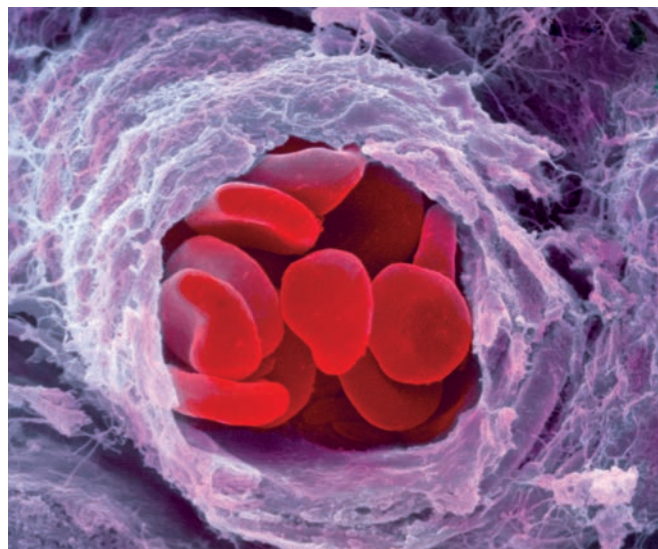


Electron microscopes (EM) produce images using electrons which have wave properties just as light does. Because the wavelength of electrons can be much smaller than that of visible light, much greater resolution and magnification can be obtained. A scanning electron microscope (SEM) can produce images with a three-dimensional quality.

All EM images are monochromatic (black and white). Artistic coloring has been added here, as is common. On the left is an SEM image of a blood clot forming (yellow-color web) due to a wound. White blood cells are colored green here for visibility. On the right, red blood cells in a small artery. A red blood cell travels about 15 km a day inside our bodies and lives roughly 4 months before damage or rupture. Humans contain 4 to 6 liters of blood, and  $2 \text{ to } 3 \times 10^{13}$  red blood cells.



# Early Quantum Theory and Models of the Atom

## CHAPTER 27

### CHAPTER-OPENING QUESTION—Guess now!

It has been found experimentally that

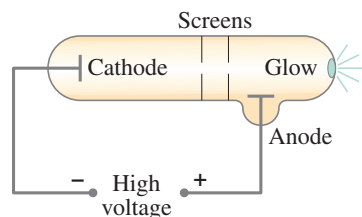
- (a) light behaves as a wave.
- (b) light behaves as a particle.
- (c) electrons behave as particles.
- (d) electrons behave as waves.
- (e) all of the above are true.
- (f) only (a) and (b) are true.
- (g) only (a) and (c) are true.
- (h) none of the above are true.

The second aspect of the revolution that shook the world of physics in the early part of the twentieth century was the quantum theory (the other was Einstein's theory of relativity). Unlike the special theory of relativity, the revolution of quantum theory required almost three decades to unfold, and many scientists contributed to its development. It began in 1900 with Planck's quantum hypothesis, and culminated in the mid-1920s with the theory of quantum mechanics of Schrödinger and Heisenberg which has been so effective in explaining the structure of matter. The discovery of the electron in the 1890s, with which we begin this Chapter, might be said to mark the beginning of modern physics, and is a sort of precursor to the quantum theory.

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## 27-1 Discovery and Properties of the Electron

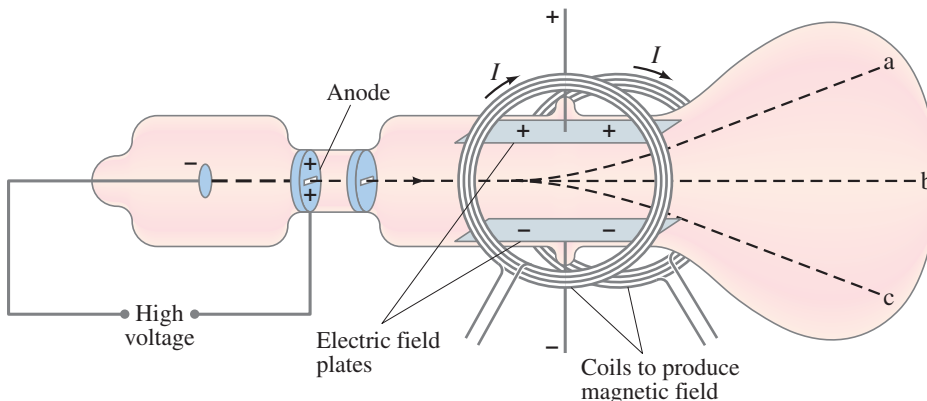


**FIGURE 27-1** Discharge tube. In some models, one of the screens is the anode (positive plate).

Toward the end of the nineteenth century, studies were being done on the discharge of electricity through rarefied gases. One apparatus, diagrammed in Fig. 27-1, was a glass tube fitted with electrodes and evacuated so only a small amount of gas remained inside. When a very high voltage was applied to the electrodes, a dark space seemed to extend outward from the cathode (negative electrode) toward the opposite end of the tube; and that far end of the tube would glow. If one or more screens containing a small hole were inserted as shown, the glow was restricted to a tiny spot on the end of the tube. It seemed as though something being emitted by the cathode traveled across to the opposite end of the tube. These “somethings” were named **cathode rays**.

