Notebook

January 24, 2018

1 LMTH 2040: Multidisciplinary Calculus

Spring 2018
Monday and Wednesday 12:00 p.m - 1:15 p.m.
261 65 West 11th St.
Email: koehlerj@newschool.edu
Office Hours: by appointment

1.0.1 Overview

This class is an introduction to basic ideas and applications of the Calculus. The content focuses on integration, differentiation, and differential equations and how these can be used in several real applications. We will use the computer consistently, introducing the Python computer language to complete our work.

1.0.2 Learning Objectives

- Understand the mathematical concepts and history of Integration
- Use integration to solve problems
- Understand the mathematical concept and history of Differentiation
- Use derivatives and differentiation to solve problems
- Connect Integration and Differentiation through Differential Equations and explore the history of these problems
- Use differential equations to solve problems

1.0.3 Course Requirements

- Complete Weekly Problem Sets
- Ask and answer questions on Piazza or Stack Exchange
- Complete 6 computer labs
- Complete three independent investigations
- Complete Final Project or Problem Set

1.0.4 Final Grade Calculuation

- Participation/Attendance 20%
- Group Problem Sets 20%
- Individual Problem Sets 20%
- Computer Labs 20%
- Final 20%

1.0.5 Course Policies

We will rely heavily on peers and open collaboration on our assignments. We will use a variety of resources for the class, described below:

- Assignments and announcements will be posted on our Canvas Page
- Course Materials from proprietary sources will also be shared through Canvas
- Course Website: http://spring-2018-calc.readthedocs.io/en/latest/
- Students are free to ask and post responses in our Piazza discussion board, available here: `https://piazza.com/newschool/spring2018/lmth2040/home`.
- Students are free to ask questions on open message boards; StackExchange in particular. Please repost any questions like this in our Piazza page.

1.0.6 Course Reading Materials

We will use two freely available textbooks this semester. Make sure you can locate and access them:

- OpenStax Calculus Textbook (pdf here)
- The Origins of Calculus by David Perkins (ebook in library)

There are many many other freely accessible resources for learning calculus such as EdX, Coursera, Khan Academy, etc. Feel free to make ample use of these through the semester.

1.0.7 Resources

IS Student Support

Rachel Gottlieb, B.A. Candidate in Interdisciplinary Science and Gender Studies gottr694@gmail.com Availability: Mondays & Wednesdays after 3:30 in Room 459 Science Lab, 65 West 11th Street

Marina Delgado, B.A. Candidate in Interdisciplinary Science and Poetry Email: delgm708@newschool.edu Mondays and Wednesdays: 3:30 PM - 5:30 PM (except for Job Talk dates) Tuesdays: 12:30 PM - 3:00 PM, Room 459 Science Lab, 65 West 11th Street Fridays are flexible!

The university provides many resources to help students achieve academic and artistic excellence. These resources include:

- The University (and associated) Libraries: http://library.newschool.edu
- The University Learning Center: http://www.newschool.edu/learning-center
- University Disabilities Service: www.newschool.edu/student-disability-services/ In keeping with the university's policy of providing equal access for students with disabilities, any student with a disability who needs academic accommodations is welcome to meet with me privately. All conversations will be kept confidential. Students requesting any accommodations will also need to contact Student Disability Service (SDS). SDS will conduct an intake and, if appropriate, the Director will provide an academic accommodation notification letter for you to bring to me. At that point, I will review the letter with you and discuss these accommodations in relation to this course.

1.0.8 Schedule

- Monday, January 22. Introduction to Class: What is a number? Investigation integers, rational numbers, irrational numbers, and related historical problems.

- Wednesday, January 24. Number and Algorithm: Continue to work on problem set in groups. Problem write-up workshop.

- **Computer Lab I** Monday, January 29: Sequences, Functions, Summations with Python

- Wednesday, January 31. Summations: Interpret problems with areas. Add rectangles, trapezoids, and parabolas, discuss accuracy.

- Monday, February 5. Definite Integral: Introduce definition of definite integral, basic rules for polynomial, trigonometric, and exponential functions. Use technology and tables.

- ** Computer Lab II** Wednesday, February 7: Discrete and Continuous Distributions with Python.

- Monday and Wednesday, February 12 - 14.

PROJECT I: Statistics of distributions, Voting Power, Work-Force-Center of Mass, Numerical Algorithms for Integration, History of Integration, Consumer/Producer Surplus

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- Wednesday, February 21. Differences and Derivatives: Investigate the discrete case of first and second differences, connect these with average rates of change.

- Monday, February 26. Slopes and a Definition: Use slopes to move to continuous case, discuss approximate and exact solutions, connect first and second derivatives with first and second differences.

- Wednesday, February 28. Lab III. Derivatives with Python.

- Monday March 5. Application of Derivatives I: Historical Problems related to Derivatives.

- Wednesday, March 7. Application of Derivatives II: Vectors, Trigonometry, and Problems from Physics.

