This snowboarder flying through the air shows an example of motion in two dimensions. In the absence of air resistance, the path would be a perfect parabola. The gold arrow represents the downward acceleration of gravity, $\vec{\mathbf{g}}$. Galileo analyzed the motion of objects in 2 dimensions under the action of gravity near the Earth's surface (now called "projectile motion") into its horizontal and vertical components.

We will discuss vectors and how to add them. Besides analyzing projectile motion, we will also see how to work with relative velocity.

CHAPTER 3

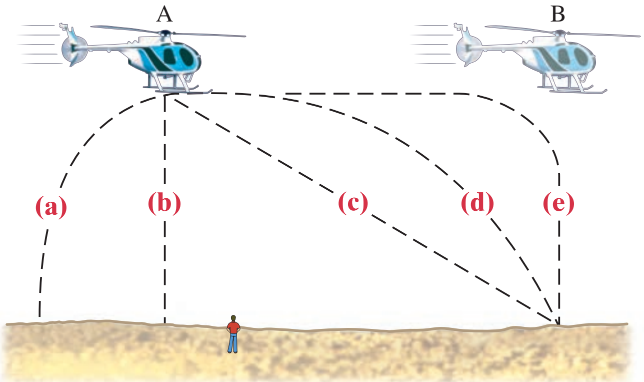
Kinematics in Two Dimensions; Vectors

CONTENTS

CHAPTER-OPENING QUESTION—Guess now!

[*Don't worry about getting the right answer now—you will get another chance later in the Chapter. See also p. 1 of Chapter 1 for more explanation.*]

A small heavy box of emergency supplies is dropped from a moving helicopter at point A as it flies at constant speed in a horizontal direction. Which path in the drawing below best describes the path of the box (neglecting air resistance) as seen by a person standing on the ground?



In Chapter 2 we dealt with motion along a straight line. We now consider the motion of objects that move in paths in two (or three) dimensions. In particular, we discuss an important type of motion known as *projectile motion*: objects projected outward near the Earth's surface, such as struck baseballs and golf balls, kicked footballs, and other projectiles. Before beginning our discussion of motion in two dimensions, we will need a new tool, vectors, and how to add them.

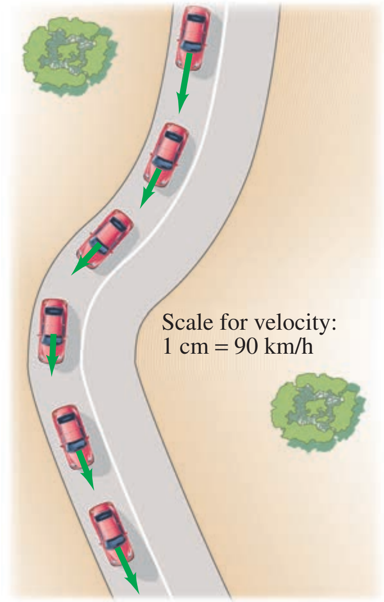


FIGURE 3–1 Car traveling on a road, slowing down to round the curve. The green arrows represent the velocity vector at each position.

3–1 Vectors and Scalars

We mentioned in Chapter 2 that the term *velocity* refers not only to how fast an object is moving but also to its direction. A quantity such as velocity, which has *direction* as well as *magnitude*, is a **vector** quantity. Other quantities that are also vectors are displacement, force, and momentum. However, many quantities have no direction associated with them, such as mass, time, and temperature. They are specified completely by a number and units. Such quantities are called **scalar** quantities.

Drawing a diagram of a particular physical situation is always helpful in physics, and this is especially true when dealing with vectors. On a diagram, each vector is represented by an arrow. The arrow is always drawn so that it points in the direction of the vector quantity it represents. The length of the arrow is drawn proportional to the magnitude of the vector quantity. For example, in Fig. 3–1, green arrows have been drawn representing the velocity of a car at various places as it rounds a curve. The magnitude of the velocity at each point can be read off Fig. 3–1 by measuring the length of the corresponding arrow and using the scale shown $(1\text{ cm} = 90\text{ km/h})$.

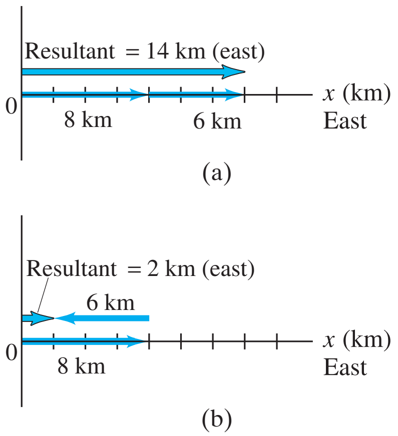
When we write the symbol for a vector, we will always use boldface type, with a tiny arrow over the symbol. Thus for velocity we write $\vec{\mathbf{v}}$. If we are concerned only with the magnitude of the vector, we will write simply v, in italics, as we do for other symbols.

3–2 Addition of Vectors—Graphical Methods

Because vectors are quantities that have direction as well as magnitude, they must be added in a special way. In this Chapter, we will deal mainly with displacement vectors, for which we now use the symbol $\vec{\mathbf{D}}$, and velocity vectors, $\vec{\mathbf{v}}$. But the results will apply for other vectors we encounter later.

