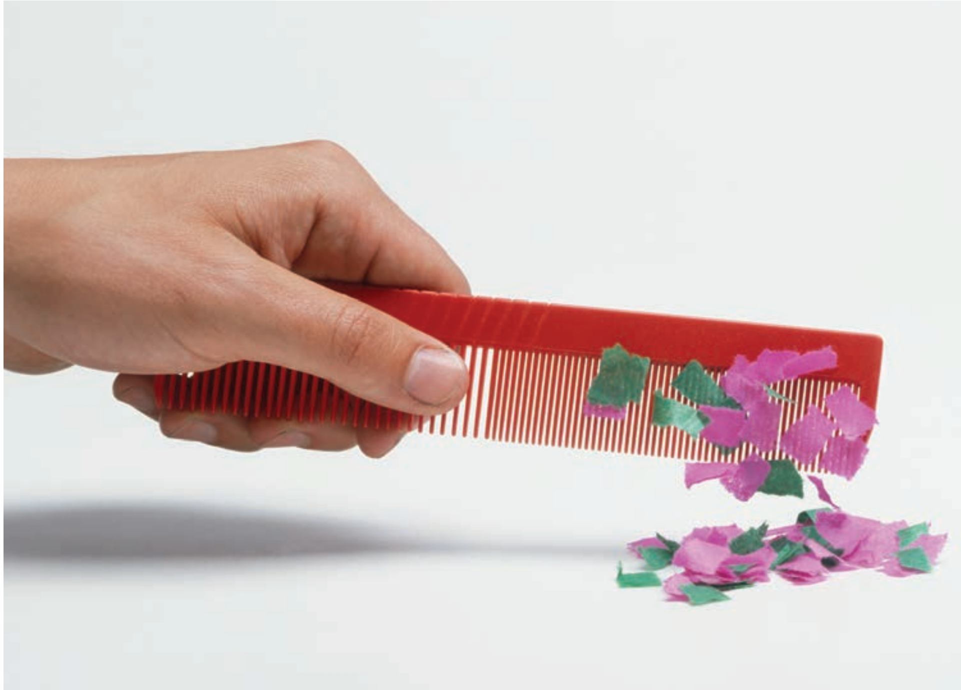

This comb has acquired a static electric charge, either from passing through hair, or being rubbed by a cloth or paper towel. The electrical charge on the comb induces a polarization (separation of charge) in scraps of paper, and thus attracts them.

Our introduction to electricity in this Chapter covers conductors and insulators, and Coulomb's law which relates the force between two point charges as a function of their distance apart. We also introduce the powerful concept of electric field.

CHAPTER 16

Electric Charge and Electric Field

CONTENTS

16–1 Static Electricity; Electric Charge and Its Conservation
16–2 Electric Charge in the Atom
16–3 Insulators and Conductors
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CHAPTER-OPENING QUESTION—Guess now!

Two identical tiny spheres have the same electric charge. If their separation is doubled, the force each exerts on the other will be

(a) half.
(b) double.
(c) four times larger.
(d) one-quarter as large.
(e) unchanged.

The word "electricity" may evoke an image of complex modern technology: lights, motors, electronics, and computers. But the electric force plays an even deeper role in our lives. According to atomic theory, electric forces between atoms and molecules hold them together to form liquids and solids, and electric forces are also involved in the metabolic processes that occur within our bodies. Many of the forces we have dealt with so far, such as elastic forces, the normal force, and friction and other contact forces (pushes and pulls), are now considered to result from electric forces acting at the atomic level. Gravity, on the other hand, is a separate force.†

†As we discussed in Section 5–9, physicists in the twentieth century came to recognize four different fundamental forces in nature: (1) gravitational force, (2) electromagnetic force (we will see later that electric and magnetic forces are intimately related), (3) strong nuclear force, and (4) weak nuclear force. The last two forces operate at the level of the nucleus of an atom. Recent theory has combined the electromagnetic and weak nuclear forces so they are now considered to have a common origin known as the electroweak force. We discuss the other forces in later Chapters.

The earliest studies on electricity date back to the ancients, but only since the late 1700s has electricity been studied in detail. We will discuss the development of ideas about electricity, including practical devices, as well as its relation to magnetism, in the next seven Chapters.

16–1 Static Electricity; Electric Charge and Its Conservation

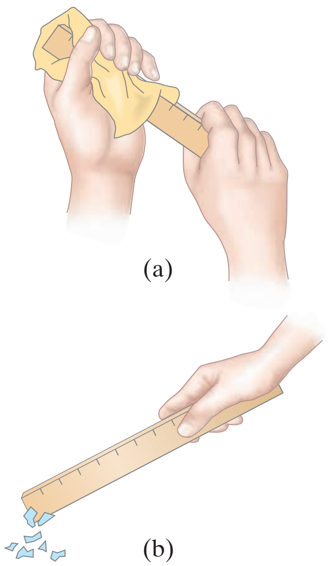
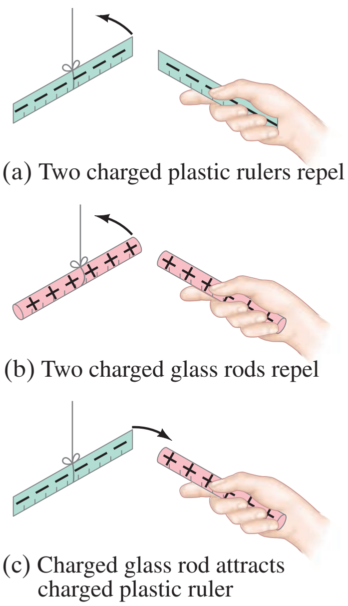


FIGURE 16–1 (a) Rub a plastic ruler with a cloth or paper towel, and (b) bring it close to some tiny pieces of paper.

The word *electricity* comes from the Greek word *elektron*, which means "amber." Amber is petrified tree resin, and the ancients knew that if you rub a piece of amber with a cloth, the amber attracts small pieces of leaves or dust. A piece of hard rubber, a glass rod, or a plastic ruler rubbed with a cloth will also display this "amber effect," or **static electricity** as we call it today. You can readily pick up small pieces of paper with a plastic comb or ruler that you have just vigorously rubbed with even a paper towel. See the photo on the previous page and Fig. 16–1. You have probably experienced static electricity when combing your hair or when taking a synthetic blouse or shirt from a clothes dryer. And you may have felt a shock when you touched a metal doorknob after sliding across a car seat or walking across a synthetic carpet. In each case, an object becomes "charged" as a result of rubbing, and is said to possess a net **electric charge**.

FIGURE 16–2 Like charges repel one another; unlike charges attract. (Note color coding: we color positive charged objects pink or red, and negative charges blue-green. We use these colors especially for point charges, but not always for real objects.)



Is all electric charge the same, or is there more than one type? In fact, there are *two* types of electric charge, as the following simple experiments show. A plastic ruler suspended by a thread is vigorously rubbed with a cloth to charge it. When a second plastic ruler, which has been charged in the same way, is brought close to the first, it is found that one ruler *repels* the other. This is shown in Fig. 16–2a. Similarly, if a rubbed glass rod is brought close to a second charged glass rod, again a repulsive force is seen to act, Fig. 16–2b. However, if the charged glass rod is brought close to the charged plastic ruler, it is found that they *attract* each other, Fig. 16–2c. The charge on the glass must therefore be different from that on the plastic. Indeed, it is found experimentally that all charged objects fall into one of two categories. Either they are attracted to the plastic and repelled by the glass; or they are repelled by the plastic and attracted to the glass. Thus there seem to be two, and only two, types of electric charge. Each type of charge repels the same type but attracts the opposite type. That is: **unlike charges attract; like charges repel**.

The two types of electric charge were referred to as **positive** and **negative** by the American statesman, philosopher, and scientist Benjamin Franklin (1706–1790). The choice of which name went with which type of charge was arbitrary. Franklin's choice set the charge on the rubbed glass rod to be positive charge, so the charge on a rubbed plastic ruler (or amber) is called negative charge. We still follow this convention today.

Franklin argued that whenever a certain amount of charge is produced on one object, an equal amount of the opposite type of charge is produced on another object. The positive and negative are to be treated *algebraically*, so during any process, the net change in the amount of charge produced is zero. For example, when a plastic ruler is rubbed with a paper towel, the plastic acquires a negative charge and the towel acquires an equal amount of positive charge. The charges are separated, but the sum of the two is zero.

This is an example of a law that is now well established: the **law of conservation of electric charge**, which states that

LAW OF CONSERVATION OF ELECTRIC CHARGE

> **the net amount of electric charge produced in any process is zero;**

or, said another way,

> **no net electric charge can be created or destroyed.**

If one object (or a region of space) acquires a positive charge, then an equal amount of negative charge will be found in neighboring areas or objects. No violations have ever been found, and the law of conservation of electric charge is as firmly established as those for energy and momentum.

16–2 Electric Charge in the Atom

Only within the past century has it become clear that an understanding of electricity originates inside the atom itself. In later Chapters we will discuss atomic structure and the ideas that led to our present view of the atom in more detail. But it will help our understanding of electricity if we discuss it briefly now.

A simplified model of an atom shows it as having a tiny but massive, positively charged nucleus surrounded by one or more negatively charged electrons (Fig. 16–3). The nucleus contains protons, which are positively charged, and neutrons, which have no net electric charge. All protons and all electrons have exactly the same magnitude of electric charge; but their signs are opposite. Hence neutral atoms, having no net charge, contain equal numbers of protons and electrons. Sometimes an atom may lose one or more of its electrons, or may gain extra electrons, in which case it will have a net positive or negative charge and is called an **ion**.

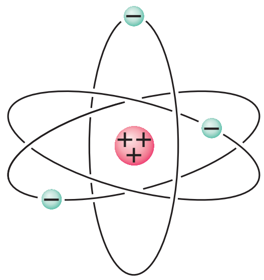
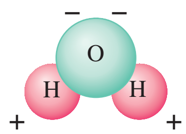


FIGURE 16–3 Simple model of the atom.

In solid materials the nuclei tend to remain close to fixed positions, whereas some of the electrons may move quite freely. When an object is *neutral*, it contains equal amounts of positive and negative charge. The charging of a solid object by rubbing can be explained by the transfer of electrons from one object to the other. When a plastic ruler becomes negatively charged by rubbing with a paper towel, electrons are transferred from the towel to the plastic, leaving the towel with a positive charge equal in magnitude to the negative charge acquired by the plastic. In liquids and gases, nuclei or ions can move as well as electrons.

Normally when objects are charged by rubbing, they hold their charge only for a limited time and eventually return to the neutral state. Where does the charge go? Usually the excess charge "leaks off" onto water molecules in the air. This is because water molecules are **polar**—that is, even though they are neutral, their charge is not distributed uniformly, Fig. 16–4. Thus the extra electrons on, say, a charged plastic ruler can "leak off" into the air because they are attracted to the positive end of water molecules. A positively charged object, on the other hand, can be neutralized by transfer of loosely held electrons from water molecules in the air. On dry days, static electricity is much more noticeable since the air contains fewer water molecules to allow leakage of charge. On humid or rainy days, it is difficult to make any object hold a net charge for long.

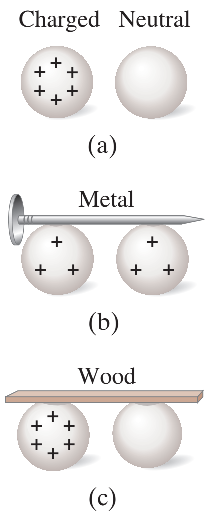
FIGURE 16–4 Diagram of a water molecule. Because it has opposite charges on different ends, it is called a "polar" molecule.



16–3 Insulators and Conductors

Suppose we have two metal spheres, one highly charged and the other electrically neutral (Fig. 16–5a). If we now place a metal object, such as a nail, so that it touches both spheres (Fig. 16–5b), the previously uncharged sphere quickly becomes charged. If, instead, we had connected the two spheres by a wooden rod or a piece of rubber (Fig. 16–5c), the uncharged ball would not become noticeably charged. Materials like the iron nail are said to be **conductors** of electricity, whereas wood and rubber are **nonconductors** or **insulators**.

FIGURE 16–5 (a) A charged metal sphere and a neutral metal sphere. (b) The two spheres connected by a conductor (a metal nail), which conducts charge from one sphere to the other. (c) The original two spheres connected by an insulator (wood); almost no charge is conducted.



Metals are generally good conductors, whereas most other materials are insulators (although even insulators conduct electricity very slightly). Nearly all natural materials fall into one or the other of these two distinct categories. However, a few materials (notably silicon and germanium) fall into an intermediate category known as **semiconductors**.

From the atomic point of view, the electrons in an insulating material are bound very tightly to the nuclei. In a good metal conductor, on the other hand, some of the electrons are bound very loosely and can move about freely within the metal (although they cannot *leave* the metal easily) and are often referred to as **free electrons** or **conduction electrons**. When a positively charged object is brought close to or touches a conductor, the free electrons in the conductor are attracted by this positively charged object and move quickly toward it. If a negatively charged object is brought close to the conductor, the free electrons in the conductor move swiftly away from it. In a semiconductor, there are many fewer free electrons, and in an insulator, almost none.

16–4 Induced Charge; the Electroscope

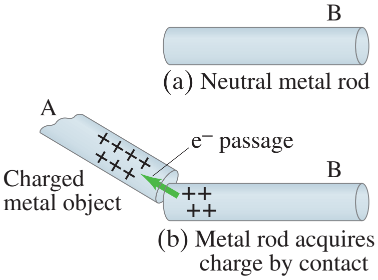


FIGURE 16–6 A neutral metal rod in (a) will acquire a positive charge if placed in contact (b) with a positively charged metal object. (Electrons move as shown by the green arrow.) This is called charging by conduction.

Suppose a positively charged metal object A is brought close to an uncharged metal object B. If the two touch, the free electrons in the neutral one are attracted to the positively charged object and some of those electrons will pass over to it, Fig. 16–6. Since object B, originally neutral, is now missing some of its negative electrons, it will have a net positive charge. This process is called **charging by conduction**, or "by contact," and the two objects end up with the same sign of charge.

Now suppose a positively charged object is brought close to a neutral metal rod, but does not touch it. Although the free electrons of the metal rod do not leave the rod, they still move within the metal toward the external positive charge, leaving a positive charge at the opposite end of the rod (Fig. 16–7b). A charge is said to have been *induced* at the two ends of the metal rod. No net charge has been created in the rod: charges have merely been *separated*. The net charge on the metal rod is still zero. However, if the metal is separated into two pieces, we would have two charged objects: one charged positively and one charged negatively. This is **charging by induction**.

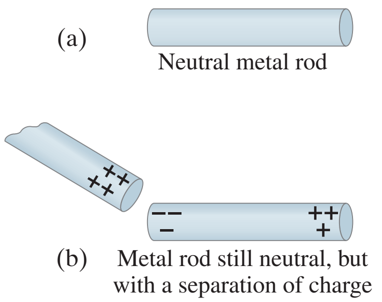


FIGURE 16–7 Charging by induction: if the rod in (b) is cut into two parts, each part will have a net charge.

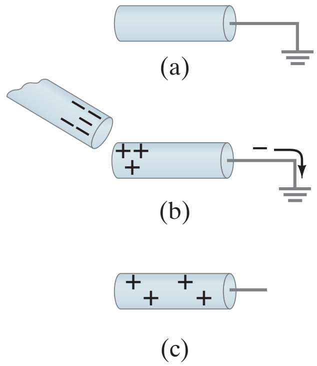
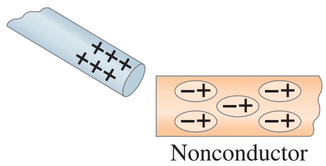


FIGURE 16–8 Inducing a charge on an object connected to ground.

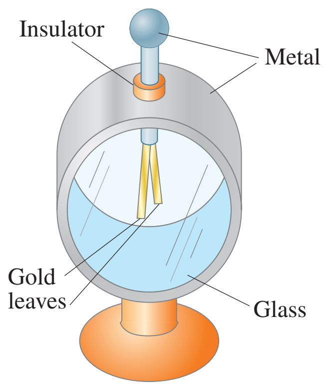
Another way to induce a net charge on a metal object is to first connect it with a conducting wire to the ground (or a conducting pipe leading into the ground) as shown in Fig. 16–8a (the symbol ⏚ means connected to "ground"). The object is then said to be **grounded** or "earthed." The Earth, because it is so large and can conduct, easily accepts or gives up electrons; hence it acts like a reservoir for charge. If a charged object—say negative this time—is brought up close to the metal object, free electrons in the metal are repelled and many of them move down the wire into the Earth, Fig. 16–8b. This leaves the metal positively charged. If the wire is now cut, the metal object will have a positive induced charge on it (Fig. 16–8c). If the wire is cut *after* the negative object is moved away, the electrons would all have moved from the ground back into the metal object and it would be neutral again.

FIGURE 16–9 A charged object brought near a nonconductor causes a charge separation within the nonconductor's molecules.