- Monday, March 12. Lab IV. Optimization in Python.
- Wednesday, March 14. Differentiation Review: Various problems involving differentiation.
- Week of March 19 and 21.

PROJECT II: Linear Regression, Arbitrating Disputes, Item Response Ideas, Motion of Projectiles, Numerical Differentiation, History of Differentiation, Elasticity of Demand

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- Monday, March 26. Recursion: Using the logistic population, focus on models built recursively using rates of change.
- Wednesday, March 28. Differential Equations: Antiderivates and solving separable differential equations.
- Monday, April 2. Lab V - Modeling Differential Equations with Python
- Wednesday, April 4. Systems: Model population change in time with Lotka-Volterra
- Monday, April 9. Qualitative Analysis of ODE's: Phase Plane Analysis
- Wednesday, April 11. Numerical Approaches: Euler and Runge-Kutta
- Monday, April 16. Lab VI: Solving and Visualizing ODE's with Python.
- Wednesday, April 18. Differential Equations Review.
- Week of April 23 - 25.

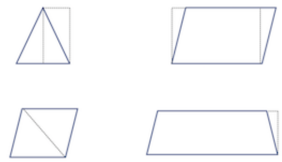
PROJECT III: Gradient Descent, Dynamical Systems and Social Theory, Differential Equations and Physics, CobWeb Diagrams in Economics, Advanced Numerical Solutions, History of Differential Equations.

- April 20 - May 9

FINAL PROJECT: Paper or Problem Set.

1.0.9 Technology

We wil use Python and Jupyter Notebooks to complete our work. You should download and install these through the Anaconda program, freely available here.



2 Problem Set I

Mathematical Goals:

- Solve problems related to areas
- Understand Base number notation
- Perform basic operations on numbers including root extraction
- Use Pythagorean Theorem to find distances
- Solve quadratic equations
- Investigate partial sums of series

The problems below come from the marvelous text *The Historical Development of Calculus* by C.H. Edwards.

Suggested Reading:

- Chapter 1: *The Historical Development of Calculus*, C.H.Edwards
- Chapter 1: *The Origins of Calculus*, David Perkins
- *History and Origins of the Calculus*, G.W. Leibniz

2.0.1 Problem I

Use the images above to explain the formulas for the areas:

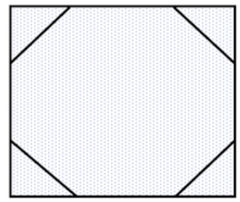
- of triangles $A = \frac{1}{2}bh$
- of parallelograms $A = bh$
- of trapezoids $A = \frac{1}{2}(b_1 + b_2)h$

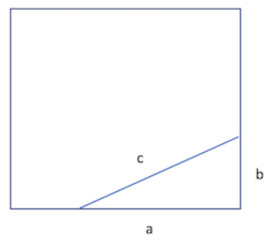
2.0.2 Problem II

An Egyptian Papyrus calculates the area of a quadrilateral by multiplying half the sum of two opposite sides times half the sum of the other two sides. Is this the correct result for a trapezoid or parallelogram that is not a rectangle?

2.0.3 Problem III

a. In one of the Rhind papyrus problems the area of a cirle is calculated by squaring 8/9 of its diameter. Compare this method with the area formula $A = \pi r^2$ to obtain the Egyptian approximation of $\pi = 3.16$.
b. This approximation to π may have been found by trisecting each side of a square circumscribed about a circle of diameter d, and cut off its 4 corners. Show that the area here would be:





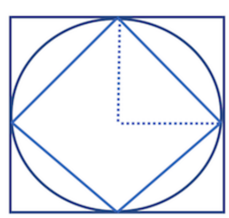
$$A = \frac{7}{9}d^2 = \frac{63}{81}d^2 \approx \frac{64}{81}d^2 = (\frac{8}{9}d)^2$$

2.0.4 Problem IV

Four copies of a right triangle with legs a and b and hypotenuse c together with a square of edge c, are assembled as in Figure 3 to form a square of edge $a + b$. Explain why the assembled figure *is* a squre, and derive the Pythagorean relation by computing its area in two different ways.

2.0.5 Problem V

The Babylonians generally used $3r^2$ for the area of a circle of radius r, corresponding to the poor approximation $\pi \approx 3$. Show that this approxiamation could have been obtained by averaging the areas of the iscribed and circumscribed squares shown below.



