Problem Set 5
GR6413 [Ng]
Miguel Acosta
December 14, 2017

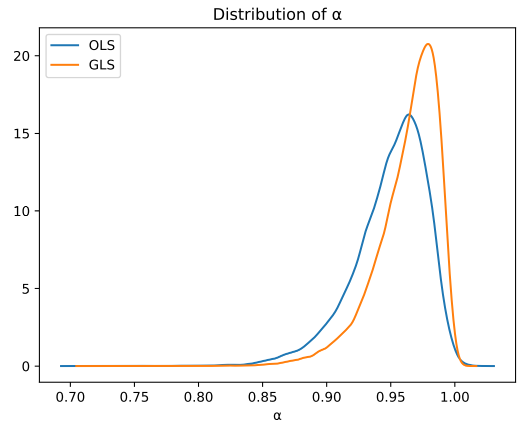
Q1. In the first panel, I plot the distribution of $\widehat{\alpha}$ (not sure what the normalization $T(\widehat{\alpha}-1)$ means). In the second panel, I plot the t-stats constructed as follows.

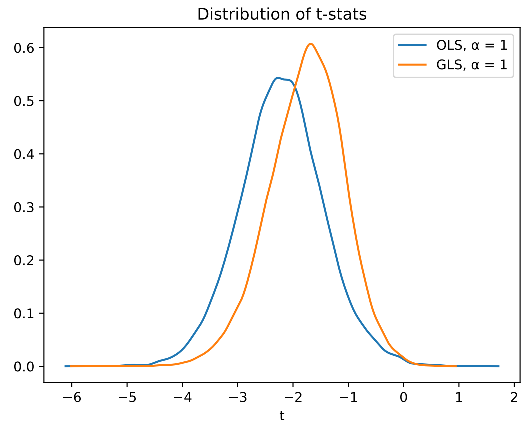
OLS: Estimate $y_t = a + \beta t + \alpha y_{t-1} + \text{error}_t$, then

$$t_{\text{OLS}} \equiv \frac{\widehat{\alpha}-1}{\sqrt{\frac{\sum_{t=1}^{T} y_t^2}{T}}}$$

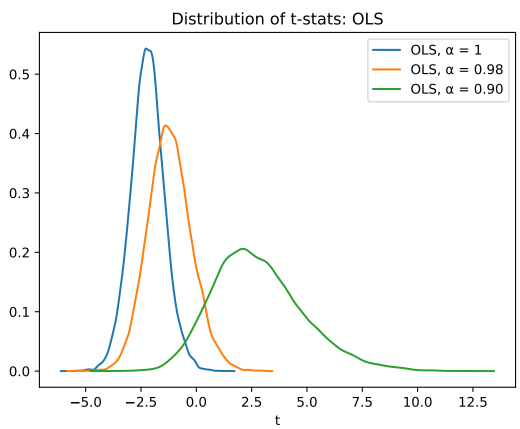
- GLS: Estimate $y_t^d = \alpha_G y_{t-1}^d + \text{error}_t$, then

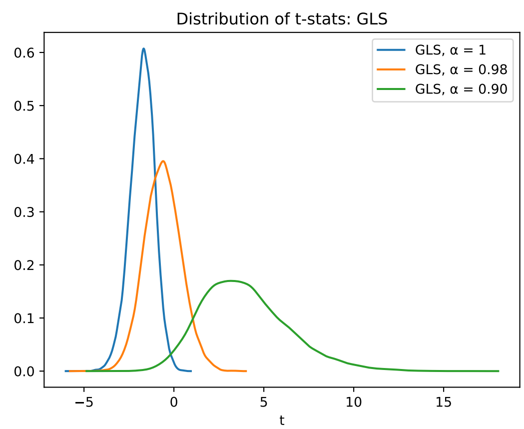
$$t_{\text{GLS}} \equiv \frac{\widehat{\alpha}_G-1}{\sqrt{\frac{\sum_{t=1}^{T} \left(y_t^d\right)^2}{T}}}$$





Here I plot the same t-stats as mentioned in the previous exercise, except I substitute 1 for 0.98 and 0.90.





i.

