Algorithmic Strategies 2025/26

Week 1 – Problem Solving

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Outline

Backtracking

Reading about problem reduction

- T. Cormen et al., Introduction to Algorithms, Chapter 34
- S.S. Skiena, The Algorithm Design Manual, Section 9.1

Introduction

Our aim is to solve (well-defined) problems in an efficient way

- Efficiency is understood in terms of CPU-time and/or memory.
- We can solve a problem if we can recast it as another similar problem for which an algorithm is known.
- Otherwise, we use a generic problem-solving strategy, such as backtracking, dynamic programming, etc..

Introduction

What is a (computational) problem?

An abstract question that can be solved by a computer.

- There are two entities in a problem:
  - Instances
  - Solutions
- Problem instances may have no solution, one solution, or multiple solutions.

Introduction

Example: The Travelling Salesman Problem (TSP)

- An instance is characterized by of a set of cities and the distances between them

- A solution is a sequence of cities, describing the order in which they should be visited.

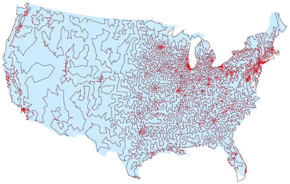


Figure credit: David Applegate, Robert Bixby, Vasek Chvatal and William Cook.

Decision vs. Optimization

Types of problems

- Counting problems: the output is the number of solutions.
  - Count the number of tours.
- Decision problems: the output is either yes or no.
  - Does exist a tour with a length $\leq k$?
- Optimization problems: the output is the minimum/maximum.
  - Find the tour with the minimum length.

An optimization problem has a corresponding decision problem.

Decision vs. Optimization

Solution strategy

- We can solve an optimization problem by solving a sequence of related decision problems (under some mild assumptions).
  - Perform binary search on k.
- We can solve a decision problem by solving the related optimization problem.
  - Check whether optimal value is $\leq k$.

Decision vs. Optimization

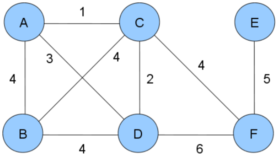
Given a knapsack of capacity W and n objects with weights $w_1, \ldots, w_n$ and values $v_1, \ldots, v_n$.

- Decision knapsack: Given k, is there a subset of objects that fits into the knapsack and has total value $\geq k$?
- Optimization knapsack: Find the subset that fits the knapsack and that maximizes the total value.

Decision vs. Optimization

- Decision spanning tree: Given a weighted graph G and k, does G have a spanning tree of total weight $\leq k$?

- Minimum spanning tree: Given a weighted graph G, find a minimum spanning tree.



Problem reduction

Reduction is a transformation of one problem into another problem (A reduces to B):

1. Transform a problem A into a problem B for which an algorithm is known.
2. Transform the solution of problem B into the solution of problem A.

We are very often interested in polynomial reductions: a reduction that takes polynomial amount of time.

Problem reduction

Partition problem

Given a set of integers $\{p_1, \ldots, p_m\}$ whose sum is S, can it be partitioned into two subsets with the same sum?

Subset sum problem

Given a set of integers $\{s_1, \ldots, s_n\}$, is there a subset whose sum is B?

Assume that you have an algorithm for the subset sum problem.

Can you solve the partition problem?

Can you reduce the partition problem to the subset sum problem?

Problem reduction

Partition problem

Given a set of integers $\{p_1, \ldots, p_m\}$ whose sum is S, can it be partitioned into two subsets with the same sum?

Subset sum problem

Given a set of integers $\{s_1, \ldots, s_n\}$, is there a subset whose sum is B?

Assume that you have an algorithm for the subset sum problem.

Can you solve the partition problem?

Can you reduce the partition problem to the subset sum problem?

Set $n = m$, $s_i = p_i$ and $B = S/2$. Solve the subset sum problem and check the output.

Problem reduction

Subset sum problem

Given a set of integers $\{s_1, \dots, s_n\}$, is there a subset whose sum is B?

Knapsack problem

Find the subset of a set of m objects that fits into a knapsack of capacity W and that maximizes the total value.

Assume that you have an algorithm for the knapsack problem.

Can you solve the subset sum problem?

Can you reduce the subset sum problem to the knapsack problem?

Problem reduction

Subset sum problem

Given a set of integers $\{s_1, \dots, s_n\}$, is there a subset whose sum is B?

Knapsack problem

Find the subset of a set of m objects that fits into a knapsack of capacity W and that maximizes the total value.

Assume that you have an algorithm for the knapsack problem.

Can you solve the subset sum problem?

Can you reduce the subset sum problem to the knapsack problem?

Set $m = n$, $w_i = v_i = s_i$ and $W = B$. Solve the knapsack problem. Then, check if the sum of the values of the chosen objects is B.

Problem reduction

- You can solve many problems by using reduction techniques. You need to recognize the underlying problem.
- Or use a general strategy: Break the problem down into smaller problems which you can solve, and devise how to recover the solution from the partial solutions found
- This is the main strategy of backtracking, dynamic programming, greedy algorithms and branch-&-bound.
- To know how to break the problem in the most effective manner requires a lot of training.