Lab 2: RC Circuit Analysis

Circuit Theory and Electronics Fundamentals

Mestrado Integrado Engenharia Física Tecnológica, Técnico, University of Lisbon

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Contents

1 Introduction

The objective of this laboratory assignment is to study a circuit containing an AC voltage source V_S and a capacitor connected to seven resistors R_{1-7}, a linear voltage-controlled current source and a linear current-controlled voltage source. The circuit can be seen in Figure 1.

In Section 2, it was used the node analysis methods to solve the circuit for $t < 0$. It was also determined the equivalent resistance Req as seen from the capacitor terminals. After that it was calculated the natural and forced solutions for the voltage in node 6. Finally, it was analysed the response to frequency variation. In Section 3, the circuit is analysed by simulation, and the results are compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 4.

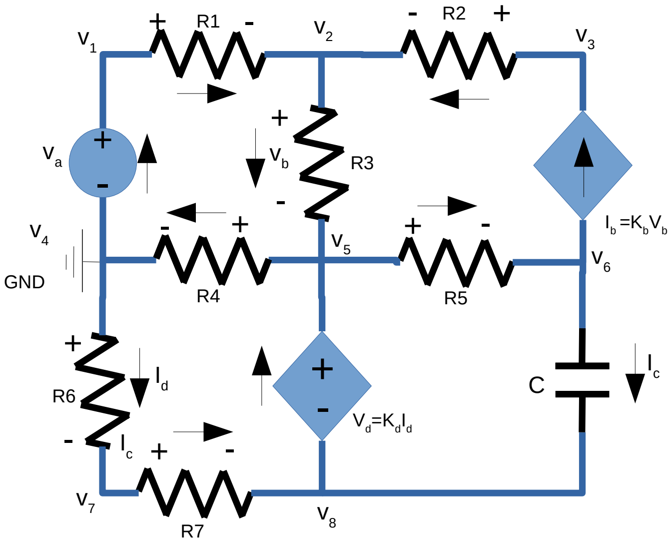

Figure 1: RC Circuit

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically, in terms of node, AC and phasor analysis.

2.1 Circuit solution for $t < 0$

After naming the nodes from 1 to 8, as shown in Figure 1, assigning the 4^{th} node as the reference node (V_4 =GND) and assuming that the circuit is open in the capacitor branch(because it's supposed that the capacitor is totally charged in $t- > 0$) the equation to some of the nodes(those that aren't connected to Voltage sources) can be derived from the Kirchhoff Current Law (KCL). For the nodes 2,3,6 and 7, respectively:

$$(V_2 - V_5)G_3 + (V_2 - V_1)G_1 + (V_2 - V_3)G_2 = 0 \tag{1}$$

$$(V_3 - V_2)G_2 - K_b(V_2 - V_5) = 0 \tag{2}$$

$$(V_5 - V_6)G_5 - K_b(V_2 - V_5) = 0 \tag{3}$$

$$(V_4 - V_7)G_6 - (V_7 - V_8)G_7 = 0 \tag{4}$$

Now looking at the nodes that are connected to Voltage Sources and knowing the definition of voltage difference, the equations are:

$$V_1 = V_S \tag{5}$$

$$V_5 = K_d \times G_6(V_4 - V_7) + V_8 \tag{6}$$

Noticing that the current through V_d is the same as the one through R_7, the equation to supernode 5-8 goes as:

$$(V_5 - V_4)G_4 + (V_5 - V_6)G_5 + (V_5 - V_2)G_3 + (V_8 - V_7)G_7 = 0 \tag{7}$$

Solving the linear system:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -G_1 & G_1 + G_2 + G_3 & -G_2 & 0 & -G_3 & 0 & 0 & 0 \\ 0 & -K_b - G_2 & G_2 & 0 & K_b & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -G_3 & 0 & -G_4 & G_4 + G_5 + G_3 & -G_5 & -G_7 & G_7 \\ 0 & -K_b & 0 & 0 & G_5 + K_b & -G_5 & 0 & 0 \\ 0 & 0 & 0 & G_6 & 0 & 0 & -G_6 - G_7 & G_7 \\ 0 & 0 & 0 & -K_d \times G_6 & 1 & 0 & K_d \times G_6 & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} V_S \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The results are:

