

# Optimal Monetary Policy with Uncertain Fundamentals and Dispersed Information

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Advanced Macro with Jennifer La'o

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# Outline

Introduction

Model

Equilibrium

Analyzing Equilibrium: Special Cases

Optimal Monetary Policy

# Motivation

- ▶ Characterizing optimal monetary policy when aggregate fluctuations are driven fundamental and noise shocks

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- ▶ If central bank (CB) can observe shocks, response is simple
- ▶ Even if CB can achieve full information level of output, should it?

# Preview of Results

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- ▶ GE counterpart to Morris & Shin: Under optimal policy, precise public signal is always welfare enhancing
- ▶ Closed-form solutions

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- ▶ Households trade in complete state-contingent asset markets
- ▶ **Government:** taxes production and provides lump-sum subsidies

# Households (Consumers and Producers)

**Consumers** eat ( $C_{it}$ ), work ( $N_{it}$ ), and maximize

$$\mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{1-\gamma} C_{it}^{1-\gamma} - \frac{1}{1+\eta} N_{it}^{1+\eta} \right\} \right], C_{it} \equiv \left( \int_{j \in J_{it}} C_{ijt}^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}}$$

They also save and trade in asset markets.

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**Producers** (in logs) maximize profits by hiring, setting prices, and producing according to:

$$\begin{aligned} y_{it} &= a_{it} + n_{it} & a_{it} &= a_t + \epsilon_{it} \\ \int_0^1 \epsilon_{it} di &= 0 & a_t &= \rho a_{t-1} + \theta_t \end{aligned}$$

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**Signals:** HH observes  $a_{t-1}$ , and public and private signals, resp

$$x_{it} \equiv \theta_t + \epsilon_{it} \qquad s_t \equiv \theta_t + e_t$$

# Consumer and Producer Matching<sup>1</sup>

Consumer  $i$  assigned a sampling shock  $v_{it}$  that determines the (properties of their) basket, i.e.,

$$\epsilon_{jt} \sim N(v_{it}, \sigma_{\epsilon|v}^2), \forall j \in J_{it}$$

► More properties

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Useful parameter: heterogeneity in consumption baskets

$$\chi \equiv \frac{\sigma_v^2}{\sigma_{\epsilon}^2} \in [0, 1]$$

$\chi = 0$ : everybody buys representative.  $\chi = 1$ : identical productivity.

► More properties

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<sup>1</sup>Or, what's with  $J_{it}$ ?

# Timing, Financial Markets, and Trading

$(t, 0)$	$(t, I)$	$(t, II)$	$(t + 1, 0)$
Everybody observes $a_{t-1}$	Household $i$ observes	Household $i$ observes price	State-contingent
Central bank sets $R_t$ .	$s_t = \theta_t + e_t$	vector $\{P_{jt}\}_{j \in J_{it}}$ and	claims are settled
Agents trade state-	$x_i = \theta_t + \epsilon_{it}$	chooses consumption	
contingent claims	Sets price $P_{it}$	vector $\{C_{ijt}\}_{j \in J_{it}}$	

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Define the state  $\omega_{it} = (\epsilon_{it}, v_{it}, \theta_t, e_t)$  and state-contingent claim  $Z_{it+1}(\omega_{it})$  with price  $Q_t(\omega_{it})$ . HH balances at the CB,  $B_{it} \geq \underline{B}$  satisfy

$$B_{it+1} = R_t \left[ B_{it} - \int_{\mathbb{R}^4} Q_t(\tilde{\omega}_{it}) Z_{it+1}(\tilde{\omega}_{it}) d\tilde{\omega}_{it} \right. \\ \left. + (1 + \tau) P_{it} Y_{it} - \int_{J_{it}} P_{jt} C_{ijt} dj - T_t \right] + Z_{it+1}(\omega_{it})$$

Identical ex-ante  $\iff$  full insurance  $\iff B_{it} = 0$  in equilibrium.

