품질공학 HW #4

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6.10. The thickness of a printed circuit board is an important quality parameter. Data on board thickness (in inches) are given in Table 6E.5 for 25 samples of three boards each.

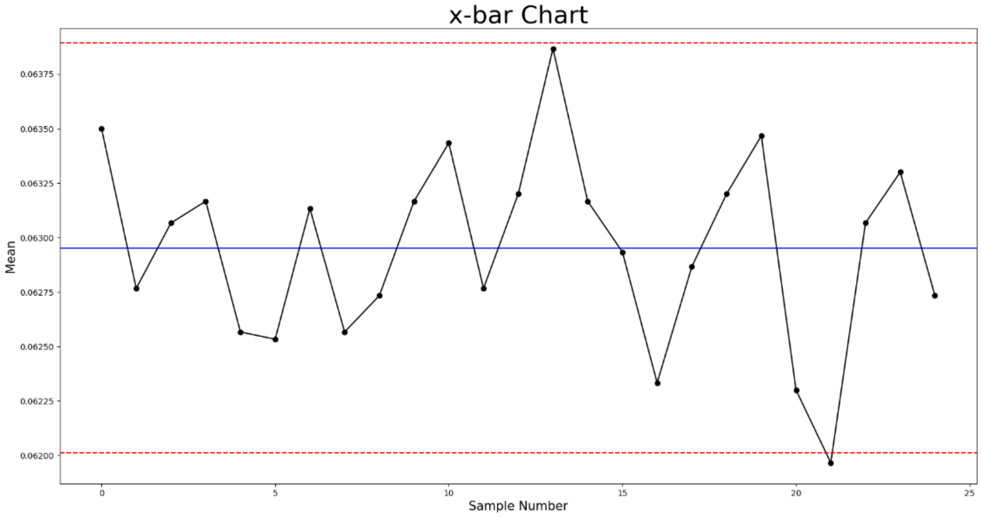
(a) Set up $\bar{x}$ and R control charts. Is the process in statistical control?

(b) Estimate the process standard deviation.

(c) What are the limits that you would expect to contain nearly all the process measurements?

(d) If the specifications are at 0.0630 in. ± 0.0015 in., what is the value of the PCR C_p?

(a)

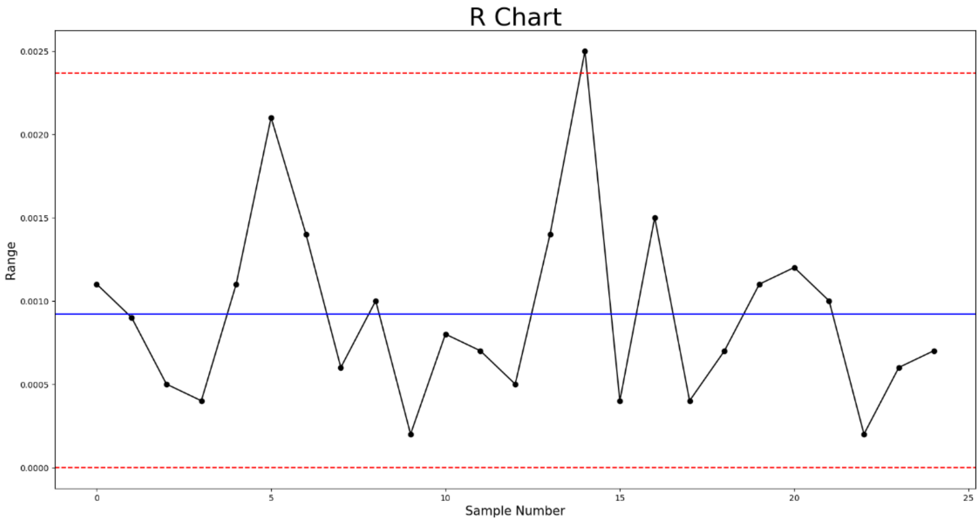


x chart

$UCL = 0.063893$

$Center\ Line = 0.062952$

$LCL = 0.062011$



R chart

$UCL = 0.002368$

$Center\ Line = 0.00092$

$LCL = 0$

blue line은 Center line을, red line은 upper, lower control limit 을 나타낸다.

$\bar{x}\ chart$, $R\ chart$ 모두 control limit 을 벗어나는 mean 이 있으므로 process가 in control 상태라고 할 수 없다.

(b) $\hat{\sigma} = \bar{R}/d_2$ 는 σ의 unbiased estimator.

$n=3$ 일 때 $d_2 = 1.693$

$\bar{R} = 0.00092$

$\therefore \hat{\sigma} = 0.00092/1.693 = \boxed{0.000543}$

(c) 3-sigma control limit를 구해야 한다.

