CCE2501: Modelling and Simulation
Assignment Description, October 2016
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Please submit the answers (including code and working used to achieved them) to the assignment in the form of **one pdf document** by not later than **10th December 2016** on VLE.

The assignment is worth 30% of the unit's final mark.

Question 1

It is desired to generate random numbers under the log-normal distribution described by

$$y(t) = \frac{1}{t\sigma\sqrt{2\pi}} e^{-\frac{(\ln t-\mu)^2}{2\sigma^2}}$$

defined over the interval $\{0 \leq t \leq 5\}$ with $\mu = 0$ and $\sigma = 0.75$.

a) Develop an algorithm in Python based on Von-Neumann's accept-reject method to generate random numbers under *y(t)*.

b) Generate 10000 points in order to plot the proper probability density function with blue scatter points, and superimpose a red curve of the log-normal distribution produced using the standard Python library.

Question 2

In this question, you will be simulating the traffic-lights controlled by the T-Junction shown in Figure 1. The two roads leading to the T-junction are one-way and the vehicle flow is in the direction of the arrows.

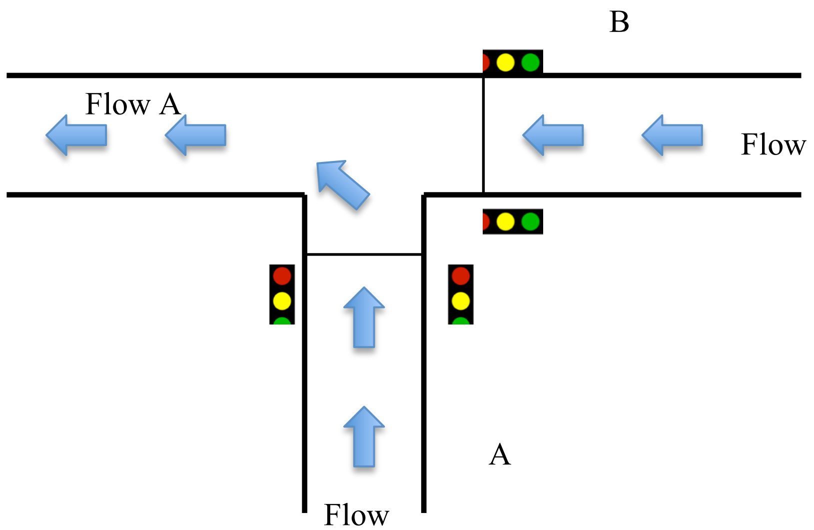

Figure 1 – T-Junction

Assume that the inter-arrival time between cars follows an exponential distribution with $\lambda = 2$, with a lower bound at t = 1 second and an upper bound at t = 10 seconds. Assume that the duty cycles for both traffic lights are the same. At time t0, road A's traffic light is green, while at time t0 + 30 seconds, road B's traffic light is green. For the duration while road A's light is green, cars in road A can pass freely with a fixed service time of 2 seconds, and vice versa. When road A's light turns to red, any car arriving at the traffic light will be queued until the light turns to green again.

Following a study, it has been noticed that the volume of cars entering the junction from the two different roads varies depending on the time in the morning when people are going to work, and the evening when people are leaving work. In particular, the number of cars entering roads A & B in the morning (6:30 am to 10:30 am) and evening (4:00 pm to 8:00 pm) follow the Gaussian distributions with the parameters:

- Morning, road A: μ = 8:30 am and σ = 1 hour.
- Morning, road B: μ = 9:00 am and σ = 0.95 hour.
- Evening, road A: μ = 6:00 pm and σ = 0.95 hour.
- Evening, road B: μ = 5:30 pm and σ = 1 hour.

The exponentially distributed inter-arrival time therefore has to be modulated (divided) by the Gaussian probability density function depending on the time of day.

a) Develop a conceptual model made up of queuing systems for the traffic-lights controlled T-Junction. Explain all your reasoning.

b) Write GSL code including the initialization and event routines to implement your conceptual model.

c) Implement your model in Python and run two separate simulations for morning and evening, plotting the queue lengths as a function of time in roads A and B in both cases. Calculate the traffic intensities for both roads and both times.

d) What difference do you notice between roads A and B in the morning and evening, and what action can the transport authorities take to reduce the overall traffic?

Question 3

Using Monte Carlo simulation with 100,000 iterations, calculate area of under the curve defined by the polynomial:

$$y(x) \quad = \quad 5x^4 - 8.7x^3 + 33x^2 + 21x + 10.8$$

over the interval: $\{0 \leq x \leq 5\}$.

(a) Compare your answer to the area achieved analytically.

(b) Run the simulation for increasing number of iterations in steps of 10,000 up to 1,000,000 and plot the area of the curve achieved through MC simulation versus number of iterations. Superimpose a constant line (in red colour) representing the area achieved analytically. What is the minimum number of iterations required to achieve a value for the area which beyond which it remains within 1% of the analytical value?