mathbb{A}_i^t = (c_i^t, s_i^t)$, where $1 \leq c_i^t \leq M$, $s_i^t = 1$ (send) or $s_i^t = 0$ (receive). We also denote C_i^t as the available channel set of the ith SU during time slot t. We define *link rendezvous* as follows.

Link Rendezvous: A successful link rendezvous between the ith SU and the jth SU during time slot t requires that (c_i^t, s_i^t) and (c_j^t, s_j^t) satisfy $c_i^t \in C_i^t$, $c_j^t \in C_j^t, c_i^t = c_j^t$, and s_i^t XOR $s_j^t = 1$.

In the following two sections, we propose two different distributed link rendezvous schemes based on two different basic ideas.

3.3.2 The Basic Link Rendezvous Scheme

Our basic idea is that besides the channel hopping sequence, considering the send-or-receive problem, there is a *transmission sequence* S_i for the ith SU. S_i only contains 0 or 1 values which represent the sending mode or receiving mode, respectively, for each SU during each time slot, i.e, If $S_i[t] = 1$, during the current time slot, the SU should tune its transceiver to the sending mode, and If $S_i[t] = 0$, during the current time slot, the SU should tune its transceiver to the receiving mode. When tuning to the sending mode during a time slot, an SU can first send an RTS packet on the current channel and then tune to the receiving mode waiting for a replied CTS packet. When tuning to the receiving mode, an SU should tune the transceiver to the receiving mode, wait for RTS packets from other SUs on the current channel, and tune to the sending to reply a CTS packet if it received an RTS packet.

Since we try to design a distributed scheme for each SU, our idea for the basic link rendezvous scheme is to make each SU apply the same basic transmission sequence S_b. We assume that the length of the basic transmission sequence S_b is L and each SU circularly hops according to S_b, i,e, if the SU hops to the end of S_b, it should hop to the first element of S_b during the next time slot. For example, if the transmission sequence is $\{1, 0, 0, 1\}$, the real transmission sequence should be $\{1, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 1 \ldots\}$. However, since two SUs are not synchronized and do not know any control information about each other before a successful link rendezvous, two SUs may follow different real transmission sequences even though they all follow the same basic transmission sequence S_b. Therefore, for the ith SU and jth SU, the real transmission sequence of the jth SU S_j should be a circular shift from the ith SU's real transmission sequence S_i. For example, if the basic transmission sequence S_b is $\{1, 0, 0, 1\}$, S_i is $\{1, 0, 0, 1, 1, 0, 0, 1, \ldots\}$, and S_j is $\{1, 1, 0, 0, 1, 1, 0, 0, \ldots\}$.

We define the element pair (S_i^t, S_j^t) as the *transmission pair* of the ith and jth SU. According to the previous analysis, only when one SU is on sending mode while the other is on receiving mode, two SUs can successfully finish a RTS and CTS exchange. Therefore, only the transmission pair $(0, 1)$ or $(1, 0)$ can result in a successful link rendezvous. We define $(0, 1)$ or $(1, 0)$ as the *valid transmission pair*. We define the *valid number* $N_v = \mathcal{V}(S_i, S_j)$ as the number of valid transmission pairs two real transmission sequences could get within the period L, where

$$\mathcal{V}(S_i, S_j) = \sum_{k=0}^{L-1} [(S_i[k] \ XOR \ S_j[k]) == 1]. \tag{3.4}$$

Furthermore, the average valid number $\overline{N_v}$ of a basic transmission sequence S_b is defined as

$$\overline{N_v} = \mathcal{B}(S_b) = \frac{1}{L} \sum_{k=0}^{L-1} \mathcal{V}(S_i, \mathcal{H}(S_j, k)), \tag{3.5}$$

where $\mathcal{H}(S_j, k)$ is defined as the function to make S_j perform k times circularly left shifts. A larger $\overline{N_v}$ means that there are more valid transmission pairs within a period which can make a successful link rendezvous happen with a higher probability. Therefore, our goal is to find the optimal basic transmission sequence S_b for a given length L, which can finally result in the largest average valid number $\overline{N_v}$.

To get the optimal basic transmission sequence S_b with length L, we first derive the following lemma:

Lemma 14. *For a basic transmission sequence with length L, if there are n 1 elements in it. The average valid number $\overline{N_v}$ is $2(nL - n^2)/L$.*

Proof. Considering all the L scenarios regarding different circular shifts regarding a basic transmission sequence, each 1 element can generate total $L-n$ such $(1,0)$ pairs which are valid transmission pairs. Therefore there are total $2n(L-n)$ $(1,0)$ pairs two basic transmission sequences can get. Finally, the average valid number considering all L different circular shifts is $2(nL - n^2)/L$. □

Lemma 15. *When the number of 1 elements in S_b is $\frac{L}{2}$, The average valid number $\overline{N_v}$ reaches its maximum value $L/2$.*

Proof. For a given L, $\overline{N_v} = \frac{2}{L}(n_r L - n_r^2) = \frac{2}{L}(-(n_r - \frac{L}{2})^2 + \frac{L^2}{4})$. Therefore, when $n = \frac{2}{L}$, $\overline{N_v}$ reaches the maximum value $\frac{2}{L} \times \frac{L^2}{4} = \frac{L}{2}$. □

Lemma 14 indicates that $\overline{N_v}$ is only related to the number of 1 elements in S_b regardless of their positions, for a given L. Therefore, each SU can just generate its own basic transmission sequence of length L which also should contains $\frac{L}{2}$ 1 elements. We propose a distributed algorithm for each SU to achieve a successful link rendezvous in **Algorithm 8** in which we let L equal to the number of total channels.

Here is an example of the basic link rendezvous scheme in Fig. 3.28, where $M = 4$, the transmission sequence of SU1 is $\{1, 0, 0, 1\}$, the transmission sequence of SU2 is $\{1, 1, 0, 0\}$, each SU hops according to its own channel hopping sequence, and

Algorithm 8 The basic distribute rendezvous algorithm for each SU

Input:

The total number of channels: M

A channel hopping sequence: S

Randomly generate a basic transmission sequence S_b with length M;
Randomly choose a starting index i on S_b;
$count = 0$; //count time slots
while Not rendezvous **do**
  Hop to the current channel according to S.
  if the current channel is available and $S_b[i] == 1$ **then**
    Tune to the sending mode and send RTS packets;
  end if
  if the current channel is available and $S_b[i] == 0$ **then**
    Tune to the receiving mode;
  end if
  i++;
  if $i > M$ **then**
    i = 1; //circularly hop
  end if
end while

the common available channels for them are channel 3 and 2. The two SUs first hop to the common available channel 3. However, they do not achieve a successful link rendezvous since they are all tuning to the sending mode during that time slot. Therefore, they just continue the channel hopping process until they all hop to the common available channel 2. Finally, they achieve a successful link rendezvous on channel 2 since SU1 is tuning to the sending mode and SU2 is tuning to the receiving mode.

					Fail							Succeed				
SU1	1	1	4	2	3	2	4	3	1	1	4	2	3	2	4	3
	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1
SU2	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0
	2	3	2	4	3	1	1	4	4	2	3	2	4	3	1	1

Figure 3.28: An example of the basic link rendezvous scheme.

Based on **Algorithm 8**, we can get the following theorem:

Theorem 14. *The expectation of time slots to achieve a link rendezvous for **Algorithm 8** is $E[T] = 2T_e$, where T_e is the expectation of the channel rendezvous time for a specific given channel hopping sequence.*

Proof. According to the design of the transmission sequence of each SU, the probability that there is one sender and one receiver when two SUs hop to a common available channel is 0.5, Therefore, if two SUs hop to a common available channel, the probability of a successful link rendezvous is 0.5. If a link rendezvous fails because of that there is not a valid transmission pair, two SUs should continue the channel hopping process until a successful link rendezvous. Finally, the expectation of time slots to achieve a link rendezvous time is:

$$\sum_{i=1}^{\infty} T_e(1-0.5)^{(i-1)}(0.5) = \frac{T_e}{0.5} = 2T_e. \tag{3.6}$$

□

3.3.3 The Enhanced Link Rendezvous Scheme

In the basic link rendezvous scheme, each SU should follow two sequences, the channel hopping sequence and the transmission sequence. In this section, we propose an enhanced link rendezvous scheme that each SU can just follow a *virtual channel hopping sequence* so each SU does not need to implement an existing channel hopping sequence and a transmission sequence simultaneously.