$UCL = \bar{\bar{x}} + 3\sigma = 0.062952 + 3 \cdot 0.000543 = \boxed{0.064581}$

$LCL = \bar{\bar{x}} - 3\sigma = 0.062952 - 3 \cdot 0.000543 = \boxed{0.061323}$

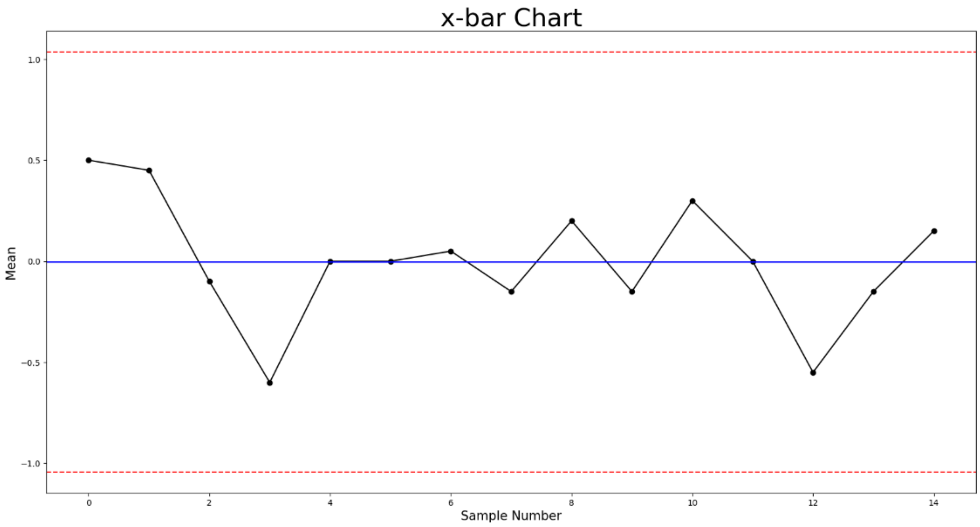
(d) PCR $C_p = \dfrac{USL - LSL}{6\sigma}$ $USL = 0.0645$ $LSL = 0.0615$

$$= \frac{0.0645 - 0.0615}{6 \cdot 0.000543} = \boxed{0.921}$$

6.11.

The fill volume of soft-drink beverage bottles is an important quality characteristic. The volume is measured (approximately) by placing a gauge over the crown and comparing the height of the liquid in the neck of the bottle against a coded scale. On this scale, a reading of zero corresponds to the correct fill height. Fifteen samples of size $n = 10$ have been analyzed, and the fill heights are shown in Table 6E.6.

(a) Set up $\overline{x}$ and s control charts on this process. Does the process exhibit statistical control? If necessary, construct revised control limits.
(b) Set up an R chart, and compare it with the s chart in part (a).
(c) Set up an s^2 chart and compare it with the s chart in part (a).

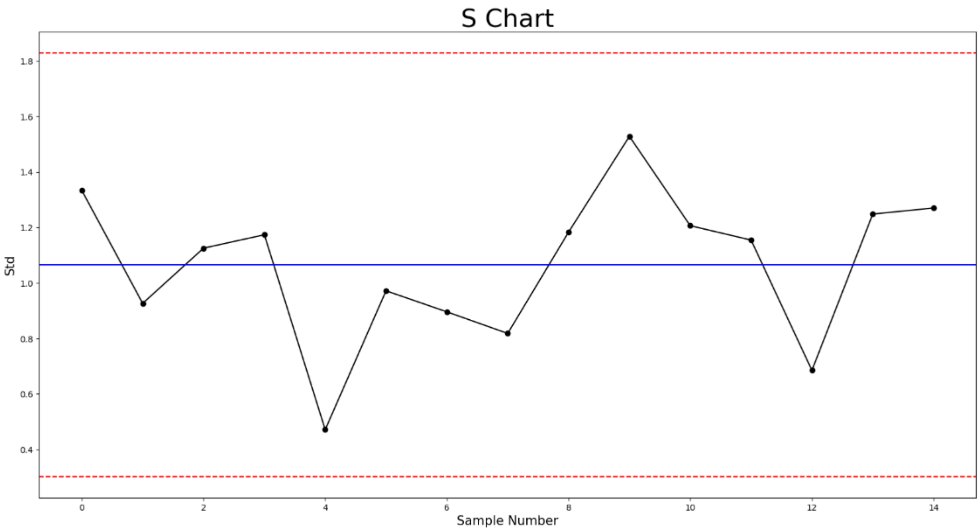


x chart

$UCL = 1.03622$

$Center\ Line(\bar{\bar{x}}) = -0.00333$

$LCL = -1.0429$



s chart

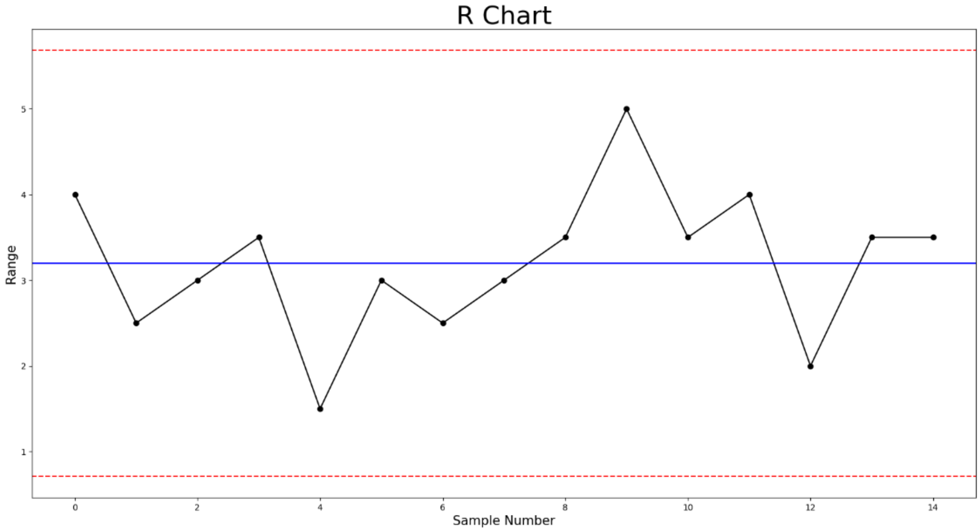
$UCL = 1.82962$

$Center\ Line(\bar{s}) = 1.06621$

$LCL = 0.3028$

모든 관측치가 control limit 안에 있다. 그러므로 process 가 in statistical control 상태라고 할 수 있다.

