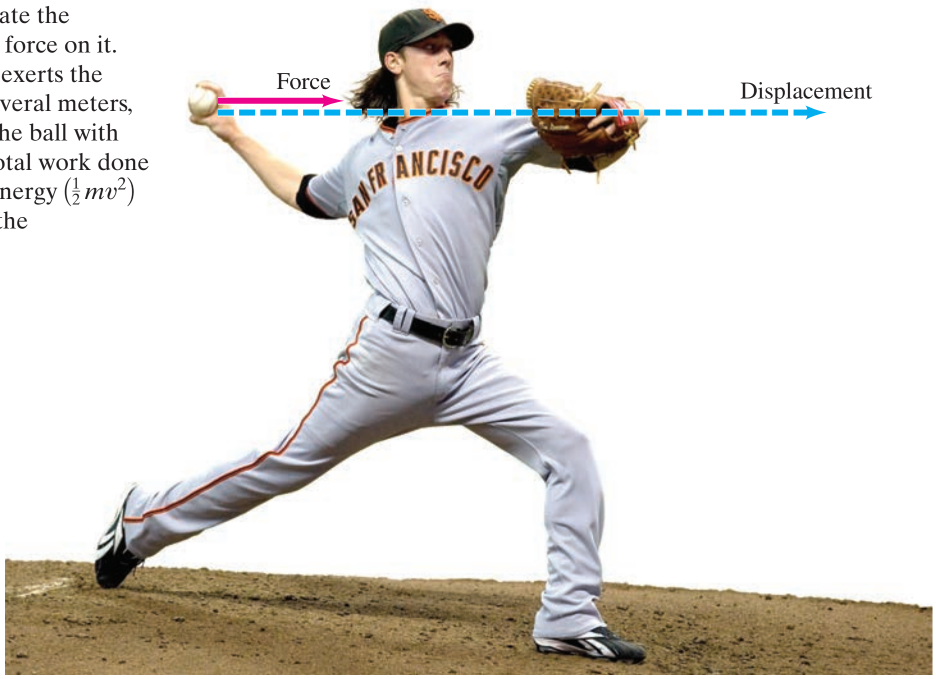
This baseball pitcher is about to accelerate the baseball to a high velocity by exerting a force on it. He will be doing work on the ball as he exerts the force over a displacement of perhaps several meters, from behind his head until he releases the ball with arm outstretched in front of him. The total work done on the ball will be equal to the kinetic energy $\left(\frac{1}{2}mv^2\right)$ acquired by the ball, a result known as the work-energy principle.

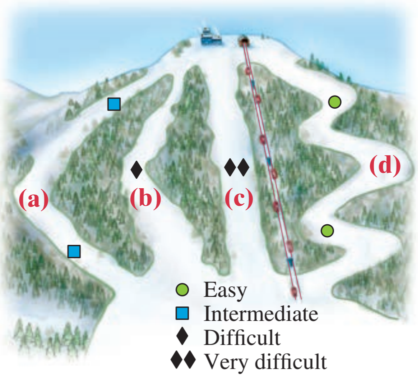


CHAPTER 6

Work and Energy

CHAPTER-OPENING QUESTION—Guess now!

A skier starts at the top of a hill. On which run does her gravitational potential energy change the most: **(a)**, **(b)**, **(c)**, or **(d)**; or are they **(e)** all the same? On which run would her speed at the bottom be the fastest if the runs are icy and we assume no friction or air resistance? Recognizing that there is always some friction, answer the above two questions again. List your four answers now.



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Until now we have been studying the translational motion of an object in terms of Newton's three laws of motion. In that analysis, *force* has played a central role as the quantity determining the motion. In this Chapter and the next, we discuss an alternative analysis of the translational motion of objects in terms of the quantities *energy* and *momentum*. The significance of energy and momentum is that they are *conserved*. That is, in quite general circumstances they remain constant. That conserved quantities exist gives us not only a deeper insight into the nature of the world, but also gives us another way to approach solving practical problems.

The conservation laws of energy and momentum are especially valuable in dealing with systems of many objects, in which a detailed consideration of the forces involved would be difficult or impossible. These laws apply to a wide range of phenomena. They even apply in the atomic and subatomic worlds, where Newton's laws are not sufficient.

This Chapter is devoted to the very important concept of *energy* and the closely related concept of *work*. These two quantities are scalars and so have no direction associated with them, which often makes them easier to work with than vector quantities such as acceleration and force.

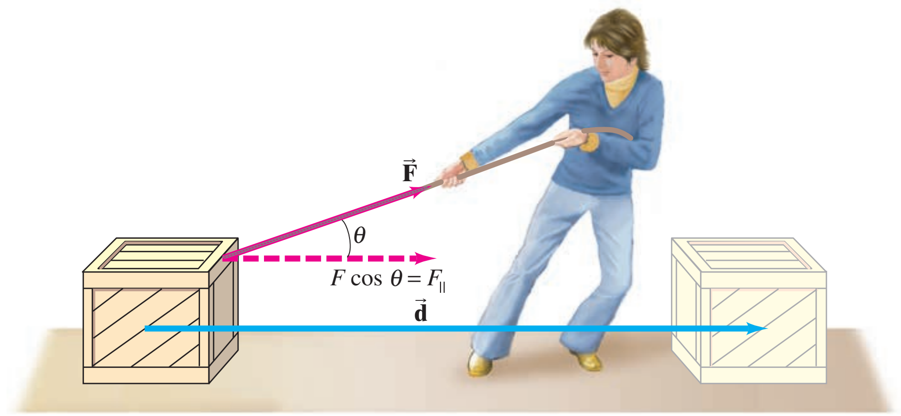


FIGURE 6–1 A person pulling a crate along the floor. The work done by the force $\vec{\mathbf{F}}$ is $W = Fd\cos\theta$, where $\vec{\mathbf{d}}$ is the displacement.

