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Explorations Into Factors Explaining Money Market Returns

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ABSTRACT

In this article, we measure and interpret the common "factors" that describe money market returns. Results are presented for both three- and four-factor models. We find that the three-factor model explains, on average, 86 percent of the total variation in most money market returns while the four-factor model explains, on average, 90 percent of this variation. Using mimicking portfolios, we provide an interpretation of the systematic risks represented by these factors.

IN THIS ARTICLE, WE attempt to measure and interpret the common "factors" that describe money market returns. The factor approach we employ assumes that the covariance matrix of a set of random variables, in this case excess returns, can be decomposed into common or systematic components and idiosyncratic or nonsystematic components. This decomposition into systematic and nonsystematic components is based on an assumption of a linear relationship between the returns of each security and a set of "common" factors. This is the assumption of linear return-generating models, which form an integral part of the structure of many asset pricing theories—for example, arbitrage pricing theory (APT) as developed by Ross (1976). The interpretation is that the common factors represent sources of systematic or nondiversifiable risk and the idiosyncratic component of returns represents diversifiable risks.[1]

Our focus here is not on testing a particular asset pricing model per se but on developing empirical evidence for the existence of stylized facts regarding money market returns. The stylized facts take the form of the existence, measurement, and interpretation of the common factors that are found in money markets. Our attempt to measure and interpret these sources of systematic risk may eventually lead to the construction of observable eco-

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[1] There is now an extensive list of articles empirically investigating APT. Beginning primarily with the work of Roll and Ross (1980) and more recently the work of Connor and Korajczyk (1988), Lehmann and Modest (1988), and Gultekin and Rogalski (1985). While our work shares the assumption of a linear return-generating model with this work, its emphasis is on conducting a test of the APT model. In addition, with the exception of Gultekin and Rogalski (1985), who examine government bonds, its focus is on equity markets rather than fixed income markets.

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nomic variables so as to guide the development of financial theory. That is, by learning what common factors characterize the returns in money markets, we can begin to develop models with observable state variables.

For this purpose, we assume a linear k-factor return-generating model and investigate the decomposition of the unconditional covariance matrix of returns into common factors. Our analysis is therefore restricted to an unconditional analysis using a fixed factor model. We consider both three- and four-factor models. In the three-factor model we find that three factors explain an average of 86 percent of the total variation in most of the money market instruments considered. These three factors can be thought of as three parameters that characterize systematic movements in the yield curve.

The first factor we call a level factor because it represents movements in yields that are approximately parallel in nature and represents, on average, 73 percent of the total explained variation. Duration is commonly used as an approximation of the change in the price of a fixed income security associated with a parallel shift in the yield curve. The problem is that this description of yield curve movements is unrepresentative of all yield curve movements.

The second factor is called steepness since it represents changes in the steepness of the yield curve. This factor accounts for 12 percent of the total explained variation in returns. The third factor we refer to as a Treasury factor. It involves movements in the yield curve characterized by the private issuer money market instruments moving uniformly away from the Treasury bill market and captures the credit risks in treasury issues. The third factor accounts for approximately 15 percent of the total variation explained by the three factors. The credit risks in the private issuer instruments we consider here combine bank risk and firm risk. To further investigate the nature of these credit risks, we expand the set of instruments to include higher credit risks and examine a four-factor model. The four-factor model is found to explain an average of 90 percent of the total variation in ex post returns. In the four-factor model, three of the four factors are essentially the same as those of the three-factor model—namely, a level, a steepness, and a Treasury factor. However, there exists an additional factor, which we refer to as a private issuer factor. This factor represents movements in the yield curve characterized by commercial paper instruments moving uniformly away from the other private sector instruments, particularly certificates of deposit. This factor accounts for approximately 4 percent of the total variation explained by the four factors.

The factors in our analysis are unobservable. We thus construct proxies for these factors by using mimicking portfolios in a manner similar to Fama and Macbeth's (1973) tests of the capital asset pricing model and derived explicitly in the context of APT by Huberman, Kandel, and Stambaugh (1987). These mimicking portfolios can then be used to investigate further the properties of the underlying factors.

The fractions of total variation explained above are computed excluding the short end of the yield curve and the 1-month T-bill. The inclusion of these short securities reduces the average explained variation to 82 percent. As will

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be shown in Section V, this effect is largely caused by the reduced variation explained by factor 1 for these instruments. These securities have large idiosyncratic noise terms. These larger idiosyncratic noise terms can in part be attributed to institutional considerations that induce demand-specific shocks to the particular security. For example, 1-month T-bills for accounting purposes can be considered "cash equivalent." This serves to increase the liquidity of a firm's balance sheet. In addition, while these instruments are liquid in the sense that they mature quickly (most large institutional investors buy and hold such instruments) the bid-ask spread tends to be large. The bid-ask spread on the 1-month T-bill can be as much as 25 to 35 basis points. It should be noted, however, that factor 1 explains a much higher percentage of the variation of one-month private issuer securities. This seems to be more consistent with the "cash equivalent" story than with the "bid-ask" spread effect.

The next section outlines the factor model we estimate and discusses some of the issues related to the identification and measurement of the model. In Section II, we present the data set used in our estimation procedure and examine some of the properties of the sample period chosen. Section III describes the estimation procedure for the factor loadings associated with each of the factors described above for the money instruments in our universe. In Section IV, we present the empirical results for a three- and a four-factor model. In particular, this section presents evidence regarding the role played by each factor for each instrument in explaining the total variation and provides an examination of the mimicking portfolios in explaining returns on diversified portfolios. Finally, in Section V, we conclude with a brief summary and a discussion of what we think the implications are for term structure theory, related work on term structure, and directions for future research.

I. The Linear Factor Model

We assume that there is a k-factor linear return-generating model of the form:

$$r_i = \mu_i + b_{i1} f_1 + b_{i2} f_2 + \cdots + b_{ik} f_k + \varepsilon_i \quad \text{for all } i = 1, \ldots, p \tag{1}$$

