A Sparsity-Based Model of Bounded Rationality

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Outline

Overview

- Want a simple model of bounded rationality that has the following properties:
  - Tractable to compute so that we can use it to understand the consequences of bounded rationality for economic problems
  - Has reasonable and intuitive foundations
- Consider a “sparse agent” who solves builds a simplified model of the world where only variables of first-order importance are considered
- Use this agent to get reconsider classic microeconomic results and see if the results are robust to sparse maximization

Psychological Underpinnings

- Basic idea is that we only consider a limited number of variables in decision making
- Limited attention
  - There are an extraordinary amount of possible variables to consider for any given decision problem, but psychological research shows us that we can only handle and consider so many variables at once
- Reliance on defaults
  - There is some literature in behavioral economics talking about the importance of defaults (remember our discussion of consideration sets), which are captured by a particular parametrization of this model
- Anchoring and Adjustment
  - This model captures the idea that we anchor on the default and partially (or fully) adjust towards the truth

Model Setup

- Want to define the "sparse max" operator as $smax_a u(a,x)$ s.t. $b(a,x) \geq 0$
- x is the (potentially large) vector of variables the agent can possibly consider and a is an action
- Define m as the attention vector and the perceived representation of x_i as $x_i^s = m_i x_i$
- Define κ as being the psychic cost the agent has to pay for paying attention to a variable (with $\kappa = 0$ we simply have rational choice)
- Define $a_{x_i} = \frac{\partial a}{\partial x_i} = -u_{aa}^{-1} u_{ax_i}$ as how much a change x_i should change the action for the traditional agent
- Define $\Lambda_{ij} = \sigma_{ij} a_{x_i} u_{aa} a_{x_j}$ as the cost of inattention factors

Sparse Max - Definition

- An agent solving maximization using *smax* first picks the optimal attention vector and then given this attention vector picks the optimal action
- First,
  $m^* = argmin_{m \in [0,1]^n} \frac{1}{2} \sum_{i,j=1 \ldots n} (1 - m_i)\Lambda_{ij}(1 - m_j) + \kappa \sum_{i=1 \ldots n} m_i^\alpha$
- Then, Given m^* and using that to define x_i^s here, choose the optimal action
  $a^s = argmax_a u(a, x^s)$
- We can think of the first-stage as being a tradeoff between the first term in the sum which we can view as a proxy for utility losses and a psychological penalty.

Example Usage

- Consider a case with just one variable, $x_1 = x$. Then attention function is given by $\mathcal{A}(\sigma^2, \alpha) = inf[argmin_m 0.5(m-1)^2\sigma^2 + |m|^\alpha]$
- Consider three cost functions
  - Fixed - $\mathcal{A}(\sigma^2, 0) = 1_{\sigma^2 \geq 2}$
  - Linear - $\mathcal{A}(\sigma^2, 1) = max(1 - \frac{1}{\sigma^2}, 0)$
  - Quadratic - $\mathcal{A}(\sigma^2, 2) = \frac{\sigma^2}{2+\sigma^2}$

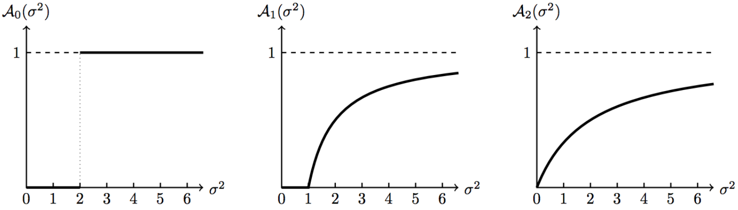

Figure 1: Three attention functions $\mathcal{A}_0, \mathcal{A}_1, \mathcal{A}_2$, corresponding to fixed cost, linear cost and quadratic cost respectively. We see that $\mathcal{A}_0$ and $\mathcal{A}_1$ induce sparsity – i.e. a range where attention is exactly 0. $\mathcal{A}_1$ and $\mathcal{A}_2$ induce a continuous reaction function. $\mathcal{A}_1$ alone induces sparsity and continuity.

Constraints?

- Consider the traditional problem of maximizing a utility function subject to a budget constraint.
- In this model, we still enforce that the budget constraint binds (even though the agent only cares about perceived prices and not actual prices) by having the agent pick some $\lambda \in \mathbb{R}$ to scale the perceived price vector so that the constraint binds.
- Under these assumptions, we have that at the optimal $\frac{\partial u / \partial c_1}{\partial u / \partial c_2} = \frac{p_1^s}{p_2^s}$ and the argument for this is the same as in traditional theory

Fiscal Illusion Application

- Consider the problem of whether to impose a wage tax that is paid by either the employee or employer.
- In economic theory (and with the traditional rational agent), it should not matter on who it is imposed since only the total tax will matter
- However, a sparse agent may pay more (or less) attention to the part paid by the employee than the employer. Then, the agent strictly prefers the tax to be paid by the employee (or employer).
- Offers a way to model an agent who sees direct effects more easily than indirect effects

Why?

- So we've built a model, but who needs it?
- Main advantages over other bounded rationaliy models
  - Sparse max has deterministic predictions (in contract to noisy signal models)
  - Sparse max predictions are continuous as a function of parameters (in contrast to models with fixed cost of attention)
  - Sparse max is applicable in many settings
- Can we obtain the same results of the model with existing models of inattention, particularly the Sims formulation we learned in class?
  - This model is tractable and thus we can easily apply it to understand its implications to standard microeconomic theory.
  - No one has been able to work out the basic consumption problem with the Sims model and modeling is very complex

Robustness of Basic Microeconomics (?)

- Money Illusion
  - Traditional theory: there is no money illusion, when the budget and prices are increased by some fixed percentage then nothing should change
  - Sparse theory: Since the agent may pay more attention to certain prices relative to others, a fixed percentage increase will cause the agent to consume less of goods with a salient price
- Slutsky matrix (each element is the compensated change in consumption of c_i as price p_j changes)
  - Traditional theory: Symmetric
  - Sparse theory: Here $S^s_{ij} = S^r_{ij} m_j$ so the consumer reacts to just a fraction m_j of the price change thus the matrix ends up being asymmetric
- Competitive equilibrium allocation
  - Traditional Theory: Independent of the price level
  - Sparse theory: Different aggregate price levels lead to materially different equilibrium allocations again due to differing levels of attention

What is actually robust?

- Efficiency of competitive equilibrium?
  - Robust if it happens at the default price level, but away from the default price competitive equilibrium is inefficient unless people have the same misperceptions
- The signs predicted under by rational choice are robust under the sparsity model
- This exercise shows that we can actually utilize the sparsity model developed to analyze basic problems in economics and understand the implications it has in these contexts (thus arguably it is tractable)
- What does all of this tell us?