

FIGURE 23-9 Example 23-3.

RESPONSE We need to draw two diagrams, one assuming the photo on p. 644 is right side up, and another assuming it is upside down. Figure 23-9 is drawn assuming the photo is upside down. In this case, the Sun blocked by the tree would be the direct view, and the full view of the Sun the reflection: the ray which reflects off the water and into the camera travels at an angle below the branch, whereas the ray that travels directly to the camera passes through the branches. This works. Try to draw a diagram assuming the photo is right side up (thus assuming that the image of the Sun in the reflection is higher above the horizon than it is as viewed directly). It won't work. The photo on p. 644 is upside down.

Also, what about the people in the photo? Try to draw a diagram showing why they don't appear in the reflection. [Hint: Assume they are not sitting at the edge of the pool, but back from the edge.] Then try to draw a diagram of the reverse (i.e., assume the photo is right side up so the people are visible only in the reflection). Reflected images are not perfect replicas when different planes (distances) are involved.

23-3 Formation of Images by Spherical Mirrors

Reflecting surfaces can also be *curved*, usually *spherical*, which means they form a section of a sphere. A **spherical mirror** is called **convex** if the reflection takes place on the outer surface of the spherical shape so that the center of the mirror surface bulges out toward the viewer, Fig. 23-10a. A mirror is called **concave** if the reflecting surface is on the inner surface of the sphere so that the mirror surface curves away from the viewer (like a “cave”), Fig. 23-10b. Concave mirrors are used as shaving or cosmetic mirrors (**magnifying mirrors**), Fig. 23-11a, because they magnify. Convex mirrors are sometimes used on cars and trucks (rearview mirrors) and in shops (to watch for theft), because they take in a wide field of view, Fig. 23-11b.

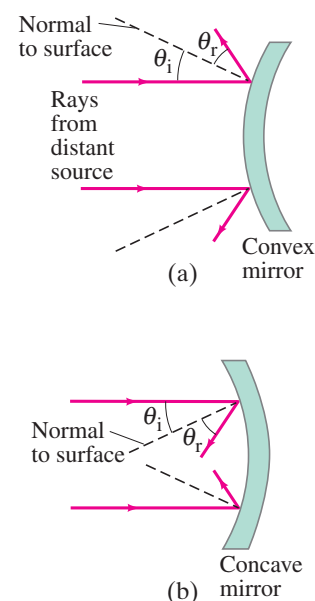


FIGURE 23-10 Mirrors with convex and concave spherical surfaces. Note that $\theta_r = \theta_i$ for each ray. (The dashed lines are perpendicular to the mirror surface at each point shown.)

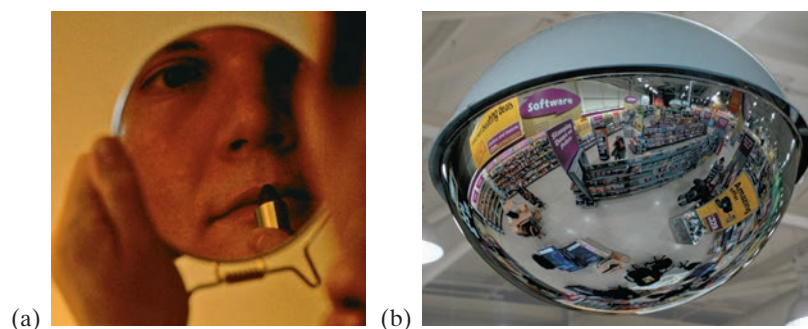


FIGURE 23-11 (a) A concave cosmetic mirror gives a magnified image. (b) A convex mirror in a store reduces image size and so includes a wide field of view. Note the extreme distortion—this mirror has a large curved surface and does not fit the “paraxial ray” approximation discussed on the next page.

Focal Point and Focal Length

To see how spherical mirrors form images, we first consider an object that is very far from a concave mirror. For a distant object, as shown in Fig. 23-12, the rays from each point on the object that strike the mirror will be nearly parallel. *For an object infinitely far away* (the Sun and stars approach this), *the rays would be precisely parallel*.

