Problem Set 5
GR6413 [Ng]
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Q1. The variable y_t is a random walk.

i. In the first panel, I plot the distribution of $\widehat{\alpha}$ (not sure what the normalization $T(\widehat{\alpha}-1)$ is supposed to show). In the second panel, I plot the t-stats constructed as follows.

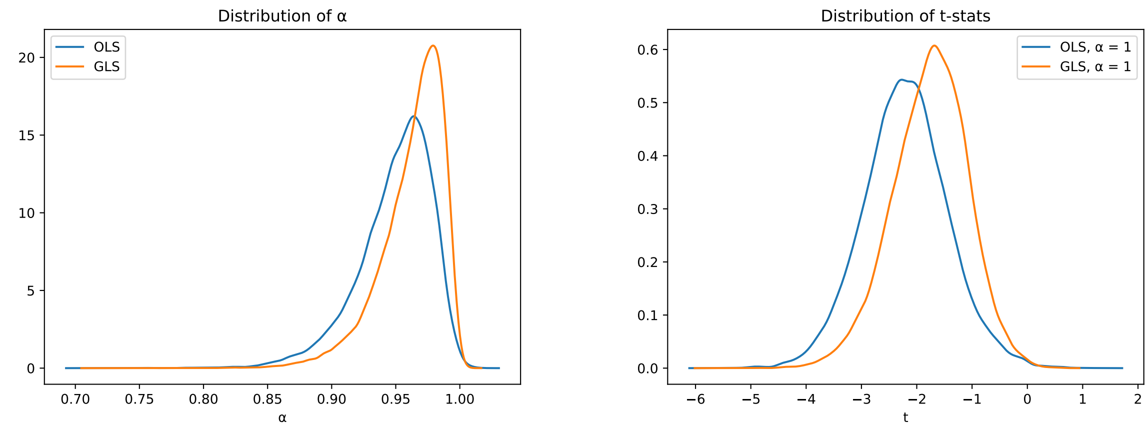
- OLS: Estimate $y_t = a + \beta t + \alpha y_{t-1} + \text{error}_t$, then

$$t_{\text{OLS}} \equiv \frac{\widehat{\alpha}-\alpha_0}{\sqrt{\frac{\sum_{t=1}^{T} y_t^2}{T}}}$$

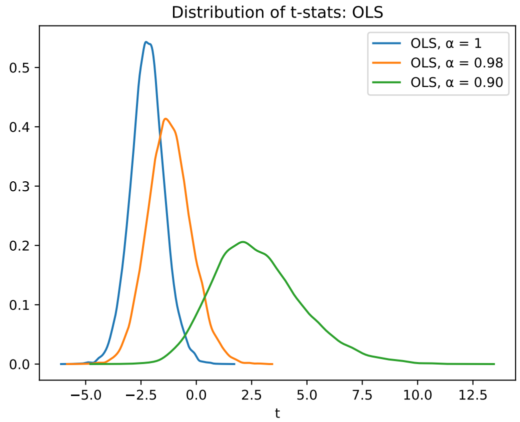
- GLS: Estimate $y_t^d = \alpha_G y_{t-1}^d + \text{error}_t$, then

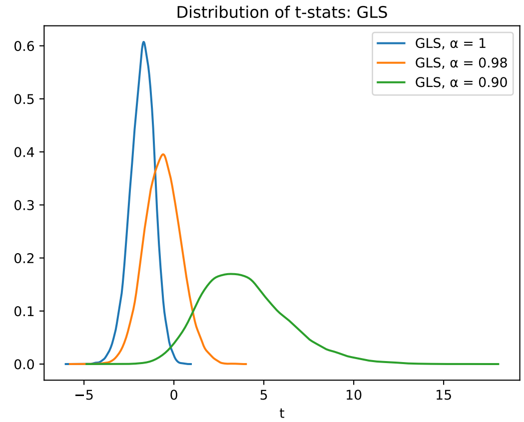
$$t_{\text{GLS}} \equiv \frac{\widehat{\alpha}_G-\alpha_0}{\sqrt{\frac{\sum_{t=1}^{T} \left(y_t^d\right)^2}{T}}}$$

It's possible that these t-stats should be multiplied by T because of super-consistency; so, one can visualize these axes inflated by 200. Here, I've set $\alpha_0 = 1$.



ii. Here I plot the same t-stats as mentioned in the previous exercise, except I substitute 1 for 0.98 and 0.90.





