This snowboarder flying through the air shows an example of motion in two dimensions. In the absence of air resistance, the path would be a perfect parabola. The gold arrow represents the downward acceleration of gravity, $\vec{\mathbf{g}}$. Galileo analyzed the motion of objects in 2 dimensions under the action of gravity near the Earth's surface (now called "projectile motion") into its horizontal and vertical components.

We will discuss vectors and how to add them. Besides analyzing projectile motion, we will also see how to work with relative velocity.

CHAPTER 3

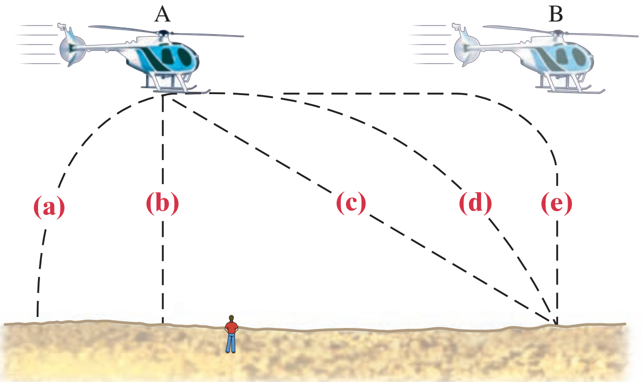
Kinematics in Two Dimensions; Vectors

CONTENTS

CHAPTER-OPENING QUESTION—Guess now!

[*Don't worry about getting the right answer now—you will get another chance later in the Chapter. See also p. 1 of Chapter 1 for more explanation.*]

A small heavy box of emergency supplies is dropped from a moving helicopter at point A as it flies at constant speed in a horizontal direction. Which path in the drawing below best describes the path of the box (neglecting air resistance) as seen by a person standing on the ground?



In Chapter 2 we dealt with motion along a straight line. We now consider the motion of objects that move in paths in two (or three) dimensions. In particular, we discuss an important type of motion known as *projectile motion*: objects projected outward near the Earth's surface, such as struck baseballs and golf balls, kicked footballs, and other projectiles. Before beginning our discussion of motion in two dimensions, we will need a new tool, vectors, and how to add them.

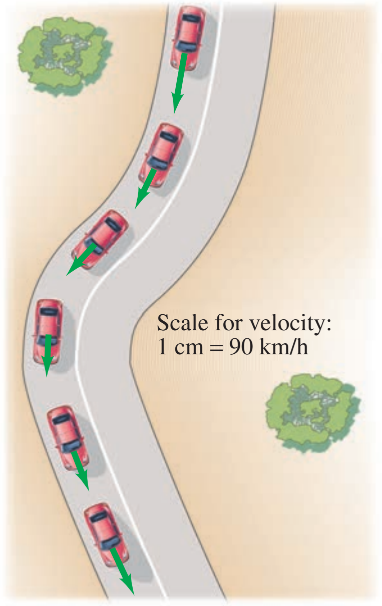


FIGURE 3–1 Car traveling on a road, slowing down to round the curve. The green arrows represent the velocity vector at each position.

3–1 Vectors and Scalars

We mentioned in Chapter 2 that the term *velocity* refers not only to how fast an object is moving but also to its direction. A quantity such as velocity, which has *direction* as well as *magnitude*, is a **vector** quantity. Other quantities that are also vectors are displacement, force, and momentum. However, many quantities have no direction associated with them, such as mass, time, and temperature. They are specified completely by a number and units. Such quantities are called **scalar** quantities.

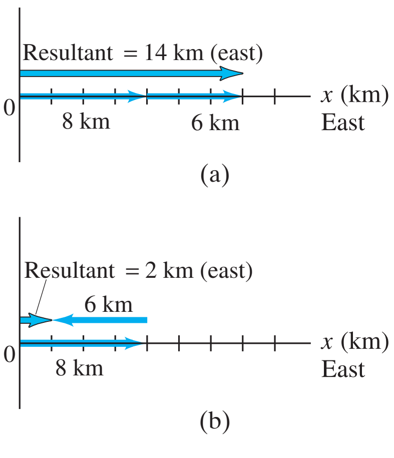
Drawing a diagram of a particular physical situation is always helpful in physics, and this is especially true when dealing with vectors. On a diagram, each vector is represented by an arrow. The arrow is always drawn so that it points in the direction of the vector quantity it represents. The length of the arrow is drawn proportional to the magnitude of the vector quantity. For example, in Fig. 3–1, green arrows have been drawn representing the velocity of a car at various places as it rounds a curve. The magnitude of the velocity at each point can be read off Fig. 3–1 by measuring the length of the corresponding arrow and using the scale shown $(1\text{ cm} = 90\text{ km/h})$.

When we write the symbol for a vector, we will always use boldface type, with a tiny arrow over the symbol. Thus for velocity we write $\vec{\mathbf{v}}$. If we are concerned only with the magnitude of the vector, we will write simply v, in italics, as we do for other symbols.

3–2 Addition of Vectors—Graphical Methods

Because vectors are quantities that have direction as well as magnitude, they must be added in a special way. In this Chapter, we will deal mainly with displacement vectors, for which we now use the symbol $\vec{\mathbf{D}}$, and velocity vectors, $\vec{\mathbf{v}}$. But the results will apply for other vectors we encounter later.

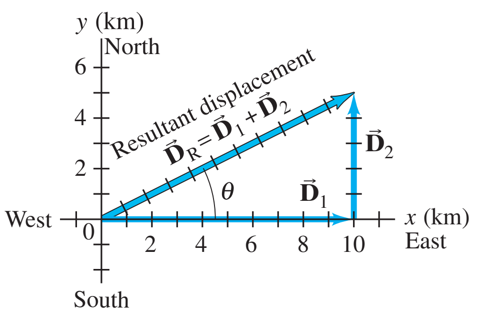
FIGURE 3–2 Combining vectors in one dimension.



We use simple arithmetic for adding scalars. Simple arithmetic can also be used for adding vectors if they are in the same direction. For example, if a person walks 8 km east one day, and 6 km east the next day, the person will be $8\text{ km} + 6\text{ km} = 14\text{ km}$ east of the point of origin. We say that the *net* or *resultant* displacement is 14 km to the east (Fig. 3–2a). If, on the other hand, the person walks 8 km east on the first day, and 6 km west (in the reverse direction) on the second day, then the person will end up 2 km from the origin (Fig. 3–2b), so the resultant displacement is 2 km to the east. In this case, the resultant displacement is obtained by subtraction: $8\text{ km} - 6\text{ km} = 2\text{ km}$.

But simple arithmetic cannot be used if the two vectors are not along the same line. For example, suppose a person walks 10.0 km east and then walks 5.0 km north. These displacements can be represented on a graph in which the positive y axis points north and the positive x axis points east, Fig. 3–3. On this graph, we draw an arrow, labeled $\vec{\mathbf{D}}_1$, to represent the 10.0-km displacement to the east. Then we draw a second arrow, $\vec{\mathbf{D}}_2$, to represent the 5.0-km displacement to the north. Both vectors are drawn to scale, as in Fig. 3–3.

FIGURE 3–3 A person walks 10.0 km east and then 5.0 km north. These two displacements are represented by the vectors $\vec{\mathbf{D}}_1$ and $\vec{\mathbf{D}}_2$, which are shown as arrows. Also shown is the resultant displacement vector, $\vec{\mathbf{D}}_R$, which is the vector sum of $\vec{\mathbf{D}}_1$ and $\vec{\mathbf{D}}_2$. Measurement on the graph with ruler and protractor shows that $\vec{\mathbf{D}}_R$ has a magnitude of 11.2 km and points at an angle $\theta = 27°$ north of east.



After taking this walk, the person is now 10.0 km east and 5.0 km north of the point of origin. The **resultant displacement** is represented by the arrow labeled $\vec{\mathbf{D}}_R$ in Fig. 3–3. (The subscript R stands for resultant.) Using a ruler and a protractor, you can measure on this diagram that the person is 11.2 km from the origin at an angle $\theta = 27°$ north of east. In other words, the resultant displacement vector has a magnitude of 11.2 km and makes an angle $\theta = 27°$ with the positive x axis. The magnitude (length) of $\vec{\mathbf{D}}_R$ can also be obtained using the theorem of Pythagoras in this case, because D_1, D_2, and D_R form a right triangle with D_R as the hypotenuse. Thus

$$D_R = \sqrt{D_1^2 + D_2^2} = \sqrt{(10.0\text{ km})^2 + (5.0\text{ km})^2}$$
$$= \sqrt{125\text{ km}^2} = 11.2\text{ km}.$$

You can use the Pythagorean theorem only when the vectors are *perpendicular* to each other.

The resultant displacement vector, $\vec{\mathbf{D}}_R$, is the sum of the vectors $\vec{\mathbf{D}}_1$ and $\vec{\mathbf{D}}_2$. That is,

$$\vec{\mathbf{D}}_R = \vec{\mathbf{D}}_1 + \vec{\mathbf{D}}_2.$$

This is a *vector* equation. An important feature of adding two vectors that are not along the same line is that the magnitude of the resultant vector is not equal to the sum of the magnitudes of the two separate vectors, but is smaller than their sum. That is,

$$D_R \leq (D_1 + D_2),$$

where the equals sign applies only if the two vectors point in the same direction. In our example (Fig. 3–3), $D_R = 11.2\text{ km}$, whereas $D_1 + D_2$ equals 15 km, which is the total distance traveled. Note also that we cannot set $\vec{\mathbf{D}}_R$ equal to 11.2 km, because we have a vector equation and 11.2 km is only a part of the resultant vector, its magnitude. We could write something like this, though: $\vec{\mathbf{D}}_R = \vec{\mathbf{D}}_1 + \vec{\mathbf{D}}_2 = (11.2\text{ km}, 27°\text{ N of E})$.

Figure 3–3 illustrates the general rules for graphically adding two vectors together, no matter what angles they make, to get their sum. The rules are as follows:

1. On a diagram, draw one of the vectors—call it $\vec{\mathbf{D}}_1$—to scale.
2. Next draw the second vector, $\vec{\mathbf{D}}_2$, to scale, placing its tail at the tip of the first vector and being sure its direction is correct.
3. The arrow drawn from the tail of the first vector to the tip of the second vector represents the *sum*, or **resultant**, of the two vectors.

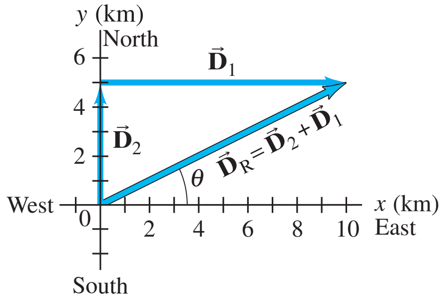
The length of the resultant vector represents its magnitude. Note that vectors can be moved parallel to themselves on paper (maintaining the same length and angle) to accomplish these manipulations. The length of the resultant can be measured with a ruler and compared to the scale. Angles can be measured with a protractor. This method is known as the **tail-to-tip method of adding vectors**.

The resultant is not affected by the order in which the vectors are added. For example, a displacement of 5.0 km north, to which is added a displacement of 10.0 km east, yields a resultant of 11.2 km and angle $\theta = 27°$ (see Fig. 3–4), the same as when they were added in reverse order (Fig. 3–3). That is, now using $\vec{\mathbf{V}}$ to represent any type of vector,

$$\vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2 = \vec{\mathbf{V}}_2 + \vec{\mathbf{V}}_1.$$

[Mathematicians call this equation the *commutative* property of vector addition.]

FIGURE 3–4 If the vectors are added in reverse order, the resultant is the same. (Compare to Fig. 3–3.)



The tail-to-tip method of adding vectors can be extended to three or more vectors. The resultant is drawn from the tail of the first vector to the tip of the last one added. An example is shown in Fig. 3–5; the three vectors could represent displacements (northeast, south, west) or perhaps three forces. Check for yourself that you get the same resultant no matter in which order you add the three vectors.

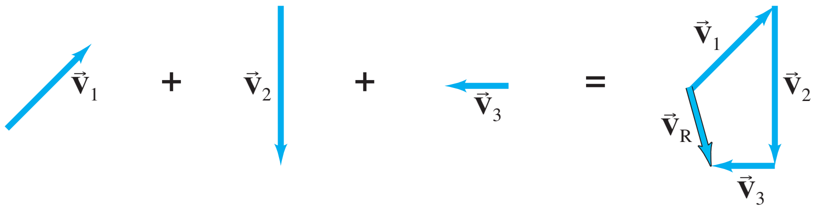


FIGURE 3–5 The resultant of three vectors: $\vec{\mathbf{V}}_R = \vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2 + \vec{\mathbf{V}}_3$.

A second way to add two vectors is the **parallelogram method**. It is fully equivalent to the tail-to-tip method. In this method, the two vectors are drawn starting from a common origin, and a parallelogram is constructed using these two vectors as adjacent sides as shown in Fig. 3–6b. The resultant is the diagonal drawn from the common origin. In Fig. 3–6a, the tail-to-tip method is shown, and we can see that both methods yield the same result.

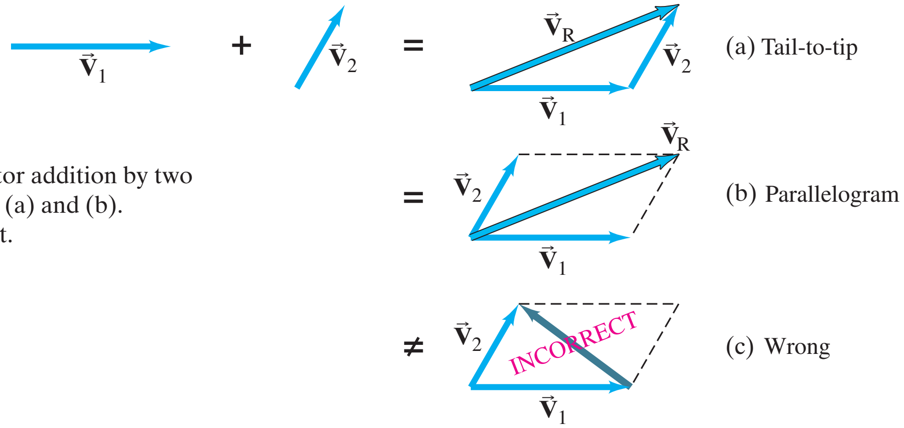


FIGURE 3–6 Vector addition by two different methods, (a) and (b). Part (c) is incorrect.

CAUTION

Be sure to use the correct diagonal on the parallelogram to get the resultant

It is a common error to draw the sum vector as the diagonal running between the tips of the two vectors, as in Fig. 3–6c. *This is incorrect*: it does not represent the sum of the two vectors. (In fact, it represents their difference, $\vec{\mathbf{V}}_2 - \vec{\mathbf{V}}_1$, as we will see in the next Section.)

CONCEPTUAL EXAMPLE 3–1 **Range of vector lengths.** Suppose two vectors each have length 3.0 units. What is the range of possible lengths for the vector representing the sum of the two?

RESPONSE The sum can take on any value from $6.0\ (= 3.0 + 3.0)$ where the vectors point in the same direction, to $0\ (= 3.0 - 3.0)$ when the vectors are antiparallel. Magnitudes between 0 and 6.0 occur when the two vectors are at an angle other than 0° and 180°.

EXERCISE A If the two vectors of Example 3–1 are perpendicular to each other, what is the resultant vector length?

3–3 Subtraction of Vectors, and Multiplication of a Vector by a Scalar

Given a vector $\vec{\mathbf{V}}$, we define the *negative* of this vector $(-\vec{\mathbf{V}})$ to be a vector with the same magnitude as $\vec{\mathbf{V}}$ but opposite in direction, Fig. 3–7. Note, however, that no vector is ever negative in the sense of its magnitude: the magnitude of every vector is positive. Rather, a minus sign tells us about its direction.

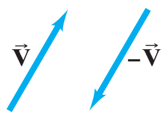


FIGURE 3–7 The negative of a vector is a vector having the same length but opposite direction.

We can now define the subtraction of one vector from another: the difference between two vectors $\vec{\mathbf{V}}_2 - \vec{\mathbf{V}}_1$ is defined as

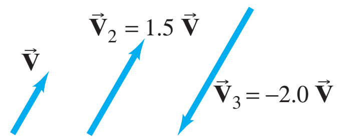
$$\vec{\mathbf{V}}_2 - \vec{\mathbf{V}}_1 = \vec{\mathbf{V}}_2 + (-\vec{\mathbf{V}}_1).$$

That is, the difference between two vectors is equal to the sum of the first plus the negative of the second. Thus our rules for addition of vectors can be applied as shown in Fig. 3–8 using the tail-to-tip method.

FIGURE 3–8 Subtracting two vectors: $\vec{\mathbf{V}}_2 - \vec{\mathbf{V}}_1$.

A vector $\vec{\mathbf{V}}$ can be multiplied by a scalar c. We define their product so that $c\vec{\mathbf{V}}$ has the same direction as $\vec{\mathbf{V}}$ and has magnitude cV. That is, multiplication of a vector by a positive scalar c changes the magnitude of the vector by a factor c but doesn't alter the direction. If c is a negative scalar (such as -2.0), the magnitude of the product $c\vec{\mathbf{V}}$ is changed by the factor $|c|$ (where $|c|$ means the magnitude of c), but the direction is precisely opposite to that of $\vec{\mathbf{V}}$. See Fig. 3–9.

FIGURE 3–9 Multiplying a vector $\vec{\mathbf{V}}$ by a scalar c gives a vector whose magnitude is c times greater and in the same direction as $\vec{\mathbf{V}}$ (or opposite direction if c is negative).



EXERCISE B What does the "incorrect" vector in Fig. 3–6c represent? (*a*) $\vec{\mathbf{V}}_2 - \vec{\mathbf{V}}_1$; (*b*) $\vec{\mathbf{V}}_1 - \vec{\mathbf{V}}_2$; (*c*) something else (specify).

3–4 Adding Vectors by Components

Adding vectors graphically using a ruler and protractor is often not sufficiently accurate and is not useful for vectors in three dimensions. We discuss now a more powerful and precise method for adding vectors. But do not forget graphical methods—they are useful for visualizing, for checking your math, and thus for getting the correct result.

Components

Consider first a vector $\vec{\mathbf{V}}$ that lies in a particular plane. It can be expressed as the sum of two other vectors, called the **components** of the original vector. The components are usually chosen to be along two perpendicular directions, such as the x and y axes. The process of finding the components is known as **resolving the vector into its components**. An example is shown in Fig. 3–10; the vector $\vec{\mathbf{V}}$ could be a displacement vector that points at an angle $\theta = 30°$ north of east, where we have chosen the positive x axis to be to the east and the positive y axis north. This vector $\vec{\mathbf{V}}$ is resolved into its x and y components by drawing dashed lines (AB and AC) out from the tip (A) of the vector, making them perpendicular to the x and y axes. Then the lines 0B and 0C represent the x and y components of $\vec{\mathbf{V}}$, respectively, as shown in Fig. 3–10b. These *vector components* are written $\vec{\mathbf{V}}_x$ and $\vec{\mathbf{V}}_y$. In this book we usually show vector components as arrows, like vectors, but dashed. The *scalar components*, V_x and V_y, are the magnitudes of the vector components, with units, accompanied by a positive or negative sign depending on whether they point along the positive or negative x or y axis. As can be seen in Fig. 3–10, $\vec{\mathbf{V}}_x + \vec{\mathbf{V}}_y = \vec{\mathbf{V}}$ by the parallelogram method of adding vectors.

Space is made up of three dimensions, and sometimes it is necessary to resolve a vector into components along three mutually perpendicular directions. In rectangular coordinates the components are $\vec{\mathbf{V}}_x$, $\vec{\mathbf{V}}_y$, and $\vec{\mathbf{V}}_z$.

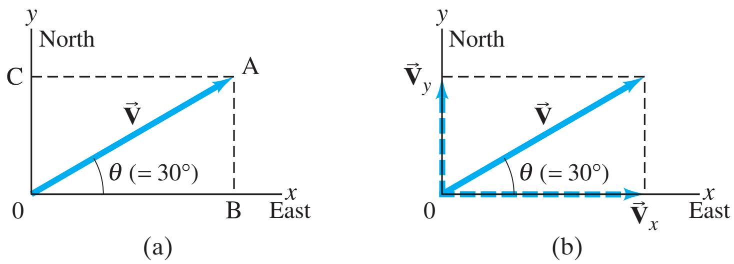


FIGURE 3–10 Resolving a vector $\vec{\mathbf{V}}$ into its components along a chosen set of x and y axes. The components, once found, themselves represent the vector. That is, the components contain as much information as the vector itself.

To add vectors using the method of components, we need to use the trigonometric functions sine, cosine, and tangent, which we now review.

Given any angle θ, as in Fig. 3–11a, a right triangle can be constructed by drawing a line perpendicular to one of its sides, as in Fig. 3–11b. The longest side of a right triangle, opposite the right angle, is called the hypotenuse, which we label h. The side opposite the angle θ is labeled o, and the side adjacent is labeled a. We let h, o, and a represent the lengths of these sides, respectively.

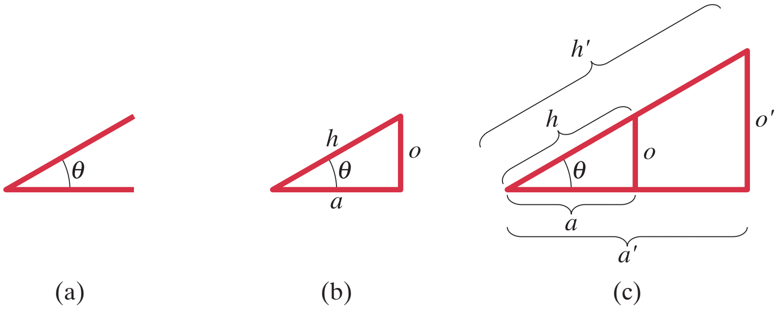


FIGURE 3–11 Starting with an angle θ as in (a), we can construct right triangles of different sizes, (b) and (c), but the ratio of the lengths of the sides does not depend on the size of the triangle.

We now define the three trigonometric functions, sine, cosine, and tangent (abbreviated sin, cos, tan), in terms of the right triangle, as follows:

$$\begin{aligned} \sin\theta &= \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{o}{h} \\ \cos\theta &= \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{a}{h} \\ \tan\theta &= \frac{\text{side opposite}}{\text{side adjacent}} = \frac{o}{a}. \end{aligned} \tag{3–1}$$

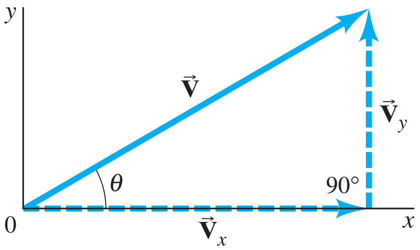
If we make the triangle bigger, but keep the same angles, then the ratio of the length of one side to the other, or of one side to the hypotenuse, remains the same. That is, in Fig. 3–11c we have: $a/h = a'/h'$; $o/h = o'/h'$; and $o/a = o'/a'$. Thus the values of sine, cosine, and tangent do not depend on how big the triangle is. They depend only on the size of the angle. The values of sine, cosine, and tangent for different angles can be found using a scientific calculator, or from the Table in Appendix A.

A useful trigonometric identity is

$$\sin^2\theta + \cos^2\theta = 1 \tag{3–2}$$

which follows from the Pythagorean theorem $(o^2 + a^2 = h^2$ in Fig. 3–11). That is:

$$\sin^2\theta + \cos^2\theta = \frac{o^2}{h^2} + \frac{a^2}{h^2} = \frac{o^2 + a^2}{h^2} = \frac{h^2}{h^2} = 1.$$



$$\sin\theta = \frac{V_y}{V}$$
$$\cos\theta = \frac{V_x}{V}$$
$$\tan\theta = \frac{V_y}{V_x}$$
$$V^2 = V_x^2 + V_y^2$$

FIGURE 3–12 Finding the components of a vector using trigonometric functions. The equations are valid only if θ is the angle $\vec{\mathbf{V}}$ makes with the positive x axis.

(See Appendix A and inside the rear cover for other details on trigonometric functions and identities.)

The use of trigonometric functions for finding the components of a vector is illustrated in Fig. 3–12, where a vector and its two components are thought of as making up a right triangle. We then see that the sine, cosine, and tangent are as given in Fig. 3–12, where θ is the angle $\vec{\mathbf{V}}$ makes with the $+x$ axis. If we multiply the definition of $\sin\theta = V_y/V$ by V on both sides, we get

$$V_y = V\sin\theta. \tag{3–3a}$$

Similarly, from the definition of $\cos\theta$, we obtain

$$V_x = V\cos\theta. \tag{3–3b}$$

Note that if θ is not the angle the vector makes with the positive x axis, Eqs. 3–3 are not valid.

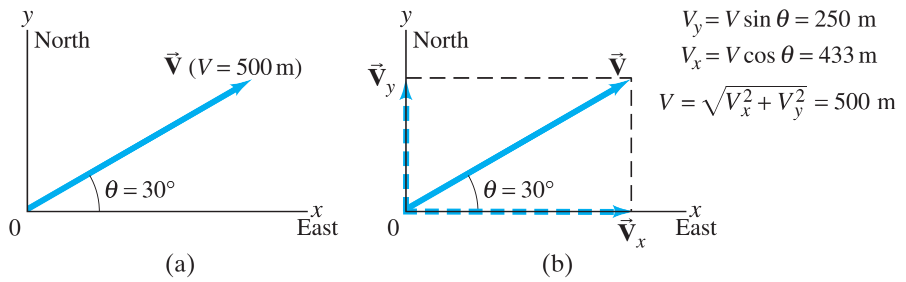


FIGURE 3–13 (a) Vector $\vec{\mathbf{V}}$ represents a displacement of 500 m at a 30° angle north of east. (b) The components of $\vec{\mathbf{V}}$ are $\vec{\mathbf{V}}_x$ and $\vec{\mathbf{V}}_y$, whose magnitudes are given on the right in the diagram.

Using Eqs. 3–3, we can calculate V_x and V_y for any vector, such as that illustrated in Fig. 3–10 or Fig. 3–12. Suppose $\vec{\mathbf{V}}$ represents a displacement of 500 m in a direction 30° north of east, as shown in Fig. 3–13. Then $V = 500$ m. From a calculator or Tables, $\sin 30° = 0.500$ and $\cos 30° = 0.866$. Then

$$\begin{aligned} V_x &= V\cos\theta = (500\text{ m})(0.866) = 433\text{ m (east)}, \\ V_y &= V\sin\theta = (500\text{ m})(0.500) = 250\text{ m (north)}. \end{aligned}$$

There are two ways to specify a vector in a given coordinate system:

1. We can give its components, V_x and V_y.
2. We can give its magnitude V and the angle θ it makes with the positive x axis.

We can shift from one description to the other using Eqs. 3–3, and, for the reverse, by using the theorem of Pythagoras[†] and the definition of tangent:

$$V = \sqrt{V_x^2 + V_y^2} \tag{3–4a}$$

$$\tan\theta = \frac{V_y}{V_x} \tag{3–4b}$$

as can be seen in Fig. 3–12.

Adding Vectors

We can now discuss how to add vectors using components. The first step is to resolve each vector into its components. Next we can see, using Fig. 3–14, that the addition of any two vectors $\vec{\mathbf{V}}_1$ and $\vec{\mathbf{V}}_2$ to give a resultant, $\vec{\mathbf{V}}_R = \vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2$, implies that

$$\begin{aligned} V_{Rx} &= V_{1x} + V_{2x} \\ V_{Ry} &= V_{1y} + V_{2y}. \end{aligned} \tag{3–5}$$

That is, the sum of the x components equals the x component of the resultant vector, and the sum of the y components equals the y component of the resultant, as can be verified by a careful examination of Fig. 3–14. Note that we do *not* add x components to y components.

If the magnitude and direction of the resultant vector are desired, they can be obtained using Eqs. 3–4.

[†]In three dimensions, the theorem of Pythagoras becomes $V = \sqrt{V_x^2 + V_y^2 + V_z^2}$, where V_z is the component along the third, or z, axis.

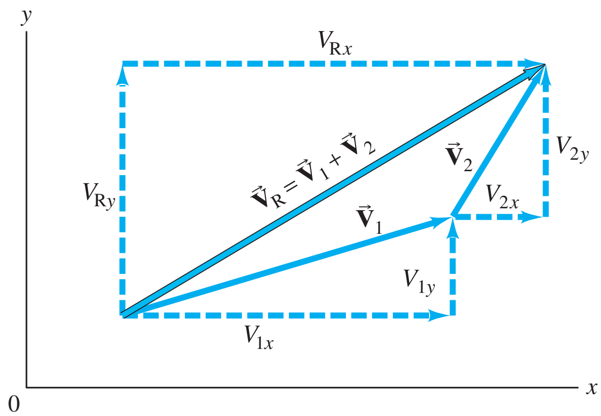


FIGURE 3–14 The components of $\vec{\mathbf{V}}_R = \vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2$ are $V_{Rx} = V_{1x} + V_{2x}$ and $V_{Ry} = V_{1y} + V_{2y}$.

The components of a given vector depend on the choice of coordinate axes. You can often reduce the work involved in adding vectors by a good choice of axes—for example, by choosing one of the axes to be in the same direction as one of the vectors. Then that vector will have only one nonzero component.

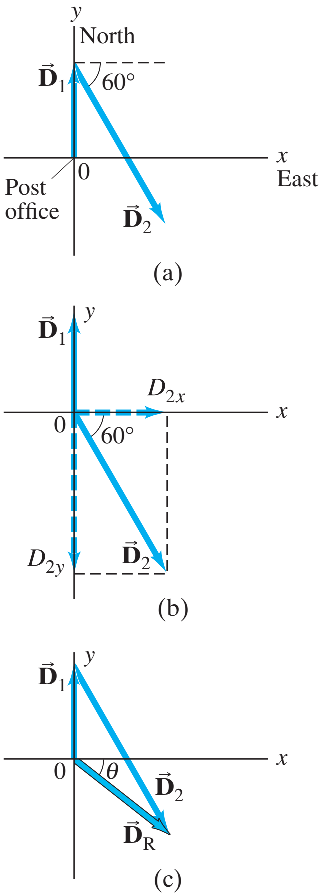


FIGURE 3–15 Example 3–2. (a) The two displacement vectors, $\vec{\mathbf{D}}_1$ and $\vec{\mathbf{D}}_2$. (b) $\vec{\mathbf{D}}_2$ is resolved into its components. (c) $\vec{\mathbf{D}}_1$ and $\vec{\mathbf{D}}_2$ are added to obtain the resultant $\vec{\mathbf{D}}_R$. The component method of adding the vectors is explained in the Example.

EXAMPLE 3–2 **Mail carrier's displacement.** A rural mail carrier leaves the post office and drives 22.0 km in a northerly direction. She then drives in a direction 60.0° south of east for 47.0 km (Fig. 3–15a). What is her displacement from the post office?

APPROACH We choose the positive x axis to be east and the positive y axis to be north, since those are the compass directions used on most maps. The origin of the xy coordinate system is at the post office. We resolve each vector into its x and y components. We add the x components together, and then the y components together, giving us the x and y components of the resultant.

SOLUTION Resolve each displacement vector into its components, as shown in Fig. 3–15b. Since $\vec{\mathbf{D}}_1$ has magnitude 22.0 km and points north, it has only a y component:

$$D_{1x} = 0, \qquad D_{1y} = 22.0\text{ km}.$$

$\vec{\mathbf{D}}_2$ has both x and y components:

$$D_{2x} = +(47.0\text{ km})(\cos 60°) = +(47.0\text{ km})(0.500) = +23.5\text{ km}$$
$$D_{2y} = -(47.0\text{ km})(\sin 60°) = -(47.0\text{ km})(0.866) = -40.7\text{ km}.$$

Notice that D_{2y} is negative because this vector component points along the negative y axis. The resultant vector, $\vec{\mathbf{D}}_R$, has components:

$$D_{Rx} = D_{1x} + D_{2x} = 0\text{ km} + 23.5\text{ km} = +23.5\text{ km}$$
$$D_{Ry} = D_{1y} + D_{2y} = 22.0\text{ km} + (-40.7\text{ km}) = -18.7\text{ km}.$$

This specifies the resultant vector completely:

$$D_{Rx} = 23.5\text{ km}, \qquad D_{Ry} = -18.7\text{ km}.$$

We can also specify the resultant vector by giving its magnitude and angle using Eqs. 3–4:

$$D_R = \sqrt{D_{Rx}^2 + D_{Ry}^2} = \sqrt{(23.5\text{ km})^2 + (-18.7\text{ km})^2} = 30.0\text{ km}$$

$$\tan\theta = \frac{D_{Ry}}{D_{Rx}} = \frac{-18.7\text{ km}}{23.5\text{ km}} = -0.796.$$

A calculator with a key labeled INV TAN, or ARC TAN, or TAN^{-1} gives $\theta = \tan^{-1}(-0.796) = -38.5°$. The negative sign means $\theta = 38.5°$ below the x axis, Fig. 3–15c. So, the resultant displacement is 30.0 km directed at 38.5° in a southeasterly direction.

NOTE Always be attentive about the quadrant in which the resultant vector lies. An electronic calculator does not fully give this information, but a good diagram does.

PROBLEM SOLVING
Identify the correct quadrant by drawing a careful diagram

As we saw in Example 3–2, any component that points along the negative x or y axis gets a minus sign. The signs of trigonometric functions depend on which "quadrant" the angle falls in: for example, the tangent is positive in the first and third quadrants (from 0° to 90°, and 180° to 270°), but negative in the second and fourth quadrants; see Appendix A, Fig. A–7. The best way to keep track of angles, and to check any vector result, is always to draw a vector diagram, like Fig. 3–15. A vector diagram gives you something tangible to look at when analyzing a problem, and provides a check on the results.

The following Problem Solving Strategy should not be considered a prescription. Rather it is a summary of things to do to get you thinking and involved in the problem at hand.

