Proceedings of the 2005 IEEE
International Conference on Robotics and Automation
Barcelona, Spain, April 2005

Bounded Smooth Time Invariant Motion Control of Unicycle Kinematic Models

Youngshik Kim and Mark A. Minor
Department of Mechanical Engineering
University of Utah
Salt Lake City, UT 84112, USA
youngshik.kim@utah.edu, minor@mech.utah.edu

Abstract – **This paper presents a new curvature based smooth time invariant control law for posture regulation and path following control of unicycle type kinematic models. The control law is developed in polar coordinates for posture regulation using nonlinear Lyapunov techniques and extended for path following via waypoint navigation techniques. The algorithm is then applied to a two-axle compliant framed modular mobile robot. To this goal the complex robot kinematics are reduced to an equivalent unicycle kinematic model where we must consider physical constraints on wheel velocity and frame curvature. Bounded curvature and velocity expressions are thus derived as control inputs. The controller is then extended to compensate for non-ideal initial conditions and drift. Simulation and experimental results evaluate algorithm performance.**

Index Term — **Lyapunov, motion control, polar coordinates, unicycle type mobile robot.**

I. INTRODUCTION

The posture regulation problem of a Compliant Framed Modular Mobile Robot (CFMMR), Fig. 1, has been studied in the previous research [1]. This wheeled mobile robotic system uses flexible frame elements to couple rigid differentially steered axles, which provides suspension and highly controllable steering capability without any additional hardware. Steering and maneuvering of the system are thus accomplished via coordinated control of the axles.

While the compliant frame provides uniqueness and simplicity, it brings out issues of curvature and velocity constraints. Physically, large frame curvature may lead to contact of front and rear wheels. An actuator of a real robot likewise has limited capabilities and realizable wheel velocity is restricted. Furthermore, unbounded velocity commands may induce wheel slip, large frame curvature, and excessive frame forces. Thus, we must consider bounded velocity and curvature as control inputs as well as nonholonomic constraints of the robot.

A curvature based smooth time invariant control law is developed to solve posture regulation of unicycle type robots and then extended for path following. The complex kinematics of the CFMMR are simplified by an equivalent curvature based model extended from a typical unicycle kinematic model. The proposed control law is thus well suited for the CFMMR as well as unicycle type robots. In order to resolve geometric constraints on the robot, maximum allowed curvature of the frame is identified and realized in an equivalent curvature based kinematic model.

In Section II we examine similar control strategies. The equivalent kinematic model is developed using a unicycle kinematic model in Section III. A curvature based smooth time invariant control law is then developed in Section IV using Lyapunov techniques. Simulation and experimental results and future work are discussed in Section V. Concluding remarks are provided in Section VI.

II. BACKGROUND

Control of nonholonomic mobile robotic systems has received a great deal of attention in recent years and many motion control schemes have been proposed to consider their nonholonomic constraints. Traditionally, Cartesian coordinates have been used to model mobile robots [2-4]. In order to deal with limited capability of a real robot, some researchers have considered physical constraints such as velocity or acceleration by introducing saturation functions in their controllers [5, 6]. The aforementioned algorithms resulted in time-varying control strategies. This is because a smooth time invariant control law cannot be facilitated for stabilization of nonholonomic robots in Cartesian coordinates as Brockett [7] proved. As indicated in [2], a time-varying control scheme may also suffer from slow convergence rate.

Astolfi [8] thus presented the use of non-smooth polar coordinates instead of Cartesian coordinates to obtain a smooth time invariant control law for posture regulation. Due to the discontinuity in the polar representation, Brockett's Theorem [7] is circumvented and a smooth time

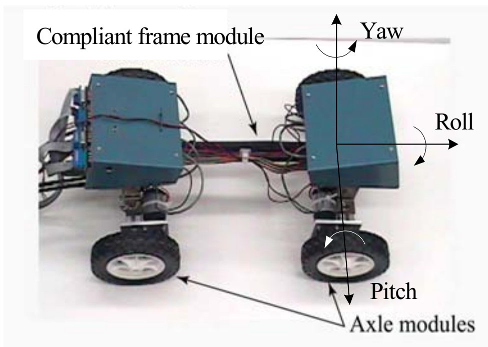

Fig. 1. Two-axle CFMMR.