6–1 Work Done by a Constant Force

The word *work* has a variety of meanings in everyday language. But in physics, work is given a very specific meaning to describe what is accomplished when a force acts on an object, and the object moves through a distance. We consider only translational motion for now and, unless otherwise explained, objects are assumed to be rigid with no complicating internal motion, and can be treated like particles. Then the **work** done on an object by a constant force (constant in both magnitude and direction) is defined to be *the product of the magnitude of the displacement times the component of the force parallel to the displacement*. In equation form, we can write

$$W = F_{||}d,$$

where $F_{||}$ is the component of the constant force $\vec{\mathbf{F}}$ parallel to the displacement $\vec{\mathbf{d}}$. We can also write

$$W = Fd\cos\theta, \tag{6–1}$$

where F is the magnitude of the constant force, d is the magnitude of the displacement of the object, and θ is the angle between the directions of the force and the displacement (Fig. 6–1). The $\cos\theta$ factor appears in Eq. 6–1 because $F\cos\theta$ $(= F_{||})$ is the component of $\vec{\mathbf{F}}$ that is parallel to $\vec{\mathbf{d}}$. Work is a scalar quantity—it has no direction, but only magnitude, which can be positive or negative.

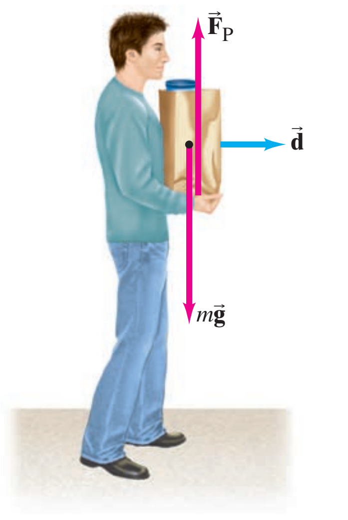
Let us consider the case in which the motion and the force are in the same direction, so $\theta = 0$ and $\cos\theta = 1$; in this case, $W = Fd$. For example, if you push a loaded grocery cart a distance of 50 m by exerting a horizontal force of 30 N on the cart, you do $30\,\mathrm{N} \times 50\,\mathrm{m} = 1500\,\mathrm{N\cdot m}$ of work on the cart.

As this example shows, in SI units work is measured in newton-meters $(\mathrm{N\cdot m})$. A special name is given to this unit, the **joule** (J): $1\,\mathrm{J} = 1\,\mathrm{N\cdot m}$.

[In the cgs system, the unit of work is called the *erg* and is defined as $1\,\mathrm{erg} = 1\,\mathrm{dyne\cdot cm}$. In British units, work is measured in foot-pounds. Their equivalence is $1\,\mathrm{J} = 10^7\,\mathrm{erg} = 0.7376\,\mathrm{ft\cdot lb}$.]

A force can be exerted on an object and yet do no work. If you hold a heavy bag of groceries in your hands at rest, you do no work on it. You do exert a force on the bag, but the displacement of the bag is zero, so the work done by you on the bag is $W = 0$. You need both a force and a displacement to do work. You also do no work on the bag of groceries if you carry it as you walk horizontally across the floor at constant velocity, as shown in Fig. 6–2. No horizontal force is required to move the bag at a constant velocity. The person shown in Fig. 6–2 exerts an upward force $\vec{\mathbf{F}}_{\mathrm{P}}$ on the bag equal to its weight. But this upward force is perpendicular to the horizontal displacement of the bag and thus is doing no work. This conclusion comes from our definition of work, Eq. 6–1: $W = 0$, because $\theta = 90°$ and $\cos 90° = 0$. Thus, when a particular force is perpendicular to the displacement, no work is done by that force. When you start or stop walking, there is a horizontal acceleration and you do briefly exert a horizontal force, and thus do work on the bag.

FIGURE 6–2 The person does no work on the bag of groceries because $\vec{\mathbf{F}}_{\mathrm{P}}$ is perpendicular to the displacement $\vec{\mathbf{d}}$.



CAUTION
Force without work

CAUTION

State that work is done on or by an object

When we deal with work, as with force, it is necessary to specify whether you are talking about work done *by* a specific object or done *on* a specific object. It is also important to specify whether the work done is due to one particular force (and which one), or the total (net) work done by the *net force* on the object.

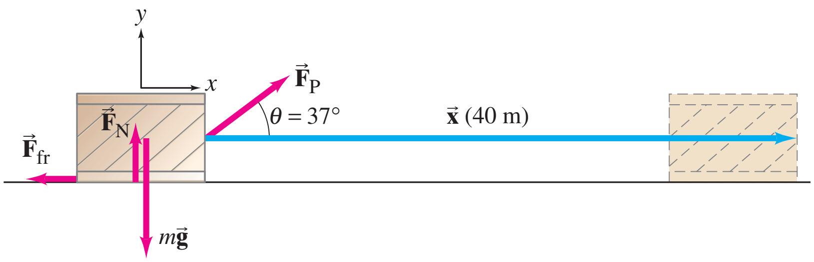


FIGURE 6–3 Example 6–1. A 50-kg crate is pulled along a floor.

EXAMPLE 6–1 Work done on a crate. A person pulls a 50-kg crate 40 m along a horizontal floor by a constant force $F_P = 100\,\text{N}$, which acts at a 37° angle as shown in Fig. 6–3. The floor is rough and exerts a friction force $\vec{\mathbf{F}}_{fr} = 50\,\text{N}$. Determine (*a*) the work done by each force acting on the crate, and (*b*) the net work done on the crate.

APPROACH We choose our coordinate system so that the vector that represents the 40-m displacement is $\vec{\mathbf{x}}$ (that is, along the x axis). Four forces act on the crate, as shown in the free-body diagram in Fig. 6–3: the force exerted by the person $\vec{\mathbf{F}}_P$; the friction force $\vec{\mathbf{F}}_{fr}$; the gravitational force exerted by the Earth, $\vec{\mathbf{F}}_G = m\vec{\mathbf{g}}$; and the normal force $\vec{\mathbf{F}}_N$ exerted upward by the floor. The net force on the crate is the vector sum of these four forces.

SOLUTION (*a*) The work done by the gravitational force $(\vec{\mathbf{F}}_G)$ and by the normal force $(\vec{\mathbf{F}}_N)$ is zero, because they are perpendicular to the displacement $\vec{\mathbf{x}}$ ($\theta = 90°$ in Eq. 6–1):

$$W_G = mgx\cos 90° = 0$$
$$W_N = F_N x\cos 90° = 0.$$

The work done by $\vec{\mathbf{F}}_P$ is

$$W_P = F_P x\cos\theta = (100\,\text{N})(40\,\text{m})\cos 37° = 3200\,\text{J}.$$

The work done by the friction force is

$$W_{fr} = F_{fr}x\cos 180° = (50\,\text{N})(40\,\text{m})(-1) = -2000\,\text{J}.$$

The angle between the displacement $\vec{\mathbf{x}}$ and $\vec{\mathbf{F}}_{fr}$ is 180° because they point in opposite directions. Since the force of friction is opposing the motion (and $\cos 180° = -1$), the work done by friction on the crate is *negative*.