There was much discussion at the time about what these rays might be. Some scientists thought they might resemble light. But the observation that the bright spot at the end of the tube could be deflected to one side by an electric or magnetic field suggested that cathode rays were charged particles; and the direction of the deflection was consistent with a negative charge. Furthermore, if the tube contained certain types of rarefied gas, the path of the cathode rays was made visible by a slight glow.

Estimates of the charge  $e$  of the cathode-ray particles, as well as of their charge-to-mass ratio  $e/m$ , had been made by 1897. But in that year, J. J. Thomson (1856–1940) was able to measure  $e/m$  directly, using the apparatus shown in Fig. 27-2. Cathode rays are accelerated by a high voltage and then pass between a pair of parallel plates built into the tube. Another voltage applied to the parallel plates produces an electric field  $\vec{E}$ , and a pair of coils produces a magnetic field  $\vec{B}$ . If  $E = B = 0$ , the cathode rays follow path b in Fig. 27-2.



**FIGURE 27-2** Cathode rays deflected by electric and magnetic fields. (See also Section 17-11 on the CRT.)

When only the electric field is present, say with the upper plate positive, the cathode rays are deflected upward as in path a in Fig. 27-2. If only a magnetic field exists, say inward, the rays are deflected downward along path c. These observations are just what is expected for a negatively charged particle. The force on the rays due to the magnetic field is  $F = evB$ , where  $e$  is the charge and  $v$  is the velocity of the cathode rays (Eq. 20-4). In the absence of an electric field, the rays are bent into a curved path, and applying Newton's second law  $F = ma$  with  $a$  = centripetal acceleration gives

$$evB = m \frac{v^2}{r},$$

and thus

$$\frac{e}{m} = \frac{v}{Br}.$$

The radius of curvature  $r$  can be measured and so can  $B$ . The velocity  $v$  can be found by applying an electric field in addition to the magnetic field. The electric

field  $E$  is adjusted so that the cathode rays are undeflected and follow path b in Fig. 27–2. In this situation the upward force due to the electric field,  $F = eE$ , is balanced by the downward force due to the magnetic field,  $F = evB$ . We equate the two forces,  $eE = evB$ , and find

$$v = \frac{E}{B}.$$

Combining this with the above equation we have

$$\frac{e}{m} = \frac{E}{B^2 r}. \quad (27-1)$$

The quantities on the right side can all be measured, and although  $e$  and  $m$  could not be determined separately, the ratio  $e/m$  could be determined. The accepted value today is  $e/m = 1.76 \times 10^{11} \text{ C/kg}$ . Cathode rays soon came to be called **electrons**.

## Discovery in Science

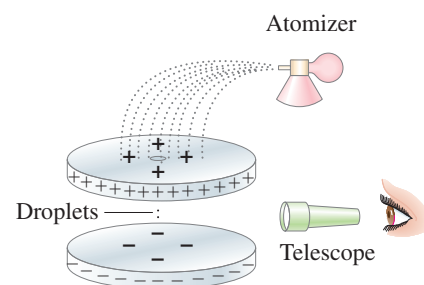
The “discovery” of the electron, like many others in science, is not quite so obvious as discovering gold or oil. Should the discovery of the electron be credited to the person who first saw a glow in the tube? Or to the person who first called them cathode rays? Perhaps neither one, for they had no conception of the electron as we know it today. In fact, the credit for the discovery is generally given to Thomson, but not because he was the first to see the glow in the tube. Rather it is because he believed that this phenomenon was due to tiny negatively charged particles and made careful measurements on them. Furthermore he argued that these particles were constituents of atoms, and not ions or atoms themselves as many thought, and he developed an electron theory of matter. His view is close to what we accept today, and this is why Thomson is credited with the “discovery.” Note, however, that neither he nor anyone else ever actually saw an electron itself. We discuss this briefly, for it illustrates the fact that discovery in science is not always a clear-cut matter. In fact some philosophers of science think the word “discovery” is often not appropriate, such as in this case.

## Electron Charge Measurement

Thomson believed that an electron was not an atom, but rather a constituent, or part, of an atom. Convincing evidence for this came soon with the determination of the charge and the mass of the cathode rays. Thomson’s student J. S. Townsend made the first direct (but rough) measurements of  $e$  in 1897. But it was the more refined **oil-drop experiment** of Robert A. Millikan (1868–1953) that yielded a precise value for the charge on the electron and showed that charge comes in discrete amounts. In this experiment, tiny droplets of mineral oil carrying an electric charge were allowed to fall under gravity between two parallel plates, Fig. 27–3. The electric field  $E$  between the plates was adjusted until the drop was suspended in midair. The downward pull of gravity,  $mg$ , was then just balanced by the upward force due to the electric field. Thus  $qE = mg$  so the charge  $q = mg/E$ . The mass of the droplet was determined by measuring its terminal velocity in the absence of the electric field. Often the droplet was charged negatively, but sometimes it was positive, suggesting that the droplet had acquired or lost electrons (by friction, leaving the atomizer). Millikan’s painstaking observations and analysis presented convincing evidence that any charge was an integral multiple of a smallest charge,  $e$ , that was ascribed to the electron, and that the value of  $e$  was  $1.6 \times 10^{-19} \text{ C}$ . This value of  $e$ , combined with the measurement of  $e/m$ , gives the mass of the electron to be  $(1.6 \times 10^{-19} \text{ C}) / (1.76 \times 10^{11} \text{ C/kg}) = 9.1 \times 10^{-31} \text{ kg}$ . This mass is less than a thousandth the mass of the smallest atom, and thus confirmed the idea that the electron is only a part of an atom. The accepted value today for the mass of the electron is

$$m_e = 9.11 \times 10^{-31} \text{ kg}.$$

The experimental result that any charge is an integral multiple of  $e$  means that electric charge is *quantized* (exists only in discrete amounts).



