Problem Set 1
GR6493 [Dean]
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Question 1 For those of us that may have forgotten what NIAS and NIAC are, define:

$$\left[P_A(a \mid \omega) \equiv \sum_{\gamma \in \Gamma(A)} \pi_A(\gamma \mid \omega) C_A(a \mid \gamma)\right] \quad \left[g(\gamma, A) \equiv \max_{a \in A} \sum_{\omega \in \Omega} \gamma(\omega) u(a, \omega)\right] \quad \left[G(\pi, A) \equiv \sum_{\gamma \in \Gamma(\pi)} P(\gamma) g(\gamma, A)\right]$$

NIAS For every chosen action a,

$$\sum \mu(\omega) P_A(a \mid \omega)[u(a(\omega)) - u(b(\omega))] \geq 0, \ \forall b \in A \tag{1}$$

NIAC For an observed sequence of decision problems $A_1, \ldots, A_K$, and associated revealed information stuctures $\overline{\pi}^1, \ldots, \overline{\pi}^K$,

$$G(A^1, \overline{\pi}^1) - G(A^1, \overline{\pi}^2) + \cdots + G(A^K, \overline{\pi}^K) - G(A^K, \overline{\pi}^1) \geq 0.$$

We claim that the data generated by Gabaix's model does not necessarily satisfy the NIAS condition. We will demonstrate this using a counterexample. The counterexample we present is a one-dimensional version of the quadratic utility example presented in section II.A of Gabaix's paper.

Let a be the agent's choice of action, x the true unobserved state, and m the agent's attention strategy. Fix any choice of $m \in (0, 1)$ (i.e., the agent is not paying full attention). Let the agent's prior belief regarding x given by $\mu(x)$. By assumption, the mean and variance of this distribution exist and are given by 0 and σ^2, respectively. Define quadratic utilities as

$$u(a, x) = -\frac{1}{2}(a - cx)^2$$

where $c > 0$ is some given constant. Define $x^s \equiv mx$According to Gabaix's model, the agent chooses

$$a = arg \max_a u(a, x^s) = cx^s = cmx.$$

Define $b \equiv cx$. Refering back to our utility function, we know $u(b, x) = 0$ for all x. Now, fix any given realization of the state x. We know that

$$u(a, \omega) = -\frac{1}{2}(cmx - cx)^2 < 0 = u(b, \omega)$$

The above inequality holds strictly since we assumed $m \in (0, 1)$. So, for any prior μ, we have

$$\int_{-\infty}^{\infty} \mu(x)[u(a(\omega) - u(b(\omega))]dx = \int_{-\infty}^{\infty} \mu(x) - \frac{1}{2}(cmx - cx)^2 dx < 0$$

Thus, we conclude that NIAS fails.

Question 2

1. First, let's establish notation. Each decision problem $D \in \{8, 9, 10, 11\}$ in this experiment has two possible actions—denote these by a_D and b_D (so, for $D = 9$, we have $a_D = 12$ and $b_D = 13$). Let the typical state be denoted by $\omega \in \Omega \equiv \{1, 2, 5, 6\}$.

   The NIAS condition (1) for this experiment requires the following conditions to hold for all D

$$\sum_{\omega\in\Omega} P(a_D \mid \omega)[u(a_D(\omega)) - u(b_D(\omega))] \geq 0 \tag{2a}$$
$$\sum_{\omega\in\Omega} P(b_D \mid \omega)[u(b_D(\omega)) - u(a_D(\omega))] \geq 0 \tag{2b}$$

Since there are only two possible actions in each decision problem, we have

$$P(a_D \mid \omega) + P(b_D \mid \omega) = 1, \forall \omega \in \Omega.$$

Using this, and the definition $\Delta_\omega \equiv u(a_D(\omega)) - u(b_D(\omega))$ allows (2) to be written as

$$\sum_{\omega\in\Omega} P(a_D \mid \omega)\Delta_\omega \geq 0$$
$$\sum_{\omega\in\Omega} (1 - P(a_D \mid \omega))(-\Delta_\omega) \geq 0,$$

i.e.,

$$\sum_{\omega\in\Omega} P(a_D \mid \omega)\Delta_\omega \geq 0$$
$$\sum_{\omega\in\Omega} P(a_D \mid \omega)\Delta_\omega \geq \sum_{\omega\in\Omega} \Delta_\omega,$$

which combine to form the (still necessary and sufficient) condition for NIAS:

$$\sum_{\omega\in\Omega} P(a_D \mid \omega)\Delta_\omega \geq \max\left\{\sum_{\omega\in\Omega} \Delta_\omega, 0\right\} = 0$$

where the last equality follows from the symmetry of the specific payoffs of this experiment.

