2021 CIMPA School
Project Proposal

I. General Information

CIMPA School title: Categories associated to the theory of representations.
Official language of the school: English.
Country: Mexico.
Name and address of the host institution (university or equivalent): Centro de ciencias matemáticas (CCM), Universidad nacional autónoma de México (UNAM). Antigua Carretera a Pátzcuaro #8701, Col. Ex Hacienda San José de la Huerta, Morelia, Michoacán, México.

Dates (option A): 02*/08/2021-13/08/2021.*
Dates (option B): *15/11/2021-26/11/2021.*

II. Administrative and scientific coordinators

Local coordinator
NAME: RAGGI CÁRDENAS
Given name (first name): Alberto Gerardo
Gender: Male
Position : Researcher
Institution: Centro de ciencias matemáticas, Universidad nacional autónoma de México.
Country: Mexico
Email address: graggi@matmor.unam.mx

External coordinator
NAME: BOUC
Given name: Serge
Gender: Male
Position: Researcher
Institution: CNRS – Université de Picardie.
Country: France
Email address: serge.bouc@u-picardie.fr

III. Description of the project

Scientific content

The aim of the school is to introduce graduate students of the region to the modern categorical methods in representation theory.

The three courses of the first week will focus on setting the basic elements required to introduce the students to further topics. Thus, these courses will consist on brief recollections and introductions to: Categories and modules, groups actions and bisets, linear representations of finite groups and block theory. For all of them there will be training sessions on the afternoons, where problems, doubts and related discussions will be held. During the second week there will be three advanced courses intended to be an exposition of the recent trends in representation theory of finite groups and related algebras. The subjects of these courses will be: Blocks of finite quasi-simple groups, cohomological methods in block theory, and fusion systems and Burnside rings.

Besides lectures, we are also planning four talks regarding complementary topics in algebra.

The organizers believe that bringing together experts from these fields will encourage students in pursuing research in these directions.

Host institution and local context in mathematics

The Centro de ciencias matemáticas (CCM) originated as the Unidad Morelia of the Mathematics Institute of UNAM, created in 1990 with the purpose of helping to the development of mathematics in the state of Michoacan. After 21 years of continuous work, significant advance in the consolidation of research groups highly qualified, and important contributions to the cultural and scientific development in the region, in 2011 the CCM is created, as an independent entity in the Sistema de la Investigación Científica of UNAM. At the present time, the CCM has 26 researchers, working on a broad spectrum of mathematics including algebra, bio-mathematics, dynamic systems and topology and set theory. Many seminars, schools and conferences have been organized at the CCM.

Together with the Facultad de Ciencias Físico-Matemáticas and the Instituto de Física y Matemáticas of the UMSNH, the Posgrado Conjunto en Ciencias Matemáticas UNAM-UMSNH (PCCM) initiated in 2008, it consists of a master's degree program and a Ph.D. program. Right now it counts on 37 full time researchers in different areas of pure and applied mathematics. From its creation to this day, at the PCCM, more than 60 students have obtained their master's degree and 41 have obtained a Ph.D.

Prior work related to the project

In recent years, workshops as well as collaborative projects regarding the subject of the school have been developed between some of the lecturers and coordinators of the school. Some of them are:

- Project ECOS M10-M01, Bisets and associated functors. Coordinators: Dr. Gerardo Raggi Cárdenas (UNAM) and Dr. Serge Bouc (CNRS-Université de Picardie). Duration: 3 years (2011-2013). The project allowed Mexican and French students and researches to visit the Mexican and French laboratories involved in the project. The researchers that participated were Luis Valero Elizondo (UMSNH), Radu Stancu (Université de Picardie) and Alexander Zimmermann (Université de Picardie). In the project participated 4 students, Mexican and French.
- Seminario interinstitucional en representaciones de grupos, UNAM – U. Gto. Coordinators: Dr. Gerardo Raggi Cárdenas (UNAM) and Dra. Nadia Romero Romero (U. Gto). During the second semester of 2015, a seminar about representations of finite groups was held both at the CCM and at Universidad de Guanajuato. Besides the coordinators, the researchers that participated were Luis Valero Elizondo (UMSNH) and Baptiste Rognerud (UNAM).
- Project UC-MEXUS-CONACYT CN-15-43, Representation rings of finite groups. Coordinators: Dr. Gerardo Raggi Cárdenas (UNAM) and Dr. Robert Boltje (UC). Duration: 18 months (July, 2015 to December, 2016). The project allowed Mexican and American students and researches to visit the Mexican and American laboratories involved in the project, it included a workshop held at the CCM in 2016. The researchers that participated were Luis Valero Elizondo (UMSNH), Nadia Romero Romero (U. Gto) and Baptiste Rognerud (UNAM). In the project participated 7 students, from Mexico, USA, Colombia and Turkey, at the levels of Master and PhD.
- Seminario especial de álgebra CCM. Coordinator: Dr. Gerardo Raggi Cárdenas (UNAM). The seminar took place at the CCM, on November 9, 2018. The researches that participated were Robert Boltje (UC), Serge Bouc (CNRS) and Nadia Romero Romero (U. Gto). It was attended by students from UNAM and UC Santa Cruz.