This work is supported by NSF Grant # IIS-0308056.

invariant state feedback control law for global asymptotic stability is possible. Exponential stability may be facilitated via discontinuous time invariant control as well [9]. For these reasons polar representation has been commonly adopted as a non-smooth coordinate transformation in the literature [1, 10, 11]. Moreover, in [12, 13] single control laws for posture regulation and path following have been proposed in the polar representation. However, their approaches focus on the controller development itself without considering physical limitations. Some papers [10, 11] consider physical constraints, but their approaches are limited to posture regulation.

To realize a new bounded control law for posture regulation and path following in this research, smooth and well bounded velocity control inputs are first derived for posture regulation in polar representation using Lyapunov techniques. These bounded velocities are then used to realize bounded path curvature via gain adjustments similar to [11]. The control strategy is finally applied to path following via waypoint navigation. While the kinematics of the CFMMR are much more complex, Albiston [1] previously indicated that they could be described in an equivalent coordinate frame that admits familiar steering algorithms for the unicycle model. In this paper we extend the equivalent coordinate frame for path following as well as posture regulation. We also extend controller dynamics to accommodate non-ideal initial conditions and drift. This is similar in spirit to the extension performed in [1, 8] that accommodates for nonholonomic systems with drift.

III. Kinematic Model

The compliant framed mobile robot has much more complex steering kinematics than unicycle type vehicles since it possesses independently steered axles with compliant coupling. Depending on the frame deflection imposed by the kinematics of the axles, the compliant frame module of the robot can be steered to maintain pure bending and constant curvature condition, which is the equivalent of front and rear steering angles $\psi_1 = -\psi_2$, as discussed in Albiston [1]. The complete kinematic configuration of the CFMMR is illustrated in Fig. 2. This kinematic configuration can provide maximum steering capability and require minimum wheel traction force for the robot. Using these kinematics, the net position and orientation of the robot may be described by an equivalent posture attached to point O located at the center of a line drawn between the axle midpoints, as shown in Fig. 2.

Before deriving the equivalent kinematic relationships, three conditions for maneuverability of the compliant wheeled mobile robot are introduced: *(i)* The robot proceeds only on paths of bounded curvature due to the kinematic constraints on the beam module in contrast to a unicycle robot; *(ii)* For simplicity the robot's motion is commanded in the forward direction; *(iii)* The pure bending condition, $\psi_1 = -\psi_2$, is maintained throughout all turning maneuvers.

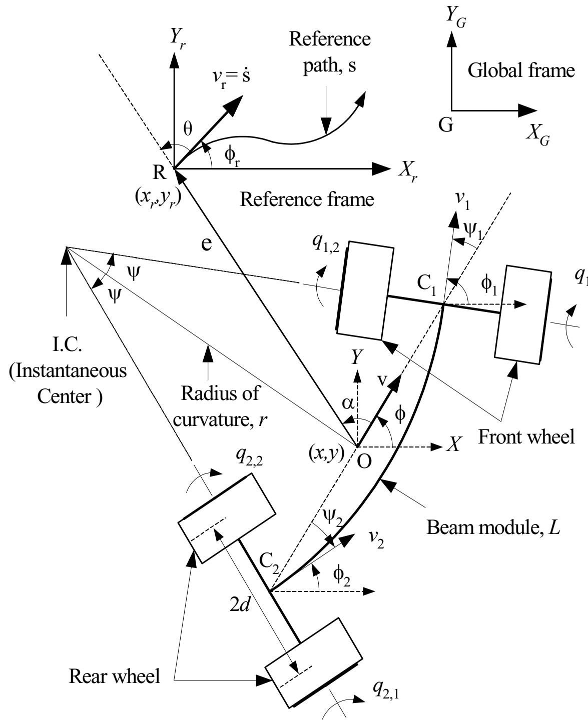

Fig. 2. Compliant framed mobile robot kinematics

Using the polar representation of an equivalent posture attached to point O, the kinematics can be written in error coordinates,

$$\begin{aligned} \dot{e} &= -v\cos\alpha + v_r\cos\theta \\ \dot{\theta} &= v\frac{\sin\alpha}{e} - v_r\frac{\sin\theta}{e} - \dot{\phi}_r \\ \dot{\alpha} &= v\frac{\sin\alpha}{e} - v_r\frac{\sin\theta}{e} - \dot{\phi} \end{aligned} \tag{1}$$

where the variable v represents the velocity of the coordinate frame O moving in a heading ϕ relative to the global frame G. The subscript r denotes the reference frame. That is, v_r and ϕ_r are reference velocity and reference heading angle of the coordinate frame R, respectively. The error states in polar representation are defined as

$$\begin{aligned} e &= \sqrt{(x-x_r)^2 + (y-y_r)^2} \\ \theta &= ATAN2\left(-(y-y_r), -(x-x_r)\right) - \phi_r \\ \alpha &= \theta - \phi + \phi_r \end{aligned} \tag{2}$$

where x and y are the Cartesian coordinates of a moving coordinate frame attached to the point, O, that describes the equivalent posture. A reference position (x_r, y_r) is attached to the moving frame R.