PROBLEM SOLVING

Adding Vectors

Here is a brief summary of how to add two or more vectors using components:

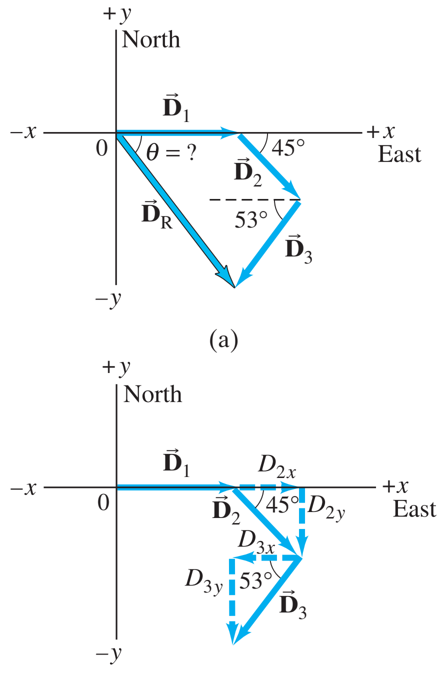
1. **Draw a diagram**, adding the vectors graphically by either the parallelogram or tail-to-tip method.
2. **Choose** x and y **axes**. Choose them in a way, if possible, that will make your work easier. (For example, choose one axis along the direction of one of the vectors, which then will have only one component.)
3. **Resolve** each vector into its x and y **components**, showing each component along its appropriate (x or y) axis as a (dashed) arrow.
4. **Calculate each component** (when not given) using sines and cosines. If θ_1 is the angle that vector $\vec{\mathbf{V}}_1$ makes with the positive x axis, then:
$$V_{1x} = V_1 \cos\theta_1, \qquad V_{1y} = V_1 \sin\theta_1 .$$
Pay careful attention to **signs**: any component that points along the negative x or y axis gets a minus sign.
5. **Add** the x **components** together to get the x component of the resultant. Similarly for y:
$$V_{Rx} = V_{1x} + V_{2x} + \text{any others}$$
$$V_{Ry} = V_{1y} + V_{2y} + \text{any others}.$$
This is the answer: the components of the resultant vector. Check signs to see if they fit the quadrant shown in your diagram (point 1 above).
6. If you want to know the **magnitude and direction** of the resultant vector, use Eqs. 3–4:
$$V_R = \sqrt{V_{Rx}^2 + V_{Ry}^2}, \qquad \tan\theta = \frac{V_{Ry}}{V_{Rx}}.$$
The vector diagram you already drew helps to obtain the correct position (quadrant) of the angle θ.

EXAMPLE 3–3 **Three short trips.** An airplane trip involves three legs, with two stopovers, as shown in Fig. 3–16a. The first leg is due east for 620 km; the second leg is southeast (45°) for 440 km; and the third leg is at 53° south of west, for 550 km, as shown. What is the plane's total displacement?

APPROACH We follow the steps in the Problem Solving Strategy above.

SOLUTION

1. **Draw a diagram** such as Fig. 3–16a, where $\vec{\mathbf{D}}_1$, $\vec{\mathbf{D}}_2$, and $\vec{\mathbf{D}}_3$ represent the three legs of the trip, and $\vec{\mathbf{D}}_R$ is the plane's total displacement.
2. **Choose axes**: Axes are also shown in Fig. 3–16a: x is east, y north.
3. **Resolve components**: It is imperative to draw a good diagram. The components are drawn in Fig. 3–16b. Instead of drawing all the vectors starting from a common origin, as we did in Fig. 3–15b, here we draw them "tail-to-tip" style, which is just as valid and may make it easier to see.
4. **Calculate the components**:
$$\begin{aligned}
\vec{\mathbf{D}}_1:\ D_{1x} &= +D_1\cos 0^\circ = D_1 = 620\text{ km}\\
D_{1y} &= +D_1\sin 0^\circ = 0\text{ km}\\
\vec{\mathbf{D}}_2:\ D_{2x} &= +D_2\cos 45^\circ = +(440\text{ km})(0.707) = +311\text{ km}\\
D_{2y} &= -D_2\sin 45^\circ = -(440\text{ km})(0.707) = -311\text{ km}\\
\vec{\mathbf{D}}_3:\ D_{3x} &= -D_3\cos 53^\circ = -(550\text{ km})(0.602) = -331\text{ km}\\
D_{3y} &= -D_3\sin 53^\circ = -(550\text{ km})(0.799) = -439\text{ km}.
\end{aligned}$$
We have given a minus sign to each component that in Fig. 3–16b points in the $-x$ or $-y$ direction. The components are shown in the Table in the margin.
5. **Add the components**: We add the x components together, and we add the y components together to obtain the x and y components of the resultant:
$$\begin{aligned}
D_{Rx} &= D_{1x} + D_{2x} + D_{3x} = 620\text{ km} + 311\text{ km} - 331\text{ km} = 600\text{ km}\\
D_{Ry} &= D_{1y} + D_{2y} + D_{3y} = 0\text{ km} - 311\text{ km} - 439\text{ km} = -750\text{ km}.
\end{aligned}$$
The x and y components of the resultant are 600 km and −750 km, and point respectively to the east and south. This is one way to give the answer.
6. **Magnitude and direction**: We can also give the answer as
$$D_R = \sqrt{D_{Rx}^2 + D_{Ry}^2} = \sqrt{(600)^2 + (-750)^2}\text{ km} = 960\text{ km}$$
$$\tan\theta = \frac{D_{Ry}}{D_{Rx}} = \frac{-750\text{ km}}{600\text{ km}} = -1.25, \qquad \text{so } \theta = -51^\circ.$$
Thus, the total displacement has magnitude 960 km and points 51° below the x axis (south of east), as was shown in our original sketch, Fig. 3–16a.

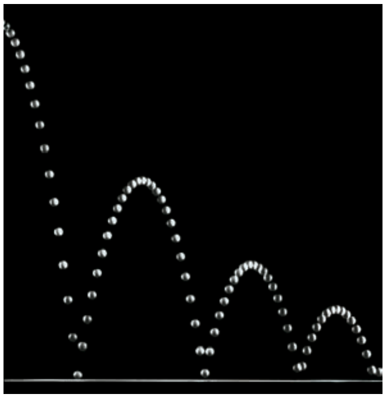


(a)
(b)

FIGURE 3–16 Example 3–3.

Vector	Components x (km)	y (km)
$\vec{\mathbf{D}}_1$	620	0
$\vec{\mathbf{D}}_2$	311	−311
$\vec{\mathbf{D}}_3$	−331	−439
$\vec{\mathbf{D}}_R$	600	−750

3–5 Projectile Motion



(a)

(b)

FIGURE 3–17 Photographs of (a) a bouncing ball and (b) a thrown basketball, each showing the characteristic "parabolic" path of projectile motion.

In Chapter 2, we studied the one-dimensional motion of an object in terms of displacement, velocity, and acceleration, including purely vertical motion of a falling object undergoing acceleration due to gravity. Now we examine the more general translational motion of objects moving through the air in two dimensions near the Earth's surface, such as a golf ball, a thrown or batted baseball, kicked footballs, and speeding bullets. These are all examples of **projectile motion** (see Fig. 3–17), which we can describe as taking place in two dimensions if there is no wind.

Although air resistance is often important, in many cases its effect can be ignored, and we will ignore it in the following analysis. We will not be concerned now with the process by which the object is thrown or projected. We consider only its motion *after* it has been projected, and *before* it lands or is caught—that is, we analyze our projected object only when it is moving freely through the air under the action of gravity alone. Then the acceleration of the object is that due to gravity, which acts downward with magnitude $g = 9.80\ \mathrm{m/s^2}$, and we assume it is constant.†

Galileo was the first to describe projectile motion accurately. He showed that it could be understood by analyzing the horizontal and vertical components of the motion separately. For convenience, we assume that the motion begins at time $t = 0$ at the origin of an xy coordinate system (so $x_0 = y_0 = 0$).

Let us look at a (tiny) ball rolling off the end of a horizontal table with an initial velocity in the horizontal (x) direction, $\vec{\mathbf{v}}_{x0}$. See Fig. 3–18, where an object falling vertically is also shown for comparison. The velocity vector $\vec{\mathbf{v}}$ at each instant points in the direction of the ball's motion at that instant and is thus always tangent to the path. Following Galileo's ideas, we treat the horizontal and vertical components of velocity and acceleration separately, and we can apply the kinematic equations (Eqs. 2–11a through 2–11c) to the x and y components of the motion.

First we examine the vertical (y) component of the motion. At the instant the ball leaves the table's top $(t = 0)$, it has only an x component of velocity. Once the ball leaves the table (at $t = 0$), it experiences a vertically downward acceleration g, the acceleration due to gravity. Thus v_y is initially zero $(v_{y0} = 0)$ but increases continually in the downward direction (until the ball hits the ground). Let us take y to be positive upward. Then the acceleration due to gravity is in the $-y$ direction, so $a_y = -g$. From Eq. 2–11a (using y in place of x) we can write $v_y = v_{y0} + a_y t = -gt$ since we set $v_{y0} = 0$. The vertical displacement is given by Eq. 2–11b written in terms of y: $y = y_0 + v_{y0} + \frac{1}{2} a_y t^2$. Given $y_0 = 0$, $v_{y0} = 0$, and $a_y = -g$, then $y = -\frac{1}{2} g t^2$.

†This restricts us to objects whose distance traveled and maximum height above the Earth are small compared to the Earth's radius (6400 km).

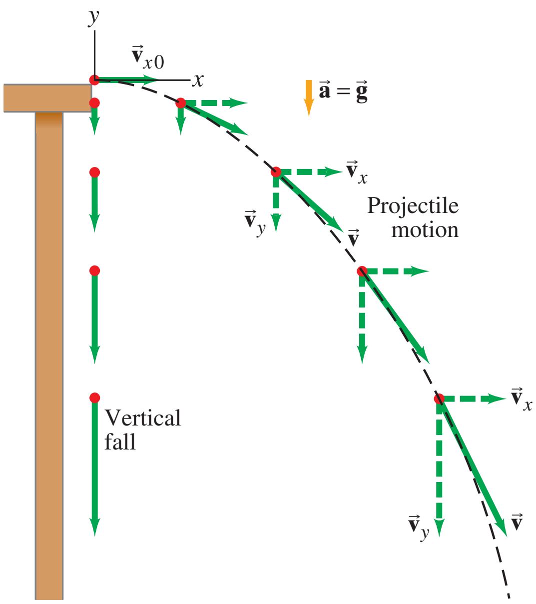


FIGURE 3–18 Projectile motion of a small ball projected horizontally with initial velocity $\vec{\mathbf{v}} = \vec{\mathbf{v}}_{x0}$. The dashed black line represents the path of the object. The velocity vector $\vec{\mathbf{v}}$ is in the direction of motion at each point, and thus is tangent to the path. The velocity vectors are green arrows, and velocity components are dashed. (A vertically falling object starting from rest at the same place and time is shown at the left for comparison; v_y is the same at each instant for the falling object and the projectile.)

In the horizontal direction, on the other hand, there is no acceleration (we are ignoring air resistance). With $a_x = 0$, the horizontal component of velocity, v_x, remains constant, equal to its initial value, v_{x0}, and thus has the same magnitude at each point on the path. The horizontal displacement (with $a_x = 0$) is given by $x = v_{x0}t + \frac{1}{2}a_x t^2 = v_{x0}t$. The two vector components, $\vec{\mathbf{v}}_x$ and $\vec{\mathbf{v}}_y$, can be added vectorially at any instant to obtain the velocity $\vec{\mathbf{v}}$ at that time (that is, for each point on the path), as shown in Fig. 3–18.

One result of this analysis, which Galileo himself predicted, is that *an object projected horizontally will reach the ground in the same time as an object dropped vertically*. This is because the vertical motions are the same in both cases, as shown in Fig. 3–18. Figure 3–19 is a multiple-exposure photograph of an experiment that confirms this.

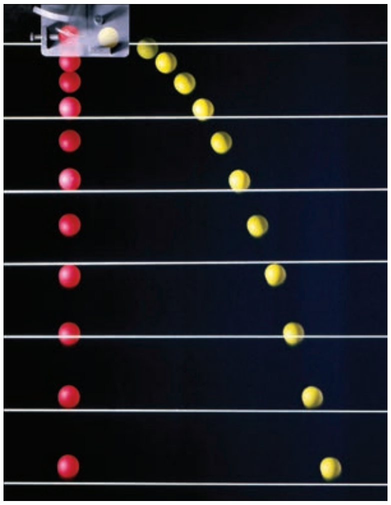


FIGURE 3–19 Multiple-exposure photograph showing positions of two balls at equal time intervals. One ball was dropped from rest at the same time the other ball was projected horizontally outward. The vertical position of each ball is seen to be the same at each instant.

EXERCISE C Two balls having different speeds roll off the edge of a horizontal table at the same time. Which hits the floor sooner, the faster ball or the slower one?

If an object is projected at an upward angle, as in Fig. 3–20, the analysis is similar, except that now there is an initial vertical component of velocity, v_{y0}. Because of the downward acceleration of gravity, the upward component of velocity v_y gradually decreases with time until the object reaches the highest point on its path, at which point $v_y = 0$. Subsequently the object moves downward (Fig. 3–20) and v_y increases in the downward direction, as shown (that is, becoming more negative). As before, v_x remains constant.

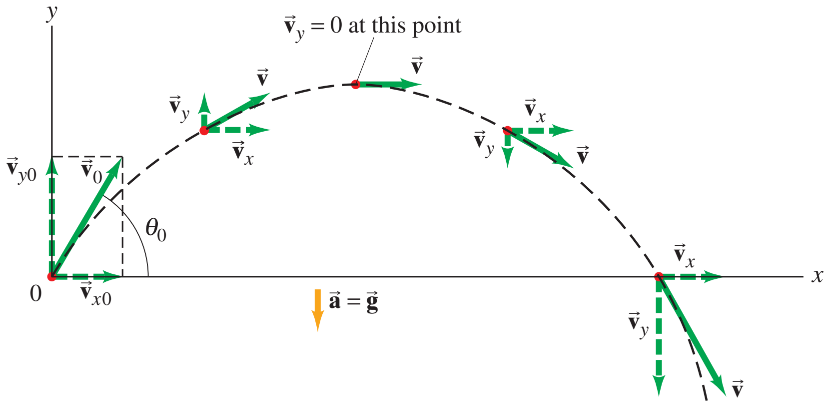


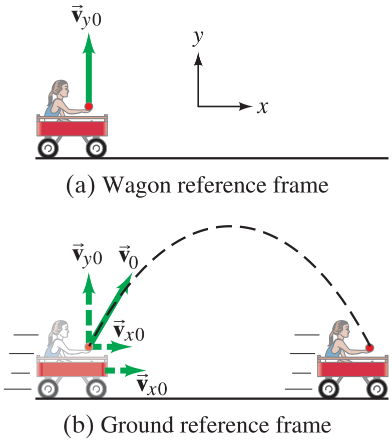
FIGURE 3–20 Path of a projectile launched with initial velocity $\vec{\mathbf{v}}_0$ at angle θ_0 to the horizontal. Path is shown dashed in black, the velocity vectors are green arrows, and velocity components are dashed. The figure does not show where the projectile hits the ground (at that point, projectile motion ceases).

EXERCISE D Where in Fig. 3–20 is (i) $\vec{\mathbf{v}} = 0$, (ii) $v_y = 0$, and (iii) $v_x = 0$?

CONCEPTUAL EXAMPLE 3–4 **Where does the apple land?** A child sits upright in a wagon which is moving to the right at constant speed as shown in Fig. 3–21. The child extends her hand and throws an apple straight upward (from her own point of view, Fig. 3–21a), while the wagon continues to travel forward at constant speed. If air resistance is neglected, will the apple land (*a*) behind the wagon, (*b*) in the wagon, or (*c*) in front of the wagon?

RESPONSE The child throws the apple straight up from her own reference frame with initial velocity $\vec{\mathbf{v}}_{y0}$ (Fig. 3–21a). But when viewed by someone on the ground, the apple also has an initial horizontal component of velocity equal to the speed of the wagon, $\vec{\mathbf{v}}_{x0}$. Thus, to a person on the ground, the apple will follow the path of a projectile as shown in Fig. 3–21b. The apple experiences no horizontal acceleration, so $\vec{\mathbf{v}}_{x0}$ will stay constant and equal to the speed of the wagon. As the apple follows its arc, the wagon will be directly under the apple at all times because they have the same horizontal velocity. When the apple comes down, it will drop right into the outstretched hand of the child. The answer is (*b*).

FIGURE 3–21 Example 3–4.



EXERCISE E Return to the Chapter-Opening Question, page 49, and answer it again now. Try to explain why you may have answered differently the first time. Describe the role of the helicopter in this example of projectile motion.

3–6 Solving Projectile Motion Problems

We now work through several Examples of projectile motion quantitatively. We use the kinematic equations (2–11a through 2–11c) separately for the vertical and horizontal components of the motion. These equations are shown separately for the x and y components of the motion in Table 3–1, for the general case of two-dimensional motion at constant acceleration. Note that x and y are the respective displacements, that v_x and v_y are the components of the velocity, and that a_x and a_y are the components of the acceleration, each of which is constant. The subscript 0 means "at $t = 0$."

TABLE 3–1 General Kinematic Equations for Constant Acceleration in Two Dimensions

x component (horizontal)		*y* component (vertical)
$v_x = v_{x0} + a_x t$	(Eq. 2–11a)	$v_y = v_{y0} + a_y t$
$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$	(Eq. 2–11b)	$y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$
$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$	(Eq. 2–11c)	$v_y^2 = v_{y0}^2 + 2a_y(y - y_0)$

We can simplify Eqs. 2–11 to use for projectile motion because we can set $a_x = 0$. See Table 3–2, which assumes y is positive upward, so $a_y = -g = -9.80\,\mathrm{m/s^2}$.

TABLE 3–2 Kinematic Equations for Projectile Motion
(y positive upward; $a_x = 0$, $a_y = -g = -9.80\,\mathrm{m/s^2}$)

Horizontal Motion ($a_x = 0$, v_x = constant)		Vertical Motion† ($a_y = -g$ = constant)
$v_x = v_{x0}$	(Eq. 2–11a)	$v_y = v_{y0} - gt$
$x = x_0 + v_{x0}t$	(Eq. 2–11b)	$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$
	(Eq. 2–11c)	$v_y^2 = v_{y0}^2 - 2g(y - y_0)$

† If y is taken positive downward, the minus ($-$) signs in front of g become $+$ signs.

If the projection angle θ_0 is chosen relative to the $+x$ axis (Fig. 3–20), then

$$v_{x0} = v_0 \cos\theta_0, \qquad \text{and} \qquad v_{y0} = v_0 \sin\theta_0 .$$

PROBLEM SOLVING
Choice of time interval

In doing Problems involving projectile motion, we must consider a time interval for which our chosen object is in the air, influenced only by gravity. We do not consider the throwing (or projecting) process, nor the time after the object lands or is caught, because then other influences act on the object, and we can no longer set $\vec{\mathbf{a}} = \vec{\mathbf{g}}$.

PROBLEM SOLVING

Projectile Motion

Our approach to solving Problems in Section 2–6 also applies here. Solving Problems involving projectile motion can require creativity, and cannot be done just by following some rules. Certainly you must avoid just plugging numbers into equations that seem to "work."

1. As always, **read** carefully; **choose** the **object** (or objects) you are going to analyze.
2. **Draw** a careful **diagram** showing what is happening to the object.
3. **Choose** an origin and an xy **coordinate system**.
4. Decide on the **time interval**, which for projectile motion can only include motion under the effect of gravity alone, not throwing or landing. The time interval must be the same for the x and y analyses. The x and y motions are connected by the common time, t.
5. **Examine** the horizontal (x) and vertical (y) **motions** separately. If you are given the initial velocity, you may want to resolve it into its x and y components.
6. List the **known** and **unknown** quantities, choosing $a_x = 0$ and $a_y = -g$ or $+g$, where $g = 9.80\,\mathrm{m/s^2}$, and using the $-$ or $+$ sign, depending on whether you choose y positive up or down. Remember that v_x never changes throughout the trajectory, and that $v_y = 0$ at the highest point of any trajectory that returns downward. The velocity just before landing is generally not zero.
7. Think for a minute before jumping into the equations. A little planning goes a long way. **Apply** the **relevant equations** (Table 3–2), combining equations if necessary. You may need to combine components of a vector to get magnitude and direction (Eqs. 3–4).

EXAMPLE 3–5 **Driving off a cliff.** A movie stunt driver on a motorcycle speeds horizontally off a 50.0-m-high cliff. How fast must the motorcycle leave the cliff top to land on level ground below, 90.0 m from the base of the cliff where the cameras are? Ignore air resistance.

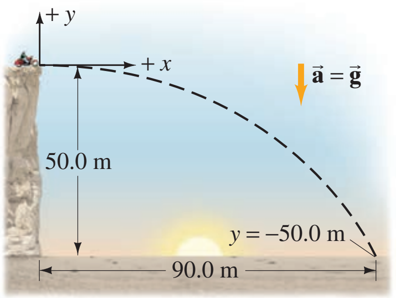


FIGURE 3–22 Example 3–5.

APPROACH We explicitly follow the steps of the Problem Solving Strategy on the previous page.

SOLUTION

1. and **2. Read, choose the object, and draw a diagram**. Our object is the motorcycle and driver, taken as a single unit. The diagram is shown in Fig. 3–22.
3. **Choose a coordinate system**. We choose the y direction to be positive upward, with the top of the cliff as $y_0 = 0$. The x direction is horizontal with $x_0 = 0$ at the point where the motorcycle leaves the cliff.
4. **Choose a time interval**. We choose our time interval to begin $(t = 0)$ just as the motorcycle leaves the cliff top at position $x_0 = 0,\ y_0 = 0$. Our time interval ends just before the motorcycle touches the ground below.
5. **Examine *x* and *y* motions**. In the horizontal (x) direction, the acceleration $a_x = 0$, so the velocity is constant. The value of x when the motorcycle reaches the ground is $x = +90.0\ \text{m}$. In the vertical direction, the acceleration is the acceleration due to gravity, $a_y = -g = -9.80\ \text{m/s}^2$. The value of y when the motorcycle reaches the ground is $y = -50.0\ \text{m}$. The initial velocity is horizontal and is our unknown, v_{x0}; the initial vertical velocity is zero, $v_{y0} = 0$.
6. **List knowns and unknowns**. See the Table in the margin. Note that in addition to not knowing the initial horizontal velocity v_{x0} (which stays constant until landing), we also do not know the time t when the motorcycle reaches the ground.

Known	Unknown
$x_0 = y_0 = 0$	v_{x0}
$x = 90.0\ \text{m}$	t
$y = -50.0\ \text{m}$	
$a_x = 0$	
$a_y = -g = -9.80\ \text{m/s}^2$	
$v_{y0} = 0$	

7. **Apply relevant equations**. The motorcycle maintains constant v_x as long as it is in the air. The time it stays in the air is determined by the y motion—when it reaches the ground. So we first find the time using the y motion, and then use this time value in the x equations. To find out how long it takes the motorcycle to reach the ground below, we use Eq. 2–11b (Tables 3–1 and 3–2) for the vertical (y) direction with $y_0 = 0$ and $v_{y0} = 0$:

$$\begin{aligned} y &= y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \\ &= 0 + \ \ 0 \ \ + \tfrac{1}{2}(-g)t^2 \end{aligned}$$

or

$$y = -\tfrac{1}{2}gt^2.$$

We solve for t and set $y = -50.0\ \text{m}$:

$$t = \sqrt{\frac{2y}{-g}} = \sqrt{\frac{2(-50.0\ \text{m})}{-9.80\ \text{m/s}^2}} = 3.19\ \text{s}.$$

To calculate the initial velocity, v_{x0}, we again use Eq. 2–11b, but this time for the horizontal (x) direction, with $a_x = 0$ and $x_0 = 0$:

$$\begin{aligned} x &= x_0 + v_{x0}t + \tfrac{1}{2}a_x t^2 \\ &= 0 + v_{x0}t + 0 \end{aligned}$$

or

$$x = v_{x0}t.$$

Then

$$v_{x0} = \frac{x}{t} = \frac{90.0\ \text{m}}{3.19\ \text{s}} = 28.2\ \text{m/s},$$

which is about 100 km/h (roughly 60 mi/h).

NOTE In the time interval of the projectile motion, the only acceleration is g in the negative y direction. The acceleration in the x direction is zero.

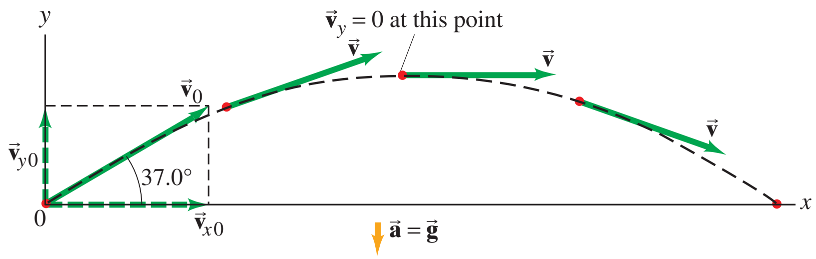


FIGURE 3–23 Example 3–6.

PHYSICS APPLIED
Sports

EXAMPLE 3–6 **A kicked football.** A kicked football leaves the ground at an angle $\theta_0 = 37.0°$ with a velocity of 20.0 m/s, as shown in Fig. 3–23. Calculate (*a*) the maximum height, (*b*) the time of travel before the football hits the ground, and (*c*) how far away it hits the ground. Assume the ball leaves the foot at ground level, and ignore air resistance and rotation of the ball.

APPROACH This may seem difficult at first because there are so many questions. But we can deal with them one at a time. We take the *y* direction as positive upward, and treat the *x* and *y* motions separately. The total time in the air is again determined by the *y* motion. The *x* motion occurs at constant velocity. The *y* component of velocity varies, being positive (upward) initially, decreasing to zero at the highest point, and then becoming negative as the football falls.

SOLUTION We resolve the initial velocity into its components (Fig. 3–23):

$$v_{x0} = v_0 \cos 37.0° = (20.0\,\mathrm{m/s})(0.799) = 16.0\,\mathrm{m/s}$$
$$v_{y0} = v_0 \sin 37.0° = (20.0\,\mathrm{m/s})(0.602) = 12.0\,\mathrm{m/s}.$$

(*a*) To find the maximum height, we consider a time interval that begins just after the football loses contact with the foot until the ball reaches its maximum height. During this time interval, the acceleration is *g* downward. At the maximum height, the velocity is horizontal (Fig. 3–23), so $v_y = 0$. This occurs at a time given by $v_y = v_{y0} - gt$ with $v_y = 0$ (see Eq. 2–11a in Table 3–2), so $v_{y0} = gt$ and

$$t = \frac{v_{y0}}{g} = \frac{(12.0\,\mathrm{m/s})}{(9.80\,\mathrm{m/s^2})} = 1.224\,\mathrm{s} \approx 1.22\,\mathrm{s}.$$

From Eq. 2–11b, with $y_0 = 0$, we can solve for *y* at this time $(t = v_{y0}/g)$:

$$y = v_{y0}t - \tfrac{1}{2}gt^2 = \frac{v_{y0}^2}{g} - \frac{1}{2}\frac{v_{y0}^2}{g} = \frac{v_{y0}^2}{2g} = \frac{(12.0\,\mathrm{m/s})^2}{2(9.80\,\mathrm{m/s^2})} = 7.35\,\mathrm{m}.$$

The maximum height is 7.35 m. [Solving Eq. 2–11c for *y* gives the same result.]

(*b*) To find the time it takes for the ball to return to the ground, we consider a different time interval, starting at the moment the ball leaves the foot $(t = 0, y_0 = 0)$ and ending just before the ball touches the ground $(y = 0$ again). We can use Eq. 2–11b with $y_0 = 0$ and also set $y = 0$ (ground level):

$$y = y_0 + v_{y0}t - \tfrac{1}{2}gt^2$$
$$0 = 0 + v_{y0}t - \tfrac{1}{2}gt^2.$$

This equation can be factored:

$$t\left(\tfrac{1}{2}gt - v_{y0}\right) = 0.$$

There are two solutions, $t = 0$ (which corresponds to the initial point, y_0), and

$$t = \frac{2v_{y0}}{g} = \frac{2(12.0\,\mathrm{m/s})}{(9.80\,\mathrm{m/s^2})} = 2.45\,\mathrm{s},$$

which is the total travel time of the football.

(*c*) The total distance traveled in the *x* direction is found by applying Eq. 2–11b with $x_0 = 0$, $a_x = 0$, $v_{x0} = 16.0\,\mathrm{m/s}$, and $t = 2.45\,\mathrm{s}$:

$$x = v_{x0}t = (16.0\,\mathrm{m/s})(2.45\,\mathrm{s}) = 39.2\,\mathrm{m}.$$

PROBLEM SOLVING
Symmetry

NOTE In (*b*), the time needed for the whole trip, $t = 2v_{y0}/g = 2.45\,\mathrm{s}$, is double the time to reach the highest point, calculated in (*a*). That is, the time to go up equals the time to come back down to the same level (ignoring air resistance).

EXERCISE F In Example 3–6, what is (*a*) the velocity vector at the maximum height, and (*b*) the acceleration vector at maximum height?

In Example 3–6, we treated the football as if it were a particle, ignoring its rotation. We also ignored air resistance. Because air resistance is significant on a football, our results are only estimates (mainly overestimates).

CONCEPTUAL EXAMPLE 3–7 **The wrong strategy.** A boy on a small hill aims his water-balloon slingshot horizontally, straight at a second boy hanging from a tree branch a distance d away, Fig. 3–24. At the instant the water balloon is released, the second boy lets go and falls from the tree, hoping to avoid being hit. Show that he made the wrong move. (He hadn't studied physics yet.) Ignore air resistance.

RESPONSE Both the water balloon and the boy in the tree start falling at the same instant, and in a time t they each fall the same vertical distance $y = \frac{1}{2}gt^2$, much like Fig. 3–19. In the time it takes the water balloon to travel the horizontal distance d, the balloon will have the same y position as the falling boy. Splat. If the boy had stayed in the tree, he would have avoided the humiliation.

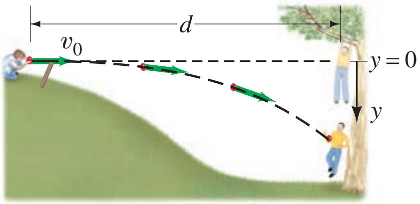


FIGURE 3–24 Example 3–7.

Level Horizontal Range

The total distance the football traveled in Example 3–6 is called the horizontal **range** R. We now derive a formula for the range, which applies to a projectile that lands at the same level it started ($= y_0$): that is, $y\text{ (final)} = y_0$ (see Fig. 3–25a). Looking back at Example 3–6 part (*c*), we see that $x = R = v_{x0}t$ where (from part *b*) $t = 2v_{y0}/g$. Thus

$$R = v_{x0}t = v_{x0}\left(\frac{2v_{y0}}{g}\right) = \frac{2v_{x0}v_{y0}}{g} = \frac{2v_0^2 \sin\theta_0 \cos\theta_0}{g}, \qquad [y = y_0]$$

where $v_{x0} = v_0 \cos\theta_0$ and $v_{y0} = v_0 \sin\theta_0$. This can be rewritten, using the trigonometric identity $2\sin\theta\cos\theta = \sin 2\theta$ (Appendix A or inside the rear cover):

$$R = \frac{v_0^2 \sin 2\theta_0}{g}. \qquad [\text{only if } y\text{ (final)} = y_0]$$

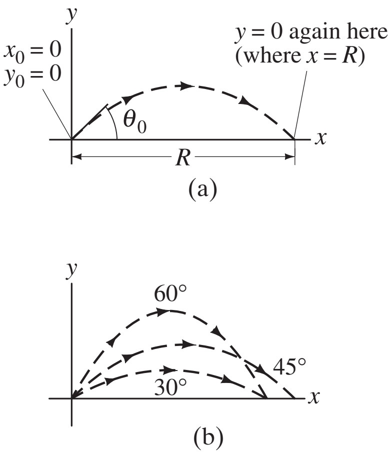
Note that the *maximum* range, for a given initial velocity v_0, is obtained when $\sin 2\theta$ takes on its maximum value of 1.0, which occurs for $2\theta_0 = 90°$; so

$$\theta_0 = 45° \text{ for maximum range, and } R_{\max} = v_0^2/g.$$

The maximum range increases by the square of v_0, so doubling the muzzle velocity of a cannon increases its maximum range by a factor of 4.

When air resistance is important, the range is less for a given v_0, and the maximum range is obtained at an angle smaller than 45°.

FIGURE 3–25 (a) The range R of a projectile. (b) There are generally two angles θ_0 that will give the same range. If one angle is θ_{01}, the other is $\theta_{02} = 90° - \theta_{01}$. Example 3–8.



EXAMPLE 3–8 **Range of a cannon ball.** Suppose one of Napoleon's cannons had a muzzle speed, v_0, of 60.0 m/s. At what angle should it have been aimed (ignore air resistance) to strike a target 320 m away?

APPROACH We use the equation just derived for the range, $R = v_0^2 \sin 2\theta_0/g$, with $R = 320\text{ m}$.

SOLUTION We solve for $\sin 2\theta_0$ in the range formula:

$$\sin 2\theta_0 = \frac{Rg}{v_0^2} = \frac{(320\text{ m})(9.80\text{ m/s}^2)}{(60.0\text{ m/s})^2} = 0.871.$$

We want to solve for an angle θ_0 that is between 0° and 90°, which means $2\theta_0$ in this equation can be as large as 180°. Thus, $2\theta_0 = 60.6°$ is a solution, so $\theta_0 = 30.3°$. But $2\theta_0 = 180° - 60.6° = 119.4°$ is also a solution (see Appendix A–7), so θ_0 can also be $\theta_0 = 59.7°$. In general we have two solutions (see Fig. 3–25b), which in the present case are given by

$$\theta_0 = 30.3° \quad \text{or} \quad 59.7°.$$

Either angle gives the same range. Only when $\sin 2\theta_0 = 1$ (so $\theta_0 = 45°$) is there a single solution (that is, both solutions are the same).

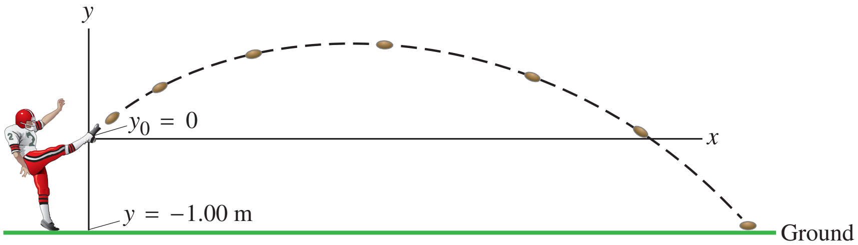


FIGURE 3–26 Example 3–9: the football leaves the punter's foot at $y = 0$, and reaches the ground where $y = -1.00$ m.

PHYSICS APPLIED
Sports

PROBLEM SOLVING
Do not use any formula unless you are sure its range of validity fits the problem; the range formula does not apply here because $y \neq y_0$

EXAMPLE 3–9 **A punt.** Suppose the football in Example 3–6 was punted, and left the punter's foot at a height of 1.00 m above the ground. How far did the football travel before hitting the ground? Set $x_0 = 0$, $y_0 = 0$.

APPROACH The only difference here from Example 3–6 is that the football hits the ground *below* its starting point of $y_0 = 0$. That is, the ball hits the ground at $y = -1.00$ m. See Fig. 3–26. Thus we cannot use the range formula which is valid only if $y\,(\text{final}) = y_0$. As in Example 3–6, $v_0 = 20.0$ m/s, $\theta_0 = 37.0°$.

