1 Problem set 4: Due 11-27-2017

Q1

i (conjugate poisson) Suppose that y_t $(t = 1, \dots, T)$ is a random sample from a Poisson distribution with mean θ. Suppose also that the prior distribution of θ is the Gamma distribution $G(\underline{\alpha}, \underline{\beta})$ where $\underline{\alpha} > 0, \underline{\beta} > 0$. Find the posterior distribution of θ.

ii (conjugate Bernoulli) We have T iid random variables y_t with density

$$p(y_t|\theta) = \begin{cases} \theta & y_t = 1 \\ 1-\theta & y_t = 0. \end{cases} \quad 0 < \theta < 1.$$

Suppose that the prior for θ is the beta distribution with hyperprameters $\underline{\alpha}, \underline{\delta}$ ie.

$$p(\theta) = \left[\frac{\Gamma(\underline{\alpha})\Gamma(\underline{\delta})}{\Gamma(\underline{\alpha}+\underline{\delta})}\right]^{-1} \theta^{\underline{\alpha}-1}(1-\theta)^{\underline{\delta}-1}.$$

Find the posterior distribution of θ.

iii Natural conjugate priors have the feature that prior information can be viewed as fictitious sample information in that it is combined with the sample in exactly the same way that additional sample information would be combined. The only difference is that the prior information is 'observed' in the mind of the researcher, not in the real world. Use the setup in (ii) to show that $\underline{\alpha}, \underline{\delta}$ can be interpreted as the information in a sample of size $\underline{T} = \underline{\alpha} + \underline{\delta} - 2$ with $\underline{\alpha} - 1$ successes from the Bernoulli process of interest.

Q2. Plotting likelihoods

i Let $n = 50, \beta = .9, \tau = 1/\sigma^2 = 1$. Draw $x_i \sim U[10, 20]$ and let $y_i = x_i\beta + e_i$, $e_i \sim N(0, \sigma^2)$. Plot the likelihood of β.

ii Let $y_1, \dots y_n$ be iid conditional on θ (probability of success). For $0 \le \theta \le 1$, $y \in [0, 1]$, the probability for each random variable is $p(y|\theta) = \theta^y(1-\theta)^{1-y}$. Let $s = \sum_{i=1}^n y_i$ be the total number of success in n trials. When s is replaced by a particular realization, the likelihood is $\ell(\theta; n, s) = \binom{n}{s} \theta^s(1-\theta)^{n-s}, \quad 0 \le \theta \le 1$. For $\theta = .2$ and $.5$, let $y \sim \text{binord}(n, \theta)$. Plot the likelihood for θ with $n = 100$. Now let $\theta = .5$, and $s = 7$ and let n be the parameter of interest. Plot the likelihood for n.

iii Consider again the Bernoulli trial example above with $\ell(\theta; n, s) \propto \theta^s(1-\theta)^{n-s}$. The conjugate prior is $p(\theta) \propto \theta^{a-1}(1-\theta)^{b-1}$. Suppose $n = 5, s = 2$. Plot the posterior distribution for (i) $a = b = 1$, (ii) $a = b = 3$, and (iii) Jeffrey's prior.

Q3. Accept-Reject/Gibbs Sampling

i Generate 200 draws of $N(0, 1)$ variates using the accept-reject method using the double exponential density $g(x) = (\alpha/2)\exp(-\alpha|x|)$.

ii Let $x \geq \underline{\mu}$ be a truncated normal. Consider (a) brute-force drawing from $N(\mu, \sigma^2)$ and keeping those draws exceeding $\underline{\mu}$; (b) Let $u \sim U[0,1]$ and make draws using classical inversion method (of the cdf) ;(c) using accept -reject method with $g(x, \underline{\mu}) = \alpha \exp^{-\alpha(z-\underline{\mu})} I_{z \geq \underline{\mu}}$.

iii Let $\theta = (\theta_1, \theta_2)$, $\theta \sim N(\mu, \Sigma)$ with support $[\underline{\theta}_i, \bar{\theta}_i]$, $i = 1, 2$. For parameters of your choice, construct a Markov chain that draws from $f(\theta_1|\theta_2)$ and $f(\theta_2|\theta_1)$ subject to the boundary constraints. (Hint: use properties of conditional multivariate normal distribution).

Q4. Consider the probit model $y_i^* = \beta_0 + \beta_1 x_i + e_i$, $y_i = 1$ if $y_i^* > 0$. Download probit.dat for data on (y_i, x_i). (i) Find the posterior mean and standard deviation of β by Gibbs sampling. (ii) What is the probability that $y_i = 1$ if $x_i = 2$?

Q5. Generate data of size $n = 100$ observations for the normal linear regression model with an intercept (set to 0) and one regressor $x \sim U[0,1]$ with $\beta = 1$.

i find the posterior mean and standard deviation using a Normal-Gamma prior $NG(\underline{\beta}, \underline{V}, \underline{s}^{-2}, \underline{\nu})$ with $\underline{\beta} = (0,1)'$, $\underline{V} = I_2$, $\underline{s}^{-2} = 1$, $\underline{\nu} = 1$. Graph the posterior for the slope parameter.

ii Repeat (i) using a flat prior.

Q6: QBE Replicate Table 1 of Chernozhukov and Hong (2003).

Optional This question is based on the An-Schorfheide (2007) model, also in Herbst and Schorfheide (2014), hereafter HS. I can send you some files if you want to try (you should!).

i Solve the model defined by (2.1) on p.12 of HS using the method taught in your macro class (ie. hxx, gxx package).

ii Put the solution in state space form and use the prediction error decomposition to build up the likelihood.

iii Use the posterior means reported in Table 4.1 (p.53) as true value to replicate the impulse response to a monetary policy shock in Figure 4.3 (p.54).

iv Use the prior reported in Table 2.2 (p.19) and the random-walk metropolis algorithm. Replicate the estimates reported in Table 4.1.

v Repeat (iv) but with HP filtered data (with $\lambda = 1600$) instead of first differencing. Are the estimates robust?

Schorfheide's website has some matlab code for this model. This code is very convoluted even for a *very* skilled programmer. It will be faster to start clean.

Note: there are many ways to solve the model and many ways to write the likelihood.

i US data for estimation: columns 1 to 3 of nkmp-data.txt (columns 4-6 are Euro data). The model is coded in nkmp-model.m (this one is from an-schorfheide)

ii The nkmp-get-num-deriv will write out the linearized model and save it to a nkmp-num-eval.m file. You only need the symbolic toolbox once. This is my hack of how to use the gxx-hxx.

iii Now the next step is put this linearized model in state space form. So use the hxx, gxx program to get all the fxp, fpy, etc.

iv build the likelihood from this state space form IN A WAY CONSISTENT WITH THE WAY YOU WRITE THE LIKELIHOOD.

The hxx-gxx produces policy functions $z_{t+1} = C + Tz_t + Re_t$, $x_{0t} = hxx(x_{0t-1}, z_t)$ and $x_{1t} = gxx(x_{0t-1}, z_t)$. Let $s_t = (x'_{1t}, x'_{0t}, z'_t)'$. You need to put this ins state space form. I use Harvey's state space formulation. $y_t = DD + ZZs_t + HH\eta_t$ and $s_t = CC + TTs_{t-1} + RRe_t$

v file for prior is nkmp-prior-us.txt. I find it easier to use the prior file for simulated data, nkmp-simdatap-prior.txt. There are so many columns because at least two parameters are fixed. The additional columns are indicators for this.