Charge separation can also be done in nonconductors. If you bring a positively charged object close to a neutral nonconductor as shown in Fig. 16–9, almost no electrons can move about freely within the nonconductor. But they can move slightly within their own atoms and molecules. Each oval in Fig. 16–9 represents a molecule (not to scale); the negatively charged electrons, attracted to the external positive charge, tend to move in its direction within their molecules. Because the negative charges in the nonconductor are nearer to the external positive charge, the nonconductor as a whole is attracted to the external positive charge (see the Chapter-Opening Photo, page 443).

FIGURE 16–10 Electroscope.



An **electroscope** is a device that can be used for detecting charge. As shown in Fig. 16–10, inside a case are two movable metal leaves, often made of gold foil, connected to a metal knob on the outside. (Sometimes only one leaf is movable.)

If a positively charged object is brought close to the knob, a separation of charge is induced: electrons are attracted up into the knob, and the leaves become positively charged, Fig. 16–11a. The two leaves repel each other as shown, because they are both positively charged. If, instead, the knob is charged by conduction (touching), the whole apparatus acquires a net charge as shown in Fig. 16–11b. In either case, the greater the amount of charge, the greater the separation of the leaves.

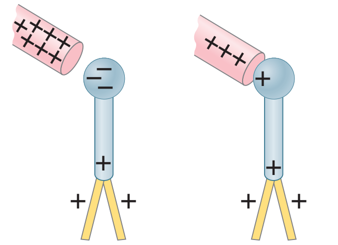
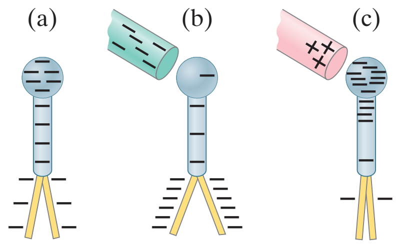


FIGURE 16–11 Electroscope charged (a) by induction, (b) by conduction.

Note that you cannot tell the sign of the charge in this way, since negative charge will cause the leaves to separate just as much as an equal amount of positive charge; in either case, the two leaves repel each other. An electroscope can, however, be used to determine the sign of the charge if it is first charged by conduction: say, negatively, as in Fig. 16–12a. Now if a negative object is brought close, as in Fig. 16–12b, more electrons are induced to move down into the leaves and they separate further. If a positive charge is brought close instead, the electrons are induced to flow upward, so the leaves are less negative and their separation is reduced, Fig. 16–12c.

FIGURE 16–12 A previously charged electroscope can be used to determine the sign of a charged object.

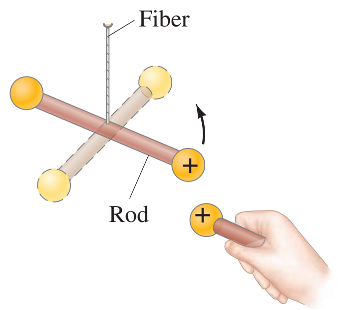


The electroscope was used in the early studies of electricity. The same principle, aided by some electronics, is used in much more sensitive modern **electrometers**.

16–5 Coulomb's Law

We have seen that an electric charge exerts a force of attraction or repulsion on other electric charges. What factors affect the magnitude of this force? To find an answer, the French physicist Charles Coulomb (1736–1806) investigated electric forces in the 1780s using a torsion balance (Fig. 16–13) much like that used by Cavendish for his studies of the gravitational force (Chapter 5).

FIGURE 16–13 Coulomb's apparatus: when an external charged sphere is placed close to the charged one on the suspended bar, the bar rotates slightly. The suspending fiber resists the twisting motion, and the angle of twist is proportional to the force applied. With this apparatus, Coulomb investigated how the electric force varies as a function of the magnitude of the charges and of the distance between them.



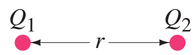
Precise instruments for the measurement of electric charge were not available in Coulomb's time. Nonetheless, Coulomb was able to prepare small spheres with different magnitudes of charge in which the *ratio* of the charges was known.† Although he had some difficulty with induced charges, Coulomb was able to argue that the electric force one tiny charged object exerts on a second tiny charged object is directly proportional to the charge on each of them. That is, if the charge on either one of the objects is doubled, the force is doubled; and if the charge on both of the objects is doubled, the force increases to four times the original value. This was the case when the distance between the two charges remained the same. If the distance between them was allowed to increase, he found that the force decreased with the *square of the distance* between them. That is, if the distance was doubled, the force fell to one-fourth of its original value. Thus, Coulomb concluded, the magnitude of the force F that one small charged object exerts on a second one is proportional to the product of the magnitude of the charge on one, Q_1, times the magnitude of the charge on the other, Q_2, and inversely proportional to the square of the distance r between them (Fig. 16–14). As an equation, we can write **Coulomb's law** as

$$F = k\frac{Q_1 Q_2}{r^2}, \qquad \text{[magnitudes]} \quad \textbf{(16–1)}$$

COULOMB'S LAW

where k is a proportionality constant.‡

FIGURE 16–14 Coulomb's law, Eq. 16–1, gives the force between two point charges, Q_1 and Q_2, a distance r apart.



†Coulomb reasoned that if a charged conducting sphere is placed in contact with an identical uncharged sphere, the charge on the first would be shared equally by the two of them because of symmetry. He thus had a way to produce charges equal to $\frac{1}{2}$, $\frac{1}{4}$, and so on, of the original charge.

‡The validity of Coulomb's law today rests on precision measurements that are much more sophisticated than Coulomb's original experiment. The exponent 2 on r in Coulomb's law has been shown to be accurate to 1 part in 10^{16} [that is, $2 \pm (1 \times 10^{-16})$].

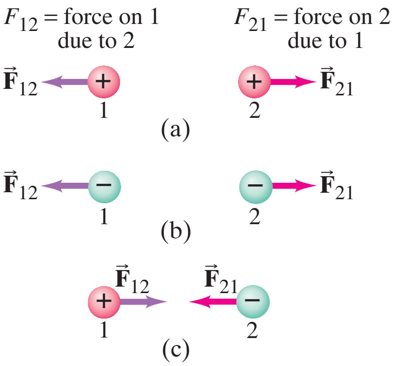


FIGURE 16–15 The direction of the static electric force one point charge exerts on another is always along the line joining the two charges, and depends on whether the charges have the same sign as in (a) and (b), or opposite signs (c).

As we just saw, Coulomb's law, Eq. 16–1,

$$F = k\frac{Q_1 Q_2}{r^2}, \qquad \text{[magnitudes]} \quad \textbf{(16–1)}$$

gives the *magnitude* of the electric force that either charge exerts on the other. The *direction* of the electric force *is always along the line joining the two charges*. If the two charges have the same sign, the force on either charge is directed away from the other (they repel each other). If the two charges have opposite signs, the force on one is directed toward the other (they attract). See Fig. 16–15. Notice that the force one charge exerts on the second is equal but opposite to that exerted by the second on the first, in accord with Newton's third law.

The SI unit of charge is the **coulomb** (C). The precise definition of the coulomb today is in terms of electric current and magnetic field, and will be discussed later (Section 20–6). In SI units, the constant k in Coulomb's law has the value

$$k = 8.988 \times 10^9\,\mathrm{N\cdot m^2/C^2}$$

or, when we only need two significant figures,

$$k \approx 9.0 \times 10^9\,\mathrm{N\cdot m^2/C^2}.$$

Thus, 1 C is that amount of charge which, if placed on each of two point objects that are 1.0 m apart, will result in each object exerting a force of $(9.0 \times 10^9\,\mathrm{N\cdot m^2/C^2})(1.0\,\mathrm{C})(1.0\,\mathrm{C})/(1.0\,\mathrm{m})^2 = 9.0 \times 10^9\,\mathrm{N}$ on the other. This would be an enormous force, equal to the weight of almost a million tons. We rarely encounter charges as large as a coulomb.[†]

Charges produced by rubbing ordinary objects (such as a comb or plastic ruler) are typically around a microcoulomb $(1\,\mu\mathrm{C} = 10^{-6}\,\mathrm{C})$ or less. Objects that carry a positive charge have a deficit of electrons, whereas negatively charged objects have an excess of electrons. The charge on one electron has been determined to have a magnitude of about 1.6022×10^{-19} C, and is negative. This is the smallest charge observed in nature,[‡] and because it is fundamental, it is given the symbol e and is often referred to as the **elementary charge**:

$$e = 1.6022 \times 10^{-19}\,\mathrm{C} \approx 1.6 \times 10^{-19}\,\mathrm{C}.$$

Note that e is defined as a positive number, so the charge on the electron is $-e$. (The charge on a proton, on the other hand, is $+e$.) Since an object cannot gain or lose a fraction of an electron, the net charge on any object must be an integral multiple of this charge. Electric charge is thus said to be **quantized** (existing only in discrete amounts: $1e$, $2e$, $3e$, etc.). Because e is so small, however, we normally do not notice this discreteness in macroscopic charges (1 μC requires about 10^{13} electrons), which thus seem continuous.

Coulomb's law looks a lot like the *law of universal gravitation*, $F = Gm_1 m_2/r^2$, which expresses the magnitude of the gravitational force a mass m_1 exerts on a mass m_2 (Eq. 5–4). Both are **inverse square laws** $(F \propto 1/r^2)$. Both also have a proportionality to a property of each object—mass for gravity, electric charge for electricity. And both act over a distance (that is, there is no need for contact). A major difference between the two laws is that gravity is always an attractive force, whereas the electric force can be either attractive or repulsive. Electric charge comes in two types, positive and negative; gravitational mass is only positive.

The constant k in Eq. 16–1 is often written in terms of another constant, ϵ_0, called the **permittivity of free space**. It is related to k by $k = 1/4\pi\epsilon_0$. Coulomb's law can be written

COULOMB'S LAW (in terms of ϵ_0)

$$F = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{r^2}, \qquad \textbf{(16–2)}$$

where

$$\epsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12}\,\mathrm{C^2/N\cdot m^2}.$$

[†]In the once common cgs system of units, k is set equal to 1, and the unit of electric charge is called the *electrostatic unit* (esu) or the statcoulomb. One esu is defined as that charge, on each of two point objects 1 cm apart, that gives rise to a force of 1 dyne.

[‡]According to the Standard Model of elementary particle physics, subnuclear particles called quarks have a smaller charge than the electron, equal to $\frac{1}{3}e$ or $\frac{2}{3}e$. Quarks have not been detected directly as isolated objects, and theory indicates that free quarks may not be detectable.

Equation 16–2 looks more complicated than Eq. 16–1, but other fundamental equations we haven't seen yet are simpler in terms of ϵ_0 rather than k. It doesn't matter which form we use since Eqs. 16–1 and 16–2 are equivalent. (The latest precise values of e and ϵ_0 are given inside the front cover.)†

Equations 16–1 and 16–2 apply to objects whose size is much smaller than the distance between them. Ideally, it is precise for **point charges** (spatial size negligible compared to other distances). For finite-sized objects, it is not always clear what value to use for r, particularly since the charge may not be distributed uniformly on the objects. If the two objects are spheres and the charge is known to be distributed uniformly on each, then r is the distance between their centers.

Coulomb's law describes the force between two charges when they are at rest. Additional forces come into play when charges are in motion, and will be discussed in later Chapters. In this Chapter we discuss only charges at rest, the study of which is called **electrostatics**, and Coulomb's law gives the **electrostatic force**.

When calculating with Coulomb's law, we usually use magnitudes, ignoring signs of the charges, and determine the direction of a force separately based on whether the force is attractive or repulsive.

PROBLEM SOLVING
Use magnitudes in Coulomb's law; find force direction from signs of charges

EXERCISE A Return to the Chapter-Opening Question, page 443, and answer it again now. Try to explain why you may have answered differently the first time.

EXAMPLE 16–1 Electric force on electron by proton. Determine the magnitude and direction of the electric force on the electron of a hydrogen atom exerted by the single proton $(Q_2 = +e)$ that is the atom's nucleus. Assume the average distance between the revolving electron and the proton is $r = 0.53 \times 10^{-10}\,\text{m}$, Fig. 16–16.

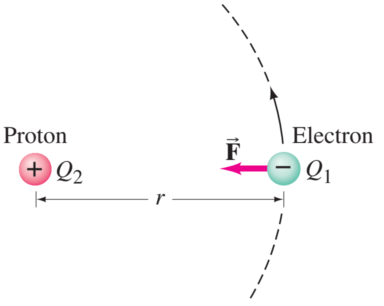


FIGURE 16–16 Example 16–1.

APPROACH To find the force magnitude we use Coulomb's law, $F = k\,Q_1 Q_2/r^2$ (Eq. 16–1), with $r = 0.53 \times 10^{-10}\,\text{m}$. The electron and proton have the same magnitude of charge, e, so $Q_1 = Q_2 = 1.6 \times 10^{-19}\,\text{C}$.

SOLUTION The magnitude of the force is

$$F = k\frac{Q_1 Q_2}{r^2} = \frac{(9.0 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(1.6 \times 10^{-19}\,\text{C})(1.6 \times 10^{-19}\,\text{C})}{(0.53 \times 10^{-10}\,\text{m})^2}$$
$$= 8.2 \times 10^{-8}\,\text{N}.$$

The direction of the force on the electron is toward the proton, because the charges have opposite signs so the force is attractive.

CONCEPTUAL EXAMPLE 16–2 Which charge exerts the greater force? Two positive point charges, $Q_1 = 50\,\mu\text{C}$ and $Q_2 = 1\,\mu\text{C}$, are separated by a distance ℓ, Fig. 16–17. Which is larger in magnitude, the force that Q_1 exerts on Q_2, or the force that Q_2 exerts on Q_1?

FIGURE 16–17 Example 16–2.


RESPONSE From Coulomb's law, the force on Q_1 exerted by Q_2 is

$$F_{12} = k\frac{Q_1 Q_2}{\ell^2}.$$

The force on Q_2 exerted by Q_1 is

$$F_{21} = k\frac{Q_2 Q_1}{\ell^2}$$

which is the same magnitude. The equation is symmetric with respect to the two charges, so $F_{21} = F_{12}$.

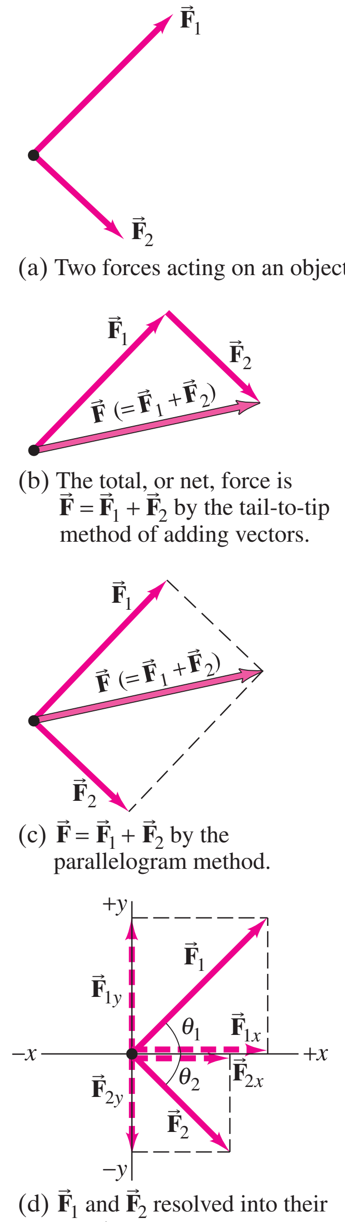
NOTE Newton's third law also tells us these two forces must have equal magnitude.

EXERCISE B In Example 16–2, how is the direction of F_{12} related to the direction of F_{21}?

†Our convention for units, such as $\text{C}^2/\text{N}\cdot\text{m}^2$ for ϵ_0, means m^2 is in the denominator. That is, $\text{C}^2/\text{N}\cdot\text{m}^2$ means $\text{C}^2/(\text{N}\cdot\text{m}^2)$ and does *not* mean $(\text{C}^2/\text{N})\cdot\text{m}^2 = \text{C}^2\cdot\text{m}^2/\text{N}$.

Keep in mind that Coulomb's law, Eq. 16–1 or 16–2, gives the force on a charge due to only *one* other charge. If several (or many) charges are present, the *net force on any one of them will be the vector sum of the forces due to each of the others*. This **principle of superposition** is based on experiment, and tells us that electric force vectors add like any other vector. For example, if you have a system of four charges, the net force on charge 1, say, is the sum of the forces exerted on charge 1 by charges 2, 3, and 4. The magnitudes of these three forces are determined from Coulomb's law, and then are added vectorially.

16–6 Solving Problems Involving Coulomb's Law and Vectors



(a) Two forces acting on an object.

(b) The total, or net, force is $\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$ by the tail-to-tip method of adding vectors.

(c) $\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$ by the parallelogram method.

(d) $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ resolved into their x and y components.

FIGURE 16–18 Review of vector addition.

