3.1 (NIAC)

$A_1: \{n, d\} \quad c_I, c_H \qquad A_2: \{n, d\} \quad \bar{c}_I, \bar{c}_H \qquad G(A_1, \pi^1) - G(A^1, \pi^2) + G(A^2, \pi^2) - G(A^2, \pi^1) \geq 0$

One condition to check since there is only one problem.

$\bar{c}_* < c_*$

$$[P_N(d|I) - P_S(d|I)]\{(-c_I + s) - (-\bar{c}_I + s)\} + [P_N(n|H) - P_S(n|H)]\{c_H - \bar{c}_H\} \geq 0$$

$$\Rightarrow [P_N(d|I) - P_S(d|I)]\{\bar{c}_I - c_I\} + [P_N(n|H) - P_S(n|H)]\{c_H - \bar{c}_H\} \geq 0$$

$$\Rightarrow [P_N(n|H) - P_S(n|H)]\{c_H - \bar{c}_H\} \geq [P_N(d|I) - P_S(d|I)]\{c_I - \bar{c}_I\}$$

true positives go up by less than true negatives

$$[1 - P_N(d|H) - (1 - P_S(d|H))]\{c_H - \bar{c}_H\} \geq [P_N(d|I) - P_S(d|I)]\{c_I - \bar{c}_I\}$$

$$\underbrace{[P_S(d|H) - P_N(d|H)]}_{\Delta \text{ in false positives}} \underbrace{\{c_H - \bar{c}_H\}}_{\substack{\text{decrease in } c_H \\ \equiv \\ \text{size of subsidy} \\ (>0)}} \geq \underbrace{[P_N(d|I) - P_S(d|I)]}_{\substack{\Delta \text{ in correct commitments} \\ (\text{probably} < 0)}} \underbrace{\{c_I - \bar{c}_I\}}_{\substack{\text{decrease in } c_I \\ \equiv \\ \text{size of subsidy} \\ (>0)}}$$

$\Updownarrow$

decrease in correct commitments

better interp

$$\underbrace{[P_S(d|I) - P_N(d|I)]}_{\Delta \text{ in correct commitments}}\{c_I - \bar{c}_I\} \geq \underbrace{[P_N(d|H) - P_S(d|H)]}_{\Delta \text{ in false positives}}\{c_H - \bar{c}_H\}$$

$\Delta \geq \Delta$ $\frac{8}{4}$

Theorem #3.2

P is consistent with rational inattention with mutual information costs **if and only if**

$$\sum_{\omega}\left[\frac{\mu(\omega)z(a,\omega)}{\sum_{c\in A}P(c)z(c,\omega)}\right] \le 1 \text{ all } a\in A \qquad (1)$$

$$\sum_{\omega}\left[\frac{\mu(\omega)z(a,\omega)}{\sum_{c\in A}P(c)z(c,\omega)}\right] = 1 \text{ all } a \text{ s.t. } P(a)>0 \qquad (2)$$

and

$$P(a|\omega)=\frac{P(a)z(a,\omega)}{\sum_{c\in A}P(c)z(c,\omega)} \qquad (3)$$

$z(a,\omega) = \exp\left\{\frac{u(a,\omega)}{\lambda}\right\}$

	I	H
d	$-c_I$	$-c_H$
n	$-s$	0

false positives go up by more than false negatives

(2)

(1)(n) $\dfrac{\mu(I)\exp(-s/\lambda)}{P(n)\exp(-s/\lambda)+P(d)\exp(-c_I/\lambda)} + \dfrac{\mu(H)\exp(0/\lambda)}{P(n)\exp(-0/\lambda)+P(d)\exp(-c_H/\lambda)} \le 1$

(1)(d) $\dfrac{\mu(I)\exp(-c_I/\lambda)}{P(n)\exp(-s/\lambda)+P(d)\exp(-c_I/\lambda)} + \dfrac{\mu(H)\exp(\frac{-c_H}{\lambda})}{P(n)\exp(-0/\lambda)+P(d)\exp(-c_H/\lambda)} \le 1$

(1)(n) holds w/ equality if $P(n)>0$

(2)(d) " $P(d)>0$

$$\boxed{\mu(I)\exp(-c_I/\lambda)+\mu(H)\exp(-c_H/\lambda) = \mu(I)\exp(-s/\lambda)+\mu(H)}$$

(:) $\Rightarrow \dfrac{\mu(I)}{\mu(H)}\exp(-c_I/\lambda)+\exp(-c_H/\lambda) =$

(3) $P(n|I) = \dfrac{P(n)\exp(-s/\lambda)}{P(n)\exp(-s/\lambda)+P(d)\exp(-c_I/\lambda)}$ (˙o˙) $\qquad P(n|H) = \dfrac{P(n)\exp(0/\lambda)}{P(n)\exp(-0/\lambda)+P(d)\exp(-c_H/\lambda)}$

