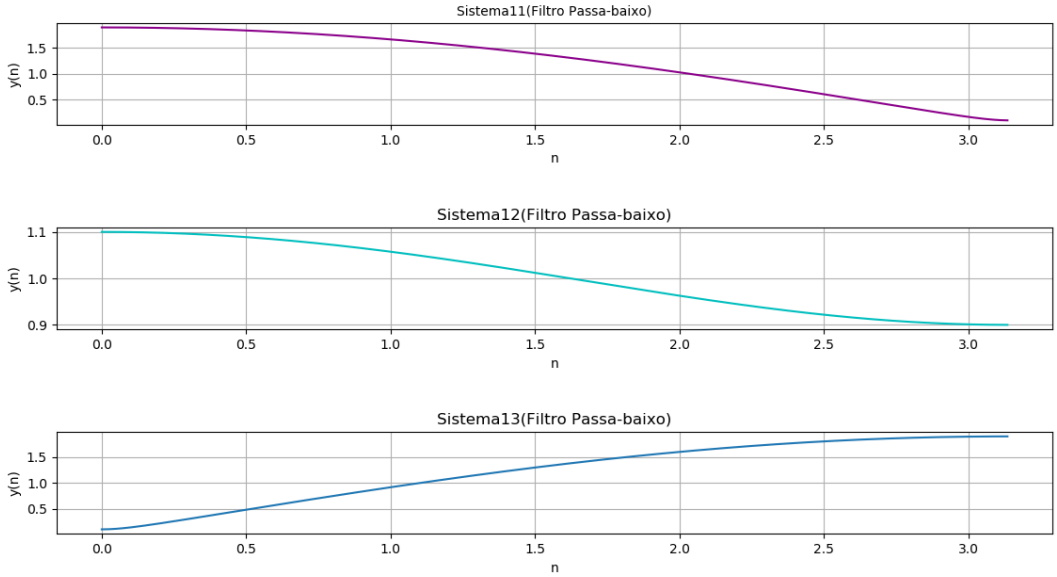
Licenciatura Engenharia Informática e Multimédia

Processamento Digital de Sinais – 1718SI

Ano letivo 2017/2018

Sistemas



Docente:

André Lourenço

Trabalho realizado por:

Miguel Távora Nº45102

Sérgio Lopes Nº43740

Turma: 21D

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Resolução de exercícios e gráficos

Grupo I:

- Exercício 1.a)

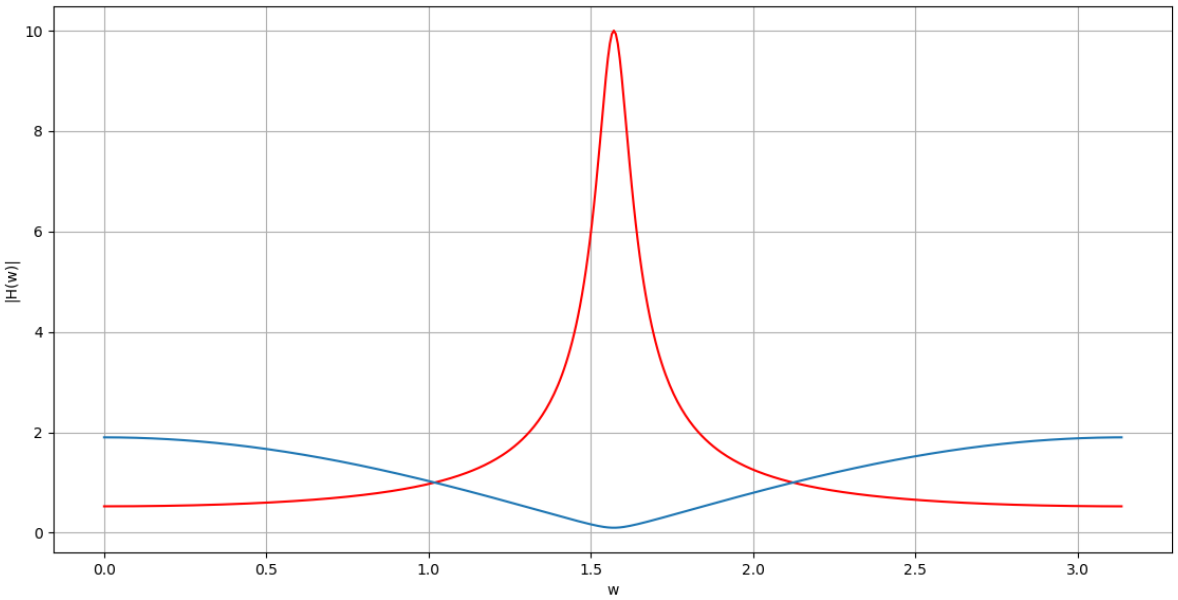
1. Considere os sistemas definidos pelas equações às diferenças:

   $y_1[n] = x[n] - rx[n-c]$,

   $y_2[n] = x[n] - ry_2[n-c]$,

   Considere sequencialmente as situações onde o atraso c toma os valores c=1 ou c=2, e o fator multiplicativo r toma os valores r=0.9 ou r=0.1 ou r=-0.9.

   a) Represente a resposta em frequência dos sistemas usando a função `y=scipy.signal.freqz(b,a)`, *onde o vector a e b representam os ganhos da função de transferência.* Qual a influência do valor de c nos sistemas?