The electric force between charged particles at rest (sometimes referred to as the **electrostatic force** or as the **Coulomb force**) is, like all forces, a vector: it has both magnitude and direction. When several forces act on an object (call them $\vec{\mathbf{F}}_1$, $\vec{\mathbf{F}}_2$, etc.), the net force $\vec{\mathbf{F}}_{\text{net}}$ on the object is the vector sum of all the forces acting on it:

$$\vec{\mathbf{F}}_{\text{net}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \cdots.$$

This is the principle of superposition for forces. We studied how to add vectors in Chapter 3; then in Chapter 4 we used the rules for adding vectors to obtain the net force on an object by adding the different vector forces acting on it. It might be useful now to review Sections 3–2, 3–3, and 3–4. Here is a brief review of vectors.

Vector Addition Review

Suppose two vector forces, $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$, act on an object (Fig. 16–18a). They can be added using the tail-to-tip method (Fig. 16–18b) or by the parallelogram method (Fig. 16–18c), as discussed in Section 3–2. These two methods are useful for *understanding* a given problem (for getting a picture in your mind of what is going on). But for *calculating* the direction and magnitude of the resultant sum, it is more precise to use the method of adding components. Figure 16–18d shows the forces $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ resolved into components along chosen x and y axes (for more details, see Section 3–4). From the definitions of the trigonometric functions (Figs. 3–11 and 3–12), we have

$$\begin{aligned} F_{1x} &= F_1 \cos\theta_1 \qquad & F_{2x} &= F_2 \cos\theta_2 \\ F_{1y} &= F_1 \sin\theta_1 \qquad & F_{2y} &= -F_2 \sin\theta_2 . \end{aligned}$$

We add up the x and y components separately to obtain the components of the resultant force $\vec{\mathbf{F}}$, which are

$$\begin{aligned} F_x &= F_{1x} + F_{2x} = F_1\cos\theta_1 + F_2\cos\theta_2 , \\ F_y &= F_{1y} + F_{2y} = F_1\sin\theta_1 - F_2\sin\theta_2 . \end{aligned}$$

The magnitude of the resultant (or *net*) force $\vec{\mathbf{F}}$ is

$$F = \sqrt{F_x^2 + F_y^2}.$$

The direction of $\vec{\mathbf{F}}$ is specified by the angle θ that $\vec{\mathbf{F}}$ makes with the x axis, which is given by

$$\tan\theta = \frac{F_y}{F_x}.$$

Adding Electric Forces; Principle of Superposition

When dealing with several charges, it is helpful to use double subscripts on each of the forces involved. The first subscript refers to the particle *on* which the force acts; the second refers to the particle that exerts the force. For example, if we have three charges, $\vec{\mathbf{F}}_{31}$ means the force exerted *on* particle 3 *by* particle 1.

As in all problem solving, it is very important to draw a diagram, in particular a free-body diagram (Chapter 4) for each object, showing all the forces acting *on* that object. In applying Coulomb's law, we can deal with charge magnitudes only (leaving out minus signs) to get the magnitude of each force. Then determine separately the direction of the force physically (along the line joining the two particles: like charges repel, unlike charges attract), and show the force on the diagram. (You could determine direction first if you like.) Finally, add all the forces on one object together as vectors to obtain the net force on that object.

EXAMPLE 16–3 **Three charges in a line.** Three charged particles are arranged in a line, as shown in Fig. 16–19a. Calculate the net electrostatic force on particle 3 (the $-4.0\,\mu$C on the right) due to the other two charges.

APPROACH The net force on particle 3 is the vector sum of the force $\vec{\mathbf{F}}_{31}$ exerted on particle 3 by particle 1 and the force $\vec{\mathbf{F}}_{32}$ exerted on 3 by particle 2:

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_{31} + \vec{\mathbf{F}}_{32}.$$

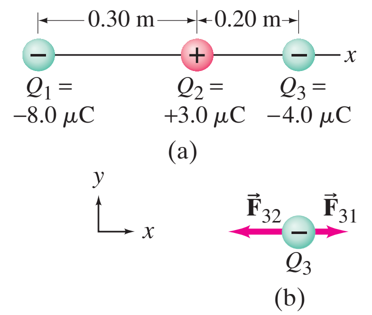


FIGURE 16–19 Example 16–3.

SOLUTION The magnitudes of these two forces are obtained using Coulomb's law, Eq. 16–1:

$$F_{31} = k\frac{Q_3 Q_1}{r_{31}^2} = \frac{(9.0 \times 10^9\,\mathrm{N\cdot m^2/C^2})(4.0 \times 10^{-6}\,\mathrm{C})(8.0 \times 10^{-6}\,\mathrm{C})}{(0.50\,\mathrm{m})^2} = 1.2\,\mathrm{N},$$

where $r_{31} = 0.50$ m is the distance from Q_3 to Q_1. Similarly,

$$F_{32} = k\frac{Q_3 Q_2}{r_{32}^2} = \frac{(9.0 \times 10^9\,\mathrm{N\cdot m^2/C^2})(4.0 \times 10^{-6}\,\mathrm{C})(3.0 \times 10^{-6}\,\mathrm{C})}{(0.20\,\mathrm{m})^2} = 2.7\,\mathrm{N}.$$

Since we were calculating the magnitudes of the forces, we omitted the signs of the charges. But we must be aware of them to get the direction of each force. Let the line joining the particles be the *x* axis, and we take it positive to the right. Then, because $\vec{\mathbf{F}}_{31}$ is repulsive and $\vec{\mathbf{F}}_{32}$ is attractive, the directions of the forces are as shown in Fig. 16–19b: F_{31} points in the positive *x* direction (away from Q_1) and F_{32} points in the negative *x* direction (toward Q_2). The net force on particle 3 is then

$$F = -F_{32} + F_{31} = -2.7\,\mathrm{N} + 1.2\,\mathrm{N} = -1.5\,\mathrm{N}.$$

The magnitude of the net force is 1.5 N, and it points to the left.

NOTE Charge Q_1 acts on charge Q_3 just as if Q_2 were not there (this is the principle of superposition). That is, the charge in the middle, Q_2, in no way blocks the effect of charge Q_1 acting on Q_3. Naturally, Q_2 exerts its own force on Q_3.

CAUTION

Each charge exerts its own force. No charge blocks the effect of the others

EXERCISE C Determine the magnitude and direction of the net force on charge Q_2 in Fig. 16–19a.

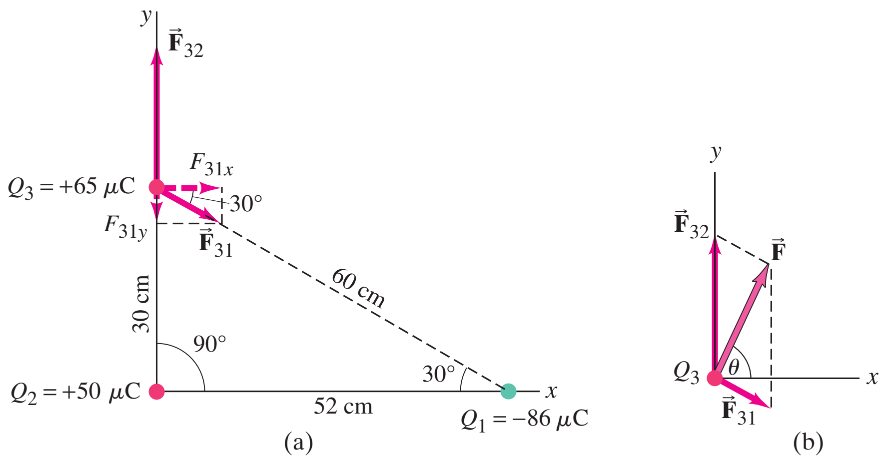


FIGURE 16–20 Determining the forces for Example 16–4. (a) The directions of the individual forces are as shown because $\vec{\mathbf{F}}_{32}$ is repulsive (the force on Q_3 is in the direction away from Q_2 because Q_3 and Q_2 are both positive) whereas $\vec{\mathbf{F}}_{31}$ is attractive (Q_3 and Q_1 have opposite signs), so $\vec{\mathbf{F}}_{31}$ points toward Q_1. (b) Adding $\vec{\mathbf{F}}_{32}$ to $\vec{\mathbf{F}}_{31}$ to obtain the net force $\vec{\mathbf{F}}$.

EXAMPLE 16–4 **Electric force using vector components.** Calculate the net electrostatic force on charge Q_3 shown in Fig. 16–20a due to the charges Q_1 and Q_2.

APPROACH We use Coulomb's law to find the magnitudes of the individual forces. The direction of each force will be along the line connecting Q_3 to Q_1 or Q_2. The forces $\vec{\mathbf{F}}_{31}$ and $\vec{\mathbf{F}}_{32}$ have the directions shown in Fig. 16–20a, since Q_1 exerts an attractive force on Q_3, and Q_2 exerts a repulsive force. The forces $\vec{\mathbf{F}}_{31}$ and $\vec{\mathbf{F}}_{32}$ are *not* along the same line, so to find the resultant force on Q_3 we resolve $\vec{\mathbf{F}}_{31}$ and $\vec{\mathbf{F}}_{32}$ into x and y components and perform the vector addition.

SOLUTION The magnitudes of $\vec{\mathbf{F}}_{31}$ and $\vec{\mathbf{F}}_{32}$ are (ignoring signs of the charges since we know the directions)

$$F_{31} = k\frac{Q_3 Q_1}{r_{31}^2} = \frac{(9.0 \times 10^9\,\mathrm{N\cdot m^2/C^2})(6.5 \times 10^{-5}\,\mathrm{C})(8.6 \times 10^{-5}\,\mathrm{C})}{(0.60\,\mathrm{m})^2} = 140\,\mathrm{N},$$

$$F_{32} = k\frac{Q_3 Q_2}{r_{32}^2} = \frac{(9.0 \times 10^9\,\mathrm{N\cdot m^2/C^2})(6.5 \times 10^{-5}\,\mathrm{C})(5.0 \times 10^{-5}\,\mathrm{C})}{(0.30\,\mathrm{m})^2} = 325\,\mathrm{N}.$$

(We keep 3 significant figures until the end, and then keep 2 because only 2 are given.) We resolve $\vec{\mathbf{F}}_{31}$ into its components along the x and y axes, as shown in Fig. 16–20a:

$$F_{31x} = F_{31}\cos 30° = (140\,\mathrm{N})\cos 30° = 120\,\mathrm{N},$$
$$F_{31y} = -F_{31}\sin 30° = -(140\,\mathrm{N})\sin 30° = -70\,\mathrm{N}.$$

The force $\vec{\mathbf{F}}_{32}$ has only a y component. So the net force $\vec{\mathbf{F}}$ on Q_3 has components

$$F_x = F_{31x} = 120\,\mathrm{N},$$
$$F_y = F_{32} + F_{31y} = 325\,\mathrm{N} - 70\,\mathrm{N} = 255\,\mathrm{N}.$$

The magnitude of the net force is

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(120\,\mathrm{N})^2 + (255\,\mathrm{N})^2} = 280\,\mathrm{N};$$

and it acts at an angle θ (see Fig. 16–20b) given by

$$\tan\theta = \frac{F_y}{F_x} = \frac{255\,\mathrm{N}}{120\,\mathrm{N}} = 2.13,$$

so $\theta = \tan^{-1}(2.13) = 65°$.

NOTE Because $\vec{\mathbf{F}}_{31}$ and $\vec{\mathbf{F}}_{32}$ are not along the same line, the magnitude of $\vec{\mathbf{F}}_3$ is not equal to the sum (or difference as in Example 16–3) of the separate magnitudes. That is, F_3 is not equal to $F_{31} + F_{32}$; nor does it equal $F_{32} - F_{31}$. Instead we had to do vector addition.

CONCEPTUAL EXAMPLE 16–5 **Make the force on Q_3 zero.** In Fig. 16–20, where could you place a fourth charge, $Q_4 = -50\,\mu C$, so that the net force on Q_3 would be zero?

RESPONSE By the principle of superposition, we need a force in exactly the opposite direction to the resultant $\vec{F}$ due to Q_2 and Q_1 that we calculated in Example 16–4, Fig. 16–20b. Our force must have magnitude 290 N, and must point down and to the left of Q_3 in Fig. 16–20b, in the direction opposite to $\vec{F}$. So Q_4 must be along this line. See Fig. 16–21.

EXERCISE D In Example 16–5, what distance r must Q_4 be from Q_3?

EXERCISE E (*a*) Consider two point charges, $+Q$ and $-Q$, which are fixed a distance d apart. Can you find a location where a third positive charge Q could be placed so that the net electric force on this third charge is zero? (*b*) What if the first two charges were both $+Q$?

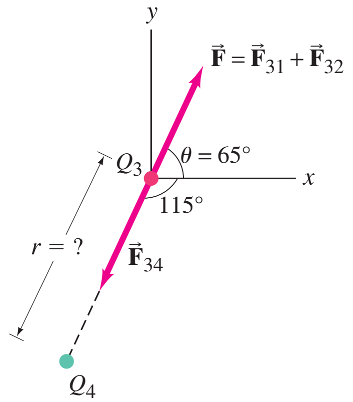


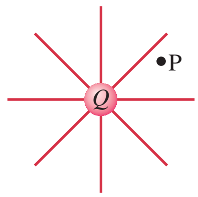
FIGURE 16–21 Example 16–5 and Exercise D: Q_4 exerts force $(\vec{F}_{34})$ that makes the net force on Q_3 zero.

16–7 The Electric Field

Many common forces might be referred to as "contact forces," such as your hands pushing or pulling a cart, or a tennis racket hitting a tennis ball.

In contrast, both the gravitational force and the electrical force act over a distance: there is a force between two objects even when the objects are not touching. The idea of a force *acting at a distance* was a difficult one for early thinkers. Newton himself felt uneasy with this idea when he published his law of universal gravitation. A helpful way to look at the situation uses the idea of the **field**, developed by the British scientist Michael Faraday (1791–1867). In the electrical case, according to Faraday, an *electric field* extends outward from every charge and permeates all of space (Fig. 16–22). If a second charge (call it Q_2) is placed near the first charge, it feels a force exerted by the electric field that is there (say, at point P in Fig. 16–22). The electric field at point P is considered to interact directly with charge Q_2 to produce the force on Q_2.

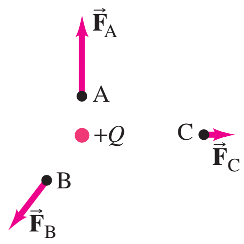
FIGURE 16–22 An electric field surrounds every charge. The red lines indicate the electric field extending out from charge Q, and P is an arbitrary point.



We can in principle investigate the electric field surrounding a charge or group of charges by measuring the force on a small positive **test charge** which is at rest. By a test charge we mean a charge so small that the force it exerts does not significantly affect the charges that create the field. If a tiny positive test charge q is placed at various locations in the vicinity of a single positive charge Q as shown in Fig. 16–23 (points A, B, C), the force exerted on q is as shown. The force at B is less than at A because B's distance from Q is greater (Coulomb's law); and the force at C is smaller still. In each case, the force on q is directed radially away from Q. The electric field is defined in terms of the force on such a positive test charge. In particular, the **electric field**, $\vec{E}$, at any point in space is defined as the force $\vec{F}$ exerted on a tiny positive test charge placed at that point divided by the magnitude of the test charge q:

$$\vec{E} = \frac{\vec{F}}{q}. \tag{16–3}$$

FIGURE 16–23 Force exerted by charge $+Q$ on a small test charge, q, placed at points A, B, and C.



More precisely, $\vec{E}$ is defined as the limit of $\vec{F}/q$ as q is taken smaller and smaller, approaching zero. That is, q is so tiny that it exerts essentially no force on the other charges which created the field. From this definition (Eq. 16–3), we see that the electric field at any point in space is a vector whose direction is the direction of the force on a tiny positive test charge at that point, and whose magnitude is the *force per unit charge*. Thus $\vec{E}$ has SI units of newtons per coulomb (N/C).

The reason for defining $\vec{E}$ as $\vec{F}/q$ (with $q \to 0$) is so that $\vec{E}$ does not depend on the magnitude of the test charge q. This means that $\vec{E}$ describes only the effect of the charges creating the electric field at that point.

The electric field at any point in space can be measured, based on the definition, Eq. 16–3. For simple situations with one or several point charges, we can calculate $\vec{\mathbf{E}}$. For example, the electric field at a distance r from a single point charge Q would have magnitude

$$E = \frac{F}{q} = \frac{kqQ/r^2}{q}$$

$$E = k\frac{Q}{r^2}; \qquad \text{[single point charge]} \quad \textbf{(16–4a)}$$

or, in terms of ϵ_0 as in Eq. 16–2 $(k = 1/4\pi\epsilon_0)$:

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}. \qquad \text{[single point charge]} \quad \textbf{(16–4b)}$$

Notice that E is independent of the test charge q—that is, E depends only on the charge Q which produces the field, and not on the value of the test charge q. Equations 16–4 are referred to as the electric field form of Coulomb's law.

If we are given the electric field $\vec{\mathbf{E}}$ at a given point in space, then we can calculate the force $\vec{\mathbf{F}}$ on any charge q placed at that point by writing (see Eq. 16–3):

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}}. \qquad \textbf{(16–5)}$$

This is valid even if q is not small as long as q does not cause the charges creating $\vec{\mathbf{E}}$ to move. If q is positive, $\vec{\mathbf{F}}$ and $\vec{\mathbf{E}}$ point in the same direction. If q is negative, $\vec{\mathbf{F}}$ and $\vec{\mathbf{E}}$ point in opposite directions. See Fig. 16–24.

