Problem Set 5
GR6413 [Ng]
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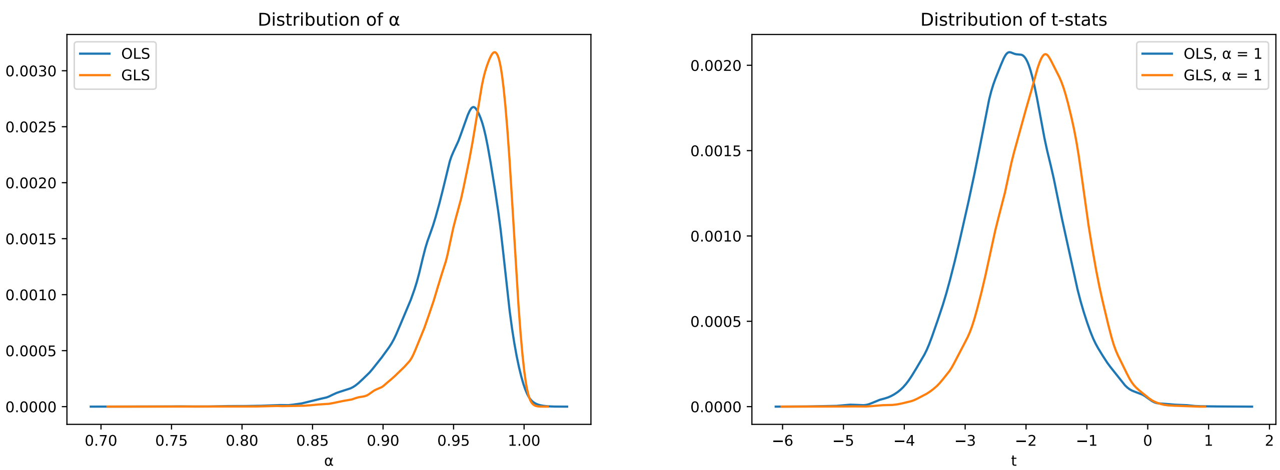
Q1. In the first panel, I plot the distribution of $\widehat{\alpha}$ (not sure what the normalization $T(\widehat{\alpha}-1)$ means). In the second panel, I plot the t-stats constructed as follows.

- OLS: Estimate $y_t = a + \beta t + \alpha y_{t-1} + \text{error}_t$, then

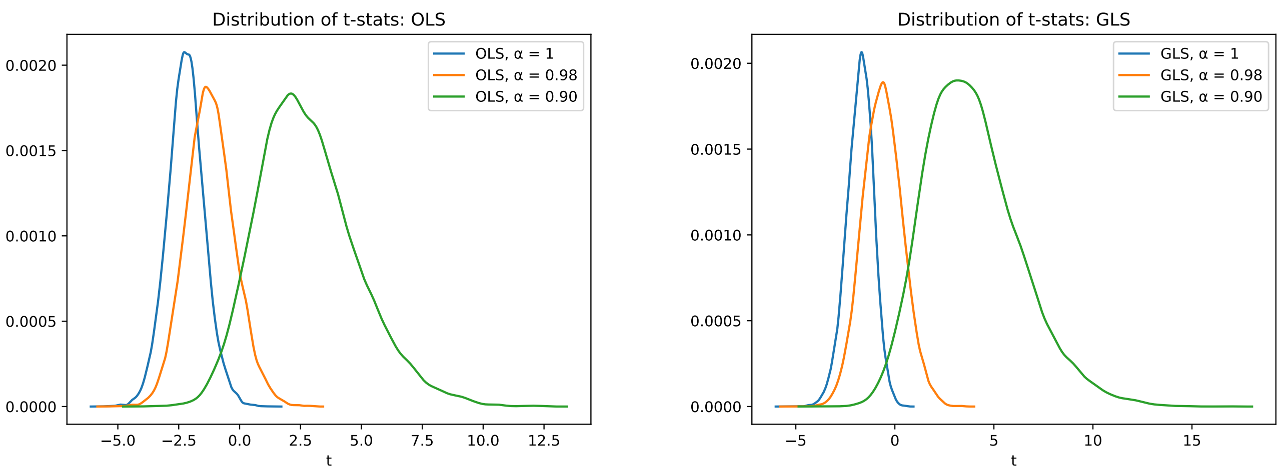
$$t_{\text{OLS}} \equiv \frac{\widehat{\alpha}-1}{\sqrt{\frac{\sum_{t=1}^{T} y_t^2}{T}}}$$

- GLS: Estimate $y_t^d = \alpha_G y_{t-1}^d + \text{error}_t$, then

$$t_{\text{GLS}} \equiv \frac{\widehat{\alpha}_G-1}{\sqrt{\frac{\sum_{t=1}^{T} \left(y_t^d\right)^2}{T}}}$$

