Q1

i. The federal funds rate enters in its normal level; percent. For $x_t = \{\text{GDP}, \text{M3}\}$, I include $400 \times \Delta \log x_t$. The estimates for μ and A are:

$$\widehat{\mu} = \begin{bmatrix} 0.88 \\ 1.9 \\ -0.26 \end{bmatrix} \qquad \widehat{A}_1 = \begin{bmatrix} 0.24 & 0.18 & -0.03 \\ -0.065 & 0.56 & -0.67 \\ 0.053 & 0.053 & 1.2 \end{bmatrix} \qquad \widehat{A}_2 = \begin{bmatrix} 0.19 & 0.055 & -0.11 \\ 0.085 & 0.076 & 0.76 \\ 0.023 & -0.022 & -0.21 \end{bmatrix}$$

The moduli of the eigenvalues of the companion matrix are:

0.33	0.14	0.14	0.96	0.65	0.65

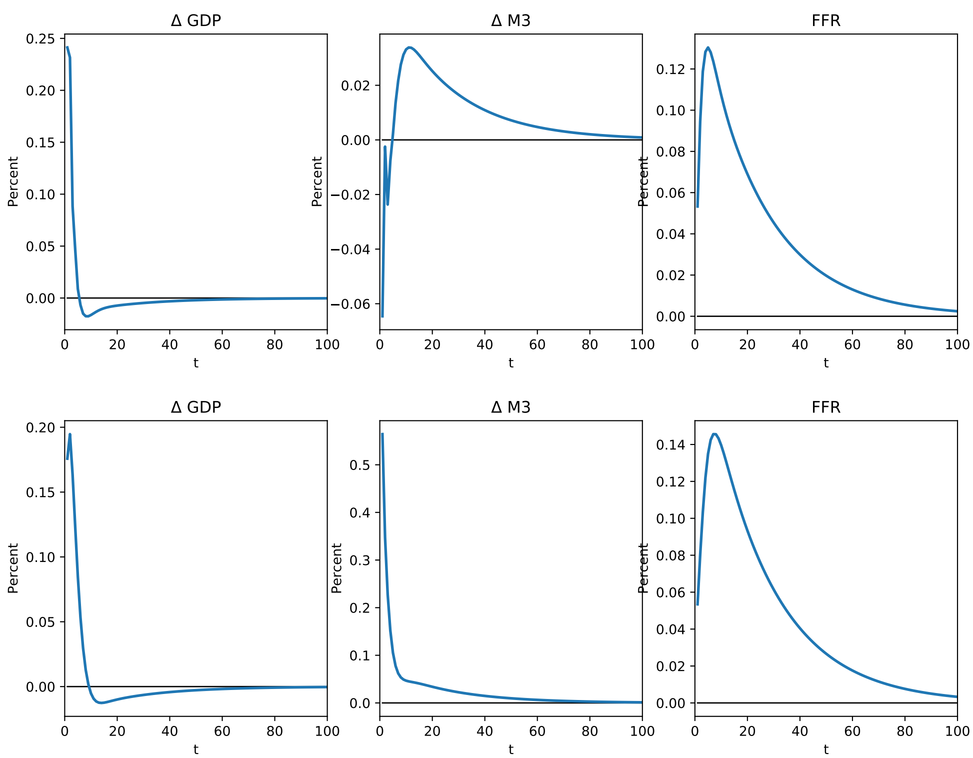
which are all less than 1; so, y_t is stable.

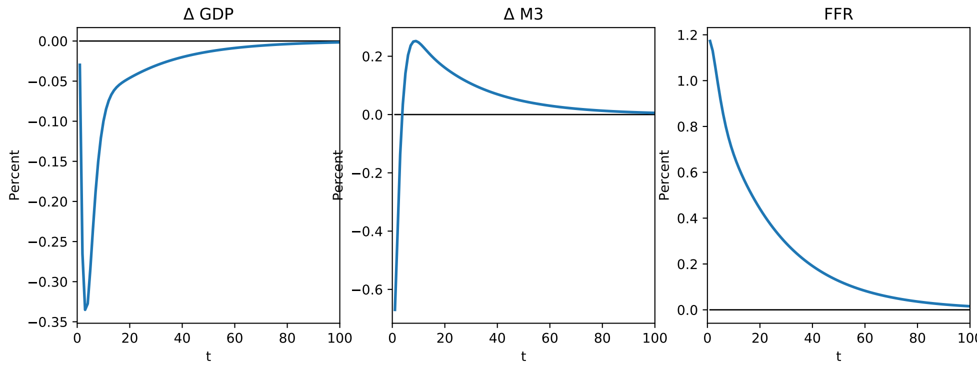
ii. The estimates are:

$$\widehat{\mathbb{E}}[y_t] = \begin{bmatrix} 3 \\ 6.7 \\ 5.2 \end{bmatrix} \qquad \widehat{\Gamma}(0) = \begin{bmatrix} 11 & 0.81 & -0.63 \\ 0.81 & 11 & 1.1 \\ -0.63 & 1.1 & 13 \end{bmatrix} \qquad \widehat{\Gamma}(1) = \begin{bmatrix} 3.8 & 2.2 & -1.6 \\ -0.55 & 6.8 & 1.6 \\ 0.26 & 1.5 & 13 \end{bmatrix}$$

iii. The function has been written.

iv. The IRFs are below. I use Δx to denote the transformation I mentioned in part i: 400 times the log difference.





v. Kilian's textbook said to use 4-8 lags as the maximum for quarterly data, so I use 8. The AIC and BIC values for each lag are

m	1	2	3	4	5	6	7	8
BIC	3.9	4	4	4.1	4.2	4.3	4.5	4.6
AIC	3.7	3.6	3.6	3.5	3.4	3.4	3.5	3.5

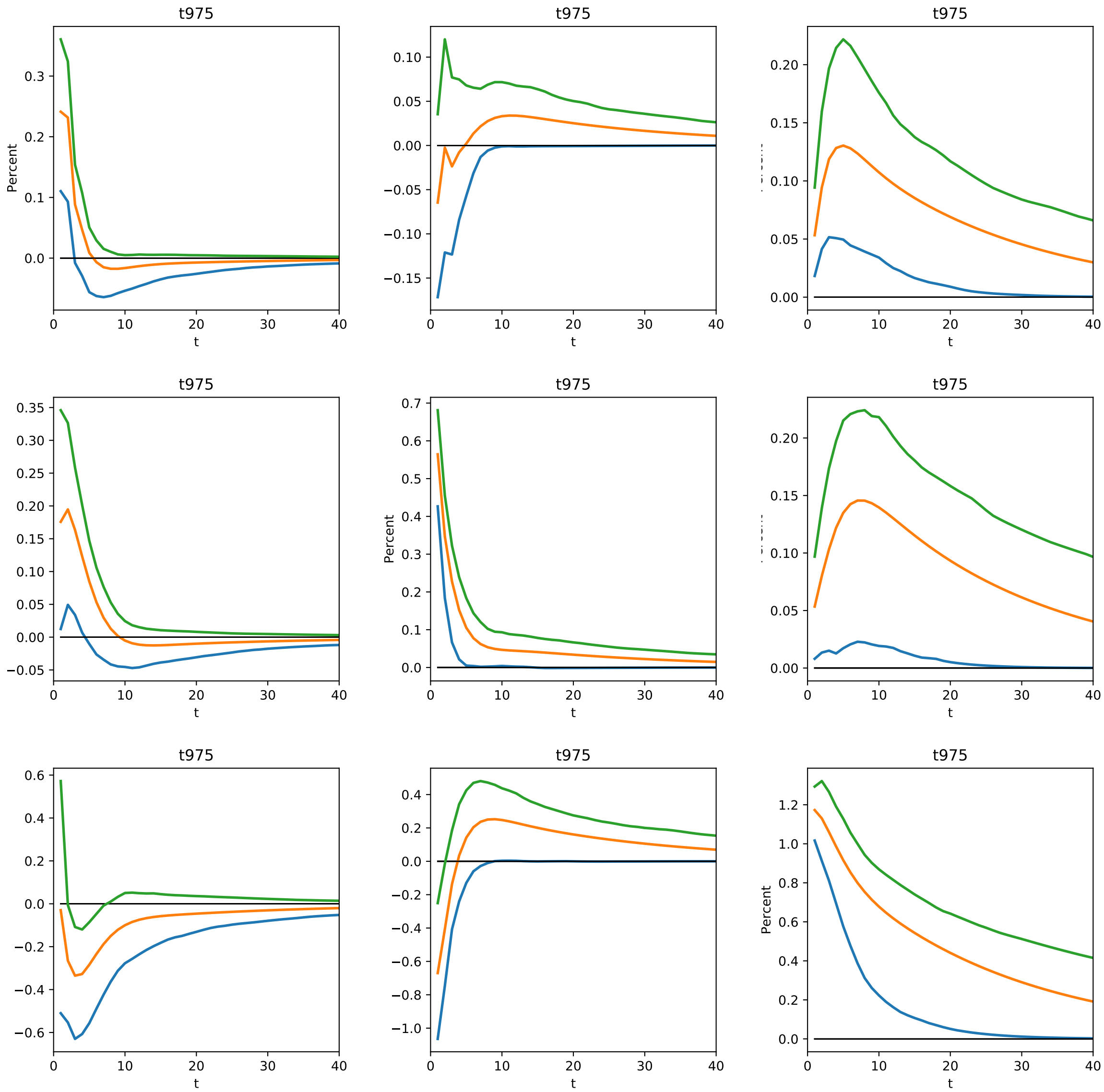
So, the optimal number of lags under BIC is 1, and 6 for AIC.