(b)



R chart

$UCL = 5.6864$

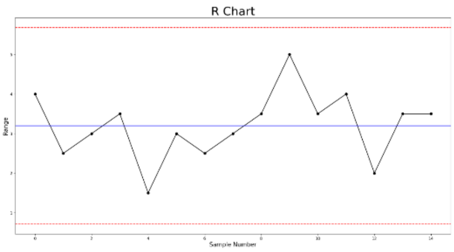
$Center\ Line(\bar{R}) = 3.2$

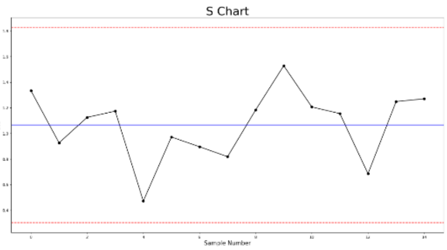
$LCL = 0.7136$

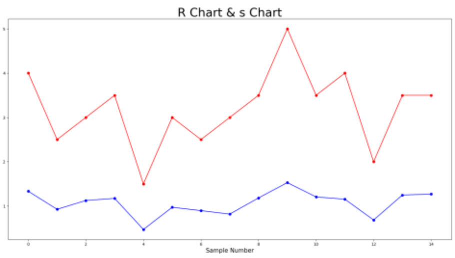
R chart 는 한 sample 내의 max, min 값의 차이인 range 가 monitoring statistic 이고, s chart는 sample의 standard deviation 이 monitoring statistic 인 chart이다.

두 관리도 모두 변동성을 모니터링한다. R chart는 range를 기반으로 하기 때문에 극단값에 의해 더 많은 변동을 보인다. 반면에 s chart는 변동성을 보다 안정적으로 나타내며, standard deviation 은 극단값에 덜 영향을 받기 때문에 전반적인 분포를 더 정확하게 반영한다.

sample size 가 커질수록 s chart 가 더 신뢰할 수 있는 관리도가 되지만 문제와 같이 size가 작을 때에는 R chart와 s chart 가 전체적으로 비슷한 개형으로 나타난다.







R chart

s chart

(c)

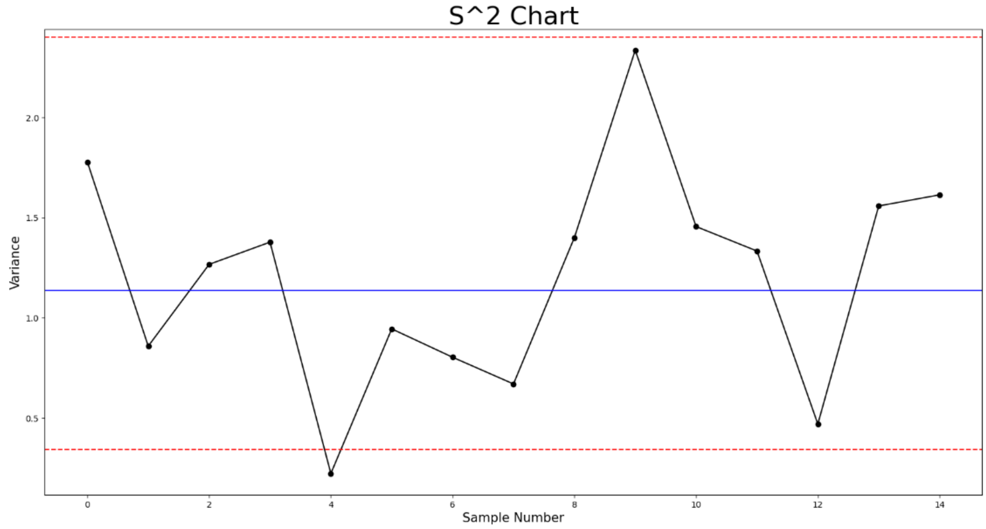
$s^2\ control\ chart$의 UCL, centerl line, LCL 은 다음과 같다.

$$\text{UCL} = \frac{\bar{s}^2}{n-1}\chi^2_{\alpha/2,n-1}$$

$$\text{Center line} = \bar{s}^2$$

$$\text{LCL} = \frac{\bar{s}^2}{n-1}\chi^2_{1-(\alpha/2),n-1}$$

이 문제에서 α 값은 0.05 로 설정하였다.



s^2 *chart*

$UCL = 2.40244$

$Center\ Line = 1.1368$

$LCL = 0.341041$

$s^2\ chart$ 는 variance를 monitoring statistic 으로 사용하여 변동성의 제곱을 보여주므로, $s\ chart$에 비해 극단적인 변동에 더 민감하게 반응한다.

그러므로 두 차트 모두 변동성의 흐름은 비슷하지만 $s^2 chart$ 가 변동성의 정도를 더 극단적으로 표현한다.

6.59. Control charts for $\bar{x}$ and s have been maintained on a process and have exhibited statistical control. The sample size is $n = 6$. The control chart parameters are as follows:

$\bar{x}$ Chart	s Chart
UCL = 708.20	UCL = 3.420
Center line = 706.00	Center line = 1.738
LCL = 703.80	LCL = 0.052

(a) Estimate the mean and standard deviation of the process.