Our basic idea is to combine the channel hopping sequence with the transmission sequence. Therefore, we expand the total M practical channels to $2M$ *virtual channels*. For real channel i, we map it to two virtual channels i and $i+M$. An example of generating the virtual channels when M equals 4 is shown in Fig. 3.29. Therefore, after this mapping process, for each SU, there are total $2M$ virtual channels. Each

SU hops according to a *virtual channel hopping sequence* generated from the total $2M$ virtual channels. When an SU hops to the virtual channel i that $i \leq M$, it should tune its transceiver to the sending mode and hop to real channel i. On the other side, when an SU hops to the virtual channel i that $M < i \leq 2M$, it should tune its transceiver to the receiving mode and hop to real channel $i - M$. Furthermore, if channel i is available to an SU during current time slot, both virtual channels i and $i + M$ are also available to the SU.

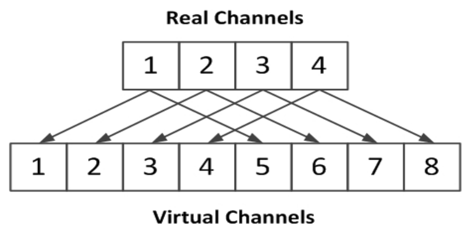

Figure 3.29: An example of generating virtual channels.

Therefore, based on the virtual channel hopping sequence and the send-or-receive problem, a successful link rendezvous is defined as follows:

Successful link rendezvous: During time slot t, if the ith SU hops to the virtual channel c_i^t and the jth SU hops to the virtual channel c_j^t, a successful link rendezvous is achieved if $|c_i^t - c_j^t| = M$, $c_i^t \in \mathcal{C}_i^t$, and $c_j^t \in \mathcal{C}_j^t$, where $\mathcal{C}_i^t$ and $\mathcal{C}_j^t$ are the current available virtual channel sets of the ith and jth SU, respectively.

According to the definition of a successful link rendezvous based on the virtual channel hopping sequence, it is quite different from a successful *channel rendezvous* that two SUs hop to a common available channel at the same time slot. Therefore, all the existing channel hopping sequences which can guarantee a successful *channel rendezvous* cannot achieve a successful *link rendezvous* based on the virtual channels.

Our goal in this section is to design an efficient *link rendezvous* scheme which can achieve a fast successful link rendezvous between two SUs. The first step is to design a virtual channel hopping sequence based on the total $2M$ virtual channels. The basic idea about the virtual channel hopping sequence is to make each SU performs channel hopping according to the same virtual channel hopping sequence $\mathcal{S}$ generated from $2M$ virtual channels. However, since two SUs are fully distributed and not synchronized, even though they follow the same virtual channel hopping sequence, they may stay on the different positions of it. Therefore, the sequence should try to achieve a successful link rendezvous regardless of different positions of two SUs on the same virtual channel hopping sequence.

The first step to generate such a virtual channel hopping sequence is to generate a basic sequence $\mathcal{B}$ such that its length is $4M$ and $\mathcal{B}[i] = \mathcal{B}[j] = j - i, 1 \leq i < j \leq 4M$, for any $1 \leq \mathcal{B}[i] \leq 2M$, where $\mathcal{B}[i]$ is the ith element of $\mathcal{B}$. Therefore, each number in the range $[1, M]$ appears exactly twice in $\mathcal{B}$.

Based on the definition of $\mathcal{B}$, we can first get the following lemma regarding the existence of $\mathcal{B}$.

Lemma 16. *If two SUs follow the same basic virtual channel hopping sequence $\mathcal{B}$, the first SU starts from the ith element , the second SU starts from the jth element, and $k = |i - j| > 0$, they could both hop to virtual channel k if $k \leq 2M$, or virtual channel $4M - K$ if $2M < k < 4M$, within $4M$ time slots.*

Proof. According to the definition of $\mathcal{B}$, if $k \leq 2M$, there are two elements in the sequence $\mathcal{B}[u] = \mathcal{B}[v] = k, v > u$, and $|v - u| = k$. Therefore, the two SUs can all hop to the virtual channel k within $4M$ time slots. If $2M < k \leq 4M$, since two SUs hop according to the same $\mathcal{B}$ circularly, the distance between them on the sequence is also $4M - K < 2M$. Therefore, according to the definition of $\mathcal{B}$, there are two elements in the sequence $\mathcal{B}[u] = \mathcal{B}[v] = 4M - k, v > u$, and $|v - u| = 4M - k$.

$\square$

Here is an example in Fig.3.30. In the example, we assume that $M = 2$, SU1 starts from the first channel in $\mathcal{B}$, and SU2 starts from the fourth channel in $\mathcal{B}$, during the fifth time slot, both SUs hop to channel 3.

SU1	1	1	4	2	3	2	4	3
SU2	2	3	2	4	3	1	1	4

Figure 3.30: An example of the basic virtual channel hopping sequence.

To get the final virtual channel hopping sequence $\mathcal{S}$, The second step is to replace some elements in $\mathcal{B}$ according to the following manipulations: for each number k that $\mathcal{B}[i] = \mathcal{B}[j] = k, 1 \leq i < j \leq 4M$, if $k \leq M$, reset $\mathcal{B}[j] = k + M$, otherwise reset $\mathcal{B}[j] = k - M$. An example is shown in Fig. 3.31, where $M = 2$ and the basic sequence $\mathcal{B}$ could be $\{1, 1, 4, 2, 3, 2, 4, 3\}$, after the replacements, the final channel hopping sequence $\mathcal{S}$ should be $\{1, 3, 4, 2, 3, 4, 2, 1\}$.

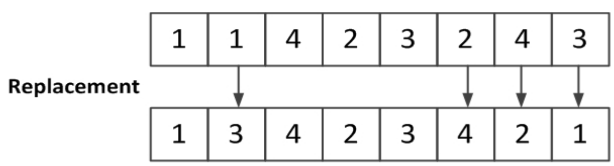

Figure 3.31: An example of the replacement manipulations.

In order to generate the virtual channel hopping sequence $\mathcal{S}$, we have the following lemmas about some important features of the basic virtual channel hopping sequence $\mathcal{B}$.

Lemma 17. *When $2M$ mod $4 = 2$, $\mathcal{B}$ does not exist.*

Proof. According to the definition of $\mathcal{B}$, designing the basic sequence is equivalent to dividing all the indexes $1, 2, \ldots, 4M$, these $4M$ numbers into two lists $\mathcal{L}_1$ and $\mathcal{L}_2$, each of which has exactly $2M$ different numbers, such that for $k = 1, 2, \ldots, 2M$, $\mathcal{L}_2[k] - \mathcal{L}_1[k] = k$. Then, we have

$$\sum_{k=1}^{2M} \mathcal{L}_2[k] - \sum_{k=1}^{2M} \mathcal{L}_1[k] = \sum_{k=1}^{2M} k = \frac{2M(2M+1)}{2}, \tag{3.7}$$

$$\sum_{k=1}^{2M} \mathcal{L}_2[k] + \sum_{k=1}^{2M} \mathcal{L}_1[k] = \sum_{k=1}^{4M} k = 2M(4M+1). \tag{3.8}$$

Combining (3.7) and (3.8), we can get

$$\sum_{k=1}^{2M} \mathcal{L}_2[k] = \frac{10M^2 + 3M}{2}. \tag{3.9}$$

When $2M \ mod \ 4 = 2$, M is an odd number that can be represented as $M = 2e+1, e \geq 0$, thus

$$\sum_{k=1}^{2M} \mathcal{L}_2[k] = \frac{10(2e+1)^2 + 3(2e+1)}{2} = 20e^2 + 23e + \frac{13}{2}. \tag{3.10}$$

the right-hand side of (3.10) is not an integer which contradicts the fact that it is a sum of $4M$ integers. □

Lemma 18. *When $M > 1$ and $2M \ mod \ 4 = 0$, $\mathcal{B}$ exists.*

Proof. When $M = 2$, sequence $\{1, 1, 4, 2, 3, 2, 4, 3\}$ satisfies the requirements of our desired basic link rendezvous sequence. When $M > 2$, the proof can be found in the second page of [43]. □

Based on the proof of **Lemma 17** and **Lemma 18**, we propose **Algorithm 9** to generate our virtual channel sequence $\mathcal{S}$.

