$$y_t = \alpha y_{t-1} + \theta e_{t-1} + e_t \qquad (\text{mean } 0!)$$

$$\gamma_1 = \mathbb{E}\left[y_t y_{t-1} \right]$$

$$= \mathbb{E}\left(\alpha y_{t-1}^2 + \theta e_{t-1} y_{t-1} + e_t y_{t-1} \right)$$

$$= \alpha \mathbb{E}\left(y_{t-1}^2 \right) + \theta\, \mathbb{E}\left(e_{t-1} \left(\alpha y_{t-2} + \theta e_{t-2} + e_{t-1} \right) \right)$$

$$= \alpha \gamma_0 + \theta \sigma^2$$

$$\gamma_0 = \mathbb{E}(y_t y_t) = \mathbb{E}\left(y_t \left(\alpha y_{t-1} + \theta e_{t-1} + e_t \right) \right)$$

$$= \alpha \gamma_1 + \theta\, \mathbb{E}\left[\left(\alpha y_{t-1} + \theta e_{t-1} + e_t \right) e_{t-1} \right] + \mathbb{E}\left[e_t \left(\alpha y_{t-1} + \theta e_{t-1} + e_t \right) \right]$$

$$= \alpha \gamma_1 + \theta \alpha \sigma^2 + \theta^2 \sigma^2 + \sigma^2$$

$$\gamma_0 = \alpha \gamma_1 + \sigma^2 \left(\theta \alpha + \theta^2 + 1 \right)$$

$$\gamma_0 = \alpha \left(\alpha \gamma_0 + \theta \sigma^2 \right) + \sigma^2 \left(\theta \alpha + \theta^2 + 1 \right)$$

$$\Rightarrow \gamma_0 \left(1 - \alpha^2 \right) = \alpha \theta \sigma^2 + \sigma^2 \left(\theta \alpha + \theta^2 + 1 \right)$$

$$\Rightarrow \boxed{\gamma_0 = \frac{\sigma^2 \left(2\alpha\theta + \theta^2 + 1 \right)}{1 - \alpha^2}} \qquad \boxed{\gamma_1 = \frac{\sigma^2}{1-\alpha^2} \left[\alpha^2 \theta + \alpha \theta^2 + \alpha + \theta \right]}$$

$$\gamma_1 = \frac{\alpha \sigma^2 \left(2\alpha\theta + \theta^2 + 1 \right)}{1-\alpha^2} + \frac{\theta \sigma^2 \left(1 - \alpha^2 \right)}{1-\alpha^2} = \frac{\sigma^2}{1-\alpha^2} \left[2\alpha^2\theta + \alpha\theta^2 + \alpha + \theta - \alpha^2\theta \right]$$