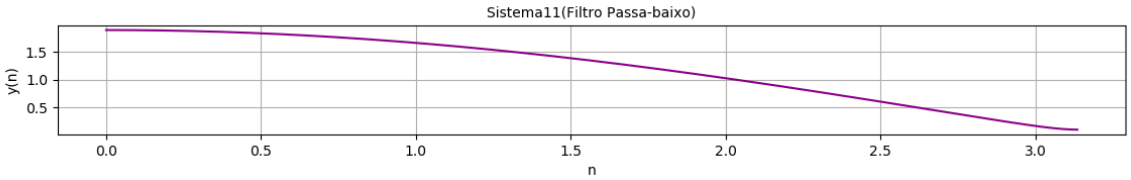


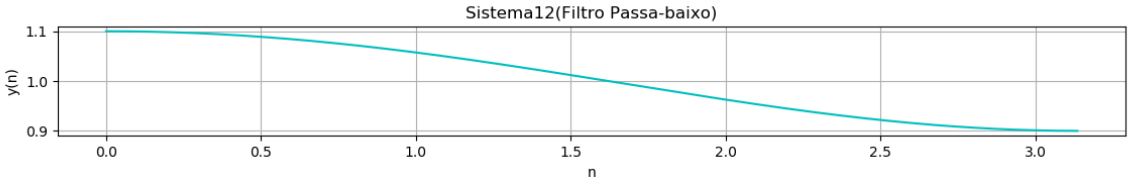
O valor c tem como função o atraso ou aceleração do sistema sendo que quando é negativo atrasa o número de unidades definido e quando é positivo acelera o número de unidades definido.

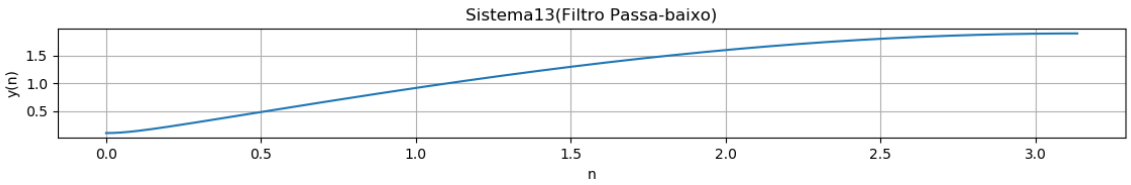
- Exercício 1.b)

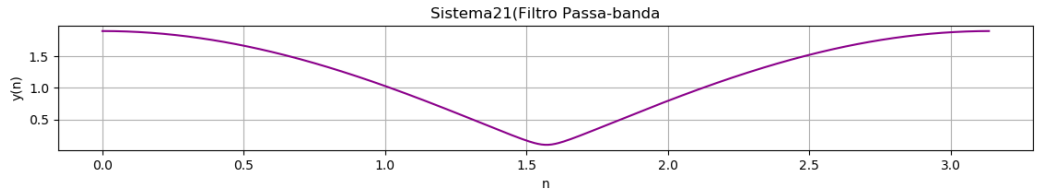
b) Qual o tipo de filtragem realizada por cada um destes filtros? (passa-baixo, pass-alto, passa-banda, outro)

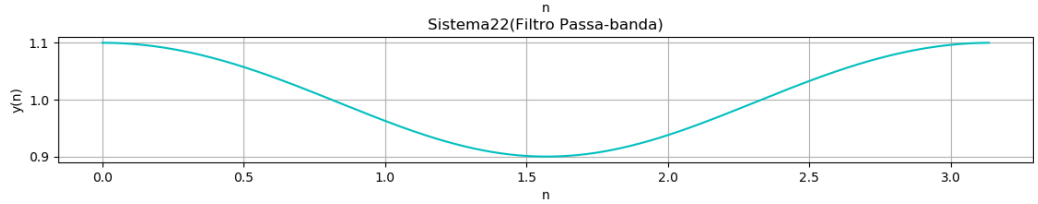
A filtragem do sistema FIR é passa-banda e a filtragem do sistema IIR é passa-baixo.

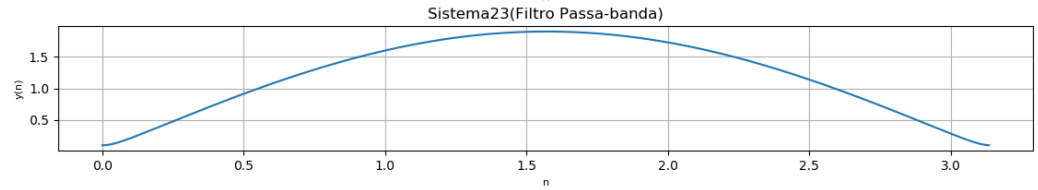








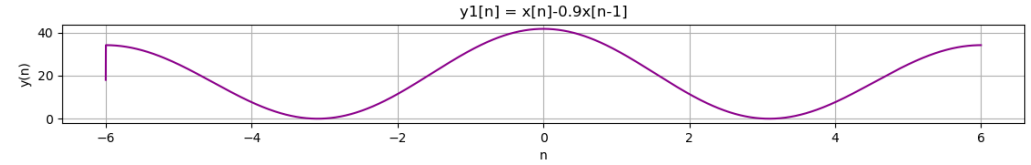


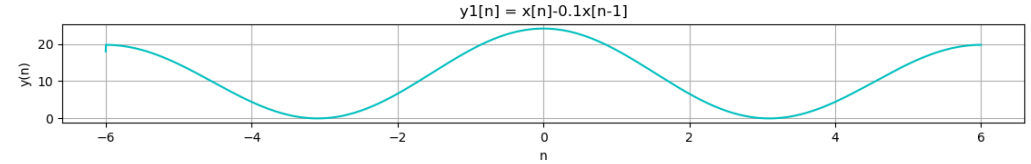


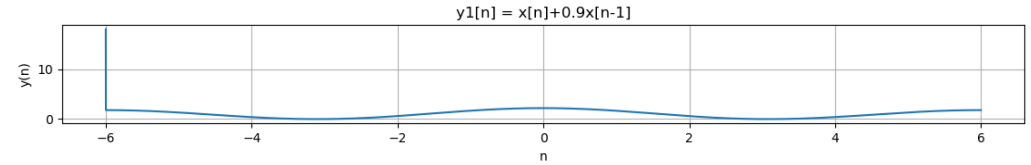
- Exercício 1.c)

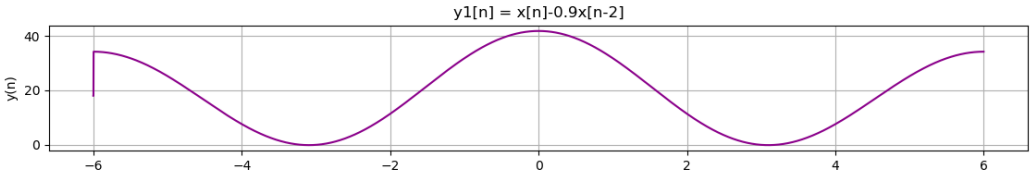
c) Qual a saída de cada um destes sistemas quando $x[n] = 10 + 2\cos(\frac{\pi}{6}n) + 10\cos(\frac{\pi}{3}n)$? Realize este cálculo teoricamente e verifique o resultado usando o python usando a função `y=scipy.signal.lfilter(b,a,x)`.

