Grammar

Base Type

$\tau ::= \sigma \mid r$

$\sigma ::= \text{float} \mid \sigma \times \sigma \mid \eta \cdot \sigma$

Natural Numbers

$\eta ::= 0 \mid 1 \mid \ldots$

Range

$r ::= \eta..\eta \mid \text{empty}$

Term

$t ::= \text{fl} \mid \eta \mid p \mid \text{for } i : r \text{ in } t \mid \text{let } x := t \text{ in } t \mid (t, t) \mid \text{if } t \subseteq t \text{ then } t \text{ else } t \mid t + t \mid t * t \mid t - t \mid t/t$

- i and x are identifiers.

Literal

$\text{fl} ::= 0.0 \mid -4.21 \mid 523.215 \mid \ldots$

Place Expression

$p ::= x \mid p[t] \mid p.\text{fst} \mid p.\text{snd}$

Environment

Type Environment

$\Gamma ::= \bullet \mid \Gamma, (x : \tau)$

Typing Rules

$$\frac{\Gamma \vdash t_l : \text{float} \quad \Gamma \vdash t_r : \text{float} \quad \text{op} \in \{+, -, *, /\}}{\Gamma \vdash t_l \text{ op } t_r : \text{float}}\ \text{T-ARITH}$$

$$\frac{x : \sigma \in \Gamma}{\Gamma \vdash x : \sigma}\ \text{T-VAR}$$

$$\frac{\Gamma \vdash t : \sigma \quad \Gamma, (x : \sigma) \vdash t_{\text{body}} : \sigma_{\text{body}}}{\Gamma \vdash \text{let } x := t \text{ in } t_{\text{body}} : \sigma_{\text{body}}}\ \text{T-LET}$$

$$\frac{r' = \mathrm{mkRng}(\eta_l..\eta_r) \quad \Gamma, (i : r') \vdash t_{\mathrm{body}} : \sigma}{\Gamma \vdash \mathrm{for}\ i : \eta_l..\eta_r\ \mathrm{in}\ t_{\mathrm{body}} : \mathrm{length}(r') \cdot \sigma}\ \text{T-FOR}$$

$$\frac{\Gamma \vdash t : \eta_t \cdot \sigma \quad \Gamma \vdash t_{\mathrm{index}} : \eta_l..\eta_r \quad \eta_r < \eta_t}{\Gamma \vdash t[t_{\mathrm{index}}] : \sigma}\ \text{T-INDEX-RANGE}$$

$$\frac{\Gamma \vdash t_{\mathrm{index}} : \mathrm{empty} \quad \Gamma \vdash t : \eta_t \cdot \sigma}{\Gamma \vdash t[t_{\mathrm{index}}] : \sigma}\ \text{T-INDEX-RANGE-EMPTY}$$

$$\frac{}{\Gamma \vdash \eta : \eta..\eta}\ \text{T-NAT}$$

$$\frac{\Gamma \vdash t : \sigma_1 \times \sigma_2}{\Gamma \vdash t.\mathrm{fst} : \sigma_1}\ \text{T-FST}$$

$$\frac{\Gamma \vdash t : \sigma_1 \times \sigma_2}{\Gamma \vdash t.\mathrm{snd} : \sigma_2}\ \text{T-SND}$$

$$\frac{\begin{array}{c}\Gamma \vdash i : r_l \quad \Gamma \vdash t_r : r_r \quad \Gamma, (i : r_l \cap r_r) \vdash t_{\mathrm{if}} : \sigma_{\mathrm{if}} \quad (r_0, r_1) = r_l / r_r \\ \Gamma, (i : r_0) \vdash t_{\mathrm{else}} : \sigma_{\mathrm{else0}} \quad \Gamma, (i : r_1) \vdash t_{\mathrm{else}} : \sigma_{\mathrm{else1}} \\ \sigma = \sigma_{\mathrm{if}} = \sigma_{\mathrm{else0}} = \sigma_{\mathrm{else1}}\end{array}}{\Gamma \vdash \mathrm{if}\ i \subseteq t_r\ \mathrm{then}\ t_{\mathrm{if}}\ \mathrm{else}\ t_{\mathrm{else}} : \sigma}\ \text{T-IF}$$

Auxillary definitions

mkRng(r) = **match** r **with**

| $\eta_l .. \eta_r \Rightarrow$ **if** $0 \leq \eta_l \leq \eta_r$ **then** $\eta_l .. \eta_r$ **else** empty

| empty $\Rightarrow$ empty

length(r) = **match** r **with**

| empty $\Rightarrow 0$

| $\eta_l .. \eta_r \Rightarrow \eta_r - \eta_l + 1$

$r_l \cap r_r$ = **match** (r_l, r_r) **with**

| (empty,_) $\Rightarrow$ empty

| (_,empty) $\Rightarrow$ empty

| $(\eta_{\mathrm{l0}} .. \eta_{\mathrm{l1}}, \eta_{\mathrm{r0}} .. \eta_{\mathrm{r1}}) \Rightarrow \mathrm{mkRng}(\max(\eta_{\mathrm{l0}}, \eta_{\mathrm{r0}}), \min(\eta_{\mathrm{l1}}, \eta_{\mathrm{r1}}))$

r_l / r_r = **match** $r_l \cap r_r$ **with**

| empty $\Rightarrow (r_l, \mathrm{empty})$

| $\eta_0 .. \eta_1 \Rightarrow$ **match** r_l **with**

  | $\eta_{\mathrm{l0}} .. \eta_{\mathrm{l1}} \Rightarrow$ (mkRng($\eta_{\mathrm{l0}}, \eta_0 - 1$), mkRng($\eta_1 + 1, \eta_{\mathrm{l1}}$))

  | _ $\Rightarrow$ **unreachable**

Examples

For expression

```
for i: (0..5) in
  for j: (0..6) in
    for k: (0..7) in
      4.2
```

This results in a value of type $5 \cdot 6 \cdot 7 \cdot \text{float}$

```
for i : 0..5 in
  for j: 0..10 in
    1.2
```

This results in a value of type $5 \cdot 10 \cdot \text{float}$

Indexing by a value of type range

```
for i: 0..5 in
  a[0][i]
```

This is equivalent to: `a[0][0:5]`

Slicing

```
for i: 0..10 in
  for j: 0..5 in
    a[i][j]
```

This is of type $10 \cdot 5 \cdot \sigma$ and equivalent to `a[0..10][0..5]` where σ is the type of $a[0][0]$

let in

```
let arr =
  for i: 0..5 in
    for j : 0..5 in
      3.14159
in
for i: 0..2 in
  for j: 0..1 in
    arr[i][j]
```

This is of type $2 \cdot 1 \cdot \text{float}$

let in, for, and tuple

tuple

```
let arr_1 =
  for i: 0..5 in
    for j: 0..5 in
      3.14159 in
let arr_2 =
  for i: 2..4 in
    for j: 1..3 in
      arr_1[i][j] in
(arr_1, arr_2)
```

This is of type $(5 \cdot 5 \cdot \text{float}) \times (2 \cdot 2 \cdot \text{float})$

Nested tuple/array

```
let tup = (3.14159, for i : 0..5 in 6.25) in
  for i : 0..10 in
    tup
```

This is of type $10 \cdot (\text{float} \times (5 \cdot \text{float}))$