Node Voltage	Value (V)
V_1	5.051442559750000
V_2	4.817046879914681
V_3	4.321913191310698
V_4	-0.000000000000000
V_5	4.849952570949478
V_6	5.575185364005538
V_7	-1.980632424391875
V_8	-2.972821675148061

Table 1: Solutions to Node Analysis Method

Branch Current	Value (mA)
I_1	0.227976653510561
I_2	-0.238717261312864
I_3	-0.010740607802307
I_4	1.191860447555040
I_5	-0.238717261312859
I_7	0.963883794044486
I_{vd}	0.963883794044486
I_b	-0.238717261312864
I_c	0.000000000000000

Table 2: Solutions to Node Analysis Method

2.2 Circuit solution for $t = 0$

To calculate the equivalent resistance, we made $V_s = 0$ (in $t = 0$, $V_s \neq 0$ but it changes in $t_0 + dt$ so it's neglected) and replaced the capacitor with a voltage source $V_x = V_6 - V_8$, where V_6 and V_8 are the voltages in nodes 6 and 8 as obtained in $t < 0$. With the nodal analysis, the current Ix supplied by Vx was calculated. Finally, the equivalent resistor as Req = Vx/Ix,and the time constant were computed. This procedure was executed so that we could get the time constant for the RC circuit, solve the natural solutions for any variable and get the voltages in $t = 0$. We used the same equations as in $t < 0$ with the following changes:

$$(V_5 - V_4)G_4 + (V_5 - V_6)G_5 + (V_5 - V_2)G_3 + (V_8 - V_7)G_7 = I_x \tag{8}$$

$$(V_5 - V_6)G_5 - K_b(V_2 - V_5) = I_x \tag{9}$$

$$V_8 = V_6 + V_x \tag{10}$$

Solving the linear system:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -G_1 & G_1+G_2+G_3 & -G_2 & 0 & -G_3 & 0 & 0 & 0 & 0 \\ 0 & -K_b-G_2 & G_2 & 0 & K_b & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -G_3 & 0 & -G_4 & G_4+G_5+G_3 & -G_5 & -G_7 & G_7 & -1 \\ 0 & -K_b & 0 & 0 & G_5+K_b & -G_5 & 0 & 0 & -1 \\ 0 & 0 & 0 & G_6 & 0 & 0 & -G_6-G_7 & G_7 & 0 \\ 0 & 0 & 0 & -K_d \times G_6 & 1 & 0 & K_d \times G_6 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ I_x \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ V_x \end{bmatrix}$$

The results are:

Variables	Value (V / Ω / $\Omega * F$)
V_1	0.000000000000000
V_2	-0.000000000000000
V_3	-0.000000000000000
V_4	-0.000000000000000
V_5	0.000000000000000
V_6	8.548007039153600
V_7	-0.000000000000000
V_8	0.000000000000000
I_x	-0.002813657696684
R_{eq}	3038.040856650000023
τ	0.003108668950621

Table 3: Solutions to Node Analysis Method

2.3 Natural Solution for Node 6 - $t > 0$

The natural solution for the voltage in node 6 is given by the general expression for RC and RL Circuits:

$$x(t) = x(\infty) + [x(0) - x(\infty)] \exp(\frac{-t}{\tau}) \tag{11}$$

Where $x(t) = v_{6n}(t)$, $x(\infty) = 0$ because the capacitor discharges and $x(0) = v_{6n}(0)$ calculated in the subsection before.