(b) Estimate the natural tolerance limits for the process.

(c) Assume that the process output is well modeled by a normal distribution. If specifications are 703 and 709, estimate the fraction nonconforming.

(d) Suppose the process mean shifts to 702.00 while the standard deviation remains constant. What is the probability of an out-of-control signal occurring on the first sample following the shift?

(e) For the shift in part (d), what is the probability of detecting the shift by at least the third subsequent sample?

$n = 6. \rightarrow A_3 = 1.287 \quad D_3 = 0 \quad D_4 = 2.004 \quad C_4 = 0.9515$

(a) mean $\hat{\mu} = \bar{\bar{x}} = \boxed{706.00}$

Standard deviation $\hat{\sigma} = \bar{s}/C_4 = 1.738/0.9515 = \boxed{1.827}$

(b) 3-sigma control limit.

$UCL = \bar{\bar{x}} + 3 \cdot \hat{\sigma} = 706 + 3 \cdot 1.827 = \boxed{711.481}$

$LCL = \bar{\bar{x}} - 3 \cdot \hat{\sigma} = 706 - 3 \cdot 1.827 = \boxed{700.519}$

(c) specifications : [703, 709]

$P(x < 703) + P(x > 709)$

$= P\left(Z < \frac{703 - 706}{1.827}\right) + P\left(Z > \frac{709 - 706}{1.827}\right)$

$= \Phi(-1.642) + 1 - \Phi(1.642) = 0.050503 \times 2$

$= \boxed{0.101006}$

(d) $\mu = 702. \quad \sigma = 1.827.$

$P(X < 703.8) + P(X > 708.2)$

$= \Phi\left(\frac{703.8 - 702}{1.827/\sqrt{6}}\right) + 1 - \Phi\left(\frac{708.2 - 702}{1.827/\sqrt{6}}\right)$

$= \Phi(2.41) + 1 - \cancel{\Phi(8.31)}^{\,1} \approx \boxed{0.992024}$

(e) $p = 0.992024$ 라 할 때

$P\{\text{detected by 3rd sample}\} = p + p(1-p) + p(1-p)^2$

$= p(1 + 1 - p + p^2 - 2p + 1) = p(p^2 - 3p + 3) = \boxed{0.992212}$

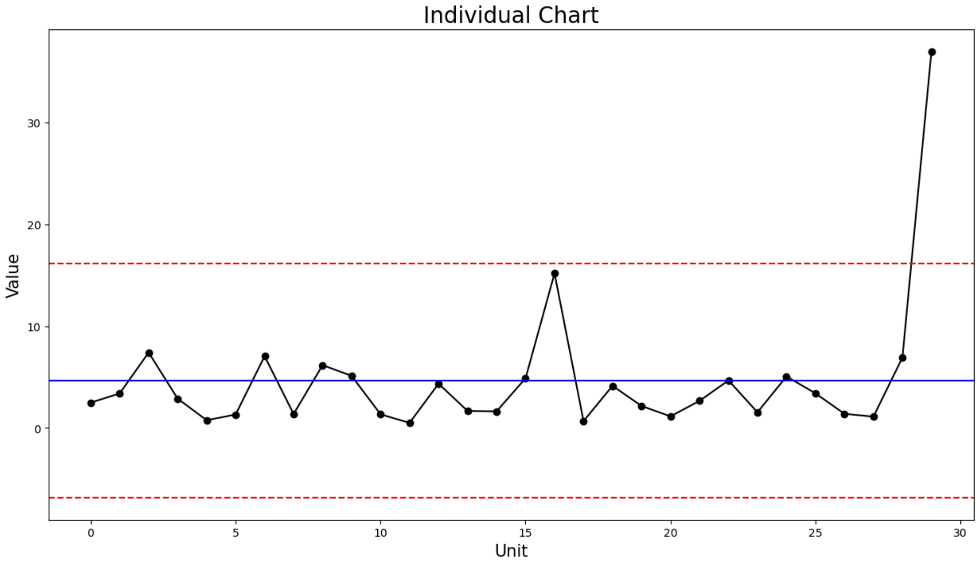
6.66. The waiting time for treatment in a "minute-clinic" located in a drugstore is monitored using control charts for individuals and the moving range. Table 6E.24 contains 30 successive measurements on waiting time.

(a) Set up individual and moving range control charts using this data.

(b) Plot these observations on the charts constructed in part (a). Interpret the results. Does the process seem to be in statistical control?

(c) Plot the waiting time data on a normal probability plot. Is it reasonable to assume normality for these data? Wouldn't a variable like waiting time often tend to have a distribution with a long tail (skewed) to the right? Why?

