Computer Exercise 1
EL2520 Control Theory and Practice

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Disturbance attenuation

How should the extra poles be chosen in exercise 4.2.1? Motivate!

The frequencies where the disturbances should be attenuated are the frequencies on the interval $[0, \omega_c]$, *where* ω_c *is the crossover frequency of* $G_d(s)$, *which is approximately 10 rad/s. The poles should be chosen to be high enough such that they have little influence on frequencies on or below the crossover frequency, as this would reduce the attenuation of disturbances. There are two additional poles needed to make the controller proper. Therefore, one can choose for example to place two poles at* $10\omega_c$

The feedback controller in exercise 4.2.2 is

$$\begin{aligned}F_y &= \frac{s+\omega_I}{s} G^{-1} G_d \frac{p_1 p_2}{(p_1+s)(p_2+s)}, \quad \omega_I = 10, \quad p_1 = 100, \quad p_2 = 100 \\ &= \frac{12.37s^3 + 370.6s^2 + 7413s + 49250}{s^3 + 199s^2 + 9900s}\end{aligned}$$

The feedback controller and prefilter in exercise 4.2.3 is

$$F_y = K * \frac{\tau_D s + 1}{\beta \tau_D s + 1} \frac{s+\omega_I}{s} G^{-1} G_d \frac{p_1 p_2}{(p_1+s)(p_2+s)}$$

$$\omega_I = 10, \quad p_1 = 100, \quad p_2 = 100 \beta = 0.5, \quad \tau_D = \frac{1}{\omega_c \sqrt{\beta}}, \quad K = 0.8$$

$$F_y = \frac{12.62s^4 + 517.7s^3 + 11740s^2 + 133900s + 555800}{s^4 + 212s^3 + 12490s^2 + 128900s}$$

$$F_r = \frac{1}{1+\tau s} = \frac{1}{1 + 0.135s}$$

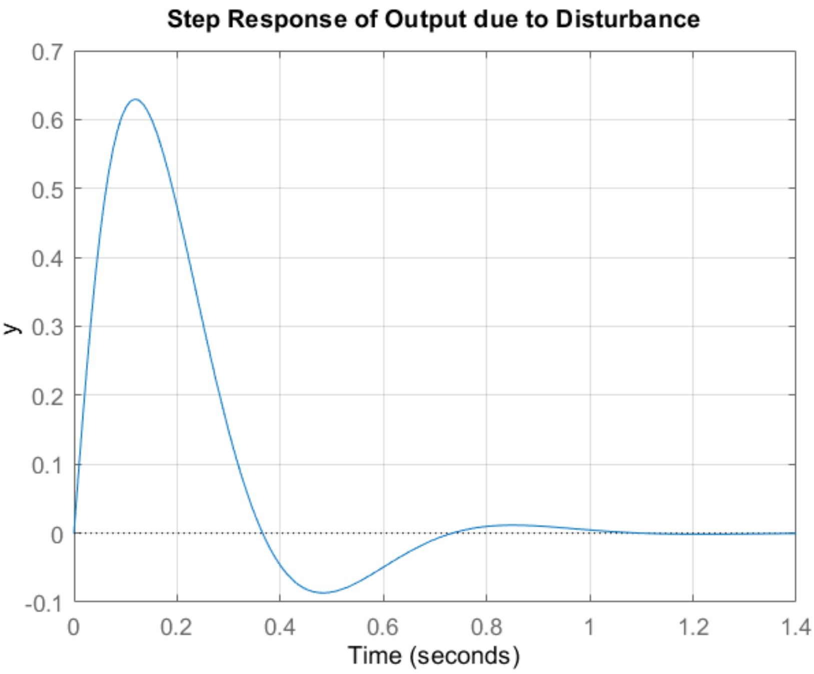

Figure 1: Step disturbance, exercise 4.2.2

Did you manage to fulfill all the specifications? If not, what do you think makes the specifications difficult to achieve?

Yes, as can be seen in the figures, the requirements are fulfilled. The results obtained, compared to the specified requirements are given below:

- $|u|_{max} = 0.995 < 1$
- Step in disturbance $|y(t)|_{max} = 0.63 < 1\ \forall t$ and $|y(t)| \leq 0.1$ for $t > 0.5$
- rise time $= 0.1867 < 0.2s$ and the overshoot $= 9.65\% < 10\%$

However, this was quite challenging, since it was difficult to balance a good step response to a reference change with a good noise attenuation while keeping within the limits of the control input magnitude. This is because the disturbance depends on the sensitivity and the reference on the complementary sensitivity, which are linked together (so a trade-off always has to be made). In order to succeed, it was necessary to reduce the crossover frequency of the sensitivity function to 9.13 rad/s, which is slightly lower than the cross-over of the disturbance transfer function. This is not desireable, however it did not seem possible to keep the cross-over frequency at 10 rad/s while still meeting the hard constraints that were given.