SOLUTION With $y = -1.00$ m and $v_{y0} = 12.0$ m/s (see Example 3–6), we use the y version of Eq. 2–11b with $a_y = -g$,

$$y = y_0 + v_{y0}t - \tfrac{1}{2}gt^2,$$

and obtain

$$-1.00\text{ m} = 0 + (12.0\text{ m/s})t - (4.90\text{ m/s}^2)t^2.$$

We rearrange this equation into standard form $(ax^2 + bx + c = 0)$ so we can use the quadratic formula:

$$(4.90\text{ m/s}^2)t^2 - (12.0\text{ m/s})t - (1.00\text{ m}) = 0.$$

The quadratic formula (Appendix A–4) gives

$$t = \frac{12.0\text{ m/s} \pm \sqrt{(-12.0\text{ m/s})^2 - 4(4.90\text{ m/s}^2)(-1.00\text{ m})}}{2(4.90\text{ m/s}^2)}$$

$$= 2.53\text{ s} \quad \text{or} \quad -0.081\text{ s}.$$

The second solution would correspond to a time prior to the kick, so it doesn't apply. With $t = 2.53$ s for the time at which the ball touches the ground, the horizontal distance the ball traveled is (using $v_{x0} = 16.0$ m/s from Example 3–6):

$$x = v_{x0}t = (16.0\text{ m/s})(2.53\text{ s}) = 40.5\text{ m}.$$

Our assumption in Example 3–6 that the ball leaves the foot at ground level would result in an underestimate of about 1.3 m in the distance our punt traveled.

*3–7 Projectile Motion Is Parabolic

We now show that the path followed by any projectile is a *parabola*, if we can ignore air resistance and can assume that $\vec{\mathbf{g}}$ is constant. To do so, we need to find y as a function of x by eliminating t between the two equations for horizontal and vertical motion (Eq. 2–11b in Table 3–2), and for simplicity we set $x_0 = y_0 = 0$:

$$x = v_{x0}t$$
$$y = v_{y0}t - \tfrac{1}{2}gt^2.$$

From the first equation, we have $t = x/v_{x0}$, and we substitute this into the second one to obtain

$$y = \left(\frac{v_{y0}}{v_{x0}}\right)x - \left(\frac{g}{2v_{x0}^2}\right)x^2. \qquad \textbf{(3–6)}$$

We see that y as a function of x has the form

$$y = Ax - Bx^2,$$

where A and B are constants for any specific projectile motion. This is the standard equation for a parabola. See Figs. 3–17 and 3–27.

The idea that projectile motion is parabolic was, in Galileo's day, at the forefront of physics research. Today we discuss it in Chapter 3 of introductory physics!

FIGURE 3–27 Examples of projectile motion: a boy jumping, and glowing lava from the volcano Stromboli.

*Some Sections of this book, such as this one, may be considered *optional* at the discretion of the instructor. See the Preface for more details.

3–8 Relative Velocity

We now consider how observations made in different frames of reference are related to each other. For example, consider two trains approaching one another, each with a speed of 80 km/h with respect to the Earth. Observers on the Earth beside the train tracks will measure 80 km/h for the speed of each of the trains. Observers on either one of the trains (a different frame of reference) will measure a speed of 160 km/h for the train approaching them.

Similarly, when one car traveling 90 km/h passes a second car traveling in the same direction at 75 km/h, the first car has a speed relative to the second car of $90\text{ km/h} - 75\text{ km/h} = 15\text{ km/h}$.

When the velocities are along the same line, simple addition or subtraction is sufficient to obtain the **relative velocity**. But if they are not along the same line, we must make use of vector addition. We emphasize, as mentioned in Section 2–1, that when specifying a velocity, it is important to specify what the reference frame is.

When determining relative velocity, it is easy to make a mistake by adding or subtracting the wrong velocities. It is important, therefore, to draw a diagram and use a careful labeling process. Each velocity is labeled by *two subscripts: the first refers to the object, the second to the reference frame in which it has this velocity*. For example, suppose a boat heads directly across a river, as shown in Fig. 3–28. We let $\vec{\mathbf{v}}_{BW}$ be the velocity of the **B**oat with respect to the **W**ater. (This is also what the boat's velocity would be relative to the shore if the water were still.) Similarly, $\vec{\mathbf{v}}_{BS}$ is the velocity of the **B**oat with respect to the **S**hore, and $\vec{\mathbf{v}}_{WS}$ is the velocity of the **W**ater with respect to the **S**hore (this is the river current). Note that $\vec{\mathbf{v}}_{BW}$ is what the boat's motor produces (against the water), whereas $\vec{\mathbf{v}}_{BS}$ is equal to $\vec{\mathbf{v}}_{BW}$ plus the effect of the current, $\vec{\mathbf{v}}_{WS}$. Therefore, the velocity of the boat relative to the shore is (see vector diagram, Fig. 3–28)

$$\vec{\mathbf{v}}_{BS} = \vec{\mathbf{v}}_{BW} + \vec{\mathbf{v}}_{WS}. \tag{3–7}$$

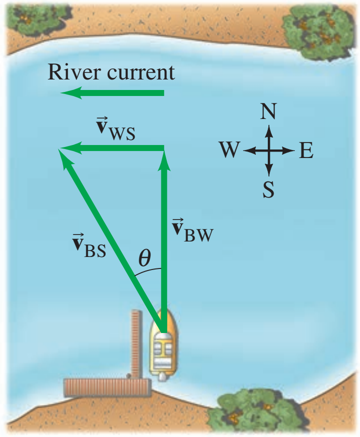


FIGURE 3–28 A boat heads north directly across a river which flows west. Velocity vectors are shown as green arrows:

$\vec{\mathbf{v}}_{BS}$ = velocity of **B**oat with respect to the **S**hore,
$\vec{\mathbf{v}}_{BW}$ = velocity of **B**oat with respect to the **W**ater,
$\vec{\mathbf{v}}_{WS}$ = velocity of **W**ater with respect to the **S**hore (river current).

As it crosses the river, the boat is dragged downstream by the current.

By writing the subscripts using this convention, we see that the inner subscripts (the two W's) on the right-hand side of Eq. 3–7 are the same; also, the outer subscripts on the right of Eq. 3–7 (the B and the S) are the same as the two subscripts for the sum vector on the left, $\vec{\mathbf{v}}_{BS}$. By following this convention (first subscript for the object, second for the reference frame), you can write down the correct equation relating velocities in different reference frames.[†]

Equation 3–7 is valid in general and can be extended to three or more velocities. For example, if a fisherman on the boat walks with a velocity $\vec{\mathbf{v}}_{FB}$ relative to the boat, his velocity relative to the shore is $\vec{\mathbf{v}}_{FS} = \vec{\mathbf{v}}_{FB} + \vec{\mathbf{v}}_{BW} + \vec{\mathbf{v}}_{WS}$. The equations involving relative velocity will be correct when adjacent inner subscripts are identical and when the outermost ones correspond exactly to the two on the velocity on the left of the equation. But this works only with plus signs (on the right), not minus signs.

It is often useful to remember that for any two objects or reference frames, A and B, the velocity of A relative to B has the same magnitude, but opposite direction, as the velocity of B relative to A:

$$\vec{\mathbf{v}}_{BA} = -\vec{\mathbf{v}}_{AB}. \tag{3–8}$$

For example, if a train is traveling 100 km/h relative to the Earth in a certain direction, objects on the Earth (such as trees) appear to an observer on the train to be traveling 100 km/h in the opposite direction.

[†]We thus can see, for example, that the equation $\vec{\mathbf{v}}_{BW} = \vec{\mathbf{v}}_{BS} + \vec{\mathbf{v}}_{WS}$ is wrong: the inner subscripts are not the same, and the outer ones on the right do not correspond to the subscripts on the left.

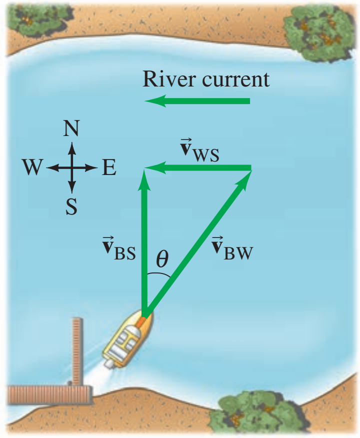


FIGURE 3–29 Example 3–10.

EXAMPLE 3–10 **Heading upstream.** A boat's speed in still water is $v_{BW} = 1.85\,\mathrm{m/s}$. If the boat is to travel north directly across a river whose westward current has speed $v_{WS} = 1.20\,\mathrm{m/s}$, at what upstream angle must the boat head? (See Fig. 3–29.)

APPROACH If the boat heads straight across the river, the current will drag the boat downstream (westward). To overcome the river's current, the boat must have an upstream (eastward) component of velocity as well as a cross-stream (northward) component. Figure 3–29 has been drawn with $\vec{\mathbf{v}}_{BS}$, the velocity of the **B**oat relative to the **S**hore, pointing directly across the river because this is where the boat is supposed to go. (Note that $\vec{\mathbf{v}}_{BS} = \vec{\mathbf{v}}_{BW} + \vec{\mathbf{v}}_{WS}$.)

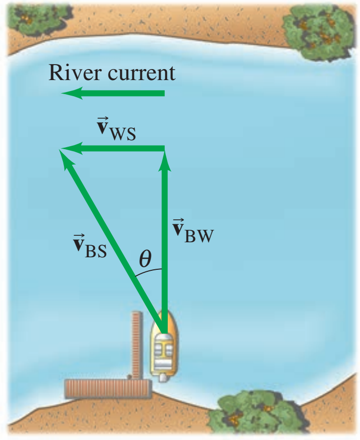
SOLUTION Vector $\vec{\mathbf{v}}_{BW}$ points upstream at angle θ as shown. From the diagram,

$$\sin\theta = \frac{v_{WS}}{v_{BW}} = \frac{1.20\,\mathrm{m/s}}{1.85\,\mathrm{m/s}} = 0.6486.$$

Thus $\theta = 40.4°$, so the boat must head upstream at a 40.4° angle.

EXAMPLE 3–11 **Heading across the river.** The same boat $(v_{BW} = 1.85\,\mathrm{m/s})$ now heads directly across the river whose current is still $1.20\,\mathrm{m/s}$. (*a*) What is the velocity (magnitude and direction) of the boat relative to the shore? (*b*) If the river is 110 m wide, how long will it take to cross and how far downstream will the boat be then?

FIGURE 3–30 Example 3–11. A boat heading directly across a river whose current moves at 1.20 m/s.



APPROACH The boat now heads directly across the river and is pulled downstream by the current, as shown in Fig. 3–30. The boat's velocity with respect to the shore, $\vec{\mathbf{v}}_{BS}$, is the sum of its velocity with respect to the water, $\vec{\mathbf{v}}_{BW}$, plus the velocity of the water with respect to the shore, $\vec{\mathbf{v}}_{WS}$: just as before,

$$\vec{\mathbf{v}}_{BS} = \vec{\mathbf{v}}_{BW} + \vec{\mathbf{v}}_{WS}.$$

SOLUTION (*a*) Since $\vec{\mathbf{v}}_{BW}$ is perpendicular to $\vec{\mathbf{v}}_{WS}$, we can get v_{BS} using the theorem of Pythagoras:

$$v_{BS} = \sqrt{v_{BW}^2 + v_{WS}^2} = \sqrt{(1.85\,\mathrm{m/s})^2 + (1.20\,\mathrm{m/s})^2} = 2.21\,\mathrm{m/s}.$$

We can obtain the angle (note how θ is defined in Fig. 3–30) from:

$$\tan\theta = v_{WS}/v_{BW} = (1.20\,\mathrm{m/s})/(1.85\,\mathrm{m/s}) = 0.6486.$$

A calculator with a key INV TAN or ARC TAN or TAN^{-1} gives $\theta = \tan^{-1}(0.6486) = 33.0°$. Note that this angle is not equal to the angle calculated in Example 3–10.

(*b*) The travel time for the boat is determined by the time it takes to cross the river. Given the river's width $D = 110\,\mathrm{m}$, we can use the velocity component in the direction of D, $v_{BW} = D/t$. Solving for t, we get $t = 110\,\mathrm{m}/1.85\,\mathrm{m/s} = 59.5\,\mathrm{s}$. The boat will have been carried downstream, in this time, a distance

$$d = v_{WS}t = (1.20\,\mathrm{m/s})(59.5\,\mathrm{s}) = 71.4\,\mathrm{m} \approx 71\,\mathrm{m}.$$

NOTE There is no acceleration in this Example, so the motion involves only constant velocities (of the boat or of the river).

Summary

A quantity such as velocity, that has both a magnitude and a direction, is called a **vector**. A quantity such as mass, that has only a magnitude, is called a **scalar**. On diagrams, vectors are represented by arrows.

Addition of vectors can be done graphically by placing the tail of each successive arrow at the tip of the previous one. The sum, or **resultant vector**, is the arrow drawn from the tail of the first vector to the tip of the last vector. Two vectors can also be added using the parallelogram method.

Vectors can be added more accurately by adding their **components** along chosen axes with the aid of trigonometric functions. A vector of magnitude V making an angle θ with the $+x$ axis has components

$$V_x = V\cos\theta, \qquad V_y = V\sin\theta. \qquad \textbf{(3–3)}$$

Given the components, we can find a vector's magnitude and direction from

$$V = \sqrt{V_x^2 + V_y^2}, \qquad \tan\theta = \frac{V_y}{V_x}. \qquad \textbf{(3–4)}$$

Projectile motion is the motion of an object in the air near the Earth's surface under the effect of gravity alone. It can be analyzed as two separate motions if air resistance can be ignored. The horizontal component of motion is at constant velocity, whereas the vertical component is at constant acceleration, $\vec{\mathbf{g}}$, just as for an object falling vertically under the action of gravity.

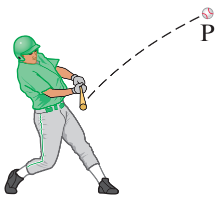
The velocity of an object relative to one frame of reference can be found by vector addition if its velocity relative to a second frame of reference, and the **relative velocity** of the two reference frames, are known.

Questions

1. One car travels due east at 40 km/h, and a second car travels north at 40 km/h. Are their velocities equal? Explain.
2. Can you conclude that a car is not accelerating if its speedometer indicates a steady 60 km/h? Explain.
3. Give several examples of an object's motion in which a great distance is traveled but the displacement is zero.
4. Can the displacement vector for a particle moving in two dimensions be longer than the length of path traveled by the particle over the same time interval? Can it be less? Discuss.
5. During baseball practice, a player hits a very high fly ball and then runs in a straight line and catches it. Which had the greater displacement, the player or the ball? Explain.
6. If $\vec{\mathbf{V}} = \vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2$, is V necessarily greater than V_1 and/or V_2? Discuss.
7. Two vectors have length $V_1 = 3.5\text{ km}$ and $V_2 = 4.0\text{ km}$. What are the maximum and minimum magnitudes of their vector sum?
8. Can two vectors, of unequal magnitude, add up to give the zero vector? Can *three* unequal vectors? Under what conditions?
9. Can the magnitude of a vector ever (*a*) equal, or (*b*) be less than, one of its components?
10. Does the odometer of a car measure a scalar or a vector quantity? What about the speedometer?
11. How could you determine the speed a slingshot imparts to a rock, using only a meter stick, a rock, and the slingshot?
12. In archery, should the arrow be aimed directly at the target? How should your angle of aim depend on the distance to the target?
13. It was reported in World War I that a pilot flying at an altitude of 2 km caught in his bare hands a bullet fired at the plane! Using the fact that a bullet slows down considerably due to air resistance, explain how this incident occurred.
14. You are on the street trying to hit a friend in his dorm window with a water balloon. He has a similar idea and is aiming at you with *his* water balloon. You aim straight at each other and throw at the same instant. Do the water balloons hit each other? Explain why or why not.
15. A projectile is launched at an upward angle of 30° to the horizontal with a speed of 30 m/s. How does the horizontal component of its velocity 1.0 s after launch compare with its horizontal component of velocity 2.0 s after launch, ignoring air resistance? Explain.
16. A projectile has the least speed at what point in its path?
17. Two cannonballs, A and B, are fired from the ground with identical initial speeds, but with θ_A larger than θ_B. (*a*) Which cannonball reaches a higher elevation? (*b*) Which stays longer in the air? (*c*) Which travels farther? Explain.
18. A person sitting in an enclosed train car, moving at constant velocity, throws a ball straight up into the air in her reference frame. (*a*) Where does the ball land? What is your answer if the car (*b*) accelerates, (*c*) decelerates, (*d*) rounds a curve, (*e*) moves with constant velocity but is open to the air?
19. If you are riding on a train that speeds past another train moving in the same direction on an adjacent track, it appears that the other train is moving backward. Why?
20. Two rowers, who can row at the same speed in still water, set off across a river at the same time. One heads straight across and is pulled downstream somewhat by the current. The other one heads upstream at an angle so as to arrive at a point opposite the starting point. Which rower reaches the opposite side first? Explain.
21. If you stand motionless under an umbrella in a rainstorm where the drops fall vertically, you remain relatively dry. However, if you start running, the rain begins to hit your legs even if they remain under the umbrella. Why?

MisConceptual Questions

1. You are adding vectors of length 20 and 40 units. Which of the following choices is a possible resultant magnitude?
   (*a*) 0.
   (*b*) 18.
   (*c*) 37.
   (*d*) 64.
   (*e*) 100.
2. The magnitude of a component of a vector must be
   (*a*) less than or equal to the magnitude of the vector.
   (*b*) equal to the magnitude of the vector.
   (*c*) greater than or equal to the magnitude of the vector.
   (*d*) less than, equal to, or greater than the magnitude of the vector.
3. You are in the middle of a large field. You walk in a straight line for 100 m, then turn left and walk 100 m more in a straight line before stopping. When you stop, you are 100 m from your starting point. By how many degrees did you turn?
   (*a*) 90°.
   (*b*) 120°.
   (*c*) 30°.
   (*d*) 180°.
   (*e*) This is impossible. You cannot walk 200 m and be only 100 m away from where you started.
4. A bullet fired from a rifle begins to fall
   (*a*) as soon as it leaves the barrel.
   (*b*) after air friction reduces its speed.
   (*c*) not at all if air resistance is ignored.
5. A baseball player hits a ball that soars high into the air. After the ball has left the bat, and while it is traveling upward (at point P in Fig. 3–31), what is the direction of acceleration? Ignore air resistance.



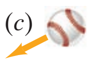
(*a*)  (*b*)  (*c*) 

FIGURE 3–31 MisConceptual Question 5.

6. One ball is dropped vertically from a window. At the same instant, a second ball is thrown horizontally from the same window. Which ball has the greater speed at ground level?
   (*a*) The dropped ball.
   (*b*) The thrown ball.
   (*c*) Neither—they both have the same speed on impact.
   (*d*) It depends on how hard the ball was thrown.

7. You are riding in an enclosed train car moving at 90 km/h. If you throw a baseball straight up, where will the baseball land?
(*a*) In front of you.
(*b*) Behind you.
(*c*) In your hand.
(*d*) Can't decide from the given information.

8. Which of the three kicks in Fig. 3–32 is in the air for the longest time? They all reach the same maximum height h. Ignore air resistance.
(*a*), (*b*), (*c*), or (*d*) all the same time.

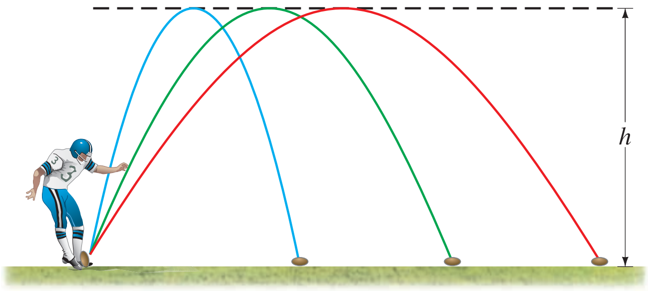


FIGURE 3–32 MisConceptual Question 8.

9. A baseball is hit high and far. Which of the following statements is true? At the highest point,
(*a*) the magnitude of the acceleration is zero.
(*b*) the magnitude of the velocity is zero.
(*c*) the magnitude of the velocity is the slowest.
(*d*) more than one of the above is true.
(*e*) none of the above are true.

10. A hunter is aiming horizontally at a monkey who is sitting in a tree. The monkey is so terrified when it sees the gun that it falls off the tree. At that very instant, the hunter pulls the trigger. What will happen?
(*a*) The bullet will miss the monkey because the monkey falls down while the bullet speeds straight forward.
(*b*) The bullet will hit the monkey because both the monkey and the bullet are falling downward at the same rate due to gravity.
(*c*) The bullet will miss the monkey because although both the monkey and the bullet are falling downward due to gravity, the monkey is falling faster.
(*d*) It depends on how far the hunter is from the monkey.

11. Which statements are *not* valid for a projectile? Take up as positive.
(*a*) The projectile has the same x velocity at any point on its path.
(*b*) The acceleration of the projectile is positive and decreasing when the projectile is moving upwards, zero at the top, and increasingly negative as the projectile descends.
(*c*) The acceleration of the projectile is a constant negative value.
(*d*) The y component of the velocity of the projectile is zero at the highest point of the projectile's path.
(*e*) The velocity at the highest point is zero.

12. A car travels 10 m/s east. Another car travels 10 m/s north. The relative speed of the first car with respect to the second is
(*a*) less than 20 m/s.
(*b*) exactly 20 m/s.
(*c*) more than 20 m/s.

For assigned homework and other learning materials, go to the MasteringPhysics website.

Problems

3–2 to 3–4 Vector Addition

1. (I) A car is driven 225 km west and then 98 km southwest (45°). What is the displacement of the car from the point of origin (magnitude and direction)? Draw a diagram.

2. (I) A delivery truck travels 21 blocks north, 16 blocks east, and 26 blocks south. What is its final displacement from the origin? Assume the blocks are equal length.

3. (I) If $V_x = 9.80$ units and $V_y = -6.40$ units, determine the magnitude and direction of $\vec{\mathbf{V}}$.

4. (II) Graphically determine the resultant of the following three vector displacements: (1) 24 m, 36° north of east; (2) 18 m, 37° east of north; and (3) 26 m, 33° west of south.

5. (II) $\vec{\mathbf{V}}$ is a vector 24.8 units in magnitude and points at an angle of 23.4° above the negative x axis. (*a*) Sketch this vector. (*b*) Calculate V_x and V_y. (*c*) Use V_x and V_y to obtain (again) the magnitude and direction of $\vec{\mathbf{V}}$. [*Note*: Part (*c*) is a good way to check if you've resolved your vector correctly.]

6. (II) Vector $\vec{\mathbf{V}}_1$ is 6.6 units long and points along the negative x axis. Vector $\vec{\mathbf{V}}_2$ is 8.5 units long and points at $+55°$ to the positive x axis. (*a*) What are the x and y components of each vector? (*b*) Determine the sum $\vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2$ (magnitude and angle).

7. (II) Figure 3–33 shows two vectors, $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, whose magnitudes are $A = 6.8$ units and $B = 5.5$ units. Determine $\vec{\mathbf{C}}$ if (*a*) $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$, (*b*) $\vec{\mathbf{C}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$, (c) $\vec{\mathbf{C}} = \vec{\mathbf{B}} - \vec{\mathbf{A}}$. Give the magnitude and direction for each.

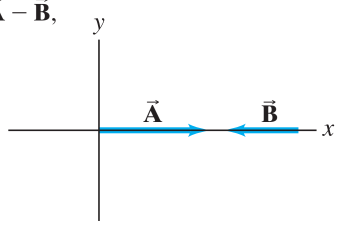


FIGURE 3–33 Problem 7.

8. (II) An airplane is traveling 835 km/h in a direction 41.5° west of north (Fig. 3–34).
(*a*) Find the components of the velocity vector in the northerly and westerly directions. (*b*) How far north and how far west has the plane traveled after 1.75 h?

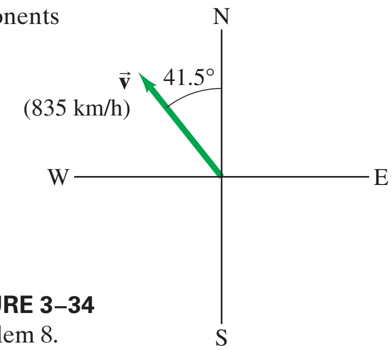


FIGURE 3–34 Problem 8.

9. (II) Three vectors are shown in Fig. 3–35. Their magnitudes are given in arbitrary units. Determine the sum of the three vectors. Give the resultant in terms of (*a*) components, (*b*) magnitude and angle with the $+x$ axis.

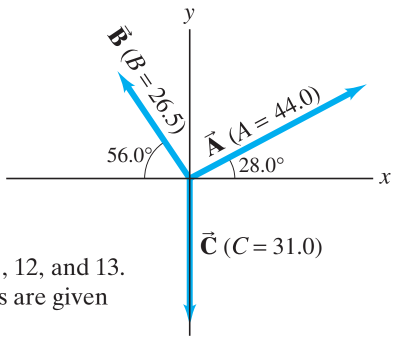


FIGURE 3–35 Problems 9, 10, 11, 12, and 13. Vector magnitudes are given in arbitrary units.

10. (II) (*a*) Given the vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ shown in Fig. 3–35, determine $\vec{\mathbf{B}} - \vec{\mathbf{A}}$. (*b*) Determine $\vec{\mathbf{A}} - \vec{\mathbf{B}}$ without using your answer in (*a*). Then compare your results and see if they are opposite.

11. (II) Determine the vector $\vec{\mathbf{A}} - \vec{\mathbf{C}}$, given the vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{C}}$ in Fig. 3–35.

12. (II) For the vectors shown in Fig. 3–35, determine (*a*) $\vec{\mathbf{B}} - 3\vec{\mathbf{A}}$, (*b*) $2\vec{\mathbf{A}} - 3\vec{\mathbf{B}} + 2\vec{\mathbf{C}}$.

13. (II) For the vectors given in Fig. 3–35, determine (*a*) $\vec{\mathbf{A}} - \vec{\mathbf{B}} + \vec{\mathbf{C}}$, (*b*) $\vec{\mathbf{A}} + \vec{\mathbf{B}} - \vec{\mathbf{C}}$, and (*c*) $\vec{\mathbf{C}} - \vec{\mathbf{A}} - \vec{\mathbf{B}}$.

14. (II) Suppose a vector $\vec{\mathbf{V}}$ makes an angle ϕ with respect to the y axis. What could be the x and y components of the vector $\vec{\mathbf{V}}$?

15. (II) The summit of a mountain, 2450 m above base camp, is measured on a map to be 4580 m horizontally from the camp in a direction 38.4° west of north. What are the components of the displacement vector from camp to summit? What is its magnitude? Choose the x axis east, y axis north, and z axis up.

16. (III) You are given a vector in the xy plane that has a magnitude of 90.0 units and a y component of -65.0 units. (*a*) What are the two possibilities for its x component? (*b*) Assuming the x component is known to be positive, specify the vector which, if you add it to the original one, would give a resultant vector that is 80.0 units long and points entirely in the $-x$ direction.

3–5 and 3–6 Projectile Motion (neglect air resistance)

17. (I) A tiger leaps horizontally from a 7.5-m-high rock with a speed of 3.0 m/s. How far from the base of the rock will she land?

18. (I) A diver running 2.5 m/s dives out horizontally from the edge of a vertical cliff and 3.0 s later reaches the water below. How high was the cliff and how far from its base did the diver hit the water?

19. (II) Estimate by what factor a person can jump farther on the Moon as compared to the Earth if the takeoff speed and angle are the same. The acceleration due to gravity on the Moon is one-sixth what it is on Earth.

20. (II) A ball is thrown horizontally from the roof of a building 7.5 m tall and lands 9.5 m from the base. What was the ball's initial speed?

21. (II) A ball thrown horizontally at 12.2 m/s from the roof of a building lands 21.0 m from the base of the building. How high is the building?

22. (II) A football is kicked at ground level with a speed of 18.0 m/s at an angle of 31.0° to the horizontal. How much later does it hit the ground?

23. (II) A fire hose held near the ground shoots water at a speed of 6.5 m/s. At what angle(s) should the nozzle point in order that the water land 2.5 m away (Fig. 3–36)? Why are there two different angles? Sketch the two trajectories.

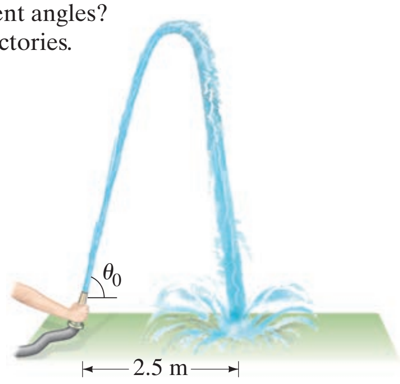


FIGURE 3–36 Problem 23.

24. (II) You buy a plastic dart gun, and being a clever physics student you decide to do a quick calculation to find its maximum horizontal range. You shoot the gun straight up, and it takes 4.0 s for the dart to land back at the barrel. What is the maximum horizontal range of your gun?

25. (II) A grasshopper hops along a level road. On each hop, the grasshopper launches itself at angle $\theta_0 = 45°$ and achieves a range $R = 0.80$ m. What is the average horizontal speed of the grasshopper as it hops along the road? Assume that the time spent on the ground between hops is negligible.

26. (II) Extreme-sports enthusiasts have been known to jump off the top of El Capitan, a sheer granite cliff of height 910 m in Yosemite National Park. Assume a jumper runs horizontally off the top of El Capitan with speed 4.0 m/s and enjoys a free fall until she is 150 m above the valley floor, at which time she opens her parachute (Fig. 3–37). (*a*) How long is the jumper in free fall? Ignore air resistance. (*b*) It is important to be as far away from the cliff as possible before opening the parachute. How far from the cliff is this jumper when she opens her chute?

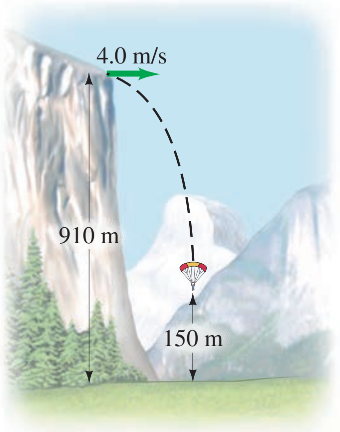


FIGURE 3–37 Problem 26.

27. (II) A projectile is fired with an initial speed of 36.6 m/s at an angle of 42.2° above the horizontal on a long flat firing range. Determine (*a*) the maximum height reached by the projectile, (*b*) the total time in the air, (*c*) the total horizontal distance covered (that is, the range), and (*d*) the speed of the projectile 1.50 s after firing.

28. (II) An athlete performing a long jump leaves the ground at a 27.0° angle and lands 7.80 m away. (*a*) What was the takeoff speed? (*b*) If this speed were increased by just 5.0%, how much longer would the jump be?

29. (II) A baseball is hit with a speed of 27.0 m/s at an angle of 45.0°. It lands on the flat roof of a 13.0-m-tall nearby building. If the ball was hit when it was 1.0 m above the ground, what horizontal distance does it travel before it lands on the building?

30. (II) A rescue plane wants to drop supplies to isolated mountain climbers on a rocky ridge 235 m below. If the plane is traveling horizontally with a speed of 250 km/h (69.4 m/s), how far in advance of the recipients (horizontal distance) must the goods be dropped (Fig. 3–38)?

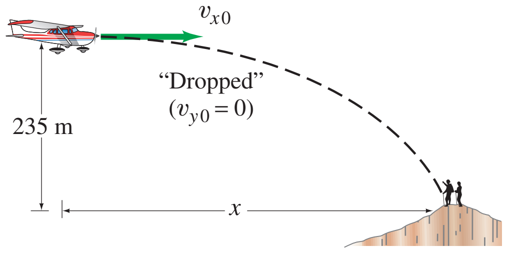


FIGURE 3–38 Problem 30.

31. (III) Suppose the rescue plane of Problem 30 releases the supplies a horizontal distance of 425 m in advance of the mountain climbers. What vertical velocity (up or down) should the supplies be given so that they arrive precisely at the climbers' position (Fig. 3–39)? With what speed do the supplies land?

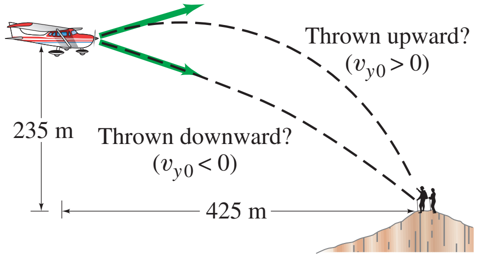


FIGURE 3–39 Problem 31.

32. (III) Show that the time required for a projectile to reach its highest point is equal to the time for it to return to its original height if air resistance is neglible.

33. (III) Suppose the kick in Example 3–6 is attempted 36.0 m from the goalposts, whose crossbar is 3.05 m above the ground. If the football is directed perfectly between the goalposts, will it pass over the bar and be a field goal? Show why or why not. If not, from what horizontal distance must this kick be made if it is to score?

34. (III) Revisit Example 3–7, and assume that the boy with the slingshot is *below* the boy in the tree (Fig. 3–40) and so aims *upward*, directly at the boy in the tree. Show that again the boy in the tree makes the wrong move by letting go at the moment the water balloon is shot.

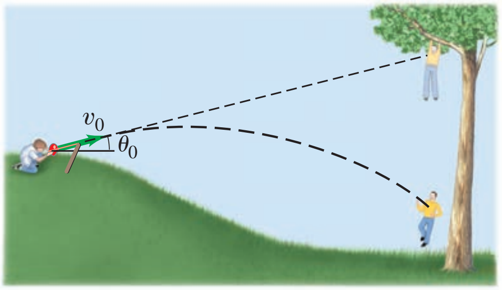


FIGURE 3–40 Problem 34.

35. (III) A stunt driver wants to make his car jump over 8 cars parked side by side below a horizontal ramp (Fig. 3–41). (*a*) With what minimum speed must he drive off the horizontal ramp? The vertical height of the ramp is 1.5 m above the cars and the horizontal distance he must clear is 22 m. (*b*) If the ramp is now tilted upward, so that "takeoff angle" is 7.0° above the horizontal, what is the new minimum speed?

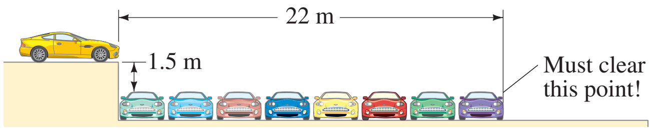


FIGURE 3–41 Problem 35.

3–8 Relative Velocity

36. (I) Huck Finn walks at a speed of 0.70 m/s across his raft (that is, he walks perpendicular to the raft's motion relative to the shore). The heavy raft is traveling down the Mississippi River at a speed of 1.50 m/s relative to the river bank (Fig. 3–42). What is Huck's velocity (speed and direction) relative to the river bank?