According to **Algorithm 9**, if $M > 0$ and $2M \ mod \ 4 = 2$, the length of $\mathcal{S}$ should be larger than $4M$ in order to guarantee the existence of $\mathcal{S}$. Furthermore, we have the following lemma about $\mathcal{S}$ regarding a successful link rendezvous. After getting the virtual channel hopping sequence $\mathcal{S}$, we can get the following lemma regarding a

Algorithm 9 The algorithm for generating the virtual channel hopping sequence

Input:

The number of channels : M

Output:

The desired rendezvous sequence: $\mathcal{S}$

$n = 2M$;
if $n\%4 == 2$ **then**
    $n = n + 2$;
end if
if $n == 4$ **then**
    $\mathcal{S} = \{1, 1, 4, 2, 3, 2, 4, 3\}$;
else
    Initialize $\mathcal{S}$ as a list of length $4n$;
    Let P be an empty list;
    $m = \lfloor n/4 \rfloor$;
    Add all pairs $(4m + r, 8m - r)$, for $r = 0, 1, \ldots, 2m - 1$, to P;
    Add pairs $(2m + 1, 6m)$ and $(2m, 4m - 1)$ to P;
    Add all pairs $(r, 4m - 1 - r)$, for $r = 1, 3, \ldots, m - 1$, to P;
    Add pair $(m, m + 1)$ to P;
    Add all pairs $(m + 2 + r, 3m - 1 - r)$, for $r = 0, 1, \ldots, m - 3$, to P;
    for $i = 1$ to n **do**
        $\mathcal{S}[P[i].first] = \mathcal{S}[P[i].second] = P[i].second - P[i].first$;
    end for
end if
for $i = 1$ to $2n$ **do**
    if $\mathcal{S}[i] > 2M$ **then**
        $\mathcal{S}[i]\ \% = 2M$; $//$limit each value to $[1, 2M]$
    end if
end for
for $i = 1$ to $2n$ **do**
    $//$replacement manipulations
    if this the second time that $k = \mathcal{S}[i]$ appears **then**
        if $k \leq M$ **then**
            $\mathcal{S}[i] = \mathcal{S}[i] + M$;
        end if
        if $k > M$ **then**
            $\mathcal{S}[i] = \mathcal{S}[i] - M$;
        end if
    end if
end for
return $\mathcal{S}$;

successful link rendezvous:

Lemma 19. *Assume that when a link rendezvous process begins, the first SU is on ith element of $\mathcal{S}$, the second SU is on the jth element of $\mathcal{S}$, and $k = |i - j| > 0$. Within $4M$ time slots, two SUs can hop to two different virtual channels c_1 and c_2 such that $|c_1 - c_2| = M$.*

Proof. According to **Lemma 16**, within $4M$ time slots, two SUs can hop to the same virtual channel c on the different positions of $\mathcal{B}$. Since we perform the replacement manipulations of $\mathcal{B}$ to get $\mathcal{S}$, the manipulation can make the same two c values in $\mathcal{B}$ change to two different values c_1 and c_2 such that $|c_1 - c_2| = M$. Therefore, within $4M$ time slots, two SUs can hop to two different virtual channels c_1 and c_2 such that $|c_1 - c_2| = M$. □

An example of Lemma 19 is shown in Fig. 3.32, where $M = 2$, the virtual channel sequence is $\{1, 3, 4, 2, 3, 4, 2, 1\}$, SU1 starts from the first element, and SU2 starts from the third element. Within $4M = 8$ time slots, two SUs first hop to channel 2 and 4 that $4 - 2 = 2 = M$ during the fourth time slot, and then hop to channel 1 and 3 that $3 - 1 = 2 = M$ during the eighth time slot.

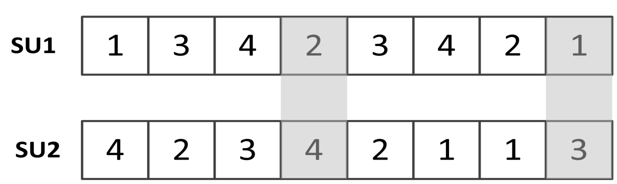

Figure 3.32: An example of the virtual channel hopping sequence.

Since a necessary condition for a successful link rendezvous is that two SUs should hop to two virtual channels c_1 and c_2, respectively, such that $c_1 > c_2, c_1 - c_2 = M$, we can design a channel hopping scheme based on **Lemma 6**. However, the problem is that if the virtual channel c_2 is not a common available virtual channel, the two

SUs still cannot achieve a successful rendezvous, even if they hop to c_1 and c_2 during the same time slot. Therefore, after every $4M$ time slots, if two SUs did not achieve a successful link rendezvous, each SU should restart a new virtual channel hopping process from a new starting element in $\mathcal{S}$ to make the two SUs can hop to new virtual channels $c_1^{'}$ and $c_2^{'}$, respectively, until $c_2^{'}$ is a common available virtual channel. Therefore, we design the following distributed link rendezvous scheme for each SU in **Algorithm 10**:

Algorithm 10 The distribute link rendezvous algorithm for each SU

Input:

The total number of channels: M

Generate the virtual channel hopping sequence $\mathcal{S}$;
Randomly choose a starting index i on $\mathcal{S}$;
count $= 0$;// count time slots
while Not rendezvous **do**
  if $\mathcal{S}[i] > M$ and $\mathcal{S}[i]$ is currently available **then**
    Hop to channel $\mathcal{S}[i] - M$ and tune to the receiving mode;
  end if
  if $\mathcal{S}[i] \leq M$ and $\mathcal{S}[i]$ is currently available **then**
    Hop to channel $\mathcal{S}[i]$, tune to the sending mode, and send RTS packets on the channel;
  end if
  count++;
  if *count* $>= 4M$ **then**
    count $= 0$;
    Randomly generate a new i;//reset i
  else
    $i++$;
    if $i > 4M$ **then**
      $i = 1$;//circular hop
    end if
  end if
end while

Based on **Algorithm10**, we can get the following lemma:

Lemma 20. *The probability that two SUs fail to rendezvous within every $4M$ time slots is $p_f \approx 1 - p_c$, where p_c is the probability that a channel is a common available channel of the two SUs.*

Proof. The probability that two SUs choose different starting indexes in $\mathcal{S}$ is $\frac{4M(4M-1)}{(4M)^2} = \frac{4M-1}{4M}$. Two SUs can achieve a successful link rendezvous only when they choose different starting indexes and the virtual channel they will hop to is available to both of them. Therefore, the probability that two SUs achieve a successful rendezvous during every $4M$ time slots is $\frac{4M-1}{4M}p_c$. Finally, the fail probability is $1-\frac{4M-1}{4M}p_c = \frac{4M-4Mp_c-p_c}{4M} = 1-p_c-\frac{p_c}{4M} \approx 1-p_c$ □

Finally, we can get the **Theorem 15** regarding the average link rendezvous time of the enhanced link rendezvous scheme in **Algorithm10**.

Theorem 15. *The expectation of link rendezvous time slots for **Algorithm10** is $E[T] = \frac{4M}{p_c}$, where M is the total number of channels and p_c is the probability that a channel is a common available channel of the two SUs.*

Proof. The expectation of link rendezvous time slots is:

$$\sum_{i=1}^{\infty} 4Mp_f^{(i-1)}(1-p_f) = \frac{4M}{1-p_f} = \frac{4M}{p_c}. \tag{3.11}$$

□

3.4 Rendezvous Process Considering Directional Antennas

In this section, we propose an efficient rendezvous scheme specific for SUs which are equipped directional antennas. Under this scenario, the rendezvous process could be more completed since each SU could be equipped with heterogeneous directional antenna and it does not know any information about the other SUs.