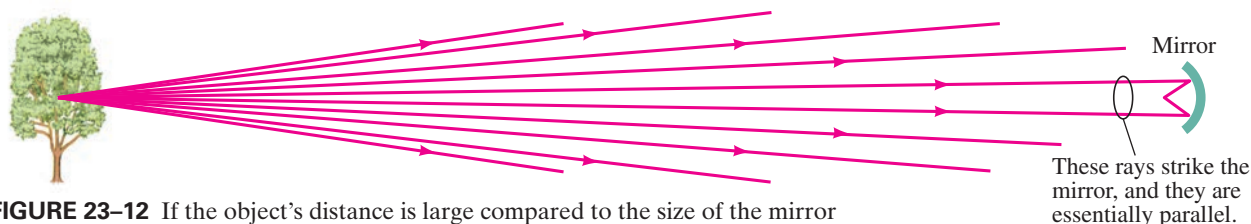


FIGURE 23-12 If the object's distance is large compared to the size of the mirror (or lens), the rays arrive nearly parallel. They are parallel for an object at infinity (∞).

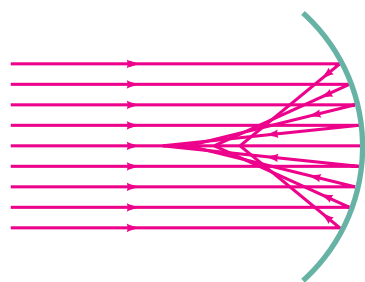
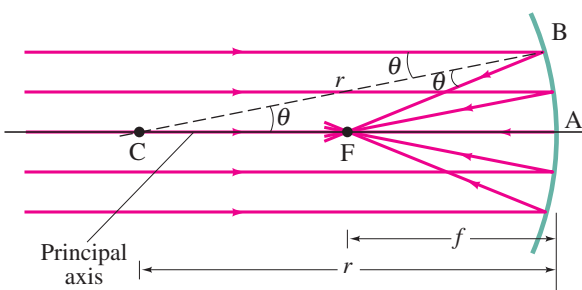


FIGURE 23-13 Parallel rays striking a concave spherical mirror do not intersect (or focus) at precisely a single point. (This “defect” is referred to as “spherical aberration.”)

Now consider such parallel rays falling on a concave mirror as in Fig. 23-13. The law of reflection holds for each of these rays at the point each strikes the mirror. As can be seen, they are not all brought to a single point. In order to form a sharp image, the rays must come to a point. Thus a spherical mirror will not make as sharp an image as a plane mirror will. However, as we show below, if the mirror is small compared to its radius of curvature, so that a reflected ray makes only a *small angle* with the incident ray (2θ in Fig. 23-14), then the rays will cross each other at very nearly a single point, or **focus**. In the case shown in Fig. 23-14, the incoming rays are parallel to the **principal axis**, which is defined as the straight line perpendicular to the curved surface at its center (line CA in Fig. 23-14). The point F, where incident parallel rays come to a focus after reflection, is called the **focal point** of the mirror. The distance between F and the center of the mirror, length FA, is called the **focal length**, f , of the mirror. The focal point is also the *image point for an object infinitely far away* along the principal axis. The image of the Sun, for example, would be at F.

FIGURE 23-14 Rays parallel to the principal axis of a concave spherical mirror come to a focus at F, the focal point, as long as the mirror is small in width as compared to its radius of curvature, r , so that the rays are “paraxial”—that is, make only small angles with the horizontal axis.



Now we will show, for a mirror whose reflecting surface is small compared to its radius of curvature, that the rays very nearly meet at a common point, F, and we will also determine the focal length f . In this approximation, we consider only rays that make a small angle with the principal axis; such rays are called **paraxial rays**, and their angles are exaggerated in Fig. 23-14 to make the labels clear. First we consider a ray that strikes the mirror at B in Fig. 23-14. The point C is the center of curvature of the mirror (the center of the sphere of which the mirror is a part). So the dashed line CB is equal to r , the radius of curvature, and CB is normal to the mirror’s surface at B. The incoming ray that hits the mirror at B makes an angle θ with this normal, and hence the reflected ray, BF, also makes an angle θ with the normal (law of reflection). The angle BCF is also θ , as shown. The triangle CBF is isosceles because two of its angles are equal. Thus length $CF = FB$. We assume the mirror surface is small compared to the mirror’s radius of curvature, so the angles are small, and the length FB is nearly equal to length FA. In this approximation, $FA = FC$. But $FA = f$, the focal length, and $CA = 2 \times FA = r$. Thus the focal length is half the radius of curvature:

$$f = \frac{r}{2}. \quad \text{[spherical mirror] (23-1)}$$

We assumed only that the angle θ was small, so this result applies for all other incident paraxial rays. Thus all paraxial rays pass through the same point F, the focal point.