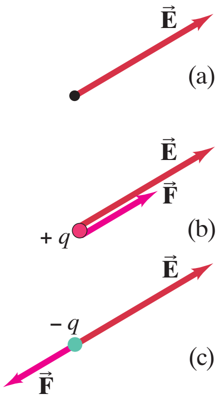


FIGURE 16–24 (a) Electric field at a given point in space. (b) Force on a positive charge at that point. (c) Force on a negative charge at that point.

PHYSICS APPLIED
Photocopier

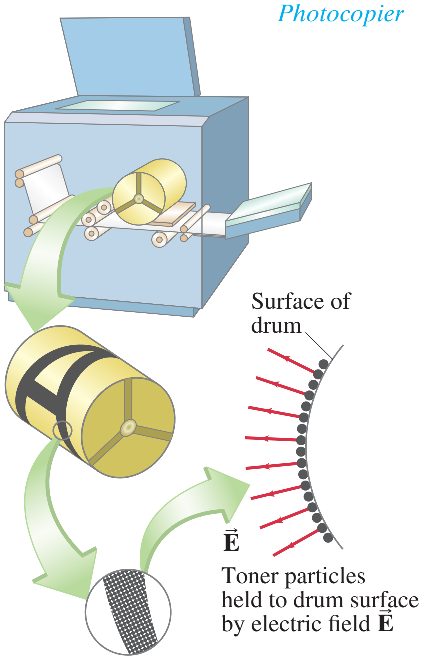


FIGURE 16–25 Example 16–6.

EXAMPLE 16–6 **Photocopy machine.** A photocopy machine works by arranging positive charges (in the pattern to be copied) on the surface of a drum, then gently sprinkling negatively charged dry toner (ink) particles onto the drum. The toner particles temporarily stick to the pattern on the drum (Fig. 16–25) and are later transferred to paper and "melted" to produce the copy. Suppose each toner particle has a mass of 9.0×10^{-16} kg and carries an average of 20 extra electrons to provide an electric charge. Assuming that the electric force on a toner particle must exceed twice its weight in order to ensure sufficient attraction, compute the required electric field strength near the surface of the drum.

APPROACH The electric force on a toner particle of charge $q = 20e$ is $F = qE$, where E is the needed electric field. This force needs to be at least as great as twice the weight (mg) of the particle.

SOLUTION The minimum value of electric field satisfies the relation

$$qE = 2mg$$

where $q = 20e$. Hence

$$E = \frac{2mg}{q} = \frac{2(9.0 \times 10^{-16}\,\text{kg})(9.8\,\text{m/s}^2)}{20(1.6 \times 10^{-19}\,\text{C})} = 5.5 \times 10^3\,\text{N/C}.$$

EXAMPLE 16–7 **Electric field of a single point charge.** Calculate the magnitude and direction of the electric field at a point P which is 30 cm to the right of a point charge $Q = -3.0 \times 10^{-6}$ C.

APPROACH The magnitude of the electric field due to a single point charge is given by Eq. 16–4. The direction is found using the sign of the charge Q.

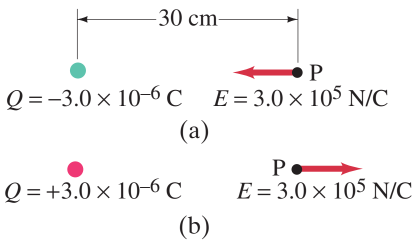
SOLUTION The magnitude of the electric field is:

$$E = k\frac{Q}{r^2} = \frac{(9.0 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(3.0 \times 10^{-6}\,\text{C})}{(0.30\,\text{m})^2} = 3.0 \times 10^5\,\text{N/C}.$$

The direction of the electric field is *toward* the charge Q, to the left as shown in Fig. 16–26a, since we defined the direction as that of the force on a positive test charge which here would be attractive. If Q had been positive, the electric field would have pointed away, as in Fig. 16–26b.

NOTE There is no electric charge at point P. But there is an electric field there. The only real charge is Q.

FIGURE 16–26 Example 16–7. Electric field at point P (a) due to a negative charge Q, and (b) due to a positive charge Q, each 30 cm from P.



This Example illustrates a general result: The electric field $\vec{\mathbf{E}}$ due to a positive charge points away from the charge, whereas $\vec{\mathbf{E}}$ due to a negative charge points toward that charge.

EXERCISE F Find the magnitude and direction of the electric field due to a $-2.5\,\mu\text{C}$ charge 50 cm below it.

If the electric field at a given point in space is due to more than one charge, the individual fields (call them $\vec{\mathbf{E}}_1$, $\vec{\mathbf{E}}_2$, etc.) due to each charge are added vectorially to get the total field at that point:

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 + \cdots.$$

The validity of this **superposition principle** for electric fields is fully confirmed by experiment.

EXAMPLE 16–8 **$\vec{\mathbf{E}}$ at a point between two charges.** Two point charges are separated by a distance of 10.0 cm. One has a charge of $-25\,\mu\text{C}$ and the other $+50\,\mu\text{C}$. (*a*) Determine the direction and magnitude of the electric field at a point P between the two charges that is 2.0 cm from the negative charge (Fig. 16–27a). (*b*) If an electron $(\text{mass} = 9.11 \times 10^{-31}\,\text{kg})$ is placed at rest at P and then released, what will be its initial acceleration (direction and magnitude)?

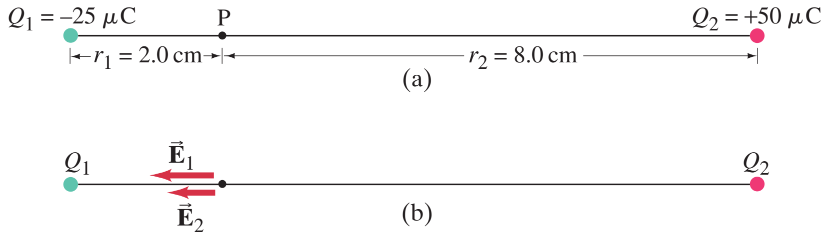


FIGURE 16–27 Example 16–8. In (b), we don't know the relative lengths of $\vec{\mathbf{E}}_1$ and $\vec{\mathbf{E}}_2$ until we do the calculation.

APPROACH The electric field at P will be the vector sum of the fields created separately by Q_1 and Q_2. The field due to the negative charge Q_1 points toward Q_1, and the field due to the positive charge Q_2 points away from Q_2. Thus both fields point to the left as shown in Fig. 16–27b, and we can add the magnitudes of the two fields together algebraically, ignoring the signs of the charges. In (*b*) we use Newton's second law $(\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}})$ to find the acceleration, where $\Sigma\vec{\mathbf{F}} = q\Sigma\vec{\mathbf{E}}$.

SOLUTION (*a*) Each field is due to a point charge as given by Eq. 16–4, $E = kQ/r^2$. The total field points to the left and has magnitude

$$\begin{aligned} E &= k\frac{Q_1}{r_1^2} + k\frac{Q_2}{r_2^2} = k\left(\frac{Q_1}{r_1^2} + \frac{Q_2}{r_2^2}\right) \\ &= \left(9.0 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{25 \times 10^{-6}\,\text{C}}{(2.0 \times 10^{-2}\,\text{m})^2} + \frac{50 \times 10^{-6}\,\text{C}}{(8.0 \times 10^{-2}\,\text{m})^2}\right) \\ &= 6.3 \times 10^8\,\text{N/C}. \end{aligned}$$

(*b*) The electric field points to the left, so the electron will feel a force to the *right* since it is negatively charged. Therefore the acceleration $a = F/m$ (Newton's second law) will be to the right. The force on a charge q in an electric field E is $F = qE$ (Eq. 16–5). Hence the magnitude of the electron's initial acceleration is

$$a = \frac{F}{m} = \frac{qE}{m} = \frac{\left(1.60 \times 10^{-19}\,\text{C}\right)\left(6.3 \times 10^8\,\text{N/C}\right)}{9.11 \times 10^{-31}\,\text{kg}} = 1.1 \times 10^{20}\,\text{m/s}^2.$$

NOTE By considering the directions of *each* field ($\vec{\mathbf{E}}_1$ and $\vec{\mathbf{E}}_2$) before doing any calculations, we made sure our calculation could be done simply and correctly.

EXERCISE G Four charges of equal magnitude, but possibly different sign, are placed on the corners of a square. What arrangement of charges will produce an electric field with the greatest magnitude at the center of the square? (*a*) All four positive charges; (*b*) all four negative charges; (*c*) three positive and one negative; (*d*) two positive and two negative; (*e*) three negative and one positive.

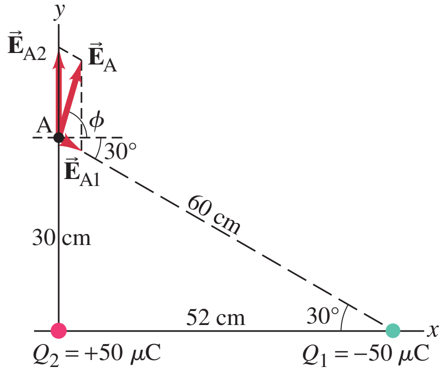


FIGURE 16–28 Calculation of the electric field at point A, Example 16–9.

PROBLEM SOLVING

Ignore signs of charges and determine direction physically, showing directions on diagram

EXAMPLE 16–9 **$\vec{\mathbf{E}}$ above two point charges.** Calculate the total electric field at point A in Fig. 16–28 due to both charges, Q_1 and Q_2.

APPROACH The calculation is much like that of Example 16–4, except now we are dealing with electric fields instead of force. The electric field at point A is the vector sum of the fields $\vec{\mathbf{E}}_{A1}$ due to Q_1, and $\vec{\mathbf{E}}_{A2}$ due to Q_2. We find the magnitude of the field produced by each point charge, then we add their components to find the total field at point A.

SOLUTION The magnitude of the electric field produced at point A by each of the charges Q_1 and Q_2 is given by $E = kQ/r^2$, so

$$E_{A1} = \frac{(9.0 \times 10^9\,\mathrm{N\cdot m^2/C^2})(50 \times 10^{-6}\,\mathrm{C})}{(0.60\,\mathrm{m})^2} = 1.25 \times 10^6\,\mathrm{N/C},$$

$$E_{A2} = \frac{(9.0 \times 10^9\,\mathrm{N\cdot m^2/C^2})(50 \times 10^{-6}\,\mathrm{C})}{(0.30\,\mathrm{m})^2} = 5.0 \times 10^6\,\mathrm{N/C}.$$

The direction of E_{A1} points from A toward Q_1 (negative charge), whereas E_{A2} points from A away from Q_2, as shown; so the total electric field at A, $\vec{\mathbf{E}}_A$, has components

$$E_{Ax} = E_{A1}\cos 30° = 1.1 \times 10^6\,\mathrm{N/C},$$
$$E_{Ay} = E_{A2} - E_{A1}\sin 30° = 4.4 \times 10^6\,\mathrm{N/C}.$$

Thus the magnitude of $\vec{\mathbf{E}}_A$ is

$$E_A = \sqrt{(1.1)^2 + (4.4)^2} \times 10^6\,\mathrm{N/C} = 4.5 \times 10^6\,\mathrm{N/C},$$

and its direction is ϕ (Fig. 16–28) given by $\tan\phi = E_{Ay}/E_{Ax} = 4.4/1.1 = 4.0$, so $\phi = 76°$.

It is worthwhile summarizing here what we have learned about solving electrostatics problems.

PROBLEM SOLVING

Electrostatics: Electric Forces and Electric Fields

Whether you use electric field or electrostatic forces, the procedure for solving electrostatics problems is similar:

1. **Draw** a careful **diagram**—namely, a free-body diagram for each object, showing all the forces acting on that object, or showing the electric field at a point due to all significant charges present. Determine the **direction** of each force or electric field physically: like charges repel each other, unlike charges attract; fields point away from a + charge, and toward a − charge. Show and label each vector force or field on your diagram.
2. **Apply Coulomb's law** to calculate the magnitude of the force that each contributing charge exerts on a charged object, or the magnitude of the electric field each charge produces at a given point. Deal only with magnitudes of charges (leaving out minus signs), and obtain the magnitude of each force or electric field.
3. **Add vectorially** all the forces on an object, or the contributing fields at a point, to get the resultant. Use **symmetry** (say, in the geometry) whenever possible.

EXAMPLE 16–10 **$\vec{\mathbf{E}}$ equidistant above two point charges.** Figure 16–29 (top of next page) is the same as Fig. 16–28 but includes point B, which is equidistant (40 cm) from Q_1 and Q_2. Calculate the total electric field at point B in Fig. 16–29 due to both charges, Q_1 and Q_2.

APPROACH We explicitly follow the steps of the Problem Solving Strategy above.

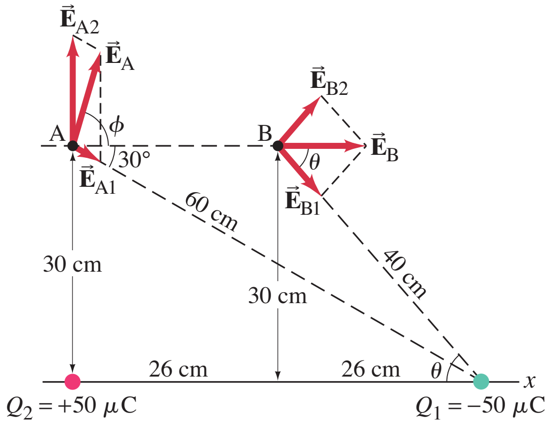


FIGURE 16–29 Same as Fig. 16–28 but with point B added. Calculation of the electric field at points A and B for Examples 16–9 and 16–10.

SOLUTION

1. **Draw** a careful **diagram**. The **directions** of the electric fields $\vec{\mathbf{E}}_{B1}$ and $\vec{\mathbf{E}}_{B2}$, as well as the net field $\vec{\mathbf{E}}_B$, are shown in Fig. 16–29. $\vec{\mathbf{E}}_{B2}$ points away from the positive charge Q_2; $\vec{\mathbf{E}}_{B1}$ points toward the negative charge Q_1.
2. **Apply Coulomb's law** to find the magnitudes of the contributing electric fields. Because B is equidistant from the two equal charges (40 cm by the Pythagorean theorem), the magnitudes of E_{B1} and E_{B2} are the same; that is,

$$E_{B1} = E_{B2} = \frac{kQ}{r^2} = \frac{(9.0 \times 10^9\ \mathrm{N\cdot m^2/C^2})(50 \times 10^{-6}\ \mathrm{C})}{(0.40\ \mathrm{m})^2} = 2.8 \times 10^6\ \mathrm{N/C}.$$

3. **Add vectorially**, and use **symmetry** when possible. The y components of $\vec{\mathbf{E}}_{B1}$ and $\vec{\mathbf{E}}_{B2}$ are equal and opposite. Because of this symmetry, the total field E_B is horizontal and equals $E_{B1}\cos\theta + E_{B2}\cos\theta = 2\,E_{B1}\cos\theta$. From Fig. 16–29, $\cos\theta = 26\ \mathrm{cm}/40\ \mathrm{cm} = 0.65$. Then

$$E_B = 2E_{B1}\cos\theta = 2(2.8 \times 10^6\ \mathrm{N/C})(0.65) = 3.6 \times 10^6\ \mathrm{N/C},$$

and the direction of $\vec{\mathbf{E}}_B$ is along the $+x$ direction.

PROBLEM SOLVING
Use symmetry to save work, when possible

16–8 Electric Field Lines

Since the electric field is a vector, it is sometimes referred to as a *vector field*. We could indicate the electric field with arrows at various points in a given situation, such as at A, B, and C in Fig. 16–30. The directions of $\vec{\mathbf{E}}_A$, $\vec{\mathbf{E}}_B$, and $\vec{\mathbf{E}}_C$ are the same as for the forces shown earlier in Fig. 16–23, but the magnitudes (arrow lengths) are different since we divide $\vec{\mathbf{F}}$ by q to get $\vec{\mathbf{E}}$. However, the relative lengths of $\vec{\mathbf{E}}_A$, $\vec{\mathbf{E}}_B$, and $\vec{\mathbf{E}}_C$ are the same as for the forces since we divide by the same q each time. To indicate the electric field in such a way at *many* points, however, would result in many arrows, which would quickly become cluttered and confusing. To avoid this, we use another technique, that of field lines.

To visualize the electric field, we draw a series of lines to indicate the direction of the electric field at various points in space. These **electric field lines** (or **lines of force**) are drawn to indicate the direction of the force due to the given field on a positive test charge. The lines of force due to a single isolated positive charge are shown in Fig. 16–31a, and for a single isolated negative charge in Fig. 16–31b. In part (a) the lines point radially outward from the charge, and in part (b) they point radially inward toward the charge because that is the direction the force would be on a positive test charge in each case (as in Fig. 16–26). Only a few representative lines are shown. We could draw lines in between those shown since the electric field exists there as well. We can draw the lines so that the *number of lines starting on a positive charge, or ending on a negative charge, is proportional to the magnitude of the charge*. Notice that nearer the charge, where the electric field is greater $(F \propto 1/r^2)$, the lines are closer together. This is a general property of electric field lines: *the closer together the lines are, the stronger the electric field in that region*. In fact, field lines can be drawn so that the number of lines crossing unit area perpendicular to $\vec{\mathbf{E}}$ is proportional to the magnitude of the electric field.