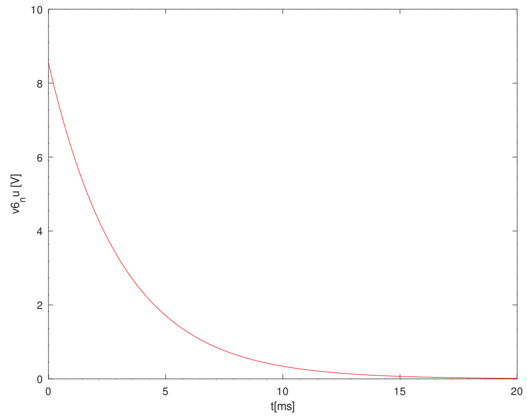

Figure 2: Natural Solution for node 6

2.4 Forced Solution - Phasors ($t > 0$)

Choosing the input frequency of $f = 1Khz$ and making use of the phasors corresponding to each variable in the circuit: $\tilde{V}_s = 1$, $\tilde{Z}_c = \frac{1}{j \times C \times 2\pi \times f}$

Running node analysis:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -G_1 & G_1+G_2+G_3 & -G_2 & 0 & -G_3 & 0 & 0 & 0 \\ 0 & -K_b-G_2 & G_2 & 0 & K_b & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -G_3 & 0 & -G_4 & G_4+G_5+G_3 & -G_5+(\frac{1}{Z_c}) & -G_7 & G_7-(\frac{1}{Z_c}) \\ 0 & -K_b & 0 & 0 & G_5+K_b & -G_5+(\frac{1}{Z_c}) & 0 & -(\frac{1}{Z_c}) \\ 0 & 0 & 0 & G_6 & 0 & 0 & -G_6-G_7 & G_7 \\ 0 & 0 & 0 & -K_d \times G_6 & 1 & 0 & K_d \times G_6 & -1 \end{bmatrix} \begin{bmatrix} \tilde{V_1} \\ \tilde{V_2} \\ \tilde{V_3} \\ \tilde{V_4} \\ \tilde{V_5} \\ \tilde{V_6} \\ \tilde{V_7} \\ \tilde{V_8} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The results are:

Phasor	
V_1	1.000000+0.000000j
V_2	0.953598+0.000000j
V_3	0.855580-0.000000j
V_4	0.000000+0.000000j
V_5	0.960112+0.000000j
V_6	-0.584086+0.086409j
V_7	-0.392092-0.000000j
V_8	-0.588509-0.000000j

Table 4: Solutions to Node Analysis Method

Specifically for node 6:

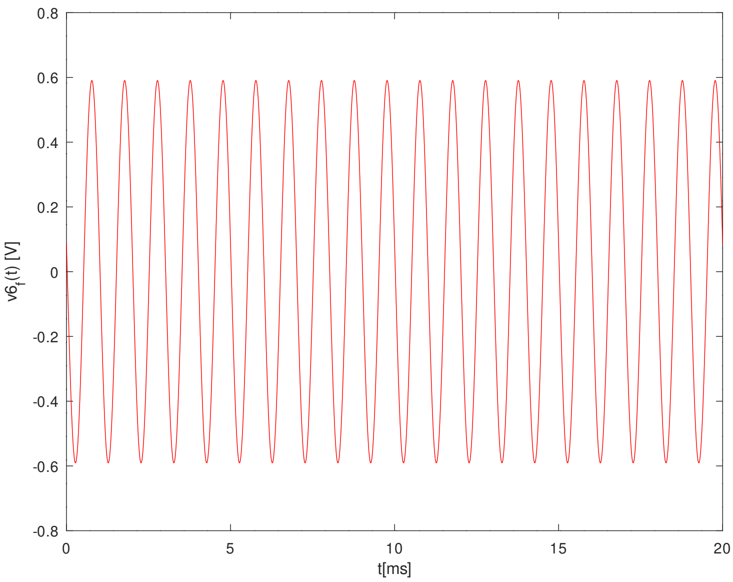

Figure 3: Forced Solution for node 6

2.5 Total Solution - $t > 0$

Superimposing the natural and forced solutions we get the total solution for $v_6(t)$. Plotting $v_6(t)$ and $v_s(t)$:

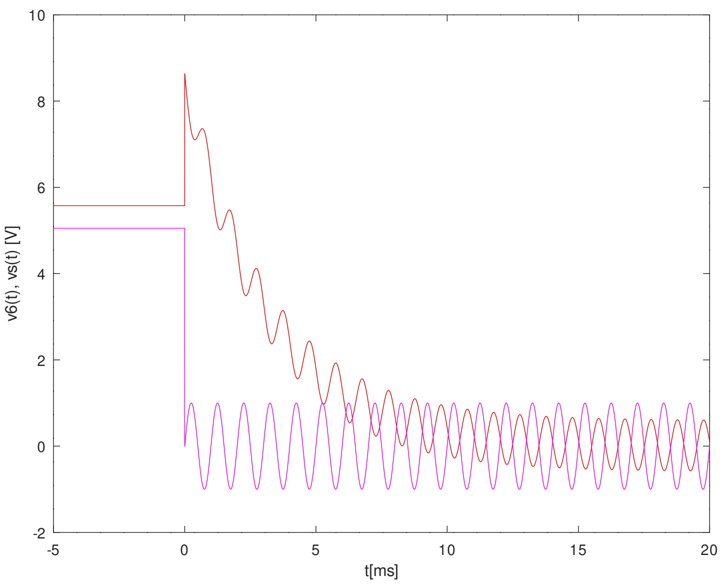

Figure 4: Total Solution for node 6

2.6 Frequency Analysis

Using the last system of equations, we solved the phasors in function of the frequency for $0.1Hz < f < 1MHz$, getting $v_6(f)$, $v_s(f)$ and $v_c(f) = v_6(f) - v_8(f)$

Comparing the magnitude, we see FALTA ESCREVER

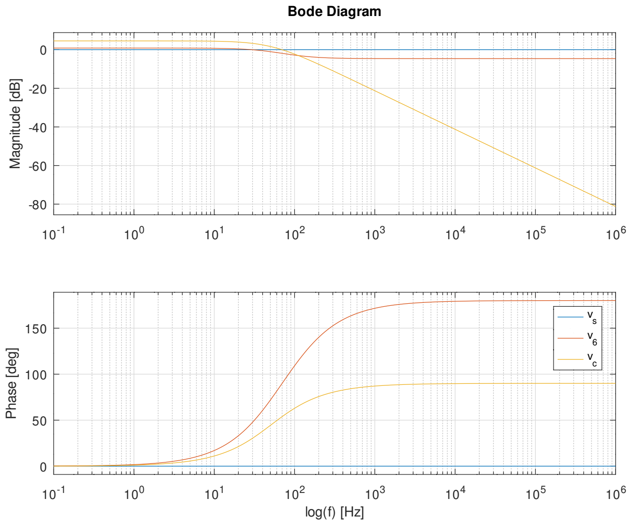

Figure 5: Frequency Analysis

3 Simulation Analysis

3.1 Operating Point Analysis for $t < 0$

Table 5 shows the simulated operating point results for the circuit under analysis. Compared to the theoretical analysis results, one notices the following differences: none.

Name	Value [A or V]
@cc[i]	0.000000e+00
@gvc[i]	-2.38717e-04
@r1[i]	2.279767e-04
@r2[i]	-2.38717e-04
@r3[i]	-1.07406e-05
@r4[i]	1.191860e-03
@r5[i]	-2.38717e-04
@r6[i]	9.638838e-04
@r7[i]	9.638838e-04
v1	5.051443e+00
v2	4.817047e+00
v3	4.321913e+00
v5	4.849953e+00
v6	5.575185e+00
v7	-1.98063e+00
v8	-2.97282e+00
vaux	-1.98063e+00

Table 5: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

3.2 Operating Point Analysis for $t = 0$

For the purpose of comparing with the theoretical analysis, the simulated operating point results in:

Name	Value [A or V]
@gvc[i]	-2.10456e-18
@r1[i]	2.009866e-18
@r2[i]	-2.10456e-18
@r3[i]	-9.46903e-20
@r4[i]	-4.36534e-19
@r5[i]	-2.81366e-03
@r6[i]	-4.33681e-19
@r7[i]	-8.59956e-19
v1	0.000000e+00
v2	-2.06646e-15
v3	-6.43161e-15
v5	-1.77636e-15
v6	8.548005e+00
v7	8.911472e-16
v8	1.776357e-15
vaux	8.911472e-16

