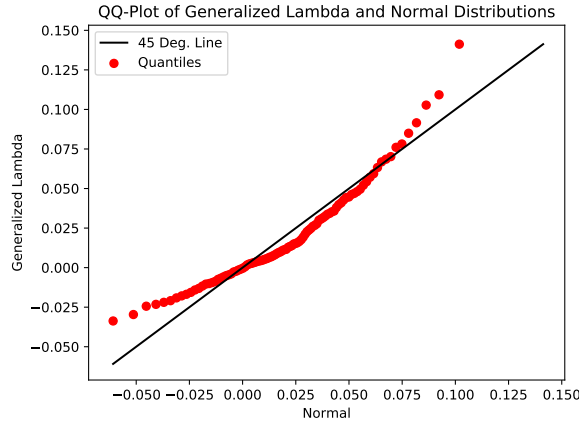


Problem Set 1
ECON 6413–Ng
Miguel Acosta
September 21, 2017

Q1. a. The sample moments and qq-plot are presented here:



Moment	Value
mean	0.0205
variance	0.0012
skewness	4.34
kurtosis	3.11
Bera-Jarque	1568.27
5% Critical Value	5.99

So, we can reject at the 1% level that the data are normal.

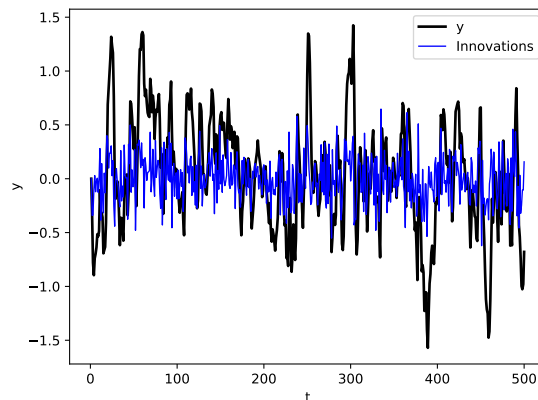
- b. Oh, math.
- c. Oh, thinking.
- d. I'll perform this estimation via GMM. For simplicity, I'll weight by the identity matrix, so that the objective function is just the sum of squared moments:

$$J_n(\mu, \sigma^2) = n\bar{g}(\mu, \sigma^2)' \mathbf{I} \bar{g}(\mu, \sigma^2)$$

The Hessian for even this simple formula gets a little out of control, so I take derivatives using finite differences. The estimates for my algorithm and Julia's LBFGS in the `Optim` package yield similar results:

	Miguel	Julia
μ	0.0205	0.0205
σ^2	0.0012	0.0012

Q2. i. Below is a picture of the simulated series, and the estimated parameters.



Moment	Value
α	0.8020
θ	0.5151
σ^2	0.0463

These estimates were similar regardless of starting value and minimizer (once I got rid of errors...). I again used Julia's LBFGS in the `Optim` package, with an analytic gradient. The objective function was just the sum of squared errors, assuming the first innovation was 0.

- ii. Maybe some math? and another computation I guess.
- iii. These results are more sensitive than others on this problem set, though the signs and magnitudes are the same across starting values and minimizers. For CLS, I use the same criterion as in part i above. Here are the estimates using two minimizers in Julia's `Optim` package, using starting values of $(0.5, 0.5)$

	Simulated Annealing	LBFGS; numerical derivative
α	0.9262	0.8992
θ	-0.2072	-0.0898
σ^2	0.0004	0.0004

For GMM, I use the following implied moments

$$g(\alpha, \theta) = \mathbb{E} \begin{bmatrix} e_t y_{t-1} \\ e_t e_{t-1} \end{bmatrix} = 0$$

where, recall,

$$y_t = \alpha y_{t-1} + e_t + \theta e_{t-1}.$$

I use optimal GMM (weighting by the inverse of the variance-covariance matrix attained from estimation using identity weighting), with my own Newton-Raphson algorithm, and Julia's LBFGS in the `Optim` package with numerical derivatives.

	Miguel	LBFGS
α	0.8953	0.8991
θ	-0.0776	-0.0900
σ^2	0.0004	0.0004