2.0.6 Problem VI

Archimedes took a similar approach to approximating π. He began by inscribing and circumscribing a circle with regular hexagons, and successively doubled the sides in order to, calculuating their perimeters to find upper and lower bounds for π. Beginning with a circle of radius 1 and compute the perimeter for the following shapes:

Polygon	Perimeter
Inscribed 6 sided	
Circumscribed 6 sided	
In 12 Sided	
Circum 12 Sided	

2.0.7 Problem VII

The Babylonians approached square root approximation in a similar iterative methodology like Archimedes use of the method of exhaustion. To start, suppose we have a guess that we think is close to $\sqrt{2}$

$$x_1 \approx \sqrt{2} \quad \rightarrow \quad x_1 \times x_1 \approx 2 \quad \rightarrow \quad x_1 \approx \frac{2}{x_1}$$

Either x_1 is a better guess or $\frac{2}{x}$, but even better still would be the average of the two:

$$x_2 = \frac{1}{2}(x_1 + \frac{2}{x_1})$$

If we continue in this manner we will get better and better approximations:

$$x_3 = \frac{1}{2}(x_2 + \frac{2}{x_2})$$

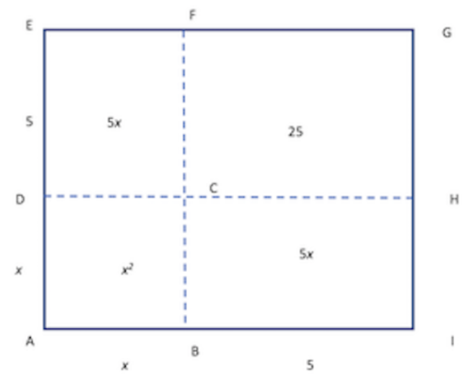
$$x_4 = \frac{1}{2}(x_3 + \frac{2}{x_3})$$

$$x_5 = \frac{1}{2}(x_4 + \frac{2}{x_4})$$

$$\vdots$$

$$x_{n+1} = \frac{1}{2}(x_n + \frac{2}{x_n})$$

Follow the Babylonians method to approximate $\sqrt{2}$ through x_5. Repeat for $\sqrt{3}$.



2.0.8 Problem VIII

The Arabic mathematician Al - Khowarizmi's introduced the base 10 numeration system to popular audiences in his writing. Here, collections of groups of 10 items were combined with positional notation and zero to make our familiar numeration system. For example, we would write 13,285 as

$$13285 = 1 * 10^4 + 3 * 10^3 + 2 * 10^2 + 8 * 10^1 + 5 * 10^0$$

Consider groupings different than base 10 for the following questions:

1. How many numerals are required for a base 5 numeration system?
2. Can you express the base 10 numbers 360, 78, 35, and 23 in base 5 notation? (Ex 10 base 10 = 20_5(base 5))
3. 0.3012 stands for
$$3/10 + 0/10^2 + 1/10^3 + 2/10^4$$
. Write the same number as a decimal in base 5.

2.0.9 Problem IX

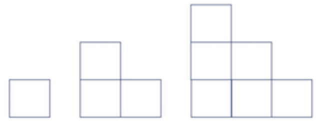
Al - Khowarizimi also discussed an approach to solving quadratic equations such as $x^2 + 10x = 39$. According to Edwards, his solution was

- Take half the number of roots, that is, five, and multply this by itself to obtain twenty-five.

- Add this to the thiry-nine, giving sixty-four.

- Take the square root, or eight, and subtract from it half the number of roots (five).
- The result, 3, is the required root.

Solve the equation $x^2 + 8x = 65$ following a similar construction to that above.

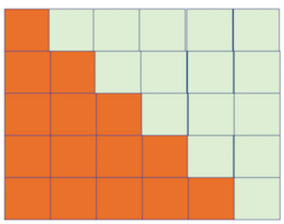


2.0.10 Problem X

The images below help to understand the value of expressions like:

$$1 + 2 + 3 + 4 + ... + (n-1) + n =$$

1. Find the sum when $n = 5$
2. Find the sum when $n = 10$
3. Is there a way to use the image below to understand the sum when $n = 100$?



2.0.11 Problem XI

In the seventeenth century, Leibniz introduced modern summation notation. The previous problems sum would be written as

$$\sum_{i=1}^{n} i$$

which indicated we are adding ($\sum$), starting with an index of 1 ($i = 1$), and the things we are adding are simply the indices (i). Sometimes, we recognize patterns in the partial sums of terms as a way to understand the general approach. Consider the summation

$$\sum_{i=1}^{n} i^3$$

Fill in the table with the following partial sums. Do you see a pattern? Describe it.

n	Partial Sum $\sum_{i=1}^{n} i^3$
1	
2	
3	
4	
10	
j	

3 Rubric for Problem Sets

We will focus on four areas to determine the quality of solutions in our work. First and most important is that you are writing your solutions to a reader, and you should be considerate of

- Clearly State the Problem in your own words
- Describe the approach to the solution
- Show solution method using multiple representations: words, tables, graphs/pictures, formulas, numbers
- Summarize and discuss

3.0.1 Goals Today

1. Establish Groups
2. Finalize Tech
3. Problems V - VII
4. Feedback - Questions - Student Information
5. Homework:

- Problem Set: Complete and write up, submit single group assignment
- Piazza: Post Information and StackExchange Handle
- Anaconda and Jupyter: Download and Open

6. Next Class:

- Introduction to Python and Jupyter