(*b*) The net work can be calculated in two equivalent ways.

(1) The net work done on an object is the algebraic sum of the work done by each force, since work is a scalar:

$$\begin{aligned} W_{net} &= W_G + W_N + W_P + W_{fr} \\ &= 0 + 0 + 3200\,\text{J} - 2000\,\text{J} = 1200\,\text{J}. \end{aligned}$$

(2) The net work can also be calculated by first determining the net force on the object and then taking the component of this net force along the displacement: $(F_{net})_x = F_P\cos\theta - F_{fr}$. Then the net work is

$$\begin{aligned} W_{net} = (F_{net})_x x &= (F_P\cos\theta - F_{fr})x \\ &= (100\,\text{N}\cos 37° - 50\,\text{N})(40\,\text{m}) = 1200\,\text{J}. \end{aligned}$$

In the vertical (y) direction, there is no displacement and no work done.

CAUTION

Negative work

In Example 6–1 we saw that friction did negative work. In general, the work done by a force is negative whenever the force (or the component of the force, $F_\parallel$) acts in the direction opposite to the direction of motion.

EXERCISE A A box is dragged a distance d across a floor by a force $\vec{\mathbf{F}}_P$ which makes an angle θ with the horizontal as in Fig. 6–1 or 6–3. If the magnitude of $\vec{\mathbf{F}}_P$ is held constant but the angle θ is increased, the work done by $\vec{\mathbf{F}}_P$ (*a*) remains the same; (*b*) increases; (*c*) decreases; (*d*) first increases, then decreases.

PROBLEM SOLVING

Work

1. **Draw a free-body diagram** showing all the forces acting on the object you choose to study.
2. **Choose** an xy **coordinate system**. If the object is in motion, it may be convenient to choose one of the coordinate directions as the direction of one of the forces, or as the direction of motion. [Thus, for an object on an incline, you might choose one coordinate axis to be parallel to the incline.]
3. **Apply Newton's laws** to determine unknown forces.
4. Find the **work done *by* a specific force** *on* the object by using $W = Fd\cos\theta$ for a constant force. The work done is negative when a force opposes the displacement.
5. To find the **net work** done on the object, either (a) find the work done by each force and add the results algebraically; or (b) find the net force on the object, F_{net}, and then use it to find the net work done, which for constant net force is:

$$W_{\text{net}} = F_{\text{net}}\,d\cos\theta.$$

EXAMPLE 6–2 **Work on a backpack.** (a) Determine the work a hiker must do on a 15.0-kg backpack to carry it up a hill of height $h = 10.0\text{ m}$, as shown in Fig. 6–4a. Determine also (b) the work done by gravity on the backpack, and (c) the net work done on the backpack. For simplicity, assume the motion is smooth and at constant velocity (i.e., acceleration is zero).

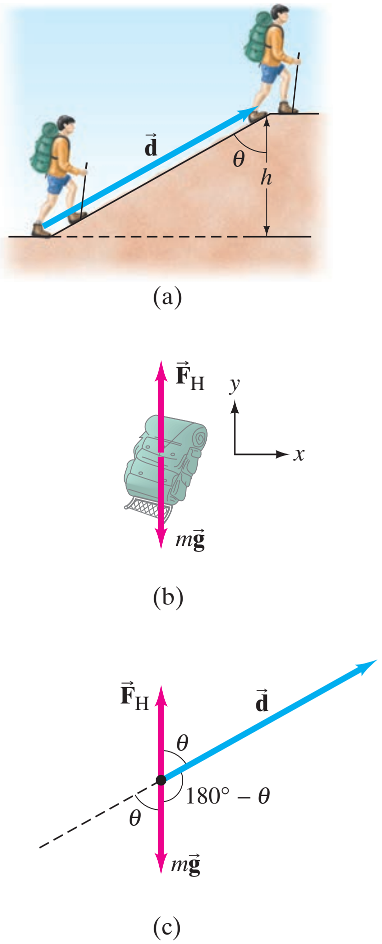


FIGURE 6–4 Example 6–2.

APPROACH We explicitly follow the steps of the Problem Solving Strategy above.

SOLUTION

1. **Draw a free-body diagram.** The forces on the backpack are shown in Fig. 6–4b: the force of gravity, $m\vec{\mathbf{g}}$, acting downward; and $\vec{\mathbf{F}}_{\text{H}}$, the force the hiker must exert upward to support the backpack. The acceleration is zero, so horizontal forces on the backpack are negligible.
2. **Choose a coordinate system.** We are interested in the vertical motion of the backpack, so we choose the y coordinate as positive vertically upward.
3. **Apply Newton's laws.** Newton's second law applied in the vertical direction to the backpack gives (with $a_y = 0$)

$$\Sigma F_y = ma_y$$
$$F_{\text{H}} - mg = 0.$$

So,

$$F_{\text{H}} = mg = (15.0\text{ kg})(9.80\text{ m/s}^2) = 147\text{ N}.$$

4. **Work done by a specific force.** (a) To calculate the work done by the hiker on the backpack, we use Eq. 6–1, where θ is shown in Fig. 6–4c,

$$W_{\text{H}} = F_{\text{H}}(d\cos\theta),$$

and we note from Fig. 6–4a that $d\cos\theta = h$. So the work done by the hiker is

$$W_{\text{H}} = F_{\text{H}}(d\cos\theta) = F_{\text{H}}h = mgh = (147\text{ N})(10.0\text{ m}) = 1470\text{ J}.$$

The work done depends only on the elevation change and not on the angle of the hill, θ. The hiker would do the same work to lift the pack vertically by height h.