A. Derivation of the kinematic equations:

In order to derive a kinematic model in polar coordinates, we first consider nonholonomic constraints of a mobile robot in Cartesian coordinates and coordinate transformation:

$$
\begin{aligned}
&x_r - x = e\cos(\alpha+\phi) = e\cos(\theta+\phi_r)\\
&y_r - y = e\sin(\alpha+\phi) = e\sin(\theta+\phi_r)\\
&\dot{x} = v\cos\phi\\
&\dot{y} = v\sin\phi\\
&\dot{x}_r = v_r\cos\phi_r\\
&\dot{y}_r = v_r\sin\phi_r\ .
\end{aligned} \tag{3}
$$

To derive a state equation for the distance error, e, square and differentiate the first equation in (2), which results in,

$$\dot{e} = \frac{(x_r - x)(\dot{x}_r - \dot{x}) + (y_r - y)(\dot{y}_r - \dot{y})}{e}\ . \tag{4}$$

Substituting (3) into (4) and applying trigonometric identities, a state equation for e is then obtained,

$$\dot{e} = -v\cos\alpha + v_r\cos\theta\ . \tag{5}$$

To derive state equations for angles, modify the second equation in (2),

$$\tan(\theta+\phi_r) = \frac{y_r - y}{x_r - x} \tag{6}$$

and differentiate (6),

$$\dot{\theta} + \dot{\phi}_r = \frac{(\dot{y}_r - \dot{y})(x_r - x) - (y_r - y)(\dot{x}_r - \dot{x})}{(x_r - x)^2\sec^2(\theta+\phi_r)}\ . \tag{7}$$

Substituting (3) into (7) and using trigonometric identities, a state equation for θ is then written,

$$\dot{\theta} = v\frac{\sin\alpha}{e} - v_r\frac{\sin\theta}{e} - \dot{\phi}_r\ . \tag{8}$$

Finally, differentiating the third equation in (2) and substituting (8) for θ, a state equation for α is derived,

$$\dot{\alpha} = \dot{\theta} - \dot{\phi} + \dot{\phi}_r = v\frac{\sin\alpha}{e} - v_r\frac{\sin\theta}{e} - \dot{\phi}\ . \tag{9}$$

The angular velocity of the equivalent posture attached to point O can be described as a function of the bounded curvature, κ and the linear velocity v, $\dot{\phi} = v\kappa$. Likewise, the reference angular velocity is expressed as $\dot{\phi}_r = v_r\kappa_r$.

The angular velocity of the robot center point O can be described as a function of the steering angles ψ_1 and ψ_2. Since $\psi = \psi_1 = -\psi_2$ by the pure bending condition, the expression for the radius and curvature of the robot center point O can be written as,

$$\kappa = \frac{1}{r} = \frac{2\psi}{L\cos\psi} \tag{10}$$

where the steering angle ψ may be solved numerically using the equation given the frame length, L. The linear and angular velocities of each axle may be found from the linear velocity v and path curvature κ of the center posture, O. Referring to the foreshortening expression presented in [14, 15], the linear and angular velocities of each axle, v_i, and $\dot{\phi}_i$, wheel angular velocities, $\dot{q}_{i,j}$, are,

$$
\begin{aligned}
&v_i = \frac{v}{\cos\psi} + \frac{(-1)^i}{6}L_f\psi\dot{\psi}\\
&\dot{\phi}_i = \dot{\phi} + (-1)^{i-1}\dot{\psi}\\
&\dot{q}_{i,j} = \frac{v_i + (-1)^{j-1}\dot{\phi}_i d}{r_w}\ .
\end{aligned}
\quad ; \quad
\begin{cases}
i = 1 \text{ for front axle}\\
i = 2 \text{ for rear axle}\\
j = 1 \text{ for right wheel}\\
j = 2 \text{ for left wheel}
\end{cases} \tag{11}
$$

where d is the half length of an axle, r_w is the wheel radius, and L_f is the shortened length of the frame,

$$L_f = L\left(1 - \frac{2\psi_1^2 - \psi_1\psi_2 + 2\psi_2^2}{30}\right) = L\left(1 - \frac{\psi^2}{6}\right). \tag{12}$$

IV. Smooth Time Invariant Control Law

In order to realize a control law, we first consider posture regulation of a unicycle model. Familiar Lyapunov-based nonlinear control techniques are utilized such that the robot posture will approach the origin of *x-axis* the system asymptotically. The control law is then extended for path following via waypoint navigation.

