Do Specialized Distress Investors Undermine Firms Ex Ante? A Simple Model of Private Bailouts*

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Abstract

We extend the canonical moral hazard model in corporate finance to include *specialized distress investors* (SDIs) who privately bail out financially-distressed firms. First, we show that SDIs can negatively impact credit rationing ex ante, even when SDIs create value and even though initial lenders are required to agree to SDI participation. Second, we show that SDI presence lowers project NPV when initial lenders can exploit a value-destroying SDI's technology to expropriate equityholders ex post. These negative results notwithstanding, we also show that firms can sometimes mitigate these effects by reducing leverage and/or by focusing on projects with certain characteristics.

JEL Classification: G21, G23, G32, G33

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1 Introduction

Bailouts have been a topic of significant interest not only for policy makers but also for academics. Much of the received wisdom suggests that bailouts can worsen moral hazard: by softening the cost of failure, bailouts may exacerbate incentives for ex ante risk shifting and/or low effort provision.[1]

The debate surrounding bailouts has focused on government interventions, mostly in the banking sector but also elsewhere.[2] In contrast, we contribute to the literature by asking whether and how *private* bailouts may similarly generate undesirable spillovers, a topic that has been hitherto ignored. By private bailouts we mean the intervention of a new investor (for example, a private equity fund)[3] who is skilled at distress investing and ends up rescuing the firm. While obviously the institutional details are very different from those associated with a public bailout, one key aspect is plausibly the same: third party involvement softens the blow associated with failure. One additional reason that motivates us to study the role of specialized distress investors is that they have been gradually becoming a more important player in corporate finance, as we document in section 2.

We develop a simple moral hazard model of entrepreneurial finance where in an interim stage a firm can become financially distressed and potentially be rescued by a *specialized distress investor* (henceforth, SDI). We find that under certain conditions the presence of

[1]See, for example, Cordella and Yeyati (2003), Farhi and Tirole (2012), Chari and Kehoe (2016), and Dell'Ariccia and Ratnovski (2019). The standard arguments notwithstanding, Philippon and Wang (2023) show that it is possible to design a bailout mechanism that does not worsen moral hazard.

[2]The Great Financial Crisis of 2008 witnessed the largest financial and non-financial bailouts in the American history and rekindled the debate about whether governments should step in to rescue large companies – so-called "too-big-to-fail". Since such companies are often in the center of a well-connected network of businesses and various stakeholders, their failure can be detrimental to the whole economy. For instance, it is estimated that the bailout of General Motors saved 1.2 million jobs and $39.4 billion in personal and social insurance taxes (McAlinden and Menk, 2013).

[3]Although for example, a non-financial firm acquiring a distressed company would also fit our definition of private bailout.

SDIs will worsen the original agency problem and, concomitantly, credit rationing. What makes the result non-obvious is that it can hold even when SDIs create value and, moreover, even though the initial lender needs to agree to the SDI entering the firm. This is different from public bailouts, which (i) do not necessarily create firm value and (ii) are sometimes imposed by governments. In addition to the credit rationing result, we also show how SDI presence can (i) lower project NPV, (ii) affect which types of projects the entrepreneur selects (e.g., in terms of duration), and (iii) impact ex ante capital structure choices. Finally, and notwithstanding our emphasis on the more negative spillovers, our model also suggests new ways in which distress investors can benefit the economy, over and above creating value ex post. For example, some types of SDIs actually alleviate the ex ante credit rationing problem, even if they are value-neutral ex post.

We now turn to the model in more detail, which basically extends the canonical moral hazard workhorse in corporate finance (Holmstrom and Tirole, 1997; Tirole, 2006) by allowing for the entry of distress investors. The model entails a project with two phases, where an entrepreneur (or "borrower") is initially funded by a lender (or "bank"), which we think of as a traditional financial institution that operates in a competitive market. At the end of the first phase, if the project is successful, then the borrower pays down the debt and enjoys the continuation value of the firm. Conversely, if the borrower becomes distressed (i.e., fails in phase 1), then an SDI potentially appears. The type of SDI we have in mind is relatively hands-on, has a long investment horizon and is involved in the management and operations of the firm. Phase-1 success depends on effort exertion by the entrepreneur and herein lies the moral hazard problem. The severity of the problem is high whenever the entrepreneur has a relatively high payoff following failure. This happens if the entrepreneur is critical for managing the firm during continuation or if she is entitled to a sizable exit pay after restructuring.

There are two different cases that we analyze. The first one, which we term "high-leverage", implies that after failure the firm becomes insolvent, i.e., the face value of debt is higher than the firm's continuation value. In this case the initial lender takes over the control of the firm and possibly sells his stake to a distress investor (formal bankruptcy proceedings). The second case, which we term "low-leverage", implies that default occurs after phase-1 failure but the firm is solvent. In such low-leverage firms, entrepreneurs remain in control and are the ones negotiating directly with distress investors (private workout). The two cases have some similarity in terms of the effect of distress investors on credit rationing. Specifically, in our model both the initial lender and the entrepreneur have no commitment, meaning that they will accept desirable SDI offers ex post even if such offers reduce pledgeable income and posssibly make initial funding unfeasible. The only difference is that in the high-leverage case, where the initial lender is in control, an SDI may actually compress the entrepreneur's payoff during restructuring (e.g., by providing an alternative to the original management team when continuing the company), which would imply an improvement with regards to the credit rationing problem instead of a worsening. Naturally this only applies to certain types of SDIs, which we term "disciplining", a trait which is not necessarily correlated with the SDI's ability to create value ex post.

Besides the effect on credit rationing, we also analyze what happens to project NPV as SDIs become more pervasive. Here there is more of a difference between high-leverage and low-leverage firms. With the latter, since the entrepreneur is in control, she only accepts those SDI offers that create value following failure/distress, which feeds back positively into NPV. In contrast, for high-leverage firms, it is the initial lender who is in control following interim failure, which sometimes leads the lender to strike deals with SDIs that are value-destroying but improve the lender's stake (the entrepreneur is expropriated). In such cases, increasing the likelihood of SDI appearance will feed back negatively into project NPV. Obviously, these NPV results also critically depend on our assumption that the initial lender has no commitment.

In addition to the first-order effects of SDI presence described above, we also show that SDIs can affect which types of projects entrepreneurs choose. In the model SDI entry is endogenous and the equilibrium frequency of SDI appearance is determined by the payoff these agents expect from becoming involved with a distressed firm. It turns out that if entrepreneurs have some control over the types of projects they look for, then they can modulate the (contingent) attractiveness of their firm to SDIs. For example, projects with higher duration (i.e., higher continuation value relative to phase 1 returns) will naturally make it more appealing for SDIs to participate. Therefore, if SDIs becoming more pervasive is problematic—e.g., by reducing pledgeable income—, then entrepreneurs could mitigate such a negative effect by picking shorter-duration projects.

Besides using project characteristics strategically, entrepreneurs who are not overly financially constrained can also decide whether it is worthwhile adding debt beyond a critical threshold, namely the threshold where following failure/distress, control is lost to the initial lenders. We show in the model that the optimal capital structure hinges on the magnitude of the (initial) loan's renegotiation costs, relative to how "disciplining" SDIs are. It is perhaps interesting to draw a parallel between this result and Jensen's classic argument (Jensen, 1986). In our case using debt beyond the aforementioned critical threshold has a desirable disciplining effect on the entrepreneur/manager not because debt service requires the firm to generate a minimum amount of cash flow, but instead because high leverage entails the potential appearance of a distress investor who has more power than the initial lender to compress the manager's payoff following failure.

Our paper relates to several strands of the literature besides bailouts. It complements theoretical and empirical literature on the topic of distressed lending (see section 2 for details), by asking a new question about potential negative spillovers associated with this economic activity. Moreover, some of our results are empirically testable and have normative implications. For example, Dou, Wang, and Wang (2022) argue that there is an undesirable under-provision of distress investors in the economy, yet do not study ex ante spillovers, which we claim are important for a broader efficiency assessment. Our work also connects to theories about the role of liquidation threats in corporate finance. It is well understood that the threat of liquidation (or, more generally, withholding financing) can play an important role in eliciting desirable behavior from borrowers. For example, Bolton and Scharfstein (1990) show how the threat of not funding a later-stage project after a report of poor performance is sometimes enough to overcome the perverse incentive to under-report interim cash flows. Dynamic agency models also employ firm size and liquidation as mechanisms that help in curbing moral hazard (e.g., DeMarzo and Fishman, 2007 and DeMarzo and Sannikov, 2006). The spirit of our model is perhaps closer to Hörner and Samuelson (2016), where the principal/lender has no commitment. Essentially, the idea is that when short-run compensation is not high-powered enough, the principal can use the fate of the firm as an additional tool to motivate the agent. We contribute to this literature by asking what happens to "interim disciplining devices" (such as liquidation following failure) when a third player intervenes in the financing and production dynamics.

The paper is organized as follows. Section 2 describes the institutional details of distressed debt investment. Section 3 presents the general framework of our corporate finance model with initial lending and specialized distress investors. Section 4 studies the severe financial distress case where in the down state the bank loan becomes undercollateralized and hence the firm has negative net worth. Section 5 presents the case where the firm has a positive net worth if the project fails and the loan is overcollateralized. Finally, section 6 concludes. The internet appendix contains all proofs.

2 Institutional details of distressed debt investment

Distressed debt refers to all publicly and privately held debt that is either distressed or has defaulted, and selling at yields significantly above U.S. Treasury yields. Since the 1990s the size of distressed debt markets more than doubled, increasing from a total face value of $300 billion to $747 billion by 2017, with the most striking expansions happening around downturns such as the dot-com bubble in 2002 and the financial crisis in 2008 (Altman, Hotchkiss, and Wang, 2019). Accordingly, investors that are specialized in distressed debt, so-called "vultures", credit or event-driven investors, have become more visible in the financing of financially troubled companies. In a sample of 288 firms that defaulted on their public debt between 1980 and 1993, Hotchkiss and Mooradian (1997) document presence of vulture investors in 60% of the cases. Similarly, Jiang, Li, and Wang (2012b) analyze 474 Chapter 11 cases between 1996 and 2007, and find publicly observable involvement by hedge funds in about 90% of the sample. Li and Wang (2016) show that the fraction of Chapter 11 firms in their sample receiving debtor-in-possession (DIP) financing from activist investors (i.e., hedge funds or private equity funds) increased from less than 10% in the late 1990s to about 40% at the beginning of 2010s. Conversely, within the same time period, the percentage of Chapter 11 firms that received DIP financing from their prepetition bank lenders was reduced. Overall, the existing empirical evidence shows that specialized investors have become more visible in the bankruptcy process of distressed firms and therefore, it is important to understand how their involvement affects traditional lending relations in credit markets.

Distress investors may vary with respect to their investment strategies and investment horizons. For instance, passive distress investors do not have any direct influence on the bankruptcy process and have relatively short investment horizons. Their strategy involves identifying underpriced distressed securities and exploiting the future price appreciation. On the other hand, active distress investors are more hands-on and have longer investment horizons (the focus of our model). A typical strategy of this group is called "loan-to-own" which involves identifying debt securities that are likely to be converted into equity in the newly restructured firm (i.e., fulcrum security). These investors do not sell their equity shares immediately after the restructuring, instead they are actively involved in the management and operations of the firm during and after reorganization. A notable example is Eddie Lampert's activist hedge fund, ESL Investments, that invested into unsecured notes and bank debt of Kmart before its bankruptcy in 2004. Once the reorganization was complete, ESL Investments emerged as Kmart's largest shareholder owning about 50 percent of the shares in the reorganized company (Gilson and Abbott, 2009). There is a third type of investor who also takes on an active role in the reorganization process but does not necessarily aim to control the company after it emerges as a healthy entity. Over a medium-term investment horizon, such an investor obtains a significant amount of the claims in an impaired class and tries to influence the outcome of the bankruptcy process, for example through her presence as a member of the creditors' committee. A typical strategy that is followed by such distress investors is called "negative control" which gives the investor some negotiation power in the form of vetoing the reorganization plan until the recovery rate of her claim is improved.[4]

[4] Under the U.S. Bankruptcy Code, a class of claims is deemed to accept the reorganization plan if it is approved by at least two-thirds in amount and one-half in number of the creditors in that class. Hence, an investor that owns just over one-third of a class's claims can block a plan.