The critical values under the three nulls are as follows:

α	OLS	GLS
1.00	0.9458	0.707
0.98	0.1488	0.07692
0.90	0.3004	0.6324

ii. Using the same data as in the previous exercises, the equation I esimate with OLS is

$$y_t = a + b \cdot \mathbb{1}\{t > 75\} + c \cdot (t \times \mathbb{1}\{t > 75\}) + d \cdot t + y_{t-1} + \text{error}_t \tag{1}$$

The critical values under the three nulls are as follows:

α	OLS
1.00	1.553
0.98	0.6185
0.90	0.1014

iii. I perform the same estimation as in equation (1), except now the data are generated from a process with a trend shift. The critical values under the three nulls are as follows:

α	OLS
1.00	3.386
0.98	2.4
0.90	0.06858

Q2. Let's try to get a handle on what this question is asking. The variable x_t is an AR(1) for which we let the AR coefficient range from 0.8 to 1, and y_t is a noisy measurement of x_t:

$$x_t = \phi x_{t-1} + u_t$$
$$y_t = \beta x_t + e_t$$

As $\phi \to 1$, x_t becomes a random walk, so we're measuring a random walk with some noise. As ϕ moves away from 1, then we are measuring some type of scaled random walk and thus making the error e_t more important. By estimating

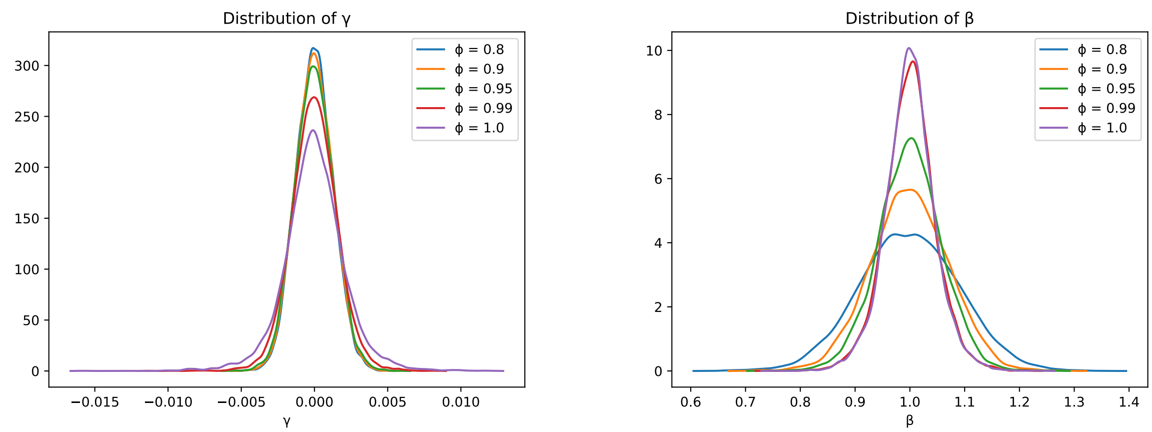
$$y_t = a + \gamma t + \beta x_t + \text{error}_t,$$

we are saying that the researcher is mistakenly confusing y_t as a variable with a trend. In what follows, I set $T = 200$ and performed 10,000 simulations for each value of ϕ.

i. The mean values of β and γ for each value of ϕ are

Mean Estimates		
ϕ	γ	β
0.8	$-7.796e-06$	0.9988
0.9	$-8.425e-06$	0.9993
0.95	$-9.593e-06$	0.9996
0.99	$-9.886e-06$	1
1.0	$-4.229e-06$	1

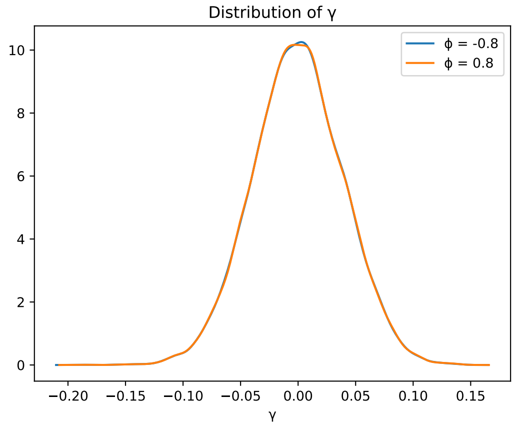
and the distributions are plotted below. As $\phi \to 1$, the mean estimate of $\beta \to 1$. The distribution also becomes much tigher around β. On the other hand, the estimates of γ become slightly less-precisely estimated about 0. So, as the process x_t becomes less and less like a random walk, it's more likely that we'll infer some type of trend, and the component of y_t coming from x_t will look less important (smaller β). Summarizing more-succinctly, the trend is more-imprecisely estimated as we get closer to a random walk.