**FIGURE 27–3** Millikan’s oil-drop experiment.

## 27-2 Blackbody Radiation; Planck's Quantum Hypothesis

### Blackbody Radiation

One of the observations that was unexplained at the end of the nineteenth century was the spectrum of light emitted by hot objects. We saw in Section 14-8 that all objects emit radiation whose total intensity is proportional to the fourth power of the Kelvin (absolute) temperature ( $T^4$ ). At normal temperatures ( $\approx 300$  K), we are not aware of this electromagnetic radiation because of its low intensity. At higher temperatures, there is sufficient infrared radiation that we can feel heat if we are close to the object. At still higher temperatures (on the order of 1000 K), objects actually glow, such as a red-hot electric stove burner or the heating element in a toaster. At temperatures above 2000 K, objects glow with a yellow or whitish color, such as white-hot iron and the filament of a lightbulb. The light emitted contains a continuous range of wavelengths or frequencies, and the spectrum is a plot of intensity vs. wavelength or frequency. As the temperature increases, the electromagnetic radiation emitted by objects not only increases in total intensity but has its peak intensity at higher and higher frequencies.

The spectrum of light emitted by a hot dense object is shown in Fig. 27-4 for an idealized **blackbody**. A blackbody is a body that, when cool, would absorb all the radiation falling on it (and so would appear black under reflection when illuminated by other sources). The radiation such an idealized blackbody would emit when hot and luminous, called **blackbody radiation** (though not necessarily black in color), approximates that from many real objects. The 6000-K curve in Fig. 27-4, corresponding to the temperature of the surface of the Sun, peaks in the visible part of the spectrum. For lower temperatures, the total intensity drops considerably and the peak occurs at longer wavelengths (or lower frequencies). This is why objects glow with a red color at around 1000 K. It is found experimentally that the wavelength at the peak of the spectrum,  $\lambda_p$ , is related to the Kelvin temperature  $T$  by

$$\lambda_p T = 2.90 \times 10^{-3} \text{ m} \cdot \text{K}. \quad (27-2)$$

This is known as **Wien's law**.

**EXAMPLE 27-1 The Sun's surface temperature.** Estimate the temperature of the surface of our Sun, given that the Sun emits light whose peak intensity occurs in the visible spectrum at around 500 nm.

**APPROACH** We assume the Sun acts as a blackbody, and use  $\lambda_p = 500$  nm in Wien's law (Eq. 27-2).

**SOLUTION** Wien's law gives

$$T = \frac{2.90 \times 10^{-3} \text{ m} \cdot \text{K}}{\lambda_p} = \frac{2.90 \times 10^{-3} \text{ m} \cdot \text{K}}{500 \times 10^{-9} \text{ m}} \approx 6000 \text{ K}.$$

**EXAMPLE 27-2 Star color.** Suppose a star has a surface temperature of 32,500 K. What color would this star appear?

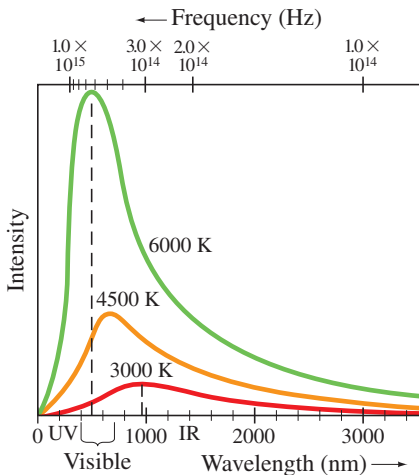
**APPROACH** We assume the star emits radiation as a blackbody, and solve for  $\lambda_p$  in Wien's law, Eq. 27-2.

**SOLUTION** From Wien's law we have

$$\lambda_p = \frac{2.90 \times 10^{-3} \text{ m} \cdot \text{K}}{T} = \frac{2.90 \times 10^{-3} \text{ m} \cdot \text{K}}{3.25 \times 10^4 \text{ K}} = 89.2 \text{ nm}.$$

The peak is in the UV range of the spectrum, and will be way to the left in Fig. 27-4. In the visible region, the curve will be descending, so the shortest visible wavelengths will be strongest. Hence the star will appear bluish (or blue-white).

**NOTE** This example helps us to understand why stars have different colors (reddish for the coolest stars; orangish, yellow, white, bluish for "hotter" stars.)