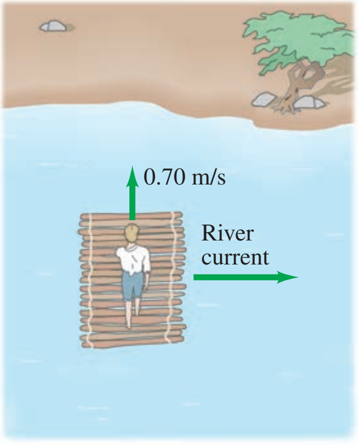


FIGURE 3–42 Problem 36.

37. (II) Two planes approach each other head-on. Each has a speed of 780 km/h, and they spot each other when they are initially 10.0 km apart. How much time do the pilots have to take evasive action?

38. (II) A passenger on a boat moving at 1.70 m/s on a still lake walks up a flight of stairs at a speed of 0.60 m/s, Fig. 3–43. The stairs are angled at 45° pointing in the direction of motion as shown. What is the velocity of the passenger relative to the water?

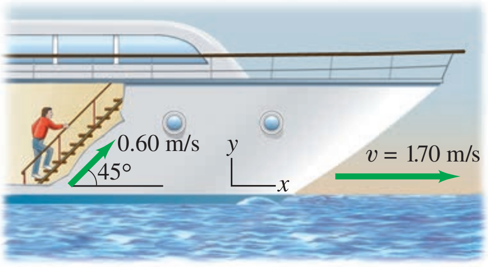


FIGURE 3–43 Problem 38.

39. (II) A person in the passenger basket of a hot-air balloon throws a ball horizontally outward from the basket with speed 10.0 m/s (Fig. 3–44). What initial velocity (magnitude and direction) does the ball have relative to a person standing on the ground (*a*) if the hot-air balloon is rising at 3.0 m/s relative to the ground during this throw, (*b*) if the hot-air balloon is descending at 3.0 m/s relative to the ground?

FIGURE 3–44 Problem 39.

40. (II) An airplane is heading due south at a speed of 688 km/h. If a wind begins blowing from the southwest at a speed of 90.0 km/h (average), calculate (*a*) the velocity (magnitude and direction) of the plane, relative to the ground, and (*b*) how far from its intended position it will be after 11.0 min if the pilot takes no corrective action. [*Hint*: First draw a diagram.]

41. (II) In what direction should the pilot aim the plane in Problem 40 so that it will fly due south?

42. (II) A swimmer is capable of swimming 0.60 m/s in still water. (*a*) If she aims her body directly across a 45-m-wide river whose current is 0.50 m/s, how far downstream (from a point opposite her starting point) will she land? (*b*) How long will it take her to reach the other side?

43. (II) A boat, whose speed in still water is 2.50 m/s, must cross a 285-m-wide river and arrive at a point 118 m upstream from where it starts (Fig. 3–45). To do so, the pilot must head the boat at a 45.0° upstream angle. What is the speed of the river's current?

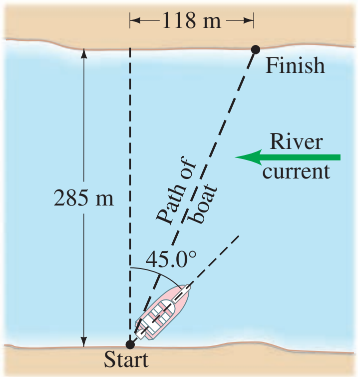


FIGURE 3–45 Problem 43.

44. (II) A child, who is 45 m from the bank of a river, is being carried helplessly downstream by the river's swift current of 1.0 m/s. As the child passes a lifeguard on the river's bank, the lifeguard starts swimming in a straight line (Fig. 3–46) until she reaches the child at a point downstream. If the lifeguard can swim at a speed of 2.0 m/s relative to the water, how long does it take her to reach the child? How far downstream does the lifeguard intercept the child?

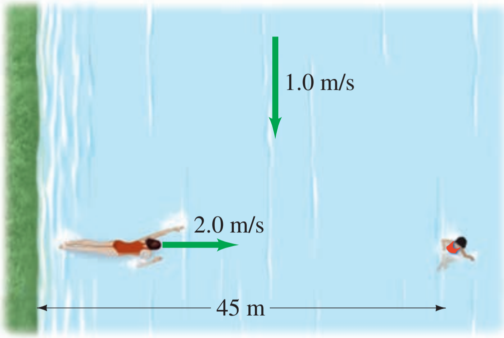


FIGURE 3–46 Problem 44.

45. (III) Two cars approach a street corner at right angles to each other (Fig. 3–47). Car 1 travels at a speed relative to Earth $v_{1E} = 35$ km/h, and car 2 at $v_{2E} = 55$ km/h. What is the relative velocity of car 1 as seen by car 2? What is the velocity of car 2 relative to car 1?

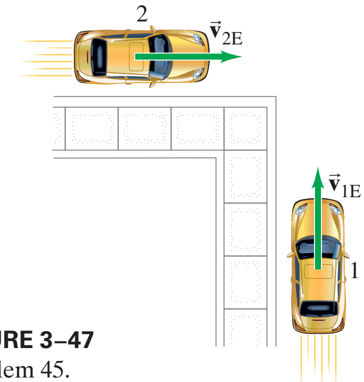


FIGURE 3–47 Problem 45.

General Problems

46. Two vectors, $\vec{V}_1$ and $\vec{V}_2$, add to a resultant $\vec{V}_R = \vec{V}_1 + \vec{V}_2$. Describe $\vec{V}_1$ and $\vec{V}_2$ if (*a*) $V_R = V_1 + V_2$, (*b*) $V_R^2 = V_1^2 + V_2^2$, (*c*) $V_1 + V_2 = V_1 - V_2$.

47. On mountainous downhill roads, escape routes are sometimes placed to the side of the road for trucks whose brakes might fail. Assuming a constant upward slope of 26°, calculate the horizontal and vertical components of the acceleration of a truck that slowed from 110 km/h to rest in 7.0 s. See Fig. 3–48.

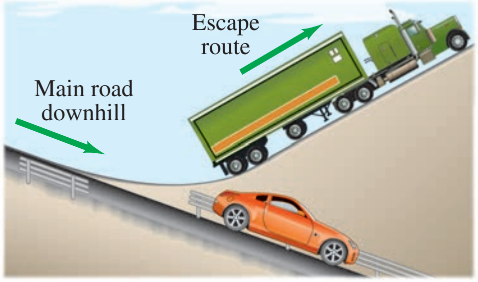


FIGURE 3–48 Problem 47.

48. A light plane is headed due south with a speed relative to still air of 185 km/h. After 1.00 h, the pilot notices that they have covered only 135 km and their direction is not south but 15.0° east of south. What is the wind velocity?

49. Romeo is throwing pebbles gently up to Juliet's window, and he wants the pebbles to hit the window with only a horizontal component of velocity. He is standing at the edge of a rose garden 8.0 m below her window and 8.5 m from the base of the wall (Fig. 3–49). How fast are the pebbles going when they hit her window?

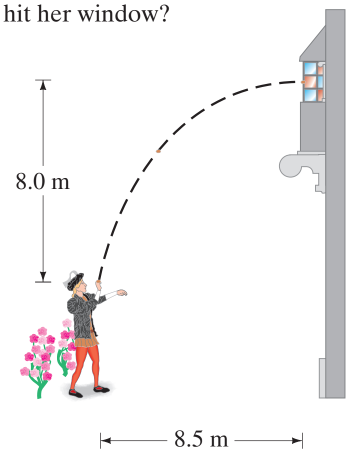


FIGURE 3–49 Problem 49.

50. *Apollo* astronauts took a "nine iron" to the Moon and hit a golf ball about 180 m. Assuming that the swing, launch angle, and so on, were the same as on Earth where the same astronaut could hit it only 32 m, estimate the acceleration due to gravity on the surface of the Moon. (We neglect air resistance in both cases, but on the Moon there is none.)

51. (*a*) A long jumper leaves the ground at 45° above the horizontal and lands 8.0 m away. What is her "takeoff" speed v_0? (*b*) Now she is out on a hike and comes to the left bank of a river. There is no bridge and the right bank is 10.0 m away horizontally and 2.5 m vertically below. If she long jumps from the edge of the left bank at 45° with the speed calculated in (*a*), how long, or short, of the opposite bank will she land (Fig. 3–50)?

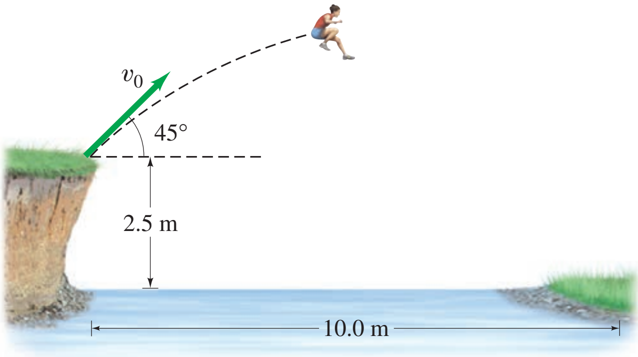


FIGURE 3–50 Problem 51.

52. A projectile is shot from the edge of a cliff 115 m above ground level with an initial speed of 65.0 m/s at an angle of 35.0° with the horizontal, as shown in Fig. 3–51. (*a*) Determine the time taken by the projectile to hit point P at ground level. (*b*) Determine the distance X of point P from the base of the vertical cliff. At the instant just before the projectile hits point P, find (*c*) the horizontal and the vertical components of its velocity, (*d*) the magnitude of the velocity, and (*e*) the angle made by the velocity vector with the horizontal. (*f*) Find the maximum height above the cliff top reached by the projectile.

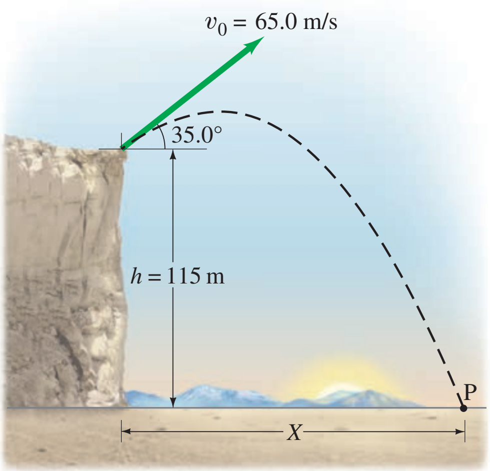


FIGURE 3–51 Problem 52.

53. Raindrops make an angle θ with the vertical when viewed through a moving train window (Fig. 3–52). If the speed of the train is v_T, what is the speed of the raindrops in the reference frame of the Earth in which they are assumed to fall vertically?

FIGURE 3–52 Problem 53.

54. A hunter aims directly at a target (on the same level) 38.0 m away. (*a*) If the arrow leaves the bow at a speed of 23.1 m/s, by how much will it miss the target? (*b*) At what angle should the bow be aimed so the target will be hit?

55. The cliff divers of Acapulco push off horizontally from rock platforms about 35 m above the water, but they must clear rocky outcrops at water level that extend out into the water 5.0 m from the base of the cliff directly under their launch point. See Fig. 3–53. What minimum pushoff speed is necessary to clear the rocks? How long are they in the air?

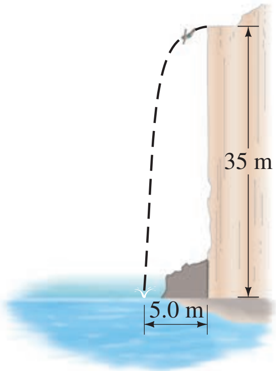


FIGURE 3–53 Problem 55.

56. When Babe Ruth hit a homer over the 8.0-m-high right-field fence 98 m from home plate, roughly what was the minimum speed of the ball when it left the bat? Assume the ball was hit 1.0 m above the ground and its path initially made a 36° angle with the ground.

57. At serve, a tennis player aims to hit the ball horizontally. What minimum speed is required for the ball to clear the 0.90-m-high net about 15.0 m from the server if the ball is "launched" from a height of 2.50 m? Where will the ball land if it just clears the net (and will it be "good" in the sense that it lands within 7.0 m of the net)? How long will it be in the air? See Fig. 3–54.

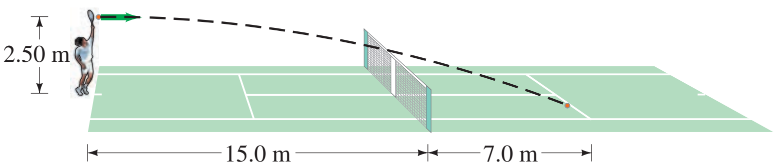


FIGURE 3–54 Problem 57.

58. Spymaster Chris, flying a constant 208 km/h horizontally in a low-flying helicopter, wants to drop secret documents into her contact's open car which is traveling 156 km/h on a level highway 78.0 m below. At what angle (with the horizontal) should the car be in her sights when the packet is released (Fig. 3–55)?

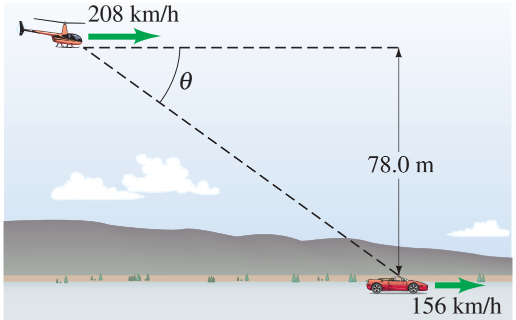


FIGURE 3–55 Problem 58.

59. A boat can travel 2.20 m/s in still water. (*a*) If the boat points directly across a stream whose current is 1.20 m/s, what is the velocity (magnitude and direction) of the boat relative to the shore? (*b*) What will be the position of the boat, relative to its point of origin, after 3.00 s?

60. A projectile is launched from ground level to the top of a cliff which is 195 m away and 135 m high (see Fig. 3–56). If the projectile lands on top of the cliff 6.6 s after it is fired, find the initial velocity of the projectile (magnitude and direction). Neglect air resistance.

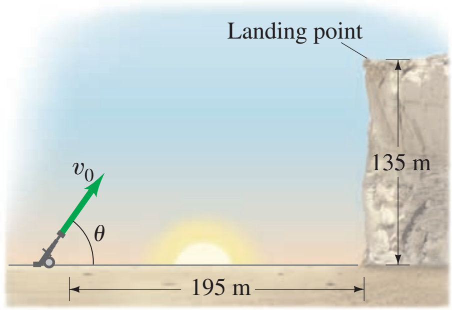


FIGURE 3–56 Problem 60.

61. A basketball is shot from an initial height of 2.40 m (Fig. 3–57) with an initial speed $v_0 = 12$ m/s directed at an angle $\theta_0 = 35°$ above the horizontal. (*a*) How far from the basket was the player if he made a basket? (*b*) At what angle to the horizontal did the ball enter the basket?

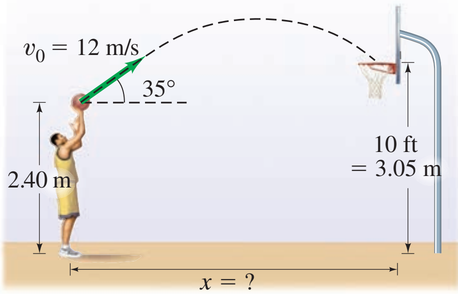


FIGURE 3–57 Problem 61.

62. A rock is kicked horizontally at 15 m/s from a hill with a 45° slope (Fig. 3–58). How long does it take for the rock to hit the ground?

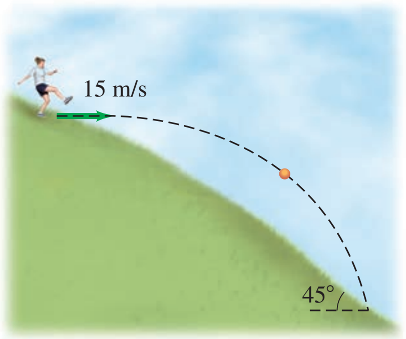


FIGURE 3–58 Problem 62.

63. A ball is shot from the top of a building with an initial velocity of 18 m/s at an angle $\theta = 42°$ above the horizontal. (*a*) What are the horizontal and vertical components of the initial velocity? (*b*) If a nearby building is the same height and 55 m away, how far below the top of the building will the ball strike the nearby building?

64 If a baseball pitch leaves the pitcher's hand horizontally at a velocity of 150 km/h, by what % will the pull of gravity change the magnitude of the velocity when the ball reaches the batter, 18 m away? For this estimate, ignore air resistance and spin on the ball.

Search and Learn

1. Here is something to try at a sporting event. Show that the maximum height h attained by an object projected into the air, such as a baseball, football, or soccer ball, is approximately given by

$$h \approx 1.2\,t^2\,\mathrm{m},$$

where t is the total time of flight for the object in seconds. Assume that the object returns to the same level as that from which it was launched, as in Fig. 3–59. For example, if you count to find that a baseball was in the air for $t = 5.0\,\mathrm{s}$, the maximum height attained was $h = 1.2 \times (5.0)^2 = 30\,\mathrm{m}$. The fun of this relation is that h can be determined without knowledge of the launch speed v_0 or launch angle θ_0. Why is that exactly? See Section 3–6.

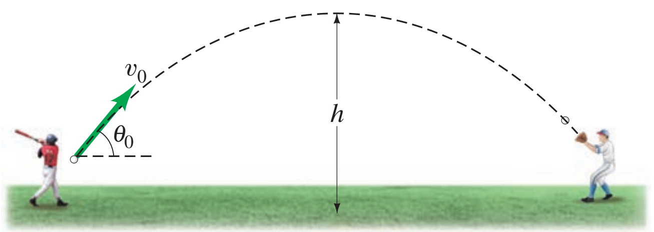


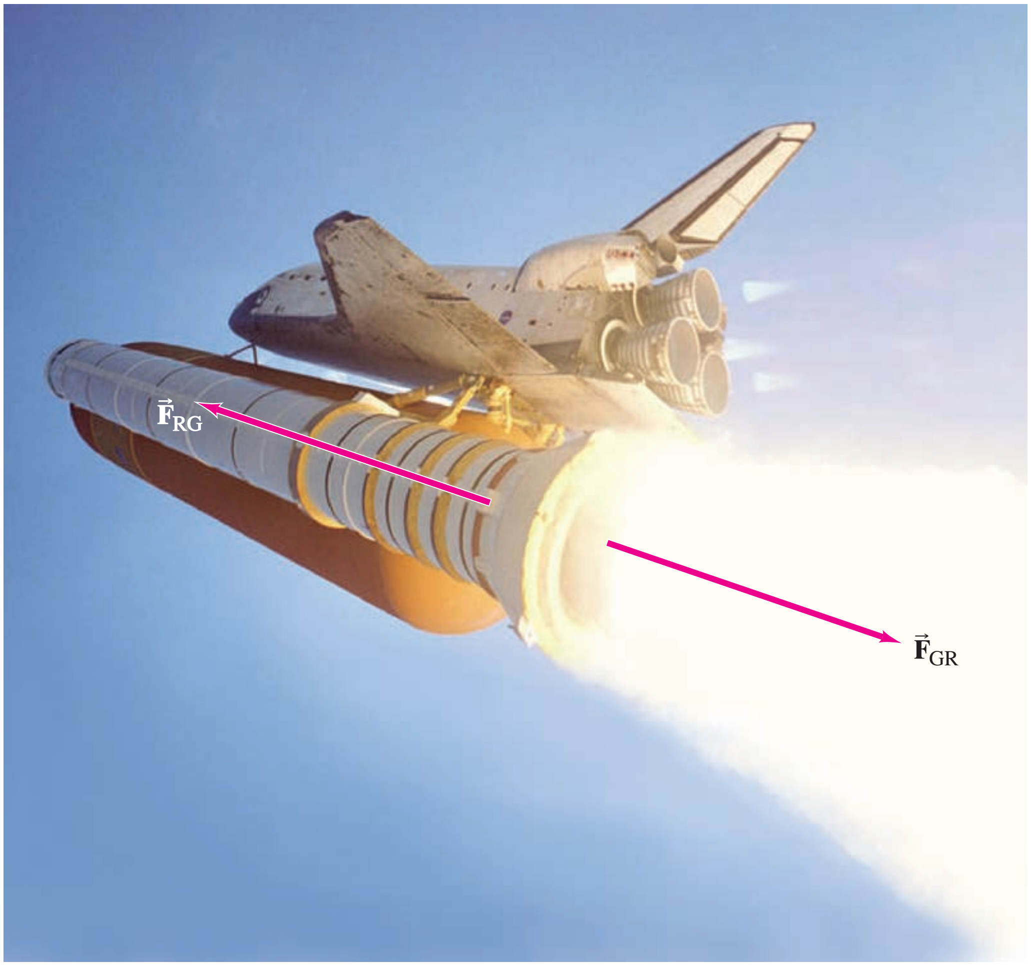
FIGURE 3–59 Search and Learn 1.

2. The initial angle of projectile A is 30°, while that of projectile B is 60°. Both have the same level horizontal range. How do the initial velocities and flight times (elapsed time from launch until landing) compare for A and B?

3. You are driving south on a highway at 12 m/s (approximately 25 mi/h) in a snowstorm. When you last stopped, you noticed that the snow was coming down vertically, but it is passing the windows of the moving car at an angle of 7.0° to the horizontal. Estimate the speed of the vertically falling snowflakes relative to the ground. [*Hint*: Construct a relative velocity diagram similar to Fig. 3–29 or 3–30. Be careful about which angle is the angle given.]

ANSWERS TO EXERCISES

A: $3.0\sqrt{2} \approx 4.2$ units.
B: (*a*).
C: They hit at the same time.
D: (i) Nowhere; (ii) at the highest point; (iii) nowhere.
E: (*d*). It provides the initial velocity of the box.
F: (*a*) $v = v_{x0} = 16.0\,\mathrm{m/s}$, horizontal; (*b*) $9.80\,\mathrm{m/s^2}$ down.



A space shuttle is carried out into space by powerful rockets. They are accelerating, increasing in speed rapidly. To do so, a force must be exerted on them according to Newton's second law, $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$. What exerts this force? The **r**ocket engines exert a force on the **g**ases they push out (expel) from the rear of the rockets (labeled $\vec{\mathbf{F}}_{GR}$). According to Newton's third law, these ejected gases exert an equal and opposite force on the rockets in the forward direction. It is this "reaction" force exerted *on* the **r**ockets *by* the **g**ases, labeled $\vec{\mathbf{F}}_{RG}$, that accelerates the rockets forward.

CHAPTER 4

Dynamics: Newton's Laws of Motion

CHAPTER-OPENING QUESTIONS—Guess now!

1. A 150-kg football player collides head-on with a 75-kg running back. During the collision, the heavier player exerts a force of magnitude F_A on the smaller player. If the smaller player exerts a force F_B back on the heavier player, which response is most accurate?

(a) $F_B = F_A$.
(b) $F_B < F_A$.
(c) $F_B > F_A$.
(d) $F_B = 0$.
(e) We need more information.

2. A line by the poet T. S. Eliot (from *Murder in the Cathedral*) has the women of Canterbury say "the earth presses up against our feet." What force is this?

(a) Gravity.
(b) The normal force.
(c) A friction force.
(d) Centrifugal force.
(e) No force—they are being poetic.

CONTENTS

We have discussed how motion is described in terms of velocity and acceleration. Now we deal with the question of *why* objects move as they do: What makes an object at rest begin to move? What causes an object to accelerate or decelerate? What is involved when an object moves in a curved path? We can answer in each case that a force is required. In this Chapter[†], we will investigate the connection between force and motion, which is the subject called **dynamics**.

4–1 Force

FIGURE 4–1 A force exerted on a grocery cart—in this case exerted by a person.

Intuitively, we experience **force** as any kind of a push or a pull on an object. When you push a stalled car or a grocery cart (Fig. 4–1), you are exerting a force on it. When a motor lifts an elevator, or a hammer hits a nail, or the wind blows the leaves of a tree, a force is being exerted. We often call these *contact forces* because the force is exerted when one object comes in contact with another object. On the other hand, we say that an object falls because of the *force of gravity* (which is not a contact force).

If an object is at rest, to start it moving requires force—that is, a force is needed to accelerate an object from zero velocity to a nonzero velocity. For an object already moving, if you want to change its velocity—either in direction or in magnitude—a force is required. In other words, to accelerate an object, a force is always required. In Section 4–4 we discuss the precise relation between acceleration and net force, which is Newton's second law.

One way to measure the magnitude (or strength) of a force is to use a spring scale (Fig. 4–2). Normally, such a spring scale is used to find the weight of an object; by weight we mean the force of gravity acting on the object (Section 4–6). The spring scale, once calibrated, can be used to measure other kinds of forces as well, such as the pulling force shown in Fig. 4–2.

A force exerted in a different direction has a different effect. Force has direction as well as magnitude, and is indeed a vector that follows the rules of vector addition discussed in Chapter 3. We can represent any force on a diagram by an arrow, just as we did with velocity. The direction of the arrow is the direction of the push or pull, and its length is drawn proportional to the magnitude of the force.

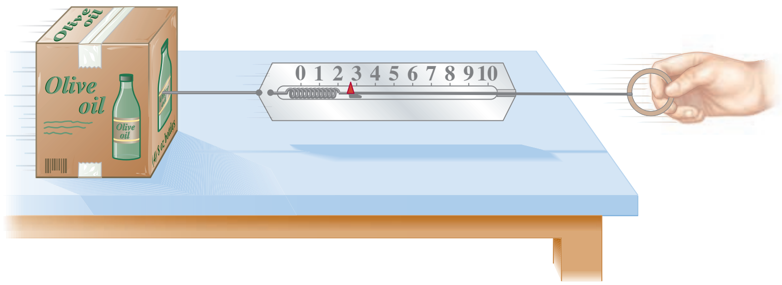


FIGURE 4–2 A spring scale used to measure a force.

4–2 Newton's First Law of Motion

What is the relationship between force and motion? Aristotle (384–322 B.C.) believed that a force was required to keep an object moving along a horizontal plane. To Aristotle, the natural state of an object was at rest, and a force was believed necessary to keep an object in motion. Furthermore, Aristotle argued, the greater the force on the object, the greater its speed.

Some 2000 years later, Galileo disagreed: he maintained that it is just as natural for an object to be in motion with a constant velocity as it is for it to be at rest.

[†]We treat everyday objects in motion here. When velocities are extremely high, close to the speed of light $(3.0 \times 10^8\,\mathrm{m/s})$, we use the theory of relativity (Chapter 26), and in the submicroscopic world of atoms and molecules we use quantum theory (Chapter 27 *ff*).

To understand Galileo's idea, consider the following observations involving motion along a horizontal plane. To push an object with a rough surface along a tabletop at constant speed requires a certain amount of force. To push an equally heavy object with a very smooth surface across the table at the same speed will require less force. If a layer of oil or other lubricant is placed between the surface of the object and the table, then almost no force is required to keep the object moving. Notice that in each successive step, less force is required. As the next step, we imagine there is no friction at all, that the object does not rub against the table—or there is a perfect lubricant between the object and the table—and theorize that once started, the object would move across the table at constant speed with *no* force applied. A steel ball bearing rolling on a hard horizontal surface approaches this situation. So does a puck on an air table, in which a thin layer of air reduces friction almost to zero.

It was Galileo's genius to imagine such an idealized world—in this case, one where there is no friction—and to see that it could lead to a more accurate and richer understanding of the real world. This idealization led him to his remarkable conclusion that if no force is applied to a moving object, it will continue to move with constant speed in a straight line. An object slows down only if a force is exerted on it. Galileo thus interpreted friction as a force akin to ordinary pushes and pulls.

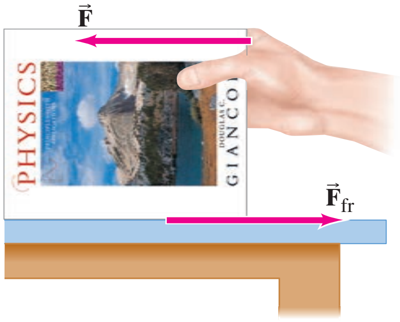


FIGURE 4–3 $\vec{\mathbf{F}}$ represents the force applied by the person and $\vec{\mathbf{F}}_{\rm fr}$ represents the force of friction.

To push an object across a table at constant speed requires a force from your hand that can balance the force of friction (Fig. 4–3). When the object moves at constant speed, your pushing force is equal in magnitude to the friction force; but these two forces are in opposite directions, so the *net* force on the object (the vector sum of the two forces) is zero. This is consistent with Galileo's viewpoint, for the object moves with constant velocity when no *net* force is exerted on it.

Upon this foundation laid by Galileo, Isaac Newton (Fig. 4–4) built his great theory of motion. Newton's analysis of motion is summarized in his famous "three laws of motion." In his great work, the *Principia* (published in 1687), Newton readily acknowledged his debt to Galileo. In fact, **Newton's first law of motion** is close to Galileo's conclusions. It states that

NEWTON'S FIRST LAW OF MOTION

> **Every object continues in its state of rest, or of uniform velocity in a straight line, as long as no net force acts on it.**

The tendency of an object to maintain its state of rest or of uniform velocity in a straight line is called **inertia**. As a result, Newton's first law is often called the **law of inertia**.

CONCEPTUAL EXAMPLE 4–1 **Newton's first law.** A school bus comes to a sudden stop, and all of the backpacks on the floor start to slide forward. What force causes them to do that?

RESPONSE It isn't "force" that does it. By Newton's first law, the backpacks continue their state of motion, maintaining their velocity. The backpacks slow down if a force is applied, such as friction with the floor.

FIGURE 4–4 Isaac Newton (1642–1727). Besides developing mechanics, including his three great laws of motion and the law of universal gravitation, he also tried to understand the nature of light.

Inertial Reference Frames

Newton's first law does not hold in every reference frame. For example, if your reference frame is an accelerating car, an object such as a cup resting on the dashboard may begin to move toward you (it stayed at rest as long as the car's velocity remained constant). The cup accelerated toward you, but neither you nor anything else exerted a force on it in that direction. Similarly, in the reference frame of the decelerating bus in Example 4–1, there was no force pushing the backpacks forward. In accelerating reference frames, Newton's first law does not hold. Physics is easier in reference frames in which Newton's first law *does* hold, and they are called **inertial reference frames** (the law of inertia is valid in them). For most purposes, we usually make the approximation that a reference frame fixed on the Earth is an inertial frame. This is not precisely true, due to the Earth's rotation, but usually it is close enough.

Any reference frame that moves with constant velocity (say, a car or an airplane) relative to an inertial frame is also an inertial reference frame. Reference frames where the law of inertia does *not* hold, such as the accelerating reference frames discussed above, are called **noninertial** reference frames. How can we be sure a reference frame is inertial or not? By checking to see if Newton's first law holds. Thus Newton's first law serves as the definition of inertial reference frames.

4–3 Mass

Newton's second law, which we come to in the next Section, makes use of the concept of mass. Newton used the term *mass* as a synonym for "quantity of matter." This intuitive notion of the mass of an object is not very precise because the concept "quantity of matter" is not very well defined. More precisely, we can say that **mass** is a *measure of the inertia* of an object. The more mass an object has, the greater the force needed to give it a particular acceleration. It is harder to start it moving from rest, or to stop it when it is moving, or to change its velocity sideways out of a straight-line path. A truck has much more inertia than a baseball moving at the same speed, and a much greater force is needed to change the truck's velocity at the same rate as the ball's. The truck therefore has much more mass.

To quantify the concept of mass, we must define a standard. In SI units, the unit of mass is the **kilogram** (kg) as we discussed in Chapter 1, Section 1–5.

CAUTION
Distinguish mass from weight

The terms *mass* and *weight* are often confused with one another, but it is important to distinguish between them. Mass is a property of an object itself (a measure of an object's inertia, or its "quantity of matter"). Weight, on the other hand, is a force, the pull of gravity acting on an object. To see the difference, suppose we take an object to the Moon. The object will weigh only about one-sixth as much as it did on Earth, since the force of gravity is weaker. But its mass will be the same. It will have the same amount of matter as on Earth, and will have just as much inertia—in the absence of friction, it will be just as hard to start it moving on the Moon as on Earth, or to stop it once it is moving. (More on weight in Section 4–6.)

4–4 Newton's Second Law of Motion

Newton's first law states that if no net force is acting on an object at rest, the object remains at rest; or if the object is moving, it continues moving with constant speed in a straight line. But what happens if a net force *is* exerted on an object? Newton perceived that the object's velocity will change (Fig. 4–5). A net force exerted on an object may make its velocity increase. Or, if the net force is in a direction opposite to the motion, that force will reduce the object's velocity. If the net force acts sideways on a moving object, the *direction* of the object's velocity changes. That change in the *direction* of the velocity is also an acceleration. So a sideways net force on an object also causes acceleration. In general, we can say that *a net force causes acceleration*.

FIGURE 4–5 The bobsled accelerates because the team exerts a force.

What precisely is the relationship between acceleration and force? Everyday experience can suggest an answer. Consider the force required to push a cart when friction is small enough to ignore. (If there is friction, consider the *net* force, which is the force you exert minus the force of friction.) If you push the cart horizontally with a gentle but constant force for a certain period of time, you will make the cart accelerate from rest up to some speed, say 3 km/h. If you push with twice the force, the cart will reach 3 km/h in half the time. The acceleration will be twice as great. If you triple the force, the acceleration is tripled, and so on. Thus, the acceleration of an object is directly proportional† to the net applied force. But the acceleration depends on the mass of the object as well. If you push an empty grocery cart with the same force as you push one that is filled with groceries, you will find that the full cart accelerates more slowly.

†A review of proportionality is given in Appendix A.

The greater the mass, the less the acceleration for the same net force. The mathematical relation, as Newton argued, is that the acceleration of an object is inversely proportional to its mass. These relationships are found to hold in general and can be summarized as follows:

The acceleration of an object is directly proportional to the net force acting on it, and is inversely proportional to the object's mass. The direction of the acceleration is in the direction of the net force acting on the object.

NEWTON'S SECOND LAW OF MOTION

This is **Newton's second law of motion**.

Newton's second law can be written as an equation:

$$\vec{\mathbf{a}} = \frac{\Sigma \vec{\mathbf{F}}}{m},$$

where $\vec{\mathbf{a}}$ stands for acceleration, m for the mass, and $\Sigma \vec{\mathbf{F}}$ for the *net force* on the object. The symbol Σ (Greek "sigma") stands for "sum of"; $\vec{\mathbf{F}}$ stands for force, so $\Sigma \vec{\mathbf{F}}$ means the *vector sum of all forces* acting on the object, which we define as the **net force**.

We rearrange this equation to obtain the familiar statement of Newton's second law:

$$\Sigma \vec{\mathbf{F}} = m\vec{\mathbf{a}}. \qquad \textbf{(4–1)}$$

NEWTON'S SECOND LAW OF MOTION

Newton's second law relates the description of motion to the cause of motion, force. It is one of the most fundamental relationships in physics. From Newton's second law we can make a more precise definition of **force** as *an action capable of accelerating an object*.