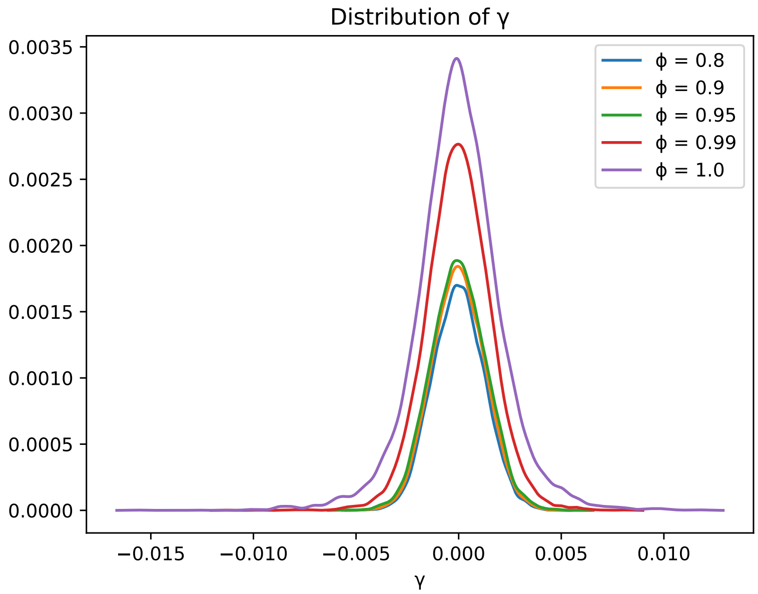


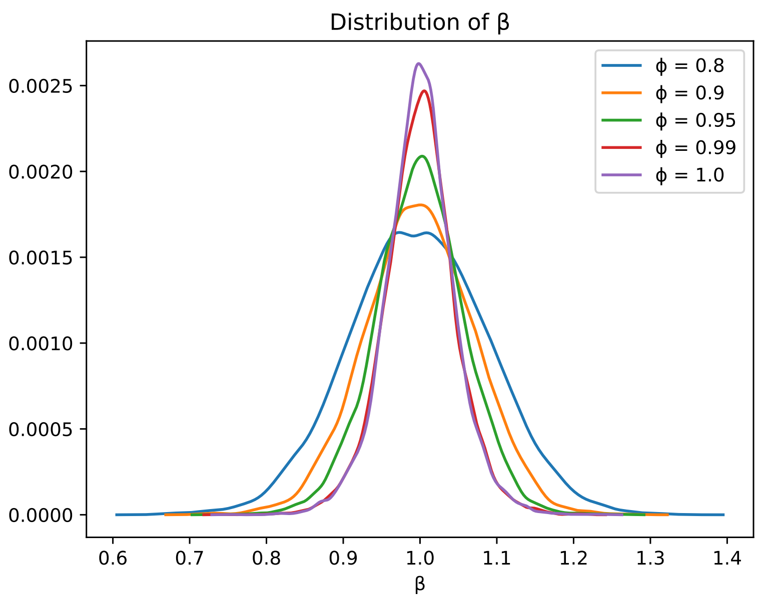
Q2. Here I plot the same t-stats as mentioned in the previous exercise, except I substitute 1 for 0.98 and 0.90.



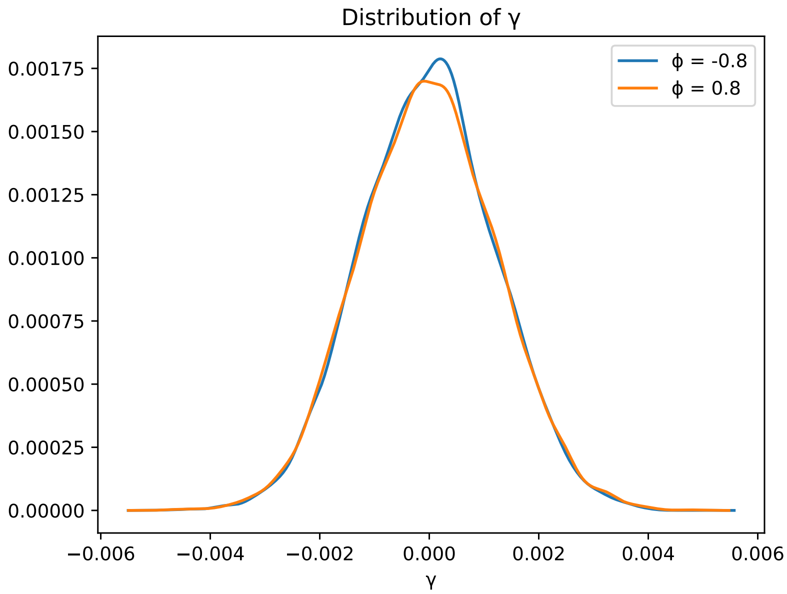
Q3. I set $T = 200$ and performed 10,000 simulations for each value of ϕ.

i.

