Foundations of Risk Management

Risk-Adjusted Return on Capital

$$\text{RAROC} = \frac{\text{Revenue} - \text{Expected Loss}}{\text{Capital}}$$

Risk Governance

Board of directors: responsible for enterprise-level risk management. If they can't understand risk, they need to recruit a *risk advisory director.*
Risk management committee: makes all risk appetite decisions and bring discussions to the board.
Compensation committee: charged with aligning managerial compensation with long-term stakeholder needs.
Audit committee: monitor compliance with accounting standards and offer opinions on the variables used in testing exposures.

Credit Risk Transfer Mechanisms

Credit Default Swaps (CDS) prices can help quantify credit risk on a real-time basis (unlike rating agencies).

Systematic Risk Measure

$$\beta_i = \frac{\text{cov}(R_i, R_M)}{\sigma_M^2} = \rho_{i,M}\frac{\sigma_i}{\sigma_M}$$

$$cov(x, y) = \rho_{x,y}\sigma_x\sigma_y$$

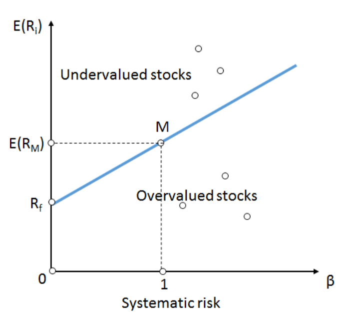
CAPM (Capital Asset Pricing Model)

Model used to determine a theoretically appropriate required rate of return of an asset, to make decisions about adding assets to a well-diversified portfolio

Minimum required return **(SML)**

$$E(R_i) = R_F + \beta_i(E(R_M) - R_F)$$

β_i: nondiversifiable or systematic risk

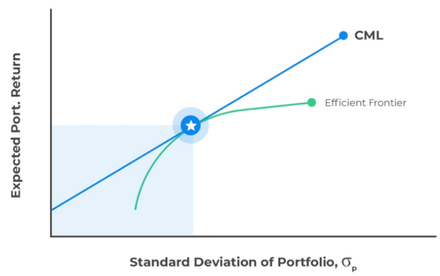


Assumptions

- Information is freely available.
- Markets are frictionless.
- Fractional investments are possible.
- There is perfect competition.
- Investors make their decisions solely based on **expected returns and variances**.
- Market participants can borrow and lend unlimited amounts at the risk-free rate.
- Expectations are homogenous.

CML (Capital Market Line)

$$E(R_P) = R_F + \frac{E(R_M) - R_F}{\sigma_M}\sigma_P$$



Measures of performance

$$Sharpe = \frac{E(R_p) - R_F}{\sigma_p} \ (slope\ of\ CML)$$

$$\text{Treynor} = \frac{E(R_p) - R_F}{\beta_p} (slope\ of\ SML)$$

$$Jensen = \alpha_p = E(R_p) - (R_F + \beta_p(E(R_M) - R_F)) = E(R_p) - CAPM$$

Performance using a benchmark (B)

$$IR = \frac{E(R_P) - E(R_B)}{tracking\ error};\quad B: Benchmark$$

$$\text{Sortino} = \frac{R_P - R_{MIN}}{downside\ deviation}$$

$$\text{tracking error} = \sqrt{\frac{\sum(R_P - R_B)^2}{n - 1}}$$

If a manager is trying to earn a return higher than the market portfolio or any other reference or benchmark, the difference will have some variability over time. **Tracking error** is the term used to describe the standard deviation of the difference between the portfolio return and the benchmark return.

Arbitrage Pricing Theory (APT)