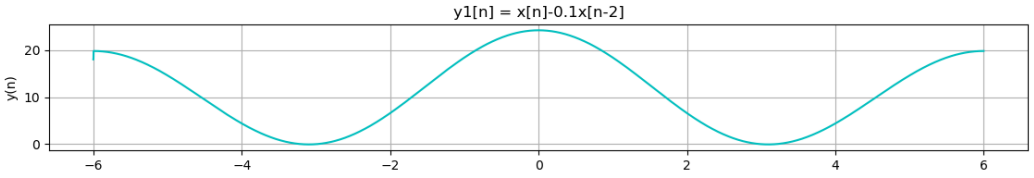
Para y1[n]:

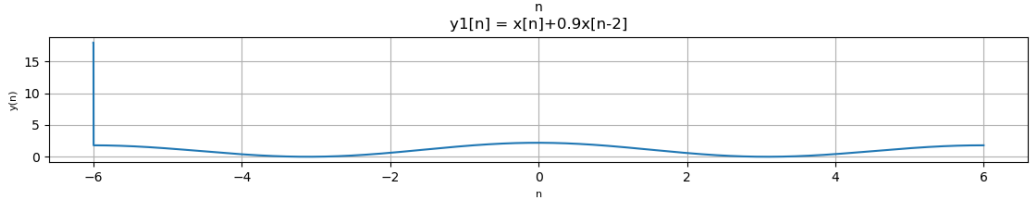

y1[n] = x[n]-0.9x[n-1]
y(n)
n


y1[n] = x[n]-0.1x[n-1]
y(n)
n


y1[n] = x[n]+0.9x[n-1]
y(n)
n


y1[n] = x[n]-0.9x[n-2]
y(n)
n


y1[n] = x[n]-0.1x[n-2]
y(n)
n


y1[n] = x[n]+0.9x[n-2]
y(n)
n

Para $r=0,9$ e $c=1$

$$y_1[m] = x[m] - 0,9x[m-1]$$

$$h(z) = \frac{Y(z)}{X(z)}$$

$$Y(z)(1-0,9z^{-1}) = X(z)(1) \Leftrightarrow$$

$$\Leftrightarrow Y(z) - 0,9z^{-1}Y(z) = X(z) \Leftrightarrow$$

$$\Leftrightarrow h(z) = \frac{1-0,9z^{-1}}{1}$$

$$h(e^{jw}) = 1-0,9e^{-jw} \longrightarrow |h(w)| = |1-0,9e^{-jw}|$$

$$\measuredangle h(w) = w$$

$w=0$ $\quad |h(0)| = 1-0,9e^{j0} = 0,1 \quad \measuredangle h(0) = 0$

$w=\frac{\pi}{6}$ $\quad |h(\frac{\pi}{6})| = 1-0,9e^{j\frac{\pi}{6}} = 0,467 \quad \measuredangle h(\frac{\pi}{6}) = \frac{\pi}{6}$

$w=\frac{\pi}{3}$ $\quad |h(\frac{\pi}{3})| = 1-0,9e^{-j\frac{\pi}{3}} = 0,61 \quad \measuredangle h(\frac{\pi}{3}) = \frac{\pi}{3}$

$$y[m] = |h(0)| \times 10 + 2|h(\tfrac{\pi}{6})| \cos(\tfrac{\pi}{6}m + \measuredangle\tfrac{\pi}{6}) + 10|h(\tfrac{\pi}{3})| \cos(\tfrac{\pi}{3}m + \measuredangle\tfrac{\pi}{3})$$

$$= 0,1\times 10 + 2\times 0,467 \cos(\tfrac{\pi}{6}m + \measuredangle\tfrac{\pi}{6}) + 10\times 0,61 \cos(\tfrac{\pi}{3}m + \measuredangle\tfrac{\pi}{3})$$

$$= 0,1 + 0,934 \cos(\tfrac{\pi}{6}m - \tfrac{\pi}{6}) + 6,1 \cos(\tfrac{\pi}{3}m - \tfrac{\pi}{3})$$

$$= 1 + 0,934 (\tfrac{\sqrt{3}}{2})^{m-1} + 6,1 (\tfrac{1}{2})^{m-1}$$

Para $r=0,1$ e $c=1$

$$h(z) = 1-0,1z^{-1}$$

$$h(e^{jw}) = 1-0,1e^{-jw}$$

$w=0$ $\quad |h(0)| = 1-0,1e^{-j0} = 0,9 \quad \measuredangle h(0) = 0$

$w=\frac{\pi}{6}$ $\quad |h(\frac{\pi}{6})| = 1-0,1e^{-j\frac{\pi}{6}} = 0,99 \quad \measuredangle h(\frac{\pi}{6}) = \frac{\pi}{6}$