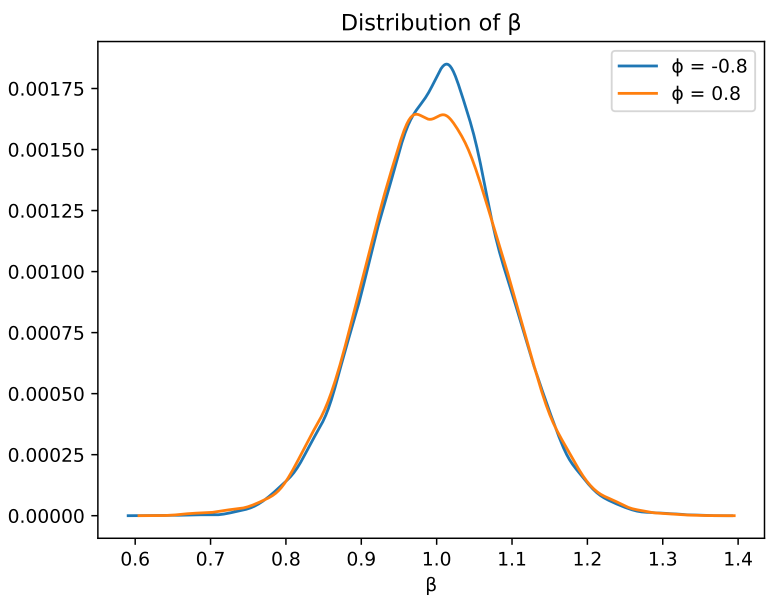




ii.



Distribution of γ
ϕ = -0.8
ϕ = 0.8
0.00175
0.00150
0.00125
0.00100
0.00075
0.00050
0.00025
0.00000
−0.006
−0.004
−0.002
0.000
0.002
0.004
0.006
γ



Distribution of β
ϕ = -0.8
ϕ = 0.8
0.00175
0.00150
0.00125
0.00100
0.00075
0.00050
0.00025
0.00000
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
β

Q4.

i. $y_{1t} - y_{1t-1} = \gamma y_{2t} - \gamma y_{2t-1} + u_{1t} - u_{1t-1}$

$\Rightarrow \Delta y_{1t} = \gamma \Delta y_{2t} + \Delta u_{1t}$

$= \gamma u_{2t} + (1-L) u_{1t}$

$$\begin{bmatrix} \Delta y_{1t} \\ \Delta y_{2t} \end{bmatrix} = \begin{bmatrix} (1-L) & \gamma \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$$

$\boxed{\text{So, the } MA(\infty) \text{ representation is } \Delta y_t = \Psi(L) u_t \text{ with } \Psi(L) \equiv \begin{bmatrix} 1-L & \gamma \\ 0 & 1 \end{bmatrix}}$

ii. $y_{1t} = \gamma y_{2t} + u_{1t}$
$y_{2t} = y_{2t-1} + u_{2t}$

$$y_t = \begin{bmatrix} 0 & \gamma \\ 0 & L \end{bmatrix} y_t + u_t$$

$$\Rightarrow u_t = \begin{bmatrix} 1 & -\gamma \\ 0 & 1-L \end{bmatrix} y_t$$

$\boxed{\text{So, the } AR(\infty) \text{ representation is } \Phi(L) y_t = u_t \text{ with } \Phi(L) \equiv \begin{bmatrix} 1 & -\gamma \\ 0 & 1-L \end{bmatrix}}$

iii $\Delta y_{1t} = \gamma \Delta y_{2t} + \Delta u_{1t} = \gamma u_{2t} + u_{1t} - (y_{1t-1} - \gamma y_{2t-1})$
$\Delta y_{2t} = u_{2t}$

$$\Rightarrow \Delta y_t = \begin{bmatrix} -1 & \gamma \\ 0 & 0 \end{bmatrix} y_t + \begin{bmatrix} 1 & \gamma \\ 0 & 1 \end{bmatrix} u_t$$

$$= \underbrace{\begin{bmatrix} -1 \\ 0 \end{bmatrix}}_{\beta} \underbrace{\begin{bmatrix} 1 & -\gamma \end{bmatrix}}_{\alpha'} y_t + \underbrace{\begin{bmatrix} 1 & \gamma \\ 0 & 1 \end{bmatrix} u_t}_{e_t}$$

$\boxed{\text{So, the VECM representation is } \Delta y_t = \beta \alpha' y_t + e_t \text{ with } \beta \equiv \begin{bmatrix} -1 \\ 0 \end{bmatrix}, \ \alpha \equiv \begin{bmatrix} 1 \\ -\gamma \end{bmatrix}, \ e_t \equiv \begin{bmatrix} 1 & \gamma \\ 0 & 1 \end{bmatrix} u_t}$