GRE Quant Cheat Sheet

Linear & Quadratic Equations

Quadratic Equations

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$\text{when } ax^2+bx+c=0$$

Quadratic Identities

- $(x+y)(x-y) = x^2 - y^2$
- $(x+y)^2 = x^2 + y^2 + 2xy$
- $(x-y)^2 = x^2 + y^2 - 2xy$

<u>Some examples</u>:

$x^2 - 9 = (x-3)(x+3)$

$4x^2 - 100 = (2x-10)(2x+10)$

$x^2y^2 - 16 = (xy-4)(xy+4)$

Properties of Numbers

Integers

$\{\ldots, -2, -1, 0, 1, 2, \ldots\}$

There are 100 integers from 1 to 100 (including both), out which 50 are odd and 50 even.

Laws of Even and Odd Numbers

- $even \cdot odd^x = even$
- $odd^x = odd$
- $even/odd = even$
- $odd/odd = odd$
- $even/even = even \text{ or } odd$

Prime Numbers

1 is not a prime. 2 is the smallest prime and the only even prime.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

Factors

If y divides evenly into x, we say y is a factor of x.

<u>Example:</u> What are the factors of 16?

$$\mathbf{1}, 2, 4, 8, \mathbf{16}$$

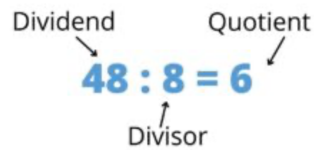
Multiples

A multiple of a number is the product of that number and an integer.

<u>Example:</u> What are the multiples of 4?

$$\mathbf{4}, 8, 12, 16, \mathbf{20}, \ldots, \mathbf{4n}$$

Division



Divisibility

- **3**: sum of digits divisible by 3
- **4**: the last two digits (combined) of number divisible by 4
- **5**: the last digit is either a 5 or zero
- **6**: even number and sum of digits is divisible by 3
- **8**: if the last three digits (combined) are divisible by 8
- **9**: sum of digits is divisible by 9

Finding the Number of Factors

Find the prime factorization of the number. Add 1 to the value of each exponent. Then multiply these results.

<u>Example:</u> 240 has 20 factors.

$240 = 2^4 \cdot 3^1 \cdot 5^1$

$(4+1)(1+1)(1+1) = 20$

Finding the LCM

1. Find the prime factorization of each integer
2. Of any repeated prime factors, take only those with the **largest** exponent
3. Of what is left, take all non-repeated prime factors.

<u>Example:</u> LCM of 24 and 60

$24 = 2^3 \cdot 3^1$

$60 = 2^2 \cdot 3^1 \cdot 5^1$

$LCM(24{,}60) = 2^3 \cdot 3^1 \cdot 5^1 = 120$

Finding the GCF

1. Find the prime factorization of each integer
2. Of any repeated prime factors, take only those with the **smallest** exponent (if there are none, GCF is 1)

<u>Example:</u> GCF of 24 and 60

$24 = 2^3 \cdot 3^1$

$60 = 2^2 \cdot 3^1 \cdot 5^1$

$LCM(24{,}60) = 2^2 \cdot 3^1 = 12$

Two consecutive integers will never share any prime factors. Thus, the GCF of two consecutive factors is 1.

LCM x GCF

$LCM(x,y) \cdot GCF(x,y) = x \cdot y$

Zeroes & Decimals

- Any factorial $\geq$ 5! Will always have zero as its units digit
- The number of trailing zeroes of a number is the number of $(5 \cdot 2)$ pairs in the prime factorization of that number
- If x is an integer with k digits, then $1/x$ will have
  - $k-1$ leading zeroes if x is not a perfect power of 10
  - $k-2$ leading zeroes otherwise

<u>Example:</u>

For 500, k=3 and $1/512 = 0.\mathbf{00}2 \ldots$

For 1000, k=4 and $1/1000 = 0.\mathbf{00}1 \ldots$

- The decimal equivalent of a fraction will terminate if the denominator of the reduced fraction has a prime factorization that contains only 2s and 5s, or both.