The critical values are

α	OLS	GLS
1	0.9458	0.707

The critical values are smaller than the asymptotic ones. The power of the two tests under the two alternatives—calculated as the share of draws for which the $|t|$ value is greater than the critical value tabulated above—are

α	Power
0.98	0.4433
0.90	0.03132

As we can see, the power of the test decreases as we get farther away from the truth.

iii. Using the same data as in the previous exercises, the equation I esimate with OLS is

$$y_t = a + b \cdot \mathbb{1}\{t > 75\} + c \cdot (t \times \mathbb{1}\{t > 75\}) + d \cdot t + y_{t-1} + \text{error}_t \tag{1}$$

The critical value becomes

α	OLS
1	1.553

which is bigger than before.

iv. I perform the same estimation as in (1), except now the data are generated from a process with a trend shift. However, we don't include a trend shift in the estimation. The size of the test with the omitted trend shift is

Size
1

That is, we essentially always reject the null of a unit root. This is evidence that it's hard to estimate a unit root even when it's present.

Q2. Let's try to get a handle on what this question is asking. The variable x_t is an AR(1) for which we let the AR coefficient range from 0.8 to 1, and y_t is a noisy measurement of x_t:

$$x_t = \phi x_{t-1} + u_t$$
$$y_t = \beta x_t + e_t$$

As $\phi \to 1$, x_t becomes a random walk, so we're measuring a random walk with some noise. As ϕ moves away from 1, then we are measuring some type of scaled random walk and thus making the error e_t more important. By estimating

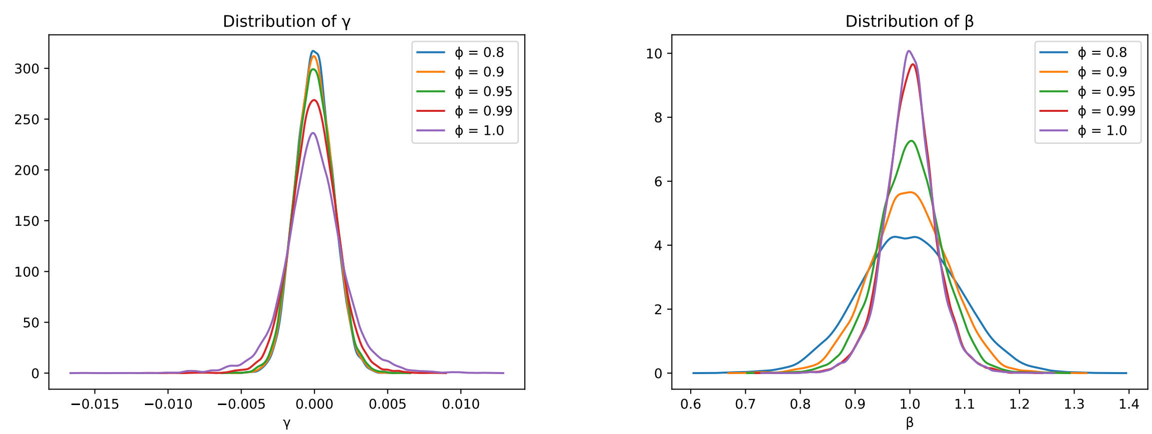
$$y_t = a + \gamma t + \beta x_t + \text{error}_t,$$

we are saying that the researcher is mistakenly confusing y_t as a variable with a trend. In what follows, I set $T = 200$ and performed 10,000 simulations for each value of ϕ.

i. The mean values of β and γ for each value of ϕ are

Mean Estimates		
ϕ	γ	β
0.8	$-7.796e-06$	0.9988
0.9	$-8.425e-06$	0.9993
0.95	$-9.593e-06$	0.9996
0.99	$-9.886e-06$	1
1.0	$-4.229e-06$	1

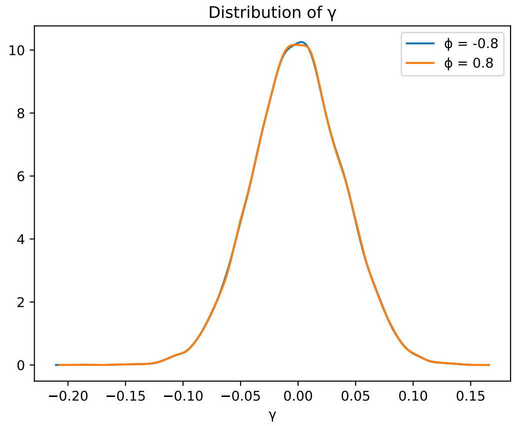
and the distributions are plotted below. As $\phi \to 1$, the mean estimate of $\beta \to 1$. The distribution also becomes much tigher around β. On the other hand, the estimates of γ become slightly less-precisely estimated about 0. So, as the process x_t becomes less and less like a random walk, it's more likely that we'll infer some type of trend, and the component of y_t coming from x_t will look less important (smaller β). Summarizing more-succinctly, the trend is more-imprecisely estimated as we get closer to a random walk.