The first term on the right-hand side of the equation is the expected excess return over a risk-free rate for asset i. The next k terms f_j, $j = 1, \ldots, k$ represent the k common factors that affect a security's returns. Each coefficient b_{ij} is referred to as the loading of the ith security on the jth factor. It represents the sensitivity of the ith security price to the jth factor. The last term in equation (1) is the error term that represents the nonsystematic risk component, which is idiosyncratic to the ith asset. In matrix notation, equation (1) can be written as

$$R = BF + \varepsilon, \tag{2}$$

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where R is the demeaned excess returns. The unobservable random vectors F and ε are assumed to satisfy:

$$E(F) = 0, \qquad \operatorname{cov}(F) = I, \qquad E(\varepsilon) = 0, \qquad \operatorname{cov}(\varepsilon) = \Psi \tag{3}$$

where Ψ is a diagonal matrix. This model implies a covariance structure for the excess returns matrix R of the form:

$$\Sigma = BB' + \Psi \tag{4}$$

Assumption (3) is used to partially identify the model. However, as is well known there is an additional ambiguity associated with the factor approach. Let T be an orthonormal matrix. Then equation (2) above can be rewritten as

$$R = BTT'F + \varepsilon \tag{5}$$

$$= \hat{B}\hat{F} + \varepsilon \tag{6}$$

Since $E[\hat{F}] = T'E(F) = 0$ and $\operatorname{cov}(\hat{F}) = T' \operatorname{cov}(F)T = T'T = I$, one cannot determine by observing the loadings R the difference between the loadings B and $\hat{B}$. As a result, even though both B and $\hat{B}$ have different loadings they both generate the same covariance matrix Σ. In addition, there are infinitely many such transformations that could be constructed to generate Σ. Rotating the factors by multiplying F by an orthonormal matrix T only changes the directions of the coordinate system for R.

In this article, we employ for the three-factor model a particular type of rotation such that the factors implied by the rotated loadings are consistent with the factors or parameters that typically describe the 30-year zero curve in the Treasury market. The factors that describe the 30-year zero curve are developed by Litterman and Scheinkman (1988). They find that three factors explain 98 percent of the variation in yields associated with coupon bonds for 1 year to 18 years in maturity. The first or "level" factor represents approximately a parallel change in yields. This accounts for 89 percent of the total explained variation. The second factor is called the "steepness" factor. The steepness factor in the Treasury-based market lowers the yields of zeroes up to 5 years and raises the yields for zeros of longer maturities. The third factor for Treasury-based securities in Litterman and Scheinkman is a "curvature" factor. That is, the effect of factor in Treasury zeroes is to increase the curvature of yield curve in the range of maturities below 20 years. In Section IV we describe the actual steps taken to perform this orthogonal rotation.

The random factors f_j are unobservable. However, we can construct portfolios that mimic a particular factor. Let s_j stand for the sensitivity with respect to the jth factor of a portfolio $(x_1, x_2, \ldots, x_n)$ with a total value of 1 dollar. Then:

$$s_j = x_1 b_{1j} + x_2 b_{2j} + \cdots + x_n b_{nj}; \qquad j = 1, \ldots, k \tag{7}$$

Finding a portfolio that is sensitive to a single factor requires solving k equations with n unknowns, which is generally possible when $n \geq k$. In fact,

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when $n > k$, there are an infinite number of such portfolios. However, each such portfolio will have its own specific variance:

$$\sigma_{s_j}^2 = (x_1)^2 \mathrm{var}(\varepsilon_1) + (x_2)^2 \mathrm{var}(\varepsilon_2) + \cdots + (x_n)^2 \mathrm{var}(\varepsilon_n) \tag{8}$$

To find the mimicking portfolio that minimizes the portfolio's specific variance, we solve the quadratic minimization problem subject to linear constraints using weighted least squares. This technique is part of the Fama and Macbeth(1973) procedure commonly used in testing asset pricing models.

Once we estimate the mimicking portfolios, they can be used to investigate the characteristics of the factors themselves. In Section IV, we explain the statistical properties of mimicking portfolios and present some regressions that help identify what type of state variables might be represented by the factors. This process of interpreting the factors themselves is particularly important, since the focus of this article is to explore the stylized facts of money markets and learn more about what type of observable state variables may be important in explaining money market returns.

II. Data

The data used to investigate the common factors in money markets were provided by the Goldman Sachs Fixed Income database over the period January 1985 to August 1988. We selected 38 instruments representing 5 different sectors. In each sector we selected between 4 and 9 generic instruments starting from the shortest maturity available to the longest available up to 1 year (Table I).

This provides a yield curve for each of the 5 sectors and allows us to search for common factors that affect all sectors equally or that distinguish some sectors from others. Across sectors the instruments differ by credit quality; within each sector they differ by maturity. The credit quality of our sample of instruments is a function of both firm risk and bank risk. We calculate

Table I

Money Market Securities

Description of the securities contained in the five money market sectors: T-bills, commercial paper (CP), certificates of deposit (CD), Eurodollar certificates of deposit (ECD), and bankers' acceptances (BA). D and SI stand for discount and simple interest respectively. I is the interest rate risk, B is the bank risk, and F is the firm risk.

Sectors	Maturities (months)	Type	Risk
T-bills	1, 2, 3, 4, 5, 6, 9, 12	D	I
CP(A1-P1)	1, 2, 3, 4, 6	D	I, B, F
CD	1, 2, 3, 5, 6	SI	I, B
ECD	1, 2, 3, 6	SI	I, B
BA	1, 2, 3, 5, 6	D	I, B

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weekly returns using Wednesday's closing yield quotes for a generic instrument of a given maturity in a given sector. We chose Wednesdays to avoid missing observations resulting from holidays, which predominately occur on Thursdays and Fridays.

The five sectors we examine are Treasury bills (TBs), A1-P1 and A2-P2. Commercial paper (CP), certificates of deposit (CDs), Eurodollar certificates of deposit (ECDs), and bankers' acceptances (BAs). To perform the statistical analysis for identifying the common factors, we are interested in the return to dollar payments at each maturity. All the instruments used are either discount securities or simple interest securities that only pay coupons at maturity. Accordingly, all the instruments are effectively zero coupon bonds.

To calculate returns for a given instrument, we first construct a time series of prices for a given type of security for a given maturity. Since we are interested in weekly returns, we then construct a time series of prices for the same type of security but with 1 less week to maturity. For example, let P_{it}^{M} be the price of security of type i (say 6-month or 180-day T-bill) in period t, then P_{it}^{M-K} is the price of a security of type i in period t with $M-K$ days to maturity. To calculate P_{it}^{M-K} when no yield is quoted for a security of that type for that maturity, we interpolate between the closest two available maturities. For example, to calculate P_{it}^{M-K} where $M-K$ is 173 days, we interpolate between the yields on a 5-month security i and 6-month security i. Since the yield curve is in general very flat, little distortion is added by this estimation procedure. The return for a security of type i in period t denoted R_i^t is defined as

$$R_t^i = \frac{P_{it-1}^{M-K} - P_{it}^{M}}{P_{it}^{M}} \tag{9}$$

We compute weekly excess returns for each instrument by taking the difference between an instrument's total return and the weekly return on the generic overnight repo rate.[2] We examine alternative covariance matrices such as yield changes to see if they remain unchanged over the sample period. Using a stability test discussed below, we find the covariance matrix of excess returns to be more stable than the covariance matrix of yield changes.