# Monetary Policy and Government

The CB follows a backward-looking rule  $\mathcal{R}$

$$R_t = \mathcal{R}(h_t)$$

with  $P_t \equiv \exp\left(\int_0^1 P_{it} di\right)$ ,  $C_t \equiv \exp\left(\int_0^1 C_{it} di\right)$ , and

$$h_t = \{C_{t-i}, P_{t-i}, \theta_{t-i}, \epsilon_{t-i}\}_{i=1}^t$$

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$$h_t = \{C_{t-i}, P_{t-i}, \theta_{t-i}, \epsilon_{t-i}\}_{i=1}^t$$

and the government runs a balanced budget

$$T_t = \tau \int_0^1 P_{it} Y_{it} di$$

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$$\rightarrow \mathbb{E}p_{t+1} > p_t \rightarrow \mathbb{E}\pi_{t+1} \uparrow \rightarrow r_t \downarrow \rightarrow c_t \uparrow.$$

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# Equilibrium

- ▶ Households act optimally!
  - ▶ Choose assets, consumption, and set prices, given exogenous LOM for  $h_t$ , other HHs, monetary policy, financial prices, and information.
- ▶ Markets clear!
  - ▶ Goods market and asset markets claim for all  $h_t$
- ▶ Everything is consistent! ( $\mathcal{D}$ )

Separable isoelastic utility and normal shocks  $\Rightarrow$  linear solution

▶ More Precision

# Linear Solutions

$$p_{it} = \phi_a a_{t-1} + \phi_s s_t + \phi_x x_{it}$$

$$c_{it} = \psi_0 + \psi_a a_{t-1} + \psi_s s_t + \psi_x x_{it} + \psi_{\bar{x}} \bar{x}_{it}$$

$$\bar{x}_{it} \equiv \int_{J_{it}} x_{jt} dj = \theta_j + \nu_{it}$$

$$p_t = \phi_a a_{t-1} + \phi_{\theta} \theta_t + \phi_s e_t$$

$$c_t = \psi_0 + \psi_a a_{t-1} + \psi_{\theta} \theta_t + \psi_s e_t$$

$$\phi_{\theta} \equiv \phi_s + \phi_x$$

$$\psi_{\theta} \equiv \psi_s + \psi_x + \psi_{\bar{x}}$$



# Optimality Conditions

$$p_{it} = \kappa_p + E_{t,I}[\bar{p}_{it} + \gamma c_{it} + \eta n_{it}] - a_{it}$$

$$c_{it} = \kappa_c + E_{t,II}[c_{it+1}] - \gamma^{-1}(r_t - E_{t,II}[\bar{p}_{it+1}] + \bar{p}_{it})$$

For a given  $\phi_a$  the optimality conditions and linear policy rules pins down a unique equilibrium.

# Monetary Policy

Interest Rate rule:

$$r_t = \xi_0 + \xi_a a_{t-1} + \xi_p (p_{t-1} - \hat{p}_{t-1})$$
$$\hat{p}_t = \mu_a a_{t-1} + \mu_\theta \theta_t + \mu_e e_t$$

At beginning of period the monetary authority announces price level target function for current period and announces current period interest rate based on previous period's price level, price level target and productivity level

# Equilibrium

In any rational expectations equilibrium the three following equations hold:

$$\psi_a = \frac{1 + \eta}{\gamma + \eta} \rho$$

$$p_t = \hat{p}_t$$

$$r_t = \xi_0 - (\mu_a + \gamma\psi_a)(1 - \rho)a_{t-1}$$

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## Benchmark Case: Full Information

$s_t$  is a noiseless signal and equilibrium allocation is independent of monetary policy

$$c_t^{fi} = \psi_0 + \frac{1 + \eta}{\gamma + \eta} a_t$$

$$p_t = p_t + \gamma c_t + \eta(c_t - a_t) - a_t$$

Equilibrium allocations are independent of monetary policy

# Imperfect Information

Simplifying assumption:  $\gamma = 1$ ,  $\eta = 0$ ,  $\chi = 0$ ,  $\rho = 1$ .