$$E(R_i) = R_F + b_{i1}RP_1 + \cdots + b_{ik}RP_k$$

Risk Factors for Financial Disasters

Interest rate risk: (measured using duration) Case study: **S&L** tries to capture spread between short-term and long-term rates, the Fed raises interest rates and they collapse. (Lesson: match duration between assets and liabilities).
Liquidity risk: loss that results from short-term funding issues. Case studies: **Lehman** Brothers, **Continental Illinois**, **Northern Rock**.
Hedging strategies: static vs. dynamic (rolling) hedge. Case study: **Metallges**.
Model risk: improper assumptions, wrong model. Case studies: **Niederhoffer**, **LTCM** (10-day VaR, short-term liquidity vacuums), **London Whale** (adjust assumptions or valuations to make bad decisions look better).
Rogue trader: misleading reporting, no separation between back- and front office. Case study: **Barings Bank**. *Nick Leeson* took speculative derivative positions (Nikkei 225 futures) in an active attempt to cover trading losses; Leeson had dual responsibilities of grading and supervising settlement operations, allowing him to hide trading losses; lessons include separation of duties and management oversight.
Financial engineering: not understanding hedging tools. Case studies: Bankers Trust, Orange County, Sachsen Landesbank.
Reputational risk: Case study: Volkswagen.
Corporate Governance: Case study: Enron (Chairman of the board and CEO were the same person). Resulted in Sarbanes-Oxley.
Cyber risk: Case study: the SWIFT system.

Financial Crisis 2007-2009

Contributing factors:

- Banks relaxed their lending standards with move to OTD. Subprime mortgages became very popular because they offered a high yield in an environment of very low interest rates.
- Institutions increasingly funded their long-term assets through short-term liabilities, when the crisis struck, the liabilities could not be rolled over.
- The Lehman Brothers default caused a loss of confidence with banks refusing to lend to each other, and ultimately requiring central banks to provide liquidity support.

Key lessons:

- The needs of all the firm's stakeholders must be considered. The board needs to have competent and independent directors.
- The board needs to take a highly proactive role in the firm's risk management process.
- The firm's risk appetite needs to be clearly articulated by the board.
- Compensation should be structured to better align management behavior with long- term stakeholder priorities as determined by the board.

Quantitative Analysis

VaR

$VaR = \mu - z\,\sigma$

scaled for daily VaR:

$\text{VaR} = \frac{\mu}{n} - z\frac{\sigma}{\sqrt{n}}$

Daily & annual VaR

$\text{VaR}_{\text{daily}} = \frac{VaR_{annual}}{\sqrt{n}}$

$VaR_{annual} = \text{VaR}_{\text{daily}}\sqrt{n}$

Bayes Rule

$P(A|B) = \frac{P(A)P(B|A)}{P(B)}$

Derived from joint probability

$P(AB) = P(B)P(A|B)$

$\Leftrightarrow P(A|B) = \frac{P(AB)}{P(B)} = \frac{P(A)P(B|A)}{P(B)}$

Total probability rule:

$P(A) = P(A|B_1)P(B_1) + \cdots + P(A|B_n)P(B_n)$

Expected Value & Variance

$E(X) = \sum P(x_i)x_i$

$\sigma^2 = E[(X-\mu)^2]$

Mean & stdev. of 2 portfolios

$\mu = w_1\mu_1 + w_2\mu_2$

$\sigma = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + \left(2w_1w_2\rho_{1,2}\sigma_1\sigma_2\right)}$

$\sigma^2 = \sum_{i=1}^{n}\sum_{j=1}^{n}\rho_{ij}(w_1\sigma_i)(w_2\sigma_j)$

Covariance & Correlation

$Cov(X,Y) = E\{[X-E(X)][Y-E(Y)]\} = E(XY) - E(X) - E(Y)$

$Corr(X,Y) = \rho_{X,Y} = \frac{Cov(X,Y)}{\sigma(X)\sigma(Y)}$

Bernoulli Distribution

Evaluates a random variable with two possible outcomes.

$P(X=x) = p^x(1-p)^{1-x}$

Binomial Distribution

Evaluates a random variable with two possible outcomes over a series of *n* trials.

$P(X=k) = \binom{n}{k}\ p^k(1-p)^{n-k}$

$E(X) = np$

$Var(X) = np(1-p)$

Poisson Distribution

X: number of successes per unit.

λ: avg. number of successes per unit.

$P(X=x) = \frac{\lambda^x e^{-\lambda}}{x!}$

$E(X) = \lambda;\quad Var(X) = \lambda$

Chi-Squared Distribution

Used for hypothesis tests concerning the variance of a normally distributed population.

$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$

F-test is used for hypotheses tests concerning the equality of the variances (s^2) of two populations.