III. Estimation of Model

In this section, we describe the procedures we use to estimate the factor model presented in Section II. The estimation procedure involves three steps. The first step is to estimate the loadings matrix B in equation (2) above using a maximum likelihood procedure. In the second step, we take the estimates for the loadings obtained in step 1 and use them to estimate the

[2] We choose the overnight repo rate as opposed to 1-week generic repo to avoid any credit risk imbedded in a 7-day term repo.

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mimicking portfolios for the set of unobservable factors F in equation (2) above. For the four-factor model mentioned above, this is the final step. For the three-factor model in the third step, we construct an orthogonal transformation using the mimicking portfolios from the 30-year Treasury zero curve described above. Using this orthogonal transformation, we rotate the original factor loading to produce a new loadings matrix $\hat{B}$. It is this loadings matrix that we then use to interpret the factors and examine explained variation in excess returns. This produces a new set of mimicking portfolios that we can then use to examine characteristics of the unobservable factors.

A. Step 1

To estimate the loadings matrix B and the error term Ψ, we employ maximum likelihood factor analysis using the algorithm developed by Joreskog (1967). The Joreskog technique is based on a maximum likelihood iterative procedure. Given that R and F_t, are joint normally distributed and the set of assumptions implied by equations (2) and (3), the log likelihood function is

$$\log L(B, \Psi) = -\frac{1}{2}N\left[\ln|\Sigma| + T_r(S\Sigma^{-1})\right] \tag{10}$$

where S is the sample covariance matrix $S = [S_{ij}]$ defined by

$$S = \left(\frac{1}{N-1}\left(\sum_{t=1}^{N} R_t R_t'\right)\right). \tag{11}$$

The S follows a Wishart distribution with $N - 1$ degrees of freedom. Joreskog shows that by making the additional identifying assumption that $B'\Psi^{-1}B$ is diagonal, it is possible to find a $\hat{B}$ that maximizes L for a given Ψ. $\hat{B}$ is given by

$$\hat{B} = \Psi^{1/2}\phi(\Theta - I)^{1/2} \tag{12}$$

where ϕ is a $p \times k$ matrix of eigenvectors corresponding to k largest eigenvalues of the matrix $\hat{S} = \Psi^{-1/2}S\Psi^{-1/2}$ represented by Θ.

The estimation process then proceeds by choosing an initial value for $\hat{\Psi}$ and then using equation (12) above to estimate a $\hat{B}$. Then, given this $\hat{B}$, use $\Sigma = BB' + \Psi$ and the first-order conditions from the log likelihood to find a new $\hat{\Psi}$. The sequence of matrix estimates $\hat{\Psi}$ and $\hat{B}$ converge quickly to final matrix of estimates. One difficulty with this technique is that the maximum likelihood function may not have a true maximum in the region when all the Ψ_i are positive. To deal with this problem, our program sets $\Psi_i \geq \varepsilon$ for all i. Thus, we are searching over a restricted region. If the solution occurs in the interior of this region, then a proper solution is obtained. If it occurs on the boundary, then an improper solution is obtained. Then the search process is restarted with Ψ_i set equal to ε.

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The above process proceeds by first specifying a fixed number of factors k and then obtaining estimates for the loading matrix B. To test for significance of the number of factors in equation (1) above, we start by specifying a single-factor model, i.e., $k = 1$ and computing the likelihood ratio test. This likelihood ratio tests the null hypothesis that k factors are sufficient to explain the return-generating process given by equation (1). If the null hypothesis is rejected, then k is set equal to 2 and a new $\hat{B}$ matrix is estimated and a new likelihood ratio test computed. For each specification of k a second value of the likelihood function is computed conditional on the sample covariance without imposing any restriction on the number of factors. A likelihood ratio is then computed by dividing the likelihood value for a specific k by the likelihood value without the restriction. Under the null hyopthesis of a specific k, twice the natural logarithm of the likelihood ratio is distributed asymptotically as a χ^2 statistic with degrees of freedom equal to $\frac{1}{2}[(p - k)^2 - (p + k)]$.

In analyzing the correlation structure of excess returns in money market securities, we have maintained the assumption that this correlation structure remains constant throughout the sample period. We find that a simple test of this assumption shows no evidence to reject it. The test of a constant correlation structure is based on a comparison of the correlation structures of the excess returns in the first and second halves of the sample period. The distance between these two estimated correlation matrices, as measured by the trace of the square of their difference, lies well within the distribution generated by a Monte Carlo simulation under the null hyopthesis of a constant covariance matrix throughout the period.

B. Step 2

Once the estimates of the loadings matrix B have been obtained, the next step is to estimate the unobservable factors themselves F_i, $i = 1, \ldots, k$. As discussed above there are an infinite number of portfolios that will mimic a given factor. However, following Barlett (1950), Fama and Macbeth (1973), and Huberman, Kandel, and Stambaugh (1987), we perform a series of cross-sectional regressions that selects the portfolio that minimizes the weighted sum of squared errors putting a loading of one on a specific factor and zero on the other factors.

To compute our weighted least square estimates, we form the sum of squared errors weighted by the reciprocal of their variances:

$$\sum_{i=1}^{P} \left(\frac{\Psi_i^2}{\varepsilon_i} \right) = \varepsilon' \Psi^{-1} \varepsilon = (R - L_f)' \varepsilon^{-1} (R - L_f). \tag{13}$$

We then choose estimates of f denoted by $\hat{f}$ to minimize equation (13). We have for each j

$$\hat{f}_j = (\hat{B}' \hat{\Psi}^{-1} \hat{B})^{-1} \hat{B}' \hat{\Psi}^{-1} (R_j). \tag{14}$$

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One recognized weakness with this estimation procedure is that estimates for L may contain measurement error.[3] Other things equal, measurement error in $\hat{L}$ should increase the error term in the estimation of equation (2) above. If the measurement error is severe, it can reduce the degree to which the mimicking portfolios are correlated with the true unobservable factors.[4]

C. Step 3

As discussed above, we want to construct an orthonormal transformation matrix T that produces loadings that imply factors that are consistent with properties of the Treasury 30-year zero curve. To accomplish this, we find the linear combination of the original three money market factors as represented by each factor's mimicking portfolio, which minimizes the residual sum of squares with the level factor in the Treasury market. Then we find the linear combination of the original money market factors that minimizes residual sum of squares with the second Treasury factor (steepness), constraining it to be independent of the new money market factor just produced in the previous step. We then take residuals from previous regressions and project them on the original money market factors. The coefficients from these regressions produce a 3×3 matrix T. Since the original mimicking portfolios are normalized to have unit variance, the T matrix is an orthonormal matrix that we can use to rotate the original factor loadings matrix B. Let,

$$\hat{B} = BT \tag{15}$$

The mimicking portfolios that are associated with the loadings matrix $\hat{B}$ we then use to represent the factors F_j, $j = 1, \ldots, k$.