$w=\frac{\pi}{3}$ $\quad |h(\frac{\pi}{3})| = 1-0,1e^{-j\frac{\pi}{3}} = 0,965 \quad \measuredangle h(\frac{\pi}{3}) = \frac{\pi}{3}$

$$y[m] = 10\times 0,9 + 2\times 0,99 \cos(\tfrac{\pi}{6}m - \tfrac{\pi}{6}) + 10\times 0,965 \cos(\tfrac{\pi}{3}m - \tfrac{\pi}{3})$$

$$= 9 + 1,98 \cos(\tfrac{\pi}{6}m - \tfrac{\pi}{6}) + 9,65 \cos(\tfrac{\pi}{3}m - \tfrac{\pi}{3})$$

$$= 9 + 1,98 (\tfrac{\sqrt{3}}{2})^{m-1} + 9,65 (\tfrac{1}{2})^{m-1}$$

Para $r=-0,9$ e $c=1$

$$h(z) = 1+0,9z^{-1}$$

$$h(e^{jw}) = 1+0,9e^{-jw}$$

$w=0$ $\quad |h(0)| = 1+0,9e^{-j0} = 1,9 \quad \measuredangle h(0) = 0$

$w=\frac{\pi}{6}$ $\quad |h(\frac{\pi}{6})| = 1+0,9e^{-j\frac{\pi}{6}} = 1,533 \quad \measuredangle h(\frac{\pi}{6}) = \frac{\pi}{6}$

$w=\frac{\pi}{3}$ $\quad |h(\frac{\pi}{3})| = 1+0,9e^{-j\frac{\pi}{3}} = 1,376 \quad \measuredangle h(\frac{\pi}{3}) = \frac{\pi}{3}$

Para $\lambda = 0,9$ e $c = 2$

$$y_1[m] = x[m] - 0,9\, x[m-2]$$

$$h(z) = \frac{y(z)}{x(z)}$$

$$y(z)(1 - 0,9z^{-2}) = x(z)(1)$$

$$h(z) = \frac{1 - 0,9z^{-2}}{1}$$

$$h(e^{jw}) = 1 - 0,9e^{-2jw} \longrightarrow |h(w)| = |1 - 0,9e^{-j2w}|$$

$$\longrightarrow \measuredangle h(w) = -w$$

$w = 0$ $\quad |h(0)| = 1 - 0,9 = 1$ $\quad \measuredangle h(0) = 0$

$w = \frac{\pi}{6}$ $\quad |h(\frac{\pi}{6})| = 1 - 0,9e^{j2\frac{\pi}{6}} = 0,684$ $\quad \measuredangle h(\frac{\pi}{6}) = \frac{\pi}{6}$

$w = \frac{\pi}{3}$ $\quad |h(\frac{\pi}{3})| = 1 - 0,9e^{j2\frac{\pi}{3}} = 0,89$ $\quad \measuredangle h(\frac{\pi}{3}) = \frac{\pi}{3}$

$$y[m] = 0,1 \times 10 + 2 \times 0,684 \cos(\tfrac{\pi}{6}m - \tfrac{\pi}{6}) + 10 \times 0,89 \cos(\tfrac{\pi}{3}m - \tfrac{\pi}{3})$$

$$= 1 + 1,3868 \cos(\tfrac{\pi}{6}(m-1)) + 8,9 \cos(\tfrac{\pi}{3}(m-1))$$

$$= 1 + 1,368 \left(\frac{\sqrt{3}}{2}\right)^{m-1} + 8,9 \left(\frac{1}{2}\right)^{m-1}$$

Para $\lambda = 0,1$ e $c = 2$

$$h(z) = 1 - 0,1z^{-2}$$

$$|h(w)| = |1 - 0,1e^{-j2w}| \quad \measuredangle h(w) = -w$$

$w = 0$ $\quad |h(0)| = 1 - 0,1 = 0,9$ $\quad \measuredangle h(0) = 0$

$w = \frac{\pi}{6}$ $\quad |h(\frac{\pi}{6})| = 1 - 0,1e^{j2\frac{\pi}{6}} = 0,965$ $\quad \measuredangle h(\frac{\pi}{6}) = \frac{\pi}{6}$

$w = \frac{\pi}{3}$ $\quad |h(\frac{\pi}{3})| = 1 - 0,1e^{j2\frac{\pi}{3}} = 0,988$ $\quad \measuredangle h(\frac{\pi}{3}) = \frac{\pi}{3}$

$$y[m] = 10 \times 0,9 + 2 \times 0,965 \cos(\tfrac{\pi}{6}m - \tfrac{\pi}{6}) + 10 \times 0,988 \cos(\tfrac{\pi}{3}m - \tfrac{\pi}{3})$$

