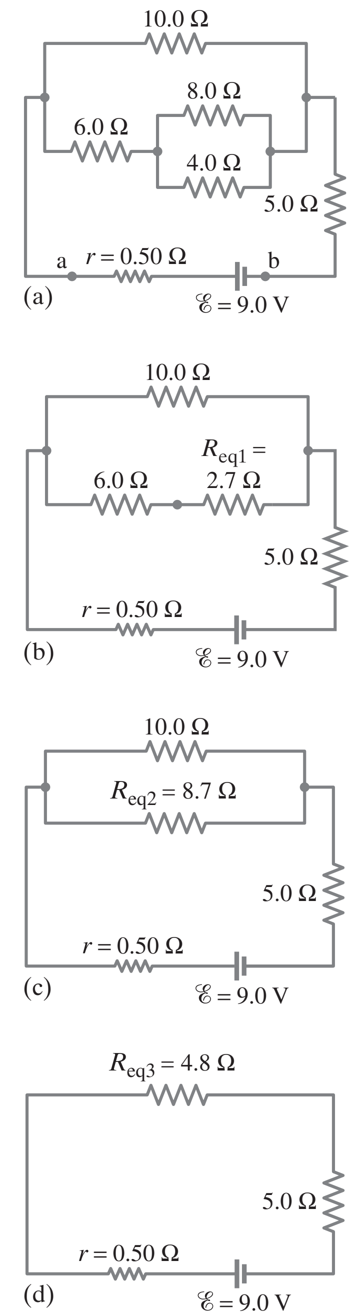


FIGURE 19–10 Circuit for Example 19–7, where r is the internal resistance of the battery.

EXAMPLE 19–7 **Analyzing a circuit.** A 9.0-V battery whose internal resistance r is 0.50 Ω is connected in the circuit shown in Fig. 19–10a. (a) How much current is drawn from the battery? (b) What is the terminal voltage of the battery? (c) What is the current in the 6.0-Ω resistor?

APPROACH To find the current out of the battery, we first need to determine the equivalent resistance R_{eq} of the entire circuit, including r, which we do by identifying and isolating simple series or parallel combinations of resistors. Once we find I from Ohm's law, $I = \mathscr{E}/R_{eq}$, we get the terminal voltage using $V_{ab} = \mathscr{E} - Ir$. For ($c$) we apply Ohm's law to the 6.0-Ω resistor.

SOLUTION (a) We want to determine the equivalent resistance of the circuit. But where do we start? We note that the 4.0-Ω and 8.0-Ω resistors are in parallel, and so have an equivalent resistance R_{eq1} given by

$$\frac{1}{R_{eq1}} = \frac{1}{8.0\,\Omega} + \frac{1}{4.0\,\Omega} = \frac{3}{8.0\,\Omega};$$

so $R_{eq1} = 2.7\,\Omega$. This 2.7 Ω is in series with the 6.0-Ω resistor, as shown in the equivalent circuit of Fig. 19–10b. The net resistance of the lower arm of the circuit is then

$$R_{eq2} = 6.0\,\Omega + 2.7\,\Omega = 8.7\,\Omega,$$

as shown in Fig. 19–10c. The equivalent resistance R_{eq3} of the 8.7-Ω and 10.0-Ω resistances in parallel is given by

$$\frac{1}{R_{eq3}} = \frac{1}{10.0\,\Omega} + \frac{1}{8.7\,\Omega} = 0.21\,\Omega^{-1},$$

so $R_{eq3} = \left(1/0.21\,\Omega^{-1}\right) = 4.8\,\Omega$. This 4.8 Ω is in series with the 5.0-Ω resistor and the 0.50-Ω internal resistance of the battery (Fig. 19–10d), so the total equivalent resistance R_{eq} of the circuit is $R_{eq} = 4.8\,\Omega + 5.0\,\Omega + 0.50\,\Omega = 10.3\,\Omega$. Hence the current drawn is

$$I = \frac{\mathscr{E}}{R_{eq}} = \frac{9.0\text{ V}}{10.3\,\Omega} = 0.87\text{ A}.$$