Finally, for instruments for which there are no data available, we estimate their loadings by interpolation. These include 4-month CDs, ECDs, and BAs and 5-month CPs and ECDs. In addition, in the T-bill market the 7-, 8-, 10-, and 11-month instruments are obtained in this fashion.

IV. Empirical Results

In this section, we present our empirical results. We estimate the loadings matrix L for $k = 1$ through $k = 10$. For $k = 10$, the likelihood ratio test shows no evidence against the model. The χ^2 statistic for the 10-factor model is 130.78 with 95 degrees of freedom and a P_b of 0.005. However, we choose to restrict our attention primarily to the case of the three- and four-factor cases. This is because the focus of the article is on the measurement and interpretation of a given set of factors to guide the theoretical development of term structure models with observable state variables rather than explicitly test-

[3] Since the estimates for $\hat{B}$ and $\hat{\Psi}$ are obtained by maximum likelihood procedure, they must satisfy the condition $\hat{B}'\hat{\Psi}^{-1}\hat{B} = \hat{\Delta}$.

[4] See Lehmann and Modest (1988) for a discussion of the impact of measurement on alternative estimation procedures.

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ing an asset pricing model. Although the three- and four-factor models are statistically rejected, they explain on average 86 and 90 percent of the variation in returns of a given money market security, respectively, and a larger percentage of the variation in diversified portfolios. Further, since the estimation procedure selects the first factor that explains most of the variation and then selects the next factor that explains the largest amount of what variation remains and so on, the marginal contribution of additional factors to the total amount of explained variation is quite low. For example, going from a three-factor to a four-factor model increases the amount of explained variation by only 4 percent. This is not meant to imply that research into the measurement and interpretation of additional factors is unimportant or uninteresting. In particular, expanding the universe of instruments to include securities that contain additional sources of systematic risk not represented fully in the existing securities will in general induce additional common factors.[5]

A. Factor Loadings

Table II presents the variance decomposition mentioned for TBs, CPs, CDs, and BAs, respectively, for the three-factor model.

This decomposition shows the relative importance of each of the factors in explaining the variation in each money market security's return. The three factors explain on average 86 percent of the variation in returns, if we exclude the 1-month T-bill. In each of the five markets, the first factor explains on average the largest percentage of the total explained variation. Among the five markets, the first factor is more important in the Treasury market than in the other money markets. For T-bills ranging from 2 to 12 months, factor 1 accounts for 89.3 percent of the total explained variation. If the 1-month T-bill is added, this average drops to 80 percent. This is because the level factor for the 1-month T-bill accounts for only 15 percent of the explained variation. For this security, factor 2 explains 60.8 percent of the total explained variation. This effectively means that steepness accounts for a large percentage of the moves in 1 month T-bills compared to the level factor. Since the level factor captures yield curve movements that represent changes in the general level of interest rates, this means that the 1-month T-bill is

[5] We examine the sensitivity of the total explained variation and the interpretation of the factors themselves to changes in the number of factors K, the universe of instruments considered, the sample period chosen, and the rotation matrix T chosen. The results show that, in general, the measurement and interpretation of the factors are robust to changes in these parameters. For example, specifying four instead of three factors does not significantly alter the measurement and interpretation of the original estimates of the three factors. In addition, the three- and four-factor models presented below represent rotated and unrotated factors, respectively, yet the measurement and interpretation of these factors is consistent across these models. In the first part of this section, we will present the three-factor model and describe its results. Since the results of the three- and four-factor models are very consistent with each other, this will serve as a lead-in to the four-factor model, which we present in the second part of this section.

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relatively insensitive to these types of movements. In the remaining four money markets, factor 1 explains an average of 71.2 percent of the total explained variation. With the exception of 1- and 2-week CP, it explains the largest percentage of explained variation for every security. In 1- and 2-week CP, factor 1 explains less of the total explained variation than both factors 2 and 3. In particular, factor 2 explains 70.1 percent of the total explained variation for 1-week CP and 59.7 percent for 2-week CP.

In the T-bill market, factor 2 explains the second largest amount of explained variation. It explains, on average, 16.3 percent of the explained variation. However, the number drops to 9.9 percent if 1-month T-bills are excluded and to 6.8 if 2-month T-bills are excluded. Thus, factor 2's contribution to explained variation is much larger at the very short end. In the remaining four money markets factor 2, on average, explains less of the variation in returns than factor 3. This is true for every security in each sector with the exception of 1- and 2-week CP, here, as discussed above, factor 2 explains more than factor 3. The average amount explained of all four markets is 10.3 percent. Notice that the amount of variation explained by factor 2 declines significantly with maturity in each of the four sectors.

In the Treasury market, factor 3 explains 3.5 percent of the total explained variation, the last of the three factors. This number drops to 0.7 percent if 1-month T-bills are excluded. In the remaining four money markets, factor 3 explains 18.5 percent on average of the total explained variation. Notice that with the exception of 1-month T-bills, factor 3, on average, explains a much larger percentage of the total explained variation.

Figure 1, Panels A to C depict the factor loadings for bond equivalent yields for factors 1, 2, and 3, respectively. The graph of each factor for each security represents the change that would be caused by a one standard deviation shock to that factor.

As seen from Figure 1, Panel A, the effect of the first factor on one-year Treasury yields is approximately constant for maturities from 2 to 12 months. Thus, a one standard deviation move in factor 1 in the Treasury bill market causes a "parallel" shift in the yield curve for maturities from 2 to 12 months. For this reason, we call the first factor the level factor. Notice that the level loading is significantly lower on the 1-month T-bill. This is probably because the 1-month T-bill provides certain liquidity services that are unavailable from other T-bills. For example, the 1-month T-bill can be considered a "cash equivalent" for accounting purposes. This induces a demand by firms for the 1-month T-bill to make their balance sheets appear more liquid. This conclusion is supported by the proportion of total explained variance accounted for by factor 1 for private issuer securities of one month maturity (shown in Table III). For the three-factor specification the proportions of total explained variance accounted for by factor 1 for 1-month CP, CDs, BAs, and ECDs are 41.6, 40.2, 43.9, and 39.7 percent, respectively, while for the 1-month T-bill it is only 15.5 percent. For the four-factor specification the results are even more dramatic (see Table III) since these represent unrotated factors. The level factor in all of the four remaining money market instruments has a

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Table II

Three-Factor Model

Total variance explained by the three-factor model for each security and the proportion of the total explained variance accounted for by each factor for each security. Explained ex post valuation is based on maximum likelihood factor analysis of weekly demeaned excess returns for the period January 1985 to August 1988.