Since it is only approximately true that the rays come to a perfect focus at F, the more curved the mirror, the worse the approximation (Fig. 23-13) and the more blurred the image. This “defect” of spherical mirrors is called **spherical aberration**; we will discuss it more with regard to lenses in Chapter 25. A **parabolic reflector**, on the other hand, will reflect the rays to a perfect focus. However, because parabolic shapes are much harder to make and thus much more expensive, spherical mirrors are used for most purposes. (Many astronomical telescopes use parabolic reflectors, as do TV satellite dish antennas which concentrate radio waves to nearly a point, Fig. 22-19.) We consider here only spherical mirrors and we will assume that they are small compared to their radius of curvature so that the image is sharp and Eq. 23-1 holds.

Image Formation—Ray Diagrams

We saw that for an object at infinity, the image is located at the focal point of a concave spherical mirror, where $f = r/2$. But where does the image lie for an object not at infinity? First consider the object shown as an arrow in Fig. 23–15a, which is placed between F and C at point O (O for object). Let us determine where the image will be for a given point O' at the top of the object, by finding the point where rays drawn from the tip of the arrow converge after reflecting from the mirror. To do this we can draw several rays and make sure these reflect from the mirror such that the angle of reflection equals the angle of incidence.

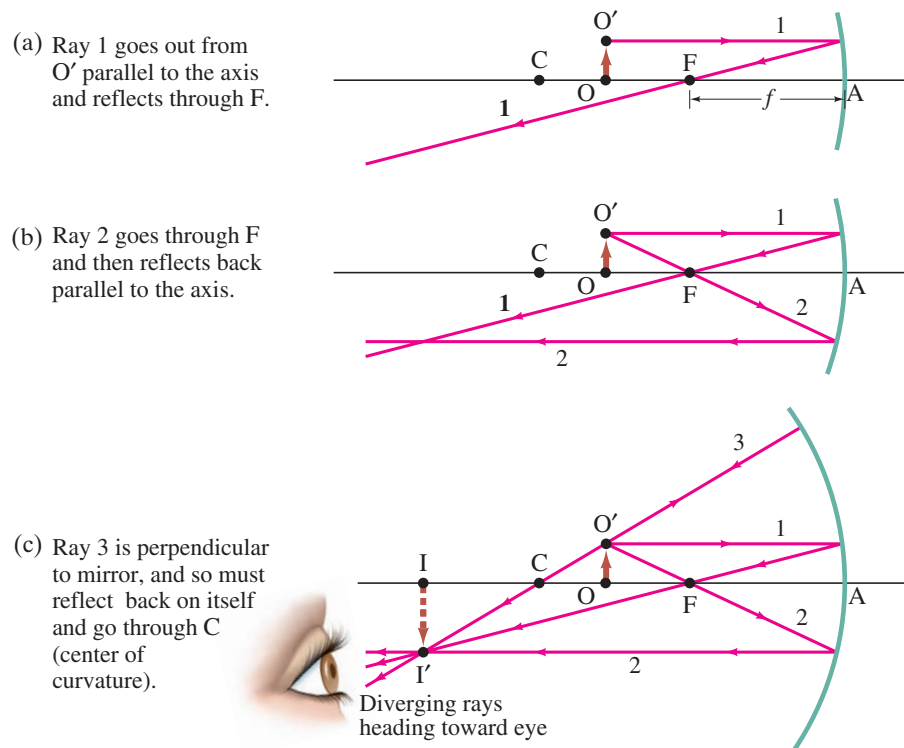


FIGURE 23–15 Rays leave point O' on the object (an arrow). Shown are the three most useful rays for determining where the image I' is formed. [Note that our mirror is not small compared to f , so our diagram will not give the precise position of the image.]