(b) The terminal voltage of the battery is

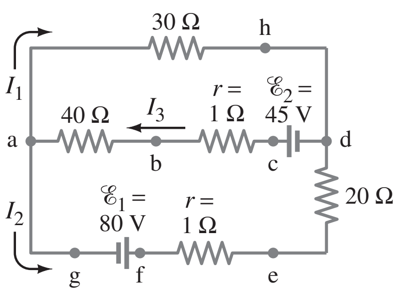
$$V_{ab} = \mathscr{E} - Ir = 9.0\text{ V} - (0.87\text{ A})(0.50\,\Omega) = 8.6\text{ V}.$$

(c) Now we can work back and get the current in the 6.0-Ω resistor. It must be the same as the current through the 8.7 Ω shown in Fig. 19–10c (why?). The voltage across that 8.7 Ω will be the emf of the battery minus the voltage drops across r and the 5.0-Ω resistor: $V_{8.7} = 9.0\text{ V} - (0.87\text{ A})(0.50\,\Omega + 5.0\,\Omega)$. Applying Ohm's law, we get the current (call it I')

$$I' = \frac{9.0\text{ V} - (0.87\text{ A})(0.50\,\Omega + 5.0\,\Omega)}{8.7\,\Omega} = 0.48\text{ A}.$$

This is the current through the 6.0-Ω resistor.

FIGURE 19–11 Currents can be calculated using Kirchhoff's rules.



19–3 Kirchhoff's Rules

In the last few Examples we have been able to find the currents in circuits by combining resistances in series and parallel, and using Ohm's law. This technique can be used for many circuits. However, some circuits are too complicated for that analysis. For example, we cannot find the currents in each part of the circuit shown in Fig. 19–11 simply by combining resistances as we did before.

To deal with complicated circuits, we use Kirchhoff's rules, devised by G. R. Kirchhoff (1824–1887) in the mid-nineteenth century. There are two rules, and they are simply convenient applications of the laws of conservation of charge and energy.

Kirchhoff's first rule or **junction rule** is based on the conservation of electric charge (we already used it to derive the equation for parallel resistors). It states that

> **at any junction point, the sum of all currents entering the junction must equal the sum of all currents leaving the junction.**

Junction rule (conservation of charge)

That is, whatever charge goes in must come out. For example, at the junction point a in Fig. 19–11, I_3 is entering whereas I_1 and I_2 are leaving. Thus Kirchhoff's junction rule states that $I_3 = I_1 + I_2$. We already saw an instance of this in the NOTE at the end of Example 19–5.

Kirchhoff's second rule or **loop rule** is based on the conservation of energy. It states that

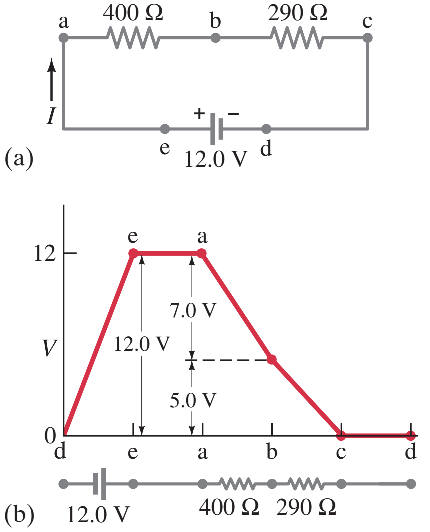
> **the sum of the changes in potential around any closed loop of a circuit must be zero.**

Loop rule (conservation of energy)

To see why this rule should hold, consider a rough analogy with the potential energy of a roller coaster on its track. When the roller coaster starts from the station, it has a particular potential energy. As it is pulled up the first hill, its gravitational potential energy increases and reaches a maximum at the top. As it descends the other side, its potential energy decreases and reaches a local minimum at the bottom of the hill. As the roller coaster continues on its up and down path, its potential energy goes through more changes. But when it arrives back at the starting point, it has exactly as much potential energy as it had when it started at this point. Another way of saying this is that there was as much uphill as there was downhill.

Similar reasoning can be applied to an electric circuit. We will analyze the circuit of Fig. 19–11 shortly, but first we consider the simpler circuit in Fig. 19–12. We have chosen it to be the same as the equivalent circuit of Fig. 19–7b already discussed. The current in this circuit is $I = (12.0\ \text{V})/(690\ \Omega) = 0.0174\ \text{A}$, as we calculated in Example 19–4. (We keep an extra digit in I to reduce rounding errors.) The positive side of the battery, point e in Fig. 19–12a, is at a high potential compared to point d at the negative side of the battery. That is, point e is like the top of a hill for a roller coaster. We follow the current around the circuit starting at any point. We choose to start at point d and follow a small positive test charge completely around this circuit. As we go, we note all changes in potential. When the test charge returns to point d, the potential will be the same as when we started (total change in potential around the circuit is zero). We plot the changes in potential around the circuit in Fig. 19–12b; point d is arbitrarily taken as zero.