$P(d|I) = \dfrac{P(d)\exp(-c_I/\lambda)}{P(n)\exp(-s/\lambda)+P(d)\exp(-c_I/\lambda)} \qquad P(d|H) = \dfrac{P(d)\exp(\frac{-c_H}{\lambda})}{P(n)\exp(-0/\lambda)+P(d)\exp(-c_H/\lambda)}$

Idea: Just consider cases:

1. $P(d)>0$, $P(n)=0$
2. ~~$P(d)>0$, $P(n)>0$~~
3. $P(d)=0$, $P(n)=0$
4. $P(d)=0$, $P(n)>0$

$P(d)=0 \Rightarrow P(d|H)=0 \Rightarrow P(n|H)=1$, $P(n|I)=1$ $\Rightarrow P(n)=1$

from (:) $\Rightarrow 1 = \dfrac{P(n)}{P(n)}$

from (˙o˙) $1 = \dfrac{P(n)\exp(-s/\lambda)}{P(n)\exp(-s/\lambda)} = 1$

way to go, Miguel...

(1)(n) holds w/ equality:

$1 = \dfrac{\mu(I)\exp(-s/\lambda)}{P(n)\exp(-s/\lambda)+\cancel{P(d)\exp(-c_I/\lambda)}} + \dfrac{\mu(H)\cancel{\exp(0/\lambda)}}{P(n)\cancel{\exp(-0/\lambda)}+\cancel{P(d)\exp(-c_H/\lambda)}}$

$1 = \dfrac{\mu(I)}{P(n)} + \dfrac{\mu(H)}{P(n)} \Rightarrow P(n) = \mu(I)+\mu(H)$
$\qquad\qquad\qquad\qquad\qquad \| $
$\qquad\qquad\qquad\qquad\qquad 1$

(1)(d)

$\dfrac{\mu(I)\exp(-c_I/\lambda)}{P(n)\exp(-s/\lambda)+\cancel{P(d)\exp(-c_I/\lambda)}} + \dfrac{\mu(H)\exp(\frac{-c_H}{\lambda})}{P(n)\cancel{\exp(-0/\lambda)}+\cancel{P(d)\exp(-c_H/\lambda)}} \le 1$

$\Rightarrow 1 \ge \dfrac{\mu(I)}{P(n)}\exp\left\{\dfrac{-c_I+s}{\lambda}\right\} + \dfrac{\mu(H)}{P(n)}\exp(-c_H/\lambda)$

$\Rightarrow 1 \ge \left(1-\dfrac{\mu(H)}{P(n)}\right)\exp\left\{\dfrac{-c_I+s}{\lambda}\right\} + \dfrac{\mu(H)}{P(n)}\exp(-c_H/\lambda)$

$1-\exp\left\{\dfrac{-c_I+s}{\lambda}\right\} \ge \dfrac{\mu(H)}{P(n)}\left(\exp\left\{-\dfrac{c_H}{\lambda}\right\} - \exp\left\{\dfrac{-c_I+s}{\lambda}\right\}\right)$

$= \mu(H)\left(\exp\left\{-\dfrac{c_H}{\lambda}\right\} - \exp\left\{\dfrac{-c_I+s}{\lambda}\right\}\right)$

if $P(d)=P(n)=0$, then nothing is defined... so, no predictions...

$P(n)=0 \qquad [P(n|I)=P(n|H)=0]$ (3)

$P(d|I)=1$, $P(d|H)=1$
$\Rightarrow P(d)=1$

(1)(d)

$\dfrac{\mu(I)\exp(-c_I/\lambda)}{P(n)\exp(-s/\lambda)+P(d)\exp(-c_I/\lambda)} + \dfrac{\mu(H)\exp(\frac{-c_H}{\lambda})}{P(n)\exp(-0/\lambda)+P(d)\exp(-c_H/\lambda)} = 1$

$\Rightarrow \dfrac{\mu(I)\exp(-c_I/\lambda)}{\exp(-c_I/\lambda)} + \dfrac{\mu(H)\cdots}{\cdots} = 1 \quad \Rightarrow \mu(I)+\mu(H)=1$ ... ok.