We use simple arithmetic for adding scalars. Simple arithmetic can also be used for adding vectors if they are in the same direction. For example, if a person walks 8 km east one day, and 6 km east the next day, the person will be $8\text{ km} + 6\text{ km} = 14\text{ km}$ east of the point of origin. We say that the *net* or *resultant* displacement is 14 km to the east (Fig. 3–2a). If, on the other hand, the person walks 8 km east on the first day, and 6 km west (in the reverse direction) on the second day, then the person will end up 2 km from the origin (Fig. 3–2b), so the resultant displacement is 2 km to the east. In this case, the resultant displacement is obtained by subtraction: $8\text{ km} - 6\text{ km} = 2\text{ km}$.

FIGURE 3–2 Combining vectors in one dimension.



But simple arithmetic cannot be used if the two vectors are not along the same line. For example, suppose a person walks 10.0 km east and then walks 5.0 km north. These displacements can be represented on a graph in which the positive y axis points north and the positive x axis points east, Fig. 3–3. On this graph, we draw an arrow, labeled $\vec{\mathbf{D}}_1$, to represent the 10.0-km displacement to the east. Then we draw a second arrow, $\vec{\mathbf{D}}_2$, to represent the 5.0-km displacement to the north. Both vectors are drawn to scale, as in Fig. 3–3.

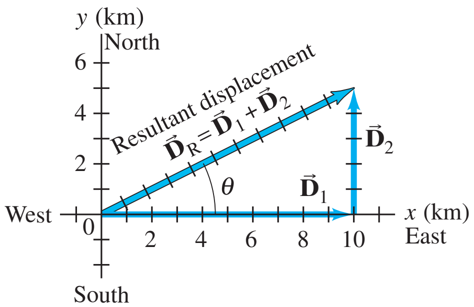


FIGURE 3–3 A person walks 10.0 km east and then 5.0 km north. These two displacements are represented by the vectors $\vec{\mathbf{D}}_1$ and $\vec{\mathbf{D}}_2$, which are shown as arrows. Also shown is the resultant displacement vector, $\vec{\mathbf{D}}_R$, which is the vector sum of $\vec{\mathbf{D}}_1$ and $\vec{\mathbf{D}}_2$. Measurement on the graph with ruler and protractor shows that $\vec{\mathbf{D}}_R$ has a magnitude of 11.2 km and points at an angle $\theta = 27°$ north of east.

After taking this walk, the person is now 10.0 km east and 5.0 km north of the point of origin. The **resultant displacement** is represented by the arrow labeled $\vec{\mathbf{D}}_R$ in Fig. 3–3. (The subscript R stands for resultant.) Using a ruler and a protractor, you can measure on this diagram that the person is 11.2 km from the origin at an angle $\theta = 27°$ north of east. In other words, the resultant displacement vector has a magnitude of 11.2 km and makes an angle $\theta = 27°$ with the positive x axis. The magnitude (length) of $\vec{\mathbf{D}}_R$ can also be obtained using the theorem of Pythagoras in this case, because D_1, D_2, and D_R form a right triangle with D_R as the hypotenuse. Thus

$$\begin{aligned} D_R &= \sqrt{D_1^2 + D_2^2} = \sqrt{(10.0\text{ km})^2 + (5.0\text{ km})^2} \\ &= \sqrt{125\text{ km}^2} = 11.2\text{ km}. \end{aligned}$$

You can use the Pythagorean theorem only when the vectors are *perpendicular* to each other.

The resultant displacement vector, $\vec{\mathbf{D}}_R$, is the sum of the vectors $\vec{\mathbf{D}}_1$ and $\vec{\mathbf{D}}_2$. That is,

$$\vec{\mathbf{D}}_R = \vec{\mathbf{D}}_1 + \vec{\mathbf{D}}_2.$$

This is a *vector* equation. An important feature of adding two vectors that are not along the same line is that the magnitude of the resultant vector is not equal to the sum of the magnitudes of the two separate vectors, but is smaller than their sum. That is,

$$D_R \leq (D_1 + D_2),$$

where the equals sign applies only if the two vectors point in the same direction. In our example (Fig. 3–3), $D_R = 11.2\text{ km}$, whereas $D_1 + D_2$ equals 15 km, which is the total distance traveled. Note also that we cannot set $\vec{\mathbf{D}}_R$ equal to 11.2 km, because we have a vector equation and 11.2 km is only a part of the resultant vector, its magnitude. We could write something like this, though: $\vec{\mathbf{D}}_R = \vec{\mathbf{D}}_1 + \vec{\mathbf{D}}_2 = (11.2\text{ km}, 27°\text{ N of E})$.

Figure 3–3 illustrates the general rules for graphically adding two vectors together, no matter what angles they make, to get their sum. The rules are as follows:

1. On a diagram, draw one of the vectors—call it $\vec{\mathbf{D}}_1$—to scale.
2. Next draw the second vector, $\vec{\mathbf{D}}_2$, to scale, placing its tail at the tip of the first vector and being sure its direction is correct.
3. The arrow drawn from the tail of the first vector to the tip of the second vector represents the *sum*, or **resultant**, of the two vectors.

