智能机器人技术第二次作业

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1. 由图,

$$egin{bmatrix} v_x \ v_y \end{bmatrix} = egin{bmatrix} -[l_1 sin heta_1 + l_2 sin(heta_1 + heta_2)] \dot{ heta_1} - l_2 sin(heta_1 + heta_2) \dot{ heta_2} \ [l_1 cos heta_1 + l_2 cos(heta_1 + heta_2)] \dot{ heta_1} + l_2 cos(heta_1 + heta_2) \dot{ heta_2} \end{bmatrix}$$

将表4.1数据代入得:

$$\begin{bmatrix} \dot{\theta_1} \\ \dot{\theta_2} \end{bmatrix}_1 = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$$
$$\begin{bmatrix} \dot{\theta_1} \\ \dot{\theta_2} \end{bmatrix}_2 = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$
$$\begin{bmatrix} \dot{\theta_1} \\ \dot{\theta_2} \end{bmatrix}_3 = \begin{bmatrix} -4 \\ 2\sqrt{3} + 6 \end{bmatrix}$$

2. 平衡状态时机械手作用力 $F = \begin{bmatrix} 0 \\ mq \end{bmatrix}$

各关节力矩
$$au = egin{bmatrix} au_1 \ au_2 \ au_3 \end{bmatrix} = J^T F$$

机械手断点位置x,y与关节变量关系为:

$$\left\{egin{aligned} x = l_1 cos heta_1 + l_2 cos (heta_1 + heta_2) + l_3 cos (heta_1 + heta_2 + heta_3) = x(heta_1, heta_2, heta_3) \ y = l_1 sin heta_1 + l_2 sin (heta_1 + heta_2) + l_3 sin (heta_1 + heta_2 + heta_3) = y(heta_1, heta_2, heta_3) \ \end{array}
ight.$$

微分得:

$$J = egin{bmatrix} rac{\partial x}{\partial heta_1} & rac{\partial x}{\partial heta_2} & rac{\partial x}{\partial heta_3} \ rac{\partial y}{\partial heta_1} & rac{\partial y}{\partial heta_2} & rac{\partial y}{\partial heta_3} \end{bmatrix}$$

其中,

$$\begin{split} \frac{\partial x}{\partial \theta_1} &= -l_1 sin\theta_1 - l_2 sin(\theta_1 + \theta_2) - l_3 sin(\theta_1 + \theta_2 + \theta_3) \\ \frac{\partial x}{\partial \theta_2} &= -l_2 sin(\theta_1 + \theta_2) - l_3 sin(\theta_1 + \theta_2 + \theta_3) \\ \frac{\partial x}{\partial \theta_3} &= -l_3 sin(\theta_1 + \theta_2 + \theta_3) \\ \frac{\partial y}{\partial \theta_1} &= l_1 cos\theta_1 + l_2 cos(\theta_1 + \theta_2) + l_3 cos(\theta_1 + \theta_2 + \theta_3) \\ \frac{\partial y}{\partial \theta_2} &= l_2 cos(\theta_1 + \theta_2) + l_3 cos(\theta_1 + \theta_2 + \theta_3) \\ \frac{\partial y}{\partial \theta_3} &= l_3 cos(\theta_1 + \theta_2 + \theta_3) \end{split}$$

将关节变量代入得:

$$au = J^T F = egin{bmatrix} -0.4\sqrt{3} - 0.4 & 0.4 + 0.8 \ -0.4 & 0.8 \ -0.4 & 0 \end{bmatrix} egin{bmatrix} 0 \ 10g \end{bmatrix} = egin{bmatrix} 12g \ 8g \ 0 \end{bmatrix}$$

3. 力矩

$$\tau_1 = \frac{d}{dt} \frac{\partial L}{\partial \dot{\theta_1}} - \frac{\partial L}{\partial \theta_1}$$

$$=(m_1p_1^2+m_2p_2^2+m_2l_1^2+2m_2l_1p_2c_2)\ddot{ heta_1}+(m_2p_2^2+m_2l_1p_2c_2)\ddot{ heta_2}+(-2m_2l_1p_2s_2)\dot{ heta_1}\dot{ heta_2}$$

$$+\left(-m_{2}l_{1}p_{2}s_{2}
ight)\dot{ heta}_{2}^{2}+\left(m_{1}p_{1}+m_{2}l_{1}
ight)gs_{1}+m_{2}gp_{2}s_{12}$$

$$=D_{11}\ddot{\theta_{1}}+D_{12}\ddot{\theta}_{2}+D_{112}\dot{\theta}_{1}\dot{\theta}_{2}+D_{122}\dot{\theta}_{2}^{2}+D_{1}$$

由此得,

$$\left\{egin{aligned} D_{11} &= m_1 p_1^2 + m_2 p_2^2 + m_2 l_1^2 + 2 m_2 l_1 p_2 c_2 \ D_{12} &= m_2 p_2^2 + m_2 l_1 p_2 c_2 \ D_{112} &= -2 m_2 l_1 p_2 s_2 \ D_{122} &= -m_2 l_1 p_2 s_2 \ D_1 &= (m_1 p_1 + m_2 l_1) g s_1 + m_2 p_2 g s_{12} \end{aligned}
ight.$$

$$au_2 = rac{d}{dt} rac{\partial L}{\partial \dot{ heta}_2} - rac{\partial L}{\partial heta_2}$$

$$=(m_2p_2^2+m_2l_1p_2c_2)\ddot{ heta}_1+m_2p_2^2\ddot{ heta}_2+(m_2l_1p_2s_2-m_2l_1p_2s_2)\dot{ heta}_1\dot{ heta}_2+(m_2l_1p_2s_2)\dot{ heta}_1^2+m_2gp_2s_{12}$$