Exercising this veto power may result in changes in the terms of the proposed plan and improve the recovery rates of certain creditor classes. For instance, during the bankruptcy of drug store chain Revco, the distress fund Magten Investments followed a similar investment strategy. The investor's goal was to reduce the senior debtholders' recovery rate by holding a blocking position in the firm's subordinated bonds (Gilson, John, and Lang, 1990).

Securities of distressed companies offer attractive returns to specialized investors who have advantages over other investors. First, distress investment is associated with a great deal of illiquidity and concentration, and often requires scaling up in order to deliver high profits. Altman and Benhenni (2019) argue that distressed securities are attractive investments particularly for hedge fund managers who can easily move in and out of securities depending on the credit cycle. Their flexibility in switching from debt to equity and ability to combine hedging with distressed and defaulted debt—which have relatively low correlation to other asset classes—allow hedge funds to take advantage of periods of high returns. Second, this segment of the debt market is not very competitive as it requires specific expertise and experience, and hence some degree of specialization. Third, banks (portfolio managers) who are concerned about meeting regulatory requirements (investment goals) may not be willing to hold distressed securities (Altman et al., 2019). Though in general commercial banks are restricted in their common stock holdings of non-financial corporations, they can still acquire stocks in loan workouts to recover their costs and reduce their risks related to the original extension of credit (James, 1995). On the other hand, Berlin (2000) argues that lenders with priority claims, such as collateralized debt, would avoid risking the priority of their claims by getting too closely involved in the management of the borrowing firm and being perceived as a controlling investor by the court. Also, lacking the expertise for managing non-financial corporations, banks may not find it optimal to tie up their capital in stocks.

There are many potential reasons for why a financial institution may not be willing to invest into risky distressed securities and/or control a distressed firm. Nonetheless, creditors with an existing relationship with the bankrupt firm (i.e., pre-petition secured bank lenders) often provide DIP financing, a strategy known as “loan-to-loan”. Besides the attractive risk-return tradeoff that DIP loans offer, pre-petition lenders may want to serve as DIP lenders to get paid off for their pre-petition debt (i.e., roll-up), to increase the security of pre-petition loans if they are under-collateralized (i.e., cross-collateralization), to protect going-concern value of their collateral or to ensure their claims remain the first in priority.

There is considerable amount of empirical work investigating the role of specialized investors in the reorganization of distressed firms. On the one hand, their involvement can result in efficiency gains by, for instance, reducing secured creditors' liquidation bias, replacing underperforming CEOs, retaining key employees, or providing additional financing. Jiang, Li, and Wang (2012a) find that hedge fund presence in a bankruptcy process increases the likelihood of a successful reorganization. In a related study, using a sample of 469 firms that attempted to resolve distress within or outside of the legal environment between 2001 and 2011, Lim (2015) finds that hedge fund involvement is associated with a higher likelihood of prepackaged restructurings, shorter duration of distress, and higher debt reduction. The study concludes that activist hedge funds can create value by enabling efficient contracting. Based on a sample of 288 firms that defaulted on public debt between 1980 and 1993, Hotchkiss and Mooradian (1997) find that active involvement of vultures in the restructured company is associated with a more pronounced improvement in post-restructuring operating performance. Furthermore, the improvement in post-restructuring operating performance is greater if the vulture investor becomes the CEO or chairman, or gains control of the target firm. Overall, these findings indicate that involvement of SDIs in the bankruptcy and restructuring of distressed firms might result in efficiency gains. On the negative side, distress investors can also lengthen the bankruptcy and reorganization process, specifically when they intend to acquire a blocking position and negotiate for a higher recovery rate of their claims (Rosenberg, 2000). Finally, if distress investors have more positions in the credit default swap (CDS) contracts of a distressed firm than their long positions in the firm's debt, then they might favor a credit event that triggers payment to the holders of CDS contracts (e.g., formal bankruptcy) rather than a private workout – the so-called "empty creditors problem" (Baird and Rasmussen, 2009).

To summarize, distressed debt is a growing market and distress investors have become more involved in the bankruptcy process over time. Distress investors have advantages over others such as being able to hold illiquid and concentrated portfolios. Depending on their investment horizon and the amount of risk that they are willing to take, distress investors may follow passive or active strategies. By using their expertise in reorganizing troubled companies, active distress investors can provide efficiency gains and increase the value of distressed companies that they invest into.

3 Model

3.1 Setup

We build on the standard model of entrepreneurial moral hazard proposed in Holmstrom and Tirole (1997) and Tirole (2006). To investigate how distress affects incentives, we add a second phase to the game, where we allow for the entry of specialized financiers who may acquire distressed claims and participate in the firm. There are three players: an entrepreneur (the borrower), an initial lender (sometimes referred to as "bank"), and a specialized distress investor (SDI). The entrepreneur, with cash on hand W, has a project that requires an initial fixed outlay of I to be paid at $t = 0$; the amount $I - W$ is financed by the bank. If the first phase of the project is successful, it yields a verifiable payoff of $R > 0$ and zero otherwise. The success probability of phase 1 is p if the entrepreneur exerts effort and $p - \Delta$ if she shirks. The entrepreneur obtains private benefit $B > 0$ from shirking.

At $t = 0$, the lender makes a competitive offer to the entrepreneur, which is determined by the lender's participation constraint. The loan contract specifies, in case of success, how phase-1 profit (R) is shared between the entrepreneur and the lender. The borrower and the lender receive R_b and $R_l = R - R_b$ (i.e., face value), respectively. The entrepreneur is protected by limited liability such that $R_b \geq 0$. At $t = 1$ we have the project's second phase (or continuation), which we represent in more reduced-form (i.e., no explicit moral hazard problem). We assume that the lender and the borrower are unable to write contracts at $t = 0$ for the sharing of phase-2 output (i.e., there is no long-run debt).[5] The overall project is assumed to be unfeasible under phase-1 shirking (see assumption 1, presented later, for the formalization).

The outcome of phase 1 determines which subgame the initial lender and the borrower play in the second phase. If phase 1 is successful, then the entrepreneur is able to fulfill her debt obligations, is fully in control of the firm, and captures the entirety of the continuation value (denoted by V). If phase 1 fails, then the borrower defaults on the loan, which does not necessarily imply that the firm is to be liquidated. Instead, the entrepreneur lacks the liquidity necessary to meet the debt obligation on time and, therefore, may lose control over

[5] Our model is only interesting if there is the possibility of default at the interim date, hence some short-term debt is necessary. We fully rule out long-term debt for expositional simplicity.

the firm.[6] While the firm is always illiquid, the transfer of control rights depends on whether the firm is solvent or not. In particular, if the firm is insolvent with negative net worth, then the control rights are fully transferred to the creditor.

In our setting, the SDI appears in phase 2, conditional on phase-1 failure, with endogenous probability q (detailed later). We assume that SDI entry takes the form of the SDI acquiring the initial lender's claim. The initial lender can accept or reject the distress investor's offer but cannot commit ex ante to a course of action with respect to said offers. If an SDI takes over the debt claim, the continuation value of the firm is given by V^e (the "entry" scenario); otherwise, the initial lender keeps the debt claim and the value of the firm is V^n (the "no entry" scenario). We assume that both V^e and V^n are net of direct and indirect costs of bankruptcy.

The SDI faces cost C_s if he attempts to enter the firm, where C_s is a uniform random variable in the interval $[0, \overline{C}_s]$; C_s is observed by the SDI prior to the decision to enter. Entry costs for the SDI might take the form of legal, transaction and information acquisition costs, as well as any additional funding that might be essential for the continuation of the firm. For instance, under Chapter 11 legal bankruptcy, an SDI may provide debtor-in-possession (DIP) financing.

We allow the entrepreneur's phase-2 payoff upon failure to differ depending on whether an SDI participates in the company or not. The entrepreneur's payoff in case of SDI entry is denoted by u_b^e and otherwise by u_b^n. We refer to these reduced-form payoffs as the *borrower's continuation rent* and they are a critical piece of our model. The borrower's continuation

[6] If the firm defaults on its debt, then there are multiple possible outcomes which for now we do not explicitly model. For instance, the initial lender might accept to exchange its existing debt for a new debt security with a different maturity, coupon, etc. Similarly, the lender might also be willing to participate in a private workout where its debt is exchanged for equity. Finally, the firm might be reorganized or liquidated through the legal bankruptcy process.

rent can be thought of as capturing the following economic effects: (i) phase-2 agency rents, i.e., minimum payments that need to be made to the entrepreneur such that she provides an appropriate amount of effort during continuation; (ii) how critical the entrepreneur's human capital is for continuation, which impacts the surplus she can bargain for; (iii) any and all entrepreneurial outside options that arise during phase 2, which also impact the entrepreneur's ability to bargain; (iv) the technology and bargaining power that is available both to the bank and the SDI; for example, the SDI can have a superior monitoring technology or his management team could have skills that are substitutes to the entrepreneur's.

The borrower's continuation rent plays an important role in shaping phase-1 incentives. Since this payoff accrues to the entrepreneur following failure, there is more scope for moral hazard during phase 1. The borrower's continuation rent thus amplifies the canonical one-period agency problem, being similar to an increased private benefit associated with shirking. The question is whether this effect is worsened or not by SDI presence.

It is important to note that what we characterize in the model as "entrepreneur" can in practice refer to any and all firm associates that are critical for the operations of the business and need to be properly incentivized. One of the important aspects of Chapter 11 bankruptcy of the U.S. is the key employee retention or incentive plans (KERPs). Management turnover during reorganization can be disruptive and costly, and such plans are designed to persuade key employees with firm-specific knowledge to stay with the bankrupt firm. According to Goyal and Wang (2015), 40% of the large public firm bankruptcies in the U.S. between 1996 and 2013 involved KERPs. Moreover, they show that KERPs are more frequently adopted by firms that have DIP financing and activist investors, which are both measures of the strength of creditor control over the bankruptcy process. In the context of our model, given that SDIs are new to the company, retaining key employees might become even more critical.

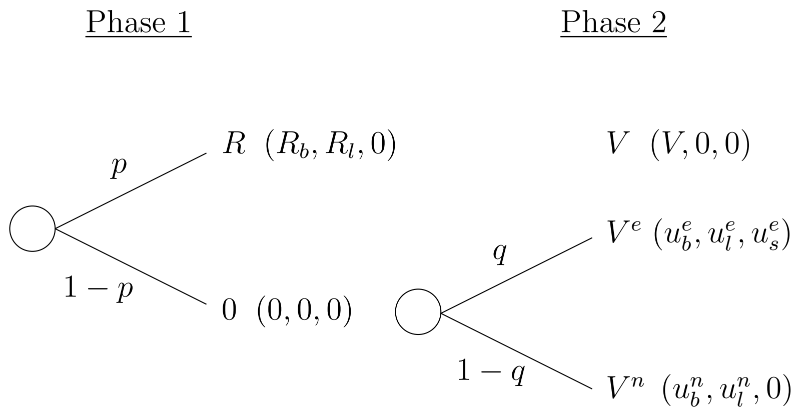

Figure 1: Timeline and payoffs. The figure depicts the two phases of the project under no-shirking. The outcomes in parenthesis represent the payoffs for the entrepreneur (first entry), the initial lender (second entry) and the SDI (third entry). The (endogenous) likelihood of SDI involvement is denoted by q. If the SDI enters the company, then the continuation value of the project is given by V^e, and the entrepreneur, the initial lender and the SDI receive u^e_b, u^e_l and u^e_s, respectively. If SDI entry does not materialize, then the continuation value of the project is given by V^n, and the entrepreneur, the initial lender and the SDI receive u^n_b, u^n_l and 0, respectively.

Nevertheless, we analyze different cases by allowing for the borrower's continuation rent to be higher or lower under SDI entry, relative to the case when the initial lender controls the company.

If there is no SDI entering the firm and the initial lender keeps the debt claim, then the lender receives a payoff of $u^n_l (= V^n - u^n_b)$. On the other hand, if an SDI enters the company, then the lender and the SDI share the surplus net of the borrower's continuation rent. In this case, we denote the lender's and the SDI's payoffs as u^e_l and u^e_s, respectively. The next section details the bargaining procedure we use to pin down these payoffs.

This concludes the description of the setup for the two phases; figure 1 summarizes the timeline and possible outcomes for the entrepreneur/borrower, the initial lender, and the distress investor.