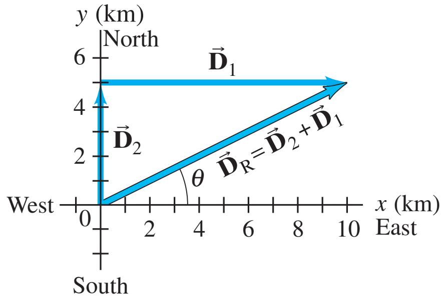
The length of the resultant vector represents its magnitude. Note that vectors can be moved parallel to themselves on paper (maintaining the same length and angle) to accomplish these manipulations. The length of the resultant can be measured with a ruler and compared to the scale. Angles can be measured with a protractor. This method is known as the **tail-to-tip method of adding vectors**.

The resultant is not affected by the order in which the vectors are added. For example, a displacement of 5.0 km north, to which is added a displacement of 10.0 km east, yields a resultant of 11.2 km and angle $\theta = 27°$ (see Fig. 3–4), the same as when they were added in reverse order (Fig. 3–3). That is, now using $\vec{\mathbf{V}}$ to represent any type of vector,

$$\vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2 = \vec{\mathbf{V}}_2 + \vec{\mathbf{V}}_1.$$

[Mathematicians call this equation the *commutative* property of vector addition.]

FIGURE 3–4 If the vectors are added in reverse order, the resultant is the same. (Compare to Fig. 3–3.)



The tail-to-tip method of adding vectors can be extended to three or more vectors. The resultant is drawn from the tail of the first vector to the tip of the last one added. An example is shown in Fig. 3–5; the three vectors could represent displacements (northeast, south, west) or perhaps three forces. Check for yourself that you get the same resultant no matter in which order you add the three vectors.

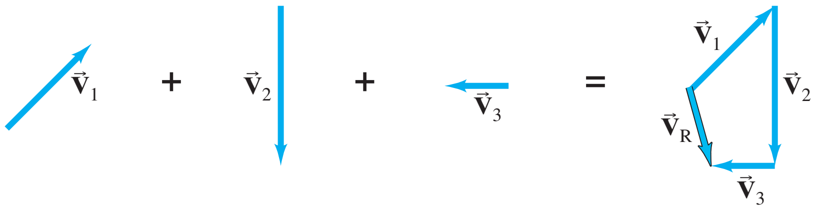


FIGURE 3–5 The resultant of three vectors: $\vec{\mathbf{V}}_R = \vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2 + \vec{\mathbf{V}}_3$.

A second way to add two vectors is the **parallelogram method**. It is fully equivalent to the tail-to-tip method. In this method, the two vectors are drawn starting from a common origin, and a parallelogram is constructed using these two vectors as adjacent sides as shown in Fig. 3–6b. The resultant is the diagonal drawn from the common origin. In Fig. 3–6a, the tail-to-tip method is shown, and we can see that both methods yield the same result.

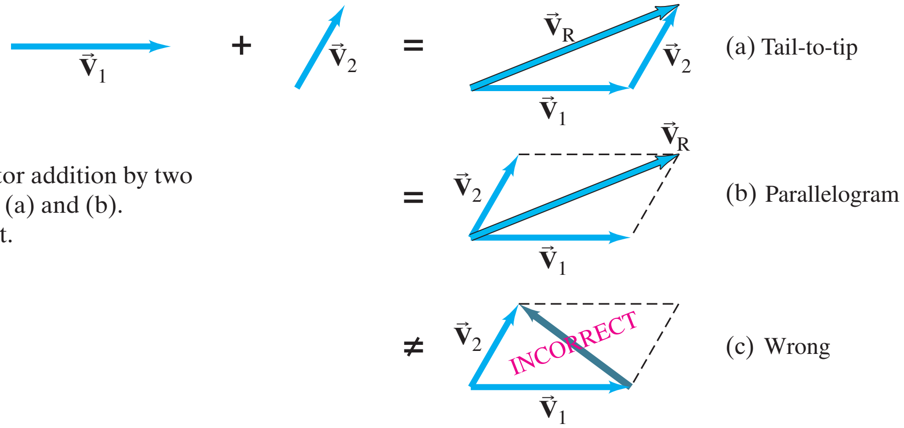


FIGURE 3–6 Vector addition by two different methods, (a) and (b). Part (c) is incorrect.

CAUTION

Be sure to use the correct diagonal on the parallelogram to get the resultant

It is a common error to draw the sum vector as the diagonal running between the tips of the two vectors, as in Fig. 3–6c. *This is incorrect*: it does not represent the sum of the two vectors. (In fact, it represents their difference, $\vec{\mathbf{V}}_2 - \vec{\mathbf{V}}_1$, as we will see in the next Section.)

CONCEPTUAL EXAMPLE 3–1 **Range of vector lengths.** Suppose two vectors each have length 3.0 units. What is the range of possible lengths for the vector representing the sum of the two?

