智能机器人技术第二次作业

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1. 由图，

   $$\begin{bmatrix} v_x \\ v_y \end{bmatrix} = \begin{bmatrix} -[l_1 sin\theta_1 + l_2 sin(\theta_1+\theta_2)]\dot{\theta_1} - l_2 sin(\theta_1+\theta_2)\dot{\theta_2} \\ [l_1 cos\theta_1 + l_2 cos(\theta_1+\theta_2)]\dot{\theta_1} + l_2 cos(\theta_1+\theta_2)\dot{\theta_2} \end{bmatrix}$$

   将表4.1数据代入得：

   $$\begin{bmatrix} \dot{\theta_1} \\ \dot{\theta_2} \end{bmatrix}_1 = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$$

   $$\begin{bmatrix} \dot{\theta_1} \\ \dot{\theta_2} \end{bmatrix}_2 = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

   $$\begin{bmatrix} \dot{\theta_1} \\ \dot{\theta_2} \end{bmatrix}_3 = \begin{bmatrix} -4 \\ 2\sqrt{3}+6 \end{bmatrix}$$

2. 平衡状态时机械手作用力$F = \begin{bmatrix} 0 \\ mg \end{bmatrix}$

   各关节力矩$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = J^T F$

   机械手断点位置x, y与关节变量关系为：

   $$\begin{cases} x = l_1 cos\theta_1 + l2cos(\theta_1+\theta_2) + l_3 cos(\theta_1+\theta_2+\theta_3) = x(\theta_1,\theta_2,\theta_3) \\ y = l_1 sin\theta_1 + l_2 sin(\theta_1+\theta_2) + l_3 sin(\theta_1+\theta_2+\theta_3) = y(\theta_1,\theta_2,\theta_3) \end{cases}$$

   微分得：

   $$J = \begin{bmatrix} \frac{\partial x}{\partial\theta_1} & \frac{\partial x}{\partial\theta_2} & \frac{\partial x}{\partial\theta_3} \\ \frac{\partial y}{\partial\theta_1} & \frac{\partial y}{\partial\theta_2} & \frac{\partial y}{\partial\theta_3} \end{bmatrix}$$

   其中，

   $$\frac{\partial x}{\partial\theta_1} = -l_1 sin\theta_1 - l_2 sin(\theta_1+\theta_2) - l_3 sin(\theta_1+\theta_2+\theta_3)$$

   $$\frac{\partial x}{\partial\theta_2} = -l_2 sin(\theta_1+\theta_2) - l_3 sin(\theta_1+\theta_2+\theta_3)$$

   $$\frac{\partial x}{\partial\theta_3} = -l_3 sin(\theta_1+\theta_2+\theta_3)$$

   $$\frac{\partial y}{\partial\theta_1} = l_1 cos\theta_1 + l_2 cos(\theta_1+\theta_2) + l_3 cos(\theta_1+\theta_2+\theta_3)$$

   $$\frac{\partial y}{\partial\theta_2} = l_2 cos(\theta_1+\theta_2) + l_3 cos(\theta_1+\theta_2+\theta_3)$$

   $$\frac{\partial y}{\partial\theta_3} = l_3 cos(\theta_1+\theta_2+\theta_3)$$

   将关节变量代入得：

$$\tau = J^T F = \begin{bmatrix} -0.4\sqrt{3}-0.4 & 0.4+0.8 \\ -0.4 & 0.8 \\ -0.4 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 10g \end{bmatrix} = \begin{bmatrix} 12g \\ 8g \\ 0 \end{bmatrix}$$

3. 力矩

$\tau_1 = \frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}_1} - \frac{\partial L}{\partial \theta_1}$

$= (m_1p_1^2 + m_2p_2^2 + m_2l_1^2 + 2m_2l_1p_2c_2)\ddot{\theta_1} + (m_2p_2^2 + m_2l_1p_2c_2)\ddot{\theta_2} + (-2m_2l_1p_2s_2)\dot{\theta}_1\dot{\theta}_2$

$+ (-m_2l_1p_2s_2)\dot{\theta}_2^2 + (m_1p_1 + m_2l_1)gs_1 + m_2gp_2s_{12}$

$= D_{11}\ddot{\theta_1} + D_{12}\ddot{\theta}_2 + D_{112}\dot{\theta}_1\dot{\theta}_2 + D_{122}\dot{\theta}_2^2 + D_1$

由此得，

$$\begin{cases} D_{11} = m_1p_1^2 + m_2p_2^2 + m_2l_1^2 + 2m_2l_1p_2c_2 \\ D_{12} = m_2p_2^2 + m_2l_1p_2c_2 \\ D_{112} = -2m_2l_1p_2s_2 \\ D_{122} = -m_2l_1p_2s_2 \\ D_1 = (m_1p_1 + m_2l_1)gs_1 + m_2p_2gs_{12} \end{cases}$$

$\tau_2 = \frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}_2} - \frac{\partial L}{\partial \theta_2}$