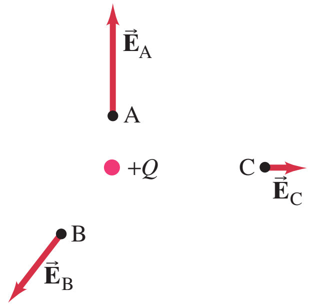
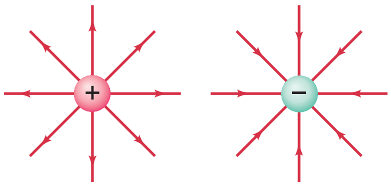


FIGURE 16–30 Electric field vector, shown at three points, due to a single point charge Q. (Compare to Fig. 16–23.)

FIGURE 16–31 Electric field lines (a) near a single positive point charge, (b) near a single negative point charge.



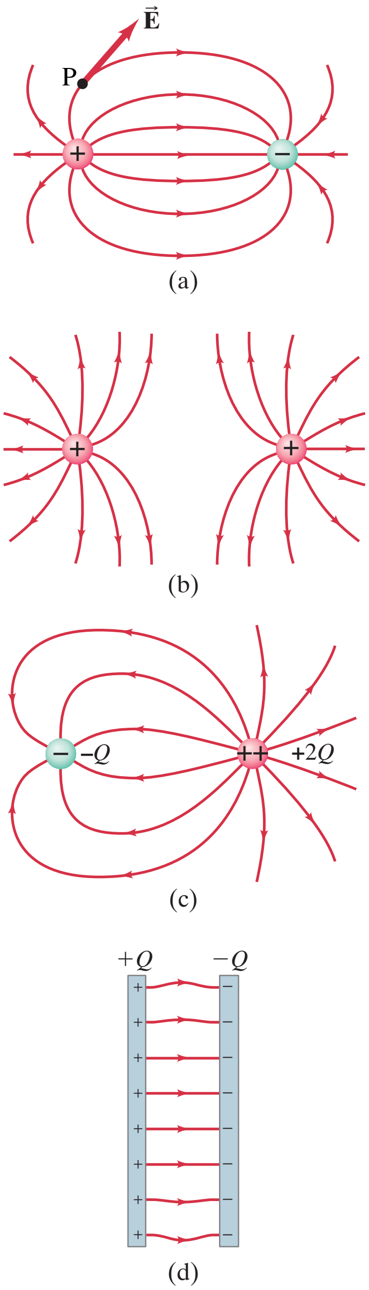


FIGURE 16–32 Electric field lines for four arrangements of charges.

FIGURE 16–33 The Earth's gravitational field, which at any point is directed toward the Earth's center (the force on any mass points toward the Earth's center).

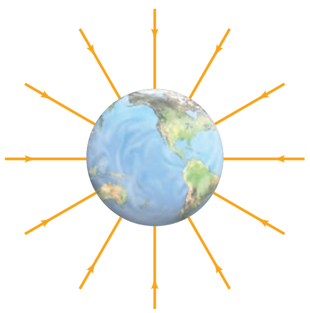


Figure 16–32a shows the electric field lines due to two equal charges of opposite sign, a combination known as an **electric dipole**. The electric field lines are curved in this case and are directed from the positive charge to the negative charge. The direction of the electric field at any point is tangent to the field line at that point as shown by the vector arrow $\vec{\mathbf{E}}$ at point P. To satisfy yourself that this is the correct pattern for the electric field lines, you can make a few calculations such as those done in Examples 16–9 and 16–10 for just this case (see Fig. 16–29). Figure 16–32b shows the electric field lines for two equal positive charges, and Fig. 16–32c for unequal charges, $-Q$ and $+2Q$. Note that twice as many lines leave $+2Q$ as enter $-Q$ (number of lines is proportional to magnitude of Q). Finally, in Fig. 16–32d, we see in cross section the field lines between two flat parallel plates carrying equal but opposite charges. Notice that the electric field lines between the two plates start out perpendicular to the surface of the metal plates (we will see why this is true in the next Section) and go directly from one plate to the other, as we expect because a positive test charge placed between the plates would feel a strong repulsion from the positive plate and a strong attraction to the negative plate. The field lines between two close plates are parallel and equally spaced in the central region, but fringe outward near the edges. Thus, in the central region, the electric field has the same magnitude at all points, and we can write

$$E = \text{constant.} \quad \begin{bmatrix} \text{between two closely spaced, oppositely} \\ \text{charged, flat parallel plates} \end{bmatrix} \qquad \textbf{(16–6)}$$

The fringing of the field near the edges can often be ignored, particularly if the separation of the plates is small compared to their height and width.† We summarize the properties of field lines as follows:

1. Electric field lines indicate the direction of the electric field; the field points in the direction tangent to the field line at any point.
2. The lines are drawn so that the magnitude of the electric field, E, is proportional to the number of lines crossing unit area perpendicular to the lines. The closer together the lines, the stronger the field.
3. Electric field lines start on positive charges and end on negative charges; and the number starting or ending is proportional to the magnitude of the charge.

Also note that field lines never cross. Why not? Because it would not make sense for the electric field to have two directions at the same point.

Gravitational Field

The field concept can also be applied to the gravitational force (Chapter 5). Thus we can say that a **gravitational field** exists for every object that has mass. One object attracts another by means of the gravitational field. The Earth, for example, can be said to possess a gravitational field (Fig. 16–33) which is responsible for the gravitational force on objects. The *gravitational field* is defined as the *force per unit mass*. The magnitude of the Earth's gravitational field at any point above the Earth's surface is thus GM_E/r^2, where M_E is the mass of the Earth, r is the distance of the point from the Earth's center, and G is the gravitational constant (Chapter 5). At the Earth's surface, r is the radius of the Earth and the gravitational field is equal to g, the acceleration due to gravity. Beyond the Earth, the gravitational field can be calculated at any point as a sum of terms due to Earth, Sun, Moon, and other bodies that contribute significantly.

†The magnitude of the constant electric field between two parallel plates is given by $E = Q/\epsilon_0 A$, where Q is the magnitude of the charge on each plate and A is the area of one plate. We show this in the optional Section 16–12 on Gauss's law.

16–9 Electric Fields and Conductors

We now discuss some properties of conductors. First, *the electric field inside a conductor is zero in the static situation*—that is, when the charges are at rest. If there were an electric field within a conductor, there would be a force on the free electrons. The electrons would move until they reached positions where the electric field, and therefore the electric force on them, did become zero.

This reasoning has some interesting consequences. For one, *any net charge on a conductor distributes itself on the surface*. For a negatively charged conductor, you can imagine that the negative charges repel one another and race to the surface to get as far from one another as possible. Another consequence is the following. Suppose that a positive charge Q is surrounded by an isolated uncharged metal conductor whose shape is a spherical shell, Fig. 16–34. Because there can be no field within the metal, the lines leaving the central positive charge must end on negative charges on the inner surface of the metal. That is, the encircled charge $+Q$ induces an equal amount of negative charge, $-Q$, on the inner surface of the spherical shell. Since the shell is neutral, a positive charge of the same magnitude, $+Q$, must exist on the outer surface of the shell. Thus, although no field exists in the metal itself, an electric field exists outside of it, as shown in Fig. 16–34, as if the metal were not even there.

A related property of static electric fields and conductors is that *the electric field is always perpendicular to the surface outside of a conductor*. If there were a component of $\vec{\mathbf{E}}$ parallel to the surface (Fig. 16–35), it would exert a force on free electrons at the surface, causing the electrons to move along the surface until they reached positions where no net force was exerted on them parallel to the surface—that is, until the electric field was perpendicular to the surface.

These properties apply only to conductors. Inside a nonconductor, which does not have free electrons, a static electric field can exist as we will see in the next Chapter. Also, the electric field outside a nonconductor does not necessarily make an angle of 90° to the surface.

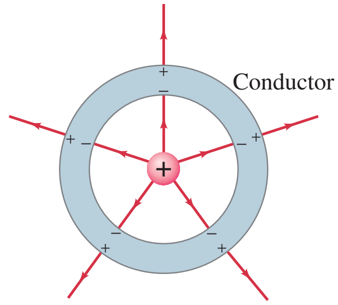
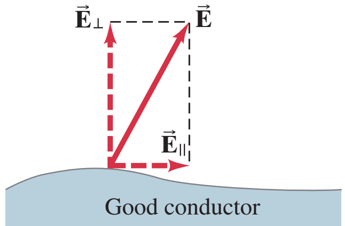


FIGURE 16–34 A charge inside a neutral spherical metal shell induces charge on its surfaces. The electric field exists even beyond the shell, but not within the conductor itself.

FIGURE 16–35 If the electric field $\vec{\mathbf{E}}$ at the surface of a conductor had a component parallel to the surface, $\vec{\mathbf{E}}_{\parallel}$, the latter would accelerate electrons into motion. In the static case, $\vec{\mathbf{E}}_{\parallel}$ must be zero, and the electric field must be perpendicular to the conductor's surface: $\vec{\mathbf{E}} = \vec{\mathbf{E}}_{\perp}$.



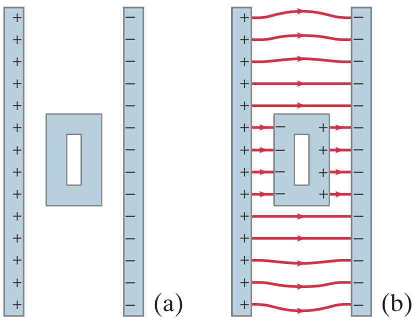


FIGURE 16–36 Example 16–11.

FIGURE 16–37 High-voltage "Van de Graaff" generators create strong electric fields in the vicinity of the "Faraday cage" below. The strong field accelerates stray electrons in the atmosphere to the KE needed to knock electrons out of air atoms, causing an avalanche of charge which flows to (or from) the metal cage. The metal cage protects the person inside it.

CONCEPTUAL EXAMPLE 16–11 **Shielding, and safety in a storm.** A neutral hollow metal box is placed between two parallel charged plates as shown in Fig. 16–36a. What is the field like inside the box?

RESPONSE If our metal box had been solid, and not hollow, free electrons in the box would have redistributed themselves along the surface until all their individual fields would have canceled each other inside the box. The net field inside the box would have been zero. For a hollow box, the external field is not changed since the electrons in the metal can move just as freely as before to the surface. Hence the field inside the hollow metal box is also zero, and the field lines are shown in Fig. 16–36b. A conducting box is an effective device for shielding delicate instruments and electronic circuits from unwanted external electric fields. We also can see that a relatively safe place to be during a lightning storm is inside a parked car, surrounded by metal. See also Fig. 16–37, where a person inside a porous "cage" is protected from a strong electric discharge. (It is not safe in a lightning storm to be near a tree which can conduct, or out in the open where you are taller than the surroundings.)

PHYSICS APPLIED
Electrical shielding

*16–10 Electric Forces in Molecular Biology: DNA Structure and Replication

PHYSICS APPLIED
Inside a cell: kinetic theory plus electrostatic force

The study of the structure and functioning of a living cell at the molecular level is known as molecular biology. It is an important area for application of physics. The interior of every biological cell is mainly water. We can imagine a cell as a thick soup of molecules continually in motion (kinetic theory, Chapter 13), colliding with one another with various amounts of kinetic energy. These molecules interact with one another because of the *electrostatic force* between molecules.

Indeed, cellular processes are now considered to be the result of *random ("thermal") molecular motion plus the ordering effect of the electrostatic force*. As an example, we look at DNA structure and replication. The picture we present is a model of what happens based on physical theories and experiment.

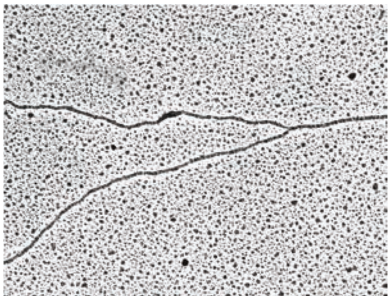
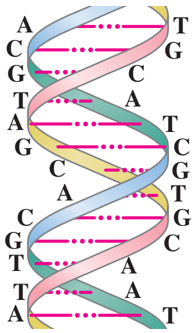


FIGURE 16–38 Image of DNA replicating, made by a transmission electron microscope.

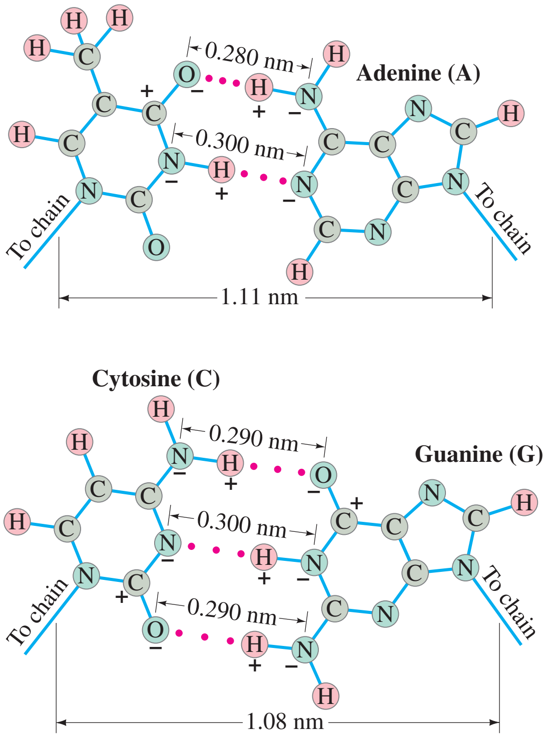
The genetic information that is passed on from generation to generation in all living cells is contained in the chromosomes, which are made up of genes. Each gene contains the information needed to produce a particular type of protein molecule, and that information is built into the principal molecule of a chromosome, DNA (deoxyribonucleic acid), Fig. 16–38. DNA molecules are made up of many small molecules known as nucleotide bases which are each *polar* (Section 16–2) due to unequal sharing of electrons. There are four types of nucleotide bases in DNA: adenine (A), cytosine (C), guanine (G), and thymine (T).

PHYSICS APPLIED
DNA structure

The DNA of a chromosome generally consists of two long DNA strands wrapped about one another in the shape of a "double helix." The genetic information is contained in the specific order of the four bases (A, C, G, T) along each strand. As shown in Fig. 16–39, the two strands are attracted by electrostatic forces—that is, by the attraction of positive charges to negative charges that exist on parts of the molecules. We see in Fig. 16–39a that an A (adenine) on one strand is always opposite a T on the other strand; similarly, a G is always opposite a C. This important ordering effect occurs because the shapes of A, T, C, and G are such that a T fits closely only into an A, and a G into a C. Only in the case of this close proximity of the charged portions is the electrostatic force great enough to hold them together even for a short time (Fig. 16–39b), forming what are referred to as "weak bonds."



(a)



(b)

FIGURE 16–39 (a) Schematic diagram of a section of DNA double helix. (b) "Close-up" view of the helix, showing how A and T attract each other and how G and C attract each other through electrostatic forces. The + and − signs indicated on certain atoms represent net charges, usually a fraction of e, due to uneven sharing of electrons. The red dots indicate the electrostatic attraction (often called a "weak bond" or "hydrogen bond"—Section 29–3). Note that there are two weak bonds between A and T, and three between C and G.

The electrostatic force between A and T, and between C and G, exists because these molecules have charged parts. These charges are due to some electrons in each of these molecules spending more time orbiting one atom than another. For example, the electron normally on the H atom of adenine (upper part of Fig. 16–39b) spends some of its time orbiting the adjacent N atom (more on this in Chapter 29), so the N has a net negative charge and the H is left with a net positive charge. This H^+ atom of adenine is then attracted to the O^- atom of thymine. These net + and − charges usually have magnitudes of a fraction of e (charge on the electron) such as $0.2e$ or $0.4e$. (This is what we mean by "polar" molecules.)

[When H^+ is involved, the weak bond it can make with a nearby negative charge, such as O^-, is relatively strong (partly because H^+ is so small) and is referred to as a **hydrogen bond** (Section 29–3).]

PHYSICS APPLIED
DNA replication

When the DNA replicates (duplicates) itself just before cell division, the arrangement of A opposite T and G opposite C is crucial for ensuring that the genetic information is passed on accurately to the next generation, Fig. 16–40. The two strands of DNA separate (with the help of enzymes, which also operate via the electrostatic force), leaving the charged parts of the bases exposed.

