GeHoJerez

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This is a simple MPM code write in C, the main purpose of this code is to understand the basics concepts of a MPM code. I also write this lines to keep some order in my ideas during this crazy years.

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1 Physical problem

1.1 Balance of momentum (Equilibrium)

$$\rho \cdot \partial_t v + \partial_x \sigma = \rho \cdot b \tag{1}$$

1.2 Compatibility

1.3 Constitutive response

$$\sigma = 2G\epsilon + \lambda tr(\epsilon) I \tag{2}$$

2 Material Point Method

2.1 Explicit MPM Scheme

1. Calculate the grid nodal mass and momentum by maping the particle mass and momentum to the corresponding grid nodes.

$$m_I^k = \sum_{p=1}^{n_p} m_p N_{Ip}^k \tag{3}$$

$$p_{I,i}^{k-1/2} = \sum_{p=1}^{n_p} m_p v_{ip}^{k-1/2} N_{Ip}^k \tag{4}$$

1. Impose essential boundary conditions on the grid nodal momentum. At the fixed boundary, set $p_{iI}^{k-1/2} = 0$.

2. For the USF only, calculate the particle strain increment $\Delta\epsilon_{ijp}^{k-1/2}$, and the update the particle density and stress as follows:

   - Calculate the grid nodal velocity $v_{iI}^{k-1/2}$

$$v_{iI}^{k-1/2} = \frac{p_{iI}^{k-1/2}}{m_I^k} \tag{5}$$

   - Calculate the particle strain increment $\Delta\epsilon_{ijp}^{k-1/2}$ with :

$$\Delta\epsilon_{ijp}^{k-1/2} = \frac{1}{2}(N_{Ip,j}^k v_{iI}^{k-1/2} + N_{Ip,i}^k v_{jI}^{k-1/2}) \tag{6}$$

- Update the particle density with :

$$\rho_p^{k+1} = \frac{\rho_p^k}{1 + \Delta\epsilon_{iip}^{k-1/2}} \tag{7}$$

- Update the particle stress stated based on $\Delta_{ijp}^{k-1/2}$ with an appropriate constitutive law.

3 Proposed tests

3.1 Simple propagation of a shock wave in a 1D media

Here we solve the transport equation with a time integrator called Two-Step Taylor-Galerkin that stabilize the solution avoiding the formation of spurious oscillations during the transport. For the spatial discretization will be used 1D li

∂_t u + c $\cdot$ ∂_x u = 0

The algorithm is as follows :

1. Transfer information to the Gauss-Points : $u_{GP}^n = \sum_{i=0}^{N} N(x_i) \cdot u_i^n$
2. Get the solution in the Gauss-Points for t = n + 1/2 : $u_{GP}^{n+1/2} = u_{GP}^n$ - Δ t/2 $\cdot$ $\sum_{i=0}^{N}$ ∂ N(x_i) $\cdot$ u_i^n
3. Get the solution in the nodes for t = n + 1 :

3.2 Simple propagation of a shock wave in a 1D elastic media using the formulation σ - v

4 Items

4.1 DONE Get the Nodal coordinates of the material points

4.1.1 23-05-2019, Madrid

4.2 DONE Get the strain increment in the material points

4.2.1 28-05-2019, Madrid

4.3 DONE Get the stress state