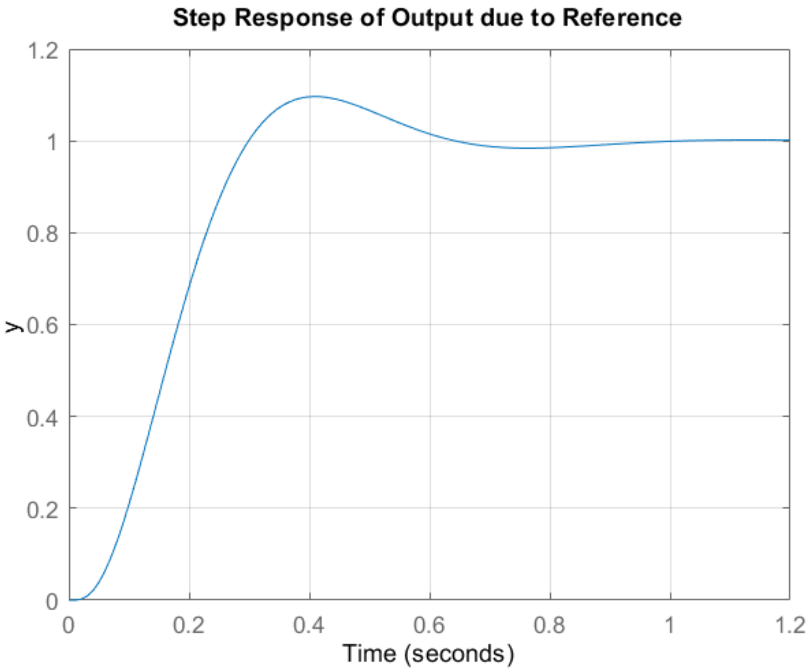

Figure 2: Reference step, exercise 4.2.3

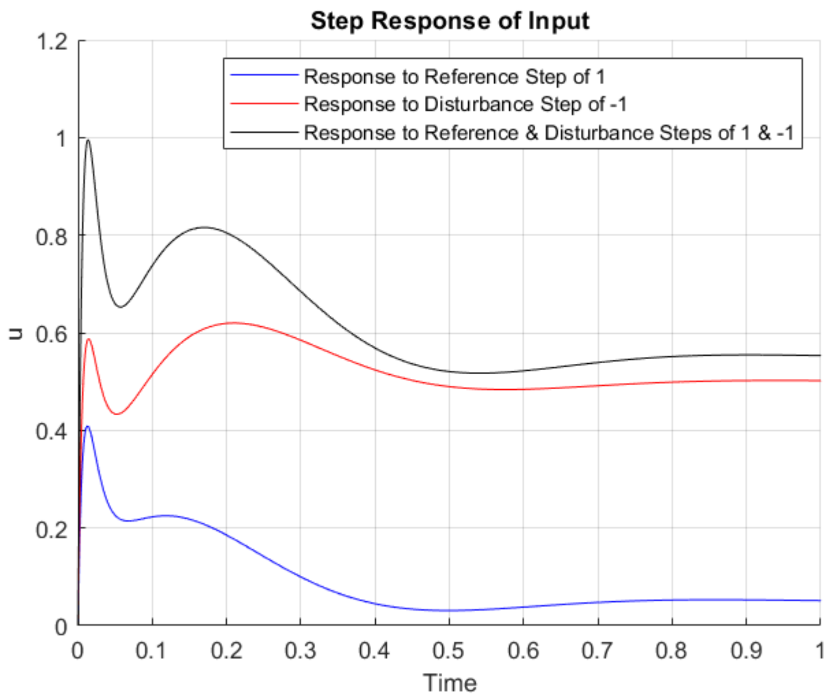

Figure 3: Control signal for a disturbance or a reference step (plus a combination of these)

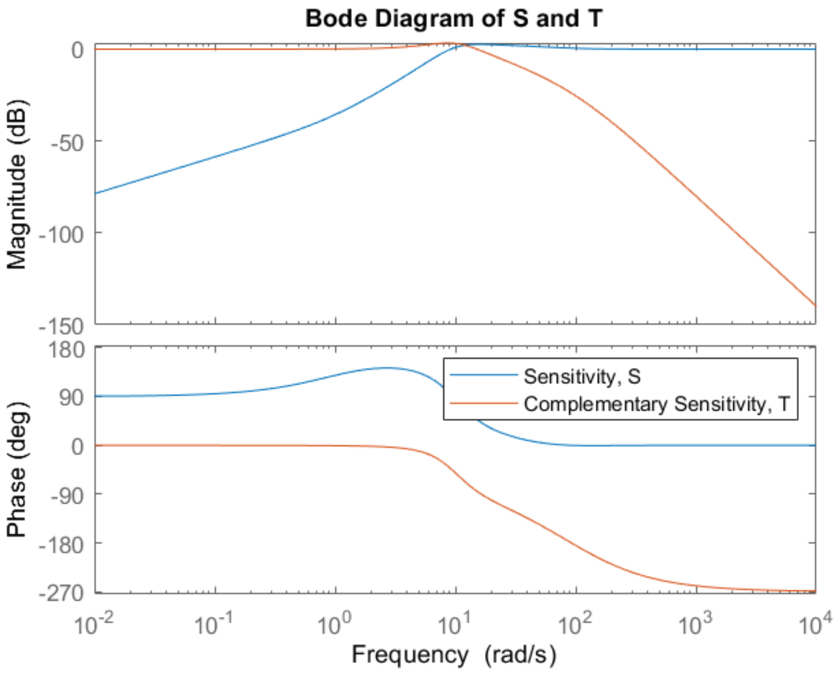

Figure 4: Bode diagram of sensitivity and complementary sensitivity functions, exercise 4.2.4