vi. Everything looks good; the estimated π and Π are:

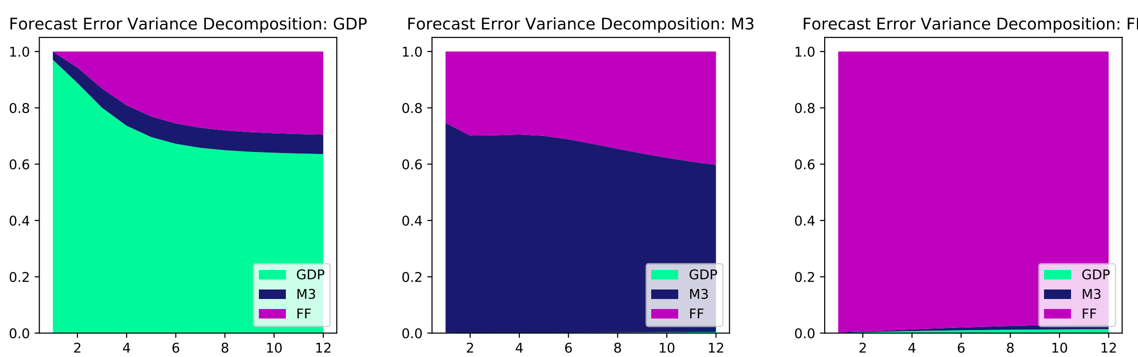
$$\widehat{\pi} = \begin{bmatrix} 0.24 \\ \text{-}0.065 \\ 0.053 \\ 0.18 \\ 0.56 \\ 0.053 \\ \text{-}0.03 \\ \text{-}0.67 \\ 1.2 \\ 0.19 \\ 0.085 \\ 0.023 \\ 0.055 \\ 0.076 \\ \text{-}0.022 \\ \text{-}0.11 \\ 0.76 \\ \text{-}0.21 \\ 0.88 \\ 1.9 \\ \text{-}0.26 \end{bmatrix} \qquad \widehat{\Pi} = \begin{bmatrix} 0.24 & 0.18 & \text{-}0.03 & 0.19 & 0.055 & \text{-}0.11 & 0.88 \\ \text{-}0.065 & 0.56 & \text{-}0.67 & 0.085 & 0.076 & 0.76 & 1.9 \\ 0.053 & 0.053 & 1.2 & 0.023 & \text{-}0.022 & \text{-}0.21 & \text{-}0.26 \end{bmatrix}.$$

vii. Again, all clear. The maximum absolute difference between the re-simulated data and the actual data is 2.1316282072803006e-14, which is pretty small.

viii.



ix.



x. We can reject this null with a p-value of 0.0002849182976748608.

Q2