FIGURE 19–12 Changes in potential around the circuit in (a) are plotted in (b).



As our positive test charge goes from point d, which is the negative or low potential side of the battery, to point e, which is the positive terminal (high potential side) of the battery, the potential increases by 12.0 V. (This is like the roller coaster being pulled up the first hill.) That is,

$$V_{ed} = +12.0\ \text{V}.$$

When our test charge moves from point e to point a, there is no change in potential because there is no source of emf and negligible resistance in the connecting wires.

Next, as the charge passes through the 400-Ω resistor to get to point b, there is a decrease in potential of $V = IR = (0.0174\ \text{A})(400\ \Omega) = 7.0\ \text{V}$. The positive test charge is flowing "downhill" since it is heading toward the negative terminal of the battery, as indicated in the graph of Fig. 19–12b. Because this is a *decrease* in potential, we use a *negative* sign:

$$V_{ba} = V_b - V_a = -7.0\ \text{V}.$$

PROBLEM SOLVING

Be consistent with signs when applying the loop rule

As the charge proceeds from b to c there is another potential decrease (a "voltage drop") of $(0.0174\ \text{A}) \times (290\ \Omega) = 5.0\ \text{V}$, and this too is a decrease in potential:

$$V_{cb} = -5.0\ \text{V}.$$

There is no change in potential as our test charge moves from c to d as we assume negligible resistance in the wires.

The sum of all the changes in potential around the circuit of Fig. 19–12 is

$$+12.0\ \text{V} - 7.0\ \text{V} - 5.0\ \text{V} = 0.$$

This is exactly what Kirchhoff's loop rule said it would be.

PROBLEM SOLVING

Kirchhoff's Rules

1. **Label the current** in each separate branch of the given circuit with a different subscript, such as I_1, I_2, I_3 (see Fig. 19–11 or 19–13). Each current refers to a segment between two junctions. Choose the direction of each current, using an arrow. The direction can be chosen arbitrarily: if the current is actually in the opposite direction, it will come out with a minus sign in the solution.

2. **Identify the unknowns**. You will need as many independent equations as there are unknowns. You may write down more equations than this, but you will find that some of the equations will be redundant (that is, not be independent in the sense of providing new information). You may use $V = IR$ for each resistor, which sometimes will reduce the number of unknowns.

3. **Apply Kirchhoff's junction rule** at one or more junctions.

4. **Apply Kirchhoff's loop rule** for one or more loops: follow each loop in one direction only. Pay careful attention to subscripts, and to signs:
   (a) For a resistor, apply Ohm's law; the potential difference is negative (a decrease) if your chosen loop direction is the same as the chosen current direction through that resistor. The potential difference is positive (an increase) if your chosen loop direction is *opposite* to the chosen current direction.
   (b) For a battery, the potential difference is positive if your chosen loop direction is from the negative terminal toward the positive terminal; the potential difference is negative if the loop direction is from the positive terminal toward the negative terminal.

5. **Solve the equations** algebraically for the unknowns. Be careful with signs. At the end, check your answers by plugging them into the original equations, or even by using any additional loop or junction rule equations not used previously.

EXAMPLE 19–8 **Using Kirchhoff's rules.** Calculate the currents I_1, I_2, and I_3 in the three branches of the circuit in Fig. 19–13 (which is the same as Fig. 19–11).

APPROACH and SOLUTION

PROBLEM SOLVING
Choose current directions arbitrarily

1. **Label the currents** and their directions. Figure 19–13 uses the labels I_1, I_2, and I_3 for the current in the three separate branches. Since (positive) current tends to move away from the positive terminal of a battery, we choose I_2 and I_3 to have the directions shown in Fig. 19–13. The direction of I_1 is not obvious in advance, so we arbitrarily chose the direction indicated. If the current actually flows in the opposite direction, our answer will have a negative sign.

2. **Identify the unknowns**. We have three unknowns (I_1, I_2, and I_3) and therefore we need three equations, which we get by applying Kirchhoff's junction and loop rules.

