Topic02a : Prelim Problem Set I

Case 1

Represent the following representations into its vectorized form using LaTeX.

> **Problem 1.a. System of Linear Equations**
>
> $$\begin{cases} -y+z=\frac{1}{32}\\ \frac{1}{2}x-2y=0\\ -x+\frac{3}{7}z=\frac{4}{5} \end{cases}$$
>
> **Problem 1.b. Linear Combination**
>
> $$\cos{(\theta)}\hat{i} + \sin{(\theta)}\hat{j} - \csc{(2\theta)}\hat{k}$$
>
> **Problem 1.c. Scenario**
>
> A conference has 200 student attendees, 45 professionals, and has 15 members of the panel. There is a team of 40 people on the organizing committee. Represent the *percent* composition of each *attendee* type of the conference in matrix form.

Express your answers in LaTeX in the answer area.

Solution

> **Problem 1.a. System of Linear Equations (Solution)**
>
> $$-y+z=\frac{1}{32}$$
> $$-y=\frac{1}{32}-z$$
> $$\frac{y}{-1}=\frac{1/32}{-1}-\frac{z}{-1}.$$
> $$y=-\frac{1-32z}{32}$$
> $$Substitute: y=-\frac{1-32z}{32}$$
> $$\frac{1}{2}x-2(-\frac{1-32z}{32})=0$$
> $$\frac{1}{2}x+(\frac{1-32z}{16})=0$$
> $$\frac{1}{2}x=(-\frac{1-32z}{16})$$
> $$2.\frac{1}{2}x=2.(-\frac{1-32z}{16})$$
> $$x=-\frac{1-32z}{8}$$
> $$Substitute: x=-\frac{1-32z}{8}$$

```
import numpy as np
```

> **Problem 1.a. System of Linear Equations**

```
A = np.array([[-1, 1],[1/2, 2],[-1, 3/7]])
A
```

```
array([[-1.        ,  1.        ],
       [ 0.5       ,  2.        ],
       [-1.        ,  0.42857143]])
```

```
B = np.array([1/32, 4/5])
B
```

```
array([0.03125, 0.8    ])
```

```
# X = np.linalg.inv(A).dot(B)
np.dot(A,B)
```

```
array([0.76875   , 1.615625  , 0.31160714])
```

> **Problem 1.b. Linear Combination**

```
theta = np.radians(30)
R = np.array([[np.cos(theta), np.sin(theta),np.arcsin(2*theta)]])
R
```

```
/usr/local/lib/python3.7/dist-packages/ipykernel_launcher.py:2: RuntimeWarning: invalid

array([[0.8660254, 0.5      ,       nan]])
```

> **Problem 1.c. Scenario**

```
Conference = np.array([200,45,15,40])
percentage = Conference/Conference.sum(axis=0)*100
print(percentage)
```

```
[66.66666667 15.          5.         13.33333333]
```

Attendees	Number of People	Percentage
$Attendees$	200	66.6%
$Professionals$	45	15%
$Panels$	15	5%
$Org\ Committee$	40	13.33%

Case 2

> **Problem 2.a: Vector Magnitude**

> The magnitude of a vector is usually computed as:
> $$||v|| = \sqrt{a_0^2 + a_1^2 + \ldots + a_n^2}$$
> Whereas v is any vector and a_k are its elements wherein k is the size of v. Re-formulate $||v||$ as a function of an inner product. Further discuss this concept and provide your user-defined function.

> **Problem 2.b: Angle Between Vectors**

> Inner products can also be related to the Law of Cosines. The property suggests that:
> $$u \cdot v = ||u|| \cdot ||v|| \cos(\theta)$$
> Whereas u and v are vectors that have the same sizes and θ is the angle between u and v.

> Explain the behavior of the dot product when the two vectors are perpendicular and when they are parallel.

Solution

> **Problem 2.a: Vector Magnitude**

```
def vector_magnitude(x):
  return np.sqrt(sum(i**2 for i in x))
```

```
a = 5*np.random.randn(6)
a
```

```
array([ 3.93964854, -7.15139717, -3.86628941, -1.8916867 , -0.48208711,
       -3.90560029])
```

```
np.linalg.norm(a)
```

```
10.03374835526016
```

```
vector_magnitude(a)
```

```
10.03374835526016
```

> **Problem 2.b: Angle Between Vectors**

```
def angle_vectors(a,b):
  inner = np.inner(a, b)
  norms = np.linalg.norm(a) * np.linalg.norm(b)

  cos = inner / norms
  rad = np.arccos(np.clip(cos, -1.0, 1.0))
  deg = np.rad2deg(rad)
  print("Radiant: ", rad)
  print("Degree: ", deg)
  return deg,rad
```

```
a = np.array([9, 5])
b = np.array([-6, 2])
print(a,b)
```

```
[9 5] [-6  2]
```

```
angle_vectors(a,b)
```

```
Radiant:  2.3127435948008137
Degree:  132.51044707800082
(132.51044707800082, 2.3127435948008137)
```

Case 3

For the final cases analysis we will be looking at series of equations building up a single feed-forward computation of a logistic regression. The case will not require you to learn fully what is logistic regression.

$$X = \begin{bmatrix} —(x^{(1)})^T— \\ —(x^{(2)})^T— \\ \vdots \\ —(x^{(m)})^T— \end{bmatrix}, Y = \begin{bmatrix} y^{(1)} \\ y^{(2)} \\ \vdots \\ y^{(m)} \end{bmatrix}, \text{ and } \theta = \begin{bmatrix} \theta^{(1)} \\ \theta^{(2)} \\ \vdots \\ \theta^{(m)} \end{bmatrix}$$