RESPONSE The sum can take on any value from $6.0\ (= 3.0 + 3.0)$ where the vectors point in the same direction, to $0\ (= 3.0 - 3.0)$ when the vectors are antiparallel. Magnitudes between 0 and 6.0 occur when the two vectors are at an angle other than 0° and 180°.

EXERCISE A If the two vectors of Example 3–1 are perpendicular to each other, what is the resultant vector length?

3–3 Subtraction of Vectors, and Multiplication of a Vector by a Scalar

Given a vector $\vec{\mathbf{V}}$, we define the *negative* of this vector $(-\vec{\mathbf{V}})$ to be a vector with the same magnitude as $\vec{\mathbf{V}}$ but opposite in direction, Fig. 3–7. Note, however, that no vector is ever negative in the sense of its magnitude: the magnitude of every vector is positive. Rather, a minus sign tells us about its direction.

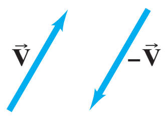


FIGURE 3–7 The negative of a vector is a vector having the same length but opposite direction.

We can now define the subtraction of one vector from another: the difference between two vectors $\vec{\mathbf{V}}_2 - \vec{\mathbf{V}}_1$ is defined as

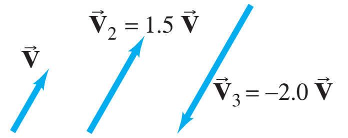
$$\vec{\mathbf{V}}_2 - \vec{\mathbf{V}}_1 = \vec{\mathbf{V}}_2 + \left(-\vec{\mathbf{V}}_1\right).$$

That is, the difference between two vectors is equal to the sum of the first plus the negative of the second. Thus our rules for addition of vectors can be applied as shown in Fig. 3–8 using the tail-to-tip method.

FIGURE 3–8 Subtracting two vectors: $\vec{\mathbf{V}}_2 - \vec{\mathbf{V}}_1$.

A vector $\vec{\mathbf{V}}$ can be multiplied by a scalar c. We define their product so that $c\vec{\mathbf{V}}$ has the same direction as $\vec{\mathbf{V}}$ and has magnitude cV. That is, multiplication of a vector by a positive scalar c changes the magnitude of the vector by a factor c but doesn't alter the direction. If c is a negative scalar (such as -2.0), the magnitude of the product $c\vec{\mathbf{V}}$ is changed by the factor $|c|$ (where $|c|$ means the magnitude of c), but the direction is precisely opposite to that of $\vec{\mathbf{V}}$. See Fig. 3–9.

FIGURE 3–9 Multiplying a vector $\vec{\mathbf{V}}$ by a scalar c gives a vector whose magnitude is c times greater and in the same direction as $\vec{\mathbf{V}}$ (or opposite direction if c is negative).



EXERCISE B What does the "incorrect" vector in Fig. 3–6c represent? (*a*) $\vec{\mathbf{V}}_2 - \vec{\mathbf{V}}_1$; (*b*) $\vec{\mathbf{V}}_1 - \vec{\mathbf{V}}_2$; (*c*) something else (specify).

3–4 Adding Vectors by Components

Adding vectors graphically using a ruler and protractor is often not sufficiently accurate and is not useful for vectors in three dimensions. We discuss now a more powerful and precise method for adding vectors. But do not forget graphical methods—they are useful for visualizing, for checking your math, and thus for getting the correct result.

Components

Consider first a vector $\vec{\mathbf{V}}$ that lies in a particular plane. It can be expressed as the sum of two other vectors, called the **components** of the original vector. The components are usually chosen to be along two perpendicular directions, such as the x and y axes. The process of finding the components is known as **resolving the vector into its components**. An example is shown in Fig. 3–10; the vector $\vec{\mathbf{V}}$ could be a displacement vector that points at an angle $\theta = 30°$ north of east, where we have chosen the positive x axis to be to the east and the positive y axis north. This vector $\vec{\mathbf{V}}$ is resolved into its x and y components by drawing dashed lines (AB and AC) out from the tip (A) of the vector, making them perpendicular to the x and y axes. Then the lines 0B and 0C represent the x and y components of $\vec{\mathbf{V}}$, respectively, as shown in Fig. 3–10b. These *vector components* are written $\vec{\mathbf{V}}_x$ and $\vec{\mathbf{V}}_y$. In this book we usually show vector components as arrows, like vectors, but dashed. The *scalar components*, V_x and V_y, are the magnitudes of the vector components, with units, accompanied by a positive or negative sign depending on whether they point along the positive or negative x or y axis. As can be seen in Fig. 3–10, $\vec{\mathbf{V}}_x + \vec{\mathbf{V}}_y = \vec{\mathbf{V}}$ by the parallelogram method of adding vectors.

Space is made up of three dimensions, and sometimes it is necessary to resolve a vector into components along three mutually perpendicular directions. In rectangular coordinates the components are $\vec{\mathbf{V}}_x$, $\vec{\mathbf{V}}_y$, and $\vec{\mathbf{V}}_z$.