<u>Example:</u>

$1/20 = 0.05$

$1/12 = 0.083333 \ldots$

Patterns in Units Digits

The units digits of all positive powers of:

- 2 end in 2-4-6-8
- 3 end in 3-9-7-1
- 4 end in 4-6
- 5 end in 5
- 6 end in 6
- 7 end in 7-9-3-1
- 8 end in 8-4-2-6
- 9 end in 9-1

Perfect Squares

A perfect square other than 0 and 1, is a number such that all of its prime factors have even exponents

<u>Example:</u>

$$12^2 = 144 = 2^4 \cdot 3^2$$

Roots & Exponents

Exponents

Rule	Powers of 2
$a^m \cdot a^n = a^{m+n}$	$2^0 = 1$
$a^m / a^n = a^{m-n}$	$2^1 = 2$
$(a^m)^n = a^{m \cdot n}$	$2^2 = 4$
$a^0 = 1$	$2^3 = 8$
$a^{-n} = 1/a^n$	$2^4 = 16$
$a^m \cdot b^m = (a \cdot b)^m$	$2^5 = 32$
$a^m / b^m = (a/b)^m$	$2^6 = 64$
$(a \cdot b)^m = a^m \cdot b^m$	$2^7 = 128$
$(a/b)^m = a^m / b^m$	$2^8 = 256$
$1/a^{-n} = a^n$	$2^9 = 512$
$a^{m/n} = \sqrt[n]{a^m}$	$2^{10} = 1024$

$11^2 = 121$ $\quad$ $14^2 = 196$
$12^2 = 144$ $\quad$ $15^2 = 225$
$13^2 = 169$ $\quad$ $16^2 = 256$
$8 \cdot 7 = 56$ $\quad$ $11 \cdot 12 = 132$

$$100x - x = 324.\overline{24} - 3.\overline{24}$$

__x * __y = __[xy%10]

Example:
$12 \cdot 23 = 27[2 \cdot 3] = 276$

Roots

Rule
$(\sqrt{a})^2 = a$
$\sqrt{a^2} = a$
$\sqrt{a}\sqrt{b} = \sqrt{ab}$
$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

$\sqrt{2} \approx 1.4$
$\sqrt{3} \approx 1.7$
$\sqrt{5} \approx 2.2$

Addition and Subtraction of:

- **Like Bases**
  $$4^{45} - 4^{43} = 4^{43}(4^2 - 4^0)$$
- **Like Radicals**
  $$\Rightarrow 10\sqrt[3]{5} + 5\sqrt[3]{5} + 6\sqrt[3]{5} + 2\sqrt[3]{5} + 2\sqrt[3]{5}$$
  $$\Rightarrow \sqrt[3]{5}(10 + 5 + 6 + 2 + 2)$$
  $$\Rightarrow \sqrt[3]{5}(25) = 5^{\frac{1}{3}} \times 5^2 = 5^{\frac{1}{3}+\frac{2}{1}} = 5^{\frac{1}{3}+\frac{6}{3}} = 5^{\frac{7}{3}}$$
- **Special Addition Rule with Exponents**
  $$3^n + 3^n + 3^n = 3^{n+1}$$

Absolute Values

Equations with One Absolute Value

Solve the equation twice:

- Case 1: Expression is positive
- Case 2: Expression is negative

Example:
$|2x + 4| = 12,\ x = ?$

$\Rightarrow 2x + 4 = 12$
$\Rightarrow 2x = 8 \rightarrow x = 4$

and

$\Rightarrow -(2x + 4) = 12$
$\Rightarrow -2x - 4 = 12$
$\Rightarrow -2x = 16 \rightarrow x = -8$

When Two Absolute Values are Equal to Each Other

Solve for the cases when the expressions are equals or opposites

Example:
$|16x + 14| = |8x + 6|\ x = ?$

Case 1: The quantities within the absolute values are equal:

$\Rightarrow 16x + 14 = 8x + 6$
$\Rightarrow 8x = -8 \rightarrow x = -1$

Case 2: The quantities within the absolute values are opposites:

$\Rightarrow 16x + 14 = -(8x + 6)$
$\Rightarrow 16x + 14 = -8x - 6$
$\Rightarrow 24x = -20 \rightarrow x = -\frac{5}{6}$

Equations with the X on Both Sides

$|x| = 4x + 3$

Calculate two scenarios:

- $x = 4x + 3 \Rightarrow x = -1$
- $-x = 4x + 3 \Rightarrow x = -3/5$

Check answers for validity:
$x = -1$ is not valid because:
$1 \neq -4 + 3 = -1$

Adding Absolute Values

It holds:

- $|a + b| \leq |a| + |b|$

If $|a + b| = |a| + |b|$, then either

- $a = 0$ and/or $b = 0$
- $sgn(a) = sgn(b)$

Subtracting Absolute Values

It holds:

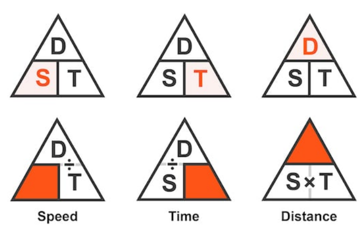
- $|a - b| \geq |a| - |b|$

Word Problems

Rates of Work

1. You can't add 'times', you can add 'rates'
2. Time and rate are always reciprocal

$$\text{avg. speed} = \frac{\text{total distance}}{\text{total time}}$$



Interest

Simple interest = (principal)(rate)(time)
Compound interest = (principal)(1 + rate)^(time)

Even/odd numbers in a range

Number of even/odd numbers in a range

$$\left(\frac{\text{Last Day Number} - \text{First Day Number}}{2}\right) + 1.$$

Rate Problems

Where do two vehicles meet?

	Rate	Time	Distance
Standard Train	**60** $\frac{\text{miles}}{\text{hour}}$	**(t + 2) hours**	
High-Speed Train	**120** $\frac{\text{miles}}{\text{hour}}$	**t hours**	

Driving to somewhere and back

	Rate	Time	Distance
To the Game	50 $\frac{\text{miles}}{\text{hour}}$	t hours	**50t miles**
From the Game	75 $\frac{\text{miles}}{\text{hour}}$	(2 – t) hours	**(150 – 75t) miles**

Ratios

It is often useful to operate with units (e.g. if they talk about two quantities in a 1:2 ratio, think about 100ml and 200ml)

Adding Ratios

Add the numerators and denominators.

<u>Example:</u>
We have two glasses of the same size with 20% and 30% coffee. If we pour both glasses into a third bigger glass, we'll have

$$\frac{20 + 30}{100 + 100} = \frac{50}{200} = 25\%$$

Coffee.

Statistics

Counting the Numbers of Multiples of an Integer in a Set of Consecutive Integers (inclusive)

$$\Rightarrow \left(\frac{\text{Highest multiple} - \text{Lowest multiple}}{\text{Given Number}}\right) + 1$$

Average in a Set of Consecutive Integers

$$\Rightarrow \frac{\text{Highest Number} + \text{Lowest Number}}{2}$$

Mean = Median

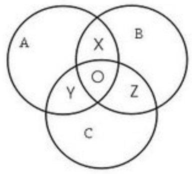
In any evenly spaced set.

Overlapping Sets

$$(A \cup B) = A + B - (A \cap B)$$

$$(A \cup B \cup C) = A + B + C - (A \cap B) - (A \cap C) - (B \cap C) + (A \cap B \cap C)$$

$$(A \cup B \cup C) = A + B + C - (\text{groups of } 2) - 2(\text{groups of } 3)$$