(b) The work done by gravity on the backpack is (from Eq. 6–1 and Fig. 6–4c)

$$W_{\text{G}} = mg\,d\cos(180° - \theta).$$

Since $\cos(180° - \theta) = -\cos\theta$ (Appendix A–7), we have

$$\begin{aligned} W_{\text{G}} &= mg(-d\cos\theta) \\ &= -mgh \\ &= -(15.0\text{ kg})(9.80\text{ m/s}^2)(10.0\text{ m}) = -1470\text{ J}. \end{aligned}$$

NOTE The work done by gravity (which is negative here) does not depend on the angle of the incline, only on the vertical height h of the hill.

PROBLEM SOLVING

Work done by gravity depends on height of hill (not on angle)

5. **Net work done.** (c) The *net* work done on the backpack is $W_{\text{net}} = 0$, because the net force on the backpack is zero (it is assumed not to accelerate significantly). We can also get the net work done by adding the work done by each force:

$$W_{\text{net}} = W_{\text{G}} + W_{\text{H}} = -1470\text{ J} + 1470\text{ J} = 0.$$

NOTE Even though the *net* work done by all the forces on the backpack is zero, the hiker *does* do work on the backpack equal to 1470 J.

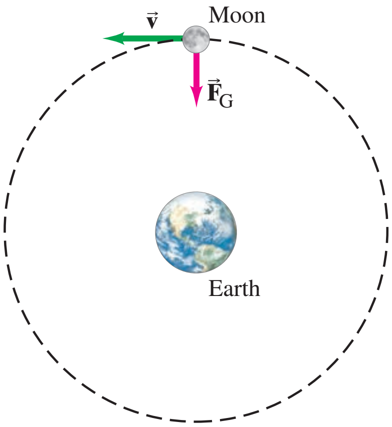
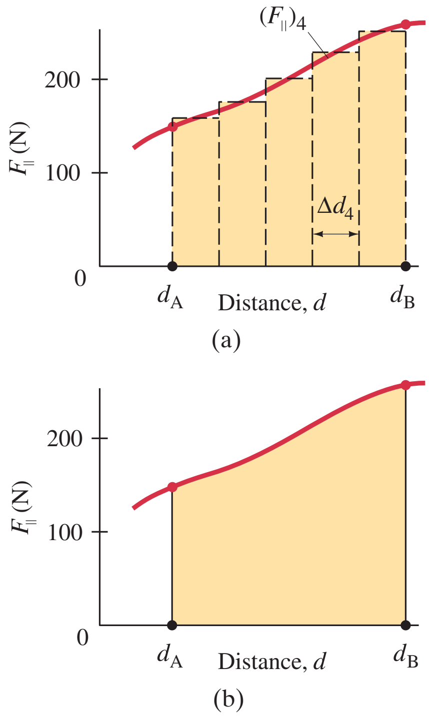


FIGURE 6–5 Example 6–3.

FIGURE 6–6 Work done by a force F is (a) approximately equal to the sum of the areas of the rectangles, (b) exactly equal to the area under the curve of $F_{\|}$ vs. d.



CONCEPTUAL EXAMPLE 6–3 **Does the Earth do work on the Moon?** The Moon revolves around the Earth in a nearly circular orbit, kept there by the gravitational force exerted by the Earth. Does gravity do (*a*) positive work, (*b*) negative work, or (*c*) no work on the Moon?

RESPONSE The gravitational force $\vec{\mathbf{F}}_G$ exerted by the Earth on the Moon (Fig. 6–5) acts toward the Earth and provides its centripetal acceleration, inward along the radius of the Moon's orbit. The Moon's displacement at any moment is tangent to the circle, in the direction of its velocity, perpendicular to the radius and perpendicular to the force of gravity. Hence the angle θ between the force $\vec{\mathbf{F}}_G$ and the instantaneous displacement of the Moon is 90°, and the work done by gravity is therefore zero $(\cos 90° = 0)$. This is why the Moon, as well as artificial satellites, can stay in orbit without expenditure of fuel: no work needs to be done against the force of gravity.

*6–2 Work Done by a Varying Force

If the force acting on an object is constant, the work done by that force can be calculated using Eq. 6–1. But in many cases, the force varies in magnitude or direction during a process. For example, as a rocket moves away from Earth, work is done to overcome the force of gravity, which varies as the inverse square of the distance from the Earth's center. Other examples are the force exerted by a spring, which increases with the amount of stretch, or the work done by a varying force that pulls a box or cart up an uneven hill.

The work done by a varying force can be determined graphically. To do so, we plot $F_{\|}$ $(= F\cos\theta,$ the component of $\vec{\mathbf{F}}$ parallel to the direction of motion at any point) as a function of distance d, as in Fig. 6–6a. We divide the distance into small segments Δd. For each segment, we indicate the average of $F_{\|}$ by a horizontal dashed line. Then the work done for each segment is $\Delta W = F_{\|}\,\Delta d$, which is the area of a rectangle Δd wide and $F_{\|}$ high. The total work done to move the object a total distance $d = d_B - d_A$ is the sum of the areas of the rectangles (five in the case shown in Fig. 6–6a). Usually, the average value of $F_{\|}$ for each segment must be estimated, and a reasonable approximation of the work done can then be made.

If we subdivide the distance into many more segments, Δd can be made smaller and our estimate of the work done would be more accurate. In the limit as Δd approaches zero, the total area of the many narrow rectangles approaches the area under the curve, Fig. 6–6b. That is, *the work done by a variable force in moving an object between two points is equal to the area under the $F_{\|}$ vs. d curve between those two points*.

6–3 Kinetic Energy, and the Work-Energy Principle

Energy is one of the most important concepts in science. Yet we cannot give a simple general definition of energy in only a few words. Nonetheless, each specific type of energy can be defined fairly simply. In this Chapter we define translational kinetic energy and some types of potential energy. In later Chapters, we will examine other types of energy, such as that related to heat and electricity. The crucial aspect of energy is that the sum of all types, the *total energy*, is the same after any process as it was before: that is, energy is a conserved quantity.

For the purposes of this Chapter, we can define energy in the traditional way as "the ability to do work." This simple definition is not always applicable,† but it is valid for mechanical energy which we discuss in this Chapter. We now define and discuss one of the basic types of energy, kinetic energy.

†Energy associated with heat is often not available to do work, as we will discuss in Chapter 15.