The dataset X has m entries with n features while Y is the vector containing the groud truths of a the entries of X, and θ are the parameters or weights of the vectors. We first compute the vector product of the dataset and the parameters as:

$$z = x^{(i)}\theta^{(i)} = X \cdot \theta$$

$$\text{Eq. 3.1}$$

We then solve for the hypothesis of the logistic regression alogrithm as:

$$h_\theta(x) = g(z)$$

$$\text{Eq. 3.2}$$

Where g is an acitvation function that maps the values of the hypothesis vector between a range of 0 and 1. We computed the activation as a sigmoid function:

$$g(z) = \frac{1}{1+e^{-z}}$$

$$\text{Eq. 3.3}$$

Finally we compute the loss of the logistic regression algorithm using J. Wheras $J(\theta)$ is a function that computes the logistic loss of the hypothesis with respect to the ground truths y. it is then computed as:

$$J(\theta) = \frac{1}{m}\sum_{i=0}^{m} = [-y^{(i)}\log(h_\theta(x^{(i)})) - (1-y^{(i)})\log(1-h_\theta(x^{(i)}))]$$

$$\text{Eq. 3.4}$$

> **Problem 3.a: Matrix Equivalences**
>
> In Eq. 1, z can also be solved as $X \cdot \theta$ which is the vectorized form of $x^{(i)}\theta^{(i)}$. However, it can also be expressed as $\theta^T \cdot X$. Prove the equality of $X \cdot \theta$ with $\theta^T \cdot X$ in this case.
>
> **Problem 3.b: Matrix Shapes**
>
> Determine the shape of h_θ if X has a shape of $(300, 5)$.
>
> **Problem 3.c: Vectorization**
>
> Express $J(\theta)$ into its vectorized form.
>
> **Problem 4.c: Computational Programming (Also Laboratory 2)**
>
> Encode Equations 3.1 to 3.4 as the class `LRegression` wherein:
>
> - `LRegression` should be instantiated with a dataset X, a ground truth vector y, and a parameter vector θ. Each parameter should have a data type of `numpy.array`.
> - It should further have `methods` reflecting to at least the four (4) aforementioned equations. Each should have a return value.

Problem 3.b

```
$$ X = 
\begin{bmatrix} 
x^1 & x^2 & x^3 & x^4 & x^5 \\ 
x^6 & x^7 & x^8 & x^9 & x^{10} \\ 
\vdots & \vdots & \vdots & \vdots & \vdots \\ 
x^{296} & x^{297} & x^{298} & x^{299} & x^{300
\end{bmatrix} 
\text { , } 
\theta = 
\begin{bmatrix} 
(\theta^1) \\
(\theta^2) \\ 
\vdots \\ 
(\theta^k)
\end{bmatrix}
$$

$$
g(z) = \frac{1}{1+e^{-z}}
$$

$$
z =X \cdot \theta \\
h_{\theta} (x) = g(X\cdot\theta) \\
h_{\theta} (x) = g(z) \\
$$

$$
h_{\theta}(x) = 
\begin{bmatrix} 
\theta^{1}(x) & \theta^{2}(x) & \theta^{3}(x) 
\\ 
\theta^{6}(x) & \theta^{7}(x) & \theta^{8}(x) 
\\
\vdots & \vdots & \vdots & \vdots & \vdots \\ 
\theta^{296}(x) & \theta^{297}(x) & \theta^{29
{300}(x) \\
\end{bmatrix} 
$$
```

$$X = \begin{bmatrix} x^1 & x^2 & x^3 & x^4 & x^5 \\ x^6 & x^7 & x^8 & x^9 & x^{10} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x^{296} & x^{297} & x^{298} & x^{299} & x^{300} \end{bmatrix} , \theta$$

$$g(z) = \frac{1}{1+e^{-z}}$$

$$z = X \cdot \theta$$
$$h_{\theta}(x) = g(X \cdot \theta)$$
$$h_{\theta}(x) = g(z)$$

$$h_{\theta}(x) = \begin{bmatrix} \theta^1(x) & \theta^2(x) & \theta^3(x) & \theta^4(\\ \theta^6(x) & \theta^7(x) & \theta^8(x) & \theta^9(\\ \vdots & \vdots & \vdots & \vdots \\ \theta^{296}(x) & \theta^{297}(x) & \theta^{298}(x) & \theta^{299} \end{bmatrix}$$

Problem 3.c

$$\begin{bmatrix} \frac{\partial J}{\partial \theta_0} \\ \frac{\partial J}{\partial \theta_1} \\ \frac{\partial J}{\partial \theta_2} \\ \vdots \\ \frac{\partial J}{\partial \theta_n} \end{bmatrix} = \frac{1}{m} \begin{bmatrix} \sum_{i=0}^{m}[-y^{(i)} \log(h_\theta(x^{(i)})) - (1-y^{(i)}) \log(1-h_\theta(x^{(i)}))] \\ \sum_{i=0}^{m}[-y^{(i)} \log(h_\theta(x^{(i)})) - (1-y^{(i)}) \log(1-h_\theta(x^{(i)}))] \\ \sum_{i=0}^{m}[-y^{(i)} \log(h_\theta(x^{(i)})) - (1-y^{(i)}) \log(1-h_\theta(x^{(i)}))] \\ \vdots \\ \sum_{i=0}^{m}[-y^{(i)} \log(h_\theta(x^{(i)})) - (1-y^{(i)}) \log(1-h_\theta(x^{(i)}))] \end{bmatrix}$$