Optimal Monetary Plicy with Uncertain Fundamentals and Dispersed Information

Guido Lorenzoni

Miguel Acosta & Joe Saia

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Motivation

- Characterizing optimal monetary policy when aggregate fluctuations are driven fundamental and noise shocks
- If central bank (CB) can observe shocks, response is simple
- Even if CB can achieve full information level of output, should it?

Preview of Results

- Heterogeneous agents with private and public info; CB with public
- By responding to past realizations of shocks, CB can affect relative responses to shocks
- CB can replicate full-information output
  - This is non-optimal
  - Cross-sectional and aggregate efficiency
- GE counterpart to Morris & Shin: Under optimal policy, precise public signal is always welfare enhancing
- Closed-form solutions

Model Overview

- Continuum of households, each with a consumer and producer
  - **Consumers**: Differentiated goods baskets
  - **Producers**: Monopolistic competitors
- Local and aggregate productivity, each with shocks and signals
- **Monetary policy** sets interest rates R_t via a rule
- Households trade in complete state-contingent asset markets
- **Government**: taxes production and provides lump-sum subsidies

Households (Consumers and Producers)

Consumers eat (C_{it}), work (N_{it}), and maximize

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{1-\gamma} C_{it}^{1-\gamma} - \frac{1}{1+\eta} N_{it}^{1+\eta} \right\} \right], C_{it} \quad \equiv \left(\int_{j \in J_{it}} C_{ijt}^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}}$$

They also save and trade in asset markets.

Producers (in logs) maximize profits by hiring, setting prices, and producing according to:

$$y_{it} = a_{it} + n_{it} \qquad a_{it} = a_t + \epsilon_{it}$$

$$\int_0^1 \epsilon_{it} di = 0 \qquad a_t = \rho a_{t-1} + \theta_t$$

Signals: HH observes a_{t-1}, and public and private signals, resp

$$x_{it} \equiv \theta_t + \epsilon_{it} \qquad s_t \equiv \theta_t + e_t$$

Consumer and Producer Matching[1]

Consumer i assigned a sampling shock v_{it} that determines the (properties of their) basket, i.e.,

$$\epsilon_{jt} \sim N(v_{it}, \sigma^2_{\epsilon|v}), \forall j \in J_{it}$$

These shocks are mean-0 and sum to 0:

$$v_{it} \sim N(0, \sigma^2_v) \qquad \int_0^1 v_{it} di = 0$$

Useful parameter: heterogeneity in consumption baskets

$$\chi \equiv \frac{\sigma^2_v}{\sigma^2_\epsilon} \in [0, 1]$$

$\chi = 0$: everybody buys representative. $\chi = 1$: identical productivity.

▸ More properties

[1]Or, what's with J_{it}?

Timing, Financial Markets, and Trading

$(t,0)$	(t,I)	(t,II)	$(t+1,0)$
Everybody observes a_{t-1} Central bank sets R_t. Agents trade state-contingent claims	Household i observes $s_t = \theta_t + e_t$ $x_i = \theta_t + \epsilon_{it}$ Sets price P_{it}	Household i observes price vector $\{P_{jt}\}_{j \in J_{it}}$ and chooses consumption vector $\{C_{ijt}\}_{j \in J_{it}}$	State-contingent claims are settled

Define the state $\omega_{it} = (\epsilon_{it}, v_{it}, \theta_t, e_t)$ and state-contingent claim $Z_{it+1}(\omega_{it})$ with price $Q_t(\omega_{it})$. HH balances at the CB, $B_{it} \geq \underline{B}$ satisfy

$$B_{it+1} = R_t \Bigg[B_{it} - \int_{\mathbb{R}^4} Q_t(\tilde{\omega}_{it}) Z_{it+1}(\tilde{\omega}_{it}) d\tilde{\omega}_{it} \\ + (1+\tau) P_{it} Y_{it} - \int_{J_{it}} P_{jt} C_{ijt} dj - T_t \Bigg] + Z_{it+1}(\omega_{it})$$

Identical ex-ante $\iff$ full insurance $\iff$ $B_{it} = 0$ in equilibrium.