3.4.1 System Model and Problem Description

In this research, we consider two SUs, coexisting with several PUs, who want to rendezvous with each other. Each SU can opportunistically access a total of M channels. Each SU is equipped with a directional antenna that can transmit or receive signals on a transmission sector with a certain angle. We assume that the directional antenna of each SU can adjust the angle of a transmission sector. The two considered SUs are located within the transmission range of each other, and they can implement the same channel rendezvous scheme which can guarantee a successful channel rendezvous within a bounded time.

The transmission area of an SU is evenly divided into N sectors without overlaps. We assume that during each time slot, an SU can only transmit in one sector and on one channel. The index of each transmission sector does not change after the initialization. When two SUs with directional antennas try to set up a communication link between them, they should tune their antennas to specific directions so that their current transmission sectors can cover each other, which is called a successful *sector rendezvous*. If the SU receiver is located in the pth transmission sector of the SU sender and the SU sender is located in the qth transmission sector of the SU sender, we denote (p, q) as a *sector rendezvous pair*. We get the following theorem about the sector rendezvous and sector rendezvous pair.

Theorem 16. *There always exists a sector rendezvous between two SUs and the rendezvous pair is unique, regardless of the number of transmission sectors of each SU.*

Proof. We assume that the SU receiver is covered by the pth transmission sector of the SU sender and the SU sender is covered by the qth transmission sector of the SU receiver. Since each sector of an SU does not overlap with another sector, both p and q are unique (if an SU is located on the border of a transmission sector, only one sector is counted). Therefore, when the SU sender is on the transmission sector p and the SU receiver is on transmission sector q simultaneously, they achieve a successful sector rendezvous. Since p and q are unique, the sector rendezvous pair (p, q) is also unique. □

After tuning its directional antenna to each transmission sector, an SU performs a predetermined channel rendezvous scheme which can guarantee a successful channel rendezvous within the maximum time to rendezvous ($MTTR$). If the SU does not rendezvous with another SU within $MTTR$ time slots in the current transmission sector, it hops to the next transmission sector and repeats the channel rendezvous process. Therefore, in this research, we only consider how to let two SUs achieve a successful sector rendezvous, given a specific channel rendezvous scheme that can guarantee a successful channel rendezvous.

We define the process that an SU hops to a transmission sector as the *sector hopping process*. If we denote the index of the sector an SU is on at time slot t as S_t, the sequence $S_0, S_1, \ldots, S_t, \ldots$ is called a *sector hopping sequence*. An example in Fig. 3.33 shows why both SUs should perform the sector hopping process by following a sector hopping sequence. If the SU receiver just stays within a sector waiting for the RTS packet, it may never hear the SU sender since their transmission sectors may never cover each other like the scenario in Fig. 3.33. Since an SU does not know others' location before a successful blind rendezvous, both SUs should perform the sector hopping process to achieve a successful sector rendezvous.

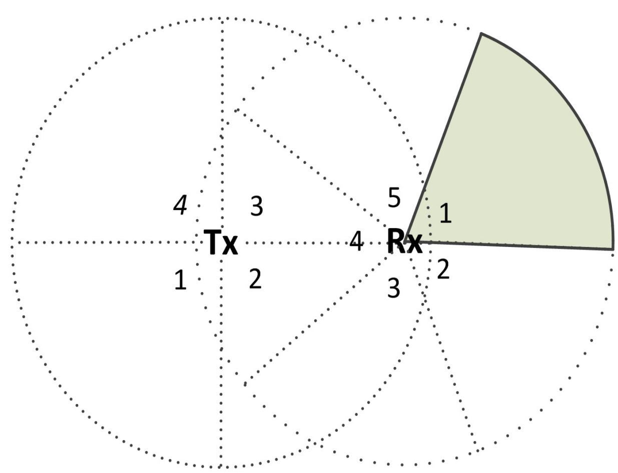

Figure 3.33: An example to show the necessity of the sector hopping process.

The *sector rendezvous problem* is defined as follows. We denote the sector hopping sequences of the SU sender and receiver as $S^s_{t_0}, S^s_{t_1}, \ldots, S^s_{t_t}$ and $S^r_{t_0}, S^r_{t_1}, \ldots, S^r_{t_t}$, respectively, where t_0 is the time the SU sender starts the rendezvous process by sending a RTS packet. Two SUs may not start the sector rendezvous process simultaneously under the blind information scenario. We assume that the rendezvous pair of the two SUs is (p, q) and they achieve sector rendezvous at time t_t if $S^s_{t_t} = p, S^r_{t_t} = q$. Our goal is to design a distributed sector hopping sequence for each SU that can guarantee a successful sector rendezvous and the time for the rendezvous $t_t - t_0$ is bounded.

The first problem about the sector rendezvous problem is the *indexing problem*. As two SUs do not know any information about each other before a successful channel rendezvous, the way they index their sectors may be quit different and unknown to each other, which may lead to $p \neq q$ for a rendezvous pair. Fig. 3.34 shows an example of the indexing problem, in which the rendezvous pair is $(2, 4)$. We denote the number of the sectors of the SU sender and SU receiver as N_s and N_r, respectively. We can notice that the indexes of all the sectors of an SU form a permutation of the numbers from 1 to N_s or N_r. Our goal is to design a distributed sector hopping sequence for each SU which can guarantee a successful sector rendezvous within a bounded time considering the indexing problem.

The second problem is called the *different sector number problem* which means that the number of sectors of each SU may be different and each SU does not know the number of sectors of the other SU before exchanging information. This is a very practical issue when two SUs' transmission parameters are different due to hardware constrains or surrounding environments. Therefore, the optimal transmission angle of each SU's directional antenna may be different. Especially, when two SUs do not know any information about each other during a blind rendezvous process, how can they configure the same transmission parameters for their antennas? An example is shown in Fig. 3.34, where $N_s = 4$ and $N_r = 5$. Under this scenario, our goal is to design a distributed sector hopping sequence for each SU which can guarantee

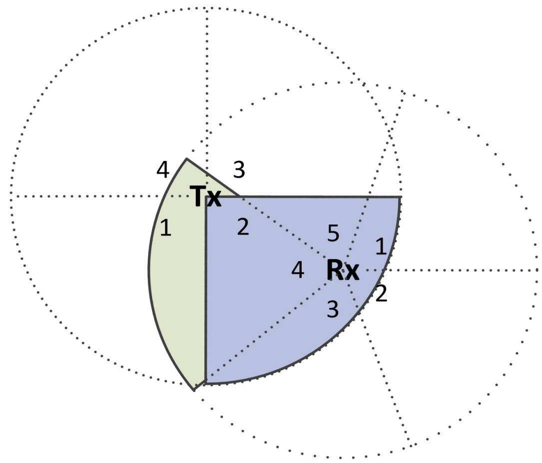

Figure 3.34: An example of the indexing problem and different sector number problem.

a successful sector rendezvous within a bounded time, regardless of different sector indexes and different total number of sectors.

In the following sections, we propose distributed schemes that can guarantee a successful sector rendezvous considering both the indexing problem and different sector number problem.

3.4.2 Rendezvous Scheme Considering the Same Number of Sectors

We first propose a sector rendezvous scheme considering the same number of sectors and the indexing problem. This may not be a general scenario. However, this scheme is a basis for the scheme considering the general scenario.

Under this scenario, we denote the number of sectors for the SU sender and SU receiver as $N_s = N_r = N$ and the rendezvous sector pair is (p, q), where $1 \leq p, q \leq N$. According to the indexing problem, (p, q) could be any element from $\{1, 2, \ldots, N\} \times \{1, 2, \ldots, N\}$, where $\times$ is the Cartesian product of the two sets. Our goal is to design a sector rendezvous scheme which can guarantee a successful sector rendezvous under this scenario.