A. Posture regulation

The system (1) is simplified for posture regulation,

$$
\begin{aligned}
&\dot{e} = -v\cos\alpha\\
&\dot{\theta} = v\frac{\sin\alpha}{e}\\
&\dot{\alpha} = v\frac{\sin\alpha}{e} - \dot{\phi}\ .
\end{aligned} \tag{13}
$$

A smooth time invariant control law is determined using two familiar quadratic Lyapunov candidate functions that may be frequently found in literature [8, 10, 13],

$$
\begin{aligned}
&V_1 \equiv \frac{1}{2}e^2\\
&V_2 \equiv \frac{1}{2}(k_2\theta^2 + \alpha^2),\ (k_2 > 0)\ .
\end{aligned} \tag{14}
$$

Consider Lyapunov function V_1 and V_2 to obtain the control velocity v and $\dot{\phi}$ (i.e. κ), respectively. The derivatives of Lyapunov functions are written as,

$$
\begin{aligned}
&\dot{V}_1 = e\dot{e} = -ev\cos\alpha\\
&\dot{V}_2 = k_2\theta\dot{\theta} + \alpha\dot{\alpha} = k_2\theta v\frac{\sin\alpha}{e} + \alpha\left(v\frac{\sin\alpha}{e} - \dot{\phi}\right).
\end{aligned} \tag{15}
$$

The proposed control law v and $\dot{\phi}$ is then determined as,

$$
\begin{aligned}
&v = v_{\max}\tanh(k_1 e);\ k_1 > 0,\ k_3 > 0\\
&\dot{\phi} = v_{\max}\left\{\left(1 + k_2\frac{\theta}{\alpha}\right)\frac{\tanh(k_1 e)}{e}\sin\alpha + k_3\tanh\alpha\right\}
\end{aligned} \tag{16}
$$

where $v_{\max}$ is the maximum linear velocity. The proposed controllers are smooth time invariant functions that asymptotically drive the state (e,θ,α) towards the origin of the system. Stability with the developed control law (16) may be proved using Lyapunov's theorem and Barbalat's Lemma [16] similarly to [10, 12, 13]. Since $\alpha \to 0$ as $t \to \infty$, the derivatives of the Lyapunov functions become negative definite except for the equilibrium point in our control strategy once $\cos(\alpha) > 0$:

$$\begin{aligned} \dot{V}_1 &= -v_{\max} e \tanh(k_1 e)\cos\alpha \\ \dot{V}_2 &= -v_{\max} k_3 \alpha \tanh\alpha \end{aligned} \tag{17}$$

Notice that continuous *tanh* functions are implemented to make the derivatives of the Lyapunov functions semi-negative definite and to guarantee bounded control inputs for large initial conditions. Approximating the angular velocity expression with large and small error states, we obtain $0 < k_3 \leq \kappa_{\max}$. Since our equivalent kinematics utilized curvature expression, the path curvature is written as a control input,

$$\kappa = \frac{\dot{\phi}}{v} = \frac{\sin\alpha}{e} + k_2 \frac{\theta}{e}\frac{\sin\alpha}{\alpha} + \frac{k_3 \tanh\alpha}{\tanh(k_1 e)\cos\alpha} \tag{18}$$

B. Boundedness

The boundedness of the control law (16) is considered here. The linear velocity is well bounded since $0 \leq \tanh(k_1 e) < 1$. Since $|\tanh(k_1 e)/e| \leq k_1$, $|\sin\alpha/\alpha| \leq 1$, $|\theta| \leq \pi$ and $|\tanh\alpha| \leq 1$, the ultimate bound of the angular velocity may be determined by,

$$\left|\dot{\phi}\right| < v_{\max}\left(k_1 + \pi k_1 k_2 + k_3\right) \tag{19}$$

