I'll perform this estimation via GMM. For simplicity, I'll weight by the identity matrix.

$$\begin{aligned}
J_n(\mu,\sigma^2) &= n\overline{g}(\mu,\sigma^2)'\,\mathbf{I}\,\overline{g}(\mu,\sigma^2) \\
&= \frac{1}{n}\left(\left[\sum(x_t-\mu)\right]^2 + \left[\sum\left\{(x_t-\mu)^2-\sigma^2\right\}\right]^2 + \left[\sum(x_t-\mu)^3\right]^2 + \left[\sum\left\{(x_t-\mu)^4-3\sigma^4\right\}\right]^2\right)
\end{aligned}$$

For now, I'm imposing that my Newton-Raphson algorithm needs analytic expressions for the gradient and Hessian. The gradient in this case is

$$\begin{aligned}
\nabla J_n(\mu,\sigma^2) &= \frac{1}{n}\begin{bmatrix}
\begin{array}{c}
-2n\sum(x_t-\mu) + 2\left[\sum\left\{(x_t-\mu)^2-\sigma^2\right\}\right](-2)\sum(x_t-\mu) \\
+2\left[\sum(x_t-\mu)^3\right](-3)\sum(x_t-\mu)^2 + 2\left[\sum\left\{(x_t-\mu)^4-3\sigma^4\right\}\right](-4)\sum(x_t-\mu)^3
\end{array} \\
\\
2\left[\sum\left\{(x_t-\mu)^2-\sigma^2\right\}\right](-1)n + 2\left[\sum\left\{(x_t-\mu)^4-3\sigma^4\right\}\right](-6)\sigma^2
\end{bmatrix} \\
&= \frac{1}{n}\begin{bmatrix}
\begin{array}{c}
-2n\sum(x_t-\mu) - 4\left[\sum\left\{(x_t-\mu)^2-\sigma^2\right\}\right]\sum(x_t-\mu) \\
-6\left[\sum(x_t-\mu)^3\right]\sum(x_t-\mu)^2 - 8\left[\sum\left\{(x_t-\mu)^4-3\sigma^4\right\}\right]\sum(x_t-\mu)^3
\end{array} \\
\\
-2n\left[\sum\left\{(x_t-\mu)^2-\sigma^2\right\}\right] - 12n\left[\sum\left\{(x_t-\mu)^4-3\sigma^4\right\}\right]\sigma^2
\end{bmatrix}
\end{aligned}$$