Many rays could be drawn leaving any point on an object, but determining the image position is faster if we deal with three particular rays. These are the rays labeled 1, 2, and 3 in Fig. 23–15 and we draw them leaving object point O' as follows:

Ray 1 leaving O' is drawn parallel to the axis; therefore after reflection it must pass along a line through F, Fig. 23–15a (just as parallel rays did in Fig. 23–14).

Ray 2 leaves O' and is made to pass through F (Fig. 23–15b); therefore it must reflect so it is parallel to the axis. (In reverse, a parallel ray passes through F.)

Ray 3 is drawn along a radius of the spherical surface (Fig. 23–15c) and is perpendicular to the mirror, so it is reflected back on itself and passes through C, the center of curvature.

All three rays leave a single point O' on the object. After reflection from a (small) mirror, the point at which these rays cross is the image point I'. All other rays from the same object point will also pass through this image point. To find the image point for any object point, only these three types of rays need to be drawn. Only two of these rays are needed, but the third serves as a check.

We have shown the image point in Fig. 23–15 only for a single point on the object. Other points on the object are imaged nearby. For instance, the bottom of the arrow, on the principal axis at point O, is imaged on the axis at point I. So a complete image of the object is formed (dashed arrow in Fig. 23–15c). Because the light actually passes through the image, this is a **real image** that will appear on a white surface or film placed there. This can be compared to the virtual image formed by a plane mirror (the light does not pass through that image, Fig. 23–7).

The image in Fig. 23–15 can be seen by the eye only when the eye is placed to the left of the image, so that some of the rays *diverging* from each point on the image (as point I') can enter the eye as shown in Fig. 23–15c (just as in Figs. 23–1 and 23–7).

RAY DIAGRAM

Finding the image position for a curved mirror



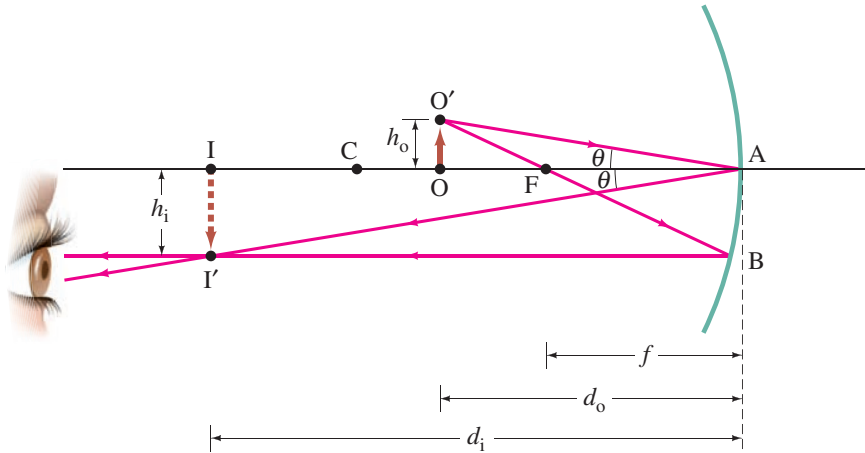
PROBLEM SOLVING

Image point is where reflected rays intersect

Mirror Equation and Magnification

Image points can be determined, roughly, by drawing the three rays as just described, Fig. 23–15. But it is difficult to draw small angles for the “paraxial” rays as we assumed. For more accurate results, we now derive an equation that gives the image distance if the object distance and radius of curvature of the mirror are known. To do this, we refer to Fig. 23–16. The **object distance**, d_o , is the distance of the object (point O) from the center of the mirror. The **image distance**, d_i , is the distance of the image (point I) from the center of the mirror. The height of the object OO' is called h_o and the height of the image, $I'I$, is h_i . Two rays leaving O' are shown: $O'FB I'$ (same as ray 2 in Fig. 23–15) and $O'AI'$, which is a fourth type of ray that reflects at the center of the mirror and can also be used to find an image point.

FIGURE 23–16 Diagram for deriving the mirror equation. For the derivation, we assume the mirror size is small compared to its radius of curvature.