In the context of these collaborations, various students obtained their PhD and were able to work with the specialists involved in the projects. Some of them have since found postdoc or teaching positions in Mexico and USA.

Expected impact of the project

Firstly, we expect the school to have students from universities in Mexico (UNAM, Universidad de Guanajuato, Universidad Michoacana, for instance) interested in the subject, so maybe later on they can chose a related research problem. Also, we believe the school will impact directly those students already working on a research problem on the subject. It will allow them to learn more on the links between the topics existing in the area of representations of finite groups, and to see this as a combined approach on the problems they are working on. It will also give them the opportunity to get in touch with experts in the area from other countries, who will share their research experience and give them some guidance. In the long term, this will have an impact in the research is currently done in the region.

Additionally, we believe the school will help to broaden the interest on the subject in the region, since we expect that students from USA and Central America could attend. Recently, the organizers of the school have gotten in touch with researchers from El Salvador and Colombia, so it would be desirable to have students from these countries coming to the school. This will also be an opportunity for them to consider doing post-graduate students in Mexico.

Infrastructure

The school will take place at the CCM, in Morelia, Mexico. The infrastructure include an auditorium, classrooms, image projectors, Wi-Fi, accommodation, food, ground transportation, and office supplies. The expected places for accommodation are hotels Casino and Howard Johnson (in the center of town, at around 9km from the CCM), also hotels Torreblanca Campestre and Comfort Inn (at around 5 km from the CCM) are considered. Breakfast will be provided by hotels and lunch will be taken at the CCM. Ground transportation between hotels and the CCM will be provided in the mornings and the afternoons.

Expected participants

We expect around 25 students from Mexico, USA, El Salvador and Colombia.

IV. Scientific committee

Member 1
NAME: BOLTJE
Given name: Robert
Gender: Male
Institution: University of California
Country: USA
Role: Lecturer

Member 2
NAME: KESSAR
Given name: Radha
Gender: Female
Institution: City University of London
Country: England
Role: Lecturer

Member 3
NAME: PÉREZ TERRAZAS
Given name: Jesús Efrén
Gender: Male
Institution: Universidad autónoma de Yucatán
Country: Mexico
Role: Expert

Member 4
NAME: ROMERO ROMERO
Given name: Nadia
Gender: Female
Institution: Universidad de Guanajuato.
Country: Mexico
Role: Lecturer

V. Organizing committee

Member 1
NAME: RAGGI CÁRDENAS
Given name: Alberto Gerardo
Gender: Male
Institution: Centro de ciencias matemáticas, Universidad nacional autónoma de México
Country: Mexico
Role: Logistics and local support for the school

Member 2
NAME: REYNOSO ALCÁNTARA
Given name: Claudia Estela
Gender: Female
Institution: Universidad de Guanajuato
Country: Mexico
Role: Course curriculum

Member 3
NAME: ROMERO ROMERO
Given name: Nadia
Gender: Female
Institution: Universidad de Guanajuato
Country: Mexico
Role: Course curriculum and lecturer

Member 4
NAME: VALERO ELIZONDO
Given name: Luis
Gender: Male
Institution: Universidad michoacana de San Nicolás de Hidalgo.
Country: Mexico
Role: Logistics and local support for the school

VI. Scientific program

Introductory courses

Course 1

Title: On modules and categories

Duration: 4.5 h

Lecturer's NAME: ROMERO ROMERO

Lecturer's given name: Nadia

Lecturer's gender: Female

Lecturer's institution: Universidad de Guanajuato

Lecturer's country: Mexico

Abstract of the course: We will begin with the basic notions of category theory and the main properties of the standard examples. Then we will focus on the category of modules over a ring. We will review the most important tools used to study this category and finally we will consider examples with particular rings appearing in the theory of representations of groups.