In our proposed scheme, the SU receiver always hops circularly according to the sector hopping sequence which is a circularly left shift of the sequence $1, 2, \ldots, N$. The shift is based on the starting index. The SU receiver can start sector hopping from any sector index. For instance, when $N = 5$, the sector hopping sequence is $3, 4, 5, 1, 2, 3, 4, 5, 1, 2, \ldots$ if it starts from sector 3 based on its indexing method. The SU sender performs the same way as the SU receiver but increasing the starting index by 1 after each round. For example, when $N = 5$, the sector hopping sequence for the SU sender will be $4, 5, 1, 2, 3, 5, 1, 2, 3, 4, \ldots,$ if it starts from sector 4 based on the SU sender's indexing method. If the rendezvous pair is $(1, 4)$, two SUs can achieve a sector rendezvous after 6 times of sector hopping. Fig. 3.35 shows an example of a successful sector rendezvous under the same sector number. The distributed algorithms for the SU receiver and SU sender to generate the sector hopping sequence are shown in **Algorithm 11** and **Algorithm 12**, respectively.

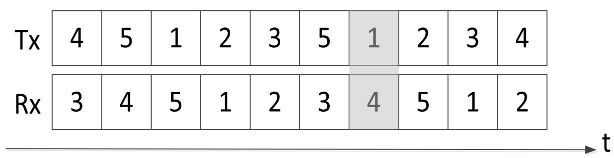

Figure 3.35: An example of a sector rendezvous under the same sector number.

Based on **Algorithm 11** and **Algorithm 12**, we can get **Theorem 17**.

Theorem 17. *By following the sector hopping sequences generated by **Algorithm 11** and **Algorithm 12**, two SUs can achieve a successful sector rendezvous within N^2 sector hops. The average sector hops for a successful sector rendezvous is $N^2/2$.*

Proof. Assume that the sector rendezvous pair is $(p, q) \in \{1, 2, \ldots, N\} \times \{1, 2, \ldots, N\}$, where $\times$ is the Cartesian product of the two sets, and during the first round, when the SU sender is on its pth sector, the SU receiver is on its $\beta_1 = \beta_0 \% N + 1$ sector, where $1 \leq \beta_0 \leq N$. According to **Algorithm 12**, during the next $N-1$ rounds, when the SU sender is on its pth sector, the SU receiver should be on sectors $\beta_2 = (\beta_0 - 1 + N)\%N + 1, \beta_3 = (\beta_0 - 2 + N)\%N + 1, \ldots, \beta_N = (\beta_0 - (N-1) + N)\%N + 1,$

Algorithm 11 Generate the sector hopping sequence for the SU receiver under the same sector number

Input:
The number of sectors: N
Current index of sector: θ_i

Output:
The index of the next sector: θ_{i+1}

```
if θ_i is null then
    // this is the first index
    θ_{i+1} = a random number from [1, N];
else
    θ_{i+1} = θ_i + 1;
    if θ_{i+1} > N then
        θ_{i+1} = 1;
    end if
end if
return θ_{i+1}
```

Algorithm 12 Generate the sector hopping sequence for the SU sender under the same sector number

Input:
The number of sectors: N
Current index of sector: θ_i
Starting index of current round: θ_s

Output:
The index of the next sector: θ_{i+1}

```
if θ_i is null then
    // this is the first index
    θ_{i+1} = a random number from [1, N];
    θ_s = θ_{i+1}; // update θ_s
else
    θ_{i+1} = θ_i + 1;
    if θ_{i+1} > N then
        θ_{i+1} = 1;
    end if
    if θ_{i+1} == θ_s then
        θ_s = θ_s + 1; // update θ_s
        if θ_s > N then
            θ_s = 1;
        end if
        θ_{i+1} = θ_s; // start a new round
    end if
end if
return θ_{i+1}
```

respectively. During the whole N sector hopping rounds, $\beta_1, \beta_2, \ldots, \beta_N$ are N different numbers in $[1, N]$. Therefore, there always exists a number $\beta_t = q$, where $1 \leq t \leq N$. Since each sector hopping round contains N sector hops, the maximum number of hops to guarantee a sector rendezvous is N^2. Since (p, q) and the starting indexes are evenly distributed, the average sector hops for a successful sector rendezvous is $N^2/2$. □

3.4.3 Rendezvous Scheme for the General Scenario

In the last section, we consider that each SU has the same number of transmission sectors. However, in practical networks, due to the hardware heterogeneity or different surrounding environments, two SUs may have different number of transmission sectors. Under this scenario, we denote the number of sectors for the SU sender and SU receiver as N_s, N_r, respectively, where N_s and N_r could be different or the same. The corresponding rendezvous sector pair is (p, q), where $1 \leq p \leq N_s$, $1 \leq q \leq N_r$. Because of the indexing problem, (p, q) could be any element from $\{1, 2, \ldots, N_s\} \times \{1, 2, \ldots, N_r\}$, where $\times$ is the Cartesian product of the two sets. Our goal is to design a distributed sector rendezvous scheme which can guarantee a successful sector rendezvous for any $(p, q) \in \{1, 2, \ldots, N_s\} \times \{1, 2, \ldots, N_r\}$ and any $N_s > 0, N_r > 0$.

The basic idea of our proposed rendezvous scheme is to let each SU adjust the number of its sectors individually to be the smallest prime number larger than current one. After this adjustment, both N_s and N_r are prime numbers.

When $N_s \neq N_r$, both the SU sender and receiver execute **Algorithm 11** to generate their sector hopping sequence. For example, when $N_s = 5$ and $N_r = 3$, the sector hopping sequences for the SU sender and receiver could be $3, 4, 5, 1, 2, 3, 4, 5, \ldots$ and $1, 2, 3, 1, 2, 3, 1, 2, 3, \ldots$, respectively. If the rendezvous pair is $(4, 1)$, two SUs can achieve a successful sector rendezvous after 7 sector hops. The example is shown in Fig. 3.36. Based on **Lemma 16**, in **Theorem 18** shown below, we can prove that when $N_s \neq N_r$, two SUs can always achieve a successful sector rendezvous by following the sector hopping sequences generated from **Algorithm 11**.

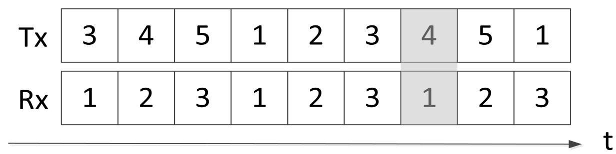

Figure 3.36: An example of a rendezvous process under different sector numbers.

Lemma 16. *If $P_1 > P_2$ are two different prime numbers, x is a random number from $[0, P_1 - 1]$, the following P_1 numbers $x\%P_1 + 1$, $(x + P_2)\%P_1 + 1$, $\ldots$, $(x + (P_1 - 1)P_2)\%P_1 + 1$ are all different.*

Proof. If there exist two different numbers $k_1 \in [0, P_1 - 1], k_2 \in [0, P_1 - 1]$ that $(x + k_1P_2)\%P_1 + 1 = (x + k_2P_2)\%P_1 + 1$. Then, $[(k1 - k2)P_2]\%P_1 = 0$. As P_2 is a prime number, P_2 and P_1 are relatively prime. Therefore, $(k1 - k2)\%P_1 = 0$. Since $0 \leq k_1 < P_1, 0 \leq k_2 < P_1$, only when $k1 = k2$, $(k1 - k2)\%P_1 = 0$, which contradicts the assumption that $k_1 \neq k_2$. □

Theorem 18. *If N_s, N_r are different prime numbers and both SUs follow the sector hopping sequence generated from **Algorithm 11**, they can achieve a successful sector rendezvous within N_sN_r sector hops.*

Proof. Without loss of generality, we assume that $N_s < N_r$, the sector rendezvous pair is (p, q), and during the first round, when the SU sender is on the pth sector, the SU receiver is on the $\beta_1 = \beta_0\%N_r + 1$ sector, where $1 \leq \beta_0 \leq N_r$. According to **Algorithm 11**, during the next $N_r - 1$ rounds, when the SU sender is on its pth sector, the SU receiver should be sequentially on $\beta_2 = (\beta_0 + N_s)\%N_r + 1, \beta_3 = (\beta_0+2N_s)\%N_r+1, \ldots, \beta_{N_r} = [\beta_0+(N_r-1)N_s]\%N_r+1$. According to **Lemma 16**, the N_r numbers $\beta_1, \beta_2, \ldots, \beta_{N_r}$ are all different. Therefore, there always exists a number

$\beta_t = q$, where $1 \leq t \leq N_r$. Since each sector hopping round contains N_s sector hops, the maximum number of hops to guarantee a sector rendezvous is N_sN_r. □

However, if the numbers of sectors of two SUs accidentally are the same prime number that $N_s = N_r$, the scheme we just proposed cannot work based on **Lemma 16**, while the scheme proposed in Section 3.4.2 can work. In practice, the two SUs do not know each other's sector number before a successful sector rendezvous. How can they choose the scheme they should execute individually to guarantee a successful sector rendezvous? The solution is to combine the idea in this section and the one in Section 3.4.2 together. For the SU receiver, it first adjusts the number of sectors N_r to be the smallest prime number larger than the current one. Then, it generates a sector hopping sequence based on **Algorithm 11**. For the SU sender, it adjusts the number of sectors N_s to be the smallest prime number larger than the current one. Next, it first generates a sector hopping sequence according to **Algorithm 12**. After N_s^2 sector hops, if there is not a successful sector rendezvous, it generates the sector hopping sequence based on **Algorithm 11**. Based on this idea, we propose two distributed algorithms **Algorithm 13** and **Algorithm 14** for the SU receiver and SU sender, respectively, considering both the indexing problem and the different sector number problem.