3. **Junction rule**: We apply Kirchhoff's junction rule to the currents at point a, where I_3 enters and I_2 and I_1 leave:

$$I_3 = I_1 + I_2. \qquad \textbf{(i)}$$

This same equation holds at point d, so we get no new information by writing an equation for point d.

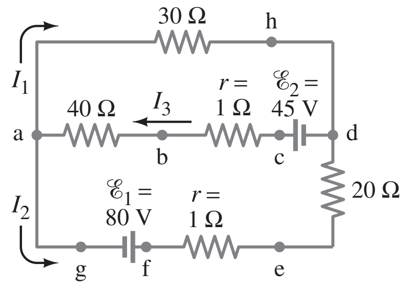


FIGURE 19–13 Currents can be calculated using Kirchhoff's rules. See Example 19–8.

4. Loop rule: We apply Kirchhoff's loop rule to two different closed loops. First we apply it to the upper loop ahdcba. We start (and end) at point a. From a to h we have a potential decrease $V_{ha} = -(I_1)(30\,\Omega)$. From h to d there is no change, but from d to c the potential increases by 45 V: that is, $V_{cd} = +45\text{ V}$. From c to a the potential decreases through the two resistances by an amount $V_{ac} = -(I_3)(40\,\Omega + 1\,\Omega) = -(41\,\Omega)I_3$. Thus we have $V_{ha} + V_{cd} + V_{ac} = 0$, or

$$-30I_1 + 45 - 41I_3 = 0, \qquad \textbf{(ii)}$$

where we have omitted the units (volts and amps) so we can more easily see the algebra. For our second loop, we take the outer loop ahdefga. (We could have chosen the lower loop abcdefga instead.) Again we start at point a, and going to point h we have $V_{ha} = -(I_1)(30\,\Omega)$. Next, $V_{dh} = 0$. But when we take our positive test charge from d to e, it actually is going uphill, against the current—or at least against the *assumed* direction of the current, which is what counts in this calculation. Thus $V_{ed} = +I_2(20\,\Omega)$ has a *positive* sign. Similarly, $V_{fe} = +I_2(1\,\Omega)$. From f to g there is a decrease in potential of 80 V because we go from the high potential terminal of the battery to the low. Thus $V_{gf} = -80\text{ V}$. Finally, $V_{ag} = 0$, and the sum of the potential changes around this loop is

$$-30I_1 + (20 + 1)I_2 - 80 = 0. \qquad \textbf{(iii)}$$

Our major work is done. The rest is algebra.

PROBLEM SOLVING
Be consistent with signs when applying the loop rule

5. Solve the equations. We have three equations—labeled (i), (ii), and (iii)—and three unknowns. From Eq. (iii) we have

$$I_2 = \frac{80 + 30I_1}{21} = 3.8 + 1.4I_1. \qquad \textbf{(iv)}$$

From Eq. (ii) we have

$$I_3 = \frac{45 - 30I_1}{41} = 1.1 - 0.73I_1. \qquad \textbf{(v)}$$

We substitute Eqs. (iv) and (v) into Eq. (i):

$$I_1 = I_3 - I_2 = 1.1 - 0.73I_1 - 3.8 - 1.4I_1.$$

We solve for I_1, collecting terms:

$$3.1I_1 = -2.7$$
$$I_1 = -0.87\text{ A}.$$

The negative sign indicates that the direction of I_1 is actually opposite to that initially assumed and shown in Fig. 19–13. The answer automatically comes out in amperes because our voltages and resistances were in volts and ohms. From Eq. (iv) we have

PROBLEM SOLVING
I_1 is in the opposite direction from that assumed in Fig. 19–13

$$I_2 = 3.8 + 1.4I_1 = 3.8 + 1.4(-0.87) = 2.6\text{ A},$$

and from Eq. (v)

$$I_3 = 1.1 - 0.73I_1 = 1.1 - 0.73(-0.87) = 1.7\text{ A}.$$

This completes the solution.

NOTE The unknowns in different situations are not necessarily currents. It might be that the currents are given and we have to solve for unknown resistance or voltage. The variables are then different, but the technique is the same.

EXERCISE D Write the Kirchhoff equation for the lower loop abcdefga of Example 19–8 and show, assuming the currents calculated in this Example, that the potentials add to zero for this lower loop.