(1)(n)

$\dfrac{\mu(I)\exp(-s/\lambda)}{\cancel{P(n)\exp(-s/\lambda)}+P(d)\exp(-c_I/\lambda)} + \dfrac{\mu(H)\cancel{\exp(0/\lambda)}}{\cancel{P(n)\exp(-0/\lambda)}+P(d)\exp(-c_H/\lambda)} \le 1$

$\dfrac{\mu(I)\exp(-s/\lambda)}{\exp(-c_I/\lambda)} + \dfrac{\mu(H)}{\exp(-c_H/\lambda)} \le 1$

$$(1-\mu(H))\exp\left(\frac{c_I - s}{\lambda}\right) + \mu(H)\exp(-c_H/\lambda) \leq 1$$

$$1 - \exp\left(\frac{c_I - s}{\lambda}\right) \geq \mu(H)\left\{\exp\left(-\frac{c_H}{\lambda}\right) - \exp\left(\frac{c_I - s}{\lambda}\right)\right\}$$

Final case $p(d) > 0$, $p(n) > 0$

$$\frac{\mu(I)\exp(-s/\lambda)}{p(n)\exp(-s/\lambda) + p(d)\exp(-c_I/\lambda)} + \frac{\mu(H)\exp(0/\lambda)}{p(n)\exp(-0/\lambda) + p(d)\exp(-c_H/\lambda)} = 1 \qquad (\star_1)$$

$$\frac{\mu(I)\exp(-c_I/\lambda)}{p(n)\exp(-s/\lambda) + p(d)\exp(-c_I/\lambda)} + \frac{\mu(H)\exp\left(\frac{-c_H}{\lambda}\right)}{p(n)\exp(-0/\lambda) + p(d)\exp(-c_H/\lambda)} = 1 \qquad (\star_2)$$

nothing really simplifies...

$\Big\downarrow$ get rid of $\exp(0)$

$$\frac{\mu(I)\exp(-s/\lambda)}{p(n)\exp(-s/\lambda) + p(d)\exp(-c_I/\lambda)} + \frac{\mu(H)}{p(n) + p(d)\exp(-c_H/\lambda)} = 1$$

$$\frac{\mu(I)\exp(-c_I/\lambda)}{p(n)\exp(-s/\lambda) + p(d)\exp(-c_I/\lambda)} + \frac{\mu(H)\exp\left(\frac{-c_H}{\lambda}\right)}{p(n) + p(d)\exp(-c_H/\lambda)} = 1$$

$\Big\downarrow$ replace $\mu(I) = 1 - \mu(H)$

$$\frac{(1-\mu(H))\exp(-s/\lambda)}{p(n)\exp(-s/\lambda) + p(d)\exp(-c_I/\lambda)} + \frac{\mu(H)}{p(n) + p(d)\exp(-c_H/\lambda)} = 1$$

$$\frac{(1-\mu(H))\exp(-c_I/\lambda)}{p(n)\exp(-s/\lambda) + p(d)\exp(-c_I/\lambda)} + \frac{\mu(H)\exp\left(\frac{-c_H}{\lambda}\right)}{p(n) + p(d)\exp(-c_H/\lambda)} = 1$$

(subtract)

$$0 = \frac{(1-\mu(H))(\exp(-s/\lambda) - \exp(-c_I/\lambda))}{p(n)\exp(-s/\lambda) + p(d)\exp(-c_I/\lambda)} + \frac{\mu(H)(1 - \exp(-c_H/\lambda))}{p(n) + p(d)\exp(-c_H/\lambda)}$$

$$\Rightarrow \frac{(1-\mu(H))}{\mu(H)}\left[\frac{\exp(-s/\lambda) - \exp(-c_I/\lambda)}{p(n)\exp(-s/\lambda) + p(d)\exp(-c_I/\lambda)}\right] = \frac{\exp(-c_H/\lambda) - 1}{p(n) + p(d)\exp(-c_H/\lambda)}$$

$$\log\left(\frac{1-\mu(H)}{\mu(H)}\right) +$$

$$\left[\mu_I - P(d)\right]\frac{z(d,I)}{z(n,I)} = \left[1 - P(d) + \mu_I z(d,H)\right]$$

$$\exp\left[(-c_I + r)/\lambda\right] > 1$$

$$\left(\mu_I z(d,I) - \mu_I z(n,I)\right)\left[P(d)\, z(d,H) + (1 - P(d))\right]$$

$$= (1 - \mu_I)\left(1 - z(d,H)\right)\left[P(d)\, z(d,I) + (1 - P(d))\, z(n,I)\right]$$

$$\mu_I z(d,I) P(d) z(d,H) + \mu_I z(d,I)(1 - P(d)) - \mu_I z(n,I) P(d) z(d,H)$$

$$- \mu_I z(n,I)(1 - P(d)) =$$

$$= P(d) z(d,I) + (1 - P(d)) z(n,I) - \mu_I P(d) z(d,I) - \mu_I (1 - P(d)) z(n,I)$$

$$+ \mu_I z(d,H) P(d) z(d,I) + \mu_I z(d,H)(1 - P(d)) z(n,I)$$

$$\mu_I z(d,I) = P(d) z(d,I) + z(n,I) - P(d) z(n,I)$$

$$+ \mu_I z(d,H) z(n,I)$$