Time Series Econometrics Glossary

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ARMA: Stationarity and Invertibility (For notation) $\alpha(L) = 1 - \alpha_1 L - \cdots - \alpha_p L^p$, $\theta(L) = 1 + \phi_1 L + \cdots + \phi_q L^q$. An ARMA$(p,q) = \alpha(L)y_t = \theta(L)u_t$.

- AR(p) is stationary if the roots of the characteristic polynomial $z_p - \alpha_1 z^{p-1} - \cdots - \alpha_p = 0$ are inside the unit circle; this can also be written as the roots of $1 - \alpha_1 L - \cdots - \alpha_p L^p = 0$ are outside the unit circle.[1]
- MA(q) are always stationary.
- AR(p) are always invertible.
- MA(q) is invertible if the roots of $\theta(z) = 0$ are inside the unit circle, or the roots of $\theta(L) = 0$ are outside.

Ergodicity

- **Stationary Ergodic** A process $\{y_t\}$ is stationary ergodic the process is "asymptotically independent" of itself. That is, for any functions—$f(y_t, \ldots, y_{t+k})$ and $g(y_{t+T}, \ldots, y_{t+T+\ell})$—which are functions of two subsamples, they become independent as the subsamples grow apart; that is, $\mathbb{E}[f \cdot g] = \mathbb{E}[f]\,\mathbb{E}[g]$ as $T \to \infty$. As an example, y_t is stationary ergodic if $\gamma_k \to 0$ as $k \to \infty$.
- **Ergodic for the Mean** The time average converges to the ensemble average as $T \to \infty$

Impulse Response Function $\frac{\partial\, E[y_{t+k}|e_t \mathcal{F}_{t-1}]}{\partial\, e_t}$ where e_t are the innovations in an MA representation.

Long- and Short-Run Variance The short-run variance for a series is γ_0. The long-run variance is

$$\lim_{T\to\infty} T \operatorname{var}(\overline{y}) = \gamma_0 + 2\sum_{k=1}^{\infty} \gamma_k \equiv \omega^2$$

which is the quantity used for the LLN for time series; $\sqrt{T}\,(\overline{y} - \mu) \sim N(0, \omega^2)$.

Observational Equivalence Two processes with the same mean, variance, and autocovariances are second order observationally equivalent. Arises with e.g. MA(1) processes in which there are two possible DGPs that are observationally equivalent, so we restrict ourselves to cases with MA parameter (absolutely) less than 1.

Martingale Difference Sequence The mean of u_t cannot be forecasted; higher moments may be. Technically, $\mathbb{E}(u_t \mid \mathcal{F}_{t-1}) = 0$.

Stationarity A random variable y_t is covariance stationary if the first two unconditional moments (mean, variance, autocovariances) do not depend on t. Strict stationarity means that the distribution is constant over time—this implies that all moments and functions of y_t are independent of t.

Wold Decomposition Any covariance stationary process can be represented as the sum of two mutually uncorrelated processes, where one is a linear forecast (deterministic) and the other is stochastic mean-zero forecast error. This implies that the process has a unique MA(∞) representation.

[1] Divide by L^p...

White Noise $\mathbb{E}(u_t) = 0$ and $\mathbb{E}(u_t u_{t+j}) = 0 \forall j \neq 0$; this means that u is linearly unpredictable—lags of u cannot help predict u. (Independent White Noise means that $u_t \sim$ iid $(0, \sigma^2)$.