Maturity	Total Variance Explained	Proportion of Total Explained Variance Accounted for by		
		Factor 1	Factor 2	Factor 3
Panel A: T-bills				
1 month	19.5	15.5	60.8	23.5
2 months	61.1	71.1	28.5	0.2
3 months	78.4	88.7	11.2	0.03
4 months	83.0	91.8	7.9	0.22
5 months	89.2	93.3	6.4	0.24
6 months	89.9	90.4	7.8	1.6
9 months	96.7	94.7	3.8	1.4
1 year	95.8	95.1	3.8	1.0
Average all securities	76.7	80.1	16.3	3.5
Average minus 1 month	84.8	89.3	9.9	0.7
Panel B: Commercial Paper				
1 week	33.0	1.1	70.1	28.7
2 week	33.8	10.1	59.7	30.0
1 month	88.4	41.6	28.4	29.9
2 months	88.2	58.8	18.6	22.4
3 months	88.8	72.4	12.8	14.6
4 months	87.7	81.1	6.5	12.3
6 months	85.6	88.4	3.2	8.4
Average all securities	63.4	50.5	28.4	23.5
Average minus 1- and 2-week securities	87.7	68.5	13.9	20.9
Panel C: Certificates of Deposit				
1 month	91.0	40.2	24.6	35.0
2 months	84.2	67.3	11.5	21.0
3 months	82.2	79.0	5.7	15.2
5 months	81.8	88.1	3.3	8.5
6 months	83.0	93.8	1.7	4.4
Average all securities	84.4	73.6	9.3	16.8
Panel D: Eurodollar Certificates of Deposit				
1 month	90.5	39.7	24.7	35.5
2 months	85.4	67.6	11.4	20.8
3 months	88.1	78.0	6.7	15.1
6 months	82.5	93.8	1.8	4.3
Average all securities	86.6	69.7	11.1	18.9

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Table II—*Continued*

Panel E: Bankers' Acceptances

Maturity	Total Variance Explained	Proportion of Total Explained Variance Accounted for by		
		Factor 1	Factor 2	Factor 3
1 month	94.5	43.9	24.2	31.8
2 months	92.7	66.6	11.3	22.0
3 months	87.4	77.9	5.9	16.1
5 months	82.5	88.2	3.2	8.5
6 months	80.6	94.1	1.7	4.1
Average all securities	87.5	74.1	9.3	16.5
Total all sectors	86.0	73.0	12.0	15.3
Money markets other than T-bills		71.2	10.3	18.5

slight steepening effect relative to the Treasury market. A one standard deviation shock to the level factor causes a much larger movement in its yield than to a 1-month CD. The magnitude of the effect increases approximately linearly with the maturity of the instrument. This effect is very consistent across all money market instruments with the exception of Treasuries. Notice too that the loadings are ordered almost uniformly in magnitude across securities, with CP having the largest loadings. This means that for a one standard deviation shock, CP securities across all maturities move more than the other money market instruments. As we will see, this result is consistent across the other factors.

Figure 1, Panel B presents factor 2 for the money market instruments. We call this factor the steepness factor. A shock to the short end of the curve causes yields to fall a proportionately larger amount than does a shock to the long end of the curve. The net effect is a steepening of each of the money market curves. The steepness factor loadings in all five money markets in the 1- to 6-month range decreases nonlinearly with maturity; as one moves lower in maturity, the effect has a larger and larger impact per unit decrease in maturity. Notice that the steepness effect is approximately uniform across all five markets in the 1- to 6-month range. That is, all five sectors' factor loadings are approximately parallel to each other. This means that for a shock to factor 2, money market yield curves steepen uniformly across sectors in each security.

Factor 3 we refer to as the Treasury factor. It represents the movement in money market yields that separate Treasury bills from the other money market instruments. A one standard deviation shock to factor 3 causes CDs, ECDs, CP, and BAs to move uniformly away from the Treasury bill market. For example, such a shock causes the yield to increase on the 6-month T-bill, while yields on all the other money market instruments decrease. This

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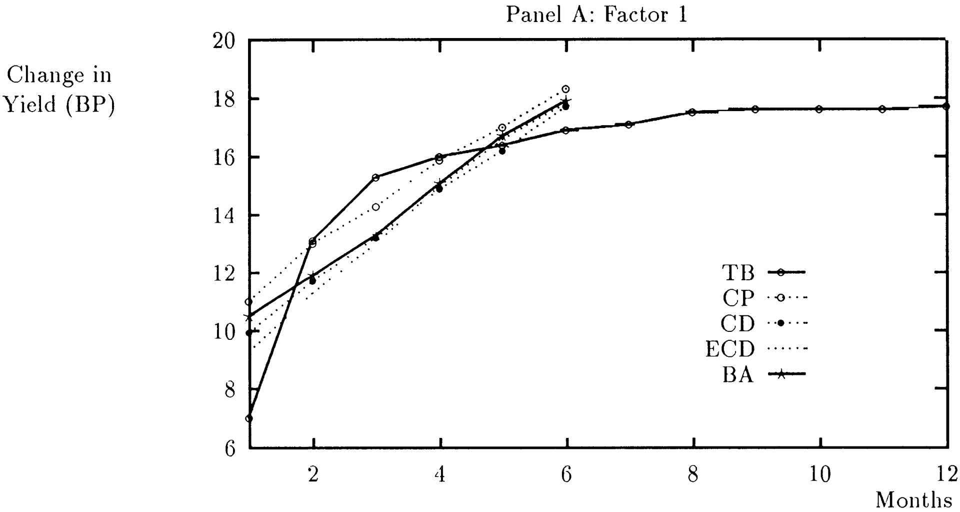

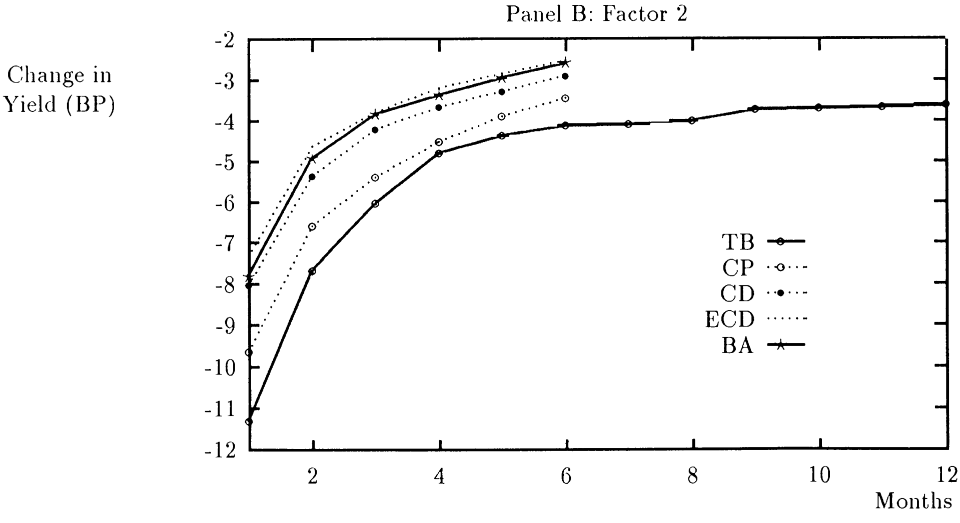

Figure 1. Loadings for three-factor model. Panels A, B, and C display the loadings in bond equivalent yield for factors 1, 2, and 3 of the three-factor model, respectively. The graph of each factor for each security represents the change that would be caused by a one standard deviation shock to that factor. The following abbreviations are used in these panels: TB, T-bills; CP, commercial paper; CD, certificates of deposit; ECD, Eurodollar certificates of deposit; and BA, bankers' acceptances.