However, it is not promising to evaluate the curvature bound directly from the control law (18) because the state variables are implicitly and dynamically dependent upon initial conditions and state equation evolution through time. As other researchers [1, 10-12] have proposed, we design the angles (θ, α) to converge to zero faster than e to guarantee curvature boundedness as the states converge to zero asymptotically. Thus, we may linearize the control law such that $\dot{\phi} \to v_{\max}\left\{k_1\left(\alpha + k_2\theta\right) + k_3\alpha\right\}$ and $v \to v_{\max} k_1 e$ as $(e,\theta,\alpha) \to (0,0,0)$. Applying the control law and linearizing the system in the neighborhood of the origin, the system is written,

$$\begin{aligned} \dot{e} &= -v_{\max} k_1 e \\ \dot{\theta} &= v_{\max} k_1 \alpha \\ \dot{\alpha} &= -v_{\max} k_3 \alpha - v_{\max} k_1 k_2 \theta \end{aligned} \tag{20}$$

To determine gains (k_1, k_2 and k_3), we utilize eigenvalues of (20). The eignevalues of the angles are determined as,

$$\lambda_{1,2} = -\frac{v_{\max}}{2}\left(k_3 \pm \sqrt{k_3^2 - 4k_1^2 k_2}\right) \tag{21}$$

The angles (θ, α) must converge to zero faster than e. This is guaranteed if the real parts in eigenvalues of θ and α are faster than that of e. Since imaginary parts of $\lambda_{1,2}$ will produce oscillations in θ and α, they are not desirable. Hence, the following inequalities should be satisfied,

$$\begin{aligned} & k_3^2 - 4k_1^2 k_2 \geq 0 \\ & \frac{v_{\max}}{2}\left(k_3 - \sqrt{k_3^2 - 4k_1^2 k_2}\right) > v_{\max} k_1 \ , \end{aligned} \tag{22}$$

which results in the following gains requirements:

$$2k_1\sqrt{k_2} < k_3 < k_1(1 + k_2);\ k_3 \leq \kappa_{\max},\ k_2 > 1 \tag{23}$$

Further, we optimize these gains with this condition to obtain bounded control inputs. This approach will guarantee the boundedness of curvature in finite time. However, since the curvature expression (18) is a function of α, large initial path curvatures may be observed for a short time depending on the initial orientation angle. Thus, a saturation algorithm must be implemented for hardware implementation to avoid this problem of transient initial curvature.

C. Path following using waypoints

Reference velocity terms in (1) may be considered as perturbations into (13) such that we extend the proposed controller for path following control. In order to resolve perturbations, we adopt the waypoint navigation strategy similarly to Aicardi [13]. A waypoint is considered as the origin of the system at any given time and the robot is stabilized along a path created by waypoints. Reference velocity is designed to generate waypoints:

$$v_r = \begin{cases} 0, & k_1 e + k_2\theta^2 + \alpha^2 > \varepsilon \\ \dot{s}, & k_1 e + k_2\theta^2 + \alpha^2 \leq \varepsilon \end{cases} \tag{24}$$

where an error tolerance, ε is a dominant parameter in manipulating system errors and reference paths. If system errors approach the inside of ε, a new waypoint is generated and the robot is stabilized to this point. Otherwise, the controller stabilizes the robot to the previous waypoint until the robot posture error reaches within ε. This procedure is iteratively executed for path following control.

For simplicity, the path curvature κ_r and linear path velocity $\dot{s}$ are determined as, $\dot{s} = v_{des}$ and $\kappa_r = \kappa_{des}$ where v_{des} and κ_{des} represent desired constant velocity and curvature of the path, respectively. The proposed control law can thus solve both the posture regulation and the path following problems with the bounded control inputs.

D. Dynamic extension

It is frequently observed that critically large control inputs are commanded at the beginning of the control action owing to initial conditions. This problem may be resolved by implementing the dynamic extension. The robot initial conditions rarely match the initial conditions of v and κ commanded by the controller. In addition, perturbations caused by non-ideal errors between the desired and actual control inputs throughout the robot motion may lead to a significant error at the final stabilization point. This drift may be resolved with a change in the feedback where the controller dynamics are extended in a cascade system with the same spirit of Bacciotti Theorem [17] by introducing new states,

$$\begin{aligned} \dot{v} &= k_v(v - v_D) + \dot{v}_D \ ; k_v < 0,\ k_c < 0 \\ \dot{\kappa} &= k_c(\kappa - \kappa_D) + \dot{\kappa}_D \end{aligned} \tag{25}$$