A moving object can do work on another object it strikes. A flying cannonball does work on a brick wall it knocks down; a moving hammer does work on a nail it drives into wood. In either case, a moving object exerts a force on a second object which undergoes a displacement. An object in motion has the ability to do work and thus can be said to have energy. The energy of motion is called **kinetic energy**, from the Greek word *kinetikos*, meaning "motion."

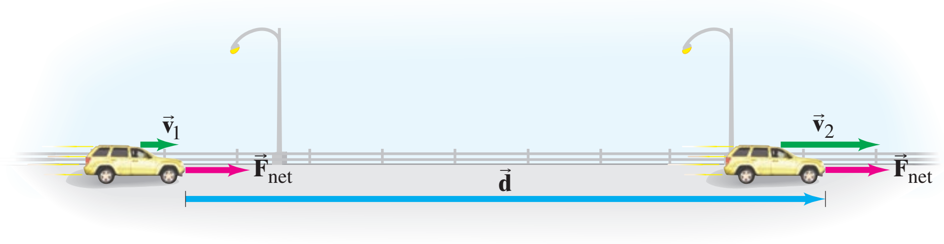


FIGURE 6–7 A constant net force F_{net} accelerates a car from speed v_1 to speed v_2 over a displacement d. The net work done is $W_{\text{net}} = F_{\text{net}}\,d$.

To obtain a quantitative definition for kinetic energy, let us consider a simple rigid object of mass m (treated as a particle) that is moving in a straight line with an initial speed v_1. To accelerate it uniformly to a speed v_2, a constant net force F_{net} is exerted on it parallel to its motion over a displacement d, Fig. 6–7. Then the net work done on the object is $W_{\text{net}} = F_{\text{net}}\,d$. We apply Newton's second law, $F_{\text{net}} = ma$, and use Eq. 2–11c $\left(v_2^2 = v_1^2 + 2ad\right)$, which we rewrite as

$$a = \frac{v_2^2 - v_1^2}{2d},$$

where v_1 is the initial speed and v_2 is the final speed. Substituting this into $F_{\text{net}} = ma$, we determine the work done:

$$W_{\text{net}} = F_{\text{net}}\,d = mad = m\left(\frac{v_2^2 - v_1^2}{2d}\right)d = m\left(\frac{v_2^2 - v_1^2}{2}\right)$$

or

$$W_{\text{net}} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2. \qquad \textbf{(6–2)}$$

We *define* the quantity $\frac{1}{2}mv^2$ to be the **translational kinetic energy (KE)** of the object:

Kinetic energy (defined)

$$\text{KE} = \tfrac{1}{2}mv^2. \qquad \textbf{(6–3)}$$

(We call this "translational" kinetic energy to distinguish it from rotational kinetic energy, which we will discuss in Chapter 8.) Equation 6–2, derived here for one-dimensional motion with a constant force, is valid in general for translational motion of an object in three dimensions and even if the force varies.

We can rewrite Eq. 6–2 as:

$$W_{\text{net}} = \text{KE}_2 - \text{KE}_1$$

or

WORK-ENERGY PRINCIPLE

$$W_{\text{net}} = \Delta\text{KE} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2. \qquad \textbf{(6–4)}$$

Equation 6–4 is a useful result known as the **work-energy principle**. It can be stated in words:

WORK-ENERGY PRINCIPLE

The net work done on an object is equal to the change in the object's kinetic energy.

CAUTION
Work-energy valid only for net work

Notice that we made use of Newton's second law, $F_{\text{net}} = ma$, where F_{net} is the *net* force—the sum of all forces acting on the object. Thus, the work-energy principle is valid only if W is the *net work* done on the object—that is, the work done by all forces acting on the object.

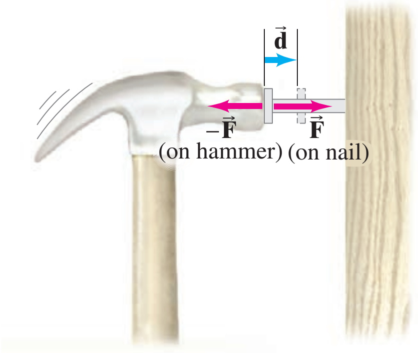


FIGURE 6–8 A moving hammer strikes a nail and comes to rest. The hammer exerts a force F on the nail; the nail exerts a force $-F$ on the hammer (Newton's third law). The work done on the nail by the hammer is positive $(W_n = Fd > 0)$. The work done on the hammer by the nail is negative $(W_h = -Fd)$.

The work-energy principle is a very useful reformulation of Newton's laws. It tells us that if (positive) net work W is done on an object, the object's kinetic energy increases by an amount W. The principle also holds true for the reverse situation: if the net work W done on an object is negative, the object's kinetic energy decreases by an amount W. That is, a net force exerted on an object opposite to the object's direction of motion decreases its speed and its kinetic energy. An example is a moving hammer (Fig. 6–8) striking a nail. The net force on the hammer ($-\vec{\mathbf{F}}$ in Fig. 6–8, where $\vec{\mathbf{F}}$ is assumed constant for simplicity) acts toward the left, whereas the displacement $\vec{\mathbf{d}}$ of the hammer is toward the right. So the net work done on the hammer, $W_h = (F)(d)(\cos 180°) = -Fd$, is negative and the hammer's kinetic energy decreases (usually to zero).

Figure 6–8 also illustrates how energy can be considered the ability to do work. The hammer, as it slows down, does positive work on the nail: $W_n = (+F)(+d) = Fd$ and is positive. The decrease in kinetic energy of the hammer ($= Fd$ by Eq. 6–4) is equal to the work the hammer can do on another object, the nail in this case.

The translational kinetic energy $\left(= \frac{1}{2}mv^2\right)$ is directly proportional to the mass of the object, and it is also proportional to the *square* of the speed. Thus, if the mass is doubled, the kinetic energy is doubled. But if the speed is doubled, the object has four times as much kinetic energy and is therefore capable of doing four times as much work.

Because of the direct connection between work and kinetic energy, energy is measured in the same units as work: joules in SI units. [The energy unit is ergs in the cgs, and foot-pounds in the British system.] Like work, kinetic energy is a scalar quantity. The kinetic energy of a group of objects is the sum of the kinetic energies of the individual objects.

The work-energy principle can be applied to a particle, and also to an object that can be approximated as a particle, such as an object that is rigid or whose internal motions are insignificant. It is very useful in simple situations, as we will see in the Examples below.