$$= 9 + 1,93 \cos(\tfrac{\pi}{6}(m-1)) + 9,88 \cos(\tfrac{\pi}{3}(m-1))$$

Para $\lambda = -0,9$ e $c = 2$

$$h(z) = 1 + 0,9z^{-2}$$

$w = 0$ $\quad |h(0)| = 1 + 0,9 = 1,9$ $\quad \measuredangle h(0) = 0$

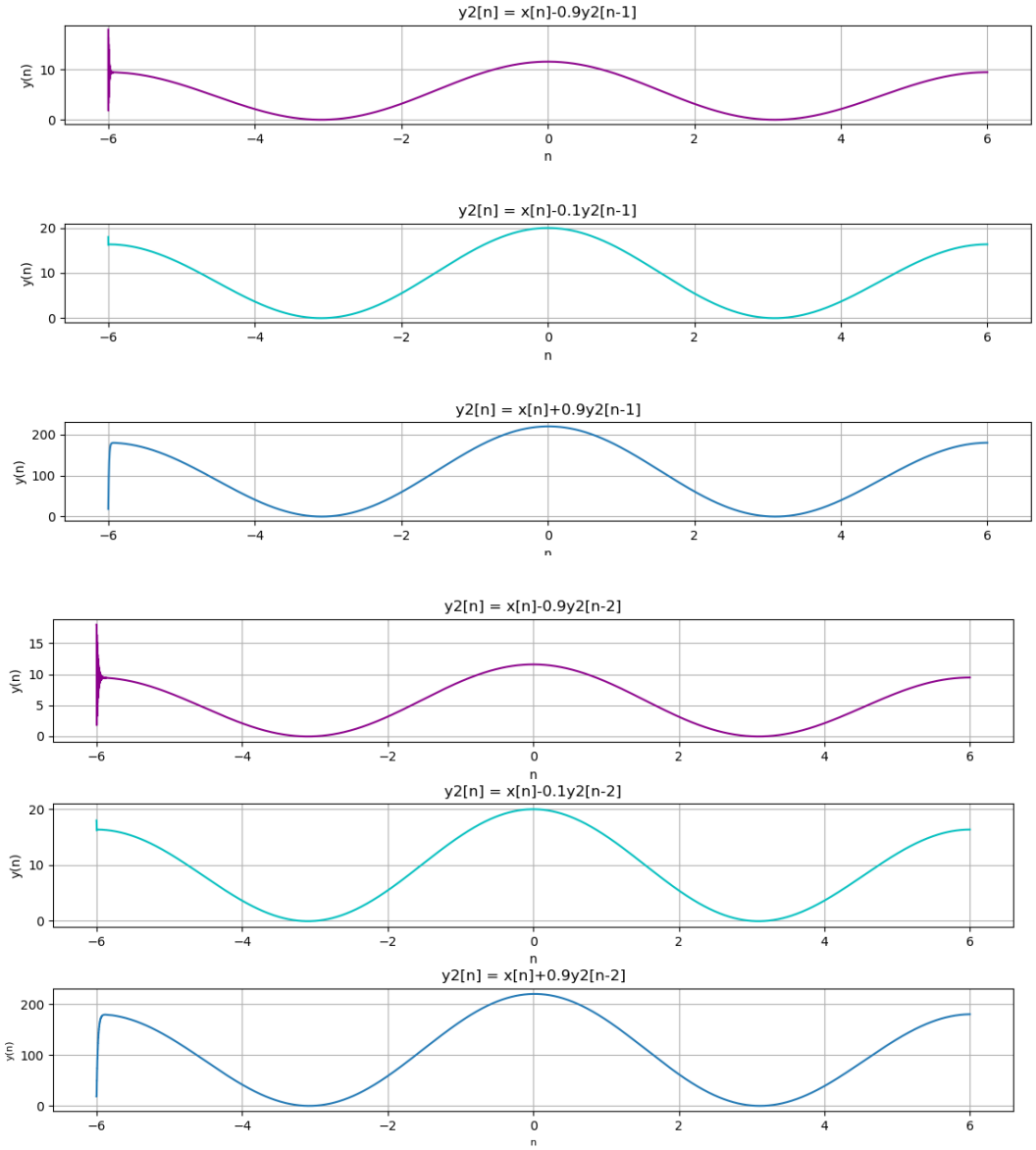
$w = \frac{\pi}{6}$ $\quad |h(\frac{\pi}{6})| = 1 + 0,9e^{-j2\frac{\pi}{6}} = 3,565$ $\quad \measuredangle h(\frac{\pi}{6}) = \frac{\pi}{6}$

$w = \frac{\pi}{3}$ $\quad |h(\frac{\pi}{3})| = 1 + 0,9e^{-j2\frac{\pi}{3}} = 1,11$ $\quad \measuredangle h(\frac{\pi}{3}) = \frac{\pi}{3}$

$$y[m] = 10 \times 1,9 + 2 \times 3,565 \cos(\tfrac{\pi}{6}m - \tfrac{\pi}{6}) + 10 \times 1,11 \cos(\tfrac{\pi}{3}m - \tfrac{\pi}{3})$$

$$= 19 + 7,13 \cos(\tfrac{\pi}{6}(m-1)) + 11,1 \cos(\tfrac{\pi}{3}(m-1))$$

Para y2[n]:


y2[n] = x[n]-0.9y2[n-1]
y2[n] = x[n]-0.1y2[n-1]
y2[n] = x[n]+0.9y2[n-1]
y2[n] = x[n]-0.9y2[n-2]
y2[n] = x[n]-0.1y2[n-2]
y2[n] = x[n]+0.9y2[n-2]
y(n)
n

~~$y_0[n] = x[n] - \alpha y_0$~~

$y_0[n] = x[n] - \alpha y_0[n-c]$

$\Rightarrow y_0[n] + \alpha y_0[n-c] = x[n]$

Para $\alpha = 0.9$ e $c = 1$

$H(z) = \frac{Y(z)}{X(z)}$

$Y(z)(1) = X(z)(1 + 0.9z^{-1})$

$Y(z) = X(z) + 0.9z^{-1}X(z)$

$H(z) = \frac{1 + 0.9z^{-1}}{1} \cdot \frac{1}{1+0.9z^{-1}}$

$H(e^{jw}) = \frac{1}{1+0.9e^{-jw}} \rightarrow \left| \frac{1}{1+0.9e^{-jw}} \right|$

$\angle H(w) = w$

$w = 0 \quad |H(0)| = \frac{1}{1+0.9e^{-j0}} = 0.526 \quad \angle H(0) = 0$

$w = \frac{\pi}{6} \quad |H(\frac{\pi}{6})| = \frac{1}{1+0.9e^{-j\frac{\pi}{6}}} = \frac{1}{1.533} = 0.652 \quad \angle H(\frac{\pi}{6}) = \frac{\pi}{6}$

$w = \frac{\pi}{3} \quad |H(\frac{\pi}{3})| = \frac{1}{1+0.9e^{-j\frac{\pi}{3}}} = 0.76 \quad \angle H(\frac{\pi}{3}) = \frac{\pi}{3}$

$y[n] = 10 \times |H(0)| + 4 \times |H(\frac{\pi}{6})| \cos(\frac{\pi}{6}n + \angle\frac{\pi}{6}) + 10 \times |H(\frac{\pi}{3})| \cos(\frac{\pi}{3}n + \angle\frac{\pi}{3})$

$= 10 \times 0.526 + 4 \times 0.652 \cos(\frac{\pi}{6}n - \frac{\pi}{6}) + 10 \times 0.76 \cos(\frac{\pi}{3}n - \frac{\pi}{3})$

$= 5.26 + 2.608 \cos(\frac{\pi}{6}n - \frac{\pi}{6}) + 7.6 \cos(\frac{\pi}{3}n - \frac{\pi}{3})$