The ray $O'AI'$ obeys the law of reflection, so the two right triangles $O'AO$ and $I'AI$ are similar. Therefore, we have

$$\frac{h_o}{h_i} = \frac{d_o}{d_i}.$$

For the other ray shown, $O'FB I'$, the triangles $O'FO$ and AFB are also similar because the angles at F are equal and we use the approximation $AB = h_i$ (mirror small compared to its radius). Furthermore $FA = f$, the focal length of the mirror, so

$$\frac{h_o}{h_i} = \frac{OF}{FA} = \frac{d_o - f}{f}.$$

The left sides of the two preceding expressions are the same, so we can equate the right sides:

$$\frac{d_o}{d_i} = \frac{d_o - f}{f}.$$

We now divide both sides by d_o and rearrange to obtain

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}. \quad (23-2)$$

This is the equation we were seeking. It is called the **mirror equation** and relates the object and image distances to the focal length f (where $f = r/2$).

The mirror equation also holds for a plane mirror: the focal length is $f = r/2 = \infty$ (Eq. 23–1), and Eq. 23–2 gives $d_i = -d_o$.

The **magnification**, m , of a mirror is defined as the height of the image divided by the height of the object. From our first set of similar triangles in Fig. 23–16, or the first equation just below Fig. 23–16, we can write:

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}. \quad (23-3)$$

The minus sign in Eq. 23–3 is inserted as a convention. Indeed, we must be careful about the signs of all quantities in Eqs. 23–2 and 23–3. Sign conventions are chosen so as to give the correct locations and orientations of images, as predicted by ray diagrams. The **sign conventions** we use are:

1. the image height h_i is positive if the image is upright, and negative if inverted, relative to the object (assuming h_o is taken as positive);
2. d_i or d_o is positive if image or object is in front of the mirror (as in Fig. 23–16); if either image or object is behind the mirror, the corresponding distance is negative. [An example of $d_i < 0$ can be seen in Fig. 23–17, Example 23–6.][†]

Thus the magnification (Eq. 23–3) is positive for an upright image and negative for an inverted image (upside down). We summarize sign conventions more fully in the Problem Solving Strategy following our discussion of convex mirrors later in this Section.

Concave Mirror Examples

EXAMPLE 23–4 Image in a concave mirror. A 1.50-cm-high object is placed 20.0 cm from a concave mirror with radius of curvature 30.0 cm. Determine (a) the position of the image, and (b) its size.

APPROACH We determine the focal length from the radius of curvature (Eq. 23–1), $f = r/2 = 15.0$ cm. The ray diagram is basically the same as Fig. 23–16, since the object is between F and C. The position and size of the image are found from Eqs. 23–2 and 23–3.

SOLUTION Referring to Fig. 23–16, we have $CA = r = 30.0$ cm, $FA = f = 15.0$ cm, and $OA = d_o = 20.0$ cm.

(a) We start with the mirror equation, Eq. 23–2, rearranging it (subtracting $(1/d_o)$ from both sides):

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{15.0 \text{ cm}} - \frac{1}{20.0 \text{ cm}} = 0.0167 \text{ cm}^{-1}.$$

So $d_i = 1/(0.0167 \text{ cm}^{-1}) = 60.0$ cm. Because d_i is positive, the image is 60.0 cm in front of the mirror, on the same side as the object.

(b) From Eq. 23–3, the magnification is

$$m = -\frac{d_i}{d_o} = -\frac{60.0 \text{ cm}}{20.0 \text{ cm}} = -3.00.$$

The image is 3.0 times larger than the object, and its height is

$$h_i = mh_o = (-3.00)(1.5 \text{ cm}) = -4.5 \text{ cm}.$$

The minus sign reminds us that the image is inverted, as shown in Fig. 23–16.

NOTE When an object is further from a concave mirror than the focal point, we can see from Fig. 23–15 or 23–16 that the image is always inverted and real.

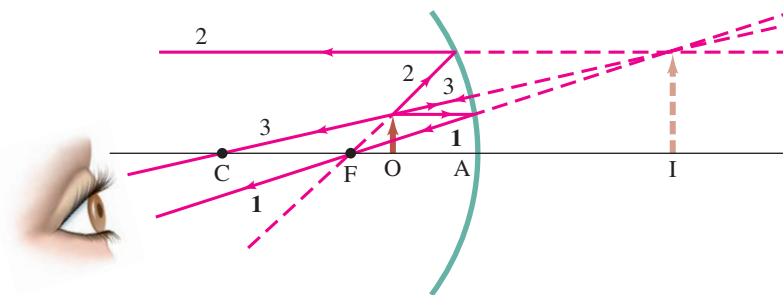
CONCEPTUAL EXAMPLE 23–5 Reversible rays. If the object in Example 23–4 is placed instead where the image is (see Fig. 23–16), where will the new image be?

