Grammar

Types

$\tau ::= \sigma \mid r \mid \alpha$

Base Types

$\sigma ::= \text{float} \mid \sigma \times \sigma \mid \eta \cdot \sigma \mid \sigma \to \sigma$

Natural Numbers

$\eta ::= 0 \mid 1 \mid \ldots$

Range

$r ::= \eta..\eta$

Term

$t ::= \text{fl} \mid p \mid \text{for } i : r \text{ in } t \mid \text{let } x := t \text{ in } t \mid (t, t) \mid \text{if } t \leq \eta \text{ then } t \text{ else } t \mid t + t \mid t * t \mid t - t \mid t/t \mid t\ t \mid \lambda x.t$

- i and x are identifiers.

Literal

$\text{fl} ::= 0.0 \mid -4.21 \mid 523.215 \mid \ldots$

Place Expression

$p ::= x \mid p[t] \mid p.\text{fst} \mid p.\text{snd}$

Environment

Type Environment

$\Gamma ::= \bullet \mid \Gamma, (x : \tau)$

Kind Environment

$\Delta = \bullet \mid \Delta, \alpha$

Constraints

$C = \bullet \mid C, (\tau \sqsubseteq \tau)$

Typing Rules

$$\frac{\alpha \text{ fresh} \quad \Delta, \alpha; \Gamma, (x : \alpha) \vdash t : \sigma_2 \mid C, (\alpha \sqsubseteq \sigma_1)}{\Delta; \Gamma \vdash \lambda x.t : \sigma_1 \to \sigma_2 \mid C} \text{T-ABS}$$

$$\frac{\Delta; \Gamma \vdash \lambda x.t_1 : \sigma_1 \to \sigma_2 \mid C \quad \Delta; \Gamma \vdash t_2 : \sigma_3 \mid C \quad \Delta; \Gamma \vdash \sigma_3 \sqsubseteq \sigma_1 \mid C}{\Delta; \Gamma \vdash (\lambda x.t_1)t_2 : \sigma_2 \mid C} \text{T-APP}$$

$$\frac{\Gamma \vdash t_l : \text{float} \quad \Gamma \vdash t_r : \text{float} \quad \text{op} \in \{+, -, *, /\}}{\Gamma \vdash t_l \text{ op } t_r : \text{float}} \text{T-ARITH}$$

$$\frac{x : \sigma \in \Gamma}{\Gamma \vdash x : \sigma} \text{T-VAR}$$

$$\frac{\Gamma \vdash t : \sigma \quad \Gamma, (x : \sigma) \vdash t_{\text{body}} : \sigma_{\text{body}}}{\Gamma \vdash \text{let } x := t \text{ in } t_{\text{body}} : \sigma_{\text{body}}} \text{T-LET}$$

$$\frac{\eta_l..\eta_r : \text{ok} \qquad \Gamma, (i : \eta_l..\eta_r) \vdash t_{\text{body}} : \sigma}{\Gamma \vdash \text{for } i : \eta_l..\eta_r \text{ in } t_{\text{body}} : \text{length}(r') \cdot \sigma} \text{T-FOR}$$

$$\frac{\Gamma \vdash t : \eta_t \cdot \sigma \quad \Gamma \vdash t_{\text{index}} : \eta_l..\eta_r \quad \eta_r < \eta_t}{\Gamma \vdash t[t_{\text{index}}] : \sigma} \text{T-INDEX-RANGE}$$

$$\frac{\Gamma \vdash t : \eta_t \cdot \sigma \quad \eta < \eta_t}{\Gamma \vdash t[\eta] : \sigma} \text{T-INDEX-NAT}$$

$$\frac{\Gamma \vdash t : \sigma_1 \times \sigma_2}{\Gamma \vdash t.\text{fst} : \sigma_1} \text{T-FST}$$

$$\frac{\Gamma \vdash t : \sigma_1 \times \sigma_2}{\Gamma \vdash t.\text{snd} : \sigma_2} \text{T-SND}$$

$$\frac{\begin{array}{c}\Gamma \vdash t : \eta_l..\eta_r \quad r_{\text{then}} = \eta_l..\min(\eta, \eta_r) \quad r_{\text{else}} = (\min(\eta, \eta_r) + 1)..\eta_r \\ r_{\text{then}} : \text{ok} \quad r_{\text{else}} : \text{ok} \quad \Gamma, (t : r_{\text{then}}) \vdash t_{\text{then}} : \sigma \quad \Gamma, (t : r_{\text{else}}) \vdash t_{\text{else}} : \sigma\end{array}}{\Gamma \vdash \text{if } t \leq \eta \text{ then } t_{\text{then}} \text{ else } t_{\text{else}} : \sigma} \text{T-IF}$$

$$\frac{\Gamma \vdash t : \eta_l..\eta_r \quad \eta_r \leq \eta \quad \Gamma \vdash t_{\text{then}} : \sigma}{\Gamma \vdash \text{if } t \leq \eta \text{ then } t_{\text{then}} \text{ else } t_{\text{else}} : \sigma} \text{T-THEN-ONLY}$$

Well-formedness Rule and Type Relation

$$\frac{\eta_0 \leq \eta_1}{\eta_0..\eta_1 : \text{ok}}\text{W-RANGE}$$

$$\frac{\eta_1 \geq \eta_2 \quad \sigma_1 \sqsubseteq \sigma_2}{\eta_1 \cdot \sigma_1 \sqsubseteq \eta_2 \cdot \sigma_2}\text{T-SUB}$$

$$\frac{}{\text{float} \sqsubseteq \text{float}}\text{T-SUB-FLOAT}$$

Auxillary Definitions

$\mathrm{length}(\eta_0..\eta_1) = \eta_1 - \eta_0 + 1$