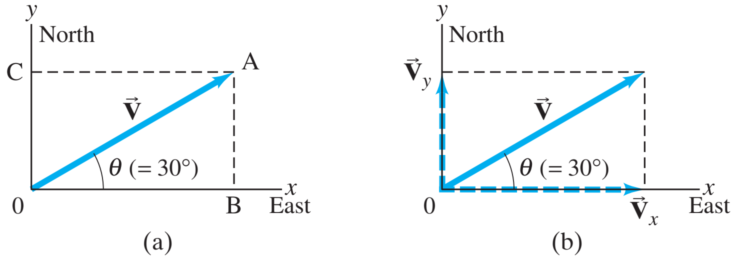


FIGURE 3–10 Resolving a vector $\vec{\mathbf{V}}$ into its components along a chosen set of x and y axes. The components, once found, themselves represent the vector. That is, the components contain as much information as the vector itself.

To add vectors using the method of components, we need to use the trigonometric functions sine, cosine, and tangent, which we now review.

Given any angle θ, as in Fig. 3–11a, a right triangle can be constructed by drawing a line perpendicular to one of its sides, as in Fig. 3–11b. The longest side of a right triangle, opposite the right angle, is called the hypotenuse, which we label h. The side opposite the angle θ is labeled o, and the side adjacent is labeled a. We let h, o, and a represent the lengths of these sides, respectively.

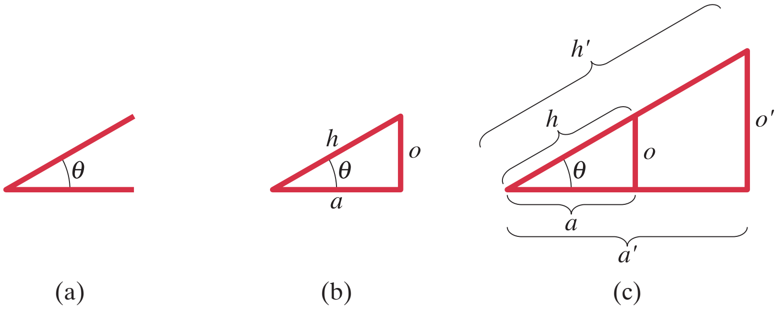


FIGURE 3–11 Starting with an angle θ as in (a), we can construct right triangles of different sizes, (b) and (c), but the ratio of the lengths of the sides does not depend on the size of the triangle.

We now define the three trigonometric functions, sine, cosine, and tangent (abbreviated sin, cos, tan), in terms of the right triangle, as follows:

$$\sin\theta = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{o}{h}$$
$$\cos\theta = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{a}{h} \qquad \textbf{(3–1)}$$
$$\tan\theta = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{o}{a}.$$

If we make the triangle bigger, but keep the same angles, then the ratio of the length of one side to the other, or of one side to the hypotenuse, remains the same. That is, in Fig. 3–11c we have: $a/h = a'/h'$; $o/h = o'/h'$; and $o/a = o'/a'$. Thus the values of sine, cosine, and tangent do not depend on how big the triangle is. They depend only on the size of the angle. The values of sine, cosine, and tangent for different angles can be found using a scientific calculator, or from the Table in Appendix A.

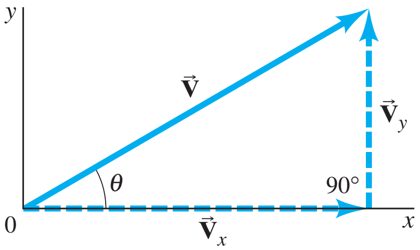
A useful trigonometric identity is

$$\sin^2\theta + \cos^2\theta = 1 \qquad \textbf{(3–2)}$$

which follows from the Pythagorean theorem $(o^2 + a^2 = h^2$ in Fig. 3–11). That is:

$$\sin^2\theta + \cos^2\theta = \frac{o^2}{h^2} + \frac{a^2}{h^2} = \frac{o^2 + a^2}{h^2} = \frac{h^2}{h^2} = 1.$$

(See Appendix A and inside the rear cover for other details on trigonometric functions and identities.)



$$\sin\theta = \frac{V_y}{V}$$
$$\cos\theta = \frac{V_x}{V}$$
$$\tan\theta = \frac{V_y}{V_x}$$
$$V^2 = V_x^2 + V_y^2$$

FIGURE 3–12 Finding the components of a vector using trigonometric functions. The equations are valid only if θ is the angle $\vec{\mathbf{V}}$ makes with the positive x axis.

The use of trigonometric functions for finding the components of a vector is illustrated in Fig. 3–12, where a vector and its two components are thought of as making up a right triangle. We then see that the sine, cosine, and tangent are as given in Fig. 3–12, where θ is the angle $\vec{\mathbf{V}}$ makes with the $+x$ axis. If we multiply the definition of $\sin\theta = V_y/V$ by V on both sides, we get

$$V_y = V\sin\theta. \qquad \textbf{(3–3a)}$$

Similarly, from the definition of $\cos\theta$, we obtain

$$V_x = V\cos\theta. \qquad \textbf{(3–3b)}$$

Note that if θ is not the angle the vector makes with the positive x axis, Eqs. 3–3 are not valid.