Algorithm 13 Sector rendezvous scheme for an SU receiver

Input:

The number of sectors: N_r

if N_r is not a prime number **then**

  Adjust N_r to be the smallest prime number larger than it;

end if

Generate a sector hopping sequence based on **Algorithm 11** to perform the sector hopping process;

Based on **Algorithm 13** and **Algorithm 14**, we can get **Theorem 19**.

Theorem 19. *Our proposed distributed schemes in **Algorithm 13** and **Algorithm 14** can guarantee a successful sector rendezvous within at most* $N_s^2 + N_sN_r$ *sector*

Algorithm 14 Sector rendezvous scheme for an SU sender

Input:

The number of sectors: N_s

1: **if** N_s is not a prime number **then**
2: Adjust N_s to be the smallest prime number larger than it;
3: **end if**
4: Generate a sector hopping sequence based on **Algorithm 12** to perform the sector hopping process;
5: **if** No successful channel rendezvous after N_s^2 sector hops **then**
6: Generate a sector hopping sequence based on **Algorithm 11** to perform the sector hopping process;
7: **end if**

hops, considering both the indexing problem and different sector number problem.

Proof. After adjustments, N_s and N_r should be prime numbers. If $N_s = N_r$ after the adjustments, the two SUs can achieve a successful sector rendezvous within N_s^2 sector hops according to **Theorem 17**. If $N_s \neq N_r$ after the adjustments, the two SUs may not achieve a successful sector rendezvous within N_s^2 sector hops. Therefore, after N_s^2 sector hops, the SU sender can realize that $N_s \neq N_r$ and then generates a sector hopping sequence based on **Algorithm 11**. According to **Theorem 18**, during the next N_sN_r sector hops, two SUs can achieve a successful sector rendezvous. Therefore, two SUs can achieve a successful sector rendezvous within at most $N_s^2 + N_sN_r$ sector hops, considering all scenarios. □

3.5 Power Control Protocol to Maximize the Number of Common Available Channels

In this section, we propose a power control protocol which can maximize the number of common available channels between two SUs.

3.5.1 Problem Description

Consider two SUs in a CRAHN: one is the sender and the other is the receiver during a transmission. For the SU sender, its transmission on a channel may generate direct interference to the PUs who are using the same channel. Thus, the sensing threshold of an SU sender should be set to guarantee the interference generated from it is low enough to the PUs outside the sensing range. However, for the SU receiver, it does not generate direct interference to any PU, but its reception may be interfered by a PU's transmission on the same channel. The SU receiver's sensing threshold should guarantee its received signal not be interfered seriously by PUs' transmission on the same channel. Therefore, the sensing threshold of the SU sender and receiver could be different because of their different roles in the communication. Now, the problem is: how to determine the optimal sensing threshold for both the SU sender and receiver? To solve this problem, we first show the relationship between the sensing threshold, sensing range, and transmission power for the SU sender and SU receiver in the following.

3.5.2 Analysis of the Sensing Range of an SU Sender

The requirement that an SU sender can sense whether there is an interfering transmission power on channel i is that the sensed power from channel i should be larger than a threshold $P_{th\_su\_s}$ [6]. The relationship between the sensing threshold and $P_{th\_su\_s}$ can be formulated by using the widely used transmission model [44,45] that

$$P_{th\_su\_s} = \frac{P_{pu}\kappa_1}{r_{su\_s}^{\alpha}}, \tag{3.12}$$

where P_{pu} is the transmission power of a PU, κ_1 is an antenna related constant, α is the path-loss factor, and r_{su_s} is the sensing range of the SU sender. Equation (1) shows that the sensing range is determined by the sensing threshold.

Moreover, for the SU sender, it cannot use channel i when its transmission power on channel i produces an unacceptable interference to a PU receiver. According to the definition of the sensing range, we can get

$$\frac{P_{su}\kappa_2}{r_{su\_s}^{\alpha}} \leq P_{pu\_min}, \tag{3.13}$$

where P_{su} is the transmission power of the SU sender and P_{pu_min} is the highest acceptable interference power on channel i for a PU receiver which can be obtained by the PU's minimum required decoding power and signal-to-interference ratio (SIR). From (3.13), we can get the minimum sensing range of an SU sender

$$r_{su\_s\_min} = \left(\frac{P_{su}\kappa_2}{P_{pu\_min}} \right)^{\frac{1}{\alpha}}. \tag{3.14}$$

For an SU, a larger sensing range means more unavailable channels and less available channels. Thus, we can use the minimum sensing range in (3.14) to set the threshold in (3.12) for the sensing process of an SU sender which can also guarantee an acceptable interference to PU receivers.

3.5.3 Analysis of the Sensing Range of an SU Receiver

For an SU receiver, its sensing range also has a similar relationship as in (3.12) with its detection threshold

$$P_{th\_su\_r} = \frac{P_{pu}\kappa_3}{r_{su\_r}^{\alpha}}, \tag{3.15}$$

where $P_{th\_su\_r}$ is the sensing detection threshold of the SU receiver, P_{pu} is the transmission power of a PU, and r_{su_r} is the sensing range of the SU receiver. However, an SU receiver cannot judge whether it can use channel i simply based on whether the sensed power on channel i is higher than $P_{th\_su\_r}$, because the received signal power from the SU sender may be much higher than the interference power. Accordingly, the SU receiver should additionally check if the received signal from the SU sender can meet the minimum required SIR that

$$\frac{P_{su\_rev}}{P_{th\_su\_r}} \geq SIR_{su\_min}, \tag{3.16}$$

where P_{su_rev} is the received SU power from channel i, $P_{th\_su\_r}$ is the sensing detection threshold for the SU receiver, and SIR_{su_min} is the minimum required SIR for correctly decoding. We ignore the environment noise here. P_{su_rev} can be obtained by

$$P_{su\_rev} = \frac{P_{su}\kappa_4}{d_{su}^{\alpha}}, \tag{3.17}$$

where d_{su} is the distance between the SU sender and SU receiver. From (3.15)(3.16)(3.17), we can get the minimum sensing range of the SU receiver

$$r_{su\_r\_min} = \left(\frac{P_{pu}\kappa_3 SIR_{su\_min} d_{su}^{\alpha}}{P_{su}\kappa_4} \right)^{\frac{1}{\alpha}}. \tag{3.18}$$

From an SU receiver's angle, in order to get more available channels, we can use the minimum sensing range in (3.18) as the SU receiver's sensing range to determine its sensing threshold by using (3.15).

The sensing range of an SU will directly affect the number of its individual available channels. Equation (3.14) and (3.18) indicate that when the transmission power of the SU sender P_{su} decreases, the sensing range of the SU sender will decrease but the sensing range of the SU receiver will increase, and vice versa. Thus, there is a trade-off between the SU transmission power and the sensing range of the SU pair which is also related to their individual available channels. How to determine the optimal SU transmission power which can result in the maximum number of common available channels for an SU pair in CRAHNs is an important issue that may affect the performance of SU transmissions. In the next section, we use the SU sensing ranges to estimate the expectation of the number of common available channels between two SUs and then determine the optimal SU transmission power to maximize the number of common available channels.