Monetary Policy and Government

The CB follows a backward-looking rule $\mathcal{R}$

$$R_t = \mathcal{R}(h_t)$$

with $P_t \equiv \exp\left(\int_0^1 P_{it} di\right)$, $C_t \equiv \exp\left(\int_0^1 C_{it} di\right)$, and

$$h_t = \{C_{t-i}, P_{t-i}, \theta_{t-i}, \epsilon_{t-i}\}_{i=1}^{t}$$

and the government runs a balanced budget

$$T_t = \tau \int_0^1 P_{it} Y_{it} di$$

Aside: How is this going to work?

A θ shock today pushes up both signals.

Producers will see higher x_{it} and s_t only.

Consumers will see higher x_{it} and the average x_{it} of goods in their basket.

So, if the CB commits to a bigger response to previous productivity,

then consumers have a pretty good idea that prices will be high tomorrow

and producers underestimate this.

$\rightarrow \mathbb{E}p_{t+1} > p_t \rightarrow \mathbb{E}\pi_{t+1} \uparrow \rightarrow r_t \downarrow \rightarrow c_t \uparrow$.

Equilibrium

- Households act optimally!
  - Choose assets, consumption, and set prices, given exogenous LOM for h_t, other HHs, monetary policy, financial prices, and information.
- Markets clear!
  - Goods market and asset markets claim for all h_t
- Everything is consistent! ($\mathcal{D}$)

Separable isoelastic utility and normal shocks $\Rightarrow$ linear solution

More Precision

Linear Solutions

$$
\begin{aligned}
p_{it} &= \phi_a a_{t-1} + \phi_s s_t + \phi_x x_{it} \\
c_{it} &= \psi_0 + \psi_a a_{t-1} + \psi_s s_t + \psi_x x_{it} + \psi_{\bar{x}} \bar{x}_{it} \\
\bar{x}_{it} &\equiv \int_{J_{it}} x_{jt} dj = \theta_j + \nu_{it} \\
p_t &= \phi_a a_{t-1} + \phi_\theta \theta_t + \phi_s e_t \\
c_t &= \psi_0 + \psi_a a_{t-1} + \psi_\theta \theta_t + \psi_s e_t \\
\phi_\theta &\equiv \phi_s + \phi_x \\
\psi_\theta &\equiv \psi_s + \psi_x + \psi_{\bar{x}}
\end{aligned}
$$

Optimality Conditions

$$p_{it} = \kappa_p + E_{t,I}[\bar{p}_{it} + \gamma c_{it} + \eta n_{it}] - a_{it}$$
$$c_{it} = \kappa_c + E_{t,II}[c_{it+1}] - \gamma^{-1}(r_t - E_{t,II}[\bar{p}_{it+1}] + \bar{p}_{it})$$

For a given ϕ_a the optimality conditions and linear policy rules pins down a unique equilibrium.

Monetary Policy

Interest Rate rule:

$$
\begin{aligned}
r_t &= \xi_0 + \xi_a a_{t-1} + \xi_p (p_{t-1} - \hat{p}_{t-1}) \\
\hat{p}_t &= \mu_a a_{t-1} + \mu_\theta \theta_t + \mu_e e_t
\end{aligned}
$$

At begining of period the monetary authority announces price level target function for current period and announces current period interest rate based on previous period's price level, price level target and productivity level

Equilibrium

In any rational expectations equilibrium the three following equations hold:

$$\begin{aligned}
\psi_a &= \frac{1+\eta}{\gamma+\eta}\rho \\
p_t &= \hat{p}_t \\
r_t &= \xi_0 - (\mu_a + \gamma\psi_a)(1-\rho)a_{t-1}
\end{aligned}$$

Benchmark Case: Full Information

s_t is a noiseless signal and equilibrium allocation is independent of monetary policy

$$c_t^{fi} = \psi_0 + \frac{1+\eta}{\gamma+\eta} a_t$$
$$p_t = p_t + \gamma c_t + \eta(c_t - a_t) - a_t$$

Equilibrium allocations are independent of monetary policy

Imperfect Information

Simplifying assumption: $\gamma = 1$, $\eta = 0$, $\chi = 0$, $\rho = 1$.