$F-test:\ F = \frac{s_1^2}{s_2^2}$

Confidence Intervals

$\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}};\quad \bar{x} \pm t\cdot SE;\quad z = \frac{x-\mu}{\sigma}$

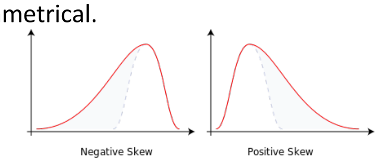
Critical z-values

Level of Significance	Two-Tailed Test	One-Tailed Test
0.10 = 10%	±1.65	+1.28 or −1.28
0.05 = 5%	±1.96	+1.65 or −1.65
0.01 = 1%	±2.58	+2.33 or −2.33

These are also used as critical values for testing betas in linear regression.

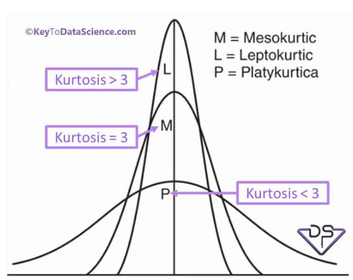
Skewness and Kurtosis

Skewness: extent to which the data is not symmetrical.



Kurtosis: measure of the degree to which a distribution is spread out compared to a normal distribution.

$excess\ kurtosis = kurtosis - 3$



The **Jarque-Bera** test statistic tests whether the sample skewness and kurtosis are compatible with an assumption that the returns are normally distributed.

Linear Regression

Ordinary Least Squares (OLS): estimate α and β in an effort to minimize the squared residuals (i.e., error terms).

Slope coefficient $\beta = \frac{Cov(X,Y)}{Var(X)}$

Intercept $\alpha = \overline{Y} - \beta\overline{X}$

(The regression line passes through a point with coordinates equal to the mean of X and Y)

Underlying Assumptions

(1) $E(\epsilon_i|X_i) = 0$. Violated by:

- Survivorship or sample selection bias
  A subset of the data is systematically excluded.
- Simultaneity bias
  X causes Y and Y causes X.
- Omitted variables
  Omitted variable is correlated with an included X and is determinant of the Y.
- Attenuation bias
  X variables are measured with error and leads to underestimation of the regression coefficients.

(2) All (X,Y) are i.i.d

(3) Variance of X is positive

(4) Variance of the errors is constant (i.e. homoskedacity)

(5) No outliers

Hypothesis Testing for Betas

$t = \frac{\hat{\beta} - \beta_0}{S_b};\quad S_b: \text{standard error} = \frac{s}{\sqrt{n}}$

$t > critical\ value \Rightarrow \beta_i \neq 0$

Regression Assumption Violations

Heteroskedacity: occurs when the variance of the residuals is not the same across all observations in the sample.

Multicollinearity: when two or more of the independent variables or linear combinations of the independent variables, are highly correlated with each other.

Coefficient of Determination

Measure of goodness of fit of the regression. % of variation explained by the model

$R^2 = \frac{ESS}{TSS};\quad TSS = ESS + RSS$

TSS: *Total sum of squares*
ESS: *Explained sum of squares*
RSS: *Residual sum of squares*

$Adjusted\ R^2 = 1 - \frac{(1-R^2)(N-1)}{N-p-1}$

p: Number of predictors
N: Sample size

F-Statistic

The F-stat tests whether at least one of the independent variables explains a significant portion of the variation of the dependent variable.

Time Series

- **AR**: ACF slowly decays.
- **MA**: PACF slowly decays.
- **ARMA**: ACF & PACF slowly decay.

Power Law

Alternative to assuming normal distributions.

$P(v > k) = kx^{-\alpha}$
Smaller alpha indicates fatter tail

Bias-Variance Tradeoff

Overfitted → low bias, high variance
Underfitted → high bias, low variance

K-Means Clustering

Inertia: measure of the distance between each data point and its centroid. A lower inertia implies a better cluster fit. However, because inertia will always fall as more centroids are added, there is a limit to which adding more centroids adds value.
Silhouette coefficient: choose *K* by comparing the distance between an observation and other points in its own cluster to its distance to data points in the next closest cluster.

Regularization

- **LASSO (L1)** $Loss = RSS + \lambda \sum \beta_i^2$
- **Ridge Reg. (L2)** $\text{Loss} = \text{RSS} + \lambda \sum |\beta_i|$

Decision Trees

Information gain measures the extent to which obtaining information about a given feature can reduce uncertainty. **Entropy** and the **Gini** coefficient are used as ways to compute information gain.

- **Pre-pruning**: when splitting stops if the training set observation count relating to a node is under a specific number.
- **Post-pruning**: when a large tree is built, and weak nodes are removed.