RESPONSE The mirror equation is *symmetric* in d_o and d_i . Thus the new image will be where the old object was. Indeed, in Fig. 23–16 we need only reverse the direction of the rays to get our new situation.

[†] d_o is always positive for a real object; $d_o < 0$ can happen only if the object is an image formed by another mirror or lens—see Example 23–16.



FIGURE 23–17 Object placed within the focal point F. The image is *behind* the mirror and is *virtual*, Example 23–6. [Note that the vertical scale (height of object = 1.0 cm) is different from the horizontal (OA = 10.0 cm) for ease of drawing, and reduces the precision of the drawing.]



EXAMPLE 23–6 Object closer to concave mirror than focal point.

A 1.00-cm-high object is placed 10.0 cm from a concave mirror whose radius of curvature is 30.0 cm. (a) Draw a ray diagram to locate (approximately) the position of the image. (b) Determine the position of the image and the magnification analytically.

APPROACH We draw the ray diagram using the rays as in Fig. 23–15, page 651. An analytic solution uses Eqs. 23–1, 23–2, and 23–3.

SOLUTION (a) Since $f = r/2 = 15.0$ cm, the object is between the mirror and the focal point. We draw the three rays as described earlier (Fig. 23–15); they are shown leaving the tip of the object in Fig. 23–17. Ray 1 leaves the tip of our object heading toward the mirror parallel to the axis, and reflects through F. Ray 2 cannot head toward F because it would not strike the mirror; so ray 2 must point as if it started at F (dashed line in Fig. 23–17) and heads to the mirror, and then is reflected parallel to the principal axis. Ray 3 is perpendicular to the mirror and reflects back on itself. The rays reflected from the mirror diverge and so never meet at a point. They appear to be coming from a point behind the mirror (dashed lines). This point locates the image of the tip of the arrow. The image is thus behind the mirror and is *virtual*.

(b) We use Eq. 23–2 to find d_i when $d_o = 10.0$ cm:

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{15.0 \text{ cm}} - \frac{1}{10.0 \text{ cm}} = \frac{2 - 3}{30.0 \text{ cm}} = -\frac{1}{30.0 \text{ cm}}.$$

Therefore, $d_i = -30.0$ cm. The minus sign means the image is behind the mirror, which our diagram also showed us. The magnification is $m = -d_i/d_o = -(-30.0 \text{ cm})/(10.0 \text{ cm}) = +3.00$. So the image is 3.00 times larger than the object. The plus sign indicates that the image is upright (same as object), which is consistent with the ray diagram, Fig. 23–17.

NOTE The image distance cannot be obtained accurately by measuring on Fig. 23–17, because our diagram violates the paraxial ray assumption (we draw rays at steeper angles to make them clearly visible).

NOTE When the object is located inside the focal point of a concave mirror ($d_o < f$), the image is always upright and virtual. If the object O in Fig. 23–17 is you, you see yourself clearly, because the reflected rays at point O (you) are diverging. Your image is upright and enlarged. This is how a shaving or cosmetic mirror is used—you must place your head closer to the mirror than the focal point if you are to see yourself right-side up (see the photograph, Fig. 23–11a). [If the object is *beyond* the focal point, as in Fig. 23–15, the image is real and inverted: upside down—and hard to use!]



PHYSICS APPLIED

*Magnifying mirror:
Seeing yourself upright and
magnified in a concave mirror*

Seeing the Image; Seeing Yourself

For a person's eye to see a sharp image, the eye must be at a place where it intercepts diverging rays from points on the image, as is the case for the eye's position in Figs. 23–15, 23–16, and 23–17. When we look at normal objects, we always detect rays diverging toward the eye as shown in Fig. 23–1. (Or, for very distant objects like stars, the rays become essentially parallel, as in Fig. 23–12.)