$$\begin{aligned} c_{it} &= E_{t,II}[c_{it+1}] + E_{t,II}[p_{t+1}] - p_t \\ E_{t,II}[p_{t+1}] &= \hat{p}_{t+1} = \mu_a E_{t,II}[a_t] \\ c_t &= \psi_0 + (1+\mu_a)a_t - p_t \\ p_{it} &= (1+\mu_a)E_{t,I}[a_t] - a_{it} \end{aligned}$$

Imperfect Information

Equilibrium depends on μ_a

$$p_{it} = \mu_a a_{t-1} + \phi_s s_t + \phi_x x_{it}$$
$$c_t = \psi_0 + \psi_a a_t + \psi_\theta \theta_t + \psi_s e_t$$

$$\frac{\partial \psi_\theta}{\partial \mu_a} > 0, \quad \frac{\partial \psi_s}{\partial \mu_a} < 0, \quad \frac{\partial \phi_x}{\partial \mu_a} > 0, \quad \frac{\partial \phi_s}{\partial \mu_a} > 0$$

Consumption Parameter Constraints

Monetary policy can only choose the relative importance of of noise shocks and foundemental shocks in the consumption function.

$$\psi_\theta \sigma_\theta^2 + \psi_s \sigma_e^2 = \frac{1+\eta}{\gamma+\eta}\sigma_\theta^2$$

Follows from the fact that price setters are required to expect consumption market to clear and first order condtions for optimal consumption.

Full Stabilization

In particular there is an imperfect information equilibrium that matches full information equilibrium with the monetary policy rule μ_a^{fi} and

$$
\begin{aligned}
c_t &= \psi_0 + \psi_a a_{t-1} + \psi_\theta \theta_t + \psi_s e_t \\
\psi_\theta &= \frac{1+\eta}{\gamma+\eta} \\
\psi_s &= 0 \\
c_t &= \psi_0 + \frac{1+\eta}{\gamma+\eta} a_t = c_t^{fi}
\end{aligned}
$$

ψ_0 can be made the same as in the full information case by the subsidy τ

Optimal Monetary Policy

Full stabilitzation is always possible but may not be possible.

1. More productive firms set lower prices
2. If monetary policy makes noise shocks more important then individual consumption is less tied to individual productivity
3. More productive households face higher expected MUC
4. In the price setting stage, productive firms will set even lower prices
5. Consumers will shift consumption towards more productive firms
6. Increases cross sectional efficiency

Welfare

$$E\left[\sum_t \beta^t U(C_{it}, N_{it})\right] = \frac{1}{1-\gamma} W_0 \exp\left((1-\gamma)\frac{1+\eta}{\gamma+\eta} w(\mu_a)\right)$$

$$\begin{aligned} w(\mu_a) = &-\frac{1}{2}(\gamma+\eta)E(c_t - c_t^{fi})^2 + \frac{1}{2}(1-\gamma)\int (c_{it} - c_t)^2 di \\ &- \frac{1}{2}(1+\eta)\int (n_{it} - n_t)^2 di + (c_t - a_t - n_t) \end{aligned}$$

Welfare Analysis

Dispersion in consumption baskets leads to second order effects being non-negligble.

$$w(\mu_a) = -\frac{1}{2}(\gamma+\eta)E(c_t - c_t^{fi})^2 + \frac{1}{2}(1-\gamma)\int (c_{it} - c_t)^2 di$$
$$-\frac{1}{2}(1+\eta)\int (n_{it} - n_t)^2 di + (c_t - a_t - n_t)$$