ROC & AUC

Receiver operating characteristic (ROC) curve is a means of illustrating the link between true positive rates and false positive rates. The model predictions improve as the **area under the ROC curve (AUC)** increases.

- AUC = 1: 100% accurate predictions.
- AUC = 0.5: no predictive value.
- AUC < 0.5: negative predictive value

Financial Markets and Products

Funds

Open-End Mutual Funds: trade at NAV, when investor buys, shares are created.
Closed-End Mutual Funds: does not trade at NAV, investor must buy shares off current investors.
ETFs: can use stop order, limit orders…

__Hedge Funds__: marketed to wealthy investors to escape some regulation. They are allowed lock-up periods, leverage, short-selling… They have incentive fees of **2 plus 20** with:

- __Hurdle rate__: benchmark that must be beaten before incentive fees can be charged.
- __High-water mark clause__: previous losses must first be recouped and hurdle rates surpassed before incentive fees once again apply.
- __Clawback clause__: enables investors to retain a portion of previously paid incentive fees to offset losses.

Biases:

- __Measurement bias__: only report good results to index vendor.
- __Backfill bias__: inconsistent reporting inflates the apparent performance.

Strategies:

- *__Long/short equity, Dedicated short__*
- *__Distressed debt__*: purchase bonds of distressed company with the potential to turn things around.
- *__Merger arbitrage__*: long purchased firm, short purchasing firm.
- *__Convertible arbitrage__*
- *__Fixed-income arbitrage__*
- *__Emerging market__*
- *__Global macro__*: leveraged bets on anticipated movements in forex, interest rates, equity, bonds…
- *__Managed futures__*: commodity futures.

Forward Prices

$F = S_0 e^{rt}$
With dividend, lease rate or convenience yield
$F = S_0 e^{(r-q)t}$
With income or yield
$\text{F} = (S_0 - I)e^{rt}$
With storage costs
$\text{F} = (S_0 + U)e^{rt}$

__Forward Prices vs Futures Prices__
Because Futures have daily mark-to-market. If asset prices are positively correlated with interest rates, futures are more expensive.

Basis

$Basis = S_t - F_0$

Hedging With Stock Index Futures

$$\#\ \text{contracts} = \beta_{\text{P,F}} \frac{\text{portfolio value}}{\text{futures} \cdot \text{multiplier}}$$

__Optimal Hedge Ratio__
$$\beta_{\text{P,F}} = \text{HR} = \rho_{\text{P,F}} \frac{\sigma_{\text{P}}}{\sigma_{\text{F}}}$$
__Adjusting Portfolio Beta__
$$\#\ \text{contracts} = (\beta * - \beta) \frac{\text{portfolio value}}{\text{asset}}$$

__Duration based hedge__
$$\#\ contracts = \frac{portfolio\ value \cdot D_P}{futures\ value \cdot D_F}$$

FOREX

__Interest rate parity (IRP)__
$$forward = \text{spot} \left(\frac{1 + \text{r}_{\text{yyy}}}{1 + \text{r}_{\text{xxx}}}\right)^{\text{T}}$$
$$forward = \text{spot} \cdot \text{e}^{(\text{r}_{\text{yyy}} - \text{r}_{\text{xxx}})\text{T}}$$

__Real Interest__
$(1 + R_{nom}) = (1 + R_{real})(1 + R_{infl.})$

__3 Main FX risks__

- Transaction risk (classic FX)
- Translation risk (financial statements)
- Economic risk (e.g., weaker currency, more competitive abroad)

Backwardation and Contango

Backwardation: futures < spot price. For this to occur there must be a significant benefit (cashflows) to holding the asset.
Contango: futures > spot price

Mortgage-Backed Securities (MBS)

Prepayments are more likely when
(1) __interest rates fall__, and borrowers wish to refinance their existing mortgages at a new/lower rate.
(2) __the borrower defaults__ and mortgage guarantors pay the outstanding.