$= 5.26 + 2.608 \cos(\frac{\pi}{6}(n-1)) + 7.6 \cos(\frac{\pi}{3}(n-1))$

Para $r=0,1$ e $C=1$

$$H(z) = \frac{1}{1+0,9z^{-1}} \rightarrow H(e^{j\omega}) = \frac{1}{1+0,1e^{j\omega}}$$

$\omega = 0 \qquad |H(0)| = \frac{1}{1+0,1e^{j0}} = 0,91$

$\omega = \frac{\pi}{6} \qquad |H(\frac{\pi}{6})| = \frac{1}{1+0,1e^{j\frac{\pi}{6}}} = 0,944 \quad \angle H(\frac{\pi}{6}) = \frac{\pi}{6}$

$\omega = \frac{\pi}{3} \qquad |H(\frac{\pi}{3})| = \frac{1}{1+0,1e^{j\frac{\pi}{3}}} = 0,966 \quad \angle H(\frac{\pi}{3}) = \frac{\pi}{3}$

$$y[n] = 10 + 9,1 + 2 \times 0,944 \cos\left(\frac{\pi}{6}n - \frac{\pi}{6}\right) + 10 \times 0,966 \cos\left(\frac{\pi}{3}n - \frac{\pi}{3}\right)$$

$$= 9,1 + 1,888 \cos\left(\frac{\pi}{6}(n-1)\right) + 9,66 \cos\left(\frac{\pi}{3}(n-1)\right)$$

$$= 9,1 + 1,888 \left(\frac{\sqrt{3}}{2}\right)^{n-1} + 9,66 \left(\frac{1}{2}\right)^{n-1}$$

Para $r=-0,9$ e $C=1$

$$H(z) = \frac{1}{1-0,9z^{-1}} \rightarrow H(e^{j\omega}) = \frac{1}{1-0,9e^{-j\omega}}$$

$\omega = 0 \qquad |H(0)| = \frac{1}{1-0,9e^{j0}} = 10$

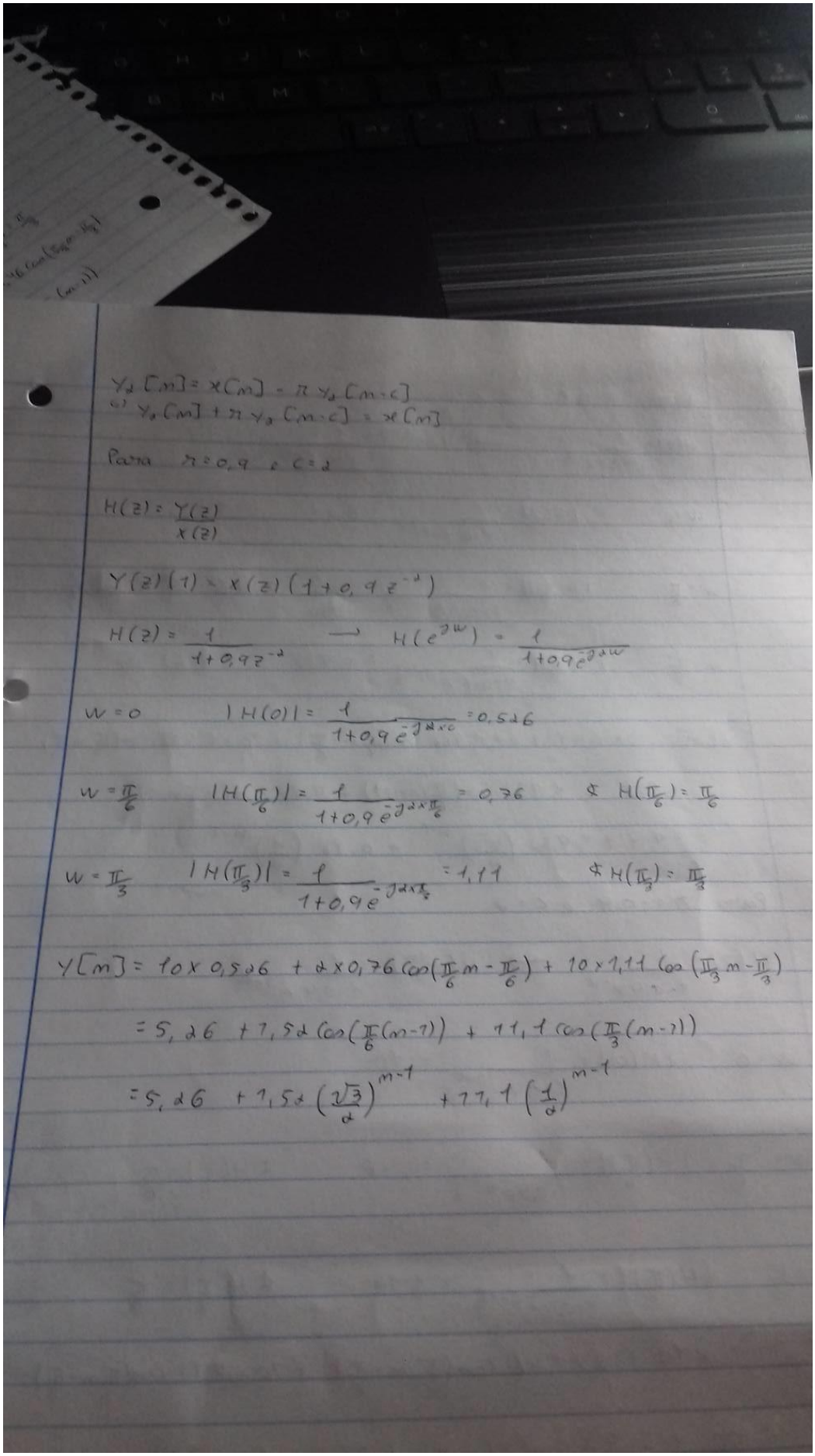
$\omega = \frac{\pi}{6} \qquad |H(\frac{\pi}{6})| = \frac{1}{1-0,9e^{j\frac{\pi}{6}}} = 2,14 \quad \angle H(\frac{\pi}{6}) = \frac{\pi}{6}$

