Problem set 2: EC 6413-2017

**Q1: HAC** As in Hamilton (2008 `http://econweb.ucsd.edu/~jhamilto/JHamilton_Engle.pdf` ), generate GARCH errors as

$$\begin{aligned} y_t &= \beta_0 + \beta_1 y_{t-1} + u_t, \quad u_t = \sqrt{h_t} v_t, v_t \sim iid(0,1) \\ h_t &= \kappa + \alpha u_{t-1}^2 + \delta h_{t-1}, \quad h_0 = \kappa/(1-\alpha-\delta) \end{aligned}$$

with $\beta_0 = \beta_1 = 0$, $\kappa = 2$, $\alpha = 0.35$, $\delta = 0.6$, The case of normal errors obtains when $\alpha = \delta = 0$. We are interested in the $t$ statistic for testing $H_0 : \beta_1 = 0$ computed using OLS, White's, and HAC standard errors. Using 5000 replications and $T = 500$. Report the size of the test (Table 2 of Hamilton.)

For Q2 to Q3 below, the DGP is an ARMA(1,1): $y_t = \alpha y_{t-1} = e_t + \beta e_{t-1}$, $e_t \sim (0, \sigma^2)$. Simulate $T = 500$ of $y_t$ with $(\alpha, \theta, \sigma^2) = (.8, 0.5, 0.5)$ and seed 6413. Let $\beta = (\alpha, \theta, \sigma^2)$. Let $\overline{g} = \widehat{\psi} - \psi(\beta)$. The GMM estimator is $\widehat{\theta} = \text{argmin}_\theta J(\theta)$ where $J(\theta) = \overline{g}(\theta)' W \overline{g}(\theta)$.

**Q2. Covariance Structure Estimation**

i Express $\gamma(\theta) = (\gamma_0, \gamma_1, \gamma_2)'$ in terms of $\beta$ and $\sigma^2$ where $\gamma_j = E[(y_t - \mu)(y_{t-j} - \mu)]$..

ii Let $\widehat{\psi} = (\widehat{\gamma}_0, \widehat{\gamma}_1, \widehat{\gamma}_2)'$ be the sample autocovariances. Estimate $\beta$.

iii $\widehat{\psi} = (\widehat{\kappa}_2, \kappa_3, \widehat{\kappa}_4)$ where $\widehat{\kappa}_j$ is an estimate of $E[(y_t - \mu)^j)]$.

**Q3. MDE with AR(2) as Auxiliary Model**

i Consider the auxiliary model $y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + u_t$. Express $\phi$ in terms of $\gamma(\theta)$.

ii Estimate the AR(2) model by OLS. Tabulate the finite sample distribution of $\widehat{\phi}(1) = \widehat{\phi}_1 + \widehat{\phi}_2$.

iii Let $\widehat{\psi} = (\widehat{\phi}_1, \widehat{\phi}_2, \widehat{\sigma}_u^2)$. Let $\overline{g}(\beta) = \widehat{\psi} - \psi(\beta)$. Estimate $\beta$ by classical *minimum distance*.

**Q4. Simulation Estimation:** Attempt at least one of the following:

i For the ARMA(1,1) model considered in Q2 and Q3, tabulate the finite sample distribution $\widehat{\beta}$ obtained by SMM and Indirect Inference, using moments and auxiliary model of your choice.

ii Gourieroux-Phillips-Yu (2010, JOE 157:1 p.68-77) uses indirect inference to estimate the dynamic panel model $y_{it} = \alpha_i + \phi y_{it-1} + e_{it}$. Consider their monte carlo experiment in Section 4 with $\phi = 0.6$, $T = 5, N = 100$. Replicate their result for bias of $\widehat{\phi}$ estimated by ML (ie. LSDV) and Indirect inference.

iii Estimate the buffer stock savings model of Deaton (Econometrica 1991) for the case of iid shocks. Matlab code for solving and simulating the model will be provided if you attempt this question.