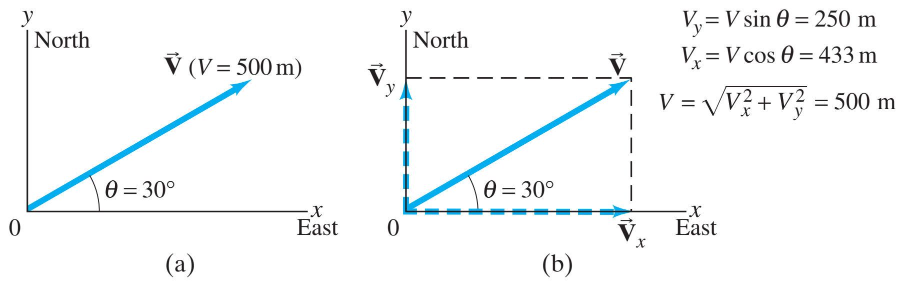


FIGURE 3–13 (a) Vector $\vec{\mathbf{V}}$ represents a displacement of 500 m at a 30° angle north of east. (b) The components of $\vec{\mathbf{V}}$ are $\vec{\mathbf{V}}_x$ and $\vec{\mathbf{V}}_y$, whose magnitudes are given on the right in the diagram.

Using Eqs. 3–3, we can calculate V_x and V_y for any vector, such as that illustrated in Fig. 3–10 or Fig. 3–12. Suppose $\vec{\mathbf{V}}$ represents a displacement of 500 m in a direction 30° north of east, as shown in Fig. 3–13. Then $V = 500$ m. From a calculator or Tables, $\sin 30° = 0.500$ and $\cos 30° = 0.866$. Then

$$V_x = V\cos\theta = (500\text{ m})(0.866) = 433\text{ m (east)},$$
$$V_y = V\sin\theta = (500\text{ m})(0.500) = 250\text{ m (north)}.$$

There are two ways to specify a vector in a given coordinate system:

1. We can give its components, V_x and V_y.
2. We can give its magnitude V and the angle θ it makes with the positive x axis.

We can shift from one description to the other using Eqs. 3–3, and, for the reverse, by using the theorem of Pythagoras† and the definition of tangent:

$$V = \sqrt{V_x^2 + V_y^2} \qquad \textbf{(3–4a)}$$

$$\tan\theta = \frac{V_y}{V_x} \qquad \textbf{(3–4b)}$$

as can be seen in Fig. 3–12.

Adding Vectors

We can now discuss how to add vectors using components. The first step is to resolve each vector into its components. Next we can see, using Fig. 3–14, that the addition of any two vectors $\vec{\mathbf{V}}_1$ and $\vec{\mathbf{V}}_2$ to give a resultant, $\vec{\mathbf{V}}_R = \vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2$, implies that

$$\begin{aligned} V_{Rx} &= V_{1x} + V_{2x} \\ V_{Ry} &= V_{1y} + V_{2y}. \end{aligned} \qquad \textbf{(3–5)}$$

That is, the sum of the x components equals the x component of the resultant vector, and the sum of the y components equals the y component of the resultant, as can be verified by a careful examination of Fig. 3–14. Note that we do *not* add x components to y components.

If the magnitude and direction of the resultant vector are desired, they can be obtained using Eqs. 3–4.

†In three dimensions, the theorem of Pythagoras becomes $V = \sqrt{V_x^2 + V_y^2 + V_z^2}$, where V_z is the component along the third, or z, axis.

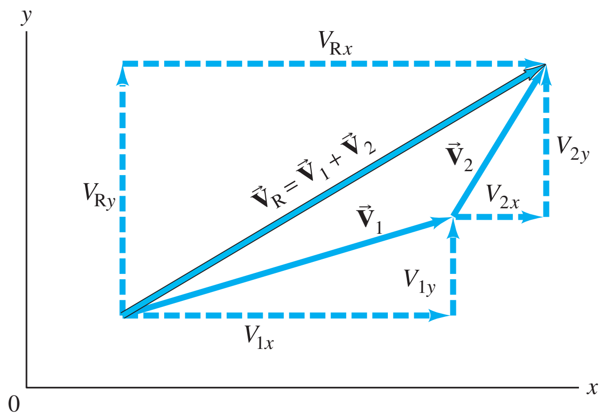


FIGURE 3–14 The components of $\vec{\mathbf{V}}_R = \vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2$ are $V_{Rx} = V_{1x} + V_{2x}$ and $V_{Ry} = V_{1y} + V_{2y}$.

The components of a given vector depend on the choice of coordinate axes. You can often reduce the work involved in adding vectors by a good choice of axes—for example, by choosing one of the axes to be in the same direction as one of the vectors. Then that vector will have only one nonzero component.

