Lab 1: Circuit analysis methods

Circuit Theory and Electronics Fundamentals

Mestrado Integrado Engenharia Física Tecnológica, Técnico, University of Lisbon

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1 Introduction

The objective of this laboratory assignment is to study a circuit containing a DC voltage source V_I and a DC current source connected to seven resistors R_{1-7}, a linear voltage-controlled current source and a linear current-controlled voltage source. The circuit can be seen if Figure 1.

In Section 2, it was used the mesh and node analysis methods to solve the circuit. In Section 3, the circuit is analysed by simulation, and the results are compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 4.

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically, in terms of node and mesh analysis.

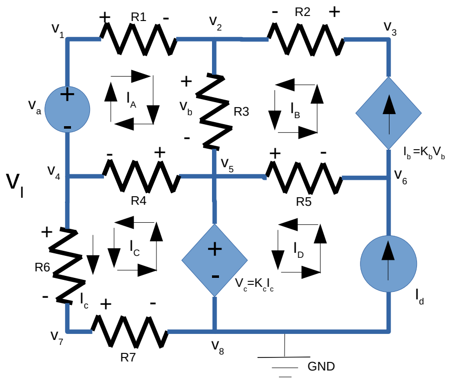

Figure 1: Circuit analysis methods

2.1 Mesh Analysis Method

The circuit consists of four meshes named A,B,C and D. Looking at mesh A and considering the Kirchhoff Voltage Law (KVL), an equation for the single loop in the circuit can be written as:

$$V_a = R_1(I_A) + R_3(I_A + I_B) + R_4(I_A + I_C) \tag{1}$$

In the calculation of voltage drop in R_3, the total current through R_3 is a combination of the currents in mesh A and mesh B. The same goes to R_4 and any element that is common to two meshes.

Continuing to mesh C, the equation is:

$$V_c = K_c \times I_C = R_4(I_A + I_C) + R_6(I_C) + R_7(I_C) \tag{2}$$

Noticing that $I_C = I_c$.

In the other two meshes, B and D, the current sources impose the current of each mesh. That being said, the equation for mesh B is:

$$I_B = K_b \times V_b = K_b \times R_3(I_A + I_B) \tag{3}$$

And for mesh D is:

$$I_D = I_d \tag{4}$$

Solving the linear system:

$$\begin{bmatrix} R_1 + R_3 + R_4 & R_3 & R_4 & 0 \\ K_b \times R_3 & K_b \times R_3 - 1 & 0 & 0 \\ R_4 & 0 & R_4 + R_6 + R_7 - K_c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_A \\ I_B \\ I_C \\ I_D \end{bmatrix} = \begin{bmatrix} V_a \\ 0 \\ 0 \\ I_d \end{bmatrix}$$

The results are:

Mesh Current	Value (mA)
I_A	1.809244100051636
I_B	-1.809325173903972
I_C	-1.040984044022358
I_D	1.017457962400000

Table 1: Solutions to Mesh Analysis Method

2.2 Node Analysis Method

After naming the nodes from 1 to 8, as shown in Figure 1, and assigning the 8^{th} node as the reference node (V_8=GND), the equation to some of the nodes(those that aren't connected to Voltage sources) can be derived from the Kirchhoff Current Law (KCL). For the nodes 2,3,6 and 7, respectively:

$$(V_1 - V_2)G_1 + (V_3 - V_2)G_2 - (V_2 - V_5)G_3 = 0 \tag{5}$$

$$K_b(V_2 - V_5) - (V_3 - V_2)G_2 = 0 \tag{6}$$

$$(V_5 - V_6)G_5 + I_d - K_b(V_2 - V_5) = 0 \tag{7}$$

$$(V_4 - V_7)G_6 - (V_7 - V_8)G_7 = 0 \tag{8}$$

Now looking at the nodes that are connected to Voltage Sources and knowing the definition of voltage difference, the equations are:

$$V_1 = V_a + V_4 \tag{9}$$

$$V_5 = K_C \times G_6(V_4 - V_7) + V_8 \tag{10}$$

Noticing that the current through V_a is the same as the one through R1, the equation to node for goes as:

$$-(V_4 - V_7)G_6 + (V_5 - V_4)G_4 - (V_1 - V_2)G_1 = 0 \tag{11}$$

Solving the linear system:

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ G_1 & -G_1 - G_2 - G_3 & G_2 & 0 & G_3 & 0 & 0 & 0 \\ 0 & K_b + G_2 & -G_2 & 0 & -K_b & 0 & 0 & 0 \\ -G_1 & G_1 & 0 & -G_6 - G_4 & G_4 & 0 & G_6 & 0 \\ 0 & 0 & 0 & K_c \times G_6 & -1 & 0 & -K_c \times G_6 & 1 \\ 0 & -K_b & 0 & 0 & G_5 + K_b & -G_5 & 0 & 0 \\ 0 & 0 & 0 & G_6 & 0 & 0 & -G_6 - G_7 & G_7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} V_a \\ 0 \\ 0 \\ 0 \\ 0 \\ -I_d \\ 0 \\ 0 \end{bmatrix}$$

The results are:

Node Voltage	Value (V)
V_1	1.807255180205026
V_2	-8.810805588482041e-03
V_3	-3.753133034617004
V_4	-3.232675342464974
V_5	-8.558265289355584e-03
V_6	8.744404418221231
V_7	-1.047445772479076
V_8	0.000000000000000

Table 2: Solutions to Mesh Analysis Method

3 Simulation Analysis

3.1 Operating Point Analysis

Table 3 shows the simulated operating point results for the circuit under analysis. Compared to the theoretical analysis results, one notices that there are no differences.

Name	Value [A or V]
@gvc[i]	-1.80933e-03
@id[current]	1.017458e-03
@r1[i]	1.809244e-03
@r2[i]	-1.80933e-03
@r3[i]	-8.10739e-08
@r4[i]	7.682601e-04
@r5[i]	-2.82678e-03
@r6[i]	-1.04098e-03
@r7[i]	-1.04098e-03
v1	1.807255e+00
v2	-8.81081e-03
v3	-3.75313e+00
v4	-3.23268e+00
v5	-8.55827e-03
v6	8.744404e+00
v7	-1.04745e+00
vaux	-3.23268e+00

Table 3: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

4 Conclusion

In this laboratory assignment the objective of analysing the circuit with the mesh and node methods has been achieved. Mesh and Node analyses have been performed both theoretically using the Octave maths tool and by circuit simulation using the Ngspice tool. The simulation results matched the theoretical results precisely. The reason for this perfect match is the fact that this is a straightforward circuit containing only linear components, so the theoretical and simulation models cannot differ. For more complex components, the theoretical and simulation models could differ but this is not the case in this work.