

Grammar

Types

$\tau ::= \sigma \mid r$

Base Types

$\sigma ::= \text{float} \mid \sigma \times \sigma \mid \eta \cdot \sigma \mid \sigma \rightarrow \sigma$

Natural Numbers

$\eta ::= 0 \mid 1 \mid \dots$

Range

$r ::= \eta.. \eta$

Term

$t ::= p \mid \text{let } x := t \text{ in } t \mid (t, t) \mid \text{if } t \leq \eta \text{ then } t \text{ else } t \mid t + t \mid t * t \mid t -$
 $t \mid t / t \mid t \ t \mid v$

$v = \text{fl} \mid \text{for } i : r \text{ in } t \mid (v, v) \mid \lambda(x : \sigma).t \mid \eta$

- i and x are identifiers.

Literal

$\text{fl} ::= 0.0 \mid -4.21 \mid 523.215 \mid \dots$

Place Expression

$p ::= x \mid p[t] \mid p.\text{fst} \mid p.\text{snd}$

Environment

Type Environment

$\Gamma ::= \bullet \mid \Gamma, (x : \tau)$

Evaluation Context

$\rho ::= \bullet \mid \rho, [x \mapsto v]$

Typing Rules

$$\boxed{\Gamma \vdash t : \sigma}$$

$$\frac{\Gamma, (x : \sigma_1) \vdash t : \sigma_2}{\Gamma \vdash \lambda(x : \sigma_1).t : \sigma_1 \rightarrow \sigma_2} \text{T-ABS}$$

$$\frac{f : \sigma_1 \rightarrow \sigma_2 \quad \Gamma \vdash t : \sigma_3 \quad \exists \sigma. \sigma_3 \sqcap \sigma_1 = \sigma}{\Gamma \vdash f \ t : \sigma_2} \text{T-APP}$$

$$\frac{\Gamma \vdash t_l : \text{float} \quad \Gamma \vdash t_r : \text{float} \quad \text{op} \in \{+, -, *, /\}}{\Gamma \vdash t_l \text{ op } t_r : \text{float}} \text{T-ARITH}$$

$$\frac{x : \sigma \in \Gamma}{\Gamma \vdash x : \sigma} \text{T-VAR}$$

$$\frac{\Gamma \vdash t : \sigma \quad \Gamma, (x : \sigma) \vdash t_{\text{body}} : \sigma_{\text{body}}}{\Gamma \vdash \text{let } x := t \text{ in } t_{\text{body}} : \sigma_{\text{body}}} \text{T-LET}$$

$$\frac{\eta_l.. \eta_r : \text{ok} \quad \Gamma, (i : \eta_l.. \eta_r) \vdash t_{\text{body}} : \sigma}{\Gamma \vdash \text{for } i : \eta_l.. \eta_r \text{ in } t_{\text{body}} : \text{length}(\eta_l.. \eta_r) \cdot \sigma} \text{T-FOR}$$

$$\frac{\Gamma \vdash t : \eta_t \cdot \sigma \quad \Gamma \vdash t_{\text{index}} : \eta_l.. \eta_r \quad \eta_r < \eta_t}{\Gamma \vdash t[t_{\text{index}}] : \sigma} \text{T-INDEX-RANGE}$$

$$\frac{\Gamma \vdash t : \eta_t \cdot \sigma \quad \eta < \eta_t}{\Gamma \vdash t[\eta] : \sigma} \text{T-INDEX-NAT}$$

$$\frac{\Gamma \vdash t : \sigma_1 \times \sigma_2}{\Gamma \vdash t.\text{fst} : \sigma_1} \text{T-FST}$$

$$\frac{\Gamma \vdash t : \sigma_1 \times \sigma_2}{\Gamma \vdash t.\text{snd} : \sigma_2} \text{T-SND}$$

$$\frac{\begin{array}{l} \Gamma \vdash t : \eta_l.. \eta_r \quad r_{\text{then}} = \eta_l.. \min(\eta, \eta_r) \quad r_{\text{else}} = (\min(\eta, \eta_r) + 1).. \eta_r \\ r_{\text{then}} : \text{ok} \quad r_{\text{else}} : \text{ok} \quad \Gamma, (t : r_{\text{then}}) \vdash t_{\text{then}} : \sigma_1 \quad \Gamma, (t : r_{\text{else}}) \vdash t_{\text{else}} : \sigma_2 \\ \sigma_1 \sqcup \sigma_2 = \sigma \end{array}}{\Gamma \vdash \text{if } t \leq \eta \text{ then } t_{\text{then}} \text{ else } t_{\text{else}} : \sigma} \text{T-IF}$$