(a)

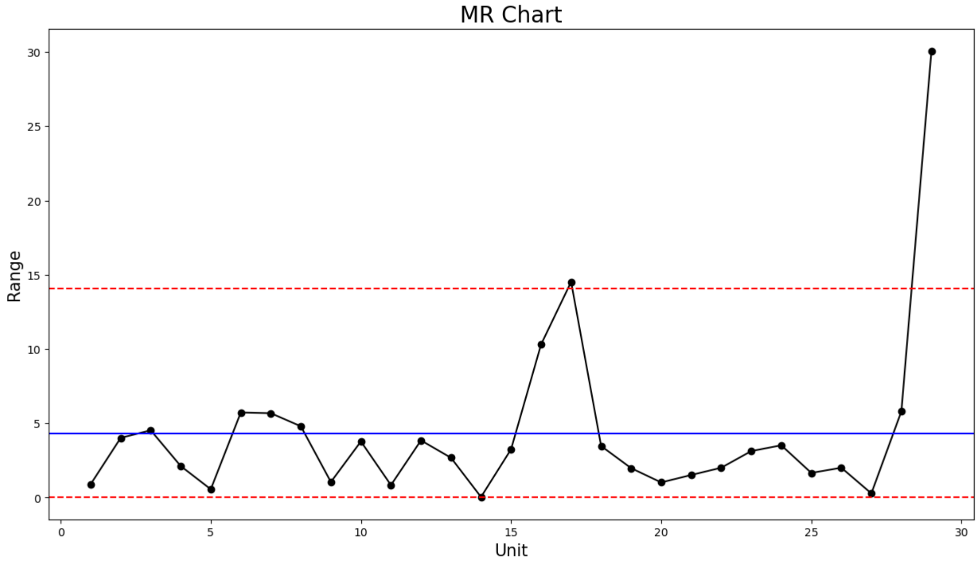


Individual chart

$UCL = 16.12219$

$Center\ Line = 4.645667$

$LCL = -6.83086$



MR chart

$UCL = 14.09767$

$Center\ Line = 4.315172$

$LCL = 0$

(b)

Individual Chart:

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...   Out of control limits -> Observation number 30
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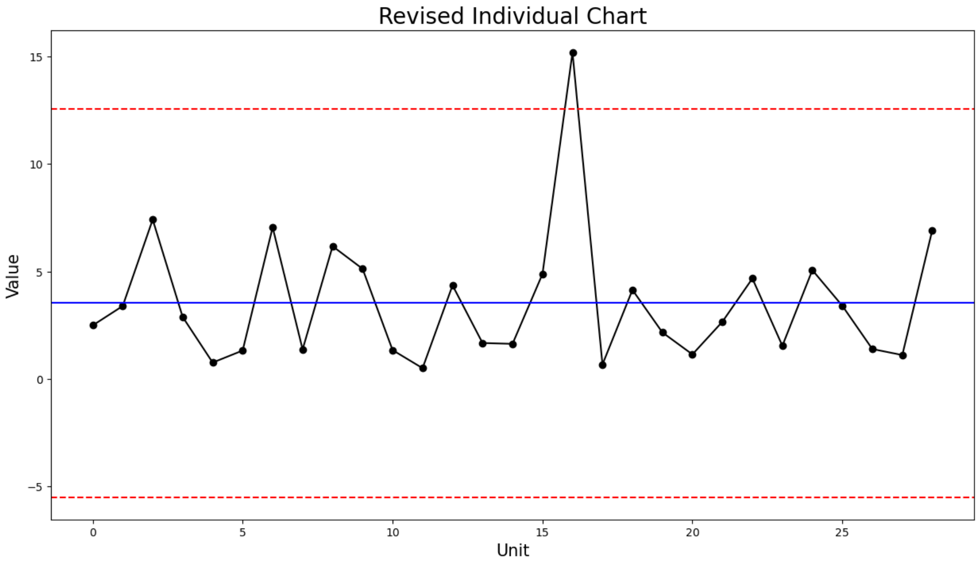

MR Chart:

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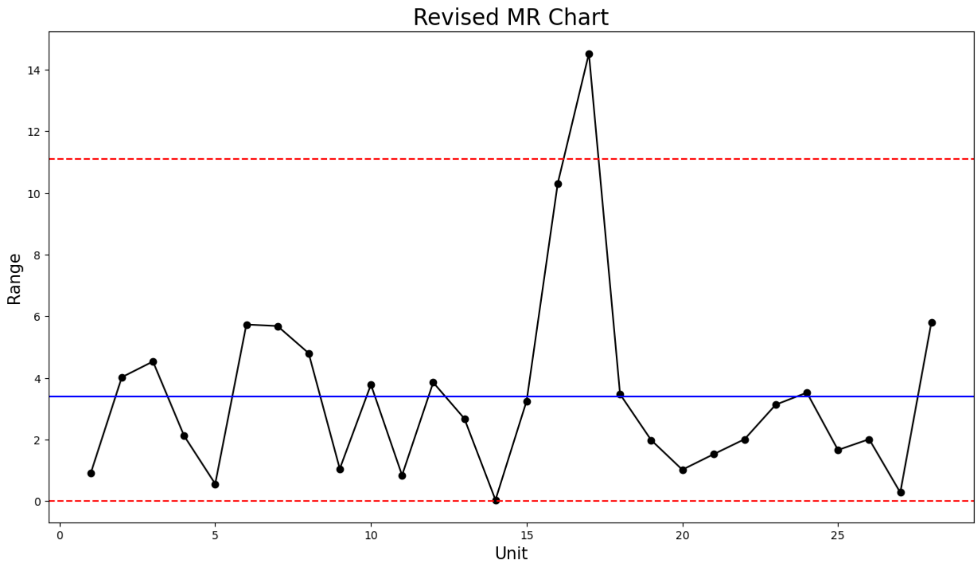
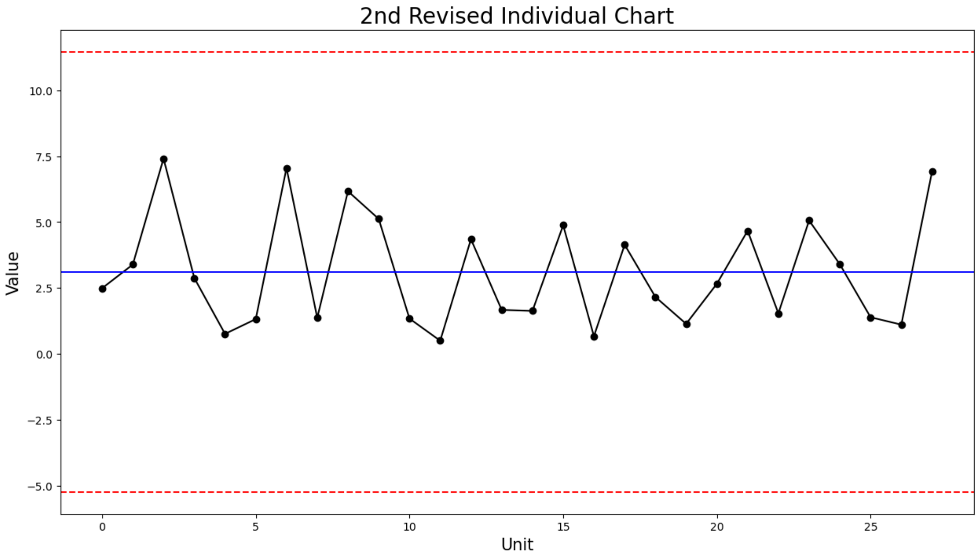
...   Out of control limits -> Observation number 18
      Out of control limits -> Observation number 30
```

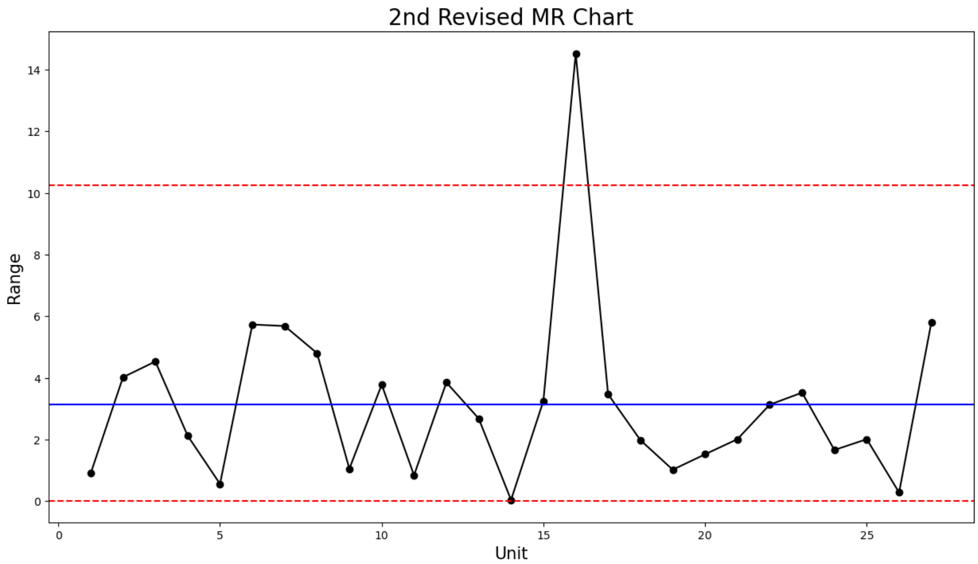


first trial Individual chart, MR chart 모두에서 이상 신호가 발견되었다. 특히, observation 30은 명백하고 차이가 큰 이상 신호를 나타낸다.

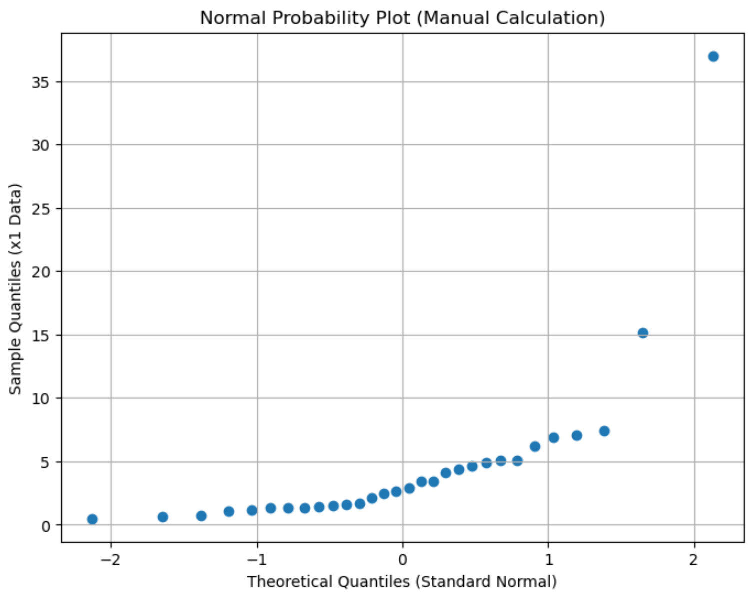
현재 in statistical control 상태라고 할 수 없다.

아직 이상값이 존재하여 다시 revise 하였다.





(c)



plot의 개형과 이상값을 고려했을 때, 이 데이터는 정규성을 따르지 않는다.

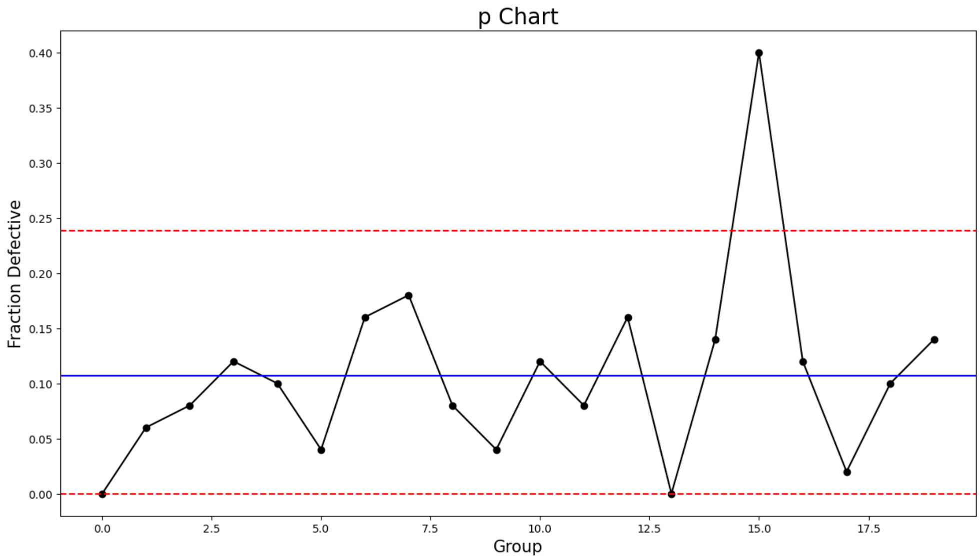
waiting time 과 같은 변수는 예상치 못한 문제 등의 상황으로 인해 대기 시간이 누적되어 더 극단적인 이상값이 나올 확률이 높다.