Conditional Prepayment Rate (CPR): annual rate at which a mortgage pool balance is assumed to be prepaid during the life of the pool.
$CPR = 1 - (1 - SMM)^{12}$

Single Mortality Rate (SMM): derived from CPR and used to estimate monthly prepayments.
$\text{SMM} = 1 - (1 - CPR)^{1/12}$

Option-Adjusted Spread (OAS): spread after the "optionality" of the cash flows is taken into account. When comparing two MBSs of similar credit quality, buy the bond with the higher OAS.
$OAS = E(MBS\ return) - Treasury\ return$

Bonds

$$\text{AI} = \text{coupon} \frac{\#\ days\ last\ coupon\ to\ settl.\ date}{\#days\ in\ coupon\ period}$$

Clean (quote) price: bond price without AI.

Dirty (cash) price: includes AI; price the seller must be paid to give up ownership.

Warrants: attachments to a corporate bond issue that give the holder the right to purchase shares of the firm at a stated price. Smaller value than regular call option due to **dilution**.

Bond Futures

Conversion factor: defines the price received by the short position of the contract.

$cash\ received = (QFP \cdot CF) + AI$

There are two embedded options associated with the delivery of a futures contract:

Cheapest to Deliver:

- Yields Up $\rightarrow$ long maturity, high coupon.
- Yields Down $\rightarrow$ short maturity, low coupon.

Wild card: the short counterparty can choose when to deliver the bond.

Corporate Bonds

Spread duration: percentage change in a bond's price for a 100- basis-point change in the credit spread.

Credit spread: difference between a corporate bond's yield and the yield on a comparable-maturity benchmark Treasury security. It increases when the economy deteriorates.

OPTIONS

For calls:

In-the-money $X < S_T$ (will be exercised)

At-the-money: $X = S_T$

Out-of-the-money $X > S_T$ (will not be exercised)

Put-call parity

$c + PV(X) = S + p$

Payoffs

- call $\rightarrow max(S_T - X, 0)$
- $put \rightarrow max(X - S_T, 0)$

Upper Bounds

- $c, C \le S_0$
  (call can't be worth more than S
- $p \le PV(X)\ ; P \le X$
  (put can't be worth more than X)

Lower Bounds

- $\max(0, S_0 - PV(X)) \le c, C$
- $max(0, PV(X) - S_0) \le p$
  $max(0, X - S_0) \le P$

Bounds of difference

$S_0 - X \le C - P \le S_0 - PV(X)$

Factor	European call	European put	American call	American put
S_t	+	-	+	-
K	-	+	-	+
T-t	?	?	+	+
σ	+	+	+	+
r	+	-	+	-
D	_	+	-	+

Trading Strategies

Covered call: sell c, buy S.

Fiduciary call: c with X, bond that pays X at T.

Protective put: p with X and S.

$Value\ at\ T = max(X, S_T)$

Because they have the same cashflows, their price is the same (-> **Put-Call Parity**)

$c + PV(X) = S + p$

With dividend

$c + PV(X) + PV(d) = S + p$

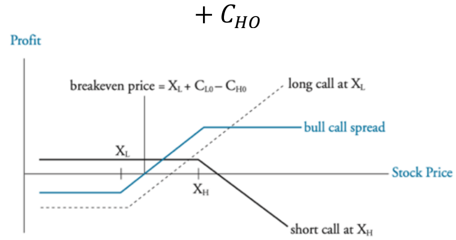
Stop-Loss strategy: a call seller purchases the underlying asset when the asset rises above the options strike price. The asset is then sold as soon as it goes below the strike price. A naked position when out-of-the-money and a covered position when in-the-money.

Spread Strategies

$(X_L < X_M < X_H)$

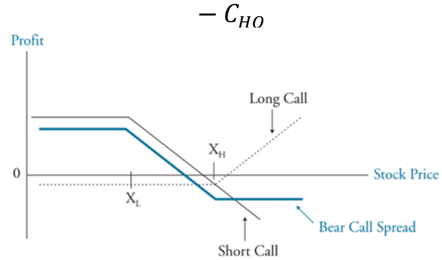
Bull Spread: sell call X_H , buy call X_L.

$$profit = max(0, S_T - X_L) - max(0, S_T - X_H) - C_{LO} + C_{HO}$$

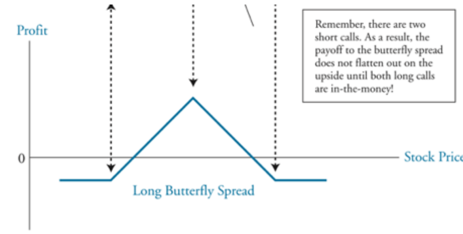


Bear Spread: buy call X_H , sell call X_L.