**FIGURE 27-4** Measured spectra of wavelengths and frequencies emitted by a blackbody at three different temperatures.



## Planck's Quantum Hypothesis

In the year 1900, Max Planck (1858–1947) proposed a theory that was able to reproduce the graphs of Fig. 27–4. His theory, still accepted today, made a new and radical assumption: that the energy of the oscillations of atoms within molecules cannot have just any value; instead each has energy which is a multiple of a minimum value related to the frequency of oscillation by

$$E = hf.$$

Here  $h$  is a new constant, now called **Planck's constant**, whose value was estimated by Planck by fitting his formula for the blackbody radiation curve to experiment. The value accepted today is

$$h = 6.626 \times 10^{-34} \text{ J}\cdot\text{s}.$$

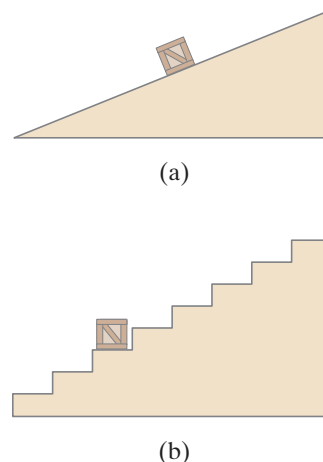
Planck's assumption suggests that the energy of any molecular vibration could be only a whole number multiple of  $hf$ :

$$E = nhf, \quad n = 1, 2, 3, \dots, \quad (27-3)$$

where  $n$  is called a **quantum number** (“quantum” means “discrete amount” as opposed to “continuous”). This idea is often called **Planck's quantum hypothesis**, although little attention was brought to this point at the time. In fact, it appears that Planck considered it more as a mathematical device to get the “right answer” rather than as an important discovery. Planck himself continued to seek a classical explanation for the introduction of  $h$ . The recognition that this was an important and radical innovation did not come until later, after about 1905 when others, particularly Einstein, entered the field.

The quantum hypothesis, Eq. 27–3, states that the energy of an oscillator can be  $E = hf$ , or  $2hf$ , or  $3hf$ , and so on, but there cannot be vibrations with energies between these values. That is, energy would not be a continuous quantity as had been believed for centuries; rather it is **quantized**—it exists only in discrete amounts. The smallest amount of energy possible ( $hf$ ) is called the **quantum of energy**. Recall from Chapter 11 that the energy of an oscillation is proportional to the amplitude squared. Another way of expressing the quantum hypothesis is that not just any amplitude of vibration is possible. The possible values for the amplitude are related to the frequency  $f$ .

A simple analogy may help. Compare a ramp, on which a box can be placed at any height, to a flight of stairs on which the box can have only certain discrete amounts of potential energy, as shown in Fig. 27–5.



**FIGURE 27-5** Ramp versus stair analogy. (a) On a ramp, a box can have continuous values of potential energy. (b) But on stairs, the box can have only discrete (quantized) values of energy.

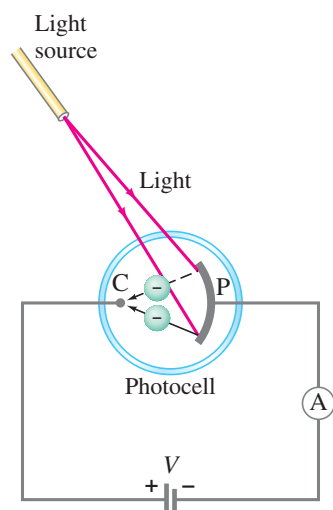
## 27–3 Photon Theory of Light and the Photoelectric Effect

In 1905, the same year that he introduced the special theory of relativity, Einstein made a bold extension of the quantum idea by proposing a new theory of light. Planck's work had suggested that the vibrational energy of molecules in a radiating object is quantized with energy  $E = nhf$ , where  $n$  is an integer and  $f$  is the frequency of molecular vibration. Einstein argued that when light is emitted by a molecular oscillator, the molecule's vibrational energy of  $nhf$  must decrease by an amount  $hf$  (or by  $2hf$ , etc.) to another integer times  $hf$ , such as  $(n - 1)hf$ . Then to conserve energy, the light ought to be emitted in packets, or *quanta*, each with an energy

$$E = hf, \quad (27-4)$$

*Photon energy*

where  $f$  is here the frequency of the emitted light. Again  $h$  is Planck's constant. Because all light ultimately comes from a radiating source, this idea suggests that *light is transmitted as tiny particles*, or **photons** as they are now called, as well as via the waves predicted by Maxwell's electromagnetic theory. The photon theory of light was also a radical departure from classical ideas. Einstein proposed a test of the quantum theory of light: quantitative measurements on the photoelectric effect.



**FIGURE 27-6** The photoelectric effect.

When light shines on a metal surface, electrons are found to be emitted from the surface. This effect is called the **photoelectric effect** and it occurs in many materials, but is most easily observed with metals. It can be observed using the apparatus shown in Fig. 27-6. A metal plate P and a smaller electrode C are placed inside an evacuated glass tube, called a **photocell**. The two electrodes are connected to an ammeter and a source of emf, as shown. When the photocell is in the dark, the ammeter reads zero. But when light of sufficiently high frequency illuminates the plate, the ammeter indicates a current flowing in the circuit. We explain completion of the circuit by imagining that electrons, ejected from the plate by the impinging light, flow across the tube from the plate to the “collector” C as indicated in Fig. 27-6.