A later-stage distress investor taking over the control of the firm upon failure can affect the project in two different ways: (i) by changing the continuation value and/or (ii) by changing the borrower's continuation rent. For instance, an SDI that has experience with distressed firms as well as the ability to run them efficiently might increase the (total) continuation value of the firm. Alternatively, an SDI who has his own management team might depend less on the human capital of the entrepreneur, reducing her bargaining power upon continuation. Let us characterize the advantage that the SDI has relative to the initial lender using two parameters: the SDI's advantage in terms of increasing total firm value is denoted by v (possibly negative, which means a disadvantage); and the SDI's advantage in reducing the borrower's continuation rent is denoted by d (also possibly negative). We can thus re-write the value of the firm under SDI management as

$$V^e = V^n + v, \quad v \geq -V^n, \tag{1}$$

where V^n, recall, is the value of the firm under initial-lender management. Similarly, the borrower's continuation rent in case of SDI participation can be re-written as

$$u_b^e = u_b^n - d, \quad d \leq u_b^n, \tag{2}$$

where u_b^n, recall, is the borrower's continuation rent without SDI entry.

Whenever $v > 0$ we will refer to the SDI as *value-enhancing* and whenever $d > 0$ we will refer to the SDI as *disciplining*. The rationale for the term "disciplining" is that with $d > 0$ the borrower payoff associated with failure is reduced upon SDI entry, which at the margin softens the moral hazard problem in phase 1. In our model, the pair (v, d) characterizes the SDI's "type" (or "style"). As we show later, the SDI's type importantly drives our results.

3.2 Equilibrium: some intermediate derivations

In this section we characterize several equilibrium outcomes that hold in general and are useful for later results. We start by formalizing our assumption stated previously that projects are not feasible under shirking:

Assumption 1 *Under shirking, the parameter configuration is such that the project's NPV is negative regardless of SDI intervention. In particular, we assume*

$$(p - \Delta)(R + V) + (1 - p + \Delta)\max(V^e, V^n) + B \leq I. \tag{3}$$

This assumption ensures that when the entrepreneur shirks, even if in the extreme case where the SDI's payoff is zero, either the entrepreneur's or the initial lender's expected payoff is negative, and hence the project is not feasible. This assumption simplifies the analysis, while maintaining the centrality of the moral hazard problem.

Next we provide some intermediate results regarding SDI entry (lemma 1) and financing feasibility (lemmas 2 and 3).

Lemma 1 *The ex ante likelihood of SDI involvement is given by*

$$q = \begin{cases} \min\left(1, \frac{u_s^e}{\overline{C}_s}\right), & \textit{if } u_s^e > 0 \\ 0, & \textit{otherwise.} \end{cases} \tag{4}$$

A necessary condition for a strictly positive probability of SDI entry is

$$u_s^e > 0 \Leftrightarrow V^e - u_l^e - u_b^e > 0 \Leftrightarrow v + d > 0. \tag{5}$$

Lemma 2 *Let $\mathcal{P}$ denote the maximum (expected) pledgeable payoff to the initial lender. Then a necessary condition for financing is*

$$\mathcal{P} \geq I - W, \tag{6}$$

where

$$\mathcal{P} \equiv p\left\{R - \frac{B}{\Delta} + V - \left[qu_b^e + (1-q)u_b^n\right]\right\} + (1-p)\left[q(V^e - u_b^e - u_s^e) + (1-q)(V^n - u_b^n)\right]. \tag{7}$$

Condition (6) corresponds then to a regular participation constraint for the initial lender, considering that the loan contract provides incentives to the borrower to exert effort such that the borrower's incentive-compatibility constraint is satisfied. We can therefore rewrite condition (6) as

$$\begin{aligned} W \geq\; & \underbrace{p\frac{B}{\Delta} + qu_b^e + (1-q)u_b^n}_{\text{Agency Rent}} + \underbrace{(1-p)q(u_s^e - \mathbb{E}\left[C_s | C_s \leq u_s^e\right])}_{\text{SDI's Rent}} \\ & - \underbrace{\left\{p(R+V) + (1-p)\left[q(V^e - \mathbb{E}\left[C_s | C_s \leq u_s^e\right]) + (1-q)V^n\right] - I\right\}}_{\text{Project's NPV}} \\ \equiv\; & \underline{W}^*, \end{aligned} \tag{8}$$

which can be interpreted as a net worth requirement such that financing is not viable if $W < \underline{W}^*$. The negative sign of the last term indicates that projects with higher NPV are more likely to be financed. Conversely, if the entrepreneur's agency rent is high and/or the SDI captures a large net (expected) surplus, then financing may be unfeasible, even if the project generates a positive NPV. We note that the entrepreneur's agency rent is a combination of the canonical one-period agency rent (pB/Δ) and the borrower's continuation rent. As explained before, the borrower's continuation rent feeds back into the moral hazard problem in phase 1 and therefore leads to a higher agency rent.

Lemma 3 *In an interior solution, where $q \in (0,1)$, the minimum amount of capital an entrepreneur must provide to secure funding, $\underline{W}^*$, becomes*

$$p\frac{B}{\Delta} + u_b^n - [p(R+V) + (1-p)V^n - I] + \underbrace{\frac{u_s^e\left[(1-p)(u_s^e - v) - d\right]}{\overline{C}_s}}_{\textit{SDI Effect}}. \tag{9}$$

If $v > 0 \wedge d > 0$, SDI entry unambiguously reduces the possibility of credit rationing (i.e., decreases $\underline{W}^*$).

The result at the end of lemma 3 shows how SDI presence can impact the availability of financing ex ante. At this stage we are only able to characterize the most benign of cases. In the next section, after endogenizing the continuation payoffs for the SDI and the initial lender, we will also show how SDIs can easily generate negative spillovers on initial finance.

Finally, lemma 4 characterizes the project's NPV and how it is affected by the presence of SDIs.

Lemma 4 *In an interior equilibrium, where $q \in (0,1)$, the overall NPV of the project is given by*

$$NPV^* = p(R+V) + (1-p)V^n - I + \underbrace{\frac{(1-p)u_s^e\left(v - \mathbb{E}\left[C_s | C_s \leq u_s^e\right]\right)}{\overline{C}_s}}_{\textit{SDI Effect}}. \tag{10}$$

Equation (10) shows that a value-destroying investor ($v < 0$) unambiguously reduces the NPV of a funded project. Nonetheless, even value-destroying SDIs may relax credit rationing, as long as the disciplining effect is strong enough; in particular, if $d > (1-p)(u_s^e - v)$ (see lemma 3).

4 The case of high-leverage firms

In this section we further formalize how surplus is shared between different agents and conduct a more detailed analysis of the model. Throughout this section, we assume that the endogenous face value of debt is high and in case of failure, regardless of whether an SDI enters or not, the firm has negative net worth.[7]

Assumption 2 *In equilibrium, the face value of the loan is high. Specifically, the continuation value of the firm net of the borrower's continuation rent is not sufficient to cover the debt payment:*

$$R_l^* > \max\left(V^n - u_b^n, V^e - u_b^e\right). \tag{11}$$

If there is no SDI entering the firm and the company is controlled by the initial lender, then after paying the entrepreneur, the lender receives a payoff of

$$u_l^n = V^n - u_b^n.$$

Furthermore, upon SDI entry we assume the initial lender and the SDI share the surplus according to an efficient bargaining procedure. The initial lender and the SDI have equal chances of being the one making a take-it-or-leave-it offer. The initial lender's expected

[7] In section 5 we will adapt the model to the complementary case where the face value of the loan is lower than the firm's net worth. Though both versions fit within the structure of the model presented in section 3, they require different institutional concerns to be addressed.

payoff under the assumed bargaining rule is then

$$u_l^e = \frac{1}{2}\left[\underbrace{V^n - u_b^n}_{\text{SDI's offer}} + \underbrace{V^e - u_b^e}_{\text{Lender's offer}}\right] \tag{12}$$

$$= V^n - u_b^n + \frac{v+d}{2}, \tag{13}$$

with the SDI retaining the remaining, given by

$$u_s^e = V^e - u_b^e - u_l^e.$$

Our non-cooperative bargaining approach is therefore equivalent to the (cooperative) Nash bargaining solution, where each player obtains the same payoff in excess of her threat point. After simplifying, the SDI's payoff is given by

$$u_s^e = \frac{v+d}{2}. \tag{14}$$

Finally, it is perhaps worthwhile to clarify that in subsequent analyses throughout this section we always treat u_b^n, the borrower's continuation rent in case the initial lender controls the firm, as exogenous.[8]

4.1 Credit rationing

Proposition 1 below formally summarizes the impact of SDI entry on financing feasibility.

Proposition 1 *In an interior equilibrium, where* $q \in (0,1)$*, (expected) pledgeable income is*

[8]This is no longer the case in section 5, where it is more natural, due to institutional details, that u_b^n is endogenous.

given by

$$\mathcal{P}^* = p\left(R - \frac{B}{\Delta} + V - u_b^n\right) + (1-p)(V^n - u_b^n) + \underbrace{\frac{v+d}{2\overline{C}_s}\left[pd + (1-p)\frac{v+d}{2}\right]}_{\text{SDI Effect}}. \tag{15}$$

SDI intervention therefore relaxes the credit rationing problem, increasing the funding probability of new projects, as long as

$$d > -\frac{1-p}{1+p}v. \tag{16}$$

It follows that the set of financing-improving SDIs is larger for long-shot projects, i.e., for lower probability of success p.

Endogenous SDI entry has two separate effects on capital rationing. First, since the initial lender only allows SDIs that increase its payoff to enter (i.e., $u_l^e \geq u_l^n$, or $v + d \geq 0$), there is a direct positive effect of SDI entry on the initial lender's continuation payoff. The last term in equation (15) captures this effect where $v + d \geq 0$ and SDI entry relaxes credit rationing. Second, by affecting the borrower's payoff under failure through d, SDI entry changes the need to use high-powered incentives for the entrepreneur to behave. More specifically, whenever $d > 0$, the entry of an SDI makes failure less appealing to the entrepreneur, reducing the agency rent and increasing credit availability. Conversely, through the same channel, an SDI that worsens the moral hazard problem (i.e., $d < 0$) reduces the credit available in the first stage. For sufficiently negative values of d, SDI entry can tighten initial financing even if the SDI increases the continuation value.

Finally, as the probability of failure increases, credit rationing becomes less of a problem with an SDI that worsens moral hazard. Intuitively, as p decreases in equation (15), the impact of SDI on the supply of credit through the first channel, namely by directly increasing the initial lender's payoff, becomes more important. Hence, in relative terms, the negative impact on credit supply of an SDI that worsens the agent's moral hazard is attenuated.

4.2 NPV: level and allocation

The initial lender provides ex ante funding competitively, which implies the project's NPV is shared between the entrepreneur and the SDI. Perhaps surprisingly, SDI (equilibrium) entry has ambiguous implications in terms of NPV and the entrepreneur's ex ante profit. These results are formalized in proposition 2 below.

Proposition 2 *Consider an interior equilibrium with $q \in (0,1)$. The ex ante equilibrium profit of the entrepreneur, denoted by π_b^*, is given by:*

$$\pi_b^* = p\,(R+V) + (1-p)V^n - I + \underbrace{(1-p)\frac{(v+d)(v-d)}{4\overline{C}_s}}_{\textit{SDI Effect}}, \tag{17}$$

which implies that SDI intervention increases the expected profit for the entrepreneur if and only if $d < v$. The project's NPV is given by

$$NPV^* = p(R+V) + (1-p)V^n - I + \underbrace{(1-p)\frac{(v+d)(3v-d)}{8\overline{C}_s}}_{\textit{SDI Effect}}, \tag{18}$$

which implies that SDI intervention increases NPV if and only if $d < 3v$.

Note that the conditions listed in the proposition for negative value effects (total NPV and/or the entrepreneur's ex ante profit) are compatible with equilibrium SDI entry, which requires only that $v+d > 0$, i.e., that the offer is attractive for the lender. In particular, under some conditions, the initial lender will accept value-destroying SDI offers that "squeeze" the entrepreneur but make the lender better off. Since there is no way to avoid this ex post value destruction ex ante, the NPV is reduced when such value-destroying SDIs are more pervasive.

4.3 Main impacts of SDI entry: summary and discussion

In this section we summarize the main regions of the parameter space and provide some economic intuition and discussion for each of these regions. Figure 2 illustrates these regions, building on the results implied by propositions 1 and 2. Throughout the discussion we refer to each region as characterizing an SDI "type".[9]

Region 1. In this region, SDI entry improves pledgeable income but decreases NPV. These SDIs are characterized by relatively low v, possibly negative (value-destroying), and a relatively high d (disciplining). The only social value that these SDIs may provide is in alleviating credit rationing. If, however, credit rationing is not a concern, then these SDIs are socially undesirable. Essentially they are rent-seeking institutions which are able to extract surplus from the entrepreneur ex post by "bribing" the initial lender, who is unable to commit ex ante not to accept said "bribe". Interestingly this type of equilibrium can occur even with a positive v, which is perhaps not obvious. The reason why NPV goes down even though $v > 0$ is that there is an inefficiently high level of entry (with concomitantly high average entry costs), motivated precisely by the payoff associated with expropriating the entrepreneur. Mapping this region to the world of distress investors, the type of SDI from region 1 is an organization that focuses on monitoring technologies and/or has power over the entrepreneur in some form.