where v and κ are the extended velocity and curvature states, and v_D and κ_D are the desired control velocity and curvature established by (16) and (18). Gains, k_v and k_c represent eigenvalues of (25). Since the dynamic extensions add additional servo-loops to the original system, we design the responses of the dynamic extension to be faster than the original system by designing k_v and k_c << $\lambda_{1,2}$. Boundedness of the actual control inputs is guaranteed if the desired control inputs are bounded. Recall that the desired control inputs (16) and (18) are designed to be bounded by $|\kappa_D(t)| \leq \kappa_{\max}$ and $0 \leq v_D(t) \leq v_{\max}$. Since the robot is considered to be at equilibrium initially, $v(0) = \kappa(0) = 0$, the expression (25) in the time domain can be written in the form,

$$\begin{aligned} v(t) &= v_D(t) - v_D(0)\exp\left(k_v t\right) \\ \kappa(t) &= \kappa_D(t) - \kappa_D(0)\exp\left(k_c t\right)\ . \end{aligned} \tag{26}$$

Notice that $v(t) \to v_D(t)$ and $\kappa(t) \to \kappa_D(t)$, as $t \to \infty$, and that typically $0 < v_D(0) \leq v_{\max}$, $|\kappa_D(0)| \leq \kappa_{\max}$. Thus, the absolute bounds of extended control inputs are obtained, $0 \leq v(t) \leq v_{\max}$ and $|\kappa(t)| \leq \kappa_{\max} - \kappa_D(0)$. Magnitudes of the actual velocity and curvature generated by dynamic extension are enforced to be zero initially to match the robot initial conditions and kept smaller than the desired values. Since excessive curvature requirements have been found to be brief, typically, these properties of dynamic extension favor boundedness requirements as well as drift free maneuverability.

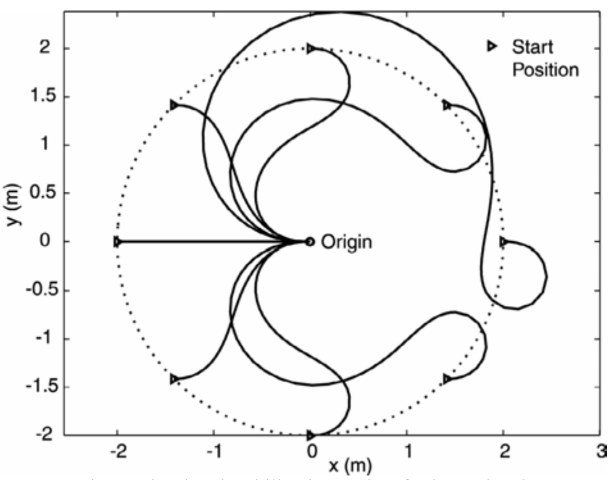

Fig. 3. Simulated stabilization paths of robot point O

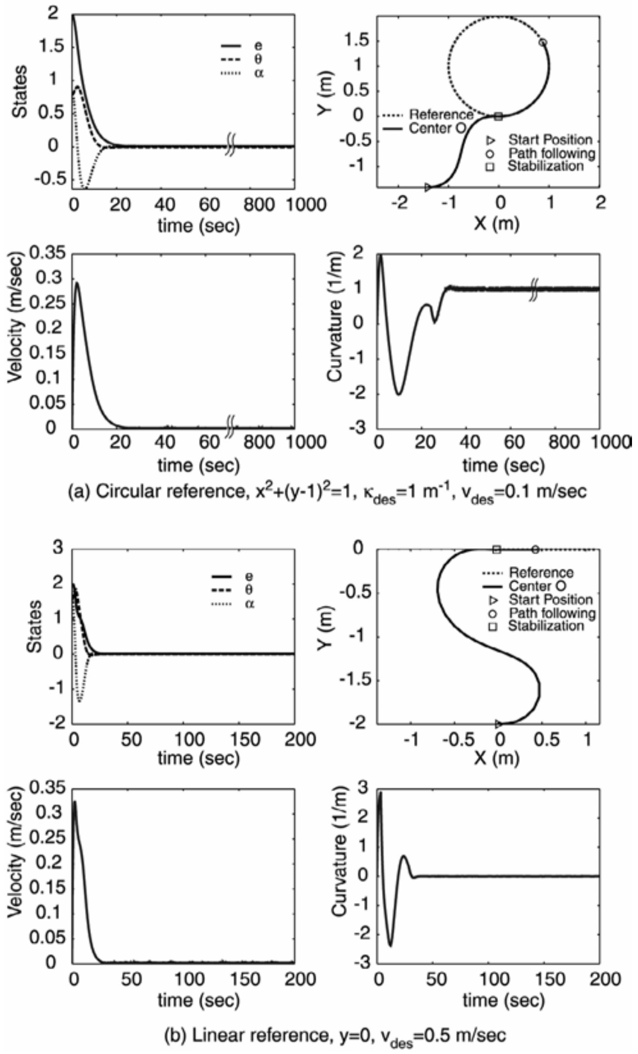

Fig. 4. Simulation results, I.C. (e,θ,α) of (a) $(2,\pi/4,\pi/4)$, (b) $(2,\pi/2,\pi/2)$