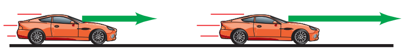


FIGURE 6–9 Example 6–4.

EXAMPLE 6–4 ESTIMATE Work on a car, to increase its kinetic energy. How much net work is required to accelerate a 1000-kg car from 20 m/s to 30 m/s (Fig. 6–9)?

APPROACH A car is a complex system. The engine turns the wheels and tires which push against the ground, and the ground pushes back (see Example 4–4). We aren't interested right now in those complications. Instead, we can get a useful result using the work-energy principle, but only if we model the car as a particle or simple rigid object.

SOLUTION The net work needed is equal to the increase in kinetic energy:

$$\begin{aligned} W &= \mathrm{KE}_2 - \mathrm{KE}_1 \\ &= \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 \\ &= \tfrac{1}{2}(1000\,\mathrm{kg})(30\,\mathrm{m/s})^2 - \tfrac{1}{2}(1000\,\mathrm{kg})(20\,\mathrm{m/s})^2 \\ &= 2.5 \times 10^5\,\mathrm{J}. \end{aligned}$$

EXERCISE B (*a*) Make a guess: will the work needed to accelerate the car in Example 6–4 from rest to 20 m/s be more than, less than, or equal to the work already calculated to accelerate it from 20 m/s to 30 m/s? (*b*) Make the calculation.

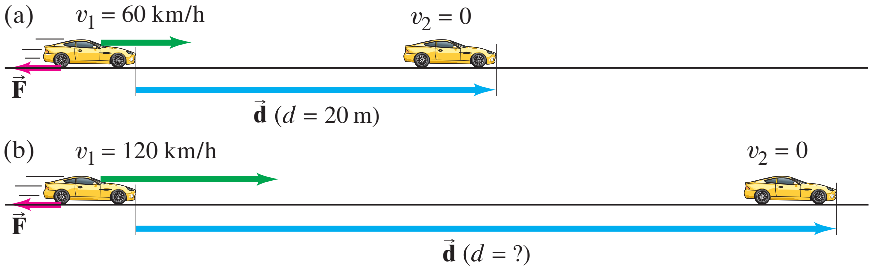


FIGURE 6–10 Example 6–5. A moving car comes to a stop. Initial velocity is (a) 60 km/h, (b) 120 km/h.

CONCEPTUAL EXAMPLE 6–5 **Work to stop a car.** A car traveling 60 km/h can brake to a stop in a distance d of 20 m (Fig. 6–10a). If the car is going twice as fast, 120 km/h, what is its stopping distance (Fig. 6–10b)? Assume the maximum braking force is approximately independent of speed.

RESPONSE Again we model the car as if it were a particle. Because the net stopping force F is approximately constant, the work needed to stop the car, Fd, is proportional to the distance traveled. We apply the work-energy principle, noting that $\vec{\mathbf{F}}$ and $\vec{\mathbf{d}}$ are in opposite directions and that the final speed of the car is zero:

$$W_{\text{net}} = Fd\cos 180^\circ = -Fd.$$

Then

$$\begin{aligned} -Fd = \Delta\text{KE} &= \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 \\ &= 0 - \tfrac{1}{2}mv_1^2. \end{aligned}$$

Thus, since the force and mass are constant, we see that the stopping distance, d, increases with the square of the speed:

$$d \propto v^2.$$

If the car's initial speed is doubled, the stopping distance is $(2)^2 = 4$ times as great, or 80 m.

PHYSICS APPLIED
Car's stopping distance $\propto$ to initial speed squared

EXERCISE C Can kinetic energy ever be negative?

EXERCISE D (a) If the kinetic energy of a baseball is doubled, by what factor has its speed increased? (b) If its speed is doubled, by what factor does its kinetic energy increase?

6–4 Potential Energy

We have just discussed how an object is said to have energy by virtue of its motion, which we call kinetic energy. But it is also possible to have **potential energy**, which is the energy associated with forces that depend on the position or configuration of an object (or objects) relative to the surroundings. Various types of potential energy (PE) can be defined, and each type is associated with a particular force.

The spring of a wind-up toy is an example of an object with potential energy. The spring acquired its potential energy because work was done *on* it by the person winding the toy. As the spring unwinds, it exerts a force and does work to make the toy move.

Gravitational Potential Energy

Perhaps the most common example of potential energy is *gravitational potential energy*. A heavy brick held high above the ground has potential energy because of its position relative to the Earth. The raised brick has the ability to do work, for if it is released, it will fall to the ground due to the gravitational force, and can do work on, say, a stake, driving it into the ground.

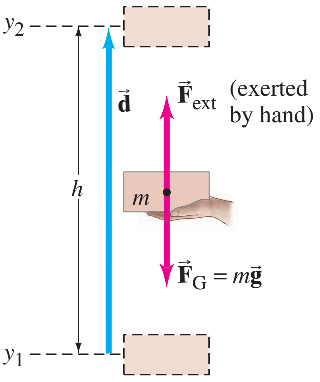


FIGURE 6–11 A person exerts an upward force $F_{\text{ext}} = mg$ to lift a brick from y_1 to y_2.

Let us seek the form for the gravitational potential energy of an object near the surface of the Earth. For an object of mass m to be lifted vertically, an upward force at least equal to its weight, mg, must be exerted on it, say by a person's hand. To lift the object without acceleration, the person exerts an "external force" $F_{\text{ext}} = mg$. If it is raised a vertical height h, from position y_1 to y_2 in Fig. 6–11 (upward direction chosen positive), a person does work equal to the product of the "external" force she exerts, $F_{\text{ext}} = mg$ upward, multiplied by the vertical displacement h. That is,

$$\begin{aligned} W_{\text{ext}} &= F_{\text{ext}}\, d \cos 0^\circ = mgh \\ &= mg(y_2 - y_1). \end{aligned} \tag{6–5a}$$

Gravity is also acting on the object as it moves from y_1 to y_2, and does work on the object equal to

$$W_{\text{G}} = F_{\text{G}}\, d \cos\theta = mgh \cos 180^\circ,$$

where $\theta = 180^\circ$ because $\vec{\mathbf{F}}_{\text{G}}$ and $\vec{\mathbf{d}}$ point in opposite directions. So

$$\begin{aligned} W_{\text{G}} &= -mgh \\ &= -mg(y_2 - y_1). \end{aligned} \tag{6–5b}$$

Next, if we allow the object to start from rest at y_2 and fall freely under the action of gravity, it acquires a velocity given by $v^2 = 2gh$ (Eq. 2–11c) after falling a height h. It then has kinetic energy $\frac{1}{2}mv^2 = \frac{1}{2}m(2gh) = mgh$, and if it strikes a stake, it can do work on the stake equal to mgh (Section 6–3).