[9] We do not discuss how contextual variables such as technology or regulation impact the prevalence of particular SDI types in a given industry. This would be relevant to empirically test the predictions of the model in the cross section of industries for example; but that is outside the scope of the paper.

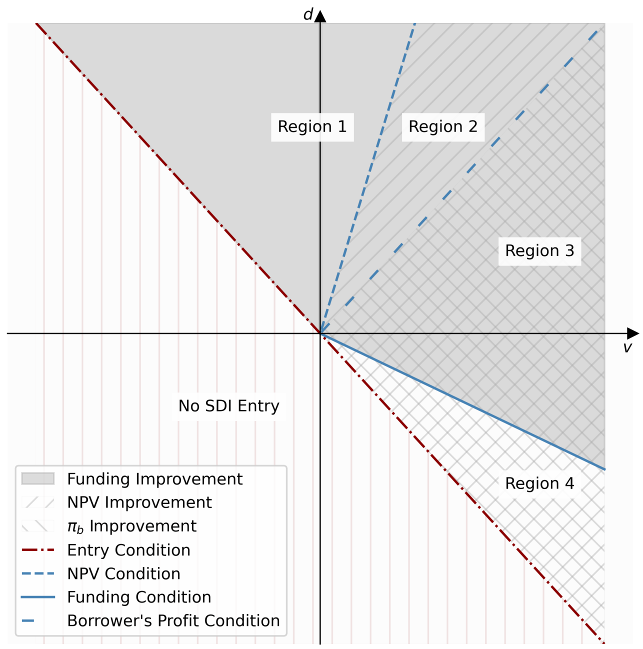

Figure 2: This figure shows the set of feasible regions, for a highly leveraged project, as a function of the advantage of the SDI relative to the initial lender in (i) running the firm efficiently, denoted by v and (ii) the second stage payoff to be paid to the entrepreneur, denoted by d.

Region 2. In this region SDIs are of a relatively nicer sort as compared to region 1, in that they create enough continuation value such that the NPV increases; one could say they have some understanding of the firm's core business. Nevertheless, for these SDIs the disciplining dimension is still relatively strong and therefore their presence hurts the entrepreneur ex post. These SDIs are socially useful in two dimensions: alleviating credit rationing if this is a concern and improving the social contribution of the project. This notwithstanding, the reduction in the entrepreneur's ex ante profit could have negative incentive effects that we do not model, for example in terms of the effort that goes into identifying project opportunities.

Region 3. Here SDIs display a bias towards v and away from d, as compared to regions

1 and 2. These SDIs are organizations with very strong business skills in the distressed firm's sector. Even if $d > 0$, the discipline dimension is not so strong that the entrepreneur is hurt ex ante. Whenever $d > 0$, meaning that the entrepreneur has a lower payoff ex post, it is still the case that her ex ante profit benefits from SDI entry. This occurs because with a high v the initial lender is better off ex post and passes these gains to the entrepreneur via the initial financing conditions (recall the initial lender provides funding competitively). In a way this region is the best of all worlds in our model, where everyone benefits from SDI presence. Interestingly, this ideal type of SDI achieves a kind of balance between value enhancement and discipline.

Region 4. Last but not least, this region represents a potentially unfortunate type of SDI, namely organizations who on average have strong business skills within the distressed firm's sector ($v > 0$) but still rely importantly on the entrepreneur ($d < 0$). Since upon entry they increase the entrepreneur's payoff significantly, this worsens the moral hazard problem and could trigger credit rationing ex ante. We believe this region is empirically plausible, as we explain next. In many cases, the initial lender, for example a traditional bank, will have very low ability in managing a distressed firm, including retaining key employees. Without skilled SDIs around, the firm may therefore end up being liquidated. Although this is not necessarily positive in itself, it has the benefit of providing the entrepreneur with strong incentives, since upon liquidation her payoff will be low. Once skilled SDIs are around and willing to rescue the company, the entrepreneur suddenly becomes more valuable following failure (negative d). And since ex post lenders will like the SDI's offer, they cannot commit ex ante not to accept it (even when doing so ex ante be beneficial). Ultimately this type of SDI might therefore *inadvertently* worsen credit rationing.

A final comment is in order. An important feature of the model is that bargaining between the SDI and the initial lender takes place after the cost of entry has been incurred. This implies that from the perspective of the SDI, entry is inefficiently infrequent (bargaining friction), because the (sunk) entry cost is not internalized. From the social perspective, this means that when SDIs are desirable, there will unfortunately be too few around. On the flip side, if SDIs are socially undesirable, fortunately not too many will enter the market.

4.4 Extension: SDI entry and ex ante project choice

In the previous section we took project characteristics as given and asked whether and how SDI presence affects financing ex ante. In this section we go one step further and ask if SDI presence can impact which types of projects entrepreneurs look for. The motivation for the analysis is as follows. In some instances, SDI presence is bad news for the entrepreneur, for example because of a reduction in pledgeable income. Given an undesirable effect of SDIs, the entrepreneur can try to reduce SDI entry by picking projects that are less attractive to later-stage investors, for example projects with lower continuation value.[10] In order to conduct the analysis we make some additional choices regarding functional forms, described below.

We now refer to V—so far just the continuation value following success—as the *baseline continuation value* and assume the following:

$$V^n = \lambda V \tag{19}$$

$$v = \gamma V, \tag{20}$$

which implies that firm value upon entry, V^e, is equal to $V(\lambda + \gamma)$. We assume $\lambda \in [0, 1]$

[10] Conversely, if SDI entry is desirable by the entrepreneur, she might choose projects that attract more SDIs.

and $\gamma > -\lambda$ (otherwise V^e could become negative). In the coming sections, we assume that V is one of the project characteristics that the entrepreneur can choose, even if only partially/locally. One can intuitively think of projects with higher V as having higher duration.

Next, we assume that the SDI's disciplining capacity is proportional to u_b^n:

$$d = \delta u_b^n, \tag{21}$$

with $\delta < 1$ (otherwise u_b^e, the borrower's continuation rent upon SDI entry, could become negative) but possibly negative. In the coming sections, we assume that u_b^n is also one of the project characteristics that the entrepreneur can choose, even if only partially/locally. One can think about projects with higher u_b^n as being more reliant on entrepreneurial human capital and/or effort during continuation.

Finally, it is useful to note that the earlier condition for the lender to (strictly) benefit from SDI entry, $v + d > 0$, can now be written as

$$\gamma V + \delta u_b^n > 0. \tag{22}$$

4.4.1 Continuation value (V)

In this section we consider if an entrepreneur would always prefer projects with higher V, as would be intuitive. The main results are presented in proposition 3.

Proposition 3 *Comparative statics on baseline continuation value V are characterized by the following:*

1. *It is always the case that $\frac{\partial \pi_b^*}{\partial V} \geq 0$ and $\frac{\partial NPV^*}{\partial V} \geq 0$.*

2. *Consider parameter configurations where in equilibrium $q = 0$ or $q = 1$. Then $\frac{\partial \mathcal{P}^*}{\partial V} \geq 0$.*

3. Suppose that $\gamma > 0$. Then there exist interior equilibria with $q \in (0, 1)$ where $\frac{\partial \mathcal{P}^*}{\partial V} < 0$. A necessary—but not sufficient—condition for this result to hold is that

$$\delta < -\frac{\gamma V(1-p)}{u_b^n(1+p)}. \tag{23}$$

The first point in the proposition is consistent with our a priori intuitions. Everything else constant, a project with higher continuation value V is socially desirable and also makes the entrepreneur better off ex ante. The second point shows that in the extreme cases where SDIs either never enter or always enter, then pledgeable income also improves with a higher continuation value. The interesting result is in the third point, which shows that under certain conditions, pledgeable income can *decrease* in continuation value.

The counter-intuitive result described in the third point is illustrated in figure 3. In this case, SDI entry creates value ($\gamma > 0$) but makes failure softer for the entrepreneur ($\delta < 0$, with δ sufficiently negative), which feeds back into the agency rent and thereby lowers pledgeable income. Notice that in figure 3, when the project's continuation value is sufficiently low, no SDI entry is allowed, since it would not increase the initial lender's payoff ($v + d < 0$). As such, in this region (in this example, $V < 59.4$) increases in V improve pledgeable income, as intuitively one would expect. Similarly, for sufficiently high values of V (in particular, $V > 67.4$), SDI entry becomes certain and pledgeable income once again increases with the project's continuation value. However, when SDI entry is uncertain, an entrepreneur who is concerned about pledgeable income (low net worth) would at least locally prefer projects with lower continuation value in order to attract fewer SDIs. Unfortunately, an improvement in pledgeable income comes at the expense of a lower NPV.[11]

[11] Unless the entrepreneur can also be strategic about other features simultaneously (e.g., higher R or p).

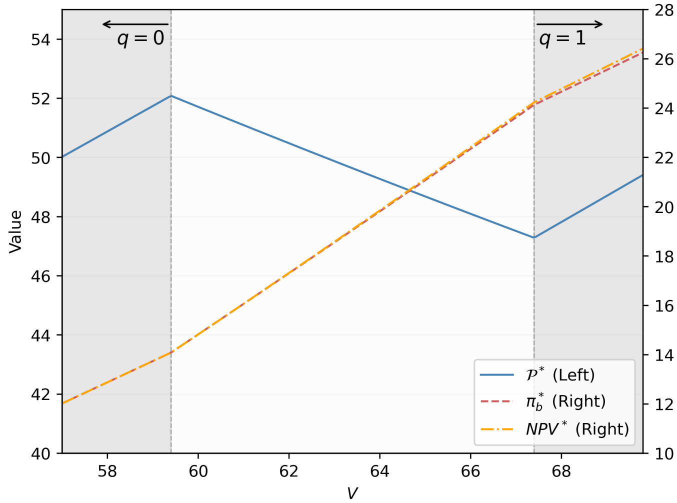

Figure 3: This figure shows several equilibrium outcomes as a function of V, the continuation value following success. These outcomes were generated for $V \in [57.5, 69.8]$ and considering the following parameter values: $R = 60$; $\lambda = 0.3$; $\gamma = 0.25$; $p = 0.8$; $\Delta = 0.75$; $u_b^n = 15$; $\delta = -0.99$; $B = 30$; $I = 85$; $W = 45$; and $\overline{C}_s = 1$. The (expected) pledgeable income, $\mathcal{P}^*$, is represented in blue (solid line; left y-axis); the ex ante profit of the entrepreneur, π_b^*, in red (dashed line; right y-axis); and the project's NPV^* in orange (dash-dotted line; right y-axis).

Note that this is an asymmetric result, in the sense that SDI presence will never per se cause entrepreneurs to look for projects with higher V.[12] This follows from point 1 in the proposition: entrepreneurs already naturally prefer the highest possible V. To be clear, suppose that SDIs improve pledgeable income and suddenly they become more pervasive (lower entry cost). The good news for financially-constrained entrepreneurs notwithstanding, the higher frequency of SDI entry would not affect the entrepreneur's project ranking along the V dimension.

[12]A caveat to this argument is that it applies fully if one thinks about *costless* project selection. The analysis would be more complex if we introduced costly project search.

4.4.2 Borrower continuation rent (u_b^n)

In this section we consider how entrepreneurs rank projects with regards to their borrower continuation rent u_b^n, which can be understood as the level of ex post reliance on entrepreneurial human capital and/or effort. Without SDIs one knows that entrepreneurs would be (weakly) better off with lower u_b^n, for two reasons: first, since this payoff feeds back into the agency rent, a higher u_b^n worsens credit rationing; second, changes in u_b^n do not impact the project's NPV and the ex ante payoff of the entrepreneur, since the initial lender provides financing competitively. What is less obvious is how SDI presence changes these economics, namely in terms of pledgeable income; a counter-intuitive result is presented in proposition 4.