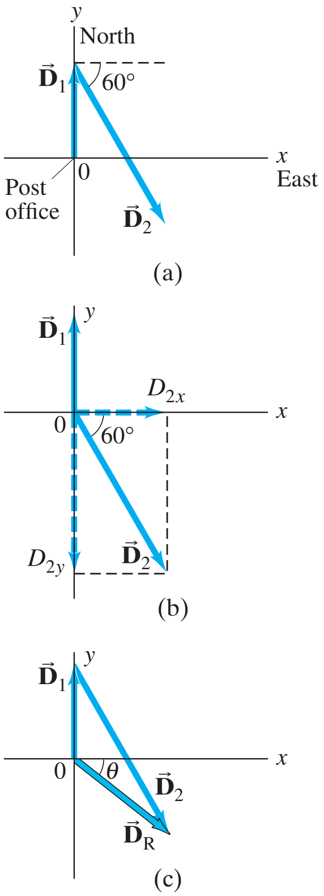


FIGURE 3–15 Example 3–2. (a) The two displacement vectors, $\vec{\mathbf{D}}_1$ and $\vec{\mathbf{D}}_2$. (b) $\vec{\mathbf{D}}_2$ is resolved into its components. (c) $\vec{\mathbf{D}}_1$ and $\vec{\mathbf{D}}_2$ are added to obtain the resultant $\vec{\mathbf{D}}_R$. The component method of adding the vectors is explained in the Example.

EXAMPLE 3–2 **Mail carrier's displacement.** A rural mail carrier leaves the post office and drives 22.0 km in a northerly direction. She then drives in a direction 60.0° south of east for 47.0 km (Fig. 3–15a). What is her displacement from the post office?

APPROACH We choose the positive x axis to be east and the positive y axis to be north, since those are the compass directions used on most maps. The origin of the xy coordinate system is at the post office. We resolve each vector into its x and y components. We add the x components together, and then the y components together, giving us the x and y components of the resultant.

SOLUTION Resolve each displacement vector into its components, as shown in Fig. 3–15b. Since $\vec{\mathbf{D}}_1$ has magnitude 22.0 km and points north, it has only a y component:

$$D_{1x} = 0, \qquad D_{1y} = 22.0\text{ km}.$$

$\vec{\mathbf{D}}_2$ has both x and y components:

$$D_{2x} = +(47.0\text{ km})(\cos 60°) = +(47.0\text{ km})(0.500) = +23.5\text{ km}$$
$$D_{2y} = -(47.0\text{ km})(\sin 60°) = -(47.0\text{ km})(0.866) = -40.7\text{ km}.$$

Notice that D_{2y} is negative because this vector component points along the negative y axis. The resultant vector, $\vec{\mathbf{D}}_R$, has components:

$$D_{Rx} = D_{1x} + D_{2x} = 0\text{ km} + 23.5\text{ km} = +23.5\text{ km}$$
$$D_{Ry} = D_{1y} + D_{2y} = 22.0\text{ km} + (-40.7\text{ km}) = -18.7\text{ km}.$$

This specifies the resultant vector completely:

$$D_{Rx} = 23.5\text{ km}, \qquad D_{Ry} = -18.7\text{ km}.$$

We can also specify the resultant vector by giving its magnitude and angle using Eqs. 3–4:

$$D_R = \sqrt{D_{Rx}^2 + D_{Ry}^2} = \sqrt{(23.5\text{ km})^2 + (-18.7\text{ km})^2} = 30.0\text{ km}$$

$$\tan\theta = \frac{D_{Ry}}{D_{Rx}} = \frac{-18.7\text{ km}}{23.5\text{ km}} = -0.796.$$

A calculator with a key labeled INV TAN, or ARC TAN, or TAN^{-1} gives $\theta = \tan^{-1}(-0.796) = -38.5°$. The negative sign means $\theta = 38.5°$ below the x axis, Fig. 3–15c. So, the resultant displacement is 30.0 km directed at 38.5° in a southeasterly direction.

NOTE Always be attentive about the quadrant in which the resultant vector lies. An electronic calculator does not fully give this information, but a good diagram does.

PROBLEM SOLVING
Identify the correct quadrant by drawing a careful diagram

As we saw in Example 3–2, any component that points along the negative x or y axis gets a minus sign. The signs of trigonometric functions depend on which "quadrant" the angle falls in: for example, the tangent is positive in the first and third quadrants (from 0° to 90°, and 180° to 270°), but negative in the second and fourth quadrants; see Appendix A, Fig. A–7. The best way to keep track of angles, and to check any vector result, is always to draw a vector diagram, like Fig. 3–15. A vector diagram gives you something tangible to look at when analyzing a problem, and provides a check on the results.

The following Problem Solving Strategy should not be considered a prescription. Rather it is a summary of things to do to get you thinking and involved in the problem at hand.

PROBLEM SOLVING

Adding Vectors

Here is a brief summary of how to add two or more vectors using components:

1. **Draw a diagram**, adding the vectors graphically by either the parallelogram or tail-to-tip method.
2. **Choose** x and y **axes**. Choose them in a way, if possible, that will make your work easier. (For example, choose one axis along the direction of one of the vectors, which then will have only one component.)
3. **Resolve** each vector into its x and y **components**, showing each component along its appropriate (x or y) axis as a (dashed) arrow.
4. **Calculate each component** (when not given) using sines and cosines. If θ_1 is the angle that vector $\vec{\mathbf{V}}_1$ makes with the positive x axis, then:
$$V_{1x} = V_1 \cos\theta_1, \qquad V_{1y} = V_1 \sin\theta_1.$$
Pay careful attention to **signs**: any component that points along the negative x or y axis gets a minus sign.
5. **Add** the x **components** together to get the x component of the resultant. Similarly for y:
$$V_{Rx} = V_{1x} + V_{2x} + \text{any others}$$
$$V_{Ry} = V_{1y} + V_{2y} + \text{any others.}$$
This is the answer: the components of the resultant vector. Check signs to see if they fit the quadrant shown in your diagram (point 1 above).
6. If you want to know the **magnitude and direction** of the resultant vector, use Eqs. 3–4:
$$V_R = \sqrt{V_{Rx}^2 + V_{Ry}^2}, \qquad \tan\theta = \frac{V_{Ry}}{V_{Rx}}.$$
The vector diagram you already drew helps to obtain the correct position (quadrant) of the angle θ.