Thus, to raise an object of mass m to a height h *requires* an amount of work equal to mgh (Eq. 6–5a). And once at height h, the object has the *ability* to do an amount of work equal to mgh. We can say that the work done in lifting the object has been stored as gravitational potential energy.

We therefore define the **gravitational potential energy** of an object, due to Earth's gravity, as the product of the object's weight mg and its height y above some reference level (such as the ground):

$$\text{PE}_{\text{G}} = mgy. \tag{6–6}$$

The higher an object is above the ground, the more gravitational potential energy it has. We combine Eq. 6–5a with Eq. 6–6:

$$\begin{aligned} W_{\text{ext}} &= mg(y_2 - y_1) \\ W_{\text{ext}} &= \text{PE}_2 - \text{PE}_1 = \Delta\text{PE}_{\text{G}}. \end{aligned} \tag{6–7a}$$

That is, the change in gravitational potential energy when an object is moved from height y_1 to height y_2 is equal to the work done by the net external force that accomplishes this without acceleration.

A more direct way to define the *change in gravitational potential energy*, $\Delta\text{PE}_{\text{G}}$, is that it is equal to the *negative of the work done by gravity* itself (Eq. 6–5b):

$$\begin{aligned} W_{\text{G}} &= -mg(y_2 - y_1) \\ W_{\text{G}} &= -(\text{PE}_2 - \text{PE}_1) = -\Delta\text{PE}_{\text{G}} \end{aligned}$$

or

$$\Delta\text{PE}_{\text{G}} = -W_{\text{G}}. \tag{6–7b}$$

CAUTION

Change in PE *is what is physically meaningful*

Gravitational potential energy depends on the *vertical height* of the object *above some reference level* (Eq. 6–6). In some situations, you may wonder from what point to measure the height y. The gravitational potential energy of a book held high above a table, for example, depends on whether we measure y from the top of the table, from the floor, or from some other reference point. What is physically important in any situation is the *change* in potential energy, ΔPE, because that is what is related to the work done, Eqs. 6–7; and it is ΔPE that can be measured. We can thus choose to measure y from any reference level that is convenient, but we must choose the reference level at the start and be consistent throughout. The *change* in potential energy between any two points does not depend on this choice.

An important result we discussed earlier (see Example 6–2 and Fig. 6–4) concerns the gravity force, which does work only in the vertical direction: the work done by gravity depends only on the vertical height h, and not on the path taken, whether it be purely vertical motion or, say, motion along an incline. Thus, from Eqs. 6–7 we see that changes in gravitational potential energy depend only on the change in vertical height and not on the path taken.

Potential energy belongs to a system, and not to a single object alone. Potential energy is associated with a force, and a force on one object is always exerted by some other object. Thus potential energy is a property of the system as a whole. For an object raised to a height y above the Earth's surface, the change in gravitational potential energy is mgy. The system here is the object plus the Earth, and properties of both are involved: object (m) and Earth (g).

CAUTION
Potential energy belongs to a system, not to a single object

EXAMPLE 6–6 Potential energy changes for a roller coaster. A 1000-kg roller-coaster car moves from point 1, Fig. 6–12, to point 2 and then to point 3. (a) What is the gravitational potential energy at points 2 and 3 relative to point 1? That is, take $y = 0$ at point 1. (b) What is the change in potential energy when the car goes from point 2 to point 3? (c) Repeat parts (a) and (b), but take the reference point $(y = 0)$ to be at point 3.

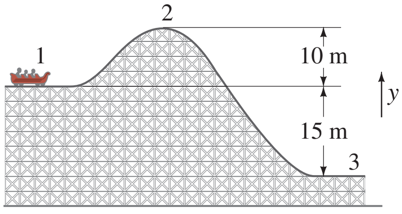


FIGURE 6–12 Example 6–6.

APPROACH We are interested in the potential energy of the car–Earth system. We take upward as the positive y direction, and use the definition of gravitational potential energy to calculate the potential energy.

SOLUTION (a) We measure heights from point 1 $(y_1 = 0)$, which means initially that the gravitational potential energy is zero. At point 2, where $y_2 = 10\text{ m}$,

$$\text{PE}_2 = mgy_2 = (1000\text{ kg})(9.8\text{ m/s}^2)(10\text{ m}) = 9.8 \times 10^4\text{ J}.$$

At point 3, $y_3 = -15\text{ m}$, since point 3 is below point 1. Therefore,

$$\text{PE}_3 = mgy_3 = (1000\text{ kg})(9.8\text{ m/s}^2)(-15\text{ m}) = -1.5 \times 10^5\text{ J}.$$

(b) In going from point 2 to point 3, the potential energy change $(\text{PE}_{\text{final}} - \text{PE}_{\text{initial}})$ is

$$\text{PE}_3 - \text{PE}_2 = (-1.5 \times 10^5\text{ J}) - (9.8 \times 10^4\text{ J}) = -2.5 \times 10^5\text{ J}.$$

The gravitational potential energy decreases by 2.5×10^5 J.

(c) Now we set $y_3 = 0$. Then $y_1 = +15\text{ m}$ at point 1, so the potential energy initially is

$$\text{PE}_1 = (1000\text{ kg})(9.8\text{ m/s}^2)(15\text{ m}) = 1.5 \times 10^5\text{ J}.$$

At point 2, $y_2 = 25\text{ m}$, so the potential energy is

$$\text{PE}_2 = 2.5 \times 10^5\text{ J}.$$

At point 3, $y_3 = 0$, so the potential energy is zero. The change in potential energy going from point 2 to point 3 is

$$\text{PE}_3 - \text{PE}_2 = 0 - 2.5 \times 10^5\text{ J} = -2.5 \times 10^5\text{ J},$$

which is the same as in part (b).