If you placed your eye between points O and I in Fig. 23–16, for example, *converging* rays from the object OO' would enter your eye and the lens of your eye could not bring them to a focus; you would see a blurry image or no perceptible image at all. [We will discuss the eye more in Chapter 25.]

If *you* are the object OO' in Fig. 23–16, situated between F and C, and are trying to see yourself in the mirror, you would see a blur; but the person whose eye is shown in Fig. 23–16 could see you clearly. If you are to the left of C in Fig. 23–16, where $d_o > 2f$, you can see yourself clearly, but upside down. Why? Because then the rays arriving from the image will be *diverging* at your position (Fig. 23–18), and your eye can then focus them. You can also see yourself clearly, and right side up, if you are closer to the mirror than its focal point ($d_o < f$), as we saw in Example 23–6, Fig. 23–17.

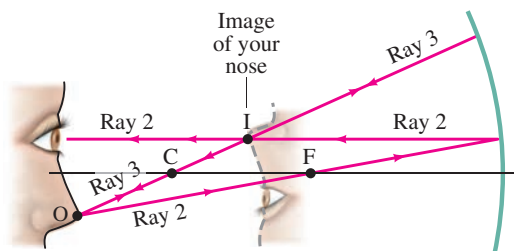


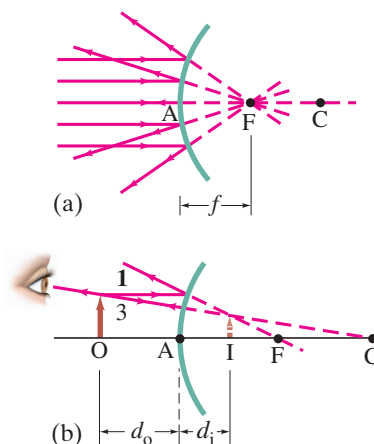
FIGURE 23–18 You can see a clear inverted image of your face in a concave mirror when you are beyond C ($d_o > 2f$), because the rays that arrive at your eye are *diverging*. Standard rays 2 and 3 are shown leaving point O on your nose. Ray 2 (and other nearby rays) enters your eye. Notice that rays are diverging as they move to the left of image point I.

Convex Mirrors

The analysis used for concave mirrors can be applied to **convex** mirrors. Even the mirror equation (Eq. 23–2) holds for a convex mirror, although the quantities involved must be carefully defined. Figure 23–19a shows parallel rays falling on a convex mirror. Again spherical aberration is significant (Fig. 23–13), unless we assume the mirror is small compared to its radius of curvature. The reflected rays diverge, but seem to come from point F behind the mirror, Fig. 23–19a. This is the **focal point**, and its distance from the center of the mirror (point A) is the **focal length**, f . The equation $f = r/2$ is valid also for a convex mirror. We see that an object at infinity produces a virtual image in a convex mirror. Indeed, no matter where the object is placed on the reflecting side of a convex mirror, the image will be virtual and upright, as indicated in Fig. 23–19b. To find the image we draw rays 1 and 3 according to the rules used before on the concave mirror, as shown in Fig. 23–19b. Note that although rays 1 and 3 don't actually pass through points F and C, the line along which each is drawn does (shown dashed).

The mirror equation, Eq. 23–2, holds for convex mirrors but the focal length f and radius of curvature must be considered negative. The proof is left as a Problem. It is also left as a Problem to show that Eq. 23–3 for the magnification is also valid.

FIGURE 23–19 Convex mirror: (a) the focal point is at F, behind the mirror; (b) the image I of the object at O is virtual, upright, and smaller than the object. [Not to scale for Example 23–7.]



PROBLEM SOLVING

Spherical Mirrors

1. Always **draw a ray diagram** even though you are going to make an analytic calculation—the diagram serves as a check, even if not precise. From one point on the object, draw at least two, preferably three, of the easy-to-draw rays using the rules described in Fig. 23–15. The image point is where the reflected rays intersect (real image) or appear to intersect (virtual).
2. Apply the **mirror equation**, Eq. 23–2, and the **magnification equation**, Eq. 23–3. It is crucially important to follow the sign conventions—see the next point.