Proposition 4 *Suppose that $\delta > 0$. Then there exist interior equilibria with $q \in (0,1)$ where $\frac{\partial \mathcal{P}^*}{\partial u_b^n} > 0$. A necessary—but not sufficient—condition for this to hold is that*

$$\delta > \frac{-1 + \sqrt{1 + 2\frac{p u_b^n}{\overline{C}_s}}}{\frac{p u_b^n}{\overline{C}_s}}. \tag{24}$$

The proposition shows that in some cases, pledgeable income is actually higher for projects with higher borrower continuation rent u_b^n. This is illustrated in figure 4 (solid blue line). This result follows from the combination of two effects: (i) a higher u_b^n encourages SDI entry, via a better bargaining position *vis-à-vis* the initial lender; (ii) SDI entry can be beneficial for pledgeable income (namely for a strongly disciplining SDI).

Figure 4 also shows two other outcomes that are not obvious. First, project NPV goes up with borrower continuation rent u_b^n. Second, the entrepreneur's ex ante profit π_b^* decreases with borrower continuation rent u_b^n. These effects are connected with the bargaining processs

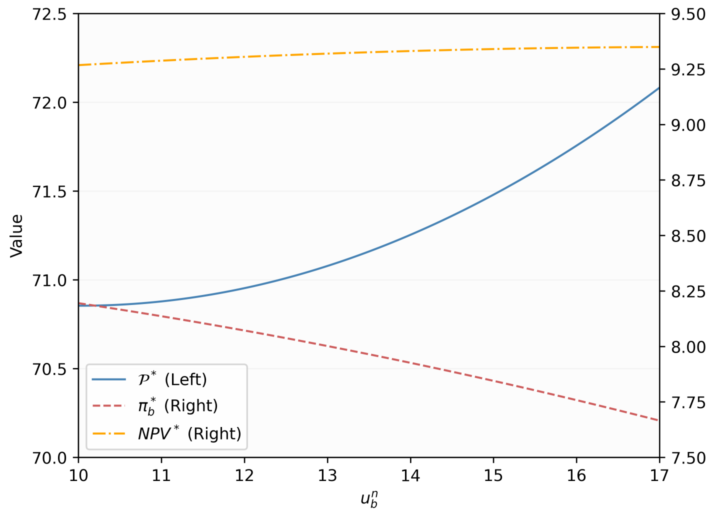

Figure 4: This figure shows several equilibrium outcomes as a function of u_b^n, the borrower's continuation rent without SDI entry. These outcomes were generated for $u_b^n \in [10, 17]$ and considering the following parameter values: $R = 50$; $V = 70$; $\lambda = 0.4$; $\gamma = 0.25$; $p = 0.8$; $\Delta = 0.55$; $\delta = 0.99$; $B = 20$; $I = 94$; $W = 50$; and $\overline{C}_s = 17.5$. The (expected) pledgeable income, $\mathcal{P}^*$, is represented in blue (solid line; left y-axis); the ex ante profit of the entrepreneur, π_b^*, in red (dashed line; right y-axis); and the project's NPV^* in orange (dash-dotted line; right axis).

between the SDI and the initial lender. With higher u_b^n the SDI is able to extract a larger fraction of the surplus when negotiating with the initial lender. And while this is socially efficient (higher NPV), due to the fact that the SDI's entry cost is not internalized during bargaining, it also implies that the entrepreneur has a lower ex ante payoff. Therefore, whenever pledgeable income is not a concern, the following counter-intuitive result holds: entrepreneurs choose projects that are somehow less problematic ex post (low u_b^n), yet this is socially inefficient.

5 The case of low-leverage firms

In this section we analyze the case where in the failure state the company is still viable such that its assets are worth more than the face value of the loan (i.e., positive net worth). If the project fails, then the entrepreneur lacks the liquidity needed to meet the debt payment but since distress is associated with a short-term liquidity problem, as opposed to the high-leverage case, it does not result in control rights being transferred to the initial lender. Instead, the entrepreneur and the lender renegotiate the terms of the existing loan. We extend the model in order to analyze the impact of a potential SDI entry on the outcome of such a private workout.

As in the case of high leverage, if the project is successful, then the entrepreneur maintains full control over the company in phase 2 where she receives V. We continue to assume that an SDI may enter only conditional on failure. The continuation value of the project upon failure is still denoted by V^e if an SDI enters and V^n if there is no SDI entry. Consistent with the high-leverage case, the difference between the continuation values of the project with and without SDI entry is denoted by v:

$$v = V^e - V^n, \quad v \geq -V^n.$$

Furthermore, assumption 1 continues to hold such that whenever the entrepreneur shirks, the project's NPV is negative regardless of SDI intervention.

First, consider the case where there is no SDI entry upon failure and the existing loan is exchanged with a new one that has a longer maturity. The new debt is due at a later stage which we do not explicitly model.[13] Restructuring the loan is costly for the initial lender and we assume that the lender imposes these costs fully on the entrepreneur in exchange for extending the maturity. More specifically, the modified claim of the initial lender generates a payoff of

$$u_l^n = R_l + C_l, \tag{25}$$

where C_l denotes the initial lender's cost of restructuring the existing loan.[14] For now, we assume that C_l is fixed and exogenous but later on we will allow C_l to vary with loan size.

In case of default, if there is no SDI entry, then the entrepreneur no longer receives a fixed payment, but becomes the residual claimant. More specifically, after the initial lender is paid u_l^n, the entrepreneur receives the remaining amount given by $V^n - u_l^n > 0$. The entrepreneur is still in control of the firm and hence keeps the firm alive as long as her payoff is positive.

Now consider the scenario where SDI entry is possible. As opposed to the high-leverage case, ex post bargaining is no longer between the SDI and the initial lender, but between the SDI and the entrepreneur. The entrepreneur allows for SDI entry only if she is not worse off with an SDI than renegotiating the loan with the initial lender. We assume that upon entry, the SDI pays down the full face value of the loan to the initial lender. One can consider this payment as short-term liquidity provided to the entrepreneur by the SDI in the form of a zero-interest loan with an extended maturity. Below we formalize the assumption that

[13]In a distressed debt exchange where the firm is typically insolvent, the initial lender might be forced to make some concessions in the form of reduced face value and lower interest payments, especially if legal bankruptcy would reduce the lender's recovery. Conversely, if under legal bankruptcy, the loan is expected to recover its full face value, then the lender's bargaining power might be higher in a private workout.

[14]We assume that the bank does not realize any losses or profits from this exchange such that the present value of the cash flows from the new contract is the same as in the original loan contract. Furthermore, C_l is contracted ex ante which eliminates the possibility of the bank ex post extracting rents from the firm by holding out from negotiations.

ensures in equilibrium the initial lender recovers the full face value.

Assumption 3 *In equilibrium, the face value of the loan is low such that if there is no entry, the continuation value of the firm (net of restructuring costs) is always above the face value of debt. In case of SDI entry, the continuation value is large enough to pay the entrepreneur her outside option and at the same time cover the full face value of the loan. Formally,*

$$R_l^* < \min\left(V^n - C_l, V^e - u_b^n\right). \tag{26}$$

Under assumption 3 if an SDI enters, upon which the entrepreneur receives at least her outside option, u_b^n, then the initial lender recovers the full face value of the original loan without any postponement or haircuts such that in net terms

$$u_l^e = R_l.$$

In equilibrium, the lender breaks even and the first stage payoff to the lender is still just

$$R_l^* = I - W.$$

This result follows from the fact that in every state the initial lender recovers R_l and whenever there is no SDI entry renegotiation costs are fully covered by the entrepreneur.

In case of SDI entry, the firm's existing debt to the initial lender is paid down and in exchange, the SDI receives newly issued securities.[15] The entrepreneur and the SDI share endogenous fractions of the firm's continuation value, net of the payment to the initial lender, $V^e - R_l$, denoted by m and $1-m$, respectively. One plausible way to think about this arrangement is that the SDI receives a debt security with a face value of R_l that is due at a later stage and at the same time some equity with an ownership share of $1-m$. Applying the bargaining rule analogous to the high-leverage case yields the following payoff to the entrepreneur:

$$\begin{aligned} u_b^e = m(V^e - R_l) &= \frac{1}{2}\Bigg[\underbrace{V^n - u_l^n}_{\text{SDI's offer}} + \underbrace{V^e - u_l^e}_{\text{entrepreneur's offer}}\Bigg] \\ &= u_b^n + \frac{v + C_l}{2}. \end{aligned} \tag{27}$$

Recall that in case of high-leverage projects, upon failure, the borrower loses her equity stake in the company and receives a fixed payment in the second stage. Conversely, when the firm has positive net worth, the entrepreneur becomes the residual claim holder and hence, her payoff depends strictly on the continuation value of the project. The wedge between the entrepreneur's payoffs with and without an SDI is

$$u_b^e - u_b^n = \frac{v + C_l}{2},$$

which implies that the entrepreneur is better off with SDI entry provided that $v + C_l > 0$.

[15] For simplicity, we assume that the existing bank loan is replaced with a new loan by the SDI with the same face value and perhaps an extended maturity. In reality, the face value of this loan might vary with the SDI's investment strategy. For instance, while an active SDI that seeks to control the firm might prefer receiving more equity in the reorganized company, a passive SDI that does not intend to do so might enter by simply taking over the existing debt as well as some of the company's equity. In the context of Chapter 11, a distress investor might also enter the firm by providing DIP financing that is to be converted into equity upon reorganization (Miller, 2023), the so-called "loan-to-own" strategy.

Interestingly, an SDI that reduces the continuation value (i.e., $v < 0$) might still be allowed to enter as long as the cost of renegotiating with the bank is high. On the other hand, the entry of a value-neutral or value-enhancing SDI is always preferred over renegotiating with the bank.

The following lemma characterizes the borrower's equilibrium ownership share.

Lemma 5 *In an interior equilibrium with $q \in (0,1)$, the entrepreneur's ownership share is given by*

$$m^* = \frac{2u_b^n + v + C_l}{2(u_b^n + v + C_l)}. \tag{28}$$

Furthermore, the entrepreneur never loses the control of the firm such that $m^* > 1/2$ always holds.

Lemma 5 shows that whenever the entrepreneur relies less on debt financing upfront, she never loses the control of the firm to the SDI. This result follows from the entrepreneur's offer being accepted half of the time and the fact that such offer allocates the full continuation value $(V^e - R_l)$ to the entrepreneur after the existing loan is paid down. As shown in the previous sections, financing the project with high leverage results in control rights being transferred to the debtholder in case of distress and the entrepreneur's second stage payoff depends solely on how important her human capital is for the continuation of the venture. On the other hand, with a more conservative capital structure, the entrepreneur never loses the control of the firm and her payoff strictly depends on the continuation value of the project.

As it is the case with a highly leveraged project, SDI entry is still costly and the SDI enters only if her payoff exceeds these costs. More specifically,

$$u_s^e = (1-m)(V^e - R_l) = \frac{v + C_l}{2} \geq C_s, \quad C_s \sim U[0, \overline{C}_s]$$

must hold for the SDI to participate in the firm which implies $v + C_l > 0$, a condition that is analogous to condition (5) in lemma 1 of the high-leverage case.

The following proposition summarizes the effect of SDI entry on the entrepreneur's ex ante payoff as well as the NPV of the project.

Proposition 5 *Consider an interior equilibrium with $q \in (0,1)$. The entrepreneur exerts effort, provided that*

$$R + V \geq \lambda V - C_l + \frac{B}{\Delta} + \underbrace{q\left(\frac{v + C_l}{2}\right)}_{\text{SDI effect}}. \tag{29}$$

The ex ante profit of the entrepreneur is given by

$$\pi_b^* = p(R+V) + (1-p)(\lambda V - C_l) - I + \underbrace{(1-p)q\left(\frac{v + C_l}{2}\right)}_{\text{SDI effect}}, \tag{30}$$

which implies that the effect of SDI entry on the entrepreneur's expected payoff is always positive. The project's NPV is given by

$$NPV^* = p(R+V) + (1-p)(\lambda V - C_l) - I + \underbrace{3(1-p)q\left(\frac{v + C_l}{4}\right)}_{\text{SDI Effect}}, \tag{31}$$

where the SDI effect is always positive.

Different than in the high-leverage case, the borrower's incentive compatibility condition is independent of R_b whenever C_l is exogenous. There are two conditions that lead to this result. First, as opposed to the case with high leverage, where the borrower receives a fixed payment in case of failure, in the low-leverage case she becomes the residual claim holder. Second, the initial lender receives R_l regardless of whether the project succeeds or fails, making R_l irrelevant for the borrower's effort decision.