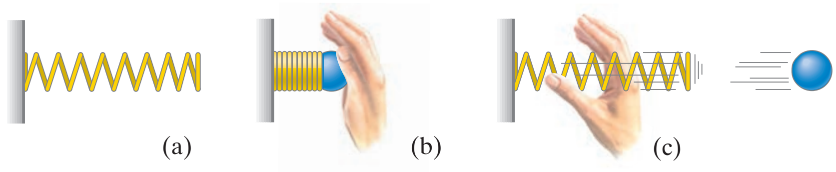
NOTE Work done by gravity depends only on the vertical height, so changes in gravitational potential energy do not depend on the path taken.

Potential Energy Defined in General

Potential Energy defined

There are other kinds of potential energy besides gravitational. Each form of potential energy is associated with a particular force, and can be defined analogously to gravitational potential energy. In general, the *change in potential energy associated with a particular force is equal to the negative of the work done by that force when the object is moved from one point to a second point* (as in Eq. 6–7b for gravity). An alternate and sometimes useful way to define the change in potential energy is: the work required of a net external force to move the object without acceleration between the two points, as in Eq. 6–7a.

FIGURE 6–13 A spring (a) can store energy (elastic PE) when compressed as in (b) and can do work when released (c).



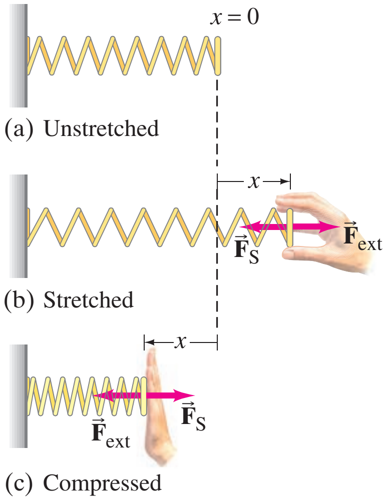
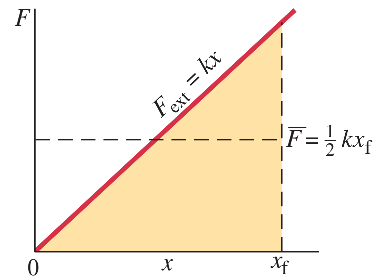


FIGURE 6–14 (a) Spring in natural (unstretched) position. (b) Spring is stretched by a person exerting a force $\vec{\mathbf{F}}_{ext}$ to the right (positive direction). The spring pulls back with a force $\vec{\mathbf{F}}_S$, where $F_S = -kx$. (c) Person compresses the spring $(x < 0)$ by exerting an external force $\vec{\mathbf{F}}_{ext}$ to the left; the spring pushes back with a force $F_S = -kx$, where $F_S > 0$ because $x < 0$.

FIGURE 6–15 As a spring is stretched (or compressed), the magnitude of the force needed increases linearly as x increases: graph of $F = kx$ vs. x from $x = 0$ to $x = x_f$.



Potential Energy of Elastic Spring

We now consider potential energy associated with elastic materials, which includes a great variety of practical applications. Consider the simple coil spring shown in Fig. 6–13. The spring has potential energy when compressed (or stretched), because when it is released, it can do work on a ball as shown. To hold a spring either stretched or compressed an amount x from its natural (unstretched) length requires the hand to exert an external force on the spring of magnitude F_{ext} which is directly proportional to x. That is,

$$F_{ext} = kx,$$

where k is a constant, called the *spring stiffness constant* (or simply **spring constant**), and is a measure of the stiffness of the particular spring. The stretched or compressed spring itself exerts a force F_S in the opposite direction on the hand, as shown in Fig. 6–14:

$$F_S = -kx. \qquad \text{[spring force]} \qquad \textbf{(6–8)}$$

This force is sometimes called a "restoring force" because the spring exerts its force in the direction opposite the displacement (hence the minus sign), acting to return it to its natural length. Equation 6–8 is known as the **spring equation** and also as **Hooke's law**, and is accurate for springs as long as x is not too great.

To calculate the potential energy of a stretched spring, let us calculate the work required to stretch it (Fig. 6–14b). We might hope to use Eq. 6–1 for the work done on it, $W = Fx$, where x is the amount it is stretched from its natural length. But this would be incorrect since the force F_{ext} $(= kx)$ is not constant but varies over the distance x, becoming greater the more the spring is stretched, as shown graphically in Fig. 6–15. So let us use the average force, $\bar{F}$. Since F_{ext} varies linearly, from zero at the unstretched position to kx when stretched to x, the average force is $\bar{F} = \frac{1}{2}[0 + kx] = \frac{1}{2}kx$, where x here is the final amount stretched (shown as x_f in Fig. 6–15 for clarity). The work done is then

$$W_{ext} = \bar{F}x = \left(\tfrac{1}{2}kx\right)(x) = \tfrac{1}{2}kx^2.$$

Hence the **elastic potential energy**, PE_{el}, is proportional to the square of the amount stretched:

$$\text{PE}_{el} = \tfrac{1}{2}kx^2. \qquad \text{[elastic spring]} \qquad \textbf{(6–9)}$$

If a spring is *compressed* a distance x from its natural ("equilibrium") length, the average force again has magnitude $\bar{F} = \frac{1}{2}kx$, and again the potential energy is given by Eq. 6–9. Thus x can be either the amount compressed or amount stretched from the spring's natural length.† Note that for a spring, we choose the reference point for zero PE at the spring's natural position.

Potential Energy as Stored Energy

In the above examples of potential energy—from a brick held at a height y, to a stretched or compressed spring—an object has the capacity or *potential* to do work even though it is not yet actually doing it. These examples show that energy can be *stored*, for later use, in the form of potential energy (as in Fig. 6–13, for a spring).

Note that there is a single universal formula for the translational kinetic energy of an object, $\frac{1}{2}mv^2$, but there is no single formula for potential energy. Instead, the mathematical form of the potential energy depends on the force involved.

†We can also obtain Eq. 6–9 using Section 6–2. The work done, and hence ΔPE, equals the area under the F vs. x graph of Fig. 6–15. This area is a triangle (colored in Fig. 6–15) of altitude kx and base x, and hence of area (for a triangle) equal to $\frac{1}{2}(kx)(x) = \frac{1}{2}kx^2$.