$\omega = \frac{\pi}{3} \qquad |H(\frac{\pi}{3})| = \frac{1}{1-0,9e^{j\frac{\pi}{3}}} = 1,46 \quad \angle H(\frac{\pi}{3}) = \frac{\pi}{3}$

$$y[n] = 10 \times 10 + 2 \times 2,14 \cos\left(\frac{\pi}{6}n - \frac{\pi}{6}\right) + 10 \times 1,46 \cos\left(\frac{\pi}{3}n - \frac{\pi}{3}\right)$$

$$= 100 + 4,28 \cos\left(\frac{\pi}{6}(n-1)\right) + 14,6 \cos\left(\frac{\pi}{3}(n-1)\right)$$

$$= 100 + 4,28 \left(\frac{\sqrt{3}}{2}\right)^{n-1} + 14,6 \left(\frac{1}{2}\right)^{n-1}$$



$y_d[n] = x[n] - r\,y_d[n-c]$

$y_d[n] + r\,y_d[n-c] = x[n]$

Para $r = 0,9$ e $c = 2$

$$H(z) = \frac{Y(z)}{X(z)}$$

$$Y(z)(1) = X(z)(1 + 0,9z^{-2})$$

$$H(z) = \frac{1}{1+0,9z^{-2}} \longrightarrow H(e^{j\omega}) = \frac{1}{1+0,9e^{-j2\omega}}$$

$\omega = 0$ $\quad |H(0)| = \frac{1}{1+0,9e^{-j2\times 0}} = 0,526$

$\omega = \frac{\pi}{6}$ $\quad |H(\frac{\pi}{6})| = \frac{1}{1+0,9e^{-j2\times\frac{\pi}{6}}} = 0,76$ $\quad \angle H(\frac{\pi}{6}) = \frac{\pi}{6}$

$\omega = \frac{\pi}{3}$ $\quad |H(\frac{\pi}{3})| = \frac{1}{1+0,9e^{-j2\times\frac{\pi}{3}}} = 1,11$ $\quad \angle H(\frac{\pi}{3}) = \frac{\pi}{3}$

$$y[n] = 10\times 0,526 + 2\times 0,76\cos\left(\frac{\pi}{6}n - \frac{\pi}{6}\right) + 10\times 1,11\cos\left(\frac{\pi}{3}n - \frac{\pi}{3}\right)$$

$$= 5,26 + 1,52\cos\left(\frac{\pi}{6}(n-1)\right) + 11,1\cos\left(\frac{\pi}{3}(n-1)\right)$$

$$= 5,26 + 1,52\left(\frac{\sqrt{3}}{2}\right)^{n-1} + 11,1\left(\frac{1}{2}\right)^{n-1}$$

Para $r = 0,1$ e $C = 2$

$H(z) = \frac{1}{1+0,1z^{-2}} \longrightarrow H(e^{j\omega}) = \frac{1}{1+0,1e^{-j2\omega}}$

$\omega = 0 \quad |H(0)| = \frac{1}{1+0,1e^{-j2\cdot 0}} = 0,91$

$\omega = \frac{\pi}{6} \quad |H(\frac{\pi}{6})| = \frac{1}{1+0,1e^{-j2\cdot\frac{\pi}{6}}} = 0,966 \quad \angle H(\frac{\pi}{6}) = \frac{\pi}{6}$

$\omega = \frac{\pi}{3} \quad |H(\frac{\pi}{3})| = \frac{1}{1+0,1e^{-j2\cdot\frac{\pi}{3}}} = 0,988 \quad \angle H(\frac{\pi}{3}) = \frac{\pi}{3}$

$y[n] = 10 \times 0,91 + 2 \times 0,966 \cos(\frac{\pi}{6}n - \frac{\pi}{6}) + 70 \times 0,988 \cos(\frac{\pi}{3}n - \frac{\pi}{3})$

$= 9,1 + 1,932 \cos(\frac{\pi}{6}(n-1)) + 9,88 \cos(\frac{\pi}{3}(n-1))$

$= 9,1 + 1,932 \left(\frac{\sqrt{3}}{2}\right)^{n-1} + 9,88 \left(\frac{1}{2}\right)^{n-1}$

Para $r = -0,9$ e $C = 2$

$H(z) = \frac{1}{1-0,9z^{-2}} \longrightarrow H(e^{j\omega}) = \frac{1}{1-0,9e^{-j2\omega}}$

$\omega = 0 \quad |H(0)| = \frac{1}{1-0,9e^{-j2\cdot 0}} = 10$

$\omega = \frac{\pi}{6} \quad |H(\frac{\pi}{6})| = \frac{1}{1-0,9e^{-j2\cdot\frac{\pi}{6}}} = 1,46 \quad \angle H(\frac{\pi}{6}) = \frac{\pi}{6}$

$\omega = \frac{\pi}{3} \quad |H(\frac{\pi}{3})| = \frac{1}{1-0,9e^{-j2\cdot\frac{\pi}{3}}} = 1,12 \quad \angle H(\frac{\pi}{3}) = \frac{\pi}{3}$

$y[n] = 10 \times 10 + 2 \times 1,46 \cos(\frac{\pi}{6}n - \frac{\pi}{6}) + 70 \times 1,12 \cos(\frac{\pi}{3}n - \frac{\pi}{3})$

$= 100 + 2,92 \cos(\frac{\pi}{6}(n-1)) + 11,2 \cos(\frac{\pi}{3}(n-1))$

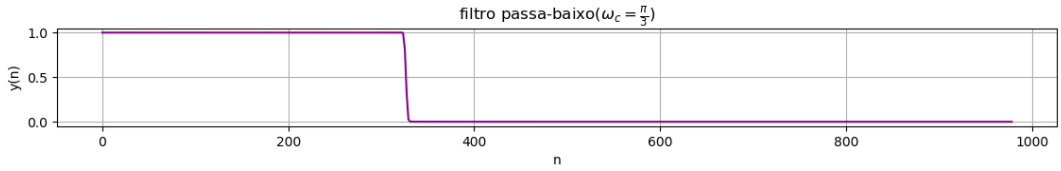
$= 100 + 2,92 \left(\frac{\sqrt{3}}{2}\right)^{n-1} + 11,2 \left(\frac{1}{2}\right)^{n-1}$