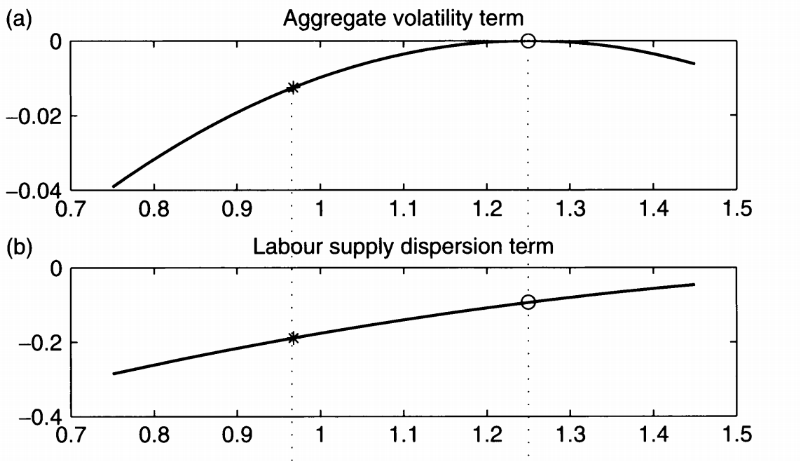
1. Aggregate volatility (-)
2. Cross sectional consumption dispersion (+/-)
3. Cross sectional labor supply dispersion (-)
4. Average production efficiency (+)

Dispersion Effects On Productivity

Increased price dispersion leads to following effects:

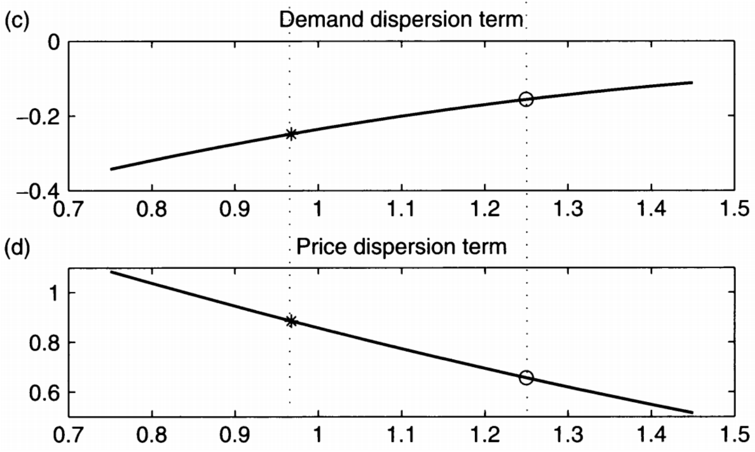
1. Higher price dispersion pushes consumers to purchase more goods from productive firms and fewer goods from unproductive firms, increasing aggregate productivity
2. On the on the production side, this leads to more volatility in log labor supply, leading to higher average labor supply

Graphs



Note: $\gamma = 1$

Graphs



Note: $\gamma = 1$

Summary

Model with aggregate and indiosyncratic productivity, noise shocks, and monetary authority with constrained information.
Introduces new friction which limits which firms consumers can buy from, introducing welfare trade offs for monetary authority.
Monetary authority can achieve constrained optimal allocation or full information allocation.

APPENDIX

More-Precise Equilibrium Characterization

A symmetric rational expectations equilibrium under the policy rule $\mathcal{R}$ is given by the following functions, which together must satisfy optimality, market clearing, and consistency.

$$
\begin{aligned}
Z_{it+1}(\omega_{it}) &= \mathcal{Z}(\omega_{it}, B_{it}, h_t) \\
P_{it} &= \mathcal{P}(B_{it}, h_t, s_t, x_{it}) \\
C_{it} &= \mathcal{C}\left(B_{it}, h_t, s_t, x_{it}, \{P_{jt}\}_{j \in J_{it}}\right) \\
Q_{it}(\omega_{it}) &= \mathcal{Q}(\omega_{it}, h_t) \\
B_{it} &\sim \mathcal{D}(\cdot \mid h_t)
\end{aligned}
$$

▸ Less Precision

More on Consumer and Producer Matching

Consumer i assigned a sampling shock v_{it} such that

$$\epsilon_{jt} \sim N(v_{it}, \sigma^2_{\epsilon|v}), \forall j \in J_{it}$$

Some properties

$$v_{it} \sim N(0, \sigma^2_v) \qquad \int_0^1 v_{it} di = 0$$

$$\sigma^2_\epsilon = \sigma^2_v + \sigma^2_{\epsilon|v} \qquad \sigma^2_v \in [0, \sigma^2_\epsilon]$$

Heterogeneity in consumption baskets: $\chi \equiv \frac{\sigma^2_v}{\sigma^2_\epsilon}$

Fewer properties