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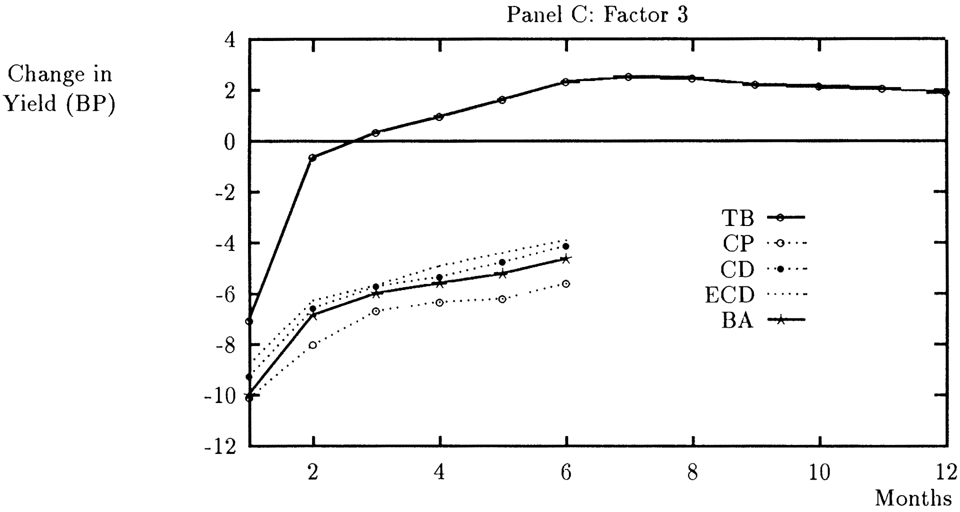

Figure 1.—*Continued*

decrease is uniform across all the money market instruments increasing uniformly as maturity decreases. Thus, this factor captures movements in yields that characterize a wedge betwen the four other money market instruments and the Treasury market. This effect captures the movement between instruments of different risk classes. Treasuries have no credit risk since they are backed by the full faith and credit of the U.S. government. All four other sectors of money market instruments have both bank and firm risk.

Table III displays the variance decomposition for the four-factor model like that presented in Table II for the three-factor model. The total explained variation in returns increases to 90.0 percent with the addition of the fourth factor. The fourth factor therefore adds an additional 4 percent to the explained variation. Comparing across factors, the order of importance of each factor in the four-factor model is consistent with the three-factor model. Factor 1 explains most of the variation in returns, accounting for, on average, 86 percent of the explained variation. As in the three-factor model, the first factor is still more important in the 3 to 12-month range than in the 1- to 2-month range. Factors 3 and 4 explain the least proportion of explained variation, accounting for 3 and 1.6 percent, respectively. Note that the total variance explained of the 1-month T-bill by factor 1 is less for the four-factor model (13.67 percent) than it is for the three-factor model (19.5 percent). This is because the three-factor model is rotated (see Step 3 in Section III) by finding a linear combination of the original three money market factors that minimize the residual sum of squares with the level factor in the 30-year treasury market. This rotation measures the total variance explained in the Treasury bill market at the expense of the total explained variance of the private issuer securities.

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Table III

Four-Factor Model

Total variance explained by the four-factor model for each security and the proportion of the total explained variance accounted for by each factor for each security. Explained ex post valuation is based on maximum likelihood factor analysis of weekly demeaned excess returns for the period January 1985 to August 1988.

Maturity	Total Variance Explained	Proportion of Total Explained Variance Accounted for by			
		Factor 1	Factor 2	Factor 3	Factor 4
Panel A: Treasury Bills					
1 month	13.67	10.6	50.0	8.3	32.0
2 months	39.46	52.0	34.0	0.2	14.0
3 months	79.2	81.3	11.8	0.004	6.9
4 months	84.4	79.1	14.8	0.07	6.0
5 months	91.5	77.3	16.5	0.2	6.0
6 months	92.3	71.5	19.7	0.003	8.8
9 months	96.4	75.6	22.2	0.1	2.1
1 year	95.4	78.2	19.6	0.2	2.0
Average all securities	73.9	65.6	23.5	4.02	0.1
Average minus 1 month	82.6	73.5	19.8	2.01	0.07
Panel B: Commercial Paper A1-P1					
1 month	92.3	82.2	13.1	0.01	4.7
2 months	93.6	93.3	5.3	0.7	0.7
3 months	97.4	96.6	1.0	2.4	0.01
4 months	97.8	96.6	0.05	2.1	1.2
6 months	96.6	94.2	0.5	2.2	3.1
Average all securities	95.4	92.5	4.0	1.6	1.9
Panel C: Commercial Paper A2-P2					
1 month	93.3	78.2	15.4	0.2	6.2
2 months	95.7	92.2	5.2	1.6	0.9
3 months	97.9	94.8	2.6	2.5	0.06
4 months	96.9	95.4	0.3	2.9	1.4
6 months	91.3	93.8	0.3	3.3	2.7
Average all securities	94.5	90.8	4.7	2.0	2.2
Panel D: Certificates of Deposit					
1 month	93.3	71.1	10.4	17.8	0.8
2 months	95.6	80.7	1.4	17.8	0.2
3 months	95.6	85.1	0.04	11.4	3.4
5 months	95.8	87.1	2.6	6.6	3.7
6 months	93.3	86.9	4.7	4.7	3.6
Average all securities	94.0	82.1	2.8	11.3	2.3

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Figure 2 Panels *A* to *D* depict the factor loadings for bond equivalent yields for factors 1, 2, 3, and 4, respectively. As in Figure 1, the graph of each factor for each security represents the yield change that would be caused by a one standard deviation shock to that factor. Notice that the effect of the first factor in the four-factor model continues to be a level effect as in the three-factor model. Factor 2 in the four-factor model also represents a steepness effect. Factor 4 of the four-factor model is consistent with the Treasury factor of the three-factor model. It represents the movement in money market yields that separate T-bills from the private issuer markets. Figure 2 Panel *C* depicts factor 3 of the four-factor model. It represents movements in money market yields that separate CP from the other private issuer markets, in particular CDs. For example, a one standard deviation shock to this factor causes CDs, BAs, and ECDs to all move uniformly away from CP. One interpretation of this factor is that it captures the changes in firm risk and bank risk that characterize the firm-issued securities, such as CP, and the bank-issued securities, such as CDs. For this reason, we call this a private issue credit factor. When moving from a three-factor to a four-factor specification and adding lower-rated CP, the essential change is the addition of a factor that represents changes between firm and bank risk. The other factors remain qualitatively unchanged.