$$profit = max(0, S_T - X_H) - max(0, S_T - X_L) + C_{LO} - C_{HO}$$



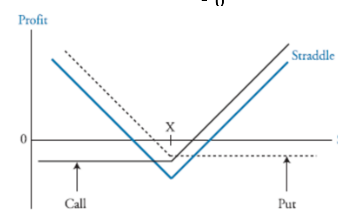
Butterfly Spread: buy calls X_L and X_H. Sell two calls X_M



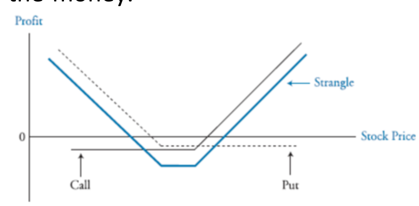
Combination Strategies

Straddle: call and put with same X and T.

$$profit = max(S_0 - X, 0) + max(X - S_0, 0) - C_0 - P_0$$

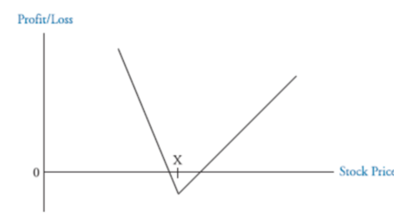


Strangle: straddle with options slightly out-of-the-money.



Strips (Straps): straddle with 2 puts 1 call (2 calls 1 put). Adds directional bias to straddle

Figure 40.9: Strip Profit/Loss

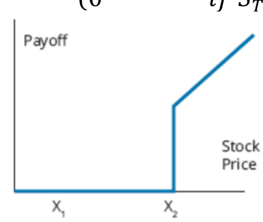


Exotic Options

Gap options:

- Strike price (X_1) determines the amount of payoff.
- Trigger price (X_2) determines whether the option will have a non-zero payoff.

$$payoff = \begin{cases} S_T - X_1 & if\ S_T > X_2 \\ 0 & if\ S_T \le X_2 \end{cases}$$



Forward Start options: options that begin their existence at some time in the future.

Compound options: options on options.

Chooser options: allows the buyer to choose whether the option is a call or a put after a certain amount of time has elapsed.

Barrier options: options that whose existence depend on whether the underlying's asset price reaches a certain barrier level. (e.g., Down-and-out)

Binary options: pays a fixed amount if the asset ends up above (below) the strike price (asset-or-nothing ...)
Lookback options: payoff depends on the maximum or minimum price of the underlying asset during the life of the option.

- **Floating**: $X \rightarrow S_{max}, S_{min}$
  - $call = max(S_T - S_{min}, 0)$
  - $put = max(S_{max} - S_T, 0)$
- **Fixed**: $S_T \rightarrow S_{max}, S_{min}$
  - $call = max(S_{max} - X, 0)$
  - $put = max(X - S_{min}, 0)$

Asian options: payoff is based on the average price of the security over the life of the option.
Basket options: options to purchase or sell multiple securities. Cheaper from a hedging perspective, because only one trade covers multiple exposures.

Valuation and Risk Models

Measures of financial risk

Coherent Risk Measure

- **Monotonicity**:$R_A \geq R_B \Rightarrow Risk(R_A) \leq Risk(R_B)$. A portfolio with greater future returns will likely have less risk.
- **Translation invariance** the risk of a portfolio is dependent on the assets within the portfolio. (cash)
- **Homogeneity** size of portfolio impacts size of risk.
- **Subadditivity**: $Risk(R_A + R_B) \leq Risk(R_A) + Risk(R_B)$. The risk of a portfolio is at most equal to the risk of the assets within the portfolio.

Value at Risk (VaR)
$\mathrm{VaR} = \mathrm{z} \cdot \sigma$
z-values:

- $95\% \rightarrow \mathrm{z} = 1.645$
- $99\% \rightarrow \mathrm{z} = 2.33$

EWMA
$\sigma_n^2 = \lambda\sigma_{n-1}^2 + (1 - \lambda)r_{n-1}^2$

λ: decay factor
r: $return$

GARCH(1, 1)
$\sigma_n^2 = \omega + \alpha r_{n-1}^2 + \beta\sigma_{n-1}^2$
α: innovation parameter
β: $variance\ parameter$
$\alpha + \beta + \gamma = 1$
$\omega = \gamma V_L$

$$V_L = \frac{\omega}{1 - \alpha - \beta}$$

Hazard Rates ($\bar{h}$)

