EC 6413-2017, Problem set 1
This version: September 7, 2017
Due: September 21, 2017

Throughout this class, you will be analyzing real and simulated data. Please set up R or Matlab (or both!) to import data from FRED using the QUANDL package.

Note: It is important to seed your random number generator or else you won't be able to replicate your work. Unless otherwise stated, please use 6413 as seed in all problem sets.

1 Week 1: Due 09-21-16

Q1. Simulate data: The generalized lambda distribution has inverse CDF (see Randles et al 1980, JASA, p.168-172):

$$F^{-1}(u) = \lambda_1 + \frac{u^{\lambda_3} - (1-u)^{\lambda_4)}}{\lambda_2}, \qquad 0 < u < 1,$$

where (λ_1, λ_2) are location/scale parameters, (λ_3, λ_4) are shape parameters that relate to skewness and kurtosis.

a Let $\lambda_1 = 0, \lambda_2 = -1, \lambda_3 = -0.0075, \lambda_4 = -0.03$. Use the inverse distribution function to generate $T = 500$ values of the random variable x_t. The population coefficient of skewness τ and kurtosis κ for these values of λ are 1.5 and 7.5. Compute the sample τ and κ. Use the Bera-Jarque statistic to test the null hypothesis that the data are normally distributed. Perform a normal *qq*-plot, ie. inverse transform of the normal distribution.

b Let $\theta = (\mu, \sigma^2)$ be the parameters of interest. Define

$$\overline{g}(\theta) = \frac{1}{n}\sum_{t=1}^{T}\begin{pmatrix} x_t - \mu \\ (x_t-\mu)^2 - \sigma^2 \\ (x_t-\mu)^3 \\ (x_t-\mu)^4 - 3\sigma^4. \end{pmatrix}$$

i What is the 4×2 matrix $G_0 = \text{plim } \frac{\partial \overline{g}}{\partial \theta}$ under f normality?

ii What is $S = Avar(\overline{g})$, the asymptotic variance of $\overline{g}$, under normality?

iii What is $Avar(\widehat{\theta}) = G_0^{-1} S G_0^{-1}$?

c Suggest an overidentifying J test for normality. What is the relation between the J test and the Bera-Jarque test?

d Estimate the parameters of the model for using your own Newton-Ralphson algorithm. Check your result with the solution from the optimizer in Matlab (or R).

Q2. Let $(\alpha, \theta) = (.8, .5)$. With $e_0, y_0 = 0$, simulate data as $y_t = \alpha y_{t-1} + e_t + \theta e_{t-1}$, where $e_t \sim N(0, \sigma^2)$, $\sigma^2 = 0.05$.

i Use non-linear least squares to estimate $(\alpha, \theta, \sigma^2)$ using the optimizer in Matlab or Julia.

ii Consider another ARMA(1,1) process $\widetilde{y}_t$ with parameters $(\alpha, \widetilde{\theta})$ and innovation $\widetilde{e}_t \sim N(0, \widetilde{\sigma}^2)$. What is $\widetilde{\sigma}^2$ that would yield an equivalent process as above? Verify by computing the autocovariances of y_t and $\widetilde{y}_t$.

iii Download monthly data CPIAUCSL and compute annual inflation. Fit an ARMA(1,1) model to inflation by (i) conditional least squares and (ii) by GMM.