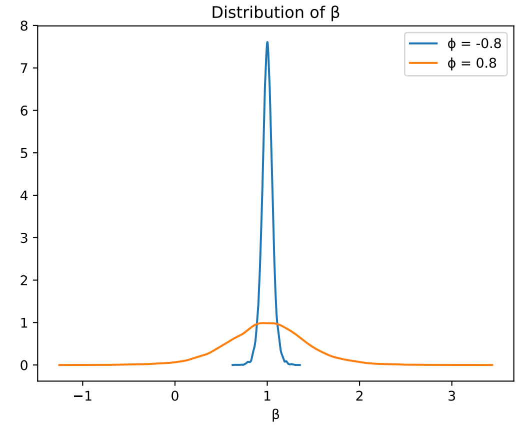


ii. Now, we let y_t be the sum of an AR(1) and a random walk, and study what happens as the AR(1) oscillates (negative AR coefficient) and doesn't.

Mean Estimates		
ϕ	γ	β
-0.8	-0.0002873	1
0.8	-0.000276	0.9987

In both cases, the trend component is estimated to be negative[1], and β is centered at about 1. But, when the AR oscillates, the estimates of β are much more precise. This is surprising to me, but maybe the point is that now that x_t is kind of crazy, OLS realizes that x_t is the primary driver of y_t.





[1] which I think makes sense... in the absence of shocks all but , an AR(1) will look like it has a negative trend.

Q3.

i. $y_{1t} - y_{1t-1} = \gamma y_{2t} - \gamma y_{2t-1} + u_{1t} - u_{1t-1}$

$\Rightarrow \Delta y_{1t} = \gamma \Delta y_{2t} + \Delta u_{1t}$

$= \gamma u_{2t} + (1-L) u_{1t}$

$$\begin{bmatrix} \Delta y_{1t} \\ \Delta y_{2t} \end{bmatrix} = \begin{bmatrix} (1-L) & \gamma \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$$

So, the $MA(\infty)$ representation is $\Delta y_t = \Psi(L) u_t$ with $\Psi(L) \equiv \begin{bmatrix} 1-L & \gamma \\ 0 & 1 \end{bmatrix}$

ii. $y_{1t} = \gamma y_{2t} + u_{1t}$

$y_{2t} = y_{2t-1} + u_{2t}$

$$y_t = \begin{bmatrix} 0 & \gamma \\ 0 & L \end{bmatrix} y_t + u_t$$

$$\Rightarrow u_t = \begin{bmatrix} 1 & -\gamma \\ 0 & 1-L \end{bmatrix} y_t$$

So, the $AR(\infty)$ representation is $\Phi(L) y_t = u_t$ with $\Phi(L) \equiv \begin{bmatrix} 1 & -\gamma \\ 0 & 1-L \end{bmatrix}$

iii $\Delta y_{1t} = \gamma \Delta y_{2t} + \Delta u_{1t} = \gamma u_{2t} + u_{1t} - (y_{1t-1} - \gamma y_{2t-1})$

$\Delta y_{2t} = u_{2t}$

$$\Rightarrow \Delta y_t = \begin{bmatrix} -1 & \gamma \\ 0 & 0 \end{bmatrix} y_t + \begin{bmatrix} 1 & \gamma \\ 0 & 1 \end{bmatrix} u_t$$

$$= \underbrace{\begin{bmatrix} -1 \\ 0 \end{bmatrix}}_{\beta} \underbrace{\begin{bmatrix} 1 & -\gamma \end{bmatrix}}_{\alpha'} y_t + \underbrace{\begin{bmatrix} 1 & \gamma \\ 0 & 1 \end{bmatrix} u_t}_{e_t}$$

So, the VECM representation is $\Delta y_t = \beta \alpha' y_t + e_t$ with $\beta \equiv \begin{bmatrix} -1 \\ 0 \end{bmatrix}$, $\alpha \equiv \begin{bmatrix} 1 \\ -\gamma \end{bmatrix}$, $e_t \equiv \begin{bmatrix} 1 & \gamma \\ 0 & 1 \end{bmatrix} u_t$