That electrons should be emitted when light shines on a metal is consistent with the electromagnetic (EM) wave theory of light: the electric field of an EM wave could exert a force on electrons in the metal and eject some of them. Einstein pointed out, however, that the wave theory and the photon theory of light give very different predictions on the details of the photoelectric effect. For example, one thing that can be measured with the apparatus of Fig. 27-6 is the maximum kinetic energy ( $KE_{\max}$ ) of the emitted electrons. This can be done by using a variable voltage source and reversing the terminals so that electrode C is negative and P is positive. The electrons emitted from P will be repelled by the negative electrode, but if this reverse voltage is small enough, the fastest electrons will still reach C and there will be a current in the circuit. If the reversed voltage is increased, a point is reached where the current reaches zero—no electrons have sufficient kinetic energy to reach C. This is called the *stopping potential*, or *stopping voltage*,  $V_0$ , and from its measurement,  $KE_{\max}$  can be determined using conservation of energy (loss of kinetic energy = gain in potential energy):

$$KE_{\max} = eV_0.$$

Now let us examine the details of the photoelectric effect from the point of view of the wave theory versus Einstein’s particle theory.

First the wave theory, assuming monochromatic light. The two important properties of a light wave are its intensity and its frequency (or wavelength). When these two quantities are varied, the wave theory makes the following predictions:

- Wave theory predictions*
1. If the light intensity is increased, the number of electrons ejected and their maximum kinetic energy should be increased because the higher intensity means a greater electric field amplitude, and the greater electric field should eject electrons with higher speed.
  2. The frequency of the light should not affect the kinetic energy of the ejected electrons. Only the intensity should affect  $KE_{\max}$ .

The photon theory makes completely different predictions. First we note that in a monochromatic beam, all photons have the same energy ( $= hf$ ). Increasing the intensity of the light beam means increasing the number of photons in the beam, but does not affect the energy of each photon as long as the frequency is not changed. According to Einstein’s theory, an electron is ejected from the metal by a collision with a single photon. In the process, all the photon energy is transferred to the electron and the photon ceases to exist. Since electrons are held in the metal by attractive forces, some minimum energy  $W_0$  is required just to get an electron out through the surface.  $W_0$  is called the **work function**, and is a few electron volts ( $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$ ) for most metals. If the frequency  $f$  of the incoming light is so low that  $hf$  is less than  $W_0$ , then the photons will not have enough energy to eject any electrons at all. If  $hf > W_0$ , then electrons will be ejected and energy will be conserved in the process. That is, the input energy (of the photon),  $hf$ , will equal the outgoing kinetic energy KE of the electron plus the energy required to get it out of the metal,  $W$ :

$$hf = KE + W. \quad (27-5a)$$

The least tightly held electrons will be emitted with the most kinetic energy ( $KE_{\max}$ ),

in which case  $W$  in this equation becomes the work function  $W_0$ , and KE becomes  $\text{KE}_{\text{max}}$ :

$$hf = \text{KE}_{\text{max}} + W_0. \quad [\text{least bound electrons}] \quad (27-5b)$$

Many electrons will require more energy than the bare minimum ( $W_0$ ) to get out of the metal, and thus the kinetic energy of such electrons will be less than the maximum.

From these considerations, the photon theory makes the following predictions:

1. An increase in intensity of the light beam means more photons are incident, so more electrons will be ejected; but since the energy of each photon is not changed, the maximum kinetic energy of electrons is not changed by an increase in intensity.
2. If the frequency of the light is increased, the maximum kinetic energy of the electrons increases linearly, according to Eq. 27-5b. That is,

$$\text{KE}_{\text{max}} = hf - W_0.$$

This relationship is plotted in Fig. 27-7.

3. If the frequency  $f$  is less than the “cutoff” frequency  $f_0$ , where  $hf_0 = W_0$ , no electrons will be ejected, no matter how great the intensity of the light.

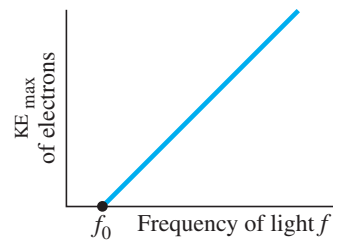
These predictions of the photon theory are very different from the predictions of the wave theory. In 1913–1914, careful experiments were carried out by R. A. Millikan. The results were fully in agreement with Einstein’s photon theory.

One other aspect of the photoelectric effect also confirmed the photon theory. If extremely low light intensity is used, the wave theory predicts a time delay before electron emission so that an electron can absorb enough energy to exceed the work function. The photon theory predicts no such delay—it only takes one photon (if its frequency is high enough) to eject an electron—and experiments showed no delay. This too confirmed Einstein’s photon theory.

Photon

theory

predictions



**FIGURE 27-7** Photoelectric effect: the maximum kinetic energy of ejected electrons increases linearly with the frequency of incident light. No electrons are emitted if  $f < f_0$ .

**EXAMPLE 27-3 Photon energy.** Calculate the energy of a photon of blue light,  $\lambda = 450 \text{ nm}$  in air (or vacuum).

**APPROACH** The photon has energy  $E = hf$  (Eq. 27-4) where  $f = c/\lambda$  (Eq. 22-4).

**SOLUTION** Since  $f = c/\lambda$ , we have

$$E = hf = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})(3.00 \times 10^8 \text{ m/s})}{(4.5 \times 10^{-7} \text{ m})} = 4.4 \times 10^{-19} \text{ J},$$

or  $(4.4 \times 10^{-19} \text{ J})/(1.60 \times 10^{-19} \text{ J/eV}) = 2.8 \text{ eV}$ . (See definition of eV in Section 17-4,  $1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$ .)