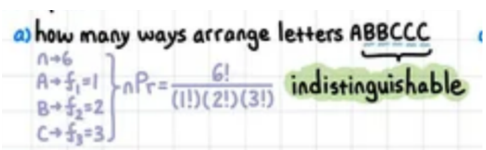
Combinatorics

n objects taken k at a time

Ordered with repetition: n^{k}

Ordered no repetition: $\frac{n!}{(n-k)!} = nPk$

No order no repetition: $\frac{n!}{k!(n-k)!} = \binom{n}{k} = nCk$ *(number of k-member subsets of a set with n members)*



Circular Arrangements

Number of ways to arrange a set of items in a circle

→ = (k – 1)!

k = number of objects to be arranged in the circle

Probability

$$P(\text{A or B}) = P(\text{A} \cup \text{B})$$
$$= P(A) + P(B) - P(A \cap B)$$

If A and B are independent:

- $P(\text{A and B}) = P(\text{A})P(B)$

If A and B are NOT independent:

- $P(\text{A and B}) = P(\text{A})P(B|A)$

Probability of choosing at least 1

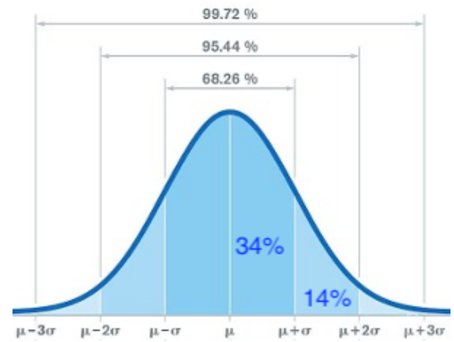
P(at least 1 item occurs)

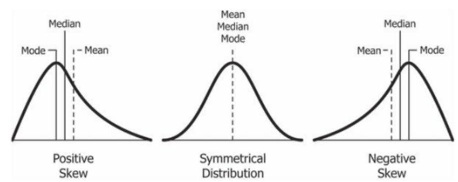
→ = 1 – P(none of these items occur)

Normal Distribution and Std. Dev.

$\mu_{\frac{1}{2}} = \mu$ *(median equals mean)*

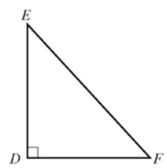
- $\mu \pm \sigma \longrightarrow \sim \mathbf{68\%}$
- $\mu \pm 2\sigma \longrightarrow \sim \mathbf{95\%}$
- $\mu \pm 3\sigma \longrightarrow \sim \mathbf{99.7\%}$





Geometry

Triangles



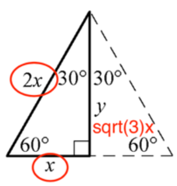
Area

$A = \frac{\mathrm{bh}}{2}$

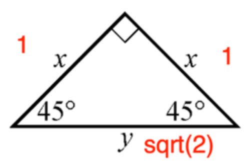
Pythagorean Theorem

$\mathrm{h}^2 = {l_1}^2 + {l_2}^2$

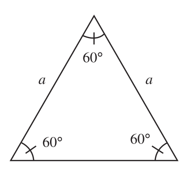
30-60-90 right triangle



Isosceles right triangle



Equilateral triangle



$h = \frac{\sqrt{3}}{2}a$

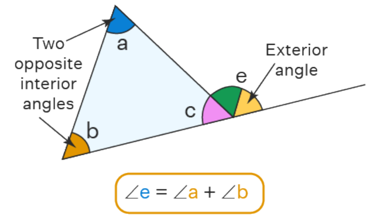
$A = \frac{\sqrt{3}}{4}a^2$

Third side rule: Any side of a triangle must be:

- greater than the difference of the other two sides
- less than their sum

Exterior angle theorem

An exterior angle of a triangle is equal to the sum of the two opposite interior angles



Angle and opposite side relationship

There is a direct relationship between the side length and the opposite angle. That is, the biggest side is opposite the biggest angle; the smallest side is opposite the smallest angle.

Super Pythagorean theorem

The diagonal of a rectangular box is:

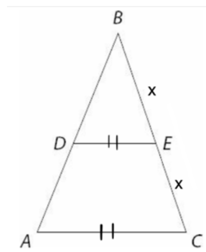
$$d^2 = l^2 + w^2 + h^2$$

Common Pythagorean triples: *side lengths of rectangular triangles*

- 3 : 4 : 5 (6 : 8 : 10, 9 : 12 : 15)
- 5 : 12 : 13
- 7 : 24 : 25
- 8 : 15 : 17

And their multiples

Similar triangles



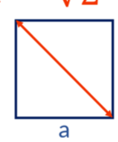
same angles but not necessarily same size.

ABC and DBE are similar. DBE's sides are double ABC's

Quadrilaterals

Square

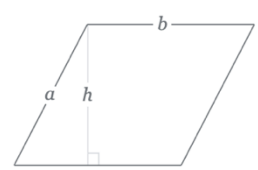
$d = \sqrt{2}\ a$



Rhombus

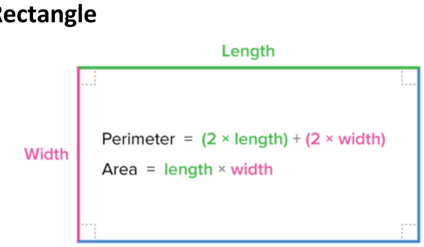
A square is a special case of a Rhombus

Parallelogram *(both pairs of opposing sides are parallel)*



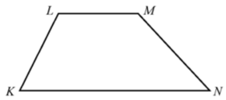
$A = b\ \cdot h$

Rectangle



- The rectangle with the **maximum area** is a **square**
- The rectangle the **minimum perimeter** is a **square**

Trapezoids *(only one pair of opposing sides are parallel)*



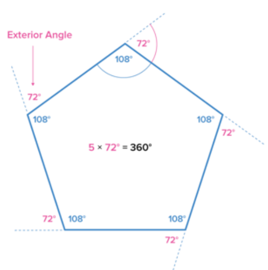
Area

$$A = \frac{1}{2}(b_1 + b_2)(h)$$

Polygons

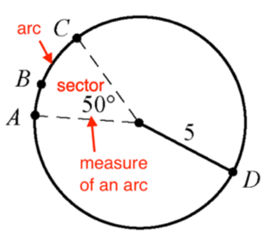
A polygon can be divided into $n - 2$ triangles, where n is the number of sides.

$$sum\ of\ interior\ angles\ = (n-2)(180)$$



$$sum\ of\ exterior\ angles\ = 360$$

Circles



Circumference
$C = 2\pi r$

Area
$A = \pi r^2$

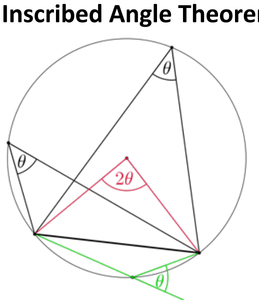
Arc Length

$$\frac{\text{central angle}}{360} = \frac{\text{arc length}}{\text{circumference}}$$

Sector Area

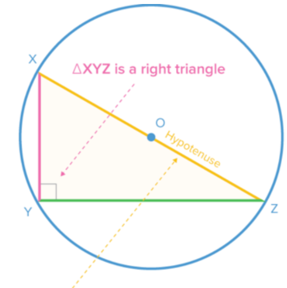
$$\frac{\text{central angle}}{360} = \frac{\text{area of sector}}{\text{area of circle}}$$

Inscribed Angle Theorem

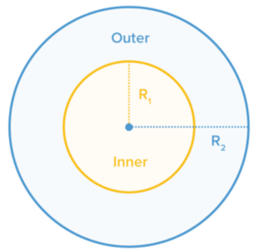


An inscribed angle is always half the central angle

Right Triangle Inscribed in a Circle



XZ is both the diameter of the circle and the hypotenuse of the triangle



Area of outer ring
$= \pi(R_2^2 - R_1^2)$

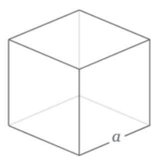
A point can be:

- Inside the circle
- On the circle (on its circumference)
- Outside of the circle

Circle Equation
$(x - h)^2 + (x - k)^2 = r^2$
For a circle with radius r, centered at (h, k)

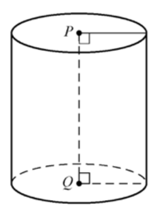
Three-Dimensional Figures

Cube



Volume
$V = a^3$
Surface area
$A = 6a^2$
Diagonal
$A = \sqrt{3}a$

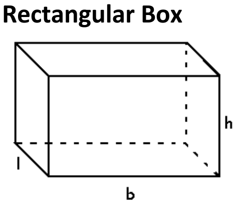
Circular Cylinder
With height *h* and a base with radius *r*.



Volume
$V = \pi r^2 h$

Surface area
$A = 2\pi r^2\ +\ 2\pi rh$

Rectangular Box



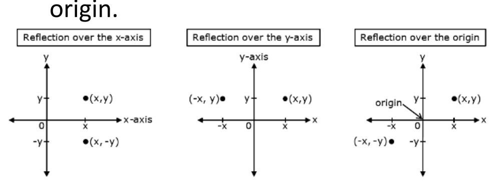
$Vol. = b \cdot h \cdot l$
$Surface\ Area = 2(\mathrm{bh} + \mathrm{bl} + \mathrm{hl})$

Coordinate Geometry

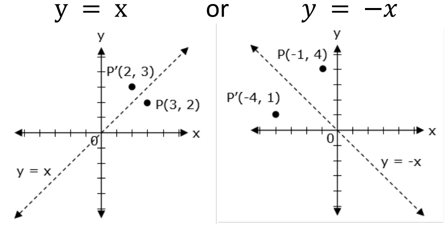
Slope (m) of line that passes through points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{Rise}{Run} = \frac{y_2 - y_1}{x_2 - x_1}$$

- For every line with **negative slope** that doesn't pass through the origin, the **x**- and **y**-intercepts are either **both positive or both negative**.
- **Perpendicular lines** have **negative reciprocal slopes**. The perpendicular line to $y = \frac{1}{2}x$ is $y = -\frac{2}{1}x$
- **Distance between points** (X_1, Y_1) and (X_2, Y_2) is $\sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2}$
- **Reflection** of point (x, y) over axis or origin.

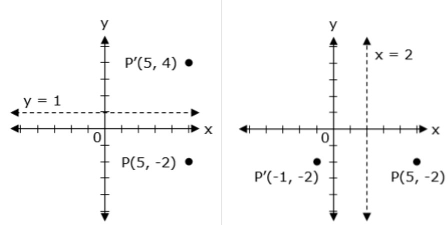


- **Reflection** of point (x, y) over lines $y = x$ or $y = -x$

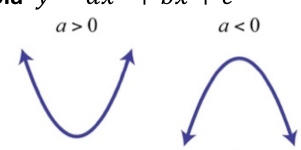


For line $y = mx + b$:
$x = my + b$ $\quad$ $-x = -my + b$

- **Reflection** of point (x, y) over lines $y = b$ or $x = a$

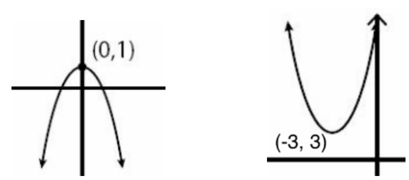


Parabola $y = ax^2 + bx + c$



When written as:
$$y = a(x - h)^2 + k$$
vertex is at $(h,\ k)$

For example:
$y = -x^2 + 1$ $\quad$ $y = (x + 3)^2 + 3$



Or find vertex by looking for what x the derivative is 0.

Functions and Sequences

Arithmetic Sequence

The difference between every pair of consecutive terms is the same

$$a_n = a_1 + (n-1)d$$

Where d is the common difference.

The sum of all terms is

$$n\frac{(a_1 + a_n)}{2}$$

Geometric Sequence

The ratio between every pair of consecutive terms is the same

$$a_n = a_1 + r^{n-1}$$

Where r is the common ratio.