$$=D_{21}\ddot{ heta}_1+D_{22}\ddot{ heta}_2+D_{212}\dot{ heta}_1\dot{ heta}_2+D_{211}\dot{ heta}_1^2+D_2 \ \begin{cases} D_{21}=m_2p_2^2+m_2l_1p_2c_2 \ D_{22}=m_2p_2^2 \ D_{212}=-m_2l_1p_2s_2+m_2l_1p_2s_2=0 \ D_{211}=m_2l_1p_2s_2 \ D_2=m_2gp_2s_{12} \end{cases}$$

$$\int D_{21} = m_2 p_2^2 + m_2 l_1 p_2 c_2 \, .$$

$$D_{22}=m_{2}p_{2}^{2}$$

$$D_{212} = -m_2 l_1 p_2 s_2 + m_2 l_1 p_2 s_2 = 0$$

$$D_{211}=m_2l_1p_2s_2$$

$$D_2 = m_2 g p_2 s_{12}$$

4. 连杆1动能:

$$E_{k1} = \frac{1}{2} m_1 l_1^2 \theta_1^2$$

连杆2质小位置:

$$x_2 = l_1 c_1 + l_2 c_{12}$$

$$y_2 = l_1 s_1 + l_2 s_1 2$$

质心速度平方:

$$\dot{x}_2 = -l_1 s_1 \dot{ heta}_1 - l_2 s_{12} (\dot{ heta}_1 + \dot{ heta}_2)$$

$$\dot{y}_2 = l_1 c_1 \dot{ heta}_1 + l_2 c_{12} (\dot{ heta}_1 + \dot{ heta}_2)$$

$$\dot{x}_{2}^{2}+\dot{y}_{2}^{2}=l_{1}^{2}\dot{ heta}_{1}^{2}+l_{2}^{2}(\dot{ heta}_{1}+\dot{ heta}_{2})^{2}+2(l_{1}l_{2}\dot{ heta}_{1}(\dot{ heta}_{1}+\dot{ heta}_{2}))c_{2}$$

$$E_{k2} = rac{1}{2} m_2 l_1^2 {\dot{ heta}}_1^2 + rac{1}{2} m_2 l_2^2 ({\dot{ heta}}_1 + {\dot{ heta}}_2)^2 + m_2 l_1 l_2 {\dot{ heta}}_1 ({\dot{ heta}}_1 + {\dot{ heta}}_2) c_2$$

连杆3质心位置:

$$\dot{x}_3 = \dot{x}_2$$

$$\dot{y}_3 = \dot{y}_2$$

速度平方:

$$\dot{x}_3^2 + \dot{y}_3^2 = \dot{x}_2^2 + \dot{y}_2^2$$

动能:

$$E_{k3} = rac{1}{2} m_3 l_1^2 \dot{ heta}_1^2 + rac{1}{2} m_3 l_2^2 (\dot{ heta}_1 + \dot{ heta}_2)^2 + m_3 l_1 l_2 \dot{ heta}_1 (\dot{ heta}_1 + \dot{ heta}_2) c_2$$

总动能:

$$E_k = \sum_{i=1}^3 E_{ki}$$

$$=rac{1}{2}(m_1+m_2+m_3)l_1^2{\dot{ heta}}_1^2+(m_2+m_3)(rac{1}{2}l_2^2({\dot{ heta}}_1+{\dot{ heta}}_2)^2+l_1l_2{\dot{ heta}}_1({\dot{ heta}}_1+{\dot{ heta}}_2)c_2)$$

系统总势能:

$$E_p = m_1 g l_1 s_1 + m_2 g (l_1 s_1 + l_2 s_{12}) + m_3 g (l_1 s_1 + l_2 s_{12})$$

代入拉格朗日函数:

$$L = E_k - E_p$$

$$rac{\partial L}{\partial \dot{ heta}_1} = (m_1 + m_2 + m_3) l_1^2 \dot{ heta}_1 + (m_2 + m_3) (l_2^2 \dot{ heta}_1 + l_2^2 \dot{ heta}_1 \dot{ heta}_2 + 2 l_1 l_2 c_2 \dot{ heta}_1 + l_1 l_2 c_2 \dot{ heta}_2)$$

$$rac{\partial L}{\partial heta_1} = (m_1 + m_2)gl_1c_1 + m_2gl_2c_{12} + m_3g(l_1c_1 + l_2c_{12})$$

动力学方程:

$$au_1 = rac{d}{dt} rac{\partial L}{\partial \dot{ heta}_1} - rac{\partial L}{\partial heta_1}$$

$$=(m_1+m_2+m_3)l_1^2\ddot{ heta}_1+(m_2+m_3)(l_2^2\ddot{ heta}_1+l_2^2\ddot{ heta}_1\dot{ heta}_2+l_2^2\dot{ heta}_1\ddot{ heta}_2+2l_1l_2c_2\ddot{ heta}_1+l_1l_2c_2\ddot{ heta}_2)$$

$$=2.8\ddot{ heta}_{1}+0.825(\ddot{ heta}_{1}\dot{ heta}_{2}+\dot{ heta}_{1}\ddot{ heta}_{2}+c_{2}\dot{ heta}_{2})+1.65c_{2}\dot{ heta}_{1}$$

$$au_2 = rac{d}{dt} rac{\partial L}{\partial \dot{ heta}_2} - rac{\partial L}{\partial heta_2}$$

$$au_3 = rac{d}{dt} rac{\partial L}{\partial \dot{ heta}_3} - rac{\partial L}{\partial heta_3}$$