$= (m_2p_2^2 + m_2l_1p_2c_2)\ddot{\theta}_1 + m_2p_2^2\ddot{\theta}_2 + (m_2l_1p_2s_2 - m_2l_1p_2s_2)\dot{\theta}_1\dot{\theta}_2 + (m_2l_1p_2s_2)\dot{\theta}_1^2 + m_2gp_2s_{12}$

$= D_{21}\ddot{\theta}_1 + D_{22}\ddot{\theta}_2 + D_{212}\dot{\theta}_1\dot{\theta}_2 + D_{211}\dot{\theta}_1^2 + D_2$

$$\begin{cases} D_{21} = m_2p_2^2 + m_2l_1p_2c_2 \\ D_{22} = m_2p_2^2 \\ D_{212} = -m_2l_1p_2s_2 + m_2l_1p_2s_2 = 0 \\ D_{211} = m_2l_1p_2s_2 \\ D_2 = m_2gp_2s_{12} \end{cases}$$

4. 连杆1动能：

$E_{k1} = \frac{1}{2}m_1l_1^2\theta_1^2$

连杆2质心位置：

$x_2 = l_1c_1 + l_2c_{12}$
$y_2 = l_1s_1 + l_2s_12$

质心速度平方：

$\dot{x}_2 = -l_1s_1\dot{\theta}_1 - l_2s_{12}(\dot{\theta}_1 + \dot{\theta}_2)$

$\dot{y}_2 = l_1c_1\dot{\theta}_1 + l_2c_{12}(\dot{\theta}_1 + \dot{\theta}_2)$

$\dot{x}_2^2 + \dot{y}_2^2 = l_1^2\dot{\theta}_1^2 + l_2^2(\dot{\theta}_1 + \dot{\theta}_2)^2 + 2(l_1l_2\dot{\theta}_1(\dot{\theta}_1 + \dot{\theta}_2))c_2$

动能：

$E_{k2} = \frac{1}{2}m_2l_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2l_2^2(\dot{\theta}_1 + \dot{\theta}_2)^2 + m_2l_1l_2\dot{\theta}_1(\dot{\theta}_1 + \dot{\theta}_2)c_2$

连杆3质心位置：

$\dot{x}_3 = \dot{x}_2$
$\dot{y}_3 = \dot{y}_2$

速度平方：

$\dot{x}_3^2 + \dot{y}_3^2 = \dot{x}_2^2 + \dot{y}_2^2$

动能：

$E_{k3} = \frac{1}{2}m_3l_1^2\dot{\theta}_1^2 + \frac{1}{2}m_3l_2^2(\dot{\theta}_1 + \dot{\theta}_2)^2 + m_3l_1l_2\dot{\theta}_1(\dot{\theta}_1 + \dot{\theta}_2)c_2$

总动能:

$E_k = \sum_{i=1}^{3} E_{ki}$

$= \frac{1}{2}(m_1 + m_2 + m_3)l_1^2\dot{\theta}_1^2 + (m_2 + m_3)(\frac{1}{2}l_2^2(\dot{\theta}_1 + \dot{\theta}_2)^2 + l_1l_2\dot{\theta}_1(\dot{\theta}_1 + \dot{\theta}_2)c_2)$

系统总势能:

$E_p = m_1gl_1s_1 + m_2g(l_1s_1 + l_2s_{12}) + m_3g(l_1s_1 + l_2s_{12})$

代入拉格朗日函数:

$L = E_k - E_p$

$\frac{\partial L}{\partial \dot{\theta}_1} = (m_1 + m_2 + m_3)l_1^2\dot{\theta}_1 + (m_2 + m_3)(l_2^2\dot{\theta}_1 + l_2^2\dot{\theta}_1\dot{\theta}_2 + 2l_1l_2c_2\dot{\theta}_1 + l_1l_2c_2\dot{\theta}_2)$

$\frac{\partial L}{\partial \theta_1} = (m_1 + m_2)gl_1c_1 + m_2gl_2c_{12} + m_3g(l_1c_1 + l_2c_{12})$

动力学方程:

$\tau_1 = \frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}_1} - \frac{\partial L}{\partial \theta_1}$

$= (m_1 + m_2 + m_3)l_1^2\ddot{\theta}_1 + (m_2 + m_3)(l_2^2\ddot{\theta}_1 + l_2^2\ddot{\theta}_1\dot{\theta}_2 + l_2^2\dot{\theta}_1\ddot{\theta}_2 + 2l_1l_2c_2\ddot{\theta}_1 + l_1l_2c_2\ddot{\theta}_2)$

$= 2.8\ddot{\theta}_1 + 0.825(\ddot{\theta}_1\dot{\theta}_2 + \dot{\theta}_1\ddot{\theta}_2 + c_2\dot{\theta}_2) + 1.65c_2\dot{\theta}_1$

$\tau_2 = \frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}_2} - \frac{\partial L}{\partial \theta_2}$

$\tau_3 = \frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}_3} - \frac{\partial L}{\partial \theta_3}$