Rate at which defaults happen at a specific time.
Survival rate:(1 – t-year cumulative def. pr.)

$$e^{-\bar{h}t}$$

Unconditional pr. of default between t_1, t_2:

$$e^{-\bar{h}_1 t_1} - e^{-\bar{h}_2 t_2}$$

Recovery Rate: (for a bond) is equal to its value just after default, expressed as a percentage of face value. $LGD = 1 - RR$

Credit Losses

Variance of losses

$$\sigma_P^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} \rho_{ij}\sigma_i\sigma_j \quad (covariances)$$

if $n = 2 \rightarrow 2\rho_{1,2}\sigma_1\sigma_2 + \sigma_1^2 + \sigma_2^2$

Standard deviation of loss from the ith loan

$$\sigma_i = \sqrt{PD_i - PD_i^2} \cdot [L_i(1 - RR_i)]$$

The standard deviation of losses as a percentage of its size

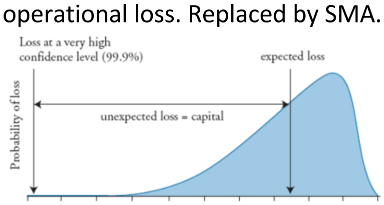
$$\alpha = \frac{\sigma\sqrt{1 + (n-1)\rho}}{\sqrt{n} \cdot L}$$

Operational Risk Capital Requirements

- **Basic indicator approach**: 15% of avg. annual gross income over 3 years.
- **Standardized approach**: avg. annual gross income for each business line, multiply by beta factors table. Highest betas: *Corporate Finance*, *Trading and sales*, *Payment and settlement*.

Business Line	Capital (% of Gross Income)
Corporate finance	18%
Trading and sales	18%
Retail banking	12%
Commercial banking	15%
Payment and settlement	18%
Agency services	15%
Asset management	12%
Retail brokerage	12%

- **AMA**: 99.9 percentile of the loss distribution minus the expected operational loss. Replaced by SMA.



- **Loss distribution approach** (economic capital): loss frequency (Poisson) & loss severity (lognormal).

New approach (regulatory capital):

- **Standardised measurement approach: BI component** (gross income adjusted by bank size) + **loss component** 7X+7Y+5Z (estimates of average losses over 10 previous years).
  - X all losses
  - Y losses > EUR 10 million
  - Z losses > EUR 100 million

  For an average bank, the calculations should ensure that the BI component and the loss component are equal.

Scaling vendor loss data

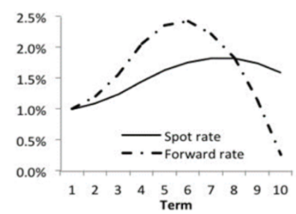
$$\text{Est. loss bank Y} = \text{loss bank X}\left(\frac{\text{Y revenue}}{X\ revenue}\right)^{0.23}$$

Forward Rates (cont. compounded)

Forward rate for the period that lies between T_1 and T_2 :

$$F = \frac{R_2T_2 - R_1T_1}{T_2 - T_1}$$

The forward curve is above the spot curve when the spot curve is rising, and below when the spot curve is declining.



Duration and Convexity

$$\mathbf{DV01} = \frac{\Delta P}{\Delta y}$$

Duration: 1st derivative of the price-yield relationship; measure of price volatility; the longer (shorter) the duration, the more (less) sensitive the bond's price is to changes in interest rates.

$$D = \frac{\Delta P/P}{\Delta y}$$

- Barbell investment: invest in bonds with short and long maturities.
- Bullet investment: invest in a single bond with an intermediate maturity.

To construct a barbell portfolio, invests in long and short-term bonds so that the weighted duration matches the bullet portfolio.

Convexity: 2nd derivative of the price-yield relationship; measures the degree of curvature; positive convexity always has a favorable impact on bond price.

$$C = \frac{P^+ + P^- - 2P}{P(\Delta y)^2}$$

Bond price changes

$$\Delta P = -D \cdot P \cdot \Delta y + \frac{1}{2} \cdot C \cdot P \cdot \Delta y^2$$

Key Rate Hedging

Bond must be covered for each key rate.