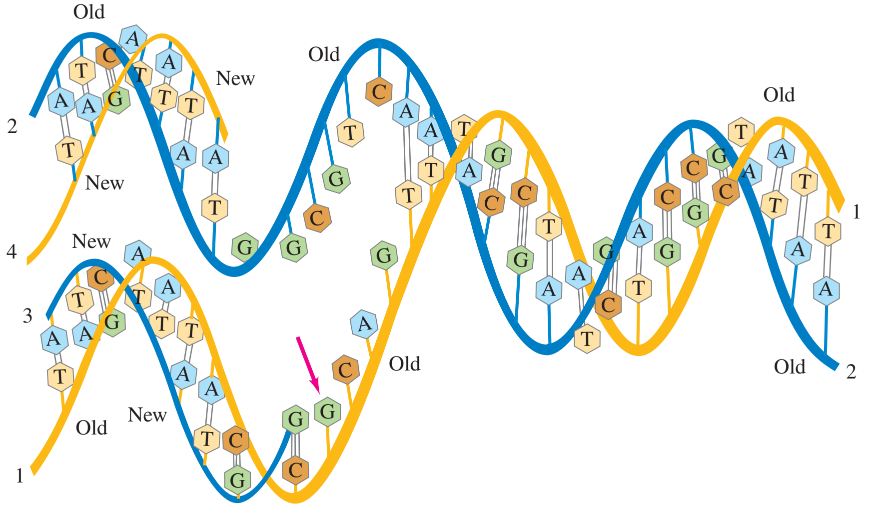


FIGURE 16–40 Replication of DNA.

Once replication starts, let us see how the correct order of bases occurs by looking at the G molecule indicated by the red arrow in Fig. 16–40. Many unattached nucleotide bases of all four kinds are bouncing around in the cellular fluid, and the only type that will experience attraction to our G, if it comes close to it, will be a C. The charges on the other three bases can not get close enough to those on the G to provide a significant attractive force—remember that the electrostatic (Coulomb) force decreases rapidly with distance $\left(\propto 1/r^2\right)$. Because the G does not attract an A, T, or G appreciably, an A, T, or G will be knocked away by collisions with other molecules before enzymes can attach it to the growing chain (number 3 in Fig. 16–40). But the electrostatic force will often hold a C opposite our G long enough so that an enzyme can attach the C to the growing end of the new chain. Thus electrostatic forces are responsible for selecting the bases in the proper order during replication. Note in Fig. 16–40 that the new number 4 strand has the same order of bases as the old number 1 strand; and the new number 3 strand is the same as the old number 2. So the two new double helixes, 1–3 and 2–4, are identical to the original 1–2 helix. Hence the genetic information is passed on accurately to the next generation.

This process of DNA replication is often presented as if it occurred in clockwork fashion—as if each molecule knew its role and went to its assigned place. But this is not the case. The forces of attraction are rather weak and become significant only when charged parts of the two molecules have "complementary shapes," meaning they can get close enough so that the electrostatic force $\left(\propto 1/r^2\right)$ is strong enough to form weak bonds. If the molecular shapes are not just right, there is almost no electrostatic attraction, which is why there are so few mistakes. Thus, out of the random motion of the molecules, the electrostatic force acts to bring order out of chaos.

The random (thermal) velocities of molecules in a cell affect *cloning*. When a bacterial cell divides, the two new bacteria have nearly identical DNA. Even if the DNA were perfectly identical, the two new bacteria would not end up behaving in exactly the same way. Long protein, DNA, and RNA molecules get bumped into different shapes, and even the expression† of genes can thus be different. Loosely held parts of large molecules such as a methyl group (CH_3) can also be knocked off by a strong collision with another molecule. Hence, cloned organisms are not identical, even if their DNA were identical. Indeed, there can not really be genetic determinism.

*16–11 Photocopy Machines and Computer Printers Use Electrostatics

Photocopy machines and laser printers use electrostatic attraction to print an image. They each use a different technique to project an image onto a special cylindrical drum (or rotating conveyor belt). The drum is typically made of aluminum, a good conductor; its surface is coated with a thin layer of selenium, which has the interesting property (called "photoconductivity") of being an electrical nonconductor in the dark, but a conductor when exposed to light.

PHYSICS APPLIED
Photocopy machines

In a **photocopier**, lenses and mirrors focus an image of the original sheet of paper onto the drum, much like a camera lens focuses an image on an electronic detector or film. Step 1, done in the dark, is the placing of a uniform positive charge on the drum's selenium layer by a charged roller or rod: see Fig. 16–41.

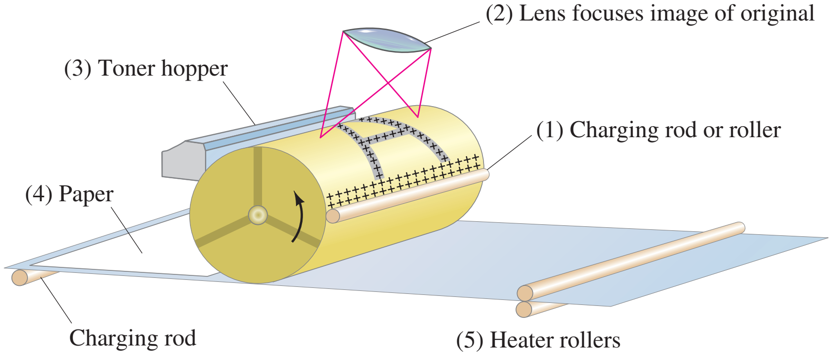


FIGURE 16–41 Inside a photocopy machine: (1) the selenium drum is given a + charge; (2) the lens focuses image on drum—only dark spots stay charged; (3) toner particles (negatively charged) are attracted to positive areas on drum; (4) the image is transferred to paper; (5) heat binds the image to the paper.

In step 2, the image to be copied is projected onto the drum. For simplicity, let us assume the image is a dark letter A on a white background (as on the page of a book) as shown in Fig. 16–41. The letter A on the drum is dark, but all around it is light. At all these light places, the selenium becomes conducting and electrons flow in from the aluminum beneath, neutralizing those positive areas. In the dark areas of the letter A, the selenium is nonconducting and so retains the positive charge already put on it, Fig. 16–41. In step 3, a fine dark powder known as *toner* is given a negative charge, and is brushed on the drum as it rotates. The negatively charged toner particles are attracted to the positive areas on the drum (the A in our case) and stick only there. In step 4, the rotating drum presses against a piece of paper which has been positively charged more strongly than the selenium, so the toner particles are transferred to the paper, forming the final image. Finally, step 5, the paper is heated to fix the toner particles firmly on the paper.

In a color copier (or printer), this process is repeated for each color—black, cyan (blue), magenta (red), and yellow. Combining these four colors in different proportions produces any desired color.

†The separate genes of a DNA double helix can be covered by protein molecules, keeping those genes from being "expressed"—that is, translated into the proteins they code for (see Section 29–3).

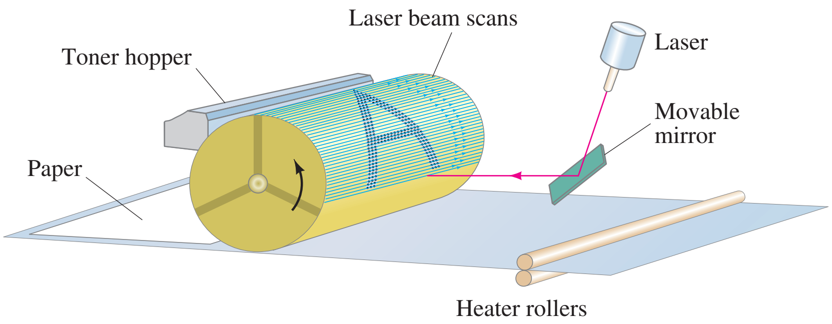


FIGURE 16–42 Inside a laser printer: a movable mirror sweeps the laser beam in horizontal lines across the drum.

PHYSICS APPLIED
Laser printer

A **laser printer** uses a computer output to program the intensity of a laser beam onto the selenium-coated drum of Fig. 16–42. The thin beam of light from the laser is scanned (by a movable mirror) from side to side across the drum in a series of horizontal lines, each line just below the previous line. As the beam sweeps across the drum, the intensity of the beam is varied by the computer output, being strong for a point that is meant to be white or bright, and weak or zero for points that are meant to come out dark. After each sweep, the drum rotates very slightly for additional sweeps, Fig. 16–42, until a complete image is formed on it. The light parts of the selenium become conducting and lose their (previously given) positive electric charge, and the toner sticks only to the dark, electrically charged areas. The drum then transfers the image to paper, as in a photocopier.

PHYSICS APPLIED
Inkjet printer

An **inkjet printer** does not use a drum. Instead nozzles spray tiny droplets of ink directly at the paper. The nozzles are swept across the paper, each sweep just above the previous one as the paper moves down. On each sweep, the ink makes dots on the paper, except for those points where no ink is desired, as directed by the computer. The image consists of a huge number of very tiny dots. The quality or resolution of a printer is usually specified in dots per inch (dpi) in each (linear) direction.

*16–12 Gauss's Law

An important relation in electricity is Gauss's law, developed by the great mathematician Karl Friedrich Gauss (1777–1855). It relates electric charge and electric field, and is a more general and elegant form of Coulomb's law.

Gauss's law involves the concept of **electric flux**, which refers to the electric field passing through a given area. For a uniform electric field $\vec{\mathbf{E}}$ passing through an area A, as shown in Fig. 16–43a, the electric flux Φ_E is defined as

$$\Phi_E = EA\cos\theta,$$

where θ is the angle between the electric field direction and a line drawn perpendicular to the area. The flux can be written equivalently as

$$\Phi_E = E_\perp A = EA_\perp = EA\cos\theta, \qquad \textbf{(16–7)}$$

where $E_\perp = E\cos\theta$ is the component of $\vec{\mathbf{E}}$ perpendicular to the area (Fig. 16–43b) and, similarly, $A_\perp = A\cos\theta$ is the projection of the area A perpendicular to the field $\vec{\mathbf{E}}$ (Fig. 16–43c).

Electric flux can be interpreted in terms of field lines. We mentioned in Section 16–8 that field lines can always be drawn so that the number (N) passing through unit area perpendicular to the field $(A_\perp)$ is proportional to the magnitude of the field (E): that is, $E \propto N/A_\perp$. Hence,

$$N \propto EA_\perp = \Phi_E, \qquad \textbf{(16–8)}$$

so the flux through an area is proportional to the number of lines passing through that area.

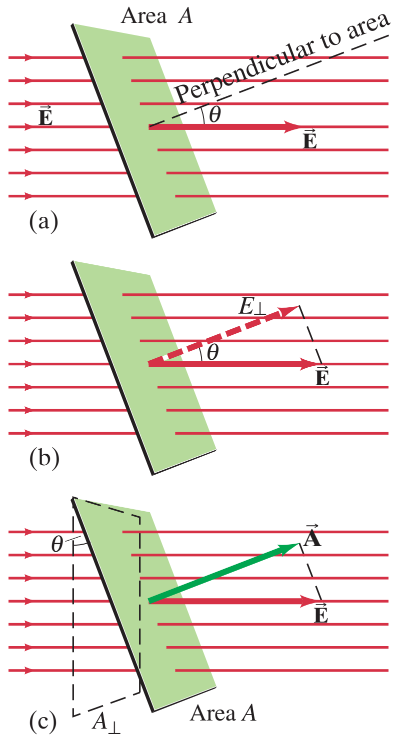


FIGURE 16–43 (a) A uniform electric field $\vec{\mathbf{E}}$ passing through a flat square area A. (b) $E_\perp = E\cos\theta$ is the component of $\vec{\mathbf{E}}$ perpendicular to the plane of area A. (c) $A_\perp = A\cos\theta$ is the projection (dashed) of the area A perpendicular to the field $\vec{\mathbf{E}}$.

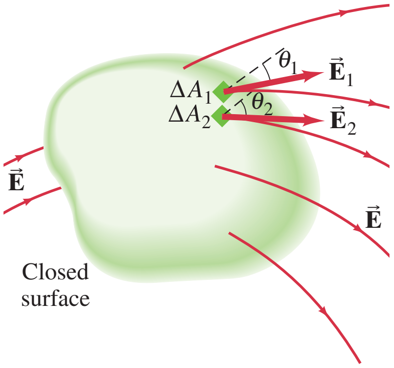


FIGURE 16–44 Electric field lines passing through a closed surface. The surface is divided up into many tiny areas, ΔA_1, ΔA_2, $\cdots$, and so on, of which only two are shown.

Gauss's law involves the *total* flux through a closed surface—a surface of any shape that encloses a volume of space. For any such surface, such as that shown in Fig. 16–44, we divide the surface up into many tiny areas, $\Delta A_1, \Delta A_2, \Delta A_3, \cdots$, and so on. We make the division so that each ΔA is small enough that it can be considered flat and so that the electric field can be considered constant through each ΔA. Then the *total* flux through the entire surface is the sum over all the individual fluxes through each of the tiny areas:

$$\begin{aligned}\Phi_E &= E_1\,\Delta A_1 \cos\theta_1 + E_2\,\Delta A_2 \cos\theta_2 + \cdots \\ &= \sum E\,\Delta A \cos\theta = \sum E_\perp\,\Delta A,\end{aligned}$$

where the symbol Σ means "sum of." We saw in Section 16–8 that the number of field lines starting on a positive charge or ending on a negative charge is proportional to the magnitude of the charge. Hence, the *net* number of lines N pointing out of any closed surface (number of lines pointing out minus the number pointing in) must be proportional to the net charge enclosed by the surface, Q_{encl}. But from Eq. 16–8, we have that the net number of lines N is proportional to the total flux Φ_E. Therefore,

$$\Phi_E = \sum_{\substack{\text{closed}\\\text{surface}}} E_\perp\,\Delta A \propto Q_{\text{encl}}.$$

The constant of proportionality, to be consistent with Coulomb's law, is $1/\epsilon_0$, so we have

GAUSS'S LAW

$$\sum_{\substack{\text{closed}\\\text{surface}}} E_\perp\,\Delta A = \frac{Q_{\text{encl}}}{\epsilon_0}, \tag{16–9}$$

where the sum (Σ) is over any closed surface, and Q_{encl} is the net charge enclosed within that surface. This is **Gauss's law**.

Coulomb's law and Gauss's law can be used to determine the electric field due to a given (static) charge distribution. Gauss's law is useful when the charge distribution is simple and symmetrical. However, we must choose the closed "gaussian" surface very carefully so we can determine $\vec{\mathbf{E}}$. We normally choose a surface that has just the **symmetry** needed so that E will be constant on all or on parts of its surface.

EXAMPLE 16–12 Charged spherical conducting shell. A thin spherical shell of radius r_0 possesses a total net charge Q that is uniformly distributed on it, Fig. 16–45. Determine the electric field at points (a) outside the shell, and (b) inside the shell.

APPROACH Because the charge is distributed symmetrically, the electric field must be *symmetric*. Thus the field outside the shell must be directed radially outward (inward if $Q < 0$) and must depend only on r.

SOLUTION (a) We choose our imaginary gaussian surface as a sphere of radius r $(r > r_0)$ concentric with the shell, shown in Fig. 16–45 as the dashed circle A_1. Then, by symmetry, the electric field will have the same magnitude at all points on this gaussian surface. Because $\vec{\mathbf{E}}$ is perpendicular to this surface, Gauss's law gives (with $Q_{\text{encl}} = Q$ in Eq. 16–9)

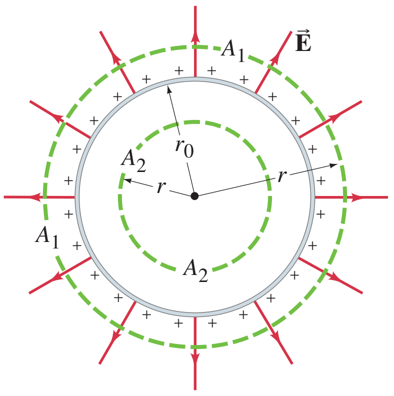
$$\sum E_\perp\,\Delta A = E \sum \Delta A = E\left(4\pi r^2\right) = \frac{Q}{\epsilon_0},$$

where $4\pi r^2$ is the surface area of our sphere (gaussian surface) of radius r. Thus

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}. \qquad [r > r_0]$$

We see that the field outside a uniformly charged spherical shell is the same as if all the charge were concentrated at the center as a point charge.

FIGURE 16–45 Cross-sectional drawing of a thin spherical shell (gray) of radius r_0, carrying a net charge Q uniformly distributed. The green circles A_1 and A_2 represent two gaussian surfaces we use to determine $\vec{\mathbf{E}}$. Example 16–12.



(*b*) Inside the shell, the electric field must also be symmetric. So E must again have the same value at all points on a spherical gaussian surface (A_2 in Fig. 16–45) concentric with the shell. Thus, E can be factored out of the sum and, with $Q_{\text{encl}} = 0$ because the charge inside surface A_2 is zero, we have

$$\begin{aligned}\sum E_{\perp}\,\Delta A &= E\sum \Delta A \\ &= E\left(4\pi r^2\right) = \frac{Q_{\text{encl}}}{\epsilon_0} = 0.\end{aligned}$$

Hence

$$E = 0 \qquad \left[r < r_0\right]$$

inside a uniform spherical shell of charge (as claimed in Section 16–9).