3. Sign Conventions

- (a) When the object, image, or focal point is on the reflecting side of the mirror (on the left in our drawings), the corresponding distance is positive. If any of these points is behind the mirror (on the right) the corresponding distance is negative.[†]
- (b) The image height h_i is positive if the image is upright, and negative if inverted, relative to the object (h_o is always taken as positive).

4. **Check** that the analytic solution is consistent with the ray diagram.

[†]Object distances are positive for material objects, but can be negative in systems with more than one mirror or lens—see Section 23–9.



FIGURE 23-20 Example 23-7.

EXAMPLE 23-7 Convex rearview mirror. An external rearview car mirror is convex with a radius of curvature of 16.0 m (Fig. 23-20). Determine the location of the image and its magnification for an object 10.0 m from the mirror.

APPROACH We follow the steps of the Problem Solving Strategy explicitly.

SOLUTION

1. Draw a ray diagram. The ray diagram will be like Fig. 23-19b, but the large object distance ($d_o = 10.0$ m) makes a precise drawing difficult. We have a convex mirror, so r is negative by convention.

2. Mirror and magnification equations. The center of curvature of a convex mirror is behind the mirror, as is its focal point, so we set $r = -16.0$ m so that the focal length is $f = r/2 = -8.0$ m. The object is in front of the mirror, $d_o = 10.0$ m. Solving the mirror equation, Eq. 23-2, for $1/d_i$ gives

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{-8.0 \text{ m}} - \frac{1}{10.0 \text{ m}} = \frac{-10.0 - 8.0}{80.0 \text{ m}} = -\frac{18}{80.0 \text{ m}}.$$

Thus $d_i = -80.0 \text{ m}/18 = -4.4$ m. Equation 23-3 gives the magnification

$$m = -\frac{d_i}{d_o} = -\frac{(-4.4 \text{ m})}{(10.0 \text{ m})} = +0.44.$$

3. Sign conventions. The image distance is negative, -4.4 m, so the image is *behind* the mirror. The magnification is $m = +0.44$, so the image is *upright* (same orientation as object, which is useful) and about half what it would be in a plane mirror.

4. Check. Our results are consistent with Fig. 23-19b.

Convex rearview mirrors on vehicles sometimes come with a warning that objects are closer than they appear in the mirror. The fact that d_i may be smaller than d_o (as in Example 23-7) seems to contradict this observation. The real reason the object seems farther away is that its image in the convex mirror is *smaller* than it would be in a plane mirror, and we judge distance of ordinary objects such as other cars mostly by their size.

23-4 Index of Refraction

We saw in Chapter 22 that the speed of light in vacuum (like other EM waves) is

$$c = 2.99792458 \times 10^8 \text{ m/s},$$

which is usually rounded off to

$$3.00 \times 10^8 \text{ m/s}$$

when extremely precise results are not required.

In air, the speed is only slightly less. In other transparent materials, such as glass and water, the speed is always less than that in vacuum. For example, in water light travels at about $\frac{3}{4}c$. The ratio of the speed of light in vacuum to the speed v in a given material is called the **index of refraction**, n , of that material:

$$n = \frac{c}{v}. \quad (23-4)$$

The index of refraction is never less than 1, and values for various materials are given in Table 23-1. For example, since $n = 1.33$ for water, the speed of light in water is

$$v = \frac{c}{n} = \frac{(3.00 \times 10^8 \text{ m/s})}{1.33} = 2.26 \times 10^8 \text{ m/s}.$$

As we shall see later, n varies somewhat with the wavelength of the light—except in vacuum—so a particular wavelength is specified in Table 23-1, that of yellow light with wavelength $\lambda = 589$ nm.

That light travels more slowly in matter than in vacuum can be explained at the atomic level as being due to the absorption and reemission of light by atoms and molecules of the material.

TABLE 23-1 Indices of Refraction[†]

Material	$n = \frac{c}{v}$
Vacuum	1.0000
Air (at STP)	1.0003
Water	1.33
Ethyl alcohol	1.36
Glass	
Fused quartz	1.46
Crown glass	1.52
Light flint	1.58
Plastic	
Acrylic, Lucite, CR-39	1.50
Polycarbonate	1.59
“High-index”	1.6–1.7
Sodium chloride	1.53
Diamond	2.42

[†] $\lambda = 589$ nm.