**EXAMPLE 27-4 ESTIMATE Photons from a lightbulb.** Estimate how many visible light photons a 100-W lightbulb emits per second. Assume the bulb has a typical efficiency of about 3% (that is, 97% of the energy goes to heat).

**APPROACH** Let’s assume an average wavelength in the middle of the visible spectrum,  $\lambda \approx 500 \text{ nm}$ . The energy of each photon is  $E = hf = hc/\lambda$ . Only 3% of the 100-W power is emitted as visible light, or  $3 \text{ W} = 3 \text{ J/s}$ . The number of photons emitted per second equals the light output of  $3 \text{ J/s}$  divided by the energy of each photon.

**SOLUTION** The energy emitted in one second ( $= 3 \text{ J}$ ) is  $E = Nhf$  where  $N$  is the number of photons emitted per second and  $f = c/\lambda$ . Hence

$$N = \frac{E}{hf} = \frac{E\lambda}{hc} = \frac{(3 \text{ J})(500 \times 10^{-9} \text{ m})}{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})(3.00 \times 10^8 \text{ m/s})} \approx 8 \times 10^{18}$$

per second, or almost  $10^{19}$  photons emitted per second, an enormous number.

**EXERCISE B** A beam contains infrared light of a single wavelength, 1000 nm, and monochromatic UV at 100 nm, both of the same intensity. Are there more 100-nm photons or more 1000-nm photons?

**EXAMPLE 27-5 Photoelectron speed and energy.** What is the kinetic energy and the speed of an electron ejected from a sodium surface whose work function is  $W_0 = 2.28 \text{ eV}$  when illuminated by light of wavelength (a) 410 nm, (b) 550 nm?

**APPROACH** We first find the energy of the photons ( $E = hf = hc/\lambda$ ). If the energy is greater than  $W_0$ , then electrons will be ejected with varying amounts of KE, with a maximum of  $\text{KE}_{\text{max}} = hf - W_0$ .

**SOLUTION** (a) For  $\lambda = 410 \text{ nm}$ ,

$$hf = \frac{hc}{\lambda} = 4.85 \times 10^{-19} \text{ J} \quad \text{or} \quad 3.03 \text{ eV}.$$

The maximum kinetic energy an electron can have is given by Eq. 27-5b,  $\text{KE}_{\text{max}} = 3.03 \text{ eV} - 2.28 \text{ eV} = 0.75 \text{ eV}$ , or  $(0.75 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV}) = 1.2 \times 10^{-19} \text{ J}$ . Since  $\text{KE} = \frac{1}{2}mv^2$  where  $m = 9.1 \times 10^{-31} \text{ kg}$ ,

$$v_{\text{max}} = \sqrt{\frac{2\text{KE}}{m}} = 5.1 \times 10^5 \text{ m/s}.$$

Most ejected electrons will have less KE and less speed than these maximum values.

(b) For  $\lambda = 550 \text{ nm}$ ,  $hf = hc/\lambda = 3.61 \times 10^{-19} \text{ J} = 2.26 \text{ eV}$ . Since this photon energy is less than the work function, no electrons are ejected.

**NOTE** In (a) we used the nonrelativistic equation for kinetic energy. If  $v$  had turned out to be more than about  $0.1c$ , our calculation would have been inaccurate by more than a percent or so, and we would probably prefer to redo it using the relativistic form (Eq. 26-5).

**EXERCISE C** Determine the lowest frequency and the longest wavelength needed to emit electrons from sodium.

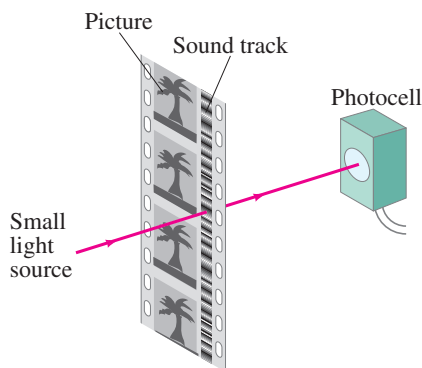
By converting units, we can show that the energy of a photon in electron volts, when given the wavelength  $\lambda$  in nm, is

$$E (\text{eV}) = \frac{1.240 \times 10^3 \text{ eV} \cdot \text{nm}}{\lambda (\text{nm})}. \quad [\text{photon energy in eV}]$$

## Applications of the Photoelectric Effect

The photoelectric effect, besides playing an important historical role in confirming the photon theory of light, also has many practical applications. Burglar alarms and automatic doors often make use of the photocell circuit of Fig. 27-6. When a person interrupts the beam of light, the sudden drop in current in the circuit activates a switch—often a solenoid—which operates a bell or opens the door. UV or IR light is sometimes used in burglar alarms because of its invisibility. Many smoke detectors use the photoelectric effect to detect tiny amounts of smoke that interrupt the flow of light and so alter the electric current. Photographic light meters use this circuit as well. Photocells are used in many other devices, such as absorption spectrophotometers, to measure light intensity. One type of film sound track is a variably shaded narrow section at the side of the film, Fig. 27-8. Light passing through the film is thus “modulated,” and the output electrical signal of the photocell detector follows the frequencies on the sound track. For many applications today, the vacuum-tube photocell of Fig. 27-6 has been replaced by a semiconductor device known as a **photodiode** (Section 29-9). In these semiconductors, the absorption of a photon liberates a bound electron so it can move freely, which changes the conductivity of the material and the current through a photodiode is altered.