The results of Example 16–12 also apply to a uniform *solid* spherical conductor that is charged, since all the charge would lie in a thin layer at the surface (Section 16–9). In particular

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$

outside a spherical conductor. Thus, the electric field outside a spherically symmetric distribution of charge is the same as for a point charge of the same magnitude at the center of the sphere. This result applies also outside a uniformly charged nonconductor, because we can use the same gaussian surface A_1 (Fig. 16–45) and the same *symmetry* argument. We can also consider this a demonstration of our statement in Chapter 5 about the **gravitational force**, which is also a perfect $1/r^2$ force: The gravitational force exerted by a uniform sphere is the same as if all the mass were at the center, as stated on page 120.

EXAMPLE 16–13 ***E* near any conducting surface.** Show that the magnitude of the electric field just outside the surface of a good conductor of any shape is given by

$$E = \frac{\sigma}{\epsilon_0},$$

where σ is defined as the surface charge density, Q/A, on the conductor's surface at that point.

APPROACH We choose as our gaussian surface a small cylindrical box, very small in height so that one of its circular ends is just above the conductor (Fig. 16–46). The other end is just below the conductor's surface, and the very short sides are perpendicular to it.

SOLUTION The electric field is zero inside a conductor and is perpendicular to the surface just outside it (Section 16–9), so electric flux passes only through the outside end of our cylindrical box; no flux passes through the very short sides or through the inside end of our gaussian box. We choose the area A (of the flat cylinder end above the conductor surface) small enough so that E is essentially uniform over it. Then Gauss's law gives

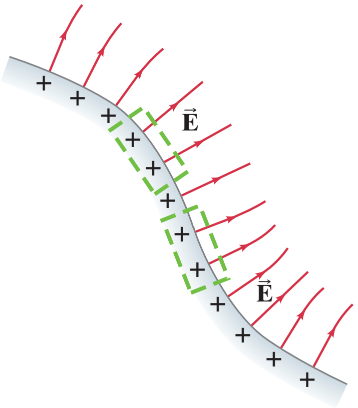
$$\sum E_{\perp}\,\Delta A = EA = \frac{Q_{\text{encl}}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0},$$

and therefore

$$E = \frac{\sigma}{\epsilon_0}. \qquad \text{[at surface of conductor]}$$

This useful result applies for any shape conductor, including a large, uniformly charged flat sheet: the electric field will be constant and equal to σ/ϵ_0.

FIGURE 16–46 Electric field near the surface of a conductor. Two small cylindrical boxes are shown dashed. Either one can serve as our gaussian surface. Example 16–13.



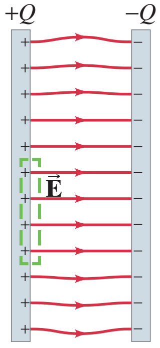


FIGURE 16–47 The electric field between two closely spaced parallel plates is uniform and equal to $E = \sigma/\epsilon_0$.

This last Example also gives us the field between the two parallel plates we discussed in Fig. 16–32d. If the plates are large compared to their separation, then the field lines are perpendicular to the plates and, except near the edges, they are parallel to each other. Therefore the electric field (see Fig. 16–47, which shows a similar very thin gaussian surface as Fig. 16–46) is also

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q/A}{\epsilon_0} = \frac{Q}{\epsilon_0 A}, \quad \begin{bmatrix}\text{between two closely spaced,} \\ \text{oppositely charged, parallel plates}\end{bmatrix} \qquad \textbf{(16–10)}$$

where $Q = \sigma A$ is the charge on one of the plates.

Summary

There are two kinds of **electric charge**, positive and negative. These designations are to be taken algebraically—that is, any charge is plus or minus so many coulombs (C), in SI units.

Electric charge is **conserved**: if a certain amount of one type of charge is produced in a process, an equal amount of the opposite type is also produced; thus the *net* charge produced is zero.

According to atomic theory, electricity originates in the atom, which consists of a positively charged nucleus surrounded by negatively charged electrons. Each electron has a charge $-e = -1.60 \times 10^{-19}$ C.

Electric **conductors** are those materials in which many electrons are relatively free to move, whereas electric **insulators** or **nonconductors** are those in which very few electrons are free to move.

An object is negatively charged when it has an excess of electrons, and positively charged when it has less than its normal number of electrons. The net charge on any object is a whole number times $+e$ or $-e$. That is, charge is **quantized**.

An object can become charged by rubbing (in which electrons are transferred from one material to another), **by conduction** (which is transfer of charge from one charged object to another by touching), or **by induction** (the separation of charge within an object because of the close approach of another charged object but without touching).

Electric charges exert a force on each other. If two charges are of opposite types, one positive and one negative, they each exert an attractive force on the other. If the two charges are the same type, each repels the other.

The magnitude of the force one point charge exerts on another is proportional to the product of their charges, and inversely proportional to the square of the distance between them:

$$F = k\frac{Q_1 Q_2}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{r^2}; \qquad \textbf{(16–1, 16–2)}$$

this is **Coulomb's law**.

We think of an **electric field** as existing in space around any charge or group of charges. The force on another charged object is then said to be due to the electric field present at its location.

The *electric field*, $\vec{\mathbf{E}}$, at any point in space due to one or more charges, is defined as the force per unit charge that would act on a tiny positive test charge q placed at that point:

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q}. \qquad \textbf{(16–3)}$$

The magnitude of the electric field a distance r from a point charge Q is

$$E = k\frac{Q}{r^2}. \qquad \textbf{(16–4a)}$$

The total electric field at a point in space is equal to the vector sum of the individual fields due to each contributing charge. This is the **principle of superposition**.

Electric fields are represented by **electric field lines** that start on positive charges and end on negative charges. Their direction indicates the direction the force would be on a tiny positive test charge placed at each point. The lines can be drawn so that the number per unit area is proportional to the magnitude of E.

The static electric field inside a conductor is zero, and the electric field lines just outside a charged conductor are perpendicular to its surface.

[*In the replication of DNA, the electrostatic force plays a crucial role in selecting the proper molecules so that the genetic information is passed on accurately from generation to generation.]

[*Photocopiers and computer printers use electric charge placed on toner particles and a drum to form an image.]

[*The **electric flux** passing through a small area A for a uniform electric field $\vec{\mathbf{E}}$ is

$$\Phi_E = E_\perp A, \qquad \textbf{(16–7)}$$

where $E_\perp$ is the component of $\vec{\mathbf{E}}$ perpendicular to the surface. The flux through a surface is proportional to the number of field lines passing through it.]

[***Gauss's law** states that the total flux summed over any closed surface (considered as made up of many small areas ΔA) is equal to the net charge Q_{encl} enclosed by the surface divided by ϵ_0:

$$\sum_{\substack{\text{closed}\\ \text{surface}}} E_\perp\, \Delta A = \frac{Q_{\text{encl}}}{\epsilon_0}. \qquad \textbf{(16–9)}$$

Gauss's law can be used to determine the electric field due to given charge distributions, but its usefulness is mainly limited to cases where the charge distribution displays much symmetry. The real importance of Gauss's law is that it is a general and elegant statement of the relation between electric charge and electric field.]

Questions

1. If you charge a pocket comb by rubbing it with a silk scarf, how can you determine if the comb is positively or negatively charged?

2. Why does a shirt or blouse taken from a clothes dryer sometimes cling to your body?

3. Explain why fog or rain droplets tend to form around ions or electrons in the air.

4. Why does a plastic ruler that has been rubbed with a cloth have the ability to pick up small pieces of paper? Why is this difficult to do on a humid day?

5. A positively charged rod is brought close to a neutral piece of paper, which it attracts. Draw a diagram showing the separation of charge in the paper, and explain why attraction occurs.

6. Contrast the *net charge* on a conductor to the "free charges" in the conductor.

7. Figures 16–7 and 16–8 show how a charged rod placed near an uncharged metal object can attract (or repel) electrons. There are a great many electrons in the metal, yet only some of them move as shown. Why not all of them?

8. When an electroscope is charged, its two leaves repel each other and remain at an angle. What balances the electric force of repulsion so that the leaves don't separate further?

9. The balloon in Fig. 16–48 was rubbed on a student's hair. Explain why the water drip curves instead of falling vertically.

FIGURE 16–48 Question 9.

10. The form of Coulomb's law is very similar to that for Newton's law of universal gravitation. What are the differences between these two laws? Compare also gravitational mass and electric charge.

11. When a charged ruler attracts small pieces of paper, sometimes a piece jumps quickly away after touching the ruler. Explain.

12. We are not normally aware of the gravitational or electric force between two ordinary objects. What is the reason in each case? Give an example where we are aware of each one and why.

13. Explain why the test charges we use when measuring electric fields must be small.

14. When determining an electric field, must we use a *positive* test charge, or would a negative one do as well? Explain.

15. Draw the electric field lines surrounding two negative electric charges a distance ℓ apart.

16. Assume that the two opposite charges in Fig. 16–32a are 12.0 cm apart. Consider the magnitude of the electric field 2.5 cm from the positive charge. On which side of this charge—top, bottom, left, or right—is the electric field the strongest? The weakest? Explain.

17. Consider the electric field at the three points indicated by the letters A, B, and C in Fig. 16–49. First draw an arrow at each point indicating the direction of the net force that a positive test charge would experience if placed at that point, then list the letters in order of *decreasing* field strength (strongest first). Explain.

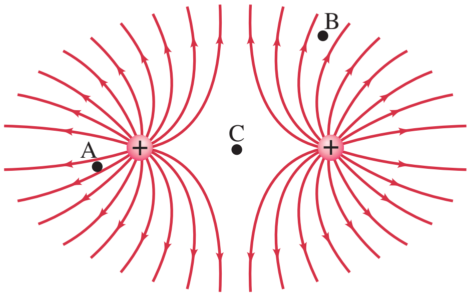


FIGURE 16–49 Question 17.

18. Why can electric field lines never cross?

19. Show, using the three rules for field lines given in Section 16–8, that the electric field lines starting or ending on a single point charge must be symmetrically spaced around the charge.

20. Given two point charges, Q and $2Q$, a distance ℓ apart, is there a point along the straight line that passes through them where $E = 0$ when their signs are (*a*) opposite, (*b*) the same? If yes, state roughly where this point will be.

21. Consider a small positive test charge located on an electric field line at some point, such as point P in Fig. 16–32a. Is the direction of the velocity and/or acceleration of the test charge along this line? Discuss.

*22. A point charge is surrounded by a spherical gaussian surface of radius r. If the sphere is replaced by a cube of side r, will Φ_E be larger, smaller, or the same? Explain.

MisConceptual Questions

1. $Q_1 = -0.10\,\mu C$ is located at the origin. $Q_2 = +0.10\,\mu C$ is located on the positive x axis at $x = 1.0$ m. Which of the following is true of the force on Q_1 due to Q_2?
(*a*) It is attractive and directed in the $+x$ direction.
(*b*) It is attractive and directed in the $-x$ direction.
(*c*) It is repulsive and directed in the $+x$ direction.
(*d*) It is repulsive and directed in the $-x$ direction.

2. Swap the positions of Q_1 and Q_2 of MisConceptual Question 1. Which of the following is true of the force on Q_1 due to Q_2?
(*a*) It does not change.
(*b*) It changes from attractive to repulsive.
(*c*) It changes from repulsive to attractive.
(*d*) It changes from the $+x$ direction to the $-x$ direction.
(*e*) It changes from the $-x$ direction to the $+x$ direction.

3. Fred the lightning bug has a mass m and a charge $+q$. Jane, his lightning-bug wife, has a mass of $\frac{3}{4}m$ and a charge $-2q$. Because they have charges of opposite sign, they are attracted to each other. Which is attracted more to the other, and by how much?
(*a*) Fred, twice as much.
(*b*) Jane, twice as much.
(*c*) Fred, four times as much.
(*d*) Jane, four times as much.
(*e*) They are attracted to each other by the same amount.

4. Figure 16–50 shows electric field lines due to a point charge. What can you say about the field at point 1 compared with the field at point 2?
(*a*) The field at point 2 is larger, because point 2 is on a field line.
(*b*) The field at point 1 is larger, because point 1 is not on a field line.
(*c*) The field at point 1 is zero, because point 1 is not on a field line.
(*d*) The field at point 1 is larger, because the field lines are closer together in that region.

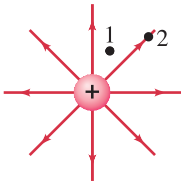


FIGURE 16–50
MisConceptual Question 4.

5. A negative point charge is in an electric field created by a positive point charge. Which of the following is true?
(*a*) The field points toward the positive charge, and the force on the negative charge is in the same direction as the field.
(*b*) The field points toward the positive charge, and the force on the negative charge is in the opposite direction to the field.
(*c*) The field points away from the positive charge, and the force on the negative charge is in the same direction as the field.
(*d*) The field points away from the positive charge, and the force on the negative charge is in the opposite direction to the field.

6. As an object acquires a positive charge, its mass usually
(*a*) decreases.
(*b*) increases.
(*c*) stays the same.
(*d*) becomes negative.

7. Refer to Fig. 16–32d. If the two charged plates were moved until they are half the distance shown without changing the charge on the plates, the electric field near the center of the plates would
(*a*) remain almost exactly the same.
(*b*) increase by a factor of 2.
(*c*) increase, but not by a factor of 2.
(*d*) decrease by a factor of 2.
(*e*) decrease, but not by a factor of 2.

8. We wish to determine the electric field at a point near a positively charged metal sphere (a good conductor). We do so by bringing a small positive test charge, q_0, to this point and measure the force F_0 on it. F_0/q_0 will be ____________ the electric field $\vec{\mathbf{E}}$ as it was at that point before the test charge was present.
(*a*) greater than
(*b*) less than
(*c*) equal to

9. We are usually not aware of the electric force acting between two everyday objects because
(*a*) the electric force is one of the weakest forces in nature.
(*b*) the electric force is due to microscopic-sized particles such as electrons and protons.
(*c*) the electric force is invisible.
(*d*) most everyday objects have as many plus charges as minus charges.

10. To be safe during a lightning storm, it is best to be
(*a*) in the middle of a grassy meadow.
(*b*) inside a metal car.
(*c*) next to a tall tree in a forest.
(*d*) inside a wooden building.
(*e*) on a metal observation tower.

11. Which are the worst places in MisConceptual Question 10?

12. Which vector best represents the direction of the electric field at the fourth corner of the square due to the three charges shown in Fig. 16–51?

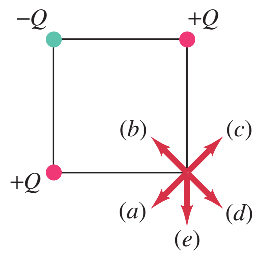


FIGURE 16–51
MisConceptual Question 12.

13. A small metal ball hangs from the ceiling by an insulating thread. The ball is attracted to a positively charged rod held near the ball. The charge of the ball must be
(*a*) positive.
(*b*) negative.
(*c*) neutral.
(*d*) positive or neutral.
(*e*) negative or neutral.

For assigned homework and other learning materials, go to the MasteringPhysics website.

Problems

16–5 and 16–6 Coulomb's Law

$[1\text{ mC} = 10^{-3}\text{ C},\ \ 1\,\mu\text{C} = 10^{-6}\text{ C},\ \ 1\text{ nC} = 10^{-9}\text{ C.}]$

1. (I) What is the magnitude of the electric force of attraction between an iron nucleus $(q = +26e)$ and its innermost electron if the distance between them is 1.5×10^{-12} m?

2. (I) How many electrons make up a charge of $-48.0\,\mu$C?

3. (I) What is the magnitude of the force a $+25\,\mu$C charge exerts on a $+2.5$ mC charge 16 cm away?

4. (I) What is the repulsive electrical force between two protons 4.0×10^{-15} m apart from each other in an atomic nucleus?

5. (II) When an object such as a plastic comb is charged by rubbing it with a cloth, the net charge is typically a few microcoulombs. If that charge is $3.0\,\mu$C, by what percentage does the mass of a 9.0-g comb change during charging?

6. (II) Two charged dust particles exert a force of 4.2×10^{-2} N on each other. What will be the force if they are moved so they are only one-eighth as far apart?

7. (II) Two small charged spheres are 6.52 cm apart. They are moved, and the force each exerts on the other is found to have tripled. How far apart are they now?

8. (II) A person scuffing her feet on a wool rug on a dry day accumulates a net charge of $-28\,\mu$C. How many excess electrons does she get, and by how much does her mass increase?

9. (II) What is the total charge of all the electrons in a 12-kg bar of gold? What is the net charge of the bar? (Gold has 79 electrons per atom and an atomic mass of 197 u.)

10. (II) Compare the electric force holding the electron in orbit $(r = 0.53 \times 10^{-10}\text{ m})$ around the proton nucleus of the hydrogen atom, with the gravitational force between the same electron and proton. What is the ratio of these two forces?

11. (II) Particles of charge $+65$, $+48$, and $-95\,\mu$C are placed in a line (Fig. 16–52). The center one is 0.35 m from each of the others. Calculate the net force on each charge due to the other two.