B. Mimicking Portfolios

Having portfolios that mimic a particular factor allows us to investigate further the properties of the unobservable common factors and the impact of these factors on other money market instruments.[6] The mimicking portfolios, by construction, have zero mean and unit variance. As a result, if we regress the demeaned excess return of a security on the three mimicking portfolios, the coefficients on the three mimicking portfolios represent the factor loadings for that particular security. Formally, we have (in the limit as measurement error goes to zero):

$$R_i = b_{i1}M_1 + b_{i2}M_2 + b_{i3}M_3 + \varepsilon_i \tag{16}$$

where R_i is the demeaned excess return on security i, b_{ij}, $j = 1, 2, 3$ is the factor loading for security i and factor j and M_1, M_2, and M_3 are the mimicking portfolios for factors 1, 2, and 3, respectively. For example, we regress the excess returns of one-, three-, and six-month generic repo rates on M_1, M_2, and M_3 over our entire sample period. We find that the factor loadings for each of the instruments are qualitatively similar to the other money markets. Factor 1 is again a level effect, and factor 2 is a steepness effect. Factor 3 for repos behaves like CP, CDs, ECDs, and BAs. A one

[6] Since we have assumed that the factors are time invariant, first-order serial correlation in the mimicking portfolio would suggest the rejection of a serially uncorrelated model in favor of an intertemporal factor model. Results from regressing each mimicking portfolio on itself lagged one period suggests that, in general, there is little or no evidence of first-order serial correlation in mimicking portfolios.

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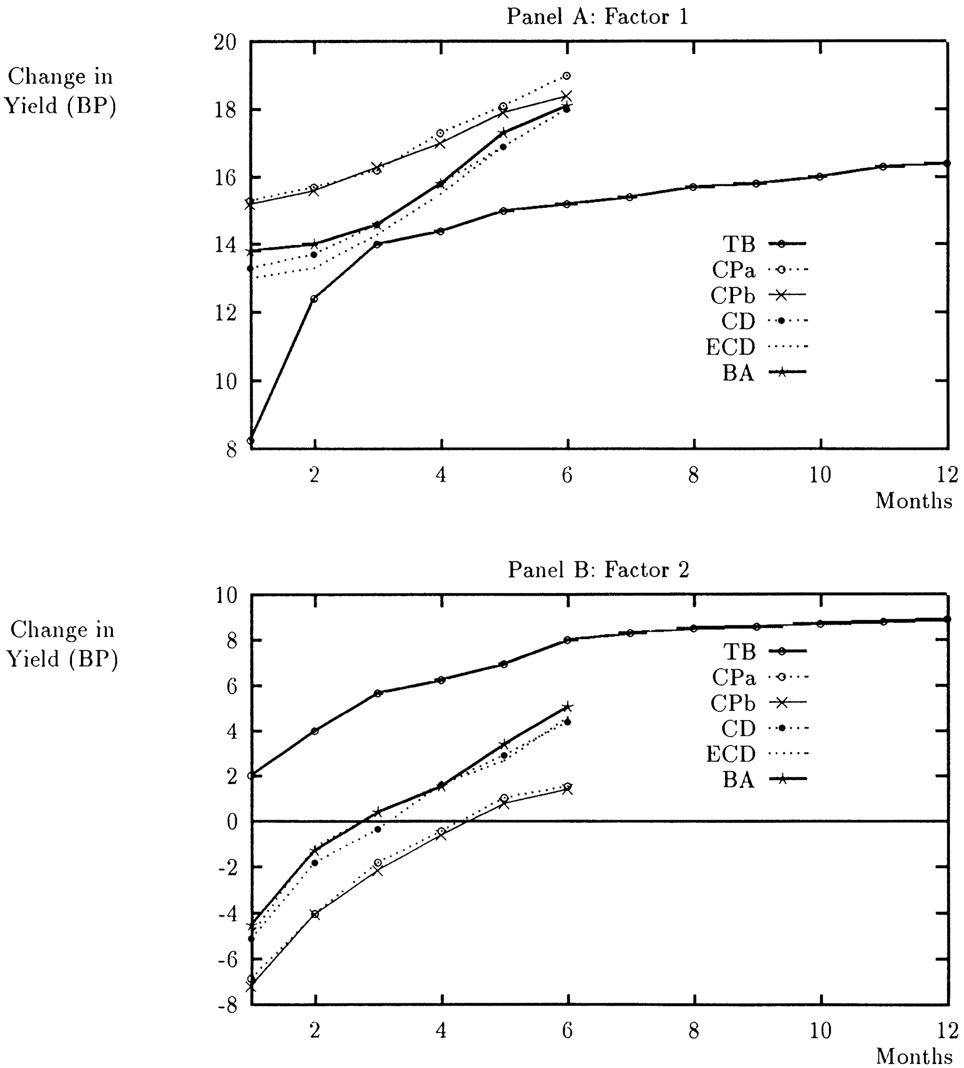

Figure 2. Loadings for four-factor model. Panels A, B, C, and D display the factor loadings in bond equivalent yield for factors 1, 2, 3, and 4 of the four-factor model, respectively. The graph of each factor for each security represents the change that would be caused by a one standard deviation shock to that factor. The following abbreviations are used in these panels: TB, T-bills; CP, commercial paper; CD, certificates of deposit; ECD, Eurodollar certificates of deposit; BA, bankers' acceptances.