I have two ideas for testing.

i. Test this condition for each person. Then, take the mean and standard deviation of those probabilities to do a standard t-test (or whatever... mean/sd)

ii. Boostrapping. Drop observations that aren't state omega. Then sample (with replacement) from this pool and calculate the condition. Then use the s.d. of those bootstrapped probabilities to do a t-test.

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Question 3

1. For concreteness, the payoffs are

$$\begin{array}{c|cc} & I & H \\ \hline d & -c_I & -c_H \\ n & -s & 0 \end{array} \quad \leq \quad \begin{array}{c|cc} & I & H \\ \hline d & -\bar{c}_I & -\bar{c}_H \\ n & -s & 0 \end{array}$$

with $\bar{c}_I < c_I < s$ and $c_H > \max\{0, \bar{c}_H\}$.

NIAS In this example, the following conditions constittue NIAS:

$$\sum_{\omega\in\{I,H\}} \mu_\omega P(d \mid \omega)[u(d(\omega)) - u(n(\omega))] \geq 0$$
$$\sum_{\omega\in\{I,H\}} \mu_\omega P(n \mid \omega)[u(n(\omega)) - u(d(\omega))] \geq 0$$

i.e.,

$$\mu_I P(d \mid I)[-c_I + s] + (1-\mu_I)P(d \mid H)[-c_H] \geq 0 \tag{3a}$$
$$\mu_I P(n \mid I)[-s + c_I] + (1-\mu_I)P(n \mid H)[c_H] \geq 0. \tag{3b}$$

Replacing $P(n \mid \omega)$ with $1 - P(d \mid \omega)$ in (3b)

$$\mu_I(1 - P(d \mid I))[-s + c_I] + (1-\mu_I)(1 - P(d \mid H))[c_H] \geq 0$$

i.e.,

$$\mu_I P(d \mid I)[-c_I + s] + (1-\mu_I)P(d \mid H)[-c_H] \geq \mu_I(s - c_I) - (1-\mu_I)c_H$$

Which combines with (3a) to form a single NIAS condition

$$\mu_I P(d \mid I)[-c_I + s] + (1-\mu_I)P(d \mid H)[-c_H] \geq \max\{\mu_I(s - c_I) - (1-\mu_I)c_H, 0\} \tag{4}$$

Let's examine the RHS:

$$\mu_I(s - c_I) - (1-\mu_I)c_H > 0 \iff \frac{\mu_I}{1-\mu_I} > \frac{c_H}{s - c_I} \tag{5}$$

When this is true, then condition (4) becomes

$$\begin{aligned}
&\mu_I P(d \mid I)[-c_I + s] + (1-\mu_I)P(d \mid H)[-c_H] \geq \mu_I(s - c_I) - (1-\mu_I)c_H \\
\iff &\frac{\mu_I}{1-\mu_I}P(d \mid I) - P(d \mid H)\frac{c_H}{s - c_I} \geq \frac{\mu_I}{1-\mu_I} - \frac{c_H}{s - c_I} > 0 \\
\implies &\frac{P(d \mid I)}{P(d \mid H)} > \frac{c_H}{s - c_I}\left[\frac{\mu_I}{1-\mu_I}\right]^{-1} \qquad (6)
\end{aligned}$$

Otherwise, when (5) is reversed, then (4) becomes

$$\begin{aligned}
&\mu_I P(d \mid I)[-c_I + s] + (1-\mu_I)P(d \mid H)[-c_H] \geq 0 \\
\iff &\frac{\mu_I}{1-\mu_I}P(d \mid I) - P(d \mid H)\frac{c_H}{s - c_I} \geq 0 \\
\iff &\frac{P(d \mid I)}{P(d \mid H)} \geq \frac{c_H}{s - c_I}\left[\frac{\mu_I}{1-\mu_I}\right]^{-1} > 1 \\
\implies &P(d \mid I) \geq P(d \mid H)
\end{aligned}$$

Which seems sensible. Not sure what to make of (6). These conditions are the same under the subsidy, just replacing the non-barred variables by their barred-counterparts.

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