The last term in inequality (29) is always positive which implies that SDI involvement encourages the entrepreneur not to exert effort by reducing ex ante expected costs of renegotiating the loan. Considering that renegotiation costs are fully (partially) paid by the entrepreneur without (with) SDI entry, increasing C_l always relaxes credit rationing by making failure less appealing to the entrepreneur. Specifically, the derivative of the right-hand-side in inequality (29) with respect to C_l is

$$-1 + \frac{v + C_l}{2\overline{C}_s} \leq 0,$$

which is always negative in an interior equilibrium.

The impact of SDI entry on the entrepreneur's expected payoff is straightforward to see: Since the entrepreneur is the one who decides on whether the SDI enters or not, SDI entry is allowed only if it increases the entrepreneur's expected payoff relative to restructuring the loan (i.e., $v + C_l \geq 0$). Also note that the SDI effect increases with the cost of restructuring the loan which provides higher direct savings to the entrepreneur and at the same time increases the ex ante likelihood of SDI entry.

The entry of an SDI that reduces the NPV of the project is always blocked by the entrepreneur. This result is different than in the highly leveraged project case where SDI entry is determined by the initial lender who breaks even in equilibrium. In the high-leverage case, even if SDI entry is costly and hence reduces the NPV of the project, SDI entry is still possible as long as it increases the second stage payoff to the initial lender. Conversely, in the low-leverage case the entrepreneur is the residual claim holder and whenever she allows for SDI entry, it is always the case that the NPV of the project also benefits from it.

5.1 The impact of C_l and W

In this subsection, we analyze the impact of restructuring costs and cash on hand on the borrower's payoff and the NPV of the project.

Proposition 6 *In an interior solution where $q \in (0, 1)$, comparative statics on the cost of restructuring the loan, C_l, are characterized by the following:*

1. *It is always the case that $\frac{\partial \pi_b^*}{\partial C_l} < 0$, but the derivative becomes less negative as the likelihood of SDI entry increases.*
2. *Whenever SDI entry is highly likely such that $q \in \left[\frac{2}{3}, 1\right)$, then $\frac{\partial NPV^*}{\partial C_l} \geq 0$.*
3. *If $q \in \left(0, \frac{2}{3}\right)$ such that SDI entry is less likely, then $\frac{\partial NPV^*}{\partial C_l} < 0$ holds.*

The cost of restructuring the loan, C_l, has an unambiguous negative effect on the borrower's ex ante payoff. Consider the extreme case where SDI entry is not possible. Then, by inspecting equation (30) it is straightforward to see that a one-dollar increase in C_l reduces the entrepreneur's ex ante payoff by \$$(1 - p)$. Now consider the case where upon failure an SDI may enter the firm with a likelihood of q. With SDI involvement being possible, increasing C_l by one dollar decreases u_b^n by exactly one dollar. Although SDI entry allows for saving on costs of restructuring the loan, since the SDI and the entrepreneur equally share the resulting surplus, an increase in C_l reduces the entrepreneur's payoff in the entry scenario as well.

Increasing restructuring costs can have opposing effects on the NPV of the project which makes the overall effect ambiguous. On the one hand, if the project fails and there is no SDI involvement, then the continuation payoff is reduced by the costs of renegotiating the loan, leaving less surplus in net terms. On the other hand, while increasing C_l reduces the probability of costly renegotiation, it also increases the expected costs of entry because high-cost SDIs who could not enter before are now able to participate in the firm. Furthermore, for $v \neq 0$ the increase in the likelihood of SDI entry has a direct impact on the expected continuation value of the firm. Ultimately, the overall effect of a change in C_l on the NPV of the project is given by the difference between the following terms: the change in expected continuation value net of change in expected entry costs,

$$(1-p)q\left(v - \frac{v + C_l}{4}\right) = (1-p)q\left(\frac{3v - C_l}{4}\right), \tag{32}$$

and the change in the expected costs of renegotiation,

$$(1-p)(1-q)C_l. \tag{33}$$

Notice that both of these terms are non-linear in C_l.[16] When SDI entry is not frequent (i.e., C_l is low), increasing C_l further does not increase SDI entry enough to justify the increase

[16] The partial derivative of $(1-p)q\left(\frac{3v-C_l}{4}\right)$ with respect to C_l is

$$(1-p)\left(\frac{v - C_l}{4\overline{C}_s}\right),$$

which is definitely negative if the SDI destroys value and is positive only if $v > C_l$. The partial derivative of $-(1-p)(1-q)C_l$ is

$$-(1-p)\left(1 - \frac{v + 2C_l}{2\overline{C}_s}\right),$$

which is positive for higher values of C_l implying that if C_l is large, increasing it further can reduce the expected costs of renegotiating the loan.

in expected renegotiation costs, and hence NPV decreases. On the other hand, if SDI entry is highly likely, then the reduction in $(1-q)$ generated by an increase in C_l overcomes the increase in the magnitude of renegotiation costs, reducing the expected value of such costs and increasing the NPV of the project.

Figure 5 illustrates an example in which the expected payoff to the entrepreneur, denoted by the blue curve on the left panel, decreases with C_l and the NPV of the project, denoted by the blue curve on the right panel, first decreases with C_l but then becomes an increasing function of it for most of the values on the x-axis. The particular SDI in this example is a value-enhancing type with $v = 12.5$ and the costs of entry are not high ($\overline{C}_s = 10$). Therefore, even for lower values of C_l, SDI entry is still likely such that $q > 2/3$. Consider a 2-unit increase in C_l from 4 to 6. The probability of SDI entry increases by 0.1 from 0.825 to 0.925 and even though C_l is now higher, the expected costs of renegotiation given in (33) decreases from 0.14 to 0.09. Similarly, the change in expected continuation value net of change in expected entry costs given in (32) increases from 1.382 to 1.456. The latter result follows from the SDI being a value-enhancing type and thus, even if entry costs increase with C_l, SDI entry becomes frequent enough to justify the increase in such costs.

5.2 Extension: optimal capital structure

When we presented the high-leverage case, we also studied an extension in section 4.4 where we asked whether entrepreneurs could adjust to SDI presence by choosing projects with certain characteristics. This section presents an analogous analysis of whether entrepreneurs can do the same, this time by altering their debt-vs-equity choices. More specifically, by varying the amount of cash on hand invested into the project, and hence the resulting share of debt in the capital structure, we study whether entrepreneurs can increase their expected

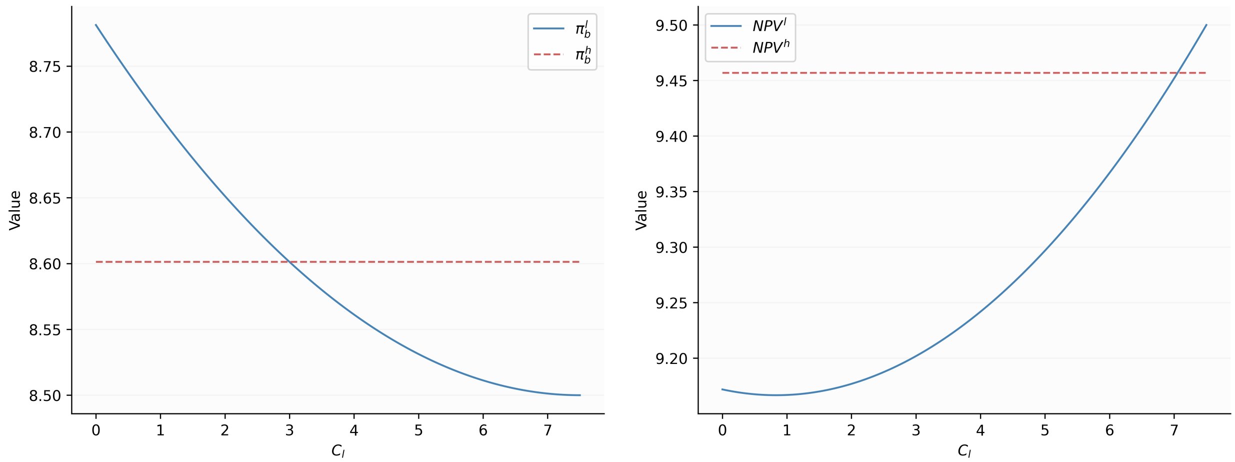

Figure 5: This figure shows the expected payoff to the entrepreneur (left panel) and the NPV of the project (right panel) as a function of C_l, the costs of renegotiating the loan, for the two different capital structures. These outcomes were generated based on the following parameter values: $R = 60$; $V = 50$; $\lambda = 0.5$; $p = 0.8$; $\Delta = 0.75$; $B = 30$; $I = 85$; $W = 70$; $v = 12.5$; $d = 6$; and $\overline{C}_s = 10$. The blue solid line on the left (right) panel represents the entrepreneur's expected payoff (the NPV of the project) with low leverage and dashed red line represents her expected payoff (the NPV of the project) in case of high leverage.

payoff.

Let us first consider the case where the entrepreneur's savings, denoted by W_l, are high enough to finance the project with low leverage but also low enough to require external financing (i.e., $W_l < I$). More specifically, assumption 3 holds:

$$I - \min\left(V^n - C_l, V^e - u_b^n\right) < W_l < I,$$

which implies that in equilibrium the upstream lender is paid the full face value of the loan upon failure regardless of an SDI enters or not (i.e., overcollateralized loan). Let the entrepreneur's ex ante payoff with a less leveraged capital structure be denoted by π_b^l.

Alternatively, consider the case where the entrepreneur invests less of her savings, $W_h <$

W_l, such that assumption 2 holds and the project is financed with high leverage. We denote the entrepreneur's ex ante payoff with high leverage by π_b^h which is independent of W_h. Proposition 7 compares the expected payoff to the entrepreneur as well as the NPV of the project if she invests W_l versus W_h.

Proposition 7 *In an interior solution where the entrepreneur can undertake the project with high or low leverage and $C_l > 0$, the optimal and efficient amounts of borrowing are characterized by the following:*

1. *The entrepreneur prefers borrowing more (i.e., $\pi_b^h > \pi_b^l$) provided that*

$$\frac{2v + C_l + d^2/C_l}{4\overline{C}_s} < 1. \tag{34}$$

2. *High leverage yields a higher NPV (i.e., $NPV_h^* > NPV_l^*$) provided that*

$$\frac{3(2v + C_l) + d^2/C_l - 2dv/C_l}{8\overline{C}_s} < 1. \tag{35}$$

The entrepreneur's decision between the two capital structures is determined by the relative magnitude of $|d|$, the disciplining impact of the SDI, and C_l, the costs of renegotiating the loan. Specifically, if $|d| < C_l$, then the inequality in (34) is satisfied unambiguously[17], and hence high leverage is preferred over an overcollateralized loan. In other words, to the extent that SDI involvement does not change the entrepreneur's agency rent significantly in either direction, the entrepreneur prefers using less of her savings and borrows more.

The economic intuition behind this result links back to the discussion in section 4.3. Consider first regions 1 and 2 in figure 2 where the SDI has a significant disciplining impact

[17] Note that $q < 1$ requires $\overline{C}_s > (v + C_l)/2$.

on the entrepreneur (i.e., d is large and positive). Recall that this is either the type of SDI that does not have much value impact and expropriates the entrepreneur by making an attractive ex post offer to the bank, or the type that is value-enhancing but at the same time disciplines the entrepreneur. In either case, such SDIs hurt the entrepreneur by reducing her ex ante payoff. Although low leverage comes with restructuring costs that are fully paid by the borrower, as long as SDI entry is likely (i.e., $v + C_l$ is high) and the cost of restructuring is low relative to the disciplining effect of an SDI entering the highly leveraged firm (i.e., $C_l < d$), the entrepreneur prefers a less levered capital structure. In other words, the entrepreneur avoids an SDI that is strict in disciplining her by using more cash on hand and borrowing less, provided that renegotiating the loan is not too costly.

Now consider region 3 in figure 2 where SDIs have strong business skills that allow them to improve continuation value without relying much on the entrepreneur's skills. With such SDIs, the entrepreneur can benefit from the increase in the continuation value without being disciplined by the SDI. Provided that renegotiation costs are relatively large (i.e., $C_l > |d|$), the entrepreneur avoids the risk of costly renegotiation by using less cash on hand and borrowing more. Finally in region 4, although SDI entry increases the entrepreneur's second stage payoff (i.e., $d < 0$), it also worsens moral hazard, and for large values of $|d|$ SDI entry becomes less likely. In this case by choosing a less levered capital structure, the entrepreneur can increase the likelihood of SDI intervention.