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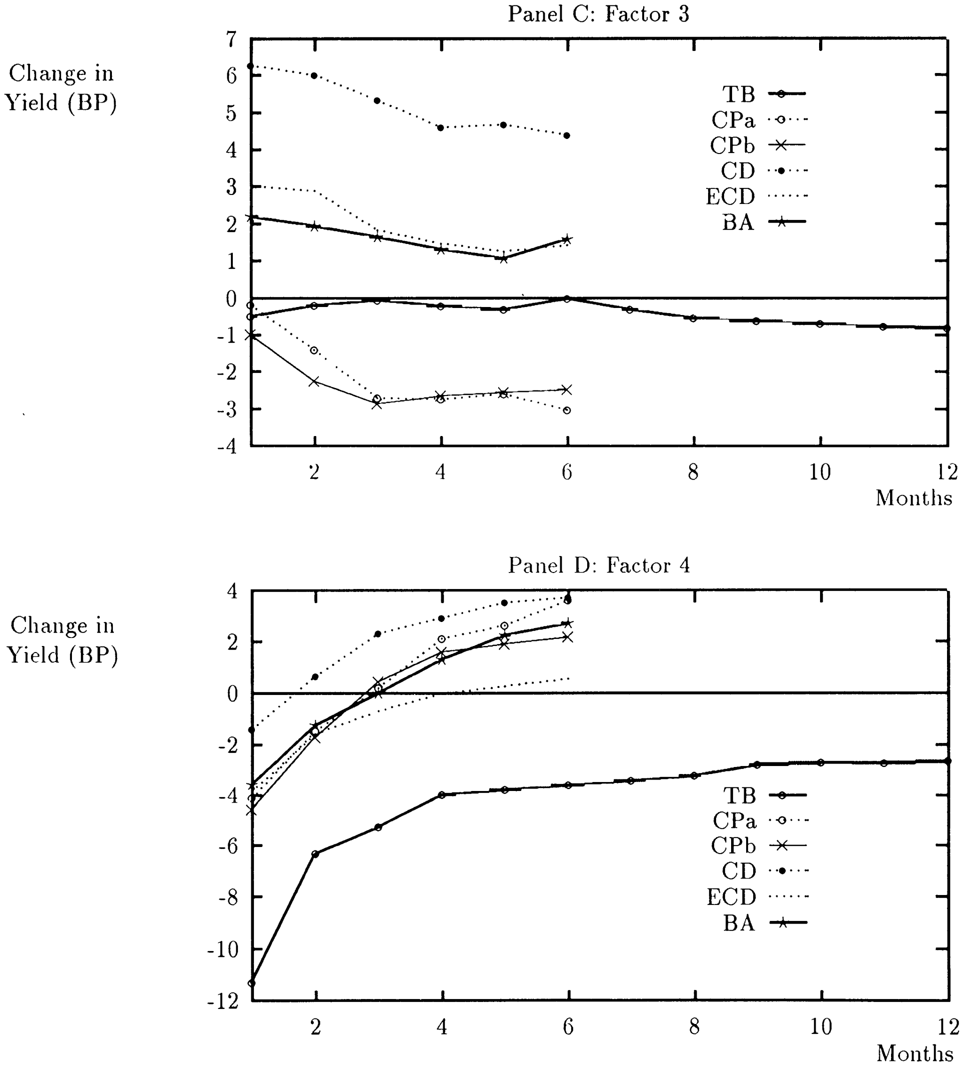

Figure 2.—*Continued.*

standard deviation shock to factor 3 causes repos to move uniformly away from the Treasury bill market. This movement captures the credit risk present in term repo that is not present in T-bills. The three factors explain on average 53 percent of the variation of these three instruments. This is significantly less than the 80 percent average for the other money market instruments. This means that the repo market has substantial movements that are "uncommon" with other moneymarket instruments. This result is consistent with the view that changes in the supply and demand for collateral

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affect generic repo rates by changing the spread between the Fed funds rate and repo rates. This effect is not found in other money market instruments.

V. Conclusion and Related Work

This article explores the common factors that describe money market returns. These common factors are interpreted as sources of systematic risk that parameterize the systematic movements in yield curves. We find that three factors explain, on average, 86 percent of the total variation in returns, and four factors explain 90 percent. In both the three- and four-factor models, three factors represent a level, a steepness, and a Treasury factor. The additional factor in the four-factor model represents a private issue credit factor. The first factor in both models accounts for an average of 71 percent of the explained variation in the two models. This may be interpreted as the change in the difference between firm and bank risks. Although factor 1 accounts for the largest percentage of explained variation, in general, factors 2 and 3 are of equal importance, on average, in the 1-week to 2-month region of the yield curve. In the T-bill market for the 1-month bill (although the amount of explained variation is low), factors 2 and 3 account for a larger percentage of the explained variation than factor 1. In particular, factor 2 accounts for approximately 60 percent of the variation in the 1-month T-bill (it is still large for the 2-month T-bill, 28.5 percent, but less than factor 1). One interpretation of this result is that factor 2 represents the impact of Federal Reserve policy on the yield curve. That is, changes in Federal Reserve policy affect the yield curve primarily through changes in the steepness of the short end of the yield curve.

These results suggest that a term structure model with three or four state variables—one that captures changes in the general level of interest rates, a second that captures changes in the steepness of the yield curve, a third that captures changes in the spread between the T-bill market and the private issuer market, and a fourth that captures changes in the credit spread between CP and the other private issuer markets—would describe almost all of the total variation in returns.

Much of the recent empirical work testing term structure models is consistent with this view. For example, Brown and Dybvig (1986), Gibbons and Ramaswamy (1993) and Stambaugh (1988) all find evidence in favor of multifactor models. This work focuses on the government bond market and investigates the empirical implications of the Cox, Ingersoll, and Ross (1985) model. As a result, a comparison to our work can only be made tenuously. Stambaugh (1988) follows Fama (1984) by investigating whether the information in forward premiums is consistent with the one- and two-factor general equilibrium models developed by Cox, Ingersoll, and Ross (1985). Using the forward premiums as linear predictors of excess returns, he constructs a rank test for the number of latent variables required to describe expected excess returns on T-bills. He finds that two, possibly three, latent variables are

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sufficient to describe the expected excess returns on T-bills. Chen and Scott (1990) also estimate a multifactor version of the Cox, Ingersoll, and Ross (1985) nominal bond pricing model. Their conclusion is that the three-factor model performs slightly better than the two-factor model and that both the two-factor and three-factor models perform much better than the one-factor model. Note, however, that this work is focused on a pricing question in the context of tightly specified general equilibrium models, while our work is directed at the factors useful in describing the variation in ex post returns. As a result, no direct comparison is warranted since, in the context of a pricing question, it may be the case that one or more of the risk premia associated with our factors is insignificant.

One desirable extension of the work discussed here consists of increasing the sample period to include at least one complete business cycle. Recently, Fama (1986) and Stambaugh (1988) claim to find that both term premia and default premia exhibit a dependence on the state of the business cycle. In addition, the expansion (or contraction) of the universe of instruments considered should be undertaken. For example, the joint estimation of the money market instruments considered here and the Treasury bond market would shed light on the interaction between the long end and short end of the yield curve that includes both government and private issuer markets.

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