Figure 57.7: KR01s for Hedging Instruments

Key Rates	Portfolio	Hedge 1	Hedge 2	Hedge 3
$KR01_1$	68	14	2	3
$KR01_2$	106	3	12	4
$KR01_3$	169	7	1	15

Hedging involves setting KR01s equal to 0:

$$68 + 14x_1 + 2x_2 + 3x_3 = 0$$
$$106 + 3x_1 + 12x_2 + 4x_3 = 0$$
$$169 + 7x_1 + x_2 + 15x_3 = 0$$

Bonds With Embedded Options

Callable bond: issuer has the right to buy it back in the future at a set price; as yields fall, bond is likely to be called.
Putable bond: bondholder has the right to sell bond back to the issuer at a set price.

Binomial Option Pricing Model

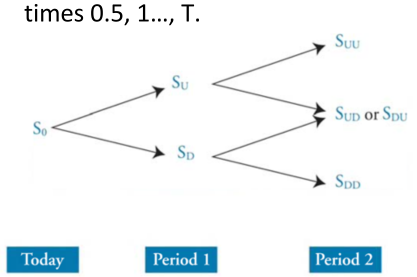
1. Calculate U and D

U, D: size of the up-move/down-move factor

2. Compute π_{up}.

$$\pi_{up} = \frac{e^{rt} - D}{U - D}; \qquad \pi_{down} = 1 - \pi_{up}$$

3. Multiply by U and D to get the value in times 0.5, 1..., T.



4. Calculate the payoff at end nodes. (e.g., $max(S_{UU} - X, 0)$)
5. Backward induction. Calculate the expected payoffs at previous nodes. $(\pi_{up} \cdot pay_{UU} + \pi_{down} \cdot pay_{UD})e^{-r \cdot 0.5}$
6. The price of the option will be the expected payoff at the first node.

π_{up} alterations

Stocks with dividends: $e^{rt} \rightarrow e^{(r-q)t}$
Currencies: $e^{rt} \rightarrow e^{(r_{DC} - r_{FC})t}$
Futures: $e^{rt} \rightarrow 1$

Black-Scholes-Merton Model

$$c = S_0 N(d_1) - Xe^{-rT} N(d_2)$$
$$p = Xe^{-rT} N(-d_2) - S_0 N(-d_1)$$

$$d_1 = \frac{ln\left(\frac{S_0}{X}\right) + (r + 0.5\sigma^2)\,T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - (\sigma\sqrt{T})$$

N: *cumulative normal probability*
X: *exercise price*
σ: stock return volatility

Stock pays dividend
$$S_0^* = S_0 - PV(dividend)$$

Some assumptions

- $S \sim Lognormal$
- Option is European
- r and σ are constant and known

The Greeks

Delta $\Delta = \frac{\partial c}{\partial s}$

Largest when options are:

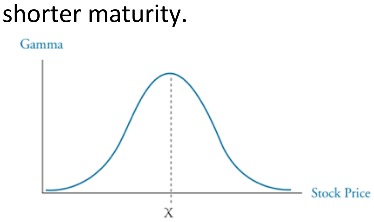
- Deep in-the-money

When gamma is large, delta will be changing rapidly.

Gamma *(curvature)* $\Gamma = \frac{\partial^2 c}{\partial s^2}$

Largest when options are:

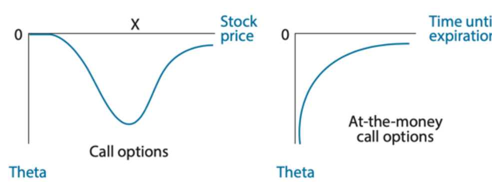
- at-the-money
- shorter maturity.



Theta *(time decay)* $\Theta = \frac{\partial c}{\partial t}$

Negative because as time passes, options decrease in value.
More negative when option is:

- at-the-money
- close to expiration.



Vega $\text{vega} = \frac{\partial c}{\partial \sigma}$

Largest when options are:

- at-the-money
- longer maturities.

Rho $\rho = \frac{\partial c}{\partial r}$

Largest when options are:

- at-the-money.

Delta-neutral hedging

$$\#\,share = \Delta \cdot \#\,options$$

The larger the delta, the more expensive it is to delta hedge.
Only valid for small changes in the asset value. To protect against larger changes → **Gamma-neutral**.

S and F generate linear payoffs and therefore have zero gamma. Gamma-neutral positions are created with options.