EXAMPLE 3–3 **Three short trips.** An airplane trip involves three legs, with two stopovers, as shown in Fig. 3–16a. The first leg is due east for 620 km; the second leg is southeast (45°) for 440 km; and the third leg is at 53° south of west, for 550 km, as shown. What is the plane's total displacement?

APPROACH We follow the steps in the Problem Solving Strategy above.

SOLUTION

1. **Draw a diagram** such as Fig. 3–16a, where $\vec{\mathbf{D}}_1$, $\vec{\mathbf{D}}_2$, and $\vec{\mathbf{D}}_3$ represent the three legs of the trip, and $\vec{\mathbf{D}}_R$ is the plane's total displacement.
2. **Choose axes**: Axes are also shown in Fig. 3–16a: x is east, y north.
3. **Resolve components**: It is imperative to draw a good diagram. The components are drawn in Fig. 3–16b. Instead of drawing all the vectors starting from a common origin, as we did in Fig. 3–15b, here we draw them "tail-to-tip" style, which is just as valid and may make it easier to see.
4. **Calculate the components**:
$$\begin{aligned}
\vec{\mathbf{D}}_1:\ D_{1x} &= +D_1\cos 0° = D_1 = 620\text{ km}\\
D_{1y} &= +D_1\sin 0° = 0\text{ km}\\
\vec{\mathbf{D}}_2:\ D_{2x} &= +D_2\cos 45° = +(440\text{ km})(0.707) = +311\text{ km}\\
D_{2y} &= -D_2\sin 45° = -(440\text{ km})(0.707) = -311\text{ km}\\
\vec{\mathbf{D}}_3:\ D_{3x} &= -D_3\cos 53° = -(550\text{ km})(0.602) = -331\text{ km}\\
D_{3y} &= -D_3\sin 53° = -(550\text{ km})(0.799) = -439\text{ km.}
\end{aligned}$$
We have given a minus sign to each component that in Fig. 3–16b points in the $-x$ or $-y$ direction. The components are shown in the Table in the margin.
5. **Add the components**: We add the x components together, and we add the y components together to obtain the x and y components of the resultant:
$$\begin{aligned}
D_{Rx} &= D_{1x} + D_{2x} + D_{3x} = 620\text{ km} + 311\text{ km} - 331\text{ km} = 600\text{ km}\\
D_{Ry} &= D_{1y} + D_{2y} + D_{3y} = 0\text{ km} - 311\text{ km} - 439\text{ km} = -750\text{ km.}
\end{aligned}$$
The x and y components of the resultant are 600 km and −750 km, and point respectively to the east and south. This is one way to give the answer.
6. **Magnitude and direction**: We can also give the answer as
$$D_R = \sqrt{D_{Rx}^2 + D_{Ry}^2} = \sqrt{(600)^2 + (-750)^2}\text{ km} = 960\text{ km}$$
$$\tan\theta = \frac{D_{Ry}}{D_{Rx}} = \frac{-750\text{ km}}{600\text{ km}} = -1.25, \qquad \text{so } \theta = -51°.$$
Thus, the total displacement has magnitude 960 km and points 51° below the x axis (south of east), as was shown in our original sketch, Fig. 3–16a.

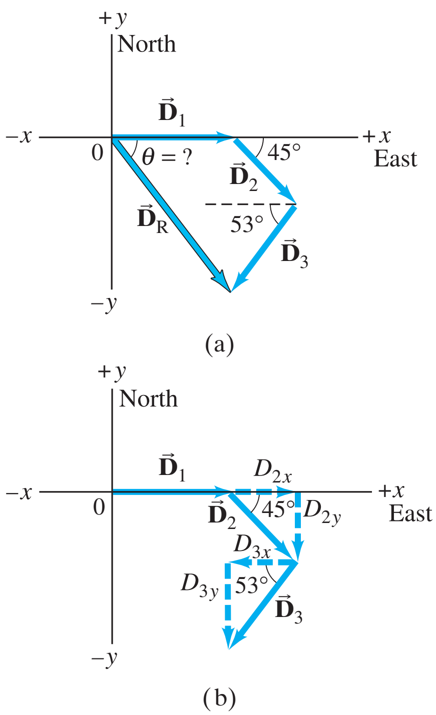


FIGURE 3–16 Example 3–3.

Vector	Components x (km)	Components y (km)
$\vec{\mathbf{D}}_1$	620	0
$\vec{\mathbf{D}}_2$	311	−311
$\vec{\mathbf{D}}_3$	−331	−439
$\vec{\mathbf{D}}_R$	600	−750