Every force $\vec{\mathbf{F}}$ is a vector, with magnitude and direction. Equation 4–1 is a vector equation valid in any inertial reference frame. It can be written in component form in rectangular coordinates as

$$\Sigma F_x = ma_x, \qquad \Sigma F_y = ma_y, \qquad \Sigma F_z = ma_z.$$

If the motion is all along a line (one-dimensional), we can leave out the subscripts and simply write $\Sigma F = ma$. Again, a is the acceleration of an object of mass m, and ΣF includes all the forces acting on that object, and *only* forces acting on that object. (Sometimes the net force ΣF is written as F_{net}, so $F_{\text{net}} = ma$.)

In SI units, with the mass in kilograms, the unit of force is called the **newton** (N). One newton is the force required to impart an acceleration of $1\ \text{m/s}^2$ to a mass of 1 kg. Thus $1\ \text{N} = 1\ \text{kg}\cdot\text{m/s}^2$.

In cgs units, the unit of mass is the gram† (g). The unit of force is the *dyne*, which is defined as the net force needed to impart an acceleration of $1\ \text{cm/s}^2$ to a mass of 1 g. Thus $1\ \text{dyne} = 1\ \text{g}\cdot\text{cm/s}^2$. Because $1\ \text{g} = 10^{-3}\ \text{kg}$ and $1\ \text{cm} = 10^{-2}\ \text{m}$, then $1\ \text{dyne} = 10^{-5}\ \text{N}$.

In the British system, which we rarely use, the unit of force is the *pound* (abbreviated lb), where $1\ \text{lb} = 4.44822\ \text{N} \approx 4.45\ \text{N}$. The unit of mass is the *slug*, which is defined as that mass which will undergo an acceleration of $1\ \text{ft/s}^2$ when a force of 1 lb is applied to it. Thus $1\ \text{lb} = 1\ \text{slug}\cdot\text{ft/s}^2$. Table 4–1 summarizes the units in the different systems.

TABLE 4–1
Units for Mass and Force

System	Mass	Force
SI	kilogram (kg)	newton (N) ($= \text{kg}\cdot\text{m/s}^2$)
cgs	gram (g)	dyne ($= \text{g}\cdot\text{cm/s}^2$)
British	slug	pound (lb)

Conversion factors: $1\ \text{dyne} = 10^{-5}\ \text{N}$; $1\ \text{lb} \approx 4.45\ \text{N}$; $1\ \text{slug} \approx 14.6\ \text{kg}$.

It is very important that only one set of units be used in a given calculation or Problem, with the SI being what we almost always use. If the force is given in, say, newtons, and the mass in grams, then before attempting to solve for the acceleration in SI units, we must change the mass to kilograms. For example, if the force is given as 2.0 N along the x axis and the mass is 500 g, we change the latter to 0.50 kg, and the acceleration will then automatically come out in m/s^2 when Newton's second law is used:

PROBLEM SOLVING
Use a consistent set of units

$$a_x = \frac{\Sigma F_x}{m} = \frac{2.0\ \text{N}}{0.50\ \text{kg}} = \frac{2.0\ \text{kg}\cdot\text{m/s}^2}{0.50\ \text{kg}} = 4.0\ \text{m/s}^2,$$

where we set $1\ \text{N} = 1\ \text{kg}\cdot\text{m/s}^2$.

†Be careful not to confuse g for gram with g for the acceleration due to gravity. The latter is always italicized (or boldface when shown as a vector).

EXAMPLE 4–2 ESTIMATE **Force to accelerate a fast car.** Estimate the net force needed to accelerate (*a*) a 1000-kg car at $\frac{1}{2}g$; (*b*) a 200-gram apple at the same rate.

APPROACH We use Newton's second law to find the net force needed for each object; we are given the mass and the acceleration. This is an estimate (the $\frac{1}{2}$ is not said to be precise) so we round off to one significant figure.

SOLUTION (*a*) The car's acceleration is $a = \frac{1}{2}g = \frac{1}{2}(9.8\,\mathrm{m/s^2}) \approx 5\,\mathrm{m/s^2}$. We use Newton's second law to get the net force needed to achieve this acceleration:

$$\Sigma F = ma \approx (1000\,\mathrm{kg})(5\,\mathrm{m/s^2}) = 5000\,\mathrm{N}.$$

(If you are used to British units, to get an idea of what a 5000-N force is, you can divide by 4.45 N/lb and get a force of about 1000 lb.)

(*b*) For the apple, $m = 200\,\mathrm{g} = 0.2\,\mathrm{kg}$, so

$$\Sigma F = ma \approx (0.2\,\mathrm{kg})(5\,\mathrm{m/s^2}) = 1\,\mathrm{N}.$$

EXAMPLE 4–3 **Force to stop a car.** What average net force is required to bring a 1500-kg car to rest from a speed of 100 km/h within a distance of 55 m?

APPROACH We use Newton's second law, $\Sigma F = ma$, to determine the force, but first we need to calculate the acceleration a. We assume the acceleration is constant so that we can use the kinematic equations, Eqs. 2–11, to calculate it.

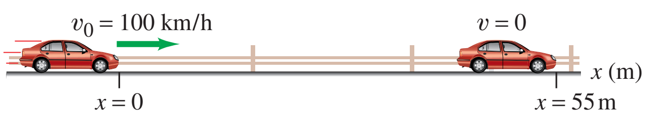


FIGURE 4–6 Example 4–3.

SOLUTION We assume the motion is along the $+x$ axis (Fig. 4–6). We are given the initial velocity $v_0 = 100\,\mathrm{km/h} = 27.8\,\mathrm{m/s}$ (Section 1–6), the final velocity $v = 0$, and the distance traveled $x - x_0 = 55\,\mathrm{m}$. From Eq. 2–11c, we have

$$v^2 = v_0^2 + 2a(x - x_0),$$

so

$$a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{0 - (27.8\,\mathrm{m/s})^2}{2(55\,\mathrm{m})} = -7.0\,\mathrm{m/s^2}.$$

The net force required is then

$$\Sigma F = ma = (1500\,\mathrm{kg})(-7.0\,\mathrm{m/s^2}) = -1.1 \times 10^4\,\mathrm{N},$$

or 11,000 N. The force must be exerted in the direction *opposite* to the initial velocity, which is what the negative sign means.

NOTE If the acceleration is not precisely constant, then we are determining an "average" acceleration and we obtain an "average" net force.

Newton's second law, like the first law, is valid only in inertial reference frames (Section 4–2). In the noninertial reference frame of a car that begins accelerating, a cup on the dashboard starts sliding—it accelerates—even though the net force on it is zero. Thus $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$ does not work in such an accelerating reference frame ($\Sigma\vec{\mathbf{F}} = 0$, but $\vec{\mathbf{a}} \neq 0$ in this noninertial frame).

EXERCISE A Suppose you watch a cup slide on the (smooth) dashboard of an accelerating car as we just discussed, but this time from an inertial reference frame outside the car, on the street. From your inertial frame, Newton's laws are valid. What force pushes the cup off the dashboard?

4–5 Newton's Third Law of Motion

FIGURE 4–7 A hammer striking a nail. The hammer exerts a force on the nail and the nail exerts a force back on the hammer. The latter force decelerates the hammer and brings it to rest.

Newton's second law of motion describes quantitatively how forces affect motion. But where, we may ask, do forces come from? Observations suggest that a force exerted on any object is always exerted *by another object*. A horse pulls a wagon, a person pushes a grocery cart, a hammer pushes on a nail, a magnet attracts a paper clip. In each of these examples, a force is exerted *on* one object, and that force is exerted *by* another object. For example, the force exerted *on* the nail is exerted *by* the hammer.

But Newton realized that things are not so one-sided. True, the hammer exerts a force on the nail (Fig. 4–7). But the nail evidently exerts a force back on the hammer as well, for the hammer's speed is rapidly reduced to zero upon contact. Only a strong force could cause such a rapid deceleration of the hammer. Thus, said Newton, the two objects must be treated on an equal basis. The hammer exerts a force on the nail, and the nail exerts a force back on the hammer. This is the essence of **Newton's third law of motion**:

NEWTON'S THIRD LAW OF MOTION

> **Whenever one object exerts a force on a second object, the second object exerts an equal force in the opposite direction on the first.**

This law is sometimes paraphrased as "to every action there is an equal and opposite reaction." This is perfectly valid. But to avoid confusion, it is very important to remember that the "action" force and the "reaction" force are acting on *different* objects.

CAUTION
Action and reaction forces act on different objects

As evidence for the validity of Newton's third law, look at your hand when you push against the edge of a desk, Fig. 4–8. Your hand's shape is distorted, clear evidence that a force is being exerted on it. You can *see* the edge of the desk pressing into your hand. You can even *feel* the desk exerting a force on your hand; it hurts! The harder you push against the desk, the harder the desk pushes back on your hand. (You only feel forces exerted *on* you; when you exert a force on another object, what you feel is that object pushing back on you.)

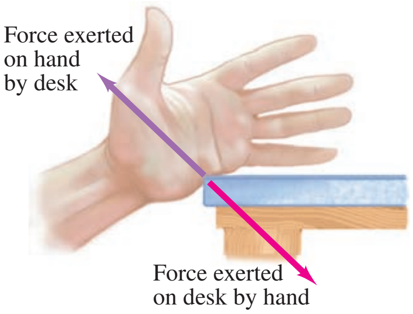


FIGURE 4–8 If your hand pushes against the edge of a desk (the force vector is shown in red), the desk pushes back against your hand (this force vector is shown in a different color, violet, to remind us that this force acts on a different object).

The force the desk exerts on your hand has the same magnitude as the force your hand exerts on the desk. This is true not only if the desk is at rest but is true even if the desk is accelerating due to the force your hand exerts.

As another demonstration of Newton's third law, consider the ice skater in Fig. 4–9. There is very little friction between her skates and the ice, so she will move freely if a force is exerted on her. She pushes against the wall; and then *she* starts moving backward. The force she exerts on the wall cannot make *her* start moving, because that force acts on the wall. Something had to exert a force *on her* to start her moving, and that force could only have been exerted by the wall. The force with which the wall pushes on her is, by Newton's third law, equal and opposite to the force she exerts on the wall.

When a person throws a package out of a small boat (initially at rest), the boat starts moving in the opposite direction. The person exerts a force on the package. The package exerts an equal and opposite force back on the person, and this force propels the person (and the boat) backward slightly.

FIGURE 4–9 An example of Newton's third law: when an ice skater pushes against the wall, the wall pushes back and this force causes her to accelerate away.

FIGURE 4–10 Another example of Newton's third law: the launch of a rocket. The rocket engine pushes the gases downward, and the gases exert an equal and opposite force upward on the rocket, accelerating it upward. (A rocket does *not* accelerate as a result of its expelled gases pushing against the ground.)

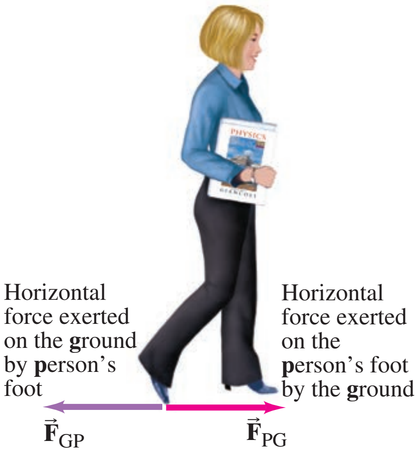
Rocket propulsion also is explained using Newton's third law (Fig. 4–10). A common misconception is that rockets accelerate because the gases rushing out the back of the engine push against the ground or the atmosphere. Not true. What happens, instead, is that a rocket exerts a strong force on the gases, expelling them; and the gases exert an equal and opposite force *on the rocket*. It is this latter force that propels the rocket forward—the force exerted *on* the rocket *by* the gases (see Chapter-Opening Photo, page 75). Thus, a space vehicle is maneuvered in empty space by firing its rockets in the direction opposite to that in which it needs to accelerate. When the rocket pushes on the gases in one direction, the gases push back on the rocket in the opposite direction. Jet aircraft too accelerate because the gases they thrust out backwards exert a forward force on the engines (Newton's third law).

Consider how we walk. A person begins walking by pushing with the foot backward against the ground. The ground then exerts an equal and opposite force forward on the person (Fig. 4–11), and it is this force, *on* the person, that moves the person forward. (If you doubt this, try walking normally where there is no friction, such as on very smooth slippery ice.) In a similar way, a bird flies forward by exerting a backward force on the air, but it is the air pushing forward (Newton's third law) on the bird's wings that propels the bird forward.

CONCEPTUAL EXAMPLE 4–4 **What exerts the force to move a car?** What makes a car go forward?

RESPONSE A common answer is that the engine makes the car move forward. But it is not so simple. The engine makes the wheels go around. But if the tires are on slick ice or wet mud, they just spin. Friction is needed. On firm ground, the tires push backward against the ground because of friction. By Newton's third law, the ground pushes on the tires in the opposite direction, accelerating the car forward.

FIGURE 4–11 We can walk forward because, when one foot pushes backward against the ground, the ground pushes forward on that foot (Newton's third law). The two forces shown *act on different objects.*



We tend to associate forces with active objects such as humans, animals, engines, or a moving object like a hammer. It is often difficult to see how an inanimate object at rest, such as a wall or a desk, or the wall of an ice rink (Fig. 4–9), can exert a force. The explanation is that every material, no matter how hard, is elastic (springy) at least to some degree. A stretched rubber band can exert a force on a wad of paper and accelerate it to fly across the room. Other materials may not stretch as readily as rubber, but they do stretch or compress when a force is applied to them. And just as a stretched rubber band exerts a force, so does a stretched (or compressed) wall, desk, or car fender.

From the examples discussed above, we can see how important it is to remember *on* what object a given force is exerted and *by* what object that force is exerted. A force influences the motion of an object only when it is applied *on* that object. A force exerted *by* an object does not influence that same object; it only influences the other object *on* which it is exerted. Thus, to avoid confusion, the two prepositions *on* and *by* must always be used—and used with care.

One way to keep clear which force acts on which object is to use double subscripts. For example, the force exerted *on* the **P**erson *by* the **G**round as the person walks in Fig. 4–11 can be labeled $\vec{\mathbf{F}}_{PG}$. And the force exerted on the ground by the person is $\vec{\mathbf{F}}_{GP}$. By Newton's third law

NEWTON'S THIRD LAW OF MOTION

$$\vec{\mathbf{F}}_{GP} = -\vec{\mathbf{F}}_{PG}. \tag{4–2}$$

$\vec{\mathbf{F}}_{GP}$ and $\vec{\mathbf{F}}_{PG}$ have the same magnitude (Newton's third law), and the minus sign reminds us that these two forces are in opposite directions.

Note carefully that the two forces shown in Fig. 4–11 act on different objects—to emphasize this we used slightly different colors for the vector arrows representing these forces. These two forces would never appear together in a sum of forces in Newton's second law, $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$. Why not? Because they act on different objects: $\vec{\mathbf{a}}$ is the acceleration of one particular object, and $\Sigma\vec{\mathbf{F}}$ must include *only* the forces on that *one* object.

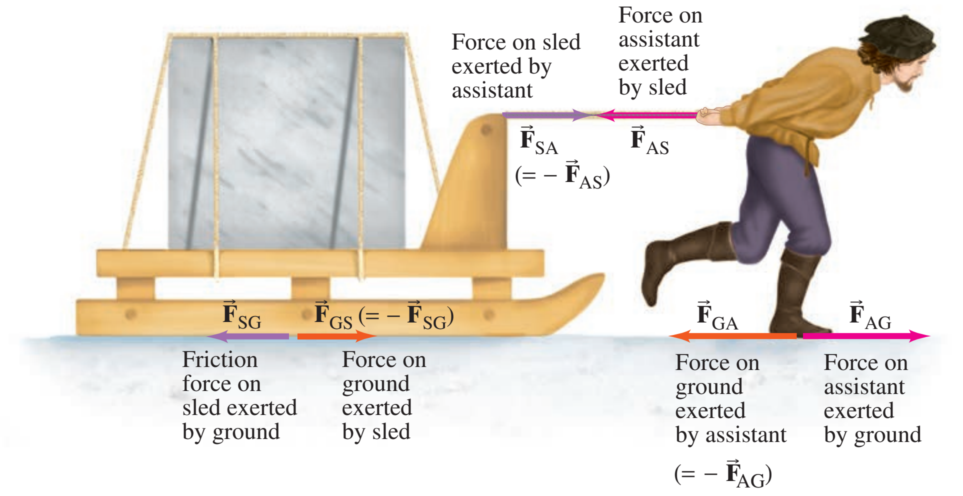


FIGURE 4–12 Example 4–5, showing only horizontal forces. Michelangelo has selected a fine block of marble for his next sculpture. Shown here is his assistant pulling it on a sled away from the quarry. Forces on the assistant are shown as red (magenta) arrows. Forces on the sled are purple arrows. Forces acting on the ground are orange arrows. Action–reaction forces that are equal and opposite are labeled by the same subscripts but reversed (such as $\vec{F}_{GA}$ and $\vec{F}_{AG}$) and are of different colors because they act on different objects.

CONCEPTUAL EXAMPLE 4–5 **Third law clarification.** Michelangelo's assistant has been assigned the task of moving a block of marble using a sled (Fig. 4–12). He says to his boss, "When I exert a forward force on the sled, the sled exerts an equal and opposite force backward. So how can I ever start it moving? No matter how hard I pull, the backward reaction force always equals my forward force, so the net force must be zero. I'll never be able to move this load." Is he correct?

RESPONSE No. Although it is true that the action and reaction forces are equal in magnitude, the assistant has forgotten that they are exerted on different objects. The forward ("action") force is exerted by the assistant on the sled (Fig. 4–12), whereas the backward "reaction" force is exerted by the sled on the assistant. To determine if the *assistant* moves or not, we must consider only the forces *on the assistant* and then apply $\Sigma\vec{F} = m\vec{a}$, where $\Sigma\vec{F}$ is the net force *on the assistant*, $\vec{a}$ is the acceleration of the assistant, and m is the assistant's mass. There are two forces on the assistant that affect his forward motion; they are shown as bright red (magenta) arrows in Figs. 4–12 and 4–13: they are (1) the horizontal force $\vec{F}_{AG}$ exerted on the assistant by the ground (the harder he pushes backward against the ground, the harder the ground pushes forward on him—Newton's third law), and (2) the force $\vec{F}_{AS}$ exerted on the assistant by the sled, pulling backward on him; see Fig. 4–13. If he pushes hard enough on the ground, the force on him exerted by the ground, $\vec{F}_{AG}$, will be larger than the sled pulling back, $\vec{F}_{AS}$, and the assistant accelerates forward (Newton's second law). The sled, on the other hand, accelerates forward when the force on *it* exerted by the assistant is greater than the frictional force exerted backward on it by the ground (that is, when $\vec{F}_{SA}$ has greater magnitude than $\vec{F}_{SG}$ in Fig. 4–12).

PROBLEM SOLVING
A study of Newton's second and third laws

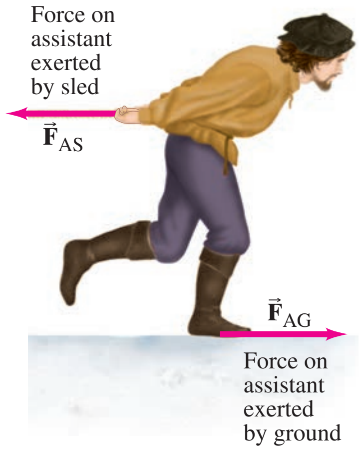


FIGURE 4–13 Example 4–5. The horizontal forces on the assistant.

Using double subscripts to clarify Newton's third law can become cumbersome, and we won't usually use them in this way. We will usually use a single subscript referring to what exerts the force on the object being discussed. Nevertheless, if there is any confusion in your mind about a given force, go ahead and use two subscripts to identify *on* what object and *by* what object the force is exerted.

EXERCISE B Return to the first Chapter-Opening Question, page 75, and answer it again now. Try to explain why you may have answered differently the first time.

EXERCISE C A tennis ball collides head-on with a more massive baseball. (i) Which ball experiences the greater force of impact? (ii) Which experiences the greater acceleration during the impact? (iii) Which of Newton's laws are useful to obtain the correct answers?

EXERCISE D If you push on a heavy desk, does it always push back on you? (*a*) No. (*b*) Yes. (*c*) Not unless someone else also pushes on it. (*d*) Yes, if it is out in space. (*e*) A desk never pushes to start with.

4–6 Weight—the Force of Gravity; and the Normal Force

As we saw in Chapter 2, Galileo claimed that all objects dropped near the surface of the Earth would fall with the same acceleration, $\vec{\mathbf{g}}$, if air resistance was negligible. The force that causes this acceleration is called the *force of gravity* or *gravitational force*. What exerts the gravitational force on an object? It is the Earth, as we will discuss in Chapter 5, and the force acts vertically[†] downward, toward the center of the Earth. Let us apply Newton's second law to an object of mass m falling freely due to gravity. For the acceleration, $\vec{\mathbf{a}}$, we use the downward acceleration due to gravity, $\vec{\mathbf{g}}$. Thus, the **gravitational force** on an object, $\vec{\mathbf{F}}_G$, can be written as

$$\vec{\mathbf{F}}_G = m\vec{\mathbf{g}}. \tag{4–3}$$

The direction of this force is down toward the center of the Earth. The magnitude of the force of gravity on an object, mg, is commonly called the object's **weight**.

In SI units, $g = 9.80\,\mathrm{m/s^2} = 9.80\,\mathrm{N/kg}$,[‡] so the weight of a 1.00-kg mass on Earth is $1.00\,\mathrm{kg} \times 9.80\,\mathrm{m/s^2} = 9.80\,\mathrm{N}$. We will mainly be concerned with the weight of objects on Earth, but we note that on the Moon, on other planets, or in space, the weight of a given mass will be different than it is on Earth. For example, on the Moon the acceleration due to gravity is about one-sixth what it is on Earth, and a 1.0-kg mass weighs only 1.6 N. Although we will not use British units, we note that for practical purposes on the Earth, a mass of 1.0 kg weighs about 2.2 lb. (On the Moon, 1 kg weighs only about 0.4 lb.)

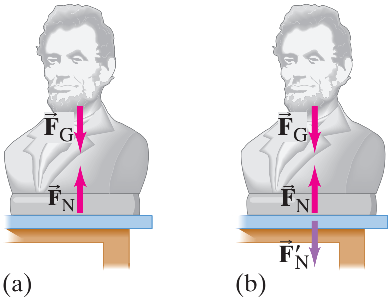


FIGURE 4–14 (a) The net force on an object at rest is zero according to Newton's second law. Therefore the downward force of gravity ($\vec{\mathbf{F}}_G$) on an object at rest must be balanced by an upward force (the normal force $\vec{\mathbf{F}}_N$) exerted by the table in this case. (b) $\vec{\mathbf{F}}'_N$ is the force exerted on the table by the statue and is the reaction force to $\vec{\mathbf{F}}_N$ by Newton's third law. ($\vec{\mathbf{F}}'_N$ is shown in a different color to remind us it acts on a different object.) The reaction force to $\vec{\mathbf{F}}_G$ is not shown.

CAUTION

Weight and normal force are ***not*** *action–reaction pairs*

The force of gravity acts on an object when it is falling. When an object is at rest on the Earth, the gravitational force on it does not disappear, as we know if we weigh it on a spring scale. The same force, given by Eq. 4–3, continues to act. Why, then, doesn't the object move? From Newton's second law, the net force on an object that remains at rest is zero. There must be another force on the object to balance the gravitational force. For an object resting on a table, the table exerts this upward force; see Fig. 4–14a. The table is compressed slightly beneath the object, and due to its elasticity, it pushes up on the object as shown. The force exerted by the table is often called a **contact force**, since it occurs when two objects are in contact. (The force of your hand pushing on a cart is also a contact force.) When a contact force acts *perpendicular* to the common surface of contact, it is referred to as the **normal force** ("normal" means perpendicular); hence it is labeled $\vec{\mathbf{F}}_N$ in Fig. 4–14a.

The two forces shown in Fig. 4–14a are both acting on the statue, which remains at rest, so the vector sum of these two forces must be zero (Newton's second law). Hence $\vec{\mathbf{F}}_G$ and $\vec{\mathbf{F}}_N$ must be of equal magnitude and in opposite directions. But they are *not* the equal and opposite forces spoken of in Newton's third law. The action and reaction forces of Newton's third law act on *different objects*, whereas the two forces shown in Fig. 4–14a act on the *same* object. For each of the forces shown in Fig. 4–14a, we can ask, "What is the reaction force?" The upward force $\vec{\mathbf{F}}_N$ on the statue is exerted by the table. The reaction to this force is a force exerted by the statue downward on the table. It is shown in Fig. 4–14b, where it is labeled $\vec{\mathbf{F}}'_N$. This force, $\vec{\mathbf{F}}'_N$, exerted on the table by the statue, is the reaction force to $\vec{\mathbf{F}}_N$ in accord with Newton's third law. What about the other force on the statue, the force of gravity $\vec{\mathbf{F}}_G$ exerted by the Earth? Can you guess what the reaction is to this force? We will see in Chapter 5 that the reaction force is also a gravitational force, exerted on the Earth by the statue.

EXERCISE E Return to the second Chapter-Opening Question, page 75, and answer it again now. Try to explain why you may have answered differently the first time.

[†]The concept of "vertical" is tied to gravity. The best definition of *vertical* is that it is the direction in which objects fall. A surface that is "horizontal," on the other hand, is a surface on which a round object won't start rolling: gravity has no effect. Horizontal is perpendicular to vertical.

[‡]Since $1\,\mathrm{N} = 1\,\mathrm{kg\cdot m/s^2}$ (Section 4–4), then $1\,\mathrm{m/s^2} = 1\,\mathrm{N/kg}$.

EXAMPLE 4–6 **Weight, normal force, and a box.** A friend has given you a special gift, a box of mass 10.0 kg with a mystery surprise inside. The box is resting on the smooth (frictionless) horizontal surface of a table (Fig. 4–15a). (a) Determine the weight of the box and the normal force exerted on it by the table. (b) Now your friend pushes down on the box with a force of 40.0 N, as in Fig. 4–15b. Again determine the normal force exerted on the box by the table. (c) If your friend pulls upward on the box with a force of 40.0 N (Fig. 4–15c), what now is the normal force exerted on the box by the table?

APPROACH The box is at rest on the table, so the net force on the box in each case is zero (Newton's first or second law). The weight of the box has magnitude mg in all three cases.

SOLUTION (a) The weight of the box is $mg = (10.0\,\mathrm{kg})(9.80\,\mathrm{m/s^2}) = 98.0\,\mathrm{N}$, and this force acts downward. The only other force on the box is the normal force exerted upward on it by the table, as shown in Fig. 4–15a. We chose the upward direction as the positive y direction; then the net force ΣF_y on the box is $\Sigma F_y = F_N - mg$; the minus sign means mg acts in the negative y direction (m and g are magnitudes). The box is at rest, so the net force on it must be zero (Newton's second law, $\Sigma F_y = ma_y$, and $a_y = 0$). Thus

$$\begin{aligned} \Sigma F_y &= ma_y \\ F_N - mg &= 0, \end{aligned}$$

so we have

$$F_N = mg.$$

The normal force on the box, exerted by the table, is 98.0 N upward, and has magnitude equal to the box's weight.

(b) Your friend is pushing down on the box with a force of 40.0 N. So instead of only two forces acting on the box, now there are three forces acting on the box, as shown in Fig. 4–15b. The weight of the box is still $mg = 98.0\,\mathrm{N}$. The net force is $\Sigma F_y = F_N - mg - 40.0\,\mathrm{N}$, and is equal to zero because the box remains at rest $(a = 0)$. Newton's second law gives

$$\Sigma F_y = F_N - mg - 40.0\,\mathrm{N} = 0.$$

We solve this equation for the normal force:

$$F_N = mg + 40.0\,\mathrm{N} = 98.0\,\mathrm{N} + 40.0\,\mathrm{N} = 138.0\,\mathrm{N},$$

which is greater than in (a). The table pushes back with more force when a person pushes down on the box. The normal force is not always equal to the weight!

(c) The box's weight is still 98.0 N and acts downward. The force exerted by your friend and the normal force both act upward (positive direction), as shown in Fig. 4–15c. The box doesn't move since your friend's upward force is less than the weight. The net force, again set to zero in Newton's second law because $a = 0$, is

$$\Sigma F_y = F_N - mg + 40.0\,\mathrm{N} = 0,$$

so

$$F_N = mg - 40.0\,\mathrm{N} = 98.0\,\mathrm{N} - 40.0\,\mathrm{N} = 58.0\,\mathrm{N}.$$

The table does not push against the full weight of the box because of the upward force exerted by your friend.

NOTE The weight of the box ($= mg$) does not change as a result of your friend's push or pull. Only the normal force is affected.

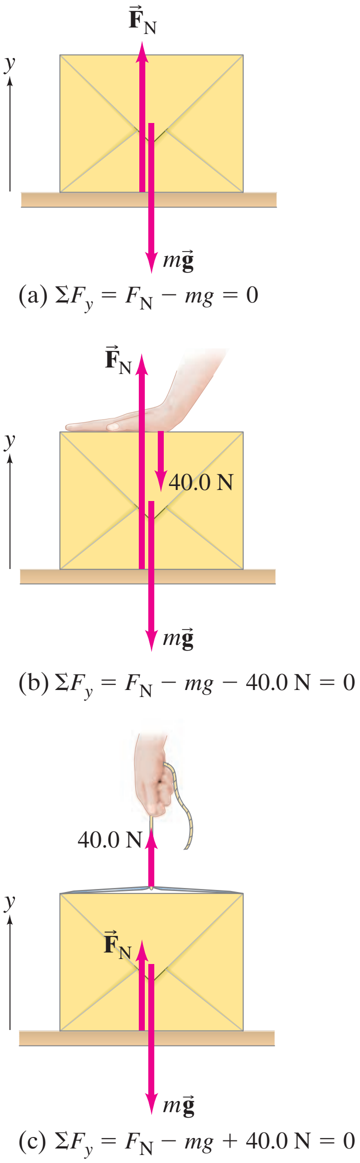


FIGURE 4–15 Example 4–6. (a) A 10-kg gift box is at rest on a table. (b) A person pushes down on the box with a force of 40.0 N. (c) A person pulls upward on the box with a force of 40.0 N. The forces are all assumed to act along a line; they are shown slightly displaced in order to be distinguishable. Only forces acting on the box are shown.

CAUTION
The normal force is not always equal to the weight

Recall that the normal force is elastic in origin (the table in Fig. 4–15 sags slightly under the weight of the box). The normal force in Example 4–6 is vertical, perpendicular to the horizontal table. The normal force is not always vertical, however. When you push against a wall, for example, the normal force with which the wall pushes back on you is horizontal (Fig. 4–9). For an object on a plane inclined at an angle to the horizontal, such as a skier or car on a hill, the normal force acts perpendicular to the plane and so is not vertical.

CAUTION
The normal force, $\vec{\mathbf{F}}_N$, is not necessarily vertical

EXAMPLE 4–7 **Accelerating the box.** What happens when a person pulls upward on the box in Example 4–6*c* with a force equal to, or greater than, the box's weight? For example, let $F_P = 100.0\,\text{N}$ (Fig. 4–16) rather than the 40.0 N shown in Fig. 4–15c.

APPROACH We can start just as in Example 4–6, but be ready for a surprise.

SOLUTION The net force on the box is

$$\begin{aligned}\Sigma F_y &= F_N - mg + F_P \\ &= F_N - 98.0\,\text{N} + 100.0\,\text{N},\end{aligned}$$

and if we set this equal to zero (thinking the acceleration might be zero), we would get $F_N = -2.0\,\text{N}$. This is nonsense, since the negative sign implies F_N points downward, and the table surely cannot *pull* down on the box (unless there's glue on the table). The least F_N can be is zero, which it will be in this case. What really happens here is that the box accelerates upward $(a \neq 0)$ because the net force is not zero. The net force (setting the normal force $F_N = 0$) is

$$\begin{aligned}\Sigma F_y = F_P - mg &= 100.0\,\text{N} - 98.0\,\text{N} \\ &= 2.0\,\text{N}\end{aligned}$$

upward. See Fig. 4–16. We apply Newton's second law and see that the box moves upward with an acceleration

$$\begin{aligned}a_y = \frac{\Sigma F_y}{m} &= \frac{2.0\,\text{N}}{10.0\,\text{kg}} \\ &= 0.20\,\text{m/s}^2.\end{aligned}$$

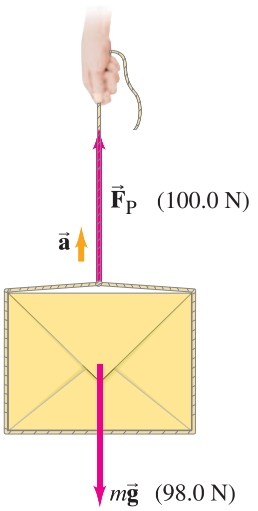


FIGURE 4–16 Example 4–7. The box accelerates upward because $F_P > mg$.

EXAMPLE 4–8 **Apparent weight loss.** A 65-kg woman descends in an elevator that briefly accelerates at 0.20*g* downward. She stands on a scale that reads in kg. (*a*) During this acceleration, what is her weight and what does the scale read? (*b*) What does the scale read when the elevator descends at a constant speed of 2.0 m/s?

APPROACH Figure 4–17 shows all the forces that act on the woman (and *only* those that act on her). The direction of the acceleration is downward, so we choose the positive direction as down (this is the opposite choice from Examples 4–6 and 4–7).

SOLUTION (*a*) From Newton's second law,

$$\begin{aligned}\Sigma F &= ma \\ mg - F_N &= m(0.20g).\end{aligned}$$

We solve for F_N:

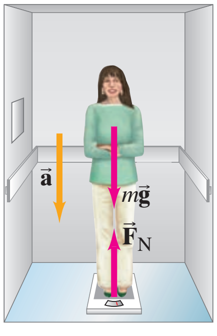
$$\begin{aligned}F_N &= mg - 0.20mg \\ &= 0.80mg,\end{aligned}$$

and it acts upward. The normal force $\vec{\mathbf{F}}_N$ is the force the scale exerts on the person, and is equal and opposite to the force she exerts on the scale: $F'_N = 0.80mg$ downward. Her weight (force of gravity on her) is still $mg = (65\,\text{kg})(9.8\,\text{m/s}^2) = 640\,\text{N}$. But the scale, needing to exert a force of only $0.80mg$, will give a reading of $0.80m = 52\,\text{kg}$.

(*b*) Now there is no acceleration, $a = 0$, so by Newton's second law, $mg - F_N = 0$ and $F_N = mg$. The scale reads her true mass of 65 kg.