Course 2

Title: Introduction to group actions and associated functors

Duration: 4.5

Lecturer's NAME: BOUC

Lecturer's given name: Serge

Lecturer's gender: Male

Lecturer's institution: CNRS – Université de Picardie

Lecturer's country: France

Abstract of the course: Starting with actions of a finite group on sets or vector spaces, I will introduce the associated Grothendieck rings, like the Burnside ring or the character ring, and consider some of their functorial properties. This will lead to a gentle presentation of biset functors for finite groups, in particular the simple ones.

Course 3

Title: Introduction to block theory

Duration: 4.5 h

Lecturer's NAME: BOLTJE

Lecturer's given name: Robert

Lecturer's gender: Male

Lecturer's institution: University of California

Lecturer's country: USA

Abstract of the course: We will introduce basic notions from modular representation theory and block theory: Vertices, sources, lifting of idempotents, blocks, defect groups, Brauer characters, Brauer's main theorems, decomposition matrix, Cartan matrix.

Advanced courses

Course 1

Title: Fusion systems and Burnside rings

Duration: 4.5 h

Lecturer’s NAME: STANCU

Lecturer’s given name: Radu

Lecturer’s gender: Male

Lecturer’s institution: Université de Picardie

Lecturer's country: France

Abstract of the course: Fix p a prime number and let S be a finite p-group. A fusion system on S is the axiomatization of the conjugation action, of a finite overgroup, on S and its subgroups. The set of isomorphism classes of finite bifree (S,S)-bisets has a natural addition given by the disjoint union and a non-commutative multiplication, making it into the free double Burnside ring at S. There is a one to one correspondence between the fusion systems on S and specific idempotents of the free double Burnside ring at S. We explain this bijection and try to understand its implications.

Course 2

Title: The blocks of finite quasi-simple groups

Duration: 4.5 h

Lecturer’s NAME: KESSAR

Lecturer’s given name: Radha

Lecturer’s gender: Female

Lecturer’s institution: City University of London

Lecturer's country: UK

Abstract of the course: The course will be an introduction to the ideas and methods underlying the distribution of ordinary irreducible characters of finite quasi-simple groups into p-blocks. The finite groups of Lie type in non-defining characteristic will be a particular focus - we will explain how Brauer's theory of p-blocks interacts with Lusztig's character theory in this context.

Course 3

Title: Cohomological methods in block theory

Duration: 4.5 h

Lecturer’s NAME: LINCKELMANN

Lecturer’s given name: Markus

Lecturer’s gender: Male

Lecturer’s institution: City University of London

Lecturer's country: UK

Abstract of the course: In any given dimension, the finite-dimensional algebras over an algebraically closed field form a variety. Only finitely many of those are expected to arise as block algebras of finite group algebras. Cohomological methods play an important role in narrowing down the potential pool of algebras that do arise as blocks. We focus on two of the most fundamental cohomology algebras associated with blocks, namely what is called the block cohomology, which is an analogue of finite group cohomology, defined in terms of the local structure of a block, and the Hochschild cohomology of a block. Both of these cohomology algebras

are closely related, and their invariants contain significant information about structural properties of the underlying block algebras. We will in particular investigate the interplay between the Lie algebra structure of the first Hochschild cohomology and the associative algebra structure of blocks.

Training sessions

During the afternoons there will be general discussions of problems about the courses taught the same day.

Talks

Talk 1
Title: Representations of partially ordered sets and their Auslander-Reiten Quiver
Duration: 1.5 h
Lecturer’s NAME: BAUTISTA RAMOS
Lecturer’s given name: Raymundo
Lecturer’s gender: Male
Lecturer’s institution: Centro de ciencias matemáticas, Universidad nacional autónoma de México
Lecturer's country: Mexico
Abstract of the course: Attached at the end of the document.

Talk 2
Title: Correspondences functors
Duration: 1.5 h
Lecturer’s NAME: BOUC
Lecturer’s given name: Serge
Lecturer’s gender: Male
Lecturer’s institution: CNRS – Université de Picardie
Lecturer's country: France
Abstract of the course: Attached at the end of the document.