Wellformedness Rules

$$\boxed{r : \text{ok}}$$

$$\frac{\eta_0 \leq \eta_1}{\eta_0.. \eta_1 : \text{ok}} \text{W-RANGE}$$

Subtyping Rules

$$\boxed{\sigma \sqcup \sigma = \sigma}$$

$$\sigma \sqcup \sigma = \sigma$$

$$\eta_1 \cdot \sigma \sqcup \eta_2 \cdot \sigma = \min(\eta_1, \eta_2) \cdot \sigma$$

$$(\sigma_1, \sigma_2) \sqcup (\sigma_3, \sigma_4) = (\sigma_1 \sqcup \sigma_3, \sigma_2 \sqcup \sigma_4)$$

$$\text{float} \sqcup \text{float} = \text{float}$$

$$\sigma_1 \rightarrow \sigma_2 \sqcup \sigma_3 \rightarrow \sigma_4 = (\sigma_1 \sqcap \sigma_3) \rightarrow (\sigma_2 \sqcup \sigma_4)$$

$$\boxed{\sigma \sqcap \sigma = \sigma}$$

$$\sigma \sqcap \sigma = \sigma$$

$$\eta_1 \cdot \sigma \sqcap \eta_2 \cdot \sigma = \max(\eta_1, \eta_2) \cdot \sigma$$

$$(\sigma_1, \sigma_2) \sqcap (\sigma_3, \sigma_4) = (\sigma_1 \sqcap \sigma_3, \sigma_2 \sqcap \sigma_4)$$

$$\text{float} \sqcap \text{float} = \text{float}$$

Evaluation Rules

$$\boxed{\rho \vdash t \longrightarrow t}$$

$$\frac{\rho \vdash t_1 \longrightarrow t'_1}{\rho \vdash t_1 t_2 \longrightarrow t'_1 t_2} \text{E-APP1}$$

$$\frac{\rho \vdash t_2 \longrightarrow t'_2}{\rho \vdash v t_2 \longrightarrow v t'_2} \text{E-APP2}$$

$$\frac{}{\rho \vdash (\lambda(x : \sigma_1). t_{\text{body}}) v \longrightarrow [x \mapsto v] t_{\text{body}}} \text{E-APPABS}$$

$$\frac{[x \mapsto v] \in \rho}{\rho \vdash x \longrightarrow v} \text{E-VAR}$$

$$\frac{\rho \vdash t \longrightarrow v \quad \rho, [x \mapsto v] \vdash t_{\text{body}} \longrightarrow t'_{\text{body}}}{\rho \vdash \text{let } x = t \text{ in } t_{\text{body}} \longrightarrow t'_{\text{body}}} \text{E-LET}$$

$$\frac{\rho \vdash t_1 \longrightarrow t'_1}{\rho \vdash t_1[v] \longrightarrow t'_1[v]} \text{E-INDEX}$$

$$\frac{\rho, [i \mapsto \eta] \vdash t \longrightarrow t'}{\rho \vdash (\text{for } i : r \text{ in } t)[\eta] \longrightarrow t'} \text{E-APPINDEX}$$

$$\frac{\rho \vdash t_1 \longrightarrow t'_1}{\rho \vdash (t_1, t_2) \longrightarrow (t'_1, t_2)} \text{E-TUP1}$$

$$\frac{\rho \vdash t_2 \longrightarrow t'_2}{\rho \vdash (t_1, t_2) \longrightarrow (t_1, t'_2)} \text{E-TUP2}$$

$$\frac{\rho \vdash t_1 \longrightarrow t'_1}{\rho \vdash t_1.\text{fst} \longrightarrow t'_1.\text{fst}} \text{E-FST}$$

$$\frac{\rho \vdash t_1 \longrightarrow t'_1}{\rho \vdash t_1.\text{snd} \longrightarrow t'_1.\text{snd}} \text{E-SND}$$

$$\frac{}{\rho \vdash (v_1, v_2).\text{fst} \longrightarrow v_1} \text{E-FSTAPP}$$

$$\frac{}{\rho \vdash (v_1, v_2).\text{snd} \longrightarrow v_2} \text{E-SNDAPP}$$

$$\frac{\rho \vdash t_1 \longrightarrow t'_1}{\rho \vdash \text{if } t_1 \leq \eta \text{ then } t_2 \text{ else } t_3 \longrightarrow \text{if } t'_1 \leq \eta \text{ then } t_2 \text{ else } t_3} \text{E-IF1}$$

$$\frac{\rho \vdash t_2 \longrightarrow t'_2}{\rho \vdash \text{if } v_1 \leq \eta \text{ then } t_2 \text{ else } t_3 \longrightarrow \text{if } v \leq \eta \text{ then } t'_2 \text{ else } t_3} \text{E-IF2}$$

$$\frac{\rho \vdash t_3 \longrightarrow t'_3}{\rho \vdash \text{if } v_1 \leq \eta \text{ then } v_2 \text{ else } t_3 \longrightarrow \text{if } v \leq \eta \text{ then } v_2 \text{ else } t'_3} \text{E-IF3}$$

$$\frac{\eta_1 \leq \eta_2}{\rho \vdash \text{if } \eta_1 \leq \eta_2 \text{ then } v_1 \text{ else } v_2 \longrightarrow v_1} \text{E-IFTRUE}$$

$$\frac{\eta_1 > \eta_2}{\rho \vdash \text{if } \eta_1 \leq \eta_2 \text{ then } v_1 \text{ else } v_2 \longrightarrow v_2} \text{E-IFFALSE}$$

Auxillary Definitions

$$\text{length}(\eta_0..\eta_1) = \eta_1 - \eta_0 + 1$$