$$\begin{aligned}c_{it} &= E_{t,II}[c_{it+1}] + E_{t,II}[p_{t+1}] - p_t \\E_{t,II}[p_{t+1}] &= \hat{p}_{t+1} = \mu_a E_{t,II}[a_t] \\c_t &= \psi_0 + (1 + \mu_a)a_t - p_t \\p_{it} &= (1 + \mu_a)E_{t,I}[a_t] - a_{it}\end{aligned}$$

# Imperfect Information

Equilibrium depends on  $\mu_a$

$$p_{it} = \mu_a a_{t-1} + \phi_s s_t + \phi_x x_{it}$$
$$c_t = \psi_0 + \psi_a a_t + \psi_\theta \theta_t + \psi_s e_t$$

$$\frac{\partial \psi_\theta}{\partial \mu_a} > 0, \quad \frac{\partial \psi_s}{\partial \mu_a} < 0, \quad \frac{\partial \phi_x}{\partial \mu_a} > 0, \quad \frac{\partial \phi_s}{\partial \mu_a} > 0$$

# Consumption Parameter Constraints

Monetary policy can only choose the relative importance of noise shocks and fundamental shocks in the consumption function.

$$\psi_{\theta}\sigma_{\theta}^2 + \psi_s\sigma_e^2 = \frac{1 + \eta}{\gamma + \eta}\sigma_{\theta}^2$$

Follows from the fact that price setters are required to expect consumption market to clear and first order conditions for optimal consumption.



# Full Stabilization

In particular there is an imperfect information equilibrium that matches full information equilibrium with the monetary policy rule  $\mu_a^{fi}$  and

$$c_t = \psi_0 + \psi_a a_{t-1} + \psi_\theta \theta_t + \psi_s e_t$$

$$\psi_\theta = \frac{1 + \eta}{\gamma + \eta}$$

$$\psi_s = 0$$

$$c_t = \psi_0 + \frac{1 + \eta}{\gamma + \eta} a_t = c_t^{fi}$$

$\psi_0$  can be made the same as in the full information case by the subsidy  $\tau$

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# Optimal Monetary Policy

Full stabiliztion is always possible but may not be possible.

1. More productive firms set lower prices
2. If monetary policy makes noise shocks more important then individual consumption is less tied to individual productivity
3. More productive households face higher expected MUC
4. In the price setting stage, productive firms will set even lower prices
5. Consumers will shift consumption towards more productive firms
6. Increases cross sectional efficiency

# Welfare

$$E \left[ \sum_t \beta^t U(C_{it}, N_{it}) \right] = \frac{1}{1-\gamma} W_0 \exp \left( (1-\gamma) \frac{1+\eta}{\gamma+\eta} w(\mu_a) \right)$$

$$\begin{aligned} w(\mu_a) = & -\frac{1}{2}(\gamma+\eta)E(c_t - c_t^{fi})^2 + \frac{1}{2}(1-\gamma) \int (c_{it} - c_t)^2 di \\ & - \frac{1}{2}(1+\eta) \int (n_{it} - n_t)^2 di + (c_t - a_t - n_t) \end{aligned}$$

# Welfare Analysis

Dispersion in consumption baskets leads to second order effects being non-negligible.

$$w(\mu_a) = -\frac{1}{2}(\gamma + \eta)E(c_t - c_t^f)^2 + \frac{1}{2}(1 - \gamma) \int (c_{it} - c_t)^2 di \\ - \frac{1}{2}(1 + \eta) \int (n_{it} - n_t)^2 di + (c_t - a_t - n_t)$$