7.3. Table 7E.1 Contains data on examination of medical insurance claims. Every day 50 claims were examined.

(a) Set up the fraction nonconforming control chart for this process. Plot the preliminary data in Table 7E.1 on the chart. Is the process in statistical control?

(b) Assume that assignable causes can be found for any out-of-control points on this chart. What center line and control limits should be used for process monitoring in the next period?

(a)



p chart

$UCL = 0.238145644$

$Center\ Line = 0.107$

$LCL = 0$

불량률은 음수가 될 수 없으므로 $LCL = 0$

(b)

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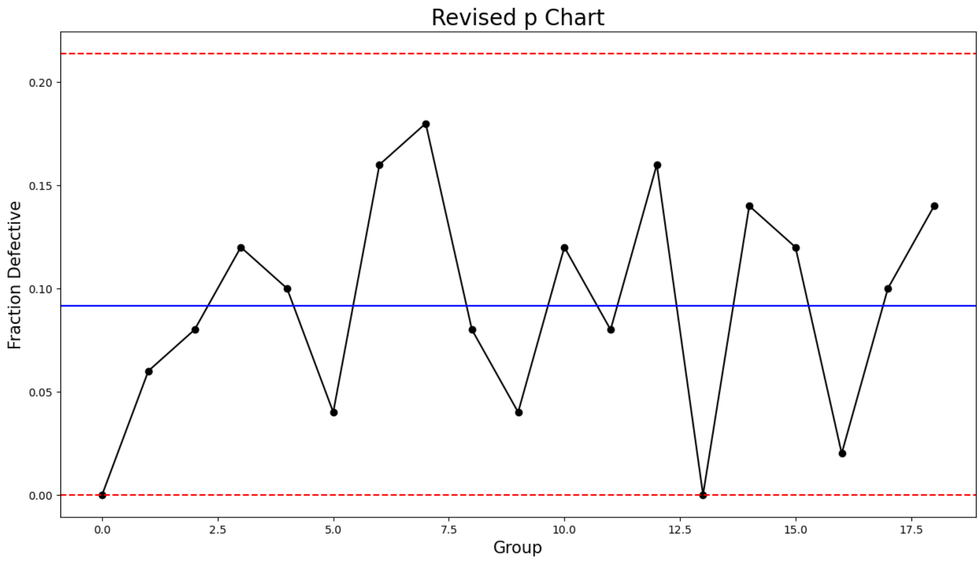
···  Out of control limits -> Day 16
```