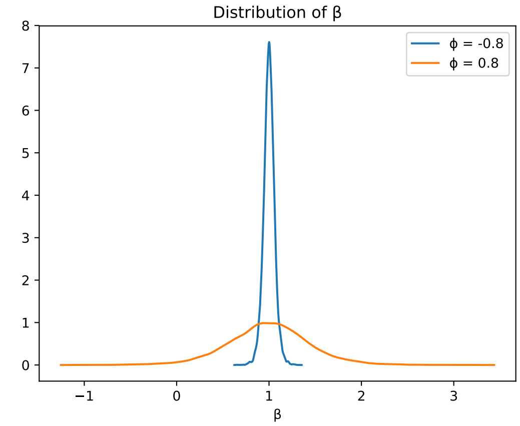


ii. Now, we let y_t be the sum of an AR(1) and a random walk, and study what happens as the AR(1) oscillates (negative AR coefficient) and doesn't.

Mean Estimates		
ϕ	γ	β
-0.8	-0.0002873	1
0.8	-0.000276	0.9987

In both cases, the trend component is estimated to be negative[1], and β is centered at about 1. But, when the AR oscillates, the estimates of β are much more precise. This is surprising to me, but maybe the point is that now that x_t is kind of crazy, OLS realizes that x_t is the primary driver of y_t.





[1]which I think makes sense... in the absence of shocks all but , an AR(1) will look like it has a negative trend.

Q3.

i. $y_{1t} - y_{1t-1} = \gamma y_{2t} - \gamma y_{2t-1} + u_{1t} - u_{1t-1}$

$\Rightarrow \Delta y_{1t} = \gamma \Delta y_{2t} + \Delta u_{1t}$

$= \gamma u_{2t} + (1-L) u_{1t}$

$$\begin{bmatrix} \Delta y_{1t} \\ \Delta y_{2t} \end{bmatrix} = \begin{bmatrix} (1-L) & \gamma \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$$

$\boxed{\text{So, the } MA(\infty) \text{ representation is } \Delta y_t = \Psi(L) u_t \text{ with } \Psi(L) \equiv \begin{bmatrix} 1-L & \gamma \\ 0 & 1 \end{bmatrix}}$

ii. $y_{1t} = \gamma y_{2t} + u_{1t}$

$y_{2t} = y_{2t-1} + u_{2t}$

$=$

$$y_t = \begin{bmatrix} 0 & \gamma \\ 0 & L \end{bmatrix} y_t + u_t$$

$$\Rightarrow u_t = \begin{bmatrix} 1 & -\gamma \\ 0 & 1-L \end{bmatrix} y_t$$

$\boxed{\text{So, the } AR(\infty) \text{ representation is } \Phi(L) y_t = u_t \text{ with } \Phi(L) \equiv \begin{bmatrix} 1 & -\gamma \\ 0 & 1-L \end{bmatrix}}$

iii $\Delta y_{1t} = \gamma \Delta y_{2t} + \Delta u_{1t} = \gamma u_{2t} + u_{1t} - (y_{1t-1} - \gamma y_{2t-1})$

$\Delta y_{2t} = u_{2t}$

$$\Rightarrow \Delta y_t = \begin{bmatrix} -1 & \gamma \\ 0 & 0 \end{bmatrix} y_t + \begin{bmatrix} 1 & \gamma \\ 0 & 1 \end{bmatrix} u_t$$

$$= \underbrace{\begin{bmatrix} -1 \\ 0 \end{bmatrix}}_{\beta} \underbrace{\begin{bmatrix} 1 & -\gamma \end{bmatrix}}_{\alpha'} y_t + \underbrace{\begin{bmatrix} 1 & \gamma \\ 0 & 1 \end{bmatrix} u_t}_{e_t}$$

$\boxed{\text{So, the VECM representation is } \Delta y_t = \beta \alpha' y_t + e_t \text{ with } \beta \equiv \begin{bmatrix} -1 \\ 0 \end{bmatrix}, \ \alpha \equiv \begin{bmatrix} 1 \\ -\gamma \end{bmatrix}, \ e_t \equiv \begin{bmatrix} 1 & \gamma \\ 0 & 1 \end{bmatrix} u_t}$