NOTE The scale in (*a*) gives a reading of 52 kg (as an "apparent mass"), but her mass doesn't change as a result of the acceleration: it stays at 65 kg.

FIGURE 4–17 Example 4–8. The acceleration vector is shown in gold to distinguish it from the red force vectors.



4–7 Solving Problems with Newton's Laws: Free-Body Diagrams

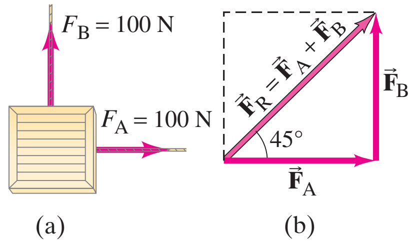
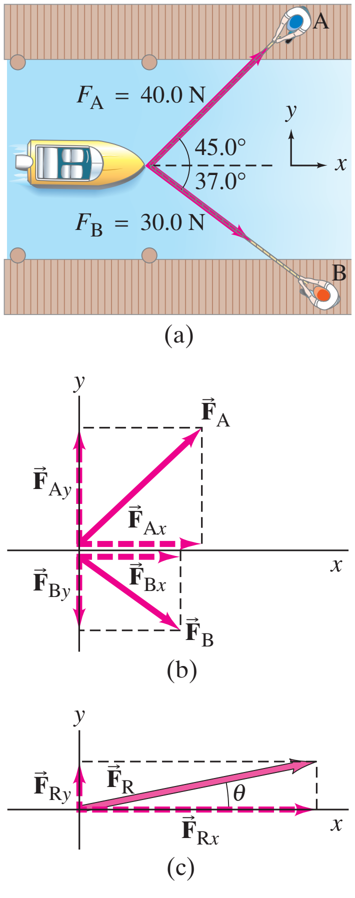


FIGURE 4–18 (a) Two horizontal forces, $\vec{F}_A$ and $\vec{F}_B$, exerted by workers A and B, act on a crate (we are looking down from above). (b) The sum, or resultant, of $\vec{F}_A$ and $\vec{F}_B$ is $\vec{F}_R$.

Newton's second law tells us that the acceleration of an object is proportional to the *net force* acting on the object. The **net force**, as mentioned earlier, is the *vector sum* of all forces acting on the object. Indeed, extensive experiments have shown that forces do add together as vectors precisely according to the rules we developed in Chapter 3. For example, in Fig. 4–18, two forces of equal magnitude (100 N each) are shown acting on an object at right angles to each other. Intuitively, we can see that the object will start moving at a 45° angle and thus the net force acts at a 45° angle. This is just what the rules of vector addition give. From the theorem of Pythagoras, the magnitude of the resultant force is $F_R = \sqrt{(100\text{ N})^2 + (100\text{ N})^2} = 141$ N.

EXAMPLE 4–9 **Adding force vectors.** Calculate the sum of the two forces exerted on the boat by workers A and B in Fig. 4–19a.

FIGURE 4–19 Example 4–9: Two force vectors act on a boat.



APPROACH We add force vectors like any other vectors as described in Chapter 3. The first step is to choose an xy coordinate system (see Fig. 4–19a), and then resolve vectors into their components.

SOLUTION The two force vectors are shown resolved into components in Fig. 4–19b. We add the forces using the method of components. The components of $\vec{F}_A$ are

$$F_{Ax} = F_A \cos 45.0° = (40.0\text{ N})(0.707) = 28.3\text{ N},$$
$$F_{Ay} = F_A \sin 45.0° = (40.0\text{ N})(0.707) = 28.3\text{ N}.$$

The components of $\vec{F}_B$ are

$$F_{Bx} = +F_B \cos 37.0° = +(30.0\text{ N})(0.799) = +24.0\text{ N},$$
$$F_{By} = -F_B \sin 37.0° = -(30.0\text{ N})(0.602) = -18.1\text{ N}.$$

F_{By} is negative because it points along the negative y axis. The components of the resultant force are (see Fig. 4–19c)

$$F_{Rx} = F_{Ax} + F_{Bx} = 28.3\text{ N} + 24.0\text{ N} = 52.3\text{ N},$$
$$F_{Ry} = F_{Ay} + F_{By} = 28.3\text{ N} - 18.1\text{ N} = 10.2\text{ N}.$$

To find the magnitude of the resultant force, we use the Pythagorean theorem,

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{(52.3)^2 + (10.2)^2}\text{ N} = 53.3\text{ N}.$$

The only remaining question is the angle θ that the net force $\vec{F}_R$ makes with the x axis. We use:

$$\tan\theta = \frac{F_{Ry}}{F_{Rx}} = \frac{10.2\text{ N}}{52.3\text{ N}} = 0.195,$$

and $\tan^{-1}(0.195) = 11.0°$. The net force on the boat has magnitude 53.3 N and acts at an 11.0° angle to the x axis.

PROBLEM SOLVING
Free-body diagram

When solving problems involving Newton's laws and force, it is very important to draw a diagram showing all the forces acting *on* each object involved. Such a diagram is called a **free-body diagram**, or **force diagram**: choose one object, and draw an arrow to represent each force acting on it. Include *every* force acting on that object. Do not show forces that the chosen object exerts on *other* objects. To help you identify each and every force that is exerted on your chosen object, ask yourself what other objects could exert a force on it. If your problem involves more than one object, a separate free-body diagram is needed for each object. For now, the likely forces that could be acting are *gravity* and *contact forces* (one object pushing or pulling another, normal force, friction). Later we will consider other types of force such as buoyancy, fluid pressure, and electric and magnetic forces.

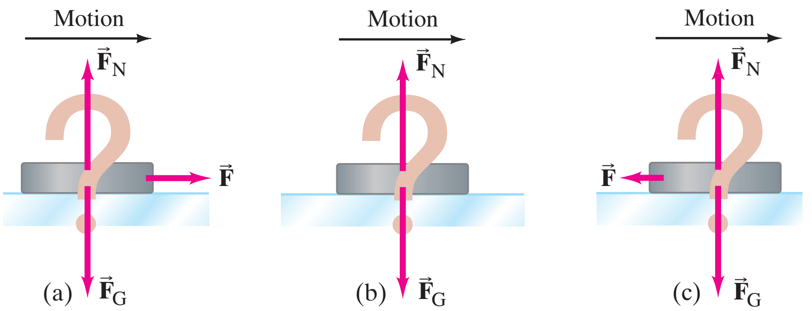


FIGURE 4–20 Example 4–10. Which is the correct free-body diagram for a hockey puck sliding across frictionless ice?

CONCEPTUAL EXAMPLE 4–10 **The hockey puck.** A hockey puck is sliding at constant velocity across a flat horizontal ice surface that is assumed to be frictionless. Which of the sketches in Fig. 4–20 is the correct free-body diagram for this puck? What would your answer be if the puck slowed down?

RESPONSE Did you choose (*a*)? If so, can you answer the question: what exerts the horizontal force labeled $\vec{\mathbf{F}}$ on the puck? If you say that it is the force needed to maintain the motion, ask yourself: what exerts this force? Remember that another object must exert any force—and there simply isn't any possibility here. Therefore, (*a*) is wrong. Besides, the force $\vec{\mathbf{F}}$ in Fig. 4–20a would give rise to an acceleration by Newton's second law. It is (*b*) that is correct. No net force acts on the puck, and the puck slides at constant velocity across the ice.

In the real world, where even smooth ice exerts at least a tiny friction force, then (*c*) is the correct answer. The tiny friction force is in the direction opposite to the motion, and the puck's velocity decreases, even if very slowly.

PROBLEM SOLVING

Newton's Laws; Free-Body Diagrams

1. **Draw a sketch** of the situation, after carefully reading the Problem at least twice.
2. Consider only one object (at a time), and draw a **free-body diagram** for that object, showing *all* the forces acting *on* that object. Include any unknown forces that you have to solve for. Do not show any forces that the chosen object exerts on other objects.

   Draw the arrow for each force vector reasonably accurately for direction and magnitude. Label each force acting on the object, including forces you must solve for, according to its source (gravity, person, friction, and so on).

   If several objects are involved, draw a free-body diagram for each object *separately*. For each object, show all the forces acting *on that object* (and *only* forces acting on that object). For each (and every) force, you must be clear about: *on* what object that force acts, and *by* what object that force is exerted. Only forces acting *on* a given object can be included in $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$ for that object.
3. Newton's second law involves vectors, and it is usually important to **resolve vectors** into components. **Choose** x and y **axes** in a way that simplifies the calculation. For example, it often saves work if you choose one coordinate axis to be in the direction of the acceleration (if known).
4. For each object, **apply Newton's second law** to the x and y components separately. That is, the x component of the net force on that object is related to the x component of that object's acceleration: $\Sigma F_x = ma_x$, and similarly for the y direction.
5. **Solve** the equation or equations for the unknown(s). Put in numerical values only at the end, and keep track of units.

This Problem Solving Strategy should not be considered a prescription. Rather it is a summary of things to do that will start you thinking and getting involved in the problem at hand.

CAUTION
Treating an object as a particle

When we are concerned only about translational motion, all the forces on a given object can be drawn as acting at the center of the object, thus treating the object as a *point particle*. However, for problems involving rotation or statics, the place *where* each force acts is also important, as we shall see in Chapters 8 and 9.

In the Examples in this Section, we assume that all surfaces are very smooth so that friction can be ignored. (Friction, and Examples using it, are discussed in Section 4–8.)

EXAMPLE 4–11 **Pulling the mystery box.** Suppose a friend asks to examine the 10.0-kg box you were given (Example 4–6, Fig. 4–15), hoping to guess what is inside; and you respond, "Sure, pull the box over to you." She then pulls the box by the attached cord, as shown in Fig. 4–21a, along the smooth surface of the table. The magnitude of the force exerted by the person is $F_P = 40.0\,\mathrm{N}$, and it is exerted at a 30.0° angle as shown. Calculate (*a*) the acceleration of the box, and (*b*) the magnitude of the upward force F_N exerted by the table on the box. Assume that friction can be neglected.

APPROACH We follow the Problem Solving Strategy on the previous page.

SOLUTION

1. **Draw a sketch**: The situation is shown in Fig. 4–21a; it shows the box and the force applied by the person, F_P.
2. **Free-body diagram**: Figure 4–21b shows the free-body diagram of the box. To draw it correctly, we show *all* the forces acting on the box and *only* the forces acting on the box. They are: the force of gravity $m\vec{\mathbf{g}}$; the normal force exerted by the table $\vec{\mathbf{F}}_N$; and the force exerted by the person $\vec{\mathbf{F}}_P$. We are interested only in translational motion, so we can show the three forces acting at a point, Fig. 4–21c.
3. **Choose axes and resolve vectors**: We expect the motion to be horizontal, so we choose the x axis horizontal and the y axis vertical. The pull of 40.0 N has components

$$F_{Px} = (40.0\,\mathrm{N})(\cos 30.0°) = (40.0\,\mathrm{N})(0.866) = 34.6\,\mathrm{N},$$
$$F_{Py} = (40.0\,\mathrm{N})(\sin 30.0°) = (40.0\,\mathrm{N})(0.500) = 20.0\,\mathrm{N}.$$

In the horizontal (x) direction, $\vec{\mathbf{F}}_N$ and $m\vec{\mathbf{g}}$ have zero components. Thus the horizontal component of the net force is F_{Px}.

4. (*a*) **Apply Newton's second law** to get the x component of the acceleration:

$$F_{Px} = ma_x.$$

5. (*a*) **Solve**:

$$a_x = \frac{F_{Px}}{m} = \frac{(34.6\,\mathrm{N})}{(10.0\,\mathrm{kg})} = 3.46\,\mathrm{m/s^2}.$$

The acceleration of the box is $3.46\,\mathrm{m/s^2}$ to the right.

(*b*) Next we want to find F_N.

4′. (*b*) **Apply Newton's second law** to the vertical (y) direction, with upward as positive:

$$\Sigma F_y = ma_y$$
$$F_N - mg + F_{Py} = ma_y.$$

5′. (*b*) **Solve**: We have $mg = (10.0\,\mathrm{kg})(9.80\,\mathrm{m/s^2}) = 98.0\,\mathrm{N}$ and, from point 3 above, $F_{Py} = 20.0\,\mathrm{N}$. Furthermore, since $F_{Py} < mg$, the box does not move vertically, so $a_y = 0$. Thus

$$F_N - 98.0\,\mathrm{N} + 20.0\,\mathrm{N} = 0,$$

so

$$F_N = 78.0\,\mathrm{N}.$$

NOTE F_N is less than mg: the table does not push against the full weight of the box because part of the pull exerted by the person is in the upward direction.

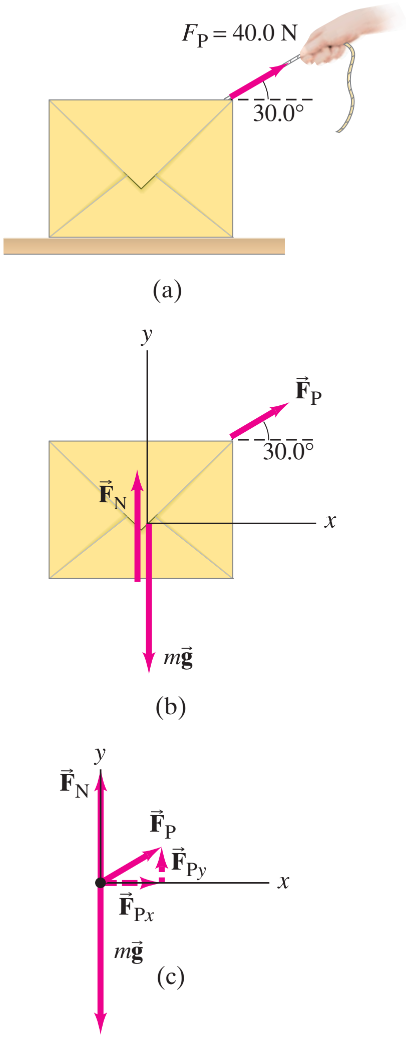


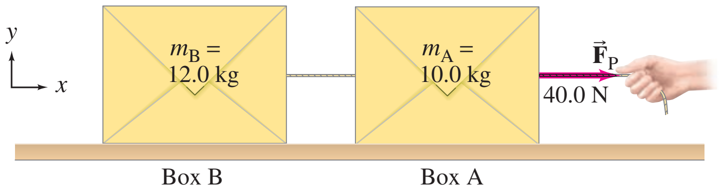
FIGURE 4–21 (a) Pulling the box, Example 4–11; (b) is the free-body diagram for the box, and (c) is the free-body diagram considering all the forces to act at a point (translational motion only, which is what we have here).

EXERCISE F A 10.0-kg box is dragged on a horizontal frictionless surface by a horizontal force of 10.0 N. If the applied force is doubled, the normal force on the box will (*a*) increase; (*b*) remain the same; (*c*) decrease.

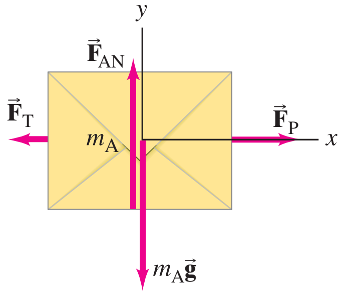
Tension in a Flexible Cord

When a flexible cord pulls on an object, the cord is said to be under **tension**, and the force it exerts on the object is the tension F_T. If the cord has negligible mass, the force exerted at one end is transmitted undiminished to each adjacent piece of cord along the entire length to the other end. Why? Because $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}} = 0$ for the cord if the cord's mass m is zero (or negligible) no matter what $\vec{\mathbf{a}}$ is. Hence the forces pulling on the cord at its two ends must add up to zero (F_T and $-F_T$). Note that flexible cords and strings can only pull. They can't push because they bend.

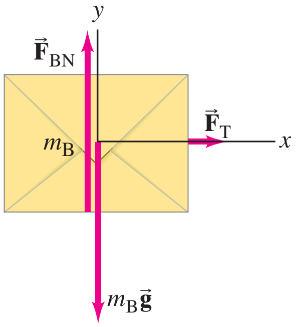
PROBLEM SOLVING
Cords can pull but can't push; tension exists throughout a taut cord



(a)



(b)



(c)

FIGURE 4–22 Example 4–12. (a) Two boxes, A and B, are connected by a cord. A person pulls horizontally on box A with force $F_P = 40.0$ N. (b) Free-body diagram for box A. (c) Free-body diagram for box B.

Our next Example involves two boxes connected by a cord. We can refer to this group of objects as a system. A **system** is any group of one or more objects we choose to consider and study.

EXAMPLE 4–12 **Two boxes connected by a cord.** Two boxes, A and B, are connected by a lightweight cord and are resting on a smooth (frictionless) table. The boxes have masses of 12.0 kg and 10.0 kg. A horizontal force F_P of 40.0 N is applied to the 10.0-kg box, as shown in Fig. 4–22a. Find (*a*) the acceleration of each box, and (*b*) the tension in the cord connecting the boxes.

APPROACH We streamline our approach by not listing each step. We have two boxes so we draw a free-body diagram for each. To draw them correctly, we must consider the forces on *each* box by itself, so that Newton's second law can be applied to each. The person exerts a force F_P on box A. Box A exerts a force F_T on the connecting cord, and the cord exerts an opposite but equal magnitude force F_T back on box A (Newton's third law). The two horizontal forces on box A are shown in Fig. 4–22b, along with the force of gravity $m_A\vec{g}$ downward and the normal force $\vec{F}_{AN}$ exerted upward by the table. The cord is light, so we neglect its mass. The tension at each end of the cord is thus the same. Hence the cord exerts a force F_T on the second box. Figure 4–22c shows the forces on box B, which are $\vec{F}_T$, $m_B\vec{g}$, and the normal force $\vec{F}_{BN}$. There will be only horizontal motion. We take the positive x axis to the right.

SOLUTION (*a*) We apply $\Sigma F_x = ma_x$ to box A:

$$\Sigma F_x = F_P - F_T = m_A a_A. \qquad \text{[box A]}$$

For box B, the only horizontal force is F_T, so

$$\Sigma F_x = F_T = m_B a_B. \qquad \text{[box B]}$$

The boxes are connected, and if the cord remains taut and doesn't stretch, then the two boxes will have the same acceleration a. Thus $a_A = a_B = a$. We are given $m_A = 10.0$ kg and $m_B = 12.0$ kg. We can add the two equations above to eliminate an unknown (F_T) and obtain

$$(m_A + m_B)a = F_P - F_T + F_T = F_P$$

or

$$a = \frac{F_P}{m_A + m_B} = \frac{40.0\text{ N}}{22.0\text{ kg}} = 1.82\text{ m/s}^2.$$

This is what we sought.

(*b*) From the equation for box B above $(F_T = m_B a_B)$, the tension in the cord is

$$F_T = m_B a = (12.0\text{ kg})(1.82\text{ m/s}^2) = 21.8\text{ N}.$$

Thus, $F_T < F_P$ $(= 40.0\text{ N})$, as we expect, since F_T acts to accelerate only m_B.

Alternate Solution to (a) We would have obtained the same result had we considered a single system, of mass $m_A + m_B$, acted on by a net horizontal force equal to F_P. (The tension forces F_T would then be considered internal to the system as a whole, and summed together would make zero contribution to the net force on the *whole* system.)

NOTE It might be tempting to say that the force the person exerts, F_P, acts not only on box A but also on box B. It doesn't. F_P acts only on box A. It affects box B via the tension in the cord, F_T, which acts on box B and accelerates it. (You could look at it this way: $F_T < F_P$ because F_P accelerates *both* boxes whereas F_T only accelerates box B.)

CAUTION

For any object, use only the forces on that object in calculating $\Sigma F = ma$

PHYSICS APPLIED
Elevator (as Atwood machine)

EXAMPLE 4–13 **Elevator and counterweight (Atwood machine).** A system of two objects suspended over a pulley by a flexible cable, as shown in Fig. 4–23a, is sometimes referred to as an *Atwood machine*. Consider the real-life application of an elevator (m_E) and its counterweight (m_C). To minimize the work done by the motor to raise and lower the elevator safely, m_E and m_C are made similar in mass. We leave the motor out of the system for this calculation, and assume that the cable's mass is negligible and that the mass of the pulley, as well as any friction, is small and ignorable. These assumptions ensure that the tension F_T in the cable has the same magnitude on both sides of the pulley. Let the mass of the counterweight be $m_C = 1000\,\text{kg}$. Assume the mass of the empty elevator is 850 kg, and its mass when carrying four passengers is $m_E = 1150\,\text{kg}$. For the latter case $(m_E = 1150\,\text{kg})$, calculate (*a*) the acceleration of the elevator and (*b*) the tension in the cable.

APPROACH Again we have two objects, and we will need to apply Newton's second law to each of them separately. Each mass has two forces acting on it: gravity downward and the cable tension pulling upward, $\vec{\mathbf{F}}_T$. Figures 4–23b and c show the free-body diagrams for the elevator (m_E) and for the counterweight (m_C). The elevator, being the heavier, will accelerate downward, whereas the counterweight will accelerate upward. The magnitudes of their accelerations will be equal (we assume the cable is massless and doesn't stretch). For the counterweight, $m_C g = (1000\,\text{kg})(9.80\,\text{m/s}^2) = 9800\,\text{N}$, so F_T must be greater than 9800 N (in order that m_C will accelerate upward). For the elevator, $m_E g = (1150\,\text{kg})(9.80\,\text{m/s}^2) = 11{,}300\,\text{N}$, which must have greater magnitude than F_T so that m_E accelerates downward. Thus our calculation must give F_T between 9800 N and 11,300 N.

SOLUTION (*a*) To find F_T as well as the acceleration a, we apply Newton's second law, $\Sigma F = ma$, to each object. We take upward as the positive y direction for both objects. With this choice of axes, $a_C = a$ because m_C accelerates upward, and $a_E = -a$ because m_E accelerates downward. Thus

$$F_T - m_E g = m_E a_E = -m_E a$$
$$F_T - m_C g = m_C a_C = +m_C a.$$

We can subtract the first equation from the second to get

$$(m_E - m_C)g = (m_E + m_C)a,$$

where a is now the only unknown. We solve this for a:

$$a = \frac{m_E - m_C}{m_E + m_C}\,g = \frac{1150\,\text{kg} - 1000\,\text{kg}}{1150\,\text{kg} + 1000\,\text{kg}}\,g = 0.070g = 0.68\,\text{m/s}^2.$$

The elevator (m_E) accelerates downward (and the counterweight m_C upward) at $a = 0.070g = 0.68\,\text{m/s}^2$.

(*b*) The tension in the cable F_T can be obtained from either of the two $\Sigma F = ma$ equations at the start of our solution, setting $a = 0.070g = 0.68\,\text{m/s}^2$:

$$\begin{aligned} F_T = m_E g - m_E a &= m_E(g - a) \\ &= 1150\,\text{kg}\,(9.80\,\text{m/s}^2 - 0.68\,\text{m/s}^2) = 10{,}500\,\text{N}, \end{aligned}$$

or

$$\begin{aligned} F_T = m_C g + m_C a &= m_C(g + a) \\ &= 1000\,\text{kg}\,(9.80\,\text{m/s}^2 + 0.68\,\text{m/s}^2) = 10{,}500\,\text{N}, \end{aligned}$$

which are consistent. As predicted, our result lies between 9800 N and 11,300 N.

NOTE We can check our equation for the acceleration a in this Example by noting that if the masses were equal $(m_E = m_C)$, then our equation above for a would give $a = 0$, as we should expect. Also, if one of the masses is zero (say, $m_C = 0$), then the other mass $(m_E \neq 0)$ would be predicted by our equation to accelerate at $a = g$, again as expected.

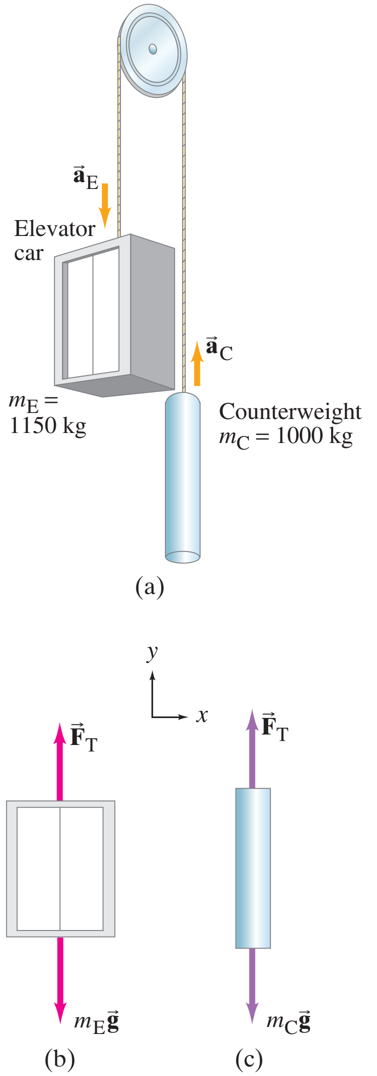


FIGURE 4–23 Example 4–13. (a) Atwood machine in the form of an elevator–counterweight system. (b) and (c) Free-body diagrams for the two objects.

PROBLEM SOLVING
Check your result by seeing if it works in situations where the answer is easily guessed

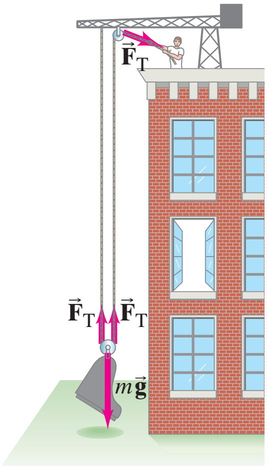


FIGURE 4–24 Example 4–14.

CONCEPTUAL EXAMPLE 4–14 **The advantage of a pulley.** A mover is trying to lift a piano (slowly) up to a second-story apartment (Fig. 4–24). He is using a rope looped over two pulleys as shown. What force must he exert on the rope to slowly lift the piano's 1600-N weight?

RESPONSE The magnitude of the tension force F_T within the rope is the same at any point along the rope if we assume we can ignore its mass. First notice the forces acting on the lower pulley at the piano. The weight of the piano $(= mg)$ pulls down on the pulley. The tension in the rope, looped through this pulley, pulls up *twice*, once on each side of the pulley. Let us apply Newton's second law to the pulley–piano combination (of mass m), choosing the upward direction as positive:

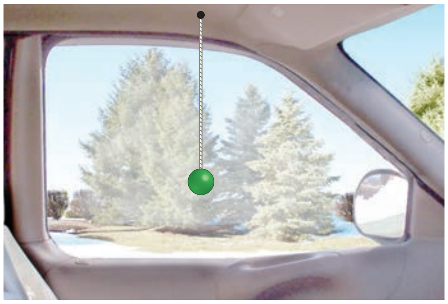
$$2F_T - mg = ma.$$

To move the piano with constant speed (set $a = 0$ in this equation) thus requires a tension in the rope, and hence a pull on the rope, of $F_T = mg/2$. The piano mover can exert a force equal to half the piano's weight.

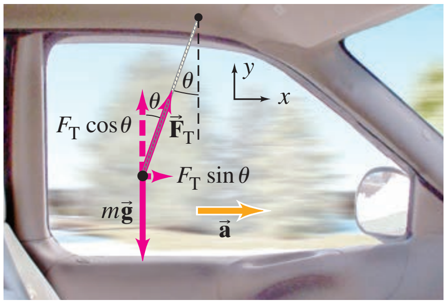
NOTE We say the pulley has given a **mechanical advantage** of 2, since without the pulley the mover would have to exert twice the force.

PHYSICS APPLIED
Accelerometer

FIGURE 4–25 Example 4–15.



(a)



(b)

EXAMPLE 4–15 **Accelerometer.** A small mass m hangs from a thin string and can swing like a pendulum. You attach it above the window of your car as shown in Fig. 4–25a. When the car is at rest, the string hangs vertically. What angle θ does the string make (*a*) when the car accelerates at a constant $a = 1.20\,\mathrm{m/s^2}$, and (*b*) when the car moves at constant velocity, $v = 90\,\mathrm{km/h}$?

APPROACH The free-body diagram of Fig. 4–25b shows the pendulum at some angle θ relative to the vertical, and the forces on it: $m\vec{\mathbf{g}}$ downward, and the tension $\vec{\mathbf{F}}_T$ in the cord (including its components). These forces do not add up to zero if $\theta \neq 0$; and since we have an acceleration a, we expect $\theta \neq 0$.

SOLUTION (*a*) The acceleration $a = 1.20\,\mathrm{m/s^2}$ is horizontal $(= a_x)$, and the only horizontal force is the x component of $\vec{\mathbf{F}}_T$, $F_T \sin\theta$ (Fig. 4–25b). Then from Newton's second law,

$$ma = F_T \sin\theta.$$

The vertical component of Newton's second law gives, since $a_y = 0$,

$$0 = F_T \cos\theta - mg.$$

So

$$mg = F_T \cos\theta.$$

Dividing these two equations, we obtain

$$\tan\theta = \frac{F_T \sin\theta}{F_T \cos\theta} = \frac{ma}{mg} = \frac{a}{g}$$

or

$$\tan\theta = \frac{1.20\,\mathrm{m/s^2}}{9.80\,\mathrm{m/s^2}} = 0.122,$$

so

$$\theta = 7.0^\circ.$$

(*b*) The velocity is constant, so $a = 0$ and $\tan\theta = 0$. Hence the pendulum hangs vertically $(\theta = 0^\circ)$.

NOTE This simple device is an **accelerometer**—it can be used to determine acceleration, by measuring the angle θ.

4–8 Problems Involving Friction, Inclines

Friction

Until now we have ignored friction, but it must be taken into account in most practical situations. Friction exists between two solid surfaces because even the smoothest looking surface is quite rough on a microscopic scale, Fig. 4–26. When we try to slide an object across a surface, these microscopic bumps impede the motion. Exactly what is happening at the microscopic level is not yet fully understood. It is thought that the atoms on a bump of one surface may come so close to the atoms of the other surface that attractive electric forces between the atoms could "bond" as a tiny weld between the two surfaces. Sliding an object across a surface is often jerky, perhaps due to the making and breaking of these bonds. Even when a round object rolls across a surface, there is still some friction, called *rolling friction*, although it is generally much less than when an object slides across a surface. We focus now on sliding friction, which is usually called **kinetic friction** (*kinetic* is from the Greek for "moving").

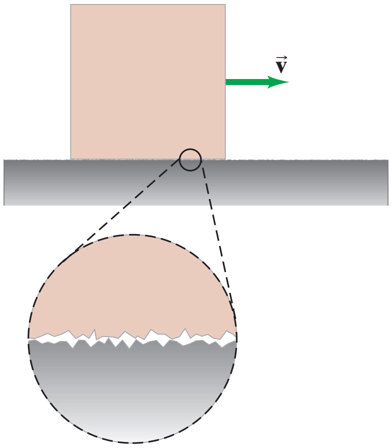
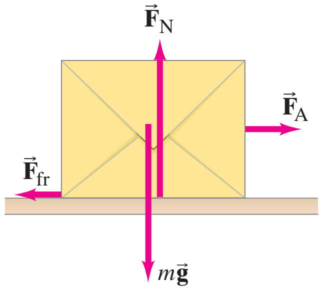


FIGURE 4–26 An object moving to the right on a table. The two surfaces in contact are assumed smooth, but are rough on a microscopic scale.

When an object slides along a rough surface, the force of kinetic friction acts opposite to the direction of the object's velocity. The magnitude of the force of kinetic friction depends on the nature of the two sliding surfaces. For given surfaces, experiment shows that the friction force is approximately proportional to the *normal force* between the two surfaces, which is the force that either object exerts on the other and is perpendicular to their common surface of contact (see Fig. 4–27). The force of friction between hard surfaces in many cases depends very little on the total surface area of contact; that is, the friction force on this book is roughly the same whether it is being slid across a table on its wide face or on its spine, assuming the surfaces have the same smoothness. We consider a simple model of friction in which we make this assumption that the friction force is independent of area. Then we write the proportionality between the magnitudes of the friction force F_{fr} and the normal force F_{N} as an equation by inserting a constant of proportionality, μ_{k}:

$$F_{\text{fr}} = \mu_{\text{k}} F_{\text{N}}. \qquad \text{[kinetic friction]}$$

FIGURE 4–27 When an object is pulled along a surface by an applied force $(\vec{\mathbf{F}}_{\text{A}})$, the force of friction $\vec{\mathbf{F}}_{\text{fr}}$ opposes the motion. The magnitude of $\vec{\mathbf{F}}_{\text{fr}}$ is proportional to the magnitude of the normal force (F_{N}).



CAUTION
$\vec{\mathbf{F}}_{\text{fr}} \perp \vec{\mathbf{F}}_{\text{N}}$

This relation is not a fundamental law; it is an experimental relation between the magnitude of the friction force F_{fr}, which acts parallel to the two surfaces, and the magnitude of the normal force F_{N}, which acts perpendicular to the surfaces. It is *not* a vector equation since the two forces have different directions, perpendicular to one another. The term μ_{k} is called the *coefficient of kinetic friction*, and its value depends on the nature of the two surfaces. Measured values for a variety of surfaces are given in Table 4–2. These are only approximate, however, since μ depends on whether the surfaces are wet or dry, on how much they have been sanded or rubbed, if any burrs remain, and other such factors. But μ_{k} (which has no units) is roughly independent of the sliding speed, as well as the area in contact.

TABLE 4–2 Coefficients of Friction†

Surfaces	Coefficient of Static Friction, μ_{s}	Coefficient of Kinetic Friction, μ_{k}
Wood on wood	0.4	0.2
Ice on ice	0.1	0.03
Metal on metal (lubricated)	0.15	0.07
Steel on steel (unlubricated)	0.7	0.6
Rubber on dry concrete	1.0	0.8
Rubber on wet concrete	0.7	0.5
Rubber on other solid surfaces	1–4	1
Teflon® on Teflon in air	0.04	0.04
Teflon on steel in air	0.04	0.04
Lubricated ball bearings	<0.01	<0.01
Synovial joints (in human limbs)	0.01	0.01

† Values are approximate and intended only as a guide.

What we have been discussing up to now is *kinetic friction*, when one object slides over another. There is also **static friction**, which refers to a force parallel to the two surfaces that can arise even when they are not sliding. Suppose an object such as a desk is resting on a horizontal floor. If no horizontal force is exerted on the desk, there also is no friction force. But now suppose you try to push the desk, and it doesn't move. You are exerting a horizontal force, but the desk isn't moving, so there must be another force on the desk keeping it from moving (the net force is zero on an object at rest). This is the force of *static friction* exerted by the floor on the desk. If you push with a greater force without moving the desk, the force of static friction also has increased. If you push hard enough, the desk will eventually start to move, and kinetic friction takes over. At this point, you have exceeded the maximum force of static friction, which is given by $(F_{fr})_{max} = \mu_s F_N$, where μ_s is the *coefficient of static friction* (Table 4–2). Because the force of static friction can vary from zero to this maximum value, we write

$$F_{fr} \leq \mu_s F_N. \qquad \text{[static friction]}$$

You may have noticed that it is often easier to keep a heavy object sliding than it is to start it sliding in the first place. This is consistent with μ_s generally being greater than μ_k (see Table 4–2).