**FIGURE 27-8** Optical sound track on movie film. In the projector, light from a small source (different from that for the picture) passes through the sound track on the moving film.





## 27-4 Energy, Mass, and Momentum of a Photon

We have just seen (Eq. 27-4) that the total energy of a single photon is given by  $E = hf$ . Because a photon always travels at the speed of light, it is truly a relativistic particle. Thus we must use relativistic formulas for dealing with its mass, energy, and momentum. The momentum of any particle of mass  $m$  is given by  $p = mv/\sqrt{1 - v^2/c^2}$ . Since  $v = c$  for a photon, the denominator is zero. To avoid having an infinite momentum, we conclude that the photon's mass must be zero:  $m = 0$ . This makes sense too because a photon can never be at rest (it always moves at the speed of light). A photon's kinetic energy is its total energy:

$$\text{KE} = E = hf. \quad [\text{photon}]$$

The momentum of a photon can be obtained from the relativistic formula (Eq. 26-9)  $E^2 = p^2c^2 + m^2c^4$  where we set  $m = 0$ , so  $E^2 = p^2c^2$  or

$$p = \frac{E}{c}. \quad [\text{photon}]$$

Since  $E = hf$  for a photon, its momentum is related to its wavelength by

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}. \quad (27-6)$$



### CAUTION

Momentum of photon is not  $mv$

**EXAMPLE 27-6 ESTIMATE Photon momentum and force.** Suppose the  $10^{19}$  photons emitted per second from the 100-W lightbulb in Example 27-4 were all focused onto a piece of black paper and absorbed. (a) Calculate the momentum of one photon and (b) estimate the force all these photons could exert on the paper.

**APPROACH** Each photon's momentum is obtained from Eq. 27-6,  $p = h/\lambda$ . Next, each absorbed photon's momentum changes from  $p = h/\lambda$  to zero. We use Newton's second law,  $F = \Delta p/\Delta t$ , to get the force. Let  $\lambda = 500$  nm.

**SOLUTION** (a) Each photon has a momentum

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34} \text{ J}\cdot\text{s}}{500 \times 10^{-9} \text{ m}} = 1.3 \times 10^{-27} \text{ kg}\cdot\text{m/s}.$$

(b) Using Newton's second law for  $N = 10^{19}$  photons (Example 27-4) whose momentum changes from  $h/\lambda$  to 0, we obtain

$$F = \frac{\Delta p}{\Delta t} = \frac{Nh/\lambda - 0}{1 \text{ s}} = N \frac{h}{\lambda} \approx (10^{19} \text{ s}^{-1})(10^{-27} \text{ kg}\cdot\text{m/s}) \approx 10^{-8} \text{ N}.$$

**NOTE** This is a tiny force, but we can see that a very strong light source could exert a measurable force, and near the Sun or a star the force due to photons in electromagnetic radiation could be considerable. See Section 22-6.

**EXAMPLE 27-7 Photosynthesis.** In *photosynthesis*, pigments such as chlorophyll in plants capture the energy of sunlight to change  $\text{CO}_2$  to useful carbohydrate. About nine photons are needed to transform one molecule of  $\text{CO}_2$  to carbohydrate and  $\text{O}_2$ . Assuming light of wavelength  $\lambda = 670$  nm (chlorophyll absorbs most strongly in the range 650 nm to 700 nm), how efficient is the photosynthetic process? The reverse chemical reaction releases an energy of 4.9 eV/molecule of  $\text{CO}_2$ , so 4.9 eV is needed to transform  $\text{CO}_2$  to carbohydrate.

**APPROACH** The efficiency is the minimum energy required (4.9 eV) divided by the actual energy absorbed, nine times the energy ( $hf$ ) of one photon.

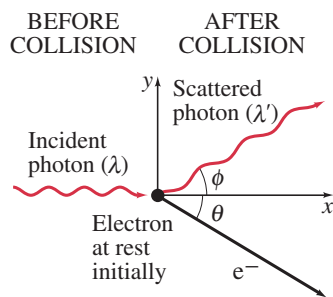
**SOLUTION** The energy of nine photons, each of energy  $hf = hc/\lambda$ , is  $(9)(6.63 \times 10^{-34} \text{ J}\cdot\text{s})(3.00 \times 10^8 \text{ m/s})/(6.7 \times 10^{-7} \text{ m}) = 2.7 \times 10^{-18} \text{ J}$  or 17 eV. Thus the process is about  $(4.9 \text{ eV}/17 \text{ eV}) = 29\%$  efficient.



### PHYSICS APPLIED

Photosynthesis

## \*27-5 Compton Effect



**FIGURE 27-9** The Compton effect. A single photon of wavelength  $\lambda$  strikes an electron in some material, knocking it out of its atom. The scattered photon has less energy (some energy is given to the electron) and hence has a longer wavelength  $\lambda'$  (shown exaggerated). Experiments found scattered X-rays of just the wavelengths predicted by conservation of energy and momentum using the photon model.

