Grammar

Types

$\tau ::= \sigma \mid r$

Base Types

$\sigma ::= \text{float} \mid \sigma \times \sigma \mid \eta \cdot \sigma \mid \sigma \to \sigma$

Natural Numbers

$\eta ::= 0 \mid 1 \mid \ldots$

Range

$r ::= \eta..\eta$

Term

$t ::= p \mid \text{let } x := t \text{ in } t \mid (t, t) \mid \text{if } t \leq \eta \text{ then } t \text{ else } t \mid t + t \mid t * t \mid t - t \mid t/t \mid t\ t \mid v$

$v = \text{fl} \mid \text{for } i : r \text{ in } t \mid (v, v) \mid \lambda(x : \sigma).t \mid \eta$

- i and x are identifiers.

Literal

$\text{fl} ::= 0.0 \mid -4.21 \mid 523.215 \mid \ldots$

Place Expression

$p ::= x \mid p[t] \mid p.\text{fst} \mid p.\text{snd}$

Environment

Type Environment

$\Gamma ::= \bullet \mid \Gamma, (x : \tau)$

Evaluation Context

$\rho ::= \bullet \mid \rho, [x \mapsto v]$

Typing Rules

$\boxed{\Gamma \vdash t : \sigma}$

$$\frac{\Gamma, (x : \sigma_1) \vdash t : \sigma_2}{\Gamma \vdash \lambda(x : \sigma_1).t : \sigma_1 \to \sigma_2}\ \text{T-ABS}$$

$$\frac{f : \sigma_1 \to \sigma_2 \quad \Gamma \vdash t : \sigma_3 \quad \exists \sigma. \sigma_3 \sqcap \sigma_1 = \sigma}{f\ t : \sigma_2}\ \text{T-APP}$$

$$\frac{\Gamma \vdash t_l : \text{float} \quad \Gamma \vdash t_r : \text{float} \quad \text{op} \in \{+, -, *, /\}}{\Gamma \vdash t_l\ \text{op}\ t_r : \text{float}}\ \text{T-ARITH}$$

$$\frac{x : \sigma \in \Gamma}{\Gamma \vdash x : \sigma}\ \text{T-VAR}$$

$$\frac{\Gamma \vdash t : \sigma \quad \Gamma, (x : \sigma) \vdash t_{\text{body}} : \sigma_{\text{body}}}{\Gamma \vdash \text{let}\ x \coloneqq t\ \text{in}\ t_{\text{body}} : \sigma_{\text{body}}}\ \text{T-LET}$$

$$\frac{\eta_l..\eta_r : \text{ok} \qquad \Gamma, (i : \eta_l..\eta_r) \vdash t_{\text{body}} : \sigma}{\Gamma \vdash \text{for}\ i : \eta_l..\eta_r\ \text{in}\ t_{\text{body}} : \text{length}(\eta_l..\eta_r) \cdot \sigma}\ \text{T-FOR}$$

$$\frac{\Gamma \vdash t : \eta_t \cdot \sigma \quad \Gamma \vdash t_{\text{index}} : \eta_l..\eta_r \quad \eta_r < \eta_t}{\Gamma \vdash t[t_{\text{index}}] : \sigma}\ \text{T-INDEX-RANGE}$$

$$\frac{\Gamma \vdash t : \eta_t \cdot \sigma \quad \eta < \eta_t}{\Gamma \vdash t[\eta] : \sigma}\ \text{T-INDEX-NAT}$$

$$\frac{\Gamma \vdash t : \sigma_1 \times \sigma_2}{\Gamma \vdash t.\text{fst} : \sigma_1}\ \text{T-FST}$$

$$\frac{\Gamma \vdash t : \sigma_1 \times \sigma_2}{\Gamma \vdash t.\text{snd} : \sigma_2}\ \text{T-SND}$$

$$\frac{\begin{array}{c}\Gamma \vdash t : \eta_l..\eta_r \quad r_{\text{then}} = \eta_l..\min(\eta, \eta_r) \quad r_{\text{else}} = (\min(\eta, \eta_r) + 1)..\eta_r \\ r_{\text{then}} : \text{ok} \quad r_{\text{else}} : \text{ok} \quad \Gamma, (t : r_{\text{then}}) \vdash t_{\text{then}} : \sigma_1 \quad \Gamma, (t : r_{\text{else}}) \vdash t_{\text{else}} : \sigma_2 \\ \sigma_1 \sqcup \sigma_2 = \sigma\end{array}}{\Gamma \vdash \text{if}\ t \le \eta\ \text{then}\ t_{\text{then}}\ \text{else}\ t_{\text{else}} : \sigma}\ \text{T-IF}$$

Wellformedness Rules

$\boxed{r : \mathrm{ok}}$

$$\frac{\eta_0 \leq \eta_1}{\eta_0 .. \eta_1 : \mathrm{ok}}\text{W-RANGE}$$

Subtyping Rules

$\boxed{\sigma \sqcup \sigma = \sigma}$

$$\sigma \sqcup \sigma = \sigma$$
$$\eta_1 \cdot \sigma \sqcup \eta_2 \cdot \sigma = \min(\eta_1, \eta_2) \cdot \sigma$$
$$(\sigma_1, \sigma_2) \sqcup (\sigma_3, \sigma_4) = (\sigma_1 \sqcup \sigma_3, \sigma_2 \sqcup \sigma_4)$$
$$\mathrm{float} \sqcup \mathrm{float} = \mathrm{float}$$
$$\sigma_1 \to \sigma_2 \sqcup \sigma_3 \to \sigma_4 = (\sigma_1 \sqcap \sigma_3) \to (\sigma_2 \sqcup \sigma_4)$$

$\boxed{\sigma \sqcap \sigma = \sigma}$

$$\sigma \sqcap \sigma = \sigma$$
$$\eta_1 \cdot \sigma \sqcap \eta_2 \cdot \sigma = \max(\eta_1, \eta_2) \cdot \sigma$$
$$(\sigma_1, \sigma_2) \sqcap (\sigma_3, \sigma_4) = (\sigma_1 \sqcap \sigma_3, \sigma_2 \sqcap \sigma_4)$$
$$\mathrm{float} \sqcap \mathrm{float} = \mathrm{float}$$

Evaluation Rules

$\boxed{\rho \vdash t \longrightarrow t}$

$$\frac{\rho \vdash t_1 \longrightarrow t_1'}{\rho \vdash t_1 t_2 \longrightarrow t_1' t_2}\text{E-APP1}$$

$$\frac{\rho \vdash t_2 \longrightarrow t_2'}{\rho \vdash v\ t_2 \longrightarrow v\ t_2'}\text{E-APP2}$$

$$\frac{}{\rho \vdash \big(\lambda(x : \sigma_1).t_{\text{body}}\big)v \longrightarrow [x \mapsto v]t_{\text{body}}}\text{E-APPABS}$$

$$\frac{[x \mapsto v] \in \rho}{\rho \vdash x \longrightarrow v}\text{E-VAR}$$

$$\frac{\rho \vdash t \longrightarrow v \quad \rho, [x \mapsto v] \vdash t_{\text{body}} \longrightarrow t_{\text{body}}'}{\rho \vdash \text{let } x = t \text{ in } t_{\text{body}} \longrightarrow t_{\text{body}}'}\text{E-LET}$$

$$\frac{\rho \vdash t_1 \longrightarrow t_1'}{\rho \vdash t_1[v] \longrightarrow t_1'[v]}\text{E-INDEX}$$

$$\frac{\rho, [i \mapsto \eta] \vdash t \longrightarrow t'}{\rho \vdash (\text{for } i : r \text{ in } t)[\eta] \longrightarrow t'}\text{E-APPINDEX}$$

$$\frac{\rho \vdash t_1 \longrightarrow t_1'}{\rho \vdash (t_1, t_2) \longrightarrow (t_1', t_2)}\text{E-TUP1}$$

$$\frac{\rho \vdash t_2 \longrightarrow t_2'}{\rho \vdash (t_1, t_2) \longrightarrow (t_1, t_2')}\text{E-TUP2}$$

$$\frac{\rho \vdash t_1 \longrightarrow t_1'}{\rho \vdash t_1.\text{fst} \longrightarrow t_1'.\text{fst}}\text{E-FST}$$

$$\frac{\rho \vdash t_1 \longrightarrow t_1'}{\rho \vdash t_1.\text{snd} \longrightarrow t_1'.\text{snd}}\text{E-SND}$$

$$\frac{}{\rho \vdash (v_1, v_2).\text{fst} \longrightarrow v_1}\text{E-FSTAPP}$$

$$\frac{}{\rho \vdash (v_1, v_2).\text{snd} \longrightarrow v_2}\text{E-SNDAPP}$$

$$\frac{\rho \vdash t_1 \longrightarrow t_1'}{\rho \vdash \text{if } t_1 \leq \eta \text{ then } t_2 \text{ else } t_3 \longrightarrow \text{if } t_1' \leq \eta \text{ then } t_2 \text{ else } t_3}\text{E-IF1}$$

$$\frac{\rho \vdash t_2 \longrightarrow t_2'}{\rho \vdash \text{if } v_1 \leq \eta \text{ then } t_2 \text{ else } t_3 \longrightarrow \text{if } v \leq \eta \text{ then } t_2' \text{ else } t_3} \text{E-IF2}$$

$$\frac{\rho \vdash t_3 \longrightarrow t_3'}{\rho \vdash \text{if } v_1 \leq \eta \text{ then } v_2 \text{ else } t_3 \longrightarrow \text{if } v \leq \eta \text{ then } v_2 \text{ else } t_3'} \text{E-IF3}$$

$$\frac{\eta_1 \leq \eta_2}{\rho \vdash \text{if } \eta_1 \leq \eta_2 \text{ then } v_1 \text{ else } v_2 \longrightarrow v_1} \text{E-IFTRUE}$$

$$\frac{\eta_1 > \eta_2}{\rho \vdash \text{if } \eta_1 \leq \eta_2 \text{ then } v_1 \text{ else } v_2 \longrightarrow v_2} \text{E-IFFALSE}$$

Auxillary Definitions

$$\text{length}(\eta_0..\eta_1) = \eta_1 - \eta_0 + 1$$

$\boxed{(x : \sigma) \in \Gamma}$

$$(x : \sigma) \in \Gamma, (x : \sigma)$$
$$(x : \sigma) \in \Gamma, (x' : \sigma') \equiv (x : \sigma) \in \Gamma$$

where $x \neq x'$

$\boxed{[x \mapsto v] \in \rho}$

$$[x \mapsto v] \in \rho, [x \mapsto v]$$
$$[x \mapsto v] \in \rho, [x' \mapsto v'] \equiv [x \mapsto v] \in \rho$$

where $x \neq x'$