V. Simulation and Experiment

Fig. 3 demonstrates simulation of posture regulation with initial e=2 m. Fig. 4 and Fig. 5 show simulation and experiment results of posture regulation and path following simultaneously. Parameters used in this research are as

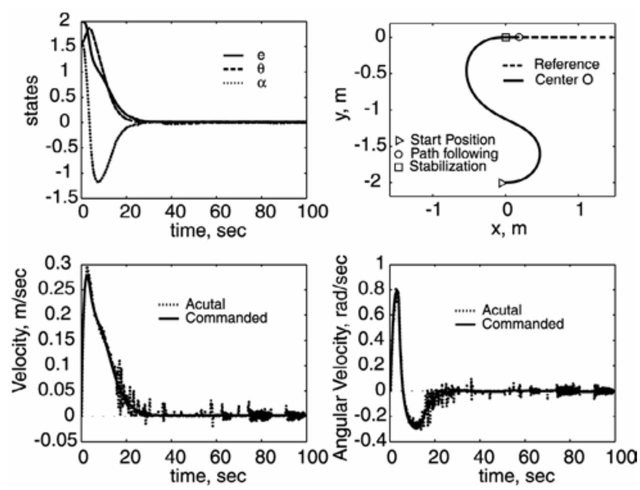

Fig. 5. Experiment for line reference, I.C. $(e,\theta,\alpha) = (2,\pi/2,\pi/2)$

Table 1. ERROR STATES AND CONTROL INPUTS AS A FUNCTION OF ε FOR THE CASE OF FIG. 4(b) AT x=0.5 m

ε	e (m)	θ (rad)	α (rad)	v (m/s, 10^{-3})	κ (1/m, 10^{-5})	Runtime (sec)
10^{-3}	0.0043	-1.45×10^{-6}	-6.96×10^{-7}	1.2	-150.0	1228.1
10^{-2}	0.0101	1.45×10^{-7}	-1.83×10^{-7}	2.4	-3.48	229.06
10^{-1}	0.102	9.02×10^{-8}	5.86×10^{-7}	24.5	1.9	44.59
1	0.996	-3.19×10^{-2}	2.24×10^{-2}	225.6	-740.0	17.15

follows; ε=0.01, $k_1 = 0.41$, $k_2 = 2.94$, $k_3 = 1.42$, $k_v=k_c = -4$, $v_{max} = 0.5$ (m/sec), $\kappa_{max} = 3\ m^{-1}$. The controller has been implemented on the two-axle CFMMR, Fig. 1. Since current research focuses on the kinematic control, we assume ideal kinematics and negligible robot dynamics.

The simulation and experimental results confirm that the paths are smooth and the proposed controller drives the robot to the origin or the target as designed. The control algorithm is efficient in the motion control of the CFMMR considering that the dynamics of the robot are not accounted for here. The path's curvature and linear velocity are well bounded for posture regulation and path following. These results thus verify the condition (*i*) and (*ii*). The front and rear heading angles also satisfy the pure bending condition (*iii*) of $\psi=\psi_1=-\psi_2$.

While the path following control by waypoint navigation takes longer time, we can drive the error relative to a reference posture to zero in 30 seconds. The path tracking speed is critically dependent upon an error tolerance. Table 1 shows relations among error tolerance, errors, control inputs and runtime. As ε decreases, *e* decreases linearly, while the runtime increases considerably. Here we selected ε=0.01 for modest runtime and errors.

Since encoder-based wheel odometry is used to evaluate the actual curvature indirectly using numerical techniques, in experiment curvature signals are very noisy. Thus, we alternatively show angular velocity, $\dot{\phi}$, instead of curvature in Fig. 5. In separate research we are compensating for the dynamics of the robot in control and implement a sensor fusion algorithm to improve posture measurements using the beam itself as an internal configuration sensor. Future work focuses on a motion control algorithm that will facilitate fast and smooth reference path with bounded curvature regardless of initial conditions

VI. CONCLUSIONS

A time invariant control law has been developed utilizing Lyapunov techniques in polar coordinates to solve posture regulation. The control strategy is then extended via waypoint navigation techniques for path following. A backstepping-based control dynamic extension is applied to compensate for non-ideal initial conditions and drift. The proposed controller is then implemented on a two-axle CFMMR. The complicated robot kinematics are simplified by an equivalent curvature based model employing the pure bending condition. Simulation and experimental results are presented and the algorithm is verified.