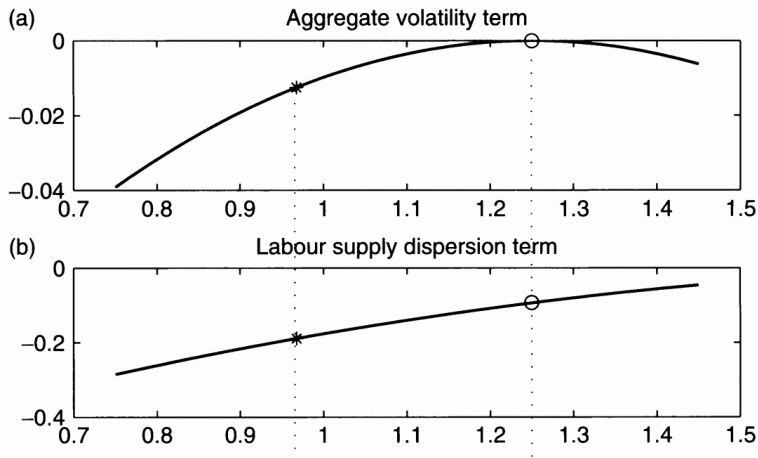
1. Aggregate volatility (-)
2. Cross sectional consumption dispersion (+/-)
3. Cross sectional labor supply dispersion (-)
4. Average production efficiency (+)

# Dispersion Effects On Productivity

Increased price dispersion leads to following effects:

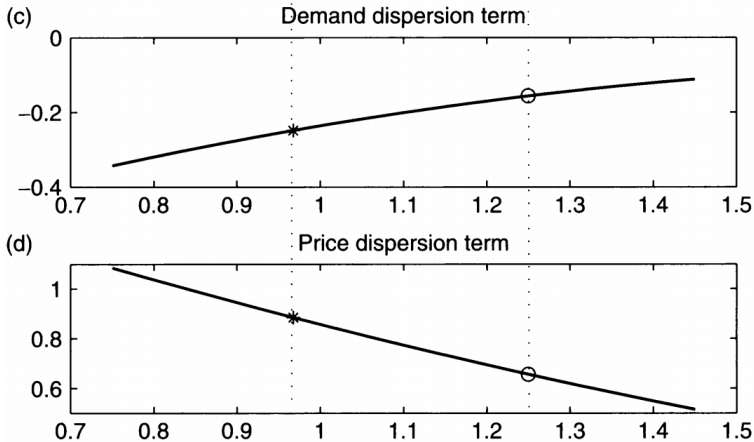
1. Higher price dispersion pushes consumers to purchase more goods from productive firms and fewer goods from unproductive firms, increasing aggregate productivity
2. On the on the production side, this leads to more volatility in log labor supply, leading to higher average labor supply

# Graphs



Note:  $\gamma = 1$

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# Summary

Model with aggregate and idiosyncratic productivity, noise shocks, and monetary authority with constrained information.

Introduces new friction which limits which firms consumers can buy from, introducing welfare trade offs for monetary authority.

Monetary authority can achieve constrained optimal allocation or full information allocation.

# APPENDIX

# More-Precise Equilibrium Characterization

A symmetric rational expectations equilibrium under the policy rule  $\mathcal{R}$  is given by the following functions, which together must satisfy optimality, market clearing, and consistency.

$$Z_{it+1}(\omega_{it}) = \mathcal{Z}(\omega_{it}, B_{it}, h_t)$$

$$P_{it} = \mathcal{P}(B_{it}, h_t, s_t, x_{it})$$

$$C_{it} = \mathcal{C}(B_{it}, h_t, s_t, x_{it}, \{P_{jt}\}_{j \in J_{it}})$$

$$Q_{it}(\omega_{it}) = \mathcal{Q}(\omega_{it}, h_t)$$

$$B_{it} \sim \mathcal{D}(\cdot \mid h_t)$$

► Less Precision

# More on Consumer and Producer Matching

Consumer  $i$  assigned a sampling shock  $v_{it}$  such that

$$\epsilon_{jt} \sim N(v_{it}, \sigma_{\epsilon|v}^2), \forall j \in J_{it}$$

Some properties

$$v_{it} \sim N(0, \sigma_v^2)$$

$$\int_0^1 v_{it} di = 0$$

$$\sigma_{\epsilon}^2 = \sigma_v^2 + \sigma_{\epsilon|v}^2$$

$$\sigma_v^2 \in [0, \sigma_{\epsilon}^2]$$

Heterogeneity in consumption baskets:  $\chi \equiv \frac{\sigma_v^2}{\sigma_{\epsilon}^2}$

► Fewer properties