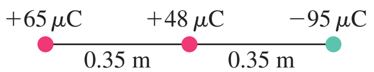


FIGURE 16–52 Problem 11.

12. (II) Three positive particles of equal charge, $+17.0\,\mu$C, are located at the corners of an equilateral triangle of side 15.0 cm (Fig. 16–53). Calculate the magnitude and direction of the net force on each particle due to the other two.

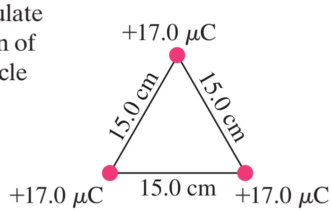


FIGURE 16–53 Problem 12.

13. (II) A charge of 6.15 mC is placed at each corner of a square 0.100 m on a side. Determine the magnitude and direction of the force on each charge.

14. (II) At each corner of a square of side ℓ there are point charges of magnitude Q, $2Q$, $3Q$, and $4Q$ (Fig. 16–54). Determine the magnitude and direction of the force on the charge $2Q$.

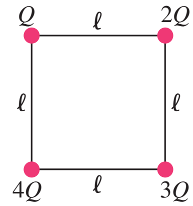


FIGURE 16–54 Problem 14.

15. (II) A large electroscope is made with "leaves" that are 78-cm-long wires with tiny 21-g spheres at the ends. When charged, nearly all the charge resides on the spheres. If the wires each make a 26° angle with the vertical (Fig. 16–55), what total charge Q must have been applied to the electroscope? Ignore the mass of the wires.

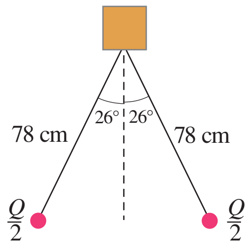


FIGURE 16–55 Problem 15.

16. (III) Two small nonconducting spheres have a total charge of $90.0\,\mu$C. (*a*) When placed 28.0 cm apart, the force each exerts on the other is 12.0 N and is repulsive. What is the charge on each? (*b*) What if the force were attractive?

17. (III) Two charges, $-Q$ and $-3Q$, are a distance ℓ apart. These two charges are free to move but do not because there is a third (fixed) charge nearby. What must be the magnitude of the third charge and its placement in order for the first two to be in equilibrium?

16–7 and 16–8 Electric Field, Field Lines

18. (I) Determine the magnitude and direction of the electric force on an electron in a uniform electric field of strength 2460 N/C that points due east.

19. (I) Determine the magnitude and direction of the electric field 21.7 cm directly above an isolated 33.0×10^{-6} C charge.

20. (I) A downward electric force of 6.4 N is exerted on a $-7.3\,\mu$C charge. Find the magnitude and direction of the electric field at the position of this charge.

21. (II) Determine the magnitude of the acceleration experienced by an electron in an electric field of 756 N/C. How does the direction of the acceleration depend on the direction of the field at that point?

22. (II) Determine the magnitude and direction of the electric field at a point midway between a $-8.0\,\mu$C and a $+5.8\,\mu$C charge 6.0 cm apart. Assume no other charges are nearby.

23. (II) Draw, approximately, the electric field lines about two point charges, $+Q$ and $-3Q$, which are a distance ℓ apart.

24. (II) An electron is released from rest in a uniform electric field and accelerates to the north at a rate of $105\ \mathrm{m/s^2}$. Find the magnitude and direction of the electric field.

25. (II) The electric field midway between two equal but opposite point charges is 386 N/C, and the distance between the charges is 16.0 cm. What is the magnitude of the charge on each?

26. (II) Calculate the electric field at one corner of a square 1.22 m on a side if the other three corners are occupied by 3.25×10^{-6} C charges.

27. (II) Calculate the electric field at the center of a square 42.5 cm on a side if one corner is occupied by a $-38.6\ \mu\mathrm{C}$ charge and the other three are occupied by $-27.0\ \mu\mathrm{C}$ charges.

28. (II) Two point charges, $Q_1 = -32\ \mu\mathrm{C}$ and $Q_2 = +45\ \mu\mathrm{C}$, are separated by a distance of 12 cm. The electric field at the point P (see Fig. 16–56) is zero. How far from Q_1 is P?

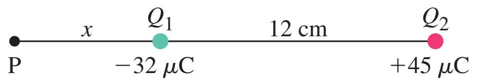


FIGURE 16–56 Problem 28.

29. (II) Determine the electric field $\vec{\mathbf{E}}$ at the origin 0 in Fig. 16–57 due to the two charges at A and B.

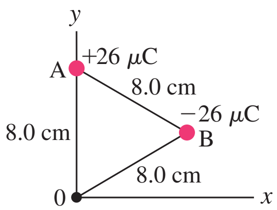


FIGURE 16–57 Problem 29.

30. (II) You are given two unknown point charges, Q_1 and Q_2. At a point on the line joining them, one-third of the way from Q_1 to Q_2, the electric field is zero (Fig. 16–58). What is the ratio Q_1/Q_2?

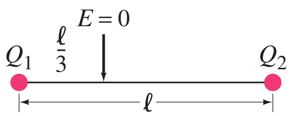


FIGURE 16–58 Problem 30.

31. (III) Use Coulomb's law to determine the magnitude and direction of the electric field at points A and B in Fig. 16–59 due to the two positive charges ($Q = 4.7\ \mu\mathrm{C}$) shown. Are your results consistent with Fig. 16–32b?

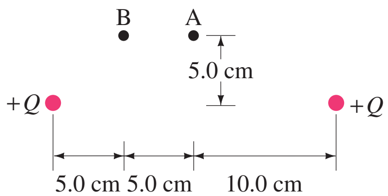


FIGURE 16–59 Problem 31.

32. (III) Determine the direction and magnitude of the electric field at the point P in Fig. 16–60. The charges are separated by a distance $2a$, and point P is a distance x from the midpoint between the two charges. Express your answer in terms of Q, x, a, and k.

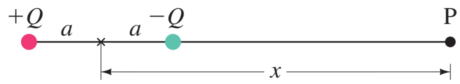


FIGURE 16–60 Problem 32.

*16–10 DNA

*33. (III) The two strands of the helix-shaped DNA molecule are held together by electrostatic forces as shown in Fig. 16–39. Assume that the net average charge (due to electron sharing) indicated on H and N atoms has magnitude $0.2e$ and on the indicated C and O atoms is $0.4e$. Assume also that atoms on each molecule are separated by 1.0×10^{-10} m. Estimate the net force between (*a*) a thymine and an adenine; and (*b*) a cytosine and a guanine. For each bond (red dots) consider only the three atoms in a line (two atoms on one molecule, one atom on the other). (*c*) Estimate the total force for a DNA molecule containing 10^5 pairs of such molecules. Assume half are A–T pairs and half are C–G pairs.

*16–12 Gauss's Law

*34. (I) The total electric flux from a cubical box of side 28.0 cm is $1.85 \times 10^3\ \mathrm{N \cdot m^2/C}$. What charge is enclosed by the box?

*35. (II) In Fig. 16–61, two objects, O_1 and O_2, have charges $+1.0\ \mu\mathrm{C}$ and $-2.0\ \mu\mathrm{C}$, respectively, and a third object, O_3, is electrically neutral. (*a*) What is the electric flux through the surface A_1 that encloses all three objects? (*b*) What is the electric flux through the surface A_2 that encloses the third object only?

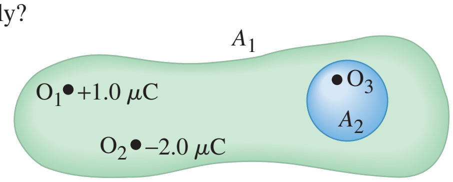


FIGURE 16–61 Problem 35.

*36. (II) A cube of side 8.50 cm is placed in a uniform field $E = 7.50 \times 10^3$ N/C with edges parallel to the field lines. (*a*) What is the net flux through the cube? (*b*) What is the flux through each of its six faces?

*37. (II) The electric field between two parallel square metal plates is 130 N/C. The plates are 0.85 m on a side and are separated by 3.0 cm. What is the charge on each plate (assume equal and opposite)? Neglect edge effects.

*38. (II) The field just outside a 3.50-cm-radius metal ball is 3.75×10^2 N/C and points toward the ball. What charge resides on the ball?

*39. (III) A point charge Q rests at the center of an uncharged thin spherical conducting shell. (See Fig. 16–34.) What is the electric field E as a function of r (*a*) for r less than the inner radius of the shell, (*b*) inside the shell, and (*c*) beyond the shell? (*d*) How does the shell affect the field due to Q alone? How does the charge Q affect the shell?

General Problems

40. How close must two electrons be if the magnitude of the electric force between them is equal to the weight of either at the Earth's surface?

41. Given that the human body is mostly made of water, estimate the total amount of positive charge in a 75-kg person.

42. A 3.0-g copper penny has a net positive charge of $32\,\mu$C. What fraction of its electrons has it lost?

43. Measurements indicate that there is an electric field surrounding the Earth. Its magnitude is about 150 N/C at the Earth's surface and points inward toward the Earth's center. What is the magnitude of the electric charge on the Earth? Is it positive or negative? [*Hint*: The electric field outside a uniformly charged sphere is the same as if all the charge were concentrated at its center.]

44. A water droplet of radius 0.018 mm remains stationary in the air. If the downward-directed electric field of the Earth is 150 N/C, how many excess electron charges must the water droplet have?

45. Estimate the net force between the CO group and the HN group shown in Fig. 16–62. The C and O have charges $\pm 0.40e$, and the H and N have charges $\pm 0.20e$, where $e = 1.6 \times 10^{-19}$ C. [*Hint*: Do not include the "internal" forces between C and O, or between H and N.]

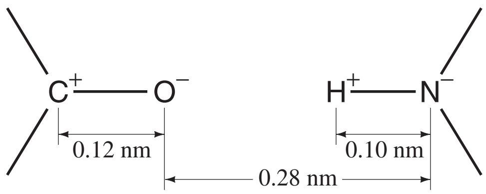


FIGURE 16–62 Problem 45.

46. In a simple model of the hydrogen atom, the electron revolves in a circular orbit around the proton with a speed of 2.2×10^6 m/s. Determine the radius of the electron's orbit. [*Hint*: See Chapter 5 on circular motion.]

47. Two small charged spheres hang from cords of equal length ℓ as shown in Fig. 16–63 and make small angles θ_1 and θ_2 with the vertical. (*a*) If $Q_1 = Q$, $Q_2 = 2Q$, and $m_1 = m_2 = m$, determine the ratio θ_1/θ_2. (*b*) Estimate the distance between the spheres.

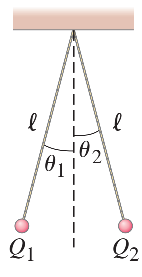


FIGURE 16–63 Problem 47.

48. A positive point charge $Q_1 = 2.5 \times 10^{-5}$ C is fixed at the origin of coordinates, and a negative point charge $Q_2 = -5.0 \times 10^{-6}$ C is fixed to the x axis at $x = +2.4$ m. Find the location of the place(s) along the x axis where the electric field due to these two charges is zero.

49. Dry air will break down and generate a spark if the electric field exceeds about 3×10^6 N/C. How much charge could be packed onto a green pea (diameter 0.75 cm) before the pea spontaneously discharges? [*Hint*: Eqs. 16–4 work outside a sphere if r is measured from its center.]

50. Two point charges, $Q_1 = -6.7\,\mu$C and $Q_2 = 1.8\,\mu$C, are located between two oppositely charged parallel plates, as shown in Fig. 16–64. The two charges are separated by a distance of $x = 0.47$ m. Assume that the electric field produced by the charged plates is uniform and equal to $E = 53{,}000$ N/C. Calculate the net electrostatic force on Q_1 and give its direction.

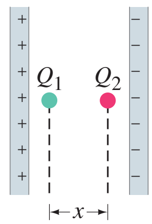


FIGURE 16–64 Problem 50.

51. Packing material made of pieces of foamed polystyrene can easily become charged and stick to each other. Given that the density of this material is about 35 kg/m^3, estimate how much charge might be on a 2.0-cm-diameter foamed polystyrene sphere, assuming the electric force between two spheres stuck together is equal to the weight of one sphere.

52. Two small, identical conducting spheres A and B are a distance R apart; each carries the same charge Q. (*a*) What is the force sphere B exerts on sphere A? (*b*) An identical sphere with zero charge, sphere C, makes contact with sphere B and is then moved very far away. What is the net force now acting on sphere A? (*c*) Sphere C is brought back and now makes contact with sphere A and is then moved far away. What is the force on sphere A in this third case?

53. For an experiment, a colleague of yours says he smeared toner particles uniformly over the surface of a sphere 1.0 m in diameter and then measured an electric field of 5000 N/C near its surface. (*a*) How many toner particles (Example 16–6) would have to be on the surface to produce these results? (*b*) What is the total mass of the toner particles?

54. A proton $(m = 1.67 \times 10^{-27}\,\text{kg})$ is suspended at rest in a uniform electric field $\vec{\mathbf{E}}$. Take into account gravity at the Earth's surface, and determine $\vec{\mathbf{E}}$.

55. A point charge of mass 0.185 kg, and net charge +0.340 μC, hangs at rest at the end of an insulating cord above a large sheet of charge. The horizontal sheet of fixed uniform charge creates a uniform vertical electric field in the vicinity of the point charge. The tension in the cord is measured to be 5.18 N. Calculate the magnitude and direction of the electric field due to the sheet of charge (Fig. 16–65).

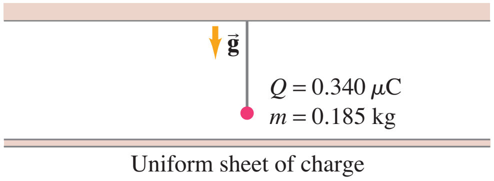


FIGURE 16–65 Problem 55.

56. An electron with speed $v_0 = 5.32 \times 10^6$ m/s is traveling parallel to an electric field of magnitude $E = 9.45 \times 10^3$ N/C. (*a*) How far will the electron travel before it stops? (*b*) How much time will elapse before it returns to its starting point?

57. What is the total charge of all the electrons in a 25-kg bar of aluminum? (Aluminum has 13 electrons per atom and an atomic mass of 27 u.)

58. Two point charges, $+Q$ and $-Q$ of mass m, are placed on the ends of a massless rod of length ℓ, which is fixed to a table by a pin through its center. If the apparatus is then subjected to a uniform electric field E parallel to the table and perpendicular to the rod, find the net torque on the system of rod plus charges.

59. Determine the direction and magnitude of the electric field at point P, Fig. 16–68. The two charges are separated by a distance of $2a$. Point P is on the perpendicular bisector of the line joining the charges, a distance x from the midpoint between them. Express your answers in terms of Q, x, a, and k.

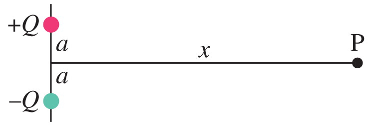


FIGURE 16–68 Problem 59.

60. A mole of carbon contains 7.22×10^{24} electrons. Two electrically neutral carbon spheres, each containing 1 mole of carbon, are separated by 15.0 cm (center to center). What fraction of electrons would have to be transferred from one sphere to the other for the electric force and the gravitational force between the spheres to be equal?

Search and Learn

1. Four equal positive point charges, each of charge 6.4 μC, are at the corners of a square of side 9.2 cm. What charge should be placed at the center of the square so that all charges are at equilibrium? Is this a stable or an unstable equilibrium (Section 9–4) in the plane?

2. Suppose electrons enter a uniform electric field midway between two plates at an angle θ_0 to the horizontal, as shown in Fig. 16–69. The path is symmetrical, so they leave at the same angle θ_0 and just barely miss the top plate. What is θ_0? Ignore fringing of the field.

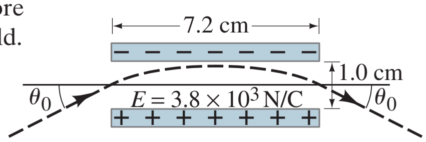


FIGURE 16–69 Search and Learn 2.

3. What experimental observations mentioned in the text rule out the possibility that the numerator in Coulomb's law contains the sum $(Q_1 + Q_2)$ rather than the product $Q_1 Q_2$?

4. Near the surface of the Earth, there is a downward electric field of 150 N/C and a downward gravitational field of 9.8 N/kg. A charged 1.0-kg mass is observed to fall with acceleration 8.0 m/s^2. What are the magnitude and sign of its charge?

5. Identical negative charges ($Q = -e$) are located at two of the three vertices of an equilateral triangle. The length of a side of the triangle is ℓ. What is the magnitude of the net electric field at the third vertex? If a third identical negative charge was located at the third vertex, then what would be the net electrostatic force on it due to the other two charges? Use symmetry and explain how you used it.

ANSWERS TO EXERCISES

A: (*d*).
B: Opposite.
C: 0.3 N, to the right.
D: 0.32 m.
E: (*a*) No; (*b*) yes, midway between them.
F: 9.0×10^4 N/C, vertically upward.
G: (*d*), if the two + charges are not at opposite corners (use symmetry).