REFERENCES

[1] B. W. Albiston and M. A. Minor, "Curvature based point stabilization for compliant framed wheeled modular mobile robots," in *Robotics and Automation, 2003. Proceedings. ICRA '03. IEEE International Conference on*, pp. 83-89, 2003.

[2] C. Samson, "Control of chained systems application to path following and time-varying point-stabilization of mobile robots," *IEEE Transactions on Automatic Control*, vol. 40, pp. 64-77, 1995.

[3] J. M. Coron and B. d'Andrea-Novel, "Smooth stabilizing time-varying control laws for a class of nonlinear systems. Application to mobile robots," in *Nonlinear Control Systems Design 1992. Selected Papers from the 2nd IFAC Symposium*, pp. 413-18, 1993.

[4] A. Tayebi, M. Tadjine, and A. Rachid, "Path-following and point-stabilization control laws for a wheeled mobile robot," in *Proceedings of the 1996 UKACC International Conference on Control. Part 2 (of 2), Sep 2-5 1996*, pp. 878-883, 1996.

[5] T.-C. Lee, K.-T. Song, C.-H. Lee, and C.-C. Teng, "Tracking control of unicycle-modeled mobile robots using a saturation feedback controller," *Control Systems Technology, IEEE Transactions on*, vol. 9, pp. 305-318, 2001.

[6] Z.-P. Jiang, E. Lefeber, and H. Nijmeijer, "Saturated stabilization and tracking of a nonholonomic mobile robot," *Systems & Control Letters*, vol. 42, pp. 327-32, 2001.

[7] R. W. Brockett, "Asymptotic stability and feedback stabilization," in *R. W. Brockett, R. S. Millman, and H. J. Sussmann, editors, Differential Geometric Control Theory, volume 27 of Progress in Mathematics*, pp. 181-191, 1983.

[8] A. Astolfi, "On the stabilization of nonholonomic systems," in *Proceedings of 1994 33rd IEEE Conference on Decision and Control*, pp. 3481-6 vol.4, 1994.

[9] A. Astolfi, "Exponential stabilization of a wheeled mobile robot via discontinuous control," *Transactions of the ASME. Journal of Dynamic Systems, Measurement and Control*, vol. 121, pp. 121-7, 1999.

[10] G. Indiveri, "Kinematic time-invariant control of a 2D nonholonomic vehicle," in *Proceedings of 1999 Conference on Decision and Control*, pp. 2112-17 vol.3, 1999.

[11] C. Webers and U. R. Zimmer, "Practical trade-offs in robust target following," in *Proceeding of the International Congress on Intelligent Systems and Applications (ISA 2000), International Symposium on Industrial Systems (IS 2000)*, pp., 2000.

[12] A. Tayebi and A. Rachid, "A unified discontinuous state feedback controller for the path-following and the point-stabilization problems of a unicycle-like mobile robot," in *Control Applications, 1997., Proceedings of the 1997 IEEE International Conference on*, pp. 31-35, 1997.

[13] M. Aicardi, G. Casalino, A. Bicchi, and A. Balestrino, "Closed loop steering of unicycle like vehicles via Lyapunov techniques," *IEEE Robotics & Automation Magazine*, vol. 2, pp. 27-35, 1995.

[14] M. A. Minor, Albiston, B.W., Swensen, C.L., "Simplified Motion Control of a Two Axle Compliant Framed Wheeled Mobile Robot," *Accepted and under revision to the IEEE Transactions on Robotics*, 2003.

[15] S. Park and M. A. Minor, "Modeling and dynamic control of compliant framed wheeled modular mobile robots," in *Robotics and Automation, 2004. Proceedings. ICRA '04. 2004 IEEE International Conference on*, pp. 3937-3943, 2004.

[16] H. K. Khalil, *Nonlinear systems*, 3rd ed. Upper Saddle River, NJ: Prentice Hall, 2002.

[17] A. Bacciotti, *Local stabilizability of nonlinear control systems*, vol. 8: World Scientific, 1991.