Grupo II:

- Exercício 2.a)

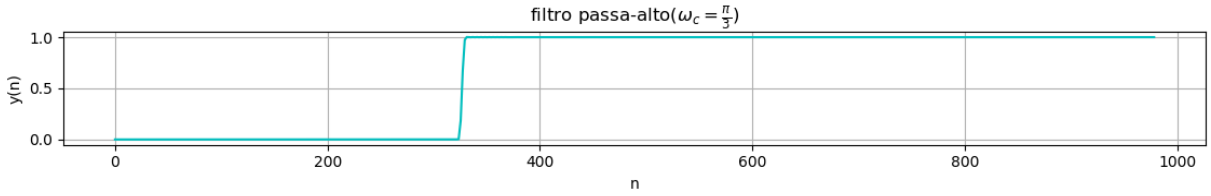
2. Pretende-se desenhar filtros com especificações conhecidas usando o python. Para tal utilize a função `y=scipy.signal.firwin(numtaps, cutoff,pass_zero=True)`, *onde numtaps defina a ordem do filtro, cutoff é uma lista que define a(s) frequência(s) de corte e pass_zero uma variável boleana (True para passa baixo e False para passa alto e passa banda). Verifique que outros parâmetros esta função permite definir.*

   a) Filtro passa-baixo com frequência de corte $\omega_c = \frac{\pi}{3}$.



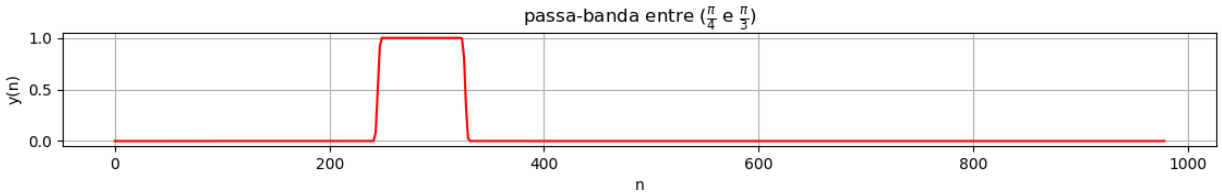
- Exercício 2.b)

b) Filtro passa-alto com frequência de corte $\omega_c = \frac{\pi}{3}$.



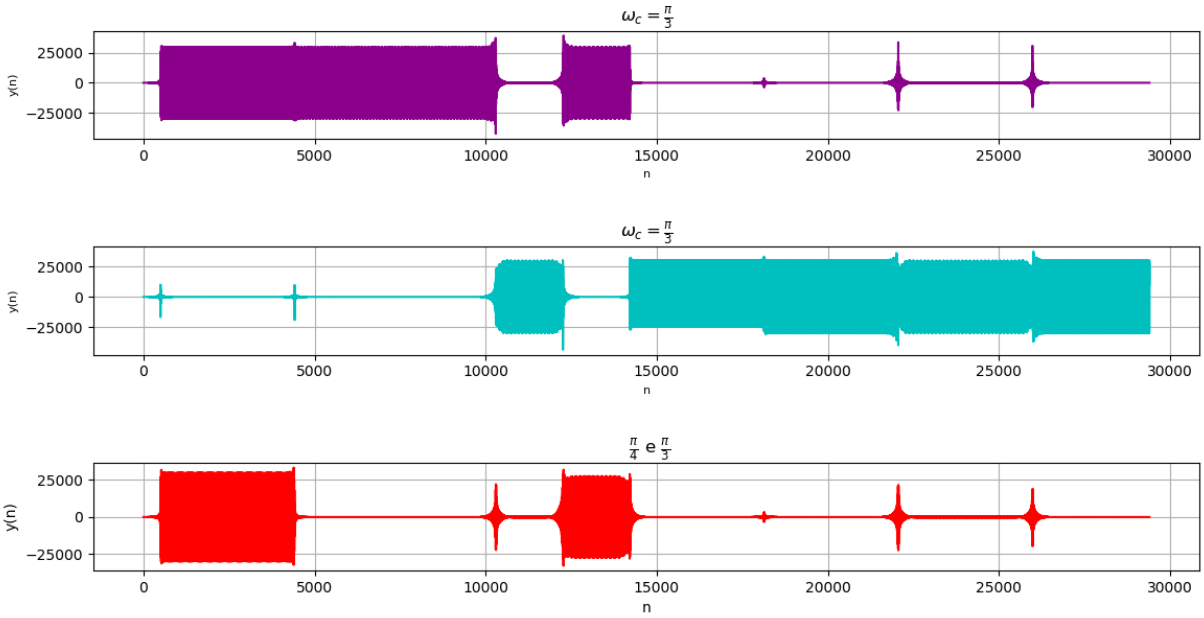
- Exercício 2.c)

c) Filtro passa-banda com frequências de corte $\omega_{c_1} = \frac{\pi}{4}$ e $\omega_{c_2} = \frac{\pi}{3}$.



- Exercício 2.d)

d) Aplique os filtros desenhados em vários sinais wav e verifique os outputs. Quais as frequências de corte e qual a relação com a frequência de amostragem?


$\omega_c = \frac{\pi}{3}$
25000
0
−25000
y(n)
0
5000
10000
15000
20000
25000
30000
n

$\omega_c = \frac{\pi}{3}$
25000
0
−25000
y(n)
0
5000
10000
15000
20000
25000
30000
n

$\frac{\pi}{4}$ e $\frac{\pi}{3}$
25000
0
−25000
y(n)
0
5000
10000
15000
20000
25000
30000
n

Conclusão

Neste trabalho aprendemos a utilizar filtros FIR e IRR no Python.