3.5.4 System Model

We consider two SUs in an area whose size is $S = L \times L$. One SU is the SU sender SU_1, while the other SU is the SU receiver SU_2. The set of all channels is $\mathcal{M} = \{1, 2, \ldots, M\}$ which has M channels. We assume that all the N PUs are evenly distributed in the whole area. Their active probability is β. Each PU randomly chooses a channel to transmit, and its traffic follows the exponential distribution.

3.5.5 The Expectation of the Number of Common Available Channels

In Fig. 1, the two circles represent the sensing range of SU_1 or SU_2. The combined area of their sensing ranges A_0 has three parts: A_1, A_2, and A_3, and their sizes are S_0, S_1, S_2, and S_3, respectively.

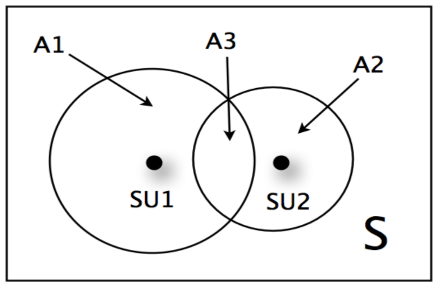

Figure 3.37: The sensing range of the SU sender and receiver.

Let $\mathcal{M}_{u\_s}$ be the set of the channels which cannot be used by the SU sender, $\mathcal{M}_{u\_r}$ be the set of channels which cannot be used by the SU receiver, and $\mathcal{M}_c$ be the set of common available channels of the SU pair. Thus, we have

$$\mathcal{M}_c = \mathcal{M} - (\mathcal{M}_{u\_s} \bigcup \mathcal{M}_{u\_r}). \tag{3.19}$$

Let $\mathcal{M}_{used} = \mathcal{M}_{u\_s} \bigcup \mathcal{M}_{u\_r}$ be the set of channels which has been used by the PUs within the sensing range of SU_1 and SU_2. Accordingly, the expectation of the number of common available channels of SU_1 and SU_2 is

$$E[|\mathcal{M}_c|] = |\mathcal{M}| - E[|\mathcal{M}_{used}|], \tag{3.20}$$

where $|\bullet|$ means the size of a set.

We use a similar method as shown in [42] to calculate $E[|\mathcal{M}_{used}|]$. Since PUs are evenly distributed in the whole area, the probability that there are k PUs in A_0 is:

$$P(n = k) = \binom{N}{k} \left(\frac{S_0}{S}\right)^k \left(1 - \frac{S_0}{S}\right)^{N-k}. \tag{3.21}$$

$$E[|\mathcal{M}_{used}|] = \sum_{k=0}^{N}\sum_{i=0}^{k}\sum_{j=0}^{i} j \binom{N}{k} \left(\frac{S_0}{S}\right)^k \left(1-\frac{S_0}{S}\right)^{N-k} \binom{k}{i}\beta^i (1-\beta)^{k-i} \frac{\binom{M}{j}S(i,j)j!}{M^i} \tag{3.24}$$

When the active probability of PUs is β, the probability that there are i concurrent active PUs among all the k PUs in A_0 is:

$$P(v=i|n=k) = \binom{k}{i}\beta^i (1-\beta)^{(k-i)}. \tag{3.22}$$

Next, we calculate the probability that there are j used channels while i active PUs in A_0. Since each PU randomly chooses a channel among all M channels with the same probability, this problem is equivalent to randomly putting i different balls into M different boxes and finding the probability of j non-empty boxes. The answer to this problem is:

$$P(u=j|v=i) = \frac{\binom{M}{j}S(i,j)j!}{M^i}, \tag{3.23}$$

where $S(i,j)$ is the Stirling number which represents the number of different ways to split i different elements into j different non-empty sets. From (3.21), (3.22), and (3.23), we can get the average number of the used channels as shown in (3.24) in the next page. Therefore, according to (3.20), in order to maximize the expectation of the number of common available channels, we should find the optimal P_{su} that minimizes $E[|\mathcal{M}_{used}|]$.

3.5.6 The Proposed Optimal Solution

From the above derivations, it is very challenging to get an explicit expression for the optimal P_{su}. We propose an approximation solution for the optimization problem. This solution is easy to implement in designing a practical power control protocol and can produce an improvement in the throughput according to the simulation results shown in Section **??**.

Let $E[|\mathcal{M}_{used}|]$ be a function of $\frac{S_0}{S}$, named $f(x)$.

Lemma 20. *$f(x_1) \geq f(x_2)$ when $x_1 \geq x_2$ and $N > 1$.*

Proof. We can rewrite $f(x)$ as:

$$
\begin{aligned}
f(x) &= \sum_{k=0}^{N} \binom{N}{k} x^k (1-x)^{N-k} g(k), \\
g(k) &= \sum_{i=0}^{k} \binom{k}{i} \beta^i (1-\beta)^{k-i} q(i), \\
q(i) &= \sum_{j=0}^{i} j p(j), \\
p(j) &= \frac{\binom{M}{j} S(i,j) j!}{M^i}.
\end{aligned}
$$

According to the definition of $p(j)$, we can get $\sum_{j=0}^{i} p(j) = 1$. Thus, we can induce that when $i > 1, q(i) = \sum_{j=0}^{i} jp(j) > \sum_{j=0}^{i} p(j) = 1$. Let $g(k) = \sum_{i=0}^{k} \binom{k}{i} \beta^i (1-\beta)^{k-i} t$, where t is a constant. If $t \leq 1$ then $\sum_{i=0}^{k} \binom{k}{i} \beta^i (1-\beta)^{k-i} t < \sum_{i=0}^{k} \binom{k}{i} \beta^i (1-\beta)^{k-i} q(i)$, because $q(i) > 1$ when $i > 1$. This is a paradox. Therefore, $t > 1$ when $k > 1$. Then, we can rewrite $g(k) = \sum_{i=0}^{k} \binom{k}{i} (t^{\frac{1}{i}} \beta)^i (1-\beta)^{k-i} = (t^{\frac{1}{i}} \beta + 1 - \beta)^k$. According to the definition, $0 < \beta < 1$, then we can get $g(k) > 1$ when $k > 1$. Repeat the same steps, we can also rewrite $f(x)$ as $f(x) = \sum_{k=0}^{N} \binom{N}{k} x^k (1-x)^{N-k} l$, where $l > 1$ when $N > 1$. Last, we can get the final expression of $f(x)$ as $f(x) = \sum_{k=0}^{N} \binom{N}{k} (l^{\frac{1}{k}} x)^k (1-x)^{N-k} = (l^{\frac{1}{k}} x + 1 - x)^N$. Since $l > 1$ when $N > 1$, we can finally conclude that $f(x)$ increases if x increases. $\square$

From Lemma 1, we can know that in order to minimize $E[|\mathcal{M}_{used}|]$, we should minimize the area size S_0. Now consider the following problem: there are two circles C_0 and C_1 whose radii are r_0 and r_1. Their distance is larger than zero and does not change. Their radii only change simultaneously with a constant rate c and $-c$. Let s be the size of their combination area, then we can get the following lemma:

Lemma 21. *When $r_0 = r_1$, s will be minimum.*

Proof. Assume r_1 increases from 0 to the maximum value r_{max}. As the radius changes with a constant but opposite rate, r_0 decreases from r_{max} to 0. Let $s = G(r_0)$. We can get that $G(x)$ is symmetric according to the axis $x = \frac{1}{2}r_{max}$, since r_0 and r_1 change simultaneously with a constant but opposite rate. When r_0 decreases from r_{max} to $\frac{1}{2}r_{max}$, $G(x)$ also decreases. Because when $r_0 > r_1$ and they change simultaneously with a constant but opposite rate, the size that C_0 decreases is larger than the size that C_1 increases. Furthermore, according to the symmetry of $G(x)$, when r_0 decreases from $\frac{1}{2}r_{max}$ to 0, $G(x)$ increases. Thus, when $r_0 = \frac{1}{2}r_{max}$ and r_1 is also $\frac{1}{2}r_{max}$, $G(x)$ will reach its minimum value. □

According to Equation (3.14) and (3.18), when P_{su} increases, $r_{su\_s\_min}$ will increase and $r_{su\_r\_min}$ will decrease, and vice versa. Using Lemma 20 and Lemma 21, we can get an approximate optimal P_{su} by assuming $r_{su\_s\_min}$ and $r_{su\_r\_min}$ change with a constant but opposite rate. Therefore, letting Equation (3.14) equal (3.18), the final expression of the approximately optimal SU transmission power is

$$P_{su\_opt} = \sqrt{\frac{P_{pu\_min} P_{pu} SIR_{su\_min} d_{su}^{\alpha} \kappa_3}{\kappa_2 \kappa_4}}. \tag{3.25}$$

Fig. 3.38 shows the minimum values of $E[|\mathcal{M}_{used}|]$ when the real optimal P_{su} is used (which is obtained by the numerical method) and the results using our approximate optimal P_{su} solution, when the distance between two SUs changes from $5m$ to $15m$. We can see that the two results coincide very well, which means that our approximate optimal solution is valid and applicable.