Talk 3
Title: (Psuedo-)monads and their (Pseudo)-Distributive Laws
Duration: 1.5 h
Lecturer’s NAME: MARMOLEJO RIVAS
Lecturer’s given name: Francisco
Lecturer’s gender: Male
Lecturer’s institution: Instituto de matemáticas, Universidad nacional autónoma de México
Lecturer's country: Mexico
Abstract of the course: Monads are the categorical version of universal algebra. The way they can be composed was explained long ago by Beck, and he called the extra structure "distributive law". In the next level up, these monads are called pseudo-monads. Their distributive laws turn out to be

quite involved mainly due to the coherence conditions necessary. This fact led to other ways to try to simplify the conditions in such a way that they could be verified in particular examples. In this talk we will first review the theory of monads and distributive laws, and we will try to give a feeling as to where exactly is the problem with the pseudo-case, and of how it is that we are trying to circumvent it.

Talk 4

Title: Biset functors for categories

Duration: 1.5 h

Lecturer's NAME: WEBB

Lecturer's given name: Peter

Lecturer's gender: Male

Lecturer's institution: University of Minnesota

Lecturer's country: USA

Abstract of the course: In the context of group theory, biset functors have been useful in various ways: for example, computing the values of group cohomology, and providing fundamental constructions such as the (torsion free part of) the Dade group. Biset functors can also be done for categories in general, with similar goals in mind. To establish this theory we must first say what a biset is. Such things have been studied for a long time under the name of profunctors, or distributors. We form a biset category, in the same way as for groups, and functors defined on this category share many of the properties of biset functors for groups. An important role is played by the Burnside ring functor, which is now defined for categories, not just groups. It turns out the that Burnside ring of a category is a more delicate construction than for groups, and we examine its structure. A key part of the work is to establish conditions under which the cohomology of a category has the structure of a biset functor. For groups, the bisets considered must be free on one side. We introduce an extension of this condition that works for categories.

VII. Tentative schedule

Day 1:

Morning: Introductory courses 1 and 2 (3h)

Afternoon: Training sessions (1.5 h)

Day 2:

Morning: Introductory courses 1 and 3 (3h)

Afternoon: Talk 1 (1.5 h)

Day 3:

Morning: Introductory courses 2 and 3 (3h)

Afternoon: Training sessions (1.5 h)

Day 4:

Morning: Introductory courses 1 and 3 (3h)

Afternoon: Training sessions (1.5 h)

Day 5:
Morning: Introductory course 2 and Talk 4 (3h)
Afternoon: Training sessions (1.5 h)

Day 6:
Morning: Advanced courses 1 and 2 (3h)
Afternoon: Training sessions (1.5 h)

Day 7:
Morning: Advanced courses 1 and 3 (3h)
Afternoon: Talk 3 (1.5 h)

Day 8:
Morning: Advanced courses 2 and 3 (3h)
Afternoon: Training sessions (1.5 h)

Day 9:
Morning: Advanced courses 1 and 3 (3h)
Afternoon: Training sessions (1.5 h)

Day 10:
Morning: Advanced course 2 and Talk 2 (3h)
Afternoon: Training sessions (1.5 h)

VIII. Provisional budget (expenses)

Transportation costs: 11,000 €

- Air and ground transportation of 6 lecturers and 2 speakers: 5,700 €
- Transportation of students from Mexico, USA and Central America : 5,300 €

Lodging: 11,120 €

- Accommodation for 25 students for 13 nights: 7,000 €
- Accommodation for 6 lecturers and 1 speaker for 13 nights: 4,000 €
- Accommodation for 1 speaker for 2 nights: 120 €

Meals: 4,200 €

- Lunch and snacks for 35 participants: 4,200 €

Logistics: 2,230 €

- Office supplies (school kits for participants, whiteboard markers and erasers, etc): 300 €
- Opening ceremony: 330 €
- Ground transportation from hotels to the school: 1,600 €

Others: 300 €

- Excursion: 300 €

Total expenses : 28,850 €

IX. Anticipated funding (resources)

Financial support requested from CIMPA: 9,600 €

Local funding:

- Expected funding from UNAM: 3,600 €
- Expected funding from Conacyt: 3,000 €

International funding:

- Expected funding from Laboratorio Solomon Lefschetz (Unité mixte international): 6,950 €

Costs covered by lecturers: 5,700 €

Total resources: 28,850 €

X. Support and involvement of local institutions

The CCM is committed to supporting school with appropriate facilities: rooms, an auditorium, wi-fi coverage, image projectors and administrative support.

XI. Additional remarks and comments