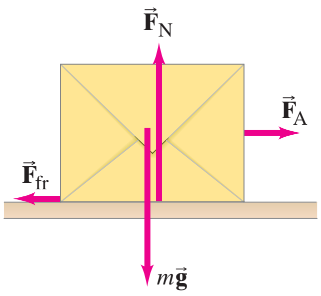
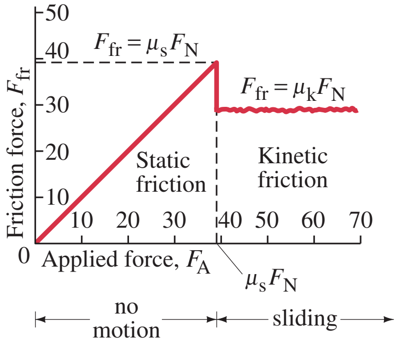


FIGURE 4–27 Repeated for Example 4–16.

FIGURE 4–28 Example 4–16. Magnitude of the force of friction as a function of the external force applied to an object initially at rest. As the applied force is increased in magnitude, the force of static friction increases in proportion until the applied force equals $\mu_s F_N$. If the applied force increases further, the object will begin to move, and the friction force drops to a roughly constant value characteristic of kinetic friction.



EXAMPLE 4–16 **Friction: static and kinetic.** Our 10.0-kg mystery box rests on a horizontal floor. The coefficient of static friction is $\mu_s = 0.40$ and the coefficient of kinetic friction is $\mu_k = 0.30$. Determine the force of friction, F_{fr}, acting on the box if a horizontal applied force F_A is exerted on it of magnitude: (*a*) 0, (*b*) 10 N, (*c*) 20 N, (*d*) 38 N, and (*e*) 40 N.

APPROACH We don't know, right off, if we are dealing with static friction or kinetic friction, nor if the box remains at rest or accelerates. We need to draw a free-body diagram, and then determine in each case whether or not the box will move: the box starts moving if F_A is greater than the maximum static friction force (Newton's second law). The forces on the box are gravity $m\vec{\mathbf{g}}$, the normal force exerted by the floor $\vec{\mathbf{F}}_N$, the horizontal applied force $\vec{\mathbf{F}}_A$, and the friction force $\vec{\mathbf{F}}_{fr}$, as shown in Fig. 4–27.

SOLUTION The free-body diagram of the box is shown in Fig. 4–27. In the vertical direction there is no motion, so Newton's second law in the vertical direction gives $\Sigma F_y = ma_y = 0$, which tells us $F_N - mg = 0$. Hence the normal force is

$$F_N = mg = (10.0\,\text{kg})(9.80\,\text{m/s}^2) = 98.0\,\text{N}.$$

(*a*) Because $F_A = 0$ in this first case, the box doesn't move, and $F_{fr} = 0$.

(*b*) The force of static friction will oppose any applied force up to a maximum of

$$\mu_s F_N = (0.40)(98.0\,\text{N}) = 39\,\text{N}.$$

When the applied force is $F_A = 10\,\text{N}$, the box will not move. Newton's second law gives $\Sigma F_x = F_A - F_{fr} = 0$, so $F_{fr} = 10\,\text{N}$.

(*c*) An applied force of 20 N is also not sufficient to move the box. Thus $F_{fr} = 20\,\text{N}$ to balance the applied force.

(*d*) The applied force of 38 N is still not quite large enough to move the box; so the friction force has now increased to 38 N to keep the box at rest.

(*e*) A force of 40 N will start the box moving since it exceeds the maximum force of static friction, $\mu_s F_N = (0.40)(98\,\text{N}) = 39\,\text{N}$. Instead of static friction, we now have kinetic friction, and its magnitude is

$$F_{fr} = \mu_k F_N = (0.30)(98.0\,\text{N}) = 29\,\text{N}.$$

There is now a net (horizontal) force on the box of magnitude $F = 40\,\text{N} - 29\,\text{N} = 11\,\text{N}$, so the box will accelerate at a rate

$$a_x = \frac{\Sigma F}{m} = \frac{11\,\text{N}}{10.0\,\text{kg}} = 1.1\,\text{m/s}^2$$

as long as the applied force is 40 N. Figure 4–28 shows a graph that summarizes this Example.

Friction can be a hindrance. It slows down moving objects and causes heating and binding of moving parts in machinery. Friction can be reduced by using lubricants such as oil. More effective in reducing friction between two surfaces is to maintain a layer of air or other gas between them. Devices using this concept, which is not practical for most situations, include air tracks and air tables in which the layer of air is maintained by forcing air through many tiny holes. Another technique to maintain the air layer is to suspend objects in air using magnetic fields ("magnetic levitation").

On the other hand, friction can be helpful. Our ability to walk depends on friction between the soles of our shoes (or feet) and the ground. (Walking involves static friction, not kinetic friction. Why?) The movement of a car, and also its stability, depend on friction. When friction is low, such as on ice, safe walking or driving becomes difficult.

CONCEPTUAL EXAMPLE 4–17 **A box against a wall.** You can hold a box against a rough wall (Fig. 4–29) and prevent it from slipping down by pressing hard horizontally. How does the application of a horizontal force keep an object from moving vertically?

RESPONSE This won't work well if the wall is slippery. You need friction. Even then, if you don't press hard enough, the box will slip. The horizontal force you apply produces a normal force on the box exerted by the wall (the net force horizontally is zero since the box doesn't move horizontally). The force of gravity mg, acting downward on the box, can now be balanced by an upward static friction force whose maximum magnitude is proportional to the normal force. The harder you push, the greater F_N is and the greater F_{fr} can be. If you don't press hard enough, then $mg > \mu_s F_N$ and the box begins to slide down.

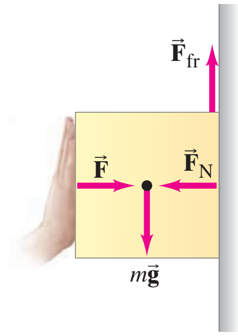


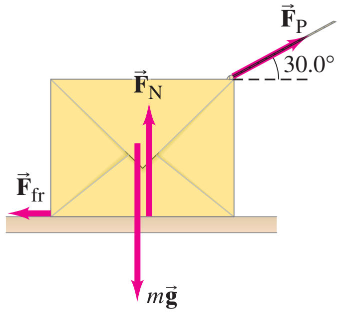
FIGURE 4–29 Example 4–17.

EXERCISE G If $\mu_s = 0.40$ and $mg = 20\,\text{N}$, what minimum force F will keep the box from falling: (*a*) 100 N; (*b*) 80 N; (*c*) 50 N; (*d*) 20 N; (*e*) 8 N?

EXAMPLE 4–18 **Pulling against friction.** A 10.0-kg box is pulled along a horizontal surface by a force F_P of 40.0 N applied at a 30.0° angle above horizontal. This is like Example 4–11 except now there is friction, and we assume a coefficient of kinetic friction of 0.30. Calculate the acceleration.

APPROACH The free-body diagram is shown in Fig. 4–30. It is much like that in Fig. 4–21b, but with one more force, friction.

FIGURE 4–30 Example 4–18.



SOLUTION The calculation for the vertical (y) direction is just the same as in Example 4–11b, $mg = (10.0\,\text{kg})(9.80\,\text{m/s}^2) = 98.0\,\text{N}$ and $F_{Py} = (40.0\,\text{N})(\sin 30.0°) = 20.0\,\text{N}$. With y positive upward and $a_y = 0$, we have

$$F_N - mg + F_{Py} = ma_y$$
$$F_N - 98.0\,\text{N} + 20.0\,\text{N} = 0,$$

so the normal force is $F_N = 78.0\,\text{N}$. Now we apply Newton's second law for the horizontal (x) direction (positive to the right), and include the friction force:

$$F_{Px} - F_{fr} = ma_x.$$

The friction force is kinetic friction as long as $F_{fr} = \mu_k F_N$ is less than $F_{Px} = (40.0\,\text{N})\cos 30.0° = 34.6\,\text{N}$, which it is:

$$F_{fr} = \mu_k F_N = (0.30)(78.0\,\text{N}) = 23.4\,\text{N}.$$

Hence the box does accelerate:

$$a_x = \frac{F_{Px} - F_{fr}}{m} = \frac{34.6\,\text{N} - 23.4\,\text{N}}{10.0\,\text{kg}} = 1.1\,\text{m/s}^2.$$

In the absence of friction, as we saw in Example 4–11, the acceleration would be much greater than this.

NOTE Our final answer has only two significant figures because our least significant input value $(\mu_k = 0.30)$ has two.

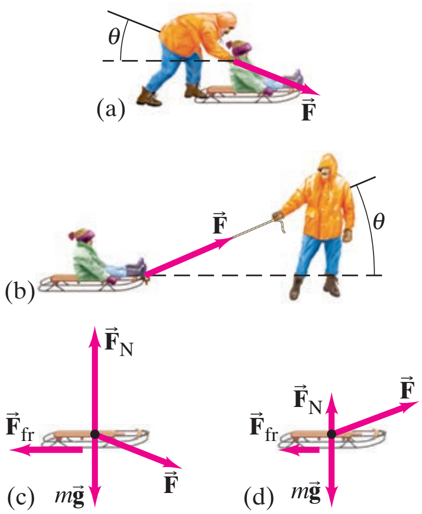
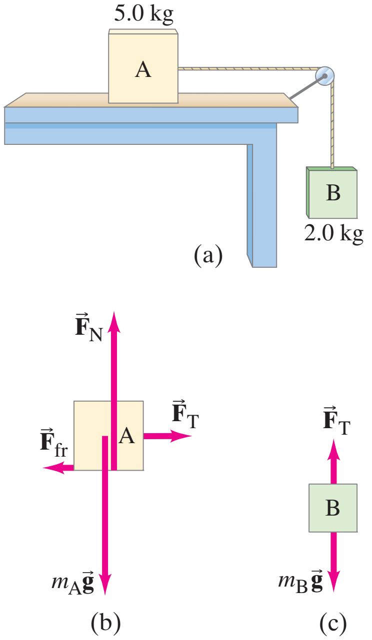


FIGURE 4–31 Example 4–19.

FIGURE 4–32 Example 4–20.



CONCEPTUAL EXAMPLE 4–19 **To push or to pull a sled?** Your little sister wants a ride on her sled. If you are on flat ground, will you exert less force if you push her or pull her? See Figs. 4–31a and b. Assume the same angle θ in each case.

RESPONSE Let us draw free-body diagrams for the sled–sister combination, as shown in Figs. 4–31c and d. They show, for the two cases, the forces exerted by you, $\vec{\mathbf{F}}$ (an unknown), by the snow, $\vec{\mathbf{F}}_{\mathrm{N}}$ and $\vec{\mathbf{F}}_{\mathrm{fr}}$, and gravity $m\vec{\mathbf{g}}$. (*a*) If you push her, and $\theta > 0$, there is a vertically downward component to your force. Hence the normal force upward exerted by the ground (Fig. 4–31c) will be larger than mg (where m is the mass of sister plus sled). (*b*) If you pull her, your force has a vertically upward component, so the normal force F_{N} will be less than mg, Fig. 4–31d. Because the friction force is proportional to the normal force, F_{fr} will be less if you pull her. So you exert less force if you pull her.

EXAMPLE 4–20 **Two boxes and a pulley.** In Fig. 4–32a, two boxes are connected by a cord running over a pulley. The coefficient of kinetic friction between box A and the table is 0.20. We ignore the mass of the cord and pulley and any friction in the pulley, which means we can assume that a force applied to one end of the cord will have the same magnitude at the other end. We wish to find the acceleration, a, of the system, which will have the same magnitude for both boxes assuming the cord doesn't stretch. As box B moves down, box A moves to the right.

APPROACH The free-body diagrams for each box are shown in Figs. 4–32b and c. The forces on box A are the pulling force of the cord F_{T}, gravity $m_{\mathrm{A}}g$, the normal force exerted by the table F_{N}, and a friction force exerted by the table F_{fr}; the forces on box B are gravity $m_{\mathrm{B}}g$, and the cord pulling up, F_{T}.

SOLUTION Box A does not move vertically, so Newton's second law tells us the normal force just balances the weight,

$$F_{\mathrm{N}} = m_{\mathrm{A}}g = (5.0\,\mathrm{kg})(9.8\,\mathrm{m/s^2}) = 49\,\mathrm{N}.$$

In the horizontal direction, there are two forces on box A (Fig. 4–32b): F_{T}, the tension in the cord (whose value we don't know), and the force of friction

$$F_{\mathrm{fr}} = \mu_{\mathrm{k}}F_{\mathrm{N}} = (0.20)(49\,\mathrm{N}) = 9.8\,\mathrm{N}.$$

The horizontal acceleration (box A) is what we wish to find; we use Newton's second law in the x direction, $\Sigma F_{\mathrm{A}x} = m_{\mathrm{A}}a_x$, which becomes (taking the positive direction to the right and setting $a_{\mathrm{A}x} = a$):

$$\Sigma F_{\mathrm{A}x} = F_{\mathrm{T}} - F_{\mathrm{fr}} = m_{\mathrm{A}}a. \qquad \text{[box A]}$$

Next consider box B. The force of gravity $m_{\mathrm{B}}g = (2.0\,\mathrm{kg})(9.8\,\mathrm{m/s^2}) = 19.6\,\mathrm{N}$ pulls downward; and the cord pulls upward with a force F_{T}. So we can write Newton's second law for box B (taking the downward direction as positive):

$$\Sigma F_{\mathrm{B}y} = m_{\mathrm{B}}g - F_{\mathrm{T}} = m_{\mathrm{B}}a. \qquad \text{[box B]}$$

[Notice that if $a \neq 0$, then F_{T} is not equal to $m_{\mathrm{B}}g$.]

We have two unknowns, a and F_{T}, and we also have two equations. We solve the box A equation for F_{T}:

$$F_{\mathrm{T}} = F_{\mathrm{fr}} + m_{\mathrm{A}}a,$$

and substitute this into the box B equation:

$$m_{\mathrm{B}}g - F_{\mathrm{fr}} - m_{\mathrm{A}}a = m_{\mathrm{B}}a.$$

Now we solve for a and put in numerical values:

$$a = \frac{m_{\mathrm{B}}g - F_{\mathrm{fr}}}{m_{\mathrm{A}} + m_{\mathrm{B}}} = \frac{19.6\,\mathrm{N} - 9.8\,\mathrm{N}}{5.0\,\mathrm{kg} + 2.0\,\mathrm{kg}} = 1.4\,\mathrm{m/s^2},$$

which is the acceleration of box A to the right, and of box B down.

If we wish, we can calculate F_{T} using the third equation up from here:

$$F_{\mathrm{T}} = F_{\mathrm{fr}} + m_{\mathrm{A}}a = 9.8\,\mathrm{N} + (5.0\,\mathrm{kg})(1.4\,\mathrm{m/s^2}) = 17\,\mathrm{N}.$$

NOTE Box B is not in free fall. It does not fall at $a = g$ because an additional force, F_{T}, is acting upward on it.

CAUTION

Tension in a cord supporting a falling object may not equal object's weight

Inclines

Now we consider what happens when an object slides down an incline, such as a hill or ramp. Such problems are interesting because gravity is the accelerating force, yet the acceleration is not vertical. Solving problems is usually easier if we choose the xy coordinate system so the x axis points along the incline (the direction of motion) and the y axis is perpendicular to the incline, as shown in Fig. 4–33. Note also that the normal force is not vertical, but is perpendicular to the sloping surface of the plane, along the y axis in Fig. 4–33.

PROBLEM SOLVING
Good choice of coordinate system simplifies the calculation

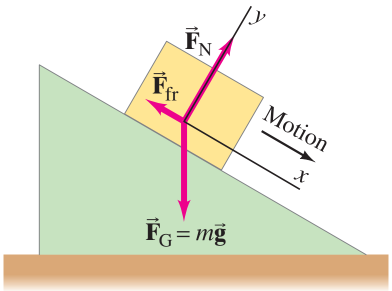


FIGURE 4–33 Forces on an object sliding down an incline.

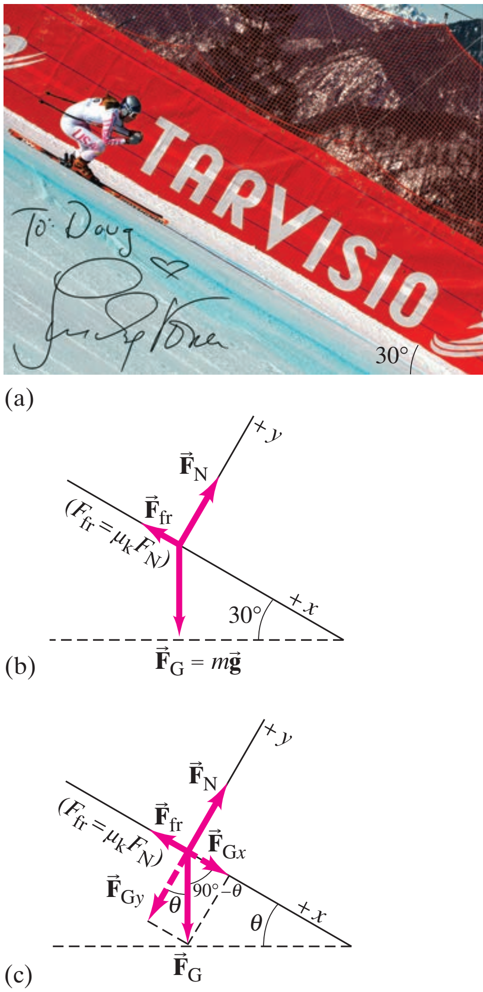
EXERCISE H Is the normal force always perpendicular to an inclined plane? Is it always vertical?

EXAMPLE 4–21 **The skier.** The skier in Fig. 4–34a has begun descending the 30° slope. If the coefficient of kinetic friction is 0.10, what is her acceleration?

PHYSICS APPLIED
Skiing

APPROACH We choose the x axis along the slope, positive downslope in the direction of the skier's motion. The y axis is perpendicular to the surface. The forces acting on the skier are gravity, $\vec{\mathbf{F}}_{\mathrm{G}} = m\vec{\mathbf{g}}$, which points vertically downward (*not* perpendicular to the slope), and the two forces exerted on her skis by the snow—the normal force perpendicular to the snowy slope (*not* vertical), and the friction force parallel to the surface. These three forces are shown acting at one point in Fig. 4–34b, which is our free-body diagram for the skier.

FIGURE 4–34 Example 4–21. Skier descending a slope; $\vec{\mathbf{F}}_{\mathrm{G}} = m\vec{\mathbf{g}}$ is the force of gravity (weight) on the skier.



SOLUTION We have to resolve only one vector into components, the weight $\vec{\mathbf{F}}_{\mathrm{G}}$, and its components are shown as dashed lines in Fig. 4–34c. To be general, we use θ rather than 30° for now. We use the definitions of sine ("side opposite") and cosine ("side adjacent") to obtain the components:

$$F_{\mathrm{G}x} = mg\sin\theta,$$
$$F_{\mathrm{G}y} = -mg\cos\theta$$

where $F_{\mathrm{G}y}$ is in the negative y direction. To calculate the skier's acceleration down the hill, a_x, we apply Newton's second law to the x direction:

$$\Sigma F_x = ma_x$$
$$mg\sin\theta - \mu_{\mathrm{k}} F_{\mathrm{N}} = ma_x$$

where the two forces are the x component of the gravity force ($+x$ direction) and the friction force ($-x$ direction). We want to find the value of a_x, but we don't yet know F_{N} in the last equation. Let's see if we can get F_{N} from the y component of Newton's second law:

$$\Sigma F_y = ma_y$$
$$F_{\mathrm{N}} - mg\cos\theta = ma_y = 0$$

where we set $a_y = 0$ because there is no motion in the y direction (perpendicular to the slope). Thus we can solve for F_{N}:

$$F_{\mathrm{N}} = mg\cos\theta$$

and we can substitute this into our equation above for ma_x:

$$mg\sin\theta - \mu_{\mathrm{k}}(mg\cos\theta) = ma_x.$$

There is an m in each term which can be canceled out. Thus (setting $\theta = 30°$ and $\mu_{\mathrm{k}} = 0.10$):

$$a_x = g\sin 30° - \mu_{\mathrm{k}}\, g\cos 30°$$
$$= 0.50g - (0.10)(0.866)g = 0.41g.$$

The skier's acceleration is 0.41 times the acceleration of gravity, which in numbers[†] is $a = (0.41)(9.8\,\mathrm{m/s^2}) = 4.0\,\mathrm{m/s^2}$.

NOTE The mass canceled out, so we have the useful conclusion that *the acceleration doesn't depend on the mass*. That such a cancellation sometimes occurs, and thus may give a useful conclusion as well as saving calculation, is a big advantage of working with the algebraic equations and putting in the numbers only at the end.

PROBLEM SOLVING
It is often helpful to put in numbers only at the end

[†]We used values rounded off to 2 significant figures to obtain $a = 4.0\,\mathrm{m/s^2}$. If we kept all the extra digits in our calculator, we would find $a = 0.4134g \approx 4.1\,\mathrm{m/s^2}$. This difference is within the expected precision (number of significant figures, Section 1–4).

CAUTION
Directions of gravity and the normal force

In Problems involving a slope or an "inclined plane," avoid making errors in the directions of the normal force and gravity. The normal force on an incline is *not* vertical: it is perpendicular to the slope or plane. And gravity is *not* perpendicular to the slope—gravity acts vertically downward toward the center of the Earth.

Summary

Newton's three laws of motion are the basic classical laws describing motion.

Newton's first law (the **law of inertia**) states that if the net force on an object is zero, an object originally at rest remains at rest, and an object in motion remains in motion in a straight line with constant velocity.

Newton's second law states that the acceleration of an object is directly proportional to the net force acting on it, and inversely proportional to its mass:

$$\Sigma \vec{\mathbf{F}} = m\vec{\mathbf{a}}. \qquad \textbf{(4–1)}$$

Newton's second law is one of the most important and fundamental laws in classical physics.

Newton's third law states that whenever one object exerts a force on a second object, the second object always exerts a force on the first object which is equal in magnitude but opposite in direction:

$$\vec{\mathbf{F}}_{AB} = -\vec{\mathbf{F}}_{BA} \qquad \textbf{(4–2)}$$

where $\vec{\mathbf{F}}_{BA}$ is the force on object B exerted by object A.

The tendency of an object to resist a change in its motion is called **inertia**. **Mass** is a measure of the inertia of an object.

Weight refers to the **gravitational force** on an object, and is equal to the product of the object's mass m and the acceleration of gravity $\vec{\mathbf{g}}$:

$$\vec{\mathbf{F}}_G = m\vec{\mathbf{g}}. \qquad \textbf{(4–3)}$$

Force, which is a vector, can be considered as a push or pull; or, from Newton's second law, force can be defined as an action capable of giving rise to acceleration. The **net force** on an object is the vector sum of all forces acting on that object.

When two objects slide over one another, the force of friction that each object exerts on the other can be written approximately as $F_{fr} = \mu_k F_N$, where F_N is the **normal force** (the force each object exerts on the other perpendicular to their contact surfaces), and μ_k is the coefficient of **kinetic friction**. If the objects are at rest relative to each other, then F_{fr} is just large enough to hold them at rest and satisfies the inequality $F_{fr} < \mu_s F_N$, where μ_s is the coefficient of **static friction**.

For solving problems involving the forces on one or more objects, it is essential to draw a **free-body diagram** for each object, showing all the forces acting on only that object. Newton's second law can be applied to the vector components for each object.

Questions

1. Why does a child in a wagon seem to fall backward when you give the wagon a sharp pull forward?
2. A box rests on the (frictionless) bed of a truck. The truck driver starts the truck and accelerates forward. The box immediately starts to slide toward the rear of the truck bed. Discuss the motion of the box, in terms of Newton's laws, as seen (*a*) by Mary standing on the ground beside the truck, and (*b*) by Chris who is riding on the truck (Fig. 4–35).

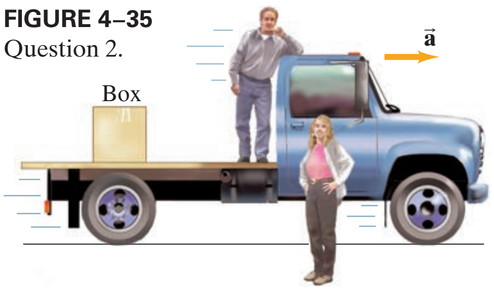


FIGURE 4–35 Question 2.

3. If an object is moving, is it possible for the net force acting on it to be zero? Explain.
4. If the acceleration of an object is zero, are no forces acting on it? Explain.
5. Only one force acts on an object. Can the object have zero acceleration? Can it have zero velocity? Explain.
6. When a golf ball is dropped to the pavement, it bounces back up. (*a*) Is a force needed to make it bounce back up? (*b*) If so, what exerts the force?
7. If you walk along a log floating on a lake, why does the log move in the opposite direction?
8. (*a*) Why do you push down harder on the pedals of a bicycle when first starting out than when moving at constant speed? (*b*) Why do you need to pedal at all when cycling at constant speed?
9. A stone hangs by a fine thread from the ceiling, and a section of the same thread dangles from the bottom of the stone (Fig. 4–36). If a person gives a sharp pull on the dangling thread, where is the thread likely to break: below the stone or above it? What if the person gives a slow and steady pull? Explain your answers.

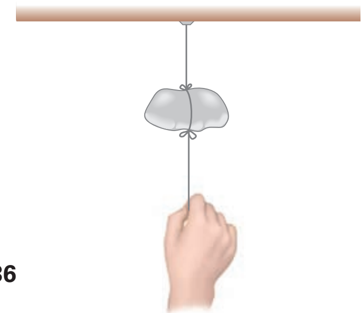


FIGURE 4–36 Question 9.

10. The force of gravity on a 2-kg rock is twice as great as that on a 1-kg rock. Why then doesn't the heavier rock fall faster?

11. (*a*) You pull a box with a constant force across a frictionless table using an attached rope held horizontally. If you now pull the rope with the same force at an angle to the horizontal (with the box remaining flat on the table), does the acceleration of the box increase, decrease, or remain the same? Explain. (*b*) What if there is friction?

12. When an object falls freely under the influence of gravity there is a net force mg exerted on it by the Earth. Yet by Newton's third law the object exerts an equal and opposite force on the Earth. Does the Earth move? Explain.

13. Compare the effort (or force) needed to lift a 10-kg object when you are on the Moon with the force needed to lift it on Earth. Compare the force needed to throw a 2-kg object horizontally with a given speed on the Moon and on Earth.

14. According to Newton's third law, each team in a tug of war (Fig. 4–37) pulls with equal force on the other team. What, then, determines which team will win?

FIGURE 4–37 Question 14. A tug of war. Describe the forces on each of the teams and on the rope.

15. When you stand still on the ground, how large a force does the ground exert on you? Why doesn't this force make you rise up into the air?

16. Whiplash sometimes results from an automobile accident when the victim's car is struck violently from the rear. Explain why the head of the victim seems to be thrown backward in this situation. Is it really?

17. Mary exerts an upward force of 40 N to hold a bag of groceries. Describe the "reaction" force (Newton's third law) by stating (*a*) its magnitude, (*b*) its direction, (*c*) *on* what object it is exerted, and (*d*) *by* what object it is exerted.

18. A father and his young daughter are ice skating. They face each other at rest and push each other, moving in opposite directions. Which one has the greater final speed? Explain.

19. A heavy crate rests on the bed of a flatbed truck. When the truck accelerates, the crate stays fixed on the truck, so it, too, accelerates. What force causes the crate to accelerate?

20. A block is given a brief push so that it slides up a ramp. After the block reaches its highest point, it slides back down, but the magnitude of its acceleration is less on the descent than on the ascent. Why?

21. Why is the stopping distance of a truck much shorter than for a train going the same speed?

22. What would your bathroom scale read if you weighed yourself on an inclined plane? Assume the mechanism functions properly, even at an angle.

MisConceptual Questions

1. A truck is traveling horizontally to the right (Fig. 4–38). When the truck starts to slow down, the crate on the (frictionless) truck bed starts to slide. In what direction could the net force be on the crate?
(*a*) No direction. The net force is zero.
(*b*) Straight down (because of gravity).
(*c*) Straight up (the normal force).
(*d*) Horizontal and to the right.
(*e*) Horizontal and to the left.

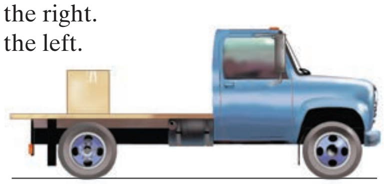


FIGURE 4–38
MisConceptual Question 1.

2. You are trying to push your stalled car. Although you apply a horizontal force of 400 N to the car, it doesn't budge, and neither do you. Which force(s) must also have a magnitude of 400 N?
(*a*) The force exerted by the car on you.
(*b*) The friction force exerted by the car on the road.
(*c*) The normal force exerted by the road on you.
(*d*) The friction force exerted by the road on you.

3. Matt, in the foreground of Fig. 4–39, is able to move the large truck because
(*a*) he is stronger than the truck.
(*b*) he is heavier in some respects than the truck.
(*c*) he exerts a greater force on the truck than the truck exerts back on him.
(*d*) the ground exerts a greater friction force on Matt than it does on the truck.
(*e*) the truck offers no resistance because its brakes are off.

FIGURE 4–39 MisConceptual Question 3.

4. A bear sling, Fig. 4–40, is used in some national parks for placing backpackers' food out of the reach of bears. As the backpacker raises the pack by pulling down on the rope, the force F needed:
(*a*) decreases as the pack rises until the rope is straight across.
(*b*) doesn't change.
(*c*) increases until the rope is straight.
(*d*) increases but the rope always sags where the pack hangs.

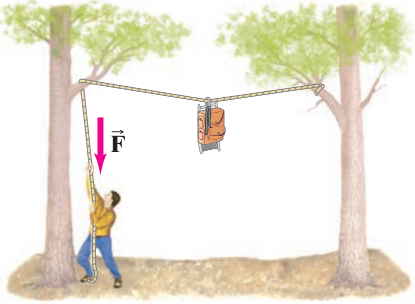


FIGURE 4–40 MisConceptual Question 4.

5. What causes the boat in Fig. 4–41 to move forward?
(*a*) The force the man exerts on the paddle.
(*b*) The force the paddle exerts on the water.
(*c*) The force the water exerts on the paddle.
(*d*) The motion of the water itself.

FIGURE 4–41 MisConceptual Question 5.

6. A person stands on a scale in an elevator. His apparent weight will be the greatest when the elevator
(*a*) is standing still.
(*b*) is moving upward at constant velocity.
(*c*) is accelerating upward.
(*d*) is moving downward at constant velocity.
(*e*) is accelerating downward.

7. When a skier skis down a hill, the normal force exerted on the skier by the hill is
(*a*) equal to the weight of the skier.
(*b*) greater than the weight of the skier.
(*c*) less than the weight of the skier.

8. A golf ball is hit with a golf club. While the ball flies through the air, which forces act on the ball? Neglect air resistance.
(*a*) The force of the golf club acting on the ball.
(*b*) The force of gravity acting on the ball.
(*c*) The force of the ball moving forward through the air.
(*d*) All of the above.
(*e*) Both (*a*) and (*c*).

9. Suppose an object is accelerated by a force of 100 N. Suddenly a second force of 100 N in the opposite direction is exerted on the object, so that the forces cancel. The object
(*a*) is brought to rest rapidly.
(*b*) decelerates gradually to rest.
(*c*) continues at the velocity it had before the second force was applied.
(*d*) is brought to rest and then accelerates in the direction of the second force.

10. You are pushing a heavy box across a rough floor. When you are initially pushing the box and it is accelerating,
(*a*) you exert a force on the box, but the box does not exert a force on you.
(*b*) the box is so heavy it exerts a force on you, but you do not exert a force on the box.
(*c*) the force you exert on the box is greater than the force of the box pushing back on you.
(*d*) the force you exert on the box is equal to the force of the box pushing back on you.
(*e*) the force that the box exerts on you is greater than the force you exert on the box.

11. A 50-N crate sits on a horizontal floor where the coefficient of static friction between the crate and the floor is 0.50. A 20-N force is applied to the crate acting to the right. What is the resulting static friction force acting on the crate?
(*a*) 20 N to the right.
(*b*) 20 N to the left.
(*c*) 25 N to the right.
(*d*) 25 N to the left.
(*e*) None of the above; the crate starts to move.

12. The normal force on an extreme skier descending a very steep slope (Fig. 4–42) can be zero if
(*a*) his speed is great enough.
(*b*) he leaves the slope (no longer touches the snow).
(*c*) the slope is greater than 75°.
(*d*) the slope is vertical (90°).

FIGURE 4–42 MisConceptual Question 12.

13. To pull an old stump out of the ground, you and a friend tie two ropes to the stump. You pull on it with a force of 500 N to the north while your friend pulls with a force of 450 N to the northwest. The total force from the two ropes is
(*a*) less than 950 N.
(*b*) exactly 950 N.
(*c*) more than 950 N.

For assigned homework and other learning materials, go to the MasteringPhysics website.

Problems

[It would be wise, before starting the Problems, to reread the Problem Solving Strategies on pages 30, 60, and 88.]

4–4 to 4–6 Newton's Laws, Gravitational Force, Normal Force [Assume no friction.]

1. (I) What force is needed to accelerate a sled (mass = 55 kg) at 1.4 m/s^2 on horizontal frictionless ice?

2. (I) What is the weight of a 68-kg astronaut (*a*) on Earth, (*b*) on the Moon ($g = 1.7\ m/s^2$), (*c*) on Mars ($g = 3.7\ m/s^2$), (*d*) in outer space traveling with constant velocity?

3. (I) How much tension must a rope withstand if it is used to accelerate a 1210-kg car horizontally along a frictionless surface at 1.20 m/s^2?

4. (II) According to a simplified model of a mammalian heart, at each pulse approximately 20 g of blood is accelerated from 0.25 m/s to 0.35 m/s during a period of 0.10 s. What is the magnitude of the force exerted by the heart muscle?

5. (II) Superman must stop a 120-km/h train in 150 m to keep it from hitting a stalled car on the tracks. If the train's mass is 3.6×10^5 kg, how much force must he exert? Compare to the weight of the train (give as %). How much force does the train exert on Superman?

6. (II) A person has a reasonable chance of surviving an automobile crash if the deceleration is no more than 30 g's. Calculate the force on a 65-kg person accelerating at this rate. What distance is traveled if brought to rest at this rate from 95 km/h?

7. (II) What average force is required to stop a 950-kg car in 8.0 s if the car is traveling at 95 km/h?