3.5.7 The Proposed Power Control Protocol

In order to implement the optimization solution, we make the following assumptions. We assume that each SU is equipped with a GPS to get its own coordinates.

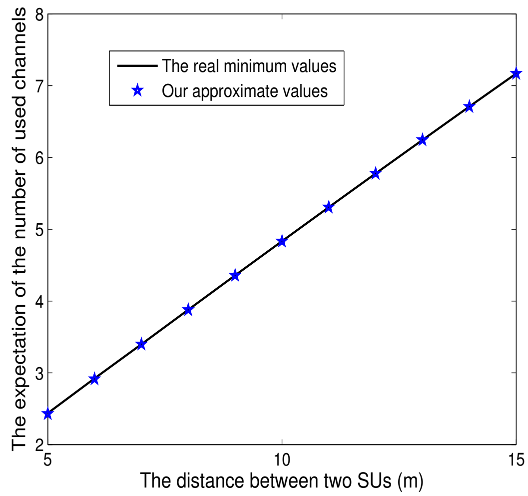

Figure 3.38: The real minimum values and our approximate values.

Moreover, according to our proposed algorithms, the SU pair should know some parameters of each other. How to get these parameters without a common control channel in CRAHNs is a challenge. In wireless ad-hoc networks, a sender and the receiver use the Request-to-Send (RTS) and Clear-to-Send (CTS) packets to inform each other before transmitting data packets. Thus, we can insert the needed information into the RTS and CTS packets. We assume that the SUs use the common hopping method in [34] to achieve successful rendezvous and the RTS and CTS packets exchange. Under this method, two SUs follow the same hopping sequence to perform channel hopping in each time slot. When one SU wants to send packets to the other, it sends the RTS packet on the currently staying channel. After receiving the CTS packet from the other SU in the later time slot, it sends data packets to the SU receiver.

In order to implement our proposed power control protocol, we design the algorithms for the SU sender and SU receiver. When the SU sender wants to send packets to the SU receiver, it will execute Algorithm 1. After receiving the RTS packet, the SU receiver will execute Algorithm 2. During the execution of the algorithms, they use the calculated optimal transmission power to get the corresponding sensing range and sensing threshold. From the pseudo code of Algorithm 1 and Algorithm 2, we can see that the complexity is $O(1)$. Thus, our algorithm has a low time cost and is easy to implement.

Algorithm 15 The algorithm for the SU sender

Input:

Its coordinates (X_{su_s}, Y_{su_s});

κ_1, κ_2, α, P_{pu}, and P_{pu_min};

Output:

Its optimal sensing threshold $P_{th\_su\_s\_opt}$;

Its optimal transmission power P_{su_opt};

1: Add X_{su_s} and Y_{su_s} into the RTS packet;

2: Send the RTS packet;

3: After receiving the returned CTS packet, get the optimal transmission power P_{su_opt} from the CTS packet;

4: Use P_{su_opt} and Equation (3.14) to determine its optimal sensing range $r_{su\_s\_opt}$;

5: Use $r_{su\_s\_opt}$ and Equation (3.12) to determine its optimal sensing threshold $P_{th\_su\_s\_opt}$;

6: Reset its sensing threshold as $P_{th\_su\_s\_opt}$ and transmission power as P_{su_opt};

Algorithm 16 The algorithm for the SU receiver

Input:

κ_2, κ_3, κ_4, α, and P_{pu};

Its minimum required SIR: SIR_{min};

The RTS packet from the SU sender;

Output:

The SU sender's optimal transmission power P_{su_opt};

Its optimal sensing threshold $P_{th\_su\_r\_opt}$;

1: Get X_{su_s} and Y_{su_s} from the RTS packet, and its own coordinates X_{su_r} and $Y_{su_r}$from its GPS;

2: Use $d_{su} = \sqrt{(X_{su\_s} - X_{su\_r})^2 + (Y_{su\_s} - Y_{su\_r})^2}$ to calculate the distance to the SU sender;

3: Use (3.25) to get the optimal transmission power P_{su_opt};

4: Use P_{su_opt} and Equation (3.18) to determine its optimal sensing range $r_{su\_r\_opt}$;

5: Use $r_{su\_r\_opt}$ and Equation (3.15) to determine its optimal sensing threshold $P_{th\_su\_r\_opt}$;

6: Reset its sensing threshold as $P_{th\_su\_r\_opt}$;

7: Insert P_{su_opt} to the CTS packet and send it;

CHAPTER 4: CONCLUSION AND FUTURE WORK

4.1 Conclusion of Completed Work

In this proposal, we proposed five schemes to realize efficient rendezvous and spectrum management considering practical scenarios. We first proposed a rendezvous and communication framework for wide-band CRNs. Furthermore, we proposed two efficient rendezvous schemes without predetermined sender and receiver. Moreover, we proposed a rendezvous scheme specifically for SUs equipped with directional antennas. Last, we proposed a power control protocol to maximize the number of common available channels. All of the proposed schemes can realize efficient rendezvous and or spectrum management with practical assumptions under different scenarios.

4.2 Future Work

The work listed below will be studied in the following months.

- Evaluate the performance of all the proposed schemes through simulations.
- In the proposed rendezvous framework for wide-band CRNs, we assume that the time slots of two SUs are aligned. However, in practical scenarios, two SUs could not be perfectly synchronized. We should consider this scenario and propose related rendezvous schemes to solve this problem in the following months.
- For the rendezvous schemes without predetermined sender, receiver, and considering multiple radios, if some radios have already rendezvoused with some other SUs and obtained some control information, for the rest radios if they try to perform rendezvous, how to use the obtained control information to avoid collisions and improve the rendezvous performance should be investigated.
- For the rendezvous scheme considering directional antennas, currently we need two hopping sequences, one is the sector hopping sequence, and the other is the channel hopping sequence. We can design just one channel hopping sequence to

improve the performance by using the idea of virtual channel as we addressed in the rendezvous scheme without predetermined sender and receiver.

4.3 Publications

- J. Li and J. Xie, "Spectrum Splitting and Channel Hopping Based Communication Framework for Wide-band Cognitive Radio Networks", submitted to IEEE Trans. Mobile Computing, 2016.
- J. Li and J. Xie, "Rendezvous Scheme Without a Predetermined Sender or Receiver in Cognitive Radio Ad-Hoc Networks", Proc. IEEE GLOBECOM, December, 2016.
- J. Li and J. Xie, "Directional Antenna Based Distributed Blind Rendezvous in Cognitive Radio Ad-Hoc Networks", Proc. IEEE GLOBECOM, December, 2015.
- J. Li and J. Xie, "Practical Fast Multiple Radio Blind Rendezvous Schemes in Ad-Hoc Cognitive Radio Networks", Proc. IEEE RESILIENCE WEEK, August, 2015.
- J. Li and J. Xie, "A New Communication Framework for Wide-band Cognitive Radio Networks", Proc. IEEE SECON, June, 2014.
- J. Li and J. Xie, "A Power Control Protocol to Maximize the Number of Common Available Channels Between Two Secondary Users in Cognitive Radio Networks ", Proc. IEEE GLOBECOM, December, 2013.

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