Figure 5 also illustrates an example of the entrepreneur's capital structure choice. The dashed horizontal line on the left panel shows the entrepreneur's expected payoff with high leverage. When renegotiation costs are low, the blue curve lies above the horizontal line (i.e., $\pi_b^l > \pi_b^h$) and the entrepreneur prefers using more of her savings and borrows less. With a more levered capital structure the entrepreneur is subject to the disciplining effect of the

SDI which reduces her payoff. In this particular example the SDI is value-enhancing with a high v, thus even if C_l is low, SDI entry is frequent enough to make low leverage preferred by the entrepreneur.

Interestingly, the values of C_l that attract the entrepreneur to a less levered capital structure do not necessarily maximize the NPV of the project. When renegotiation costs are low, SDIs participate more frequently in a firm with more debt in its capital structure than a less levered firm because they can reduce the entrepreneur's second stage payoff ($d = 6 > 0$) which in turn increases the NPV of the project. To summarize, when renegotiation costs are low, the entrepreneur prefers avoiding a disciplining SDI by borrowing less and renegotiating the loan even though it may not be the most efficient alternative. For intermediate levels of C_l, high leverage is both the optimal and the most efficient capital structure. Finally, when C_l is high enough such that SDI entry is almost certain, even if the entrepreneur prefers borrowing more, less leverage yields a higher NPV.

5.3 Extension: C_l as a function of R_l

So far we have assumed that renegotiation costs were exogenously given and did not depend on the parameters of the model. Next, we will relax this assumption by allowing for renegotiation costs to vary with loan size. While some of these costs might be independent of the loan size (e.g., transaction costs), restructuring large loans can also be more costly due to their more prominent impact on the bank's balance sheet. Therefore, we assume that the costs associated with extending the loan both have a fixed component as well as a part that is proportional to the size of the loan. More specifically,

$$C_l = \mu + \beta R_l, \quad \mu \geq 0 \ \text{ and } \ \beta \geq 0. \tag{36}$$

Given this functional form of restructuring costs, in an interior equilibrium, where $q \in (0,1)$, the following condition must hold

$$v + \mu + \beta(I - W) < 2\overline{C}_s \Rightarrow v + \mu \leq 2\overline{C}_s,$$

provided that $W < I$.

When renegotiation costs are exogenous, given that the upstream lender is competitive, neither the ex ante payoff of the entrepreneur nor the NPV of the project depends on how much cash on hand the entrepreneur invested into the project initially. Conversely, whenever C_l is an increasing function of loan size, given $R_l^* = I - W$, there is a direct relationship between cash on hand and the cost of renegotiating with the upstream lender. Given the linear form of renegotiation costs, the borrower's incentive compatibility constraint becomes

$$R + V \geq V^n - \mu - \beta(I - W) + \frac{B}{\Delta} + \frac{(v + \mu + \beta(I - W))^2}{4\overline{C}_s}.$$

In an interior solution, the right-hand-side of the inequality is an increasing function of W indicating that if the entrepreneur invests more of her savings, then renegotiation becomes less costly, making failure more appealing to the entrepreneur.

Proposition 8 summarizes the impact of cash on hand on the equilibrium outcomes.

Proposition 8 *In an interior solution, comparative statics on cash on hand, W, are characterized by the following:*

1. *It is always the case that $\frac{\partial \pi_b^*}{\partial W} \geq 0$, but the derivative is reduced as the likelihood of SDI entry increases.*

2. *Whenever SDI entry is highly likely such that $q \in \left[\frac{2}{3}, 1\right)$, then $\frac{\partial NPV^*}{\partial W} < 0$.*

3. If $q \in \left(0, \frac{2}{3}\right)$ such that SDI entry is less likely, then $\frac{\partial NPV^*}{\partial W} \geq 0$.

The first point shows that external financing is costly and an entrepreneur that has enough savings to cover the initial outlay would not raise any debt if low leverage is the only feasible source of financing. However, the possibility of SDI entry reduces the expected cost of external financing as well as the benefit to financing the project internally. The second and third points are perhaps the most interesting results. When there is no SDI involvement or when there is under-provision of distress lenders, having more cash on hand reduces the restructuring costs and increases the NPV. Therefore, in a market where there is inefficiently low SDI entry, it is both optimal and efficient to have financially less constrained entrepreneurs. On the contrary, if SDI involvement is highly likely, then increasing cash on hand discourages SDI entry and reduces the NPV. Consequently, in a market where SDI entry is not costly, financially constrained entrepreneurs can increase welfare even if they personally benefit from having more cash on hand.

6 Conclusion

Over the last decades distress investors have become more visible in credit markets but yet research about them is scarce. In this paper, we aim to partly fill this gap by studying the impact of distress investors on the traditional lending relationship between banks and firms. More specifically, we extend the canonical moral hazard model to incorporate the possibility of specialized investors appearing in case of distress.

We begin our analysis with a severe distress case where the firm becomes insolvent if the project fails and the control of the firm is transferred from the entrepreneur to the bank. We show that SDIs might worsen borrower moral hazard and reduce the availability of upstream financing. Furthermore, some SDIs can reduce project NPV. We also study a less severe distress case where the entrepreneur is still the owner of the firm even if the project fails but she renegotiates with the bank to extend the duration of the loan. Since the entrepreneur controls the firm, negative ex ante effects are absent.

Finally, we show that SDI presence can shape the entrepreneur's choice of project characteristics and capital structure.

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Internet Appendix for

Do Specialized Distress Investors Undermine Firms Ex Ante? A Simple Model of Private Bailouts

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This appendix complements the paper by providing the proofs of the formal results stated in the paper.

Proof of Lemma 1. Suppose that the initial lender has full control over the acceptance of SDI offers. Then, it must be the case that $u_l^e \in [u_l^n, V^e - u_b^e)$. If the SDI pays the minimum possible amount to the initial lender, then the SDI payoff is given by:

$$\begin{aligned} u_s^e &= V^e - u_l^n - u_b^e \\ &= V^e - V^n - (u_b^e - u_b^n) \\ &= v + d. \end{aligned}$$

For a strictly positive probability of SDI intervention, it must be then the case that $v+d > 0$. Therefore, for $v < 0 \wedge d < 0$, SDI intervention is not possible.
Alternatively,

$$\begin{aligned} u_s^e &= V^e - u_l^e - u_b^e \\ &= V^n + v - u_l^e - u_b^e \\ &= V^n + v - u_l^e - (u_b^n - d) \\ &= u_l^n - u_l^e + v + d. \end{aligned}$$

Since the initial lender only allows SDIs that increase its payoff, in equilibrium $u_l^n - u_l^e < 0$ holds. Therefore, $u_s^e > 0$ is true if and only if $v + d > 0$. This concludes the proof.■

Proof of Lemma 2. The entrepreneur's IC constraint is given by

$$p(R_b + V) + (1-p)[qu_b^e + (1-q)u_b^n] \geq (p - \Delta)(R_b + V) + (1 - p + \Delta)[qu_b^e + (1-q)u_b^n] + B,$$

which implies the following lower bound for the entrepreneur's first stage payoff

$$R_b \geq \frac{B}{\Delta} - V + qu_b^e + (1-q)u_b^n \equiv \underline{R_b}.$$

The lender's participation constraint is

$$pR_l + (1-p)[qu_l^e + (1-q)u_l^n] \geq I - W.$$

Substituting R_l with $R - \underline{R_b}$ yields

$$\mathcal{P} = p\left[R - \frac{B}{\Delta} + V - qu_b^e - (1-q)u_b^n\right] + (1-p)[qu_l^e + (1-q)u_l^n] \geq I - W.$$

This concludes the proof.■

Proof of Lemma 3. In an interior solution, where $q \in (0,1)$, the minimum amount of capital provided by the entrepreneur for financing to be feasible is given by

$$\begin{aligned}\underline{W}^* = I - \Bigg(& p\left\{R - \frac{B}{\Delta} + V - [qu_b^e + (1-q)u_b^n]\right\} \\ & + (1-p)\left[q(V^e - u_b^e - u_s^e) + (1-q)(V^n - u_b^n)\right]\Bigg),\end{aligned}$$

which can be re-written as

$$\begin{aligned}\underline{W}^* &= I - p\left(R - \frac{B}{\Delta} + V\right) + qu_b^e + (1-q)u_b^n + (1-p)[qu_s^e - qV^e - (1-q)V^n] \\ &= I - p\left(R - \frac{B}{\Delta} + V - u_b^n\right) - (1-p)(V^n - u_b^n) + q(u_b^e - u_b^n) + (1-p)q(u_s^e - v) \\ &= I - p\left(R - \frac{B}{\Delta} + V - u_b^n\right) - (1-p)(V^n - u_b^n) + \frac{u_s^e}{\overline{C}_s}\left[(1-p)(u_s^e - v) - d\right].\end{aligned}$$

Finally, it is clear that the contribution of an SDI is given by

$$\underline{W}^* - \lim_{\overline{C}_s \to \infty} \underline{W}^* = \frac{u_s^e}{\overline{C}_s} \left[(1-p)u_s^e - (1-p)(v+d) - pd\right]. \tag{A.1}$$

Suppose now that $v > 0 \wedge d > 0$. Since we are assuming that $u_l^e \in [u_l^n, V^e - u_b^e)$, u_s^e must be bounded between 0 and $V^e - u_b^e - u_l^n$, as the initial lender has full control over the acceptance of offers and thus demands at least u_l^n. Therefore, $u_s^e \in (0, V^e - u_b^e - u_l^n]$ and

$$\begin{aligned} u_s^e \leq V^e - u_b^e - u_l^n = V^n + v - u_b^e - u_l^n = v + d \\ \Leftrightarrow (1-p)u_s^e \leq (1-p)(v+d) \leq (1-p)(v+d) + pd. \end{aligned}$$

Consequently, for $v > 0 \wedge d > 0$ and if the initial lender has full control over the acceptance of the SDI offer, SDI intervention unambiguously ameliorates credit rationing. This concludes the proof.■

Proof of Lemma 4. Firstly, consider the formula for the overall NPV of the project:

$$\begin{aligned} NPV^* &= p(R+V) + (1-p)\left[q(V^e - \mathbb{E}\left[C_s | C_s \leq u_s^e\right]) + (1-q)V^n\right] - I \\ &= p(R+V) + (1-p)V^n + (1-p)q\left(v - \mathbb{E}\left[C_s | C_s \leq u_s^e\right]\right) - I. \end{aligned} \tag{A.2}$$

Thus, in an interior solution the SDI contribution is given by

$$\frac{(1-p)u_s^e \left(v - \mathbb{E}\left[C_s | C_s \leq u_s^e\right]\right)}{\overline{C}_s},$$

which is necessarily negative if $v < 0$. However, the overall impact of SDI participation may nonetheless be ambiguous, as long as the value-destroying distress investor relaxes credit rationing, i.e.,

$$\frac{u_s^e\left[(1-p)(u_s^e-v)-d\right]}{\overline{C}_s}<0 \;\Leftrightarrow d>(1-p)(u_s^e-v).$$

This concludes the proof.■

Proof of Proposition 1. One can re-write the definition of pledgeable income given in equation (7) as

$$\mathcal{P}=p\left(R-\frac{B}{\Delta}+V-u_b^n\right)+(1-p)(V^n-u_b^n)+q\left[pd+(1-p)(v+d-u_s^e)\right].$$

Furthermore, substituting for q and u_s^e yields that pledgeable income in an interior solution where $q\in(0,1)$ is given by the following relationship

$$\mathcal{P}=p\left(R-\frac{B}{\Delta}+V-u_b^n\right)+(1-p)(V^n-u_b^n)+\frac{v+d}{2\overline{C}_s}\left[pd+(1-p)\frac{v+d}{2}\right]. \tag{A.3}$$

Finally, the impact of SDI intervention is positive if and only if

$$\begin{gathered}pd+(1-p)\frac{v+d}{2}>0\\ d>-\frac{1-p}{1+p}v.\end{gathered}$$

This concludes the proof.■

Proof of Proposition 2. In the general version of the model, the expected profit of the entrepreneur is

$$\pi_b^*=p(R_b^*+V)+(1-p)\left[qu_b^e+(1-q)u_b^n\right]-W.$$

Since in equilibrium the initial lender breaks even it must be the case that

$$\begin{aligned} pR_l^* + (1-p)\left[qu_l^e + (1-q)u_l^n\right] - (I-W) = 0 \Leftrightarrow \\ R_l^* = \frac{I - W - (1-p)\left[qu_l^e + (1-q)u_l^n\right]}{p}. \end{aligned}$$

Considering that $R_b^* = R - R_l^*$, we may re-write the ex ante profit of the entrepreneur in an interior solution where $q \in (0,1)$ as

$$\begin{aligned} \pi_b^* &= p(R+V) - I + (1-p)\left[qu_l^e + (1-q)u_l^n\right] + (1-p)\left[qu_b^e + (1-q)u_b^n\right] \\ &= p(R+V) + (1-p)V^n - I + (1-p)\frac{(v+d)(v-d)}{4\overline{C}_s}. \end{aligned} \tag{A.4}$$

Given this result, it is clear that the SDI's contribution to the profit of the entrepreneur is positive as long as

$$v > d.$$

The NPV of the project given in equation (A.2) can be re-written as

$$\begin{aligned} NPV^* &= p(R+V) + (1-p)V^n - I + (1-p)\frac{u_s^e}{\overline{C}_s}\left(v - \frac{v+d}{4}\right) \\ &= p(R+V) + (1-p)V^n - I + (1-p)\frac{(v+d)(3v-d)}{8\overline{C}_s}. \end{aligned}$$

Clearly the last term is positive as long as $3v > d$. This concludes the proof.■

Proof of Proposition 3. Incorporating the assumptions $V^n = \lambda V$, $v = \gamma V$, and $d = \delta u_b^n$ in equation (17), the partial derivative of π_b^* with respect to V can be written, for an interior solution with $q \in (0,1)$, as

$$\left.\frac{\partial \pi_b^*}{\partial V}\right|_{q\in(0,1)} = p + (1-p)\lambda + \frac{(1-p)\gamma^2 V}{2\overline{C}_s}. \tag{A.5}$$

It follows that this partial derivative is always positive, independently from the sign of γ. In fact, even in both corner solutions, for $q \in \{0,1\}$ this must be the case. Notice that, for $q = 0$,

$$\left.\frac{\partial \pi_b^*}{\partial V}\right|_{q=0} = p + (1-p)\lambda \geq 0, \tag{A.6}$$

and for $q = 1$,

$$\left.\frac{\partial \pi_b^*}{\partial V}\right|_{q=1} = p + (1-p)\lambda + (1-p)\frac{\gamma}{2} \geq 0, \tag{A.7}$$

which is necessarily positive, since by assumption $\lambda > -\gamma$.

Moreover, for $q \in (0,1)$ the partial derivative of the project's NPV with respect to the baseline continuation value is given by

$$\left.\frac{\partial NPV^*}{\partial V}\right|_{q\in(0,1)} = p + (1-p)\lambda + \frac{(1-p)\gamma\left[3\gamma V + \delta u_b^n\right]}{4\overline{C}_s}. \tag{A.8}$$

This partial derivative must necessarily be positive as well. Suppose first that $\gamma > 0$. Then all terms are positive, since $2\gamma V + (\gamma V + \delta u_b^n) > 0$.[18]

Consider now that $\gamma < 0$: for the last term to be negative it must be the case that

$$\frac{(\gamma V + \delta u_b^n) - (-2\gamma V)}{4\overline{C}_s} > 0.$$

However, since the probability of SDI entry is bounded, this term is also majorized at 0.5.

[18]Recall that in an interior solution, with $q \in (0,1)$, $v + d = \gamma V + \delta u_b^n > 0$.

Given that by assumption $\lambda > -\gamma$, this partial derivative is necessarily positive.

Consider now this partial derivative in corner solutions. If $q = 0$, then

$$\left.\frac{\partial NPV^*}{\partial V}\right|_{q=0} = p + (1-p)\lambda \geq 0, \tag{A.9}$$

and if $q = 1$, by the same reasoning applied above,

$$\left.\frac{\partial NPV^*}{\partial V}\right|_{q=1} = p + (1-p)\lambda + (1-p)\frac{3}{4}\gamma \geq 0. \tag{A.10}$$

Now, consider the partial derivative of the project's pledgeable income with respect to V. Starting with a corner solution, if $q = 0$, this partial derivative is

$$\left.\frac{\partial \mathcal{P}^*}{\partial V}\right|_{q=0} = p + (1-p)\lambda \geq 0. \tag{A.11}$$

On the other hand, if $q = 1$:

$$\left.\frac{\partial \mathcal{P}^*}{\partial V}\right|_{q=1} = p + (1-p)\lambda + (1-p)\frac{\gamma}{2} \geq 0, \tag{A.12}$$

since again by assumption $\lambda > -\gamma$.

Finally, in an interior solution, this partial derivative is given by

$$\left.\frac{\partial \mathcal{P}^*}{\partial V}\right|_{q\in(0,1)} = p + (1-p)\lambda + \frac{\gamma\left[(1-p)\gamma V + \delta u_b^n\right]}{2\overline{C}_s}. \tag{A.13}$$

It is clear that for this partial derivative to be negative, then $\gamma < 0 \vee \delta < 0$. Consider the case where $\gamma > 0 \wedge \delta < 0$. For this partial derivative to be negative δ must be sufficiently negative and a necessary condition is that

$$\delta < -\frac{\gamma V(1-p)}{u_b^n} \Rightarrow \delta < -\frac{\gamma V(1-p)}{u_b^n(1+p)}. \tag{A.14}$$

This concludes the proof.■

Proof of Proposition 4. In an interior solution, where $q \in (0,1)$, the partial derivative of pledgeable income with respect to the entrepreneur's second stage payoff without SDI entry, u_b^n, is

$$\left.\frac{\partial \mathcal{P}^*}{\partial u_b^n}\right|_{q\in(0,1)} = -1 + \frac{(\gamma V + \delta u_b^n)}{2\overline{C}_s}\delta + \frac{p u_b^n}{2\overline{C}_s}\delta^2. \tag{A.15}$$

Suppose that $\delta > 0$. Since it must be the case that

$$\frac{\gamma V + \delta u_b^n}{2\overline{C}_s}\delta < \delta,$$

then for this partial derivative to be positive, a necessary condition when $\delta > 0$ is

$$\frac{p u_b^n}{2\overline{C}_s}\delta^2 > 1 - \delta.$$

Finally, this inequality implies that

$$\delta > \frac{-1 + \sqrt{1 + 2\frac{p u_b^n}{\overline{C}_s}}}{\frac{p u_b^n}{\overline{C}_s}}. \tag{A.16}$$

This concludes the proof.■

Proof of Lemma 5. Substituting for $V^e = V^n + v$ and $V^n = u_b^n + R_l + C_l$ in equation (27) yields

$$m^* = \frac{2u_b^n + v + C_l}{2(V^e - R_l)} = \frac{2u_b^n + v + C_l}{2(V^n + v - R_l)} = \frac{2u_b^n + v + C_l}{2(u_b^n + v + C_l)}.$$

It is straightforward to see that the numerator is more than half of the denominator such that

$$2u_b^n + v + C_l > u_b^n + v + C_l \Rightarrow \frac{2u_b^n + v + C_l}{2(u_b^n + v + C_l)} > \frac{u_b^n + v + C_l}{2(u_b^n + v + C_l)} = \frac{1}{2}.$$

This concludes the proof.■

Proof of Proposition 5. First, consider the case where there is no SDI entry. The entrepreneur's IC constraint is given by

$$\Delta(R_b + V) \geq \Delta(V^n - R_l - C_l) + B.$$

$R - R_l = R_b$ still holds and substituting it in the IC constraint yields

$$R + V \geq V^n - C_l + \frac{B}{\Delta}.$$

The borrower's IC constraint is independent of R_b. Now consider the IC constraint when SDI entry is possible:

$$\Delta(R_b + V) \geq \Delta[qu_b^e + (1-q)u_b^n] + B.$$

Substituting for $R_l = R - R_b$, $u_b^n = V^n - R_l - C_l$ and $u_b^e = u_b^n + \frac{v+C_l}{2}$ yield

$$R + V \geq \lambda V - C_l + \frac{B}{\Delta} + q\frac{v + C_l}{2}.$$

Considering that $R_b^* = R - R_l^*$, we may re-write the ex ante profit of the entrepreneur in an interior solution, where $q \in (0,1)$, as

$$\begin{aligned}
\pi_b^* &= p(R_b + V) - W + (1-p)\left[q u_b^e + (1-q)u_b^n\right] \\
&= p(R - I + W + V) - W + (1-p)\left[V^n - I + W - C_l + q\frac{v + C_l}{2}\right] \\
&= p(R+V) + (1-p)(V^n - C_l) - I + (1-p)q\left(\frac{v + C_l}{2}\right). \qquad \text{(A.17)}
\end{aligned}$$

Finally, the NPV of the project is given by

$$\begin{aligned}
NPV^* &= p(R+V) - I + (1-p)q(V^e - \mathbb{E}\left[C_s | C_s \leq u_s^e\right]) + (1-p)(1-q)(V^n - C_l) \\
&= p(R+V) + (1-p)(V^n - C_l) - I + (1-p)q(v + C_l - \mathbb{E}\left[C_s | C_s \leq u_s^e\right]) \\
&= p(R+V) + (1-p)(\lambda V - C_l) - I + 3(1-p)q\left(\frac{v + C_l}{4}\right). \qquad \text{(A.18)}
\end{aligned}$$

This concludes the proof.■

Proof of Proposition 6. The partial derivative of equation (30) with respective to C_l is

$$\frac{\partial \pi_b^*}{\partial C_l} = -(1-p) + (1-p)\frac{v + C_l}{2\overline{C}_s} = -(1-p) + (1-p)q.$$

In an interior equilibrium, where $q \in (0,1)$, the derivative is always negative implying that the borrower's ex ante payoff is reduced by the cost of renegotiating the loan.

Similarly, taking the derivative of equation (31) with respect to C_l yields

$$\frac{\partial NPV^*}{\partial C_l} = -(1-p) + 3(1-p)\frac{v + C_l}{4\overline{C}_s} = -(1-p) + \frac{3}{2}(1-p)q.$$

Furthermore, for the values of the probability of SDI entry that satisfy $q \in \left(\frac{2}{3}, 1\right)$, $\frac{\partial NPV^*}{\partial C_l} > 0$

holds. This concludes the proof.■

Proof of Proposition 7. The entrepreneur's expected payoff using more cash on hand is

$$\pi_b^l = p(R+V) + (1-p)(\lambda V - C_l) - I + (1-p)\frac{(v+C_l)^2}{4\overline{C}_s}. \tag{A.19}$$

Recall the borrower's ex ante payoff in a highly levered capital structure given in equation (17):

$$\pi_b^h = p\,(R+V) + (1-p)\lambda V - I + (1-p)\frac{(v+d)(v-d)}{4\overline{C}_s}. \tag{A.20}$$

It follows from equation (A.19) and (A.20) that

$$\pi_b^h > \pi_b^l \;\Rightarrow\; 4\overline{C}_s > d^2/C_l + C_l + 2v.$$

Similarly, the NPV of the project with low leverage is

$$NPV^l = p(R+V) + (1-p)(\lambda V - C_l) - I + 3(1-p)\frac{(v+C_l)^2}{8\overline{C}_s}, \tag{A.21}$$

and high leverage it is

$$NPV^h = p(R+V) + (1-p)\lambda V - I + (1-p)\frac{(v+d)(3v-d)}{8\overline{C}_s}. \tag{A.22}$$

Equation (A.21) and (A.22) together imply that

$$NPV^h > NPV^l \;\Rightarrow\; 8\overline{C}_s > d^2/C_l + 3C_l + 6v - 2dv/C_l.$$

This concludes the proof.■

Proof of Proposition 8. Substituting for $C_l = \mu + \beta(I - W)$ in equation (30) and taking the partial derivative with respective to W yields

$$\frac{\partial \pi_b^*}{\partial W} = (1-p) - \beta(1-p)\frac{v + \mu + \beta(I-W)}{2\overline{C}_s} = (1-p) - (1-p)q \geq 0.$$

Note that the derivative decreases as the likelihood of SDI entry increases.

Similarly, substituting for $C_l = \mu + \beta(I - W)$ in equation (31) and taking the partial derivative with respective to W yields

$$\frac{\partial NPV^*}{\partial W} = (1-p) - 3\beta(1-p)\frac{v + \mu + \beta(I-W)}{4\overline{C}_s} = (1-p) - (1-p)\frac{3}{2}q.$$

Furthermore, for the values of the probability of SDI entry that satisfy $q \in \left(\frac{2}{3}, 1\right]$, $\frac{\partial NPV^*}{\partial W} < 0$ holds. This concludes the proof.■