8. (II) Estimate the average force exerted by a shot-putter on a 7.0-kg shot if the shot is moved through a distance of 2.8 m and is released with a speed of 13 m/s.

9. (II) A 0.140-kg baseball traveling 35.0 m/s strikes the catcher's mitt, which, in bringing the ball to rest, recoils backward 11.0 cm. What was the average force applied by the ball on the glove?

10. (II) How much tension must a cable withstand if it is used to accelerate a 1200-kg car vertically upward at 0.70 m/s^2?

11. (II) A 20.0-kg box rests on a table. (*a*) What is the weight of the box and the normal force acting on it? (*b*) A 10.0-kg box is placed on top of the 20.0-kg box, as shown in Fig. 4–43. Determine the normal force that the table exerts on the 20.0-kg box and the normal force that the 20.0-kg box exerts on the 10.0-kg box.

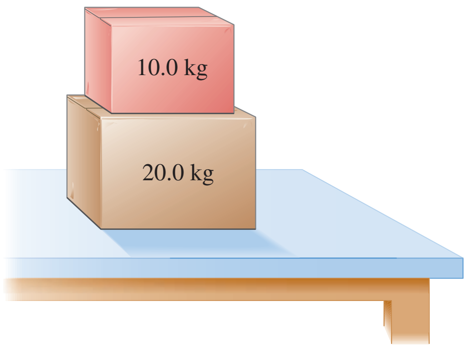


FIGURE 4–43 Problem 11.

12. (II) A 14.0-kg bucket is lowered vertically by a rope in which there is 163 N of tension at a given instant. What is the acceleration of the bucket? Is it up or down?

13. (II) A 75-kg petty thief wants to escape from a third-story jail window. Unfortunately, a makeshift rope made of sheets tied together can support a mass of only 58 kg. How might the thief use this "rope" to escape? Give a quantitative answer.

14. (II) An elevator (mass 4850 kg) is to be designed so that the maximum acceleration is 0.0680g. What are the maximum and minimum forces the motor should exert on the supporting cable?

15. (II) Can cars "stop on a dime"? Calculate the acceleration of a 1400-kg car if it can stop from 35 km/h on a dime (diameter = 1.7 cm). How many g's is this? What is the force felt by the 68-kg occupant of the car?

16. (II) A woman stands on a bathroom scale in a motionless elevator. When the elevator begins to move, the scale briefly reads only 0.75 of her regular weight. Calculate the acceleration of the elevator, and find the direction of acceleration.

17. (II) (*a*) What is the acceleration of two falling sky divers (total mass = 132 kg including parachute) when the upward force of air resistance is equal to one-fourth of their weight? (*b*) After opening the parachute, the divers descend leisurely to the ground at constant speed. What now is the force of air resistance on the sky divers and their parachute? See Fig. 4–44.

FIGURE 4–44 Problem 17.

18. (II) The cable supporting a 2125-kg elevator has a maximum strength of 21,750 N. What maximum upward acceleration can it give the elevator without breaking?

19. (III) A person jumps from the roof of a house 2.8 m high. When he strikes the ground below, he bends his knees so that his torso decelerates over an approximate distance of 0.70 m. If the mass of his torso (excluding legs) is 42 kg, find (*a*) his velocity just before his feet strike the ground, and (*b*) the average force exerted on his torso by his legs during deceleration.

4–7 Newton's Laws and Vectors [Ignore friction.]

20. (I) A box weighing 77.0 N rests on a table. A rope tied to the box runs vertically upward over a pulley and a weight is hung from the other end (Fig. 4–45). Determine the force that the table exerts on the box if the weight hanging on the other side of the pulley weighs (*a*) 30.0 N, (*b*) 60.0 N, and (*c*) 90.0 N.

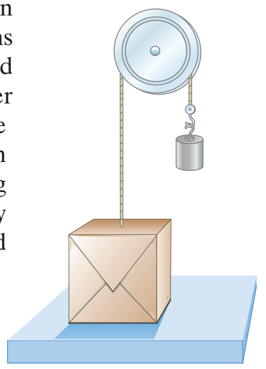


FIGURE 4–45 Problem 20.

21. (I) Draw the free-body diagram for a basketball player (*a*) just before leaving the ground on a jump, and (*b*) while in the air. See Fig. 4–46.

FIGURE 4–46 Problem 21.

22. (I) Sketch the free-body diagram of a baseball (*a*) at the moment it is hit by the bat, and again (*b*) after it has left the bat and is flying toward the outfield. Ignore air resistance.

23. (II) Arlene is to walk across a "high wire" strung horizontally between two buildings 10.0 m apart. The sag in the rope when she is at the midpoint is 10.0°, as shown in Fig. 4–47. If her mass is 50.0 kg, what is the tension in the rope at this point?

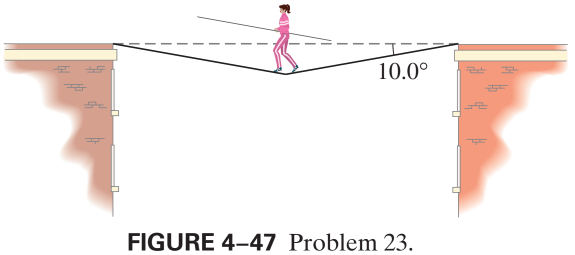


FIGURE 4–47 Problem 23.

24. (II) A window washer pulls herself upward using the bucket–pulley apparatus shown in Fig. 4–48. (*a*) How hard must she pull downward to raise herself slowly at constant speed? (*b*) If she increases this force by 15%, what will her acceleration be? The mass of the person plus the bucket is 72 kg.

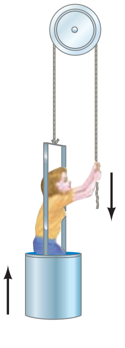


FIGURE 4–48 Problem 24.

25. (II) One 3.2-kg paint bucket is hanging by a massless cord from another 3.2-kg paint bucket, also hanging by a massless cord, as shown in Fig. 4–49. (*a*) If the buckets are at rest, what is the tension in each cord? (*b*) If the two buckets are pulled upward with an acceleration of 1.25 m/s^2 by the upper cord, calculate the tension in each cord.

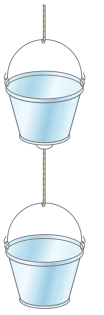


FIGURE 4–49 Problem 25.

26. (II) Two snowcats in Antarctica are towing a housing unit north, as shown in Fig. 4–50. The sum of the forces $\vec{\mathbf{F}}_A$ and $\vec{\mathbf{F}}_B$ exerted on the unit by the horizontal cables is north, parallel to the line L, and $F_A = 4500$ N. Determine F_B and the magnitude of $\vec{\mathbf{F}}_A + \vec{\mathbf{F}}_B$.

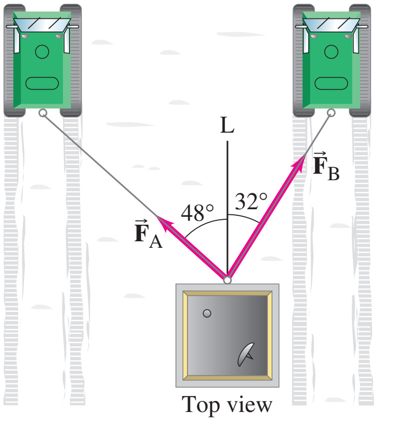


FIGURE 4–50 Problem 26.

27. (II) A train locomotive is pulling two cars of the same mass behind it, Fig. 4–51. Determine the ratio of the tension in the coupling (think of it as a cord) between the locomotive and the first car (F_{T1}), to that between the first car and the second car (F_{T2}), for any nonzero acceleration of the train.

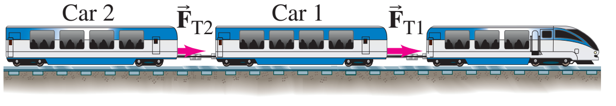


FIGURE 4–51 Problem 27.

28. (II) The two forces $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ shown in Fig. 4–52a and b (looking down) act on an 18.5-kg object on a frictionless tabletop. If $F_1 = 10.2$ N and $F_2 = 16.0$ N, find the net force on the object and its acceleration for (a) and (b).

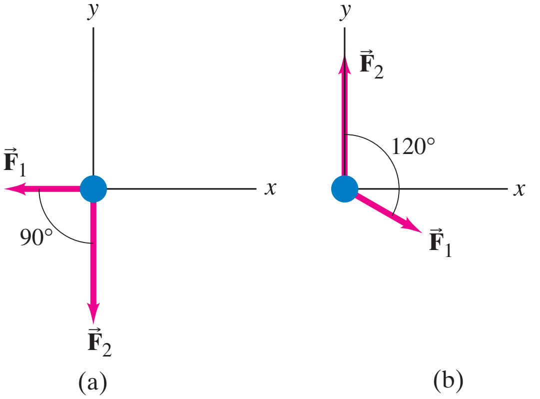


FIGURE 4–52 Problem 28.

29. (II) At the instant a race began, a 65-kg sprinter exerted a force of 720 N on the starting block at a 22° angle with respect to the ground. (*a*) What was the horizontal acceleration of the sprinter? (*b*) If the force was exerted for 0.32 s, with what speed did the sprinter leave the starting block?

30. (II) An object is hanging by a string from your rearview mirror. While you are decelerating at a constant rate from 25 m/s to rest in 6.0 s, (*a*) what angle does the string make with the vertical, and (*b*) is it toward the windshield or away from it? [*Hint*: See Example 4–15.]

31. (II) Figure 4–53 shows a block (mass m_A) on a smooth horizontal surface, connected by a thin cord that passes over a pulley to a second block (m_B), which hangs vertically. (*a*) Draw a free-body diagram for each block, showing the force of gravity on each, the force (tension) exerted by the cord, and any normal force. (*b*) Apply Newton's second law to find formulas for the acceleration of the system and for the tension in the cord. Ignore friction and the masses of the pulley and cord.

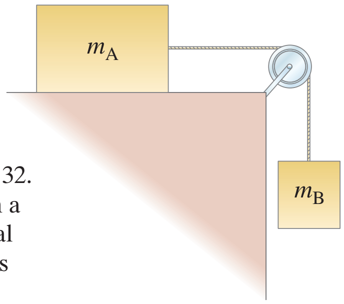


FIGURE 4–53 Problems 31 and 32. Mass m_A rests on a smooth horizontal surface; m_B hangs vertically.

32. (II) (*a*) If $m_A = 13.0\,\text{kg}$ and $m_B = 5.0\,\text{kg}$ in Fig. 4–53, determine the acceleration of each block. (*b*) If initially m_A is at rest 1.250 m from the edge of the table, how long does it take to reach the edge of the table if the system is allowed to move freely? (*c*) If $m_B = 1.0\,\text{kg}$, how large must m_A be if the acceleration of the system is to be kept at $\frac{1}{100}\,g$?

33. (III) Three blocks on a frictionless horizontal surface are in contact with each other as shown in Fig. 4–54. A force $\vec{\mathbf{F}}$ is applied to block A (mass m_A). (*a*) Draw a free-body diagram for each block. Determine (*b*) the acceleration of the system (in terms of m_A, m_B, and m_C), (*c*) the net force on each block, and (*d*) the force of contact that each block exerts on its neighbor. (*e*) If $m_A = m_B = m_C = 10.0\,\text{kg}$ and $F = 96.0\,\text{N}$, give numerical answers to (*b*), (*c*), and (*d*). Explain how your answers make sense intuitively.

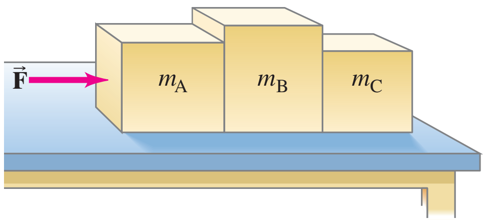


FIGURE 4–54 Problem 33.

34. (III) Suppose the pulley in Fig. 4–55 is suspended by a cord C. Determine the tension in this cord after the masses are released and before one hits the ground. Ignore the mass of the pulley and cords.

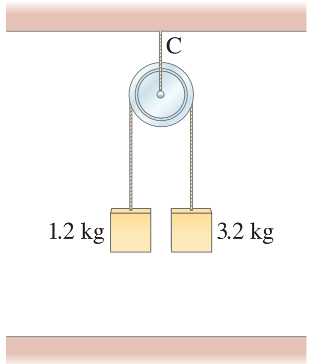


FIGURE 4–55 Problem 34.

4–8 Newton's Laws with Friction, Inclines

35. (I) If the coefficient of kinetic friction between a 22-kg crate and the floor is 0.30, what horizontal force is required to move the crate at a steady speed across the floor? What horizontal force is required if μ_k is zero?

36. (I) A force of 35.0 N is required to start a 6.0-kg box moving across a horizontal concrete floor. (*a*) What is the coefficient of static friction between the box and the floor? (*b*) If the 35.0-N force continues, the box accelerates at $0.60\,\text{m/s}^2$. What is the coefficient of kinetic friction?

37. (I) Suppose you are standing on a train accelerating at $0.20\,g$. What minimum coefficient of static friction must exist between your feet and the floor if you are not to slide?

38. (II) The coefficient of static friction between hard rubber and normal street pavement is about 0.90. On how steep a hill (maximum angle) can you leave a car parked?

39. (II) A flatbed truck is carrying a heavy crate. The coefficient of static friction between the crate and the bed of the truck is 0.75. What is the maximum rate at which the driver can decelerate and still avoid having the crate slide against the cab of the truck?

40. (II) A 2.0-kg silverware drawer does not slide readily. The owner gradually pulls with more and more force, and when the applied force reaches 9.0 N, the drawer suddenly opens, throwing all the utensils to the floor. What is the coefficient of static friction between the drawer and the cabinet?

41. (II) A box is given a push so that it slides across the floor. How far will it go, given that the coefficient of kinetic friction is 0.15 and the push imparts an initial speed of 3.5 m/s?

42. (II) A 1280-kg car pulls a 350-kg trailer. The car exerts a horizontal force of 3.6×10^3 N against the ground in order to accelerate. What force does the car exert on the trailer? Assume an effective friction coefficient of 0.15 for the trailer.

43. (II) Drag-race tires in contact with an asphalt surface have a very high coefficient of static friction. Assuming a constant acceleration and no slipping of tires, estimate the coefficient of static friction needed for a drag racer to cover 1.0 km in 12 s, starting from rest.

44. (II) For the system of Fig. 4–32 (Example 4–20), how large a mass would box A have to have to prevent any motion from occurring? Assume $\mu_s = 0.30$.

45. (II) In Fig. 4–56 the coefficient of static friction between mass m_A and the table is 0.40, whereas the coefficient of kinetic friction is 0.20. (*a*) What minimum value of m_A will keep the system from starting to move? (*b*) What value(s) of m_A will keep the system moving at constant speed? [Ignore masses of the cord and the (frictionless) pulley.]

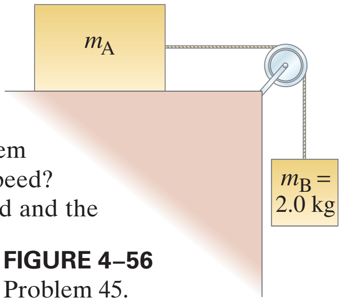


FIGURE 4–56 Problem 45.

46. (II) A small box is held in place against a rough vertical wall by someone pushing on it with a force directed upward at 28° above the horizontal. The coefficients of static and kinetic friction between the box and wall are 0.40 and 0.30, respectively. The box slides down unless the applied force has magnitude 23 N. What is the mass of the box?

47. (II) Two crates, of mass 65 kg and 125 kg, are in contact and at rest on a horizontal surface (Fig. 4–57). A 650-N force is exerted on the 65-kg crate. If the coefficient of kinetic friction is 0.18, calculate (*a*) the acceleration of the system, and (*b*) the force that each crate exerts on the other. (*c*) Repeat with the crates reversed.

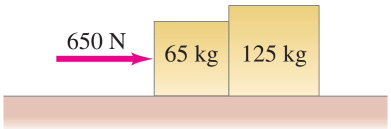


FIGURE 4–57 Problem 47.

48. (II) A person pushes a 14.0-kg lawn mower at constant speed with a force of $F = 88.0$ N directed along the handle, which is at an angle of 45.0° to the horizontal (Fig. 4–58). (*a*) Draw the free-body diagram showing all forces acting on the mower. Calculate (*b*) the horizontal friction force on the mower, then (*c*) the normal force exerted vertically upward on the mower by the ground. (*d*) What force must the person exert on the lawn mower to accelerate it from rest to 1.5 m/s in 2.5 seconds, assuming the same friction force?

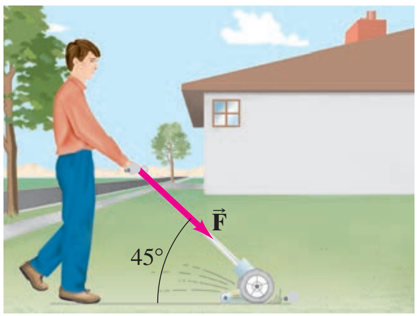


FIGURE 4–58 Problem 48.

49. (II) A wet bar of soap slides down a ramp 9.0 m long inclined at 8.0°. How long does it take to reach the bottom? Assume $\mu_k = 0.060$.

50. (II) A skateboarder, with an initial speed of 2.0 m/s, rolls virtually friction free down a straight incline of length 18 m in 3.3 s. At what angle θ is the incline oriented above the horizontal?

51. (II) Uphill escape ramps are sometimes provided to the side of steep downhill highways for trucks with overheated brakes. For a simple 11° upward ramp, what minimum length would be needed for a runaway truck traveling 140 km/h? Note the large size of your calculated length. (If sand is used for the bed of the ramp, its length can be reduced by a factor of about 2.)

52. (II) The block shown in Fig. 4–59 has mass $m = 7.0$ kg and lies on a fixed smooth frictionless plane tilted at an angle $\theta = 22.0°$ to the horizontal. (*a*) Determine the acceleration of the block as it slides down the plane. (*b*) If the block starts from rest 12.0 m up the plane from its base, what will be the block's speed when it reaches the bottom of the incline?

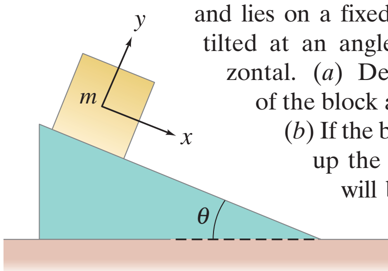


FIGURE 4–59 Block on inclined plane. Problems 52 and 53.

53. (II) A block is given an initial speed of 4.5 m/s up the 22.0° plane shown in Fig. 4–59. (*a*) How far up the plane will it go? (*b*) How much time elapses before it returns to its starting point? Ignore friction.

54. (II) The crate shown in Fig. 4–60 lies on a plane tilted at an angle $\theta = 25.0°$ to the horizontal, with $\mu_k = 0.19$. (*a*) Determine the acceleration of the crate as it slides down the plane. (*b*) If the crate starts from rest 8.15 m up along the plane from its base, what will be the crate's speed when it reaches the bottom of the incline?

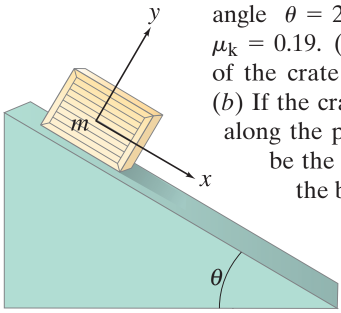


FIGURE 4–60 Crate on inclined plane. Problems 54 and 55.

55. (II) A crate is given an initial speed of 3.0 m/s up the 25.0° plane shown in Fig. 4–60. (*a*) How far up the plane will it go? (*b*) How much time elapses before it returns to its starting point? Assume $\mu_k = 0.12$.

56. (II) A car can decelerate at $-3.80\ \mathrm{m/s^2}$ without skidding when coming to rest on a level road. What would its deceleration be if the road is inclined at 9.3° and the car moves uphill? Assume the same static friction coefficient.

57. (II) A skier moves down a 12° slope at constant speed. What can you say about the coefficient of friction, μ_k? Assume the speed is low enough that air resistance can be ignored.

58. (II) The coefficient of kinetic friction for a 22-kg bobsled on a track is 0.10. What force is required to push it down along a 6.0° incline and achieve a speed of 60 km/h at the end of 75 m?

59. (III) A child slides down a slide with a 34° incline, and at the bottom her speed is precisely half what it would have been if the slide had been frictionless. Calculate the coefficient of kinetic friction between the slide and the child.

60. (III) Two masses $m_A = 2.0\,\text{kg}$ and $m_B = 5.0\,\text{kg}$ are on inclines and are connected together by a string as shown in Fig. 4–61. The coefficient of kinetic friction between each mass and its incline is $\mu_k = 0.30$. If m_A moves up, and m_B moves down, determine their acceleration. [Ignore masses of the (frictionless) pulley and the cord.]

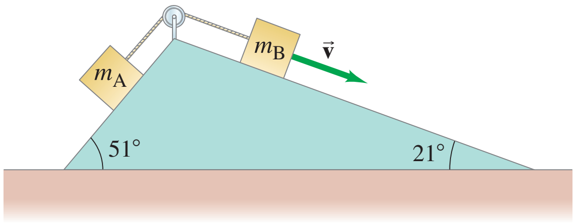


FIGURE 4–61 Problem 60.

61. (III) (*a*) Suppose the coefficient of kinetic friction between m_A and the plane in Fig. 4–62 is $\mu_k = 0.15$, and that $m_A = m_B = 2.7\,\text{kg}$. As m_B moves down, determine the magnitude of the acceleration of m_A and m_B, given $\theta = 34°$. (*b*) What smallest value of μ_k will keep the system from accelerating? [Ignore masses of the (frictionless) pulley and the cord.]

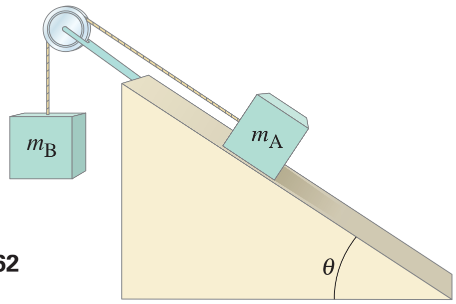


FIGURE 4–62 Problem 61.

General Problems

62. A 2.0-kg purse is dropped from the top of the Leaning Tower of Pisa and falls 55 m before reaching the ground with a speed of 27 m/s. What was the average force of air resistance?

63. A crane's trolley at point P in Fig. 4–63 moves for a few seconds to the right with constant acceleration, and the 870-kg load hangs on a light cable at a 5.0° angle to the vertical as shown. What is the acceleration of the trolley and load?

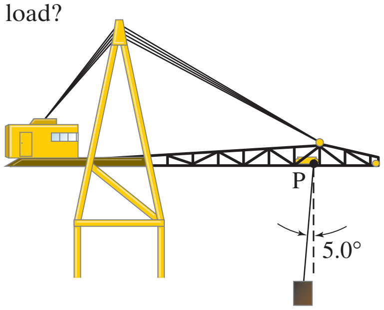


FIGURE 4–63 Problem 63.

64. A 75.0-kg person stands on a scale in an elevator. What does the scale read (in N and in kg) when (*a*) the elevator is at rest, (*b*) the elevator is climbing at a constant speed of 3.0 m/s, (*c*) the elevator is descending at 3.0 m/s, (*d*) the elevator is accelerating upward at $3.0\,\text{m/s}^2$, (*e*) the elevator is accelerating downward at $3.0\,\text{m/s}^2$?

65. If a bicyclist of mass 65 kg (including the bicycle) can coast down a 6.5° hill at a steady speed of 6.0 km/h because of air resistance, how much force must be applied to climb the hill at the same speed (and the same air resistance)?

66. A city planner is working on the redesign of a hilly portion of a city. An important consideration is how steep the roads can be so that even low-powered cars can get up the hills without slowing down. A particular small car, with a mass of 920 kg, can accelerate on a level road from rest to 21 m/s (75 km/h) in 12.5 s. Using these data, calculate the maximum steepness of a hill.

67. Francesca dangles her watch from a thin piece of string while the jetliner she is in accelerates for takeoff, which takes about 16 s. Estimate the takeoff speed of the aircraft if the string makes an angle of 25° with respect to the vertical, Fig. 4–64.

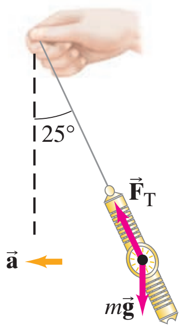


FIGURE 4–64 Problem 67.

68. (*a*) What minimum force F is needed to lift the piano (mass M) using the pulley apparatus shown in Fig. 4–65? (*b*) Determine the tension in each section of rope: F_{T1}, F_{T2}, F_{T3}, and F_{T4}. Assume pulleys are massless and frictionless, and that ropes are massless.

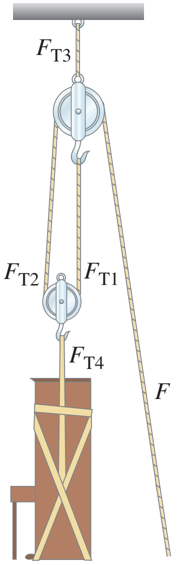


FIGURE 4–65 Problem 68.

69. In the design of a supermarket, there are to be several ramps connecting different parts of the store. Customers will have to push grocery carts up the ramps and it is desirable that this not be too difficult. The engineer has done a survey and found that almost no one complains if the force required is no more than 18 N. Ignoring friction, at what maximum angle θ should the ramps be built, assuming a full 25-kg cart?

70. A jet aircraft is accelerating at $3.8\ \mathrm{m/s^2}$ as it climbs at an angle of 18° above the horizontal (Fig. 4–66). What is the total force that the cockpit seat exerts on the 75-kg pilot?

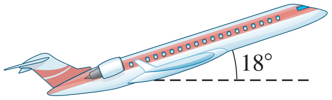


FIGURE 4–66 Problem 70.

71. A 7180-kg helicopter accelerates upward at $0.80\ \mathrm{m/s^2}$ while lifting a 1080-kg frame at a construction site, Fig. 4–67. (*a*) What is the lift force exerted by the air on the helicopter rotors? (*b*) What is the tension in the cable (ignore its mass) which connects the frame to the helicopter? (*c*) What force does the cable exert on the helicopter?

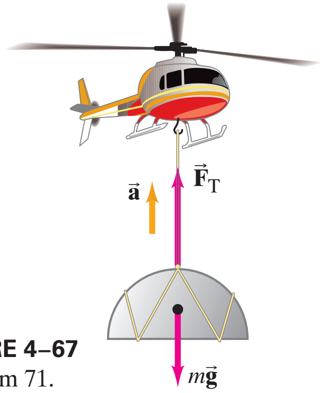


FIGURE 4–67 Problem 71.

72. A "doomsday" asteroid with a mass of 1.0×10^{10} kg is hurtling through space. Unless the asteroid's speed is changed by about 0.20 cm/s, it will collide with Earth and cause tremendous damage. Researchers suggest that a small "space tug" sent to the asteroid's surface could exert a gentle constant force of 2.5 N. For how long must this force act?

73. Three mountain climbers who are roped together in a line are ascending an icefield inclined at 31.0° to the horizontal (Fig. 4–68). The last climber slips, pulling the second climber off his feet. The first climber is able to hold them both. If each climber has a mass of 75 kg, calculate the tension in each of the two sections of rope between the three climbers. Ignore friction between the ice and the fallen climbers.

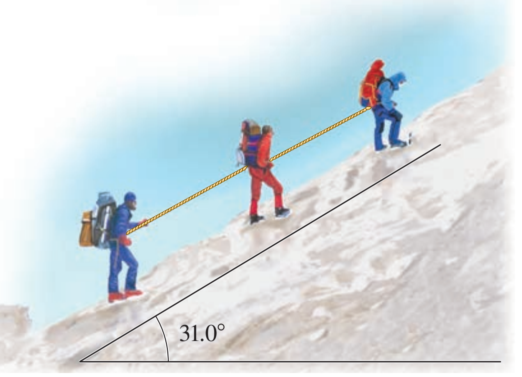


FIGURE 4–68 Problem 73.

74. As shown in Fig. 4–69, five balls (masses 2.00, 2.05, 2.10, 2.15, 2.20 kg) hang from a crossbar. Each mass is supported by "5-lb test" fishing line which will break when its tension force exceeds 22.2 N (= 5.00 lb). When this device is placed in an elevator, which accelerates upward, only the lines attached to the 2.05 and 2.00 kg masses do not break. Within what range is the elevator's acceleration?

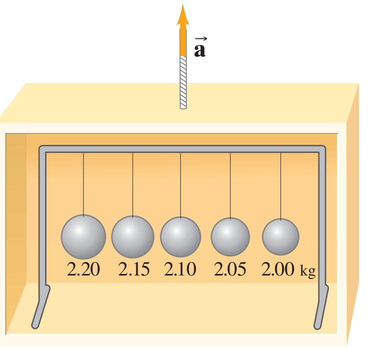


FIGURE 4–69 Problem 74.

75. A coffee cup on the horizontal dashboard of a car slides forward when the driver decelerates from 45 km/h to rest in 3.5 s or less, but not if she decelerates in a longer time. What is the coefficient of static friction between the cup and the dash? Assume the road and the dashboard are level (horizontal).

76. A roller coaster reaches the top of the steepest hill with a speed of 6.0 km/h. It then descends the hill, which is at an average angle of 45° and is 45.0 m long. What will its speed be when it reaches the bottom? Assume $\mu_k = 0.12$.

77. A motorcyclist is coasting with the engine off at a steady speed of 20.0 m/s but enters a sandy stretch where the coefficient of kinetic friction is 0.70. Will the cyclist emerge from the sandy stretch without having to start the engine if the sand lasts for 15 m? If so, what will be the speed upon emerging?

78. The 70.0-kg climber in Fig. 4–70 is supported in the "chimney" by the friction forces exerted on his shoes and back. The static coefficients of friction between his shoes and the wall, and between his back and the wall, are 0.80 and 0.60, respectively. What is the minimum normal force he must exert? Assume the walls are vertical and that the static friction forces are both at their maximum. Ignore his grip on the rope.

FIGURE 4–70 Problem 78.

79. A 28.0-kg block is connected to an empty 2.00-kg bucket by a cord running over a frictionless pulley (Fig. 4–71). The coefficient of static friction between the table and the block is 0.45 and the coefficient of kinetic friction between the table and the block is 0.32. Sand is gradually added to the bucket until the system just begins to move. (*a*) Calculate the mass of sand added to the bucket. (*b*) Calculate the acceleration of the system. Ignore mass of cord.

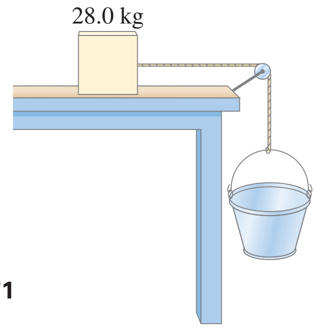


FIGURE 4–71 Problem 79.

80. A 72-kg water skier is being accelerated by a ski boat on a flat ("glassy") lake. The coefficient of kinetic friction between the skier's skis and the water surface is $\mu_k = 0.25$ (Fig. 4–72). (*a*) What is the skier's acceleration if the rope pulling the skier behind the boat applies a horizontal tension force of magnitude $F_T = 240\text{ N}$ to the skier $(\theta = 0°)$? (*b*) What is the skier's horizontal acceleration if the rope pulling the skier exerts a force of $F_T = 240\text{ N}$ on the skier at an upward angle $\theta = 12°$? (*c*) Explain why the skier's acceleration in part (*b*) is greater than that in part (*a*).

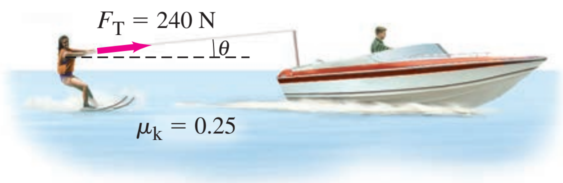


FIGURE 4–72 Problem 80.

81. (*a*) If the horizontal acceleration produced briefly by an earthquake is a, and if an object is going to "hold its place" on the ground, show that the coefficient of static friction with the ground must be at least $\mu_s = a/g$. (*b*) The famous Loma Prieta earthquake that stopped the 1989 World Series produced ground accelerations of up to $4.0\,\mathrm{m/s^2}$ in the San Francisco Bay Area. Would a chair have started to slide on a floor with coefficient of static friction 0.25?

82. Two blocks made of different materials, connected by a thin cord, slide down a plane ramp inclined at an angle θ to the horizontal, Fig. 4–73 (block B is above block A). The masses of the blocks are m_A and m_B, and the coefficients of friction are μ_A and μ_B. If $m_A = m_B = 5.0$ kg, and $\mu_A = 0.20$ and $\mu_B = 0.30$, determine (*a*) the acceleration of the blocks and (*b*) the tension in the cord, for an angle $\theta = 32°$.

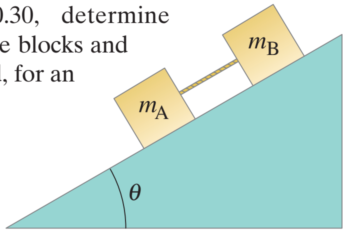


FIGURE 4–73
Problem 82.

83. A car starts rolling down a 1-in-4 hill (1-in-4 means that for each 4 m traveled along the sloping road, the elevation change is 1 m). How fast is it going when it reaches the bottom after traveling 55 m? (*a*) Ignore friction. (*b*) Assume an effective coefficient of friction equal to 0.10.

84. A 65-kg ice skater coasts with no effort for 75 m until she stops. If the coefficient of kinetic friction between her skates and the ice is $\mu_k = 0.10$, how fast was she moving at the start of her coast?

Search and Learn

1. In the equation for static friction in Section 4–8, what is the significance of the $<$ sign? When should you use the equals sign in the static friction equation?

2. Referring to Example 4–21, show that if a skier moves at constant speed straight down a slope of angle θ, then the coefficient of kinetic friction between skis and snow is $\mu_k = \tan\theta$.

3. (*a*) Show that the minimum stopping distance for an automobile traveling on a level road at speed v is equal to $v^2/(2\,\mu_s\,g)$, where μ_s is the coefficient of static friction between the tires and the road, and g is the acceleration of gravity. (*b*) What is this distance for a 1200-kg car traveling 95 km/h if $\mu_s = 0.65$? (*c*) What would it be if the car were on the Moon (the acceleration of gravity on the Moon is about $g/6$) but all else stayed the same?

ANSWERS TO EXERCISES

A: No force is needed. The car accelerates out from under the cup, which tends to remain at rest. Think of Newton's first law (see Example 4–1).
B: (*a*).
C: (i) The same; (ii) the tennis ball; (iii) Newton's third law for part (i), second law for part (ii).
D: (*b*).
E: (*b*).
F: (*b*).
G: (*c*).
H: Yes; no.