Day 16 data를 eliminate 하고 reviesed p chart를 그렸다.



revised p chart

$UCL = 0.213949711$

$Center\ Line = 0.091578947$

$LCL = 0$

7.4. The fraction nonconforming control chart in Exercise 7.3 has an LCL of zero. Assume that the revised control chart in part (b) of that exercise has a reliable estimate of the process fraction nonconforming. What sample size should be used if you want to ensure that the LCL > 0?

day 16 데이터를 제외하여 만든 revised control chart가 reliable estimate을 제공한다고 가정하였다.

이때, $\bar{p} = 0.091578947$를 활용하여 $LCL > 0$ 이 되게 하는 sample size를 구해야한다,

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} > 0$$

$$n > \frac{9 \times (1-\bar{p})}{\bar{p}} = \frac{9 \times (1 - 0.0916)}{0.0916} \cong 89.276$$

$\therefore n \geq 90$

n 은 90 이상이어야한다.

7.19. A control chart for the fraction nonconforming is to be established using a center line of $p = 0.10$. What sample size is required if we wish to detect a shift in the process fraction nonconforming to 0.20 with probability 0.50?

기존 p 값, $p = 0.10$

$\delta = magnitude\ of\ a\ process\ shift = 0.2 - 0.1 = 0.1$

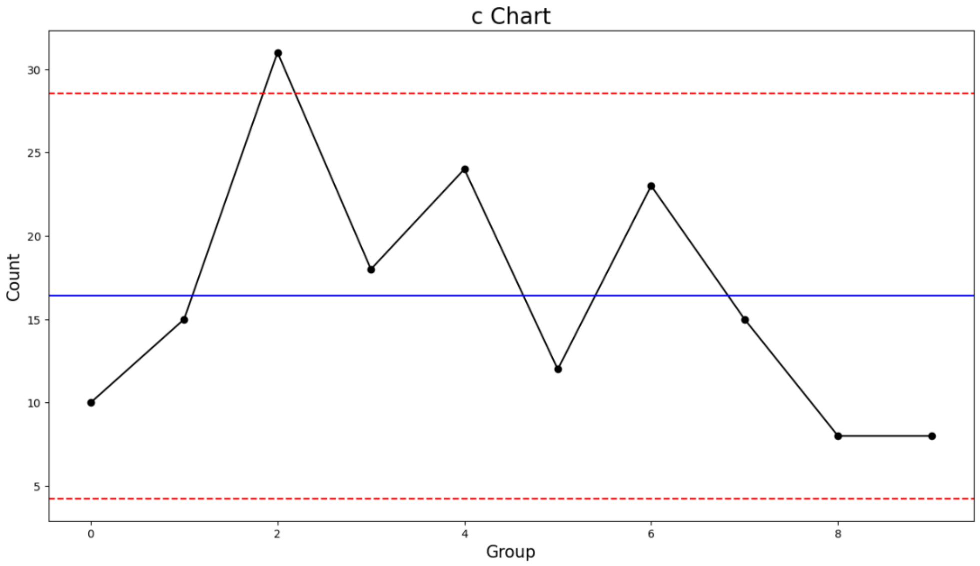
$$n = \left(\frac{L}{\delta}\right)^2 p(1-p) = \left(\frac{3}{0.1}\right)^2 0.1 \cdot 0.9 = 81$$

$\therefore \boldsymbol{n = 81}$

7.23. A control chart is used to control the fraction nonconforming for a plastic part manufactured in an injection molding process. Ten subgroups yield the data in Table 7E.9.

(a) Set up a control chart for the number nonconforming in samples of $n = 100$.

(b) For the chart established in part (a), what is the probability of detecting a shift in the process fraction nonconforming to 0.30 on the first sample after the shift has occurred?



c chart

$UCL = 28.54907404$

$Center\ Line = 16.4$

$LCL = 4.250925961$

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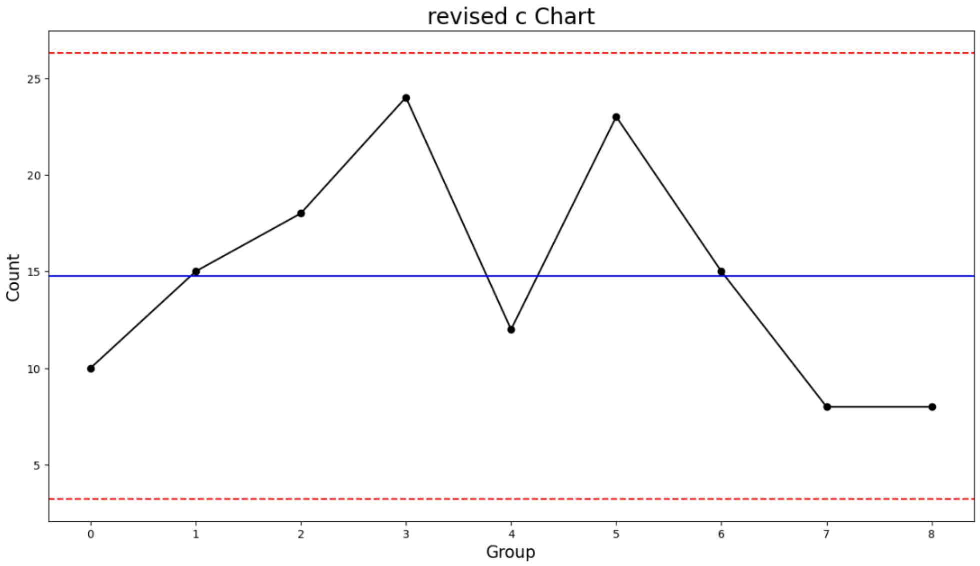
...   Out of control limits -> Sample number 3
```

sample number 3의 데이이터를 제외하고 revised c chart를 만든다.



revised c chart

$UCL = 26.3103$

$Center\ Line = 14.778$

$LCL = 3.2452$

(b) probability of detecting a shift $= 1-\beta$

$$\beta = \{ LCL \le \bar{x} \le UCL \mid \mu = \mu_0 + k\sigma \}$$

$$= \Phi\left(\frac{UCL - 30}{\sqrt{14.78}}\right) - \Phi\left(\frac{LCL - 30}{\sqrt{14.78}}\right)$$

$UCL = 26.3103$
$LCL = 3.2452$

$$= \Phi(-0.96) - \Phi(\cancel{-6.96}) \; 0$$

$$= 0.168528$$

$$\boxed{1-\beta \simeq 0.831}$$

2. 산업경영공학부 영문명칭을 쓰시오.

Industrial and Management Engineering