Table 6: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

3.3 Transient Analysis - $t > 0$

3.3.1 Natural Response

Figure 6 shows the simulated transient analysis results for the circuit under analysis in the natural state ($V_s = 0$). The boundary conditions were set by the command .ic for the nodes 6 and 8. Compared to the theoretical analysis results, one notices no differences:

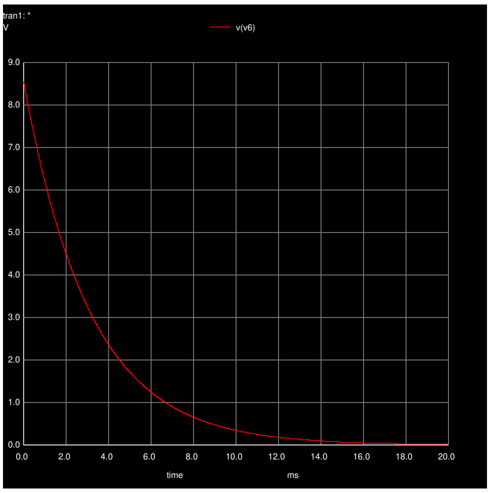

Figure 6: Transient output voltage

3.3.2 Total Response

Figure 7 shows the simulated transient analysis results for the circuit under analysis in the stimulated state ($V_s = 1$ - AC). The boundary conditions were set by the command .ic for the nodes 6 and 8. Compared to the theoretical analysis results, one notices no differences:

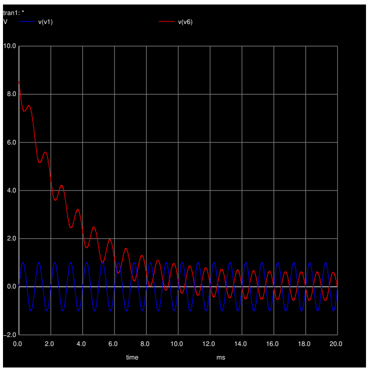

Figure 7: Transient output voltage

3.4 Frequency Analysis

3.4.1 Magnitude Response

Figure 8 shows the magnitude of the frequency response for the circuit under analysis. Compared to the theoretical analysis results, one notices no differences:

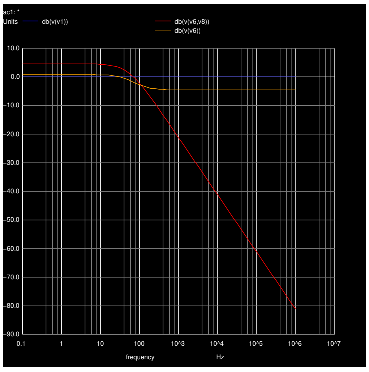

Figure 8: Magnitude response

3.4.2 Phase Response

Figure 9 shows the magnitude of the frequency response for the circuit under analysis. Compared to the theoretical analysis results, one notices the following differences: they are symmetric.

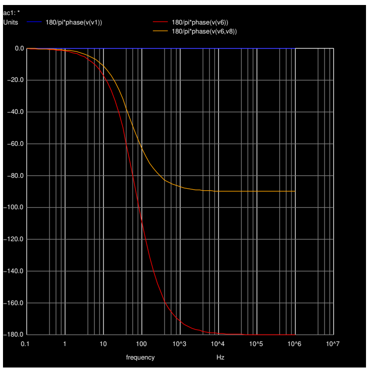

Figure 9: Phase response

4 Conclusion

In this laboratory assignment the objective of analysing an RC circuit has been achieved. Static, time and frequency analyses have been performed both theoretically using the Octave maths tool and by circuit simulation using the Ngspice tool. The simulation results matched the theoretical results precisely. The reason for this perfect match is the fact that this is a straightforward circuit containing only linear components, so the theoretical and simulation models cannot differ. For more complex components, the theoretical and simulation models could differ but this is not the case in this work.