Besides the photoelectric effect, a number of other experiments were carried out in the early twentieth century which also supported the photon theory. One of these was the **Compton effect** (1923) named after its discoverer, A. H. Compton (1892–1962). Compton aimed short-wavelength light (actually X-rays) at various materials, and detected light scattered at various angles. He found that the scattered light had a slightly longer wavelength than did the incident light, and therefore a slightly lower frequency indicating a loss of energy. He explained this result on the basis of the photon theory as incident photons colliding with electrons of the material, Fig. 27–9. Using Eq. 27–6 for momentum of a photon, Compton applied the laws of conservation of momentum and energy to the collision of Fig. 27–9 and derived the following equation for the wavelength of the scattered photons:

$$\lambda' = \lambda + \frac{h}{m_e c} (1 - \cos \phi), \quad (27-7)$$

where  $m_e$  is the mass of the electron. (The quantity  $h/m_e c$ , which has the dimensions of length, is called the **Compton wavelength** of the electron.) We see that the predicted wavelength of scattered photons depends on the angle  $\phi$  at which they are detected. Compton's measurements of 1923 were consistent with this formula. The wave theory of light predicts no such shift: an incoming electromagnetic wave of frequency  $f$  should set electrons into oscillation at frequency  $f$ ; and such oscillating electrons would reemit EM waves of this same frequency  $f$  (Section 22–2), which would not change with angle ( $\phi$ ). Hence the Compton effect adds to the firm experimental foundation for the photon theory of light.

**EXERCISE D** When a photon scatters off an electron by the Compton effect, which of the following increases: its energy, frequency, wavelength?

**EXAMPLE 27-8 X-ray scattering.** X-rays of wavelength 0.140 nm are scattered from a very thin slice of carbon. What will be the wavelengths of X-rays scattered at (a)  $0^\circ$ , (b)  $90^\circ$ , (c)  $180^\circ$ ?

**APPROACH** This is an example of the Compton effect, and we use Eq. 27–7 to find the wavelengths.

**SOLUTION** (a) For  $\phi = 0^\circ$ ,  $\cos \phi = 1$  and  $1 - \cos \phi = 0$ . Then Eq. 27–7 gives  $\lambda' = \lambda = 0.140$  nm. This makes sense since for  $\phi = 0^\circ$ , there really isn't any collision as the photon goes straight through without interacting.

(b) For  $\phi = 90^\circ$ ,  $\cos \phi = 0$ , and  $1 - \cos \phi = 1$ . So

$$\begin{aligned} \lambda' &= \lambda + \frac{h}{m_e c} = 0.140 \text{ nm} + \frac{6.63 \times 10^{-34} \text{ J}\cdot\text{s}}{(9.11 \times 10^{-31} \text{ kg})(3.00 \times 10^8 \text{ m/s})} \\ &= 0.140 \text{ nm} + 2.4 \times 10^{-12} \text{ m} = 0.142 \text{ nm}; \end{aligned}$$

that is, the wavelength is longer by one Compton wavelength ( $= h/m_e c^2 = 0.0024$  nm for an electron).

(c) For  $\phi = 180^\circ$ , which means the photon is scattered backward, returning in the direction from which it came (a direct “head-on” collision),  $\cos \phi = -1$ , and  $1 - \cos \phi = 2$ . So

$$\lambda' = \lambda + 2 \frac{h}{m_e c} = 0.140 \text{ nm} + 2(0.0024 \text{ nm}) = 0.145 \text{ nm}.$$

**NOTE** The maximum shift in wavelength occurs for backward scattering, and it is twice the Compton wavelength.



### PHYSICS APPLIED

Measuring bone density

The Compton effect has been used to diagnose bone disease such as osteoporosis. Gamma rays, which are photons of even shorter wavelength than X-rays, coming from a radioactive source are scattered off bone material. The total intensity of the scattered radiation is proportional to the density of electrons, which is in turn proportional to the bone density. A low bone density may indicate osteoporosis.

## 27–6 Photon Interactions; Pair Production

When a photon passes through matter, it interacts with the atoms and electrons. There are four important types of interactions that a photon can undergo:

1. The *photoelectric effect*: A photon may knock an electron out of an atom and in the process the photon disappears.
2. The photon may knock an atomic electron to a higher energy state in the atom if its energy is not sufficient to knock the electron out altogether. In this process the photon also disappears, and all its energy is given to the atom. Such an atom is then said to be in an *excited state*, and we shall discuss it more later.
3. The photon can be scattered from an electron (or a nucleus) and in the process lose some energy; this is the *Compton effect* (Fig. 27–9). But notice that the photon is not slowed down. It still travels with speed  $c$ , but its frequency will be lower because it has lost some energy.
4. *Pair production*: A photon can actually create matter, such as the production of an electron and a positron, Fig. 27–10. (A positron has the same mass as an electron, but the opposite charge,  $+e$ .)

In process 4, **pair production**, the photon disappears in the process of creating the electron–positron pair. This is an example of mass being created from pure energy, and it occurs in accord with Einstein’s equation  $E = mc^2$ . Notice that a photon cannot create an electron alone since electric charge would not then be conserved. The inverse of pair production also occurs: if a positron comes close to an electron, the two quickly **annihilate** each other and their energy, including their mass, appears as electromagnetic energy of photons. Because positrons are not as plentiful in nature as electrons, they usually do not last long.

Electron–positron annihilation is the basis for the type of medical imaging known as PET, as discussed in Section 31–8.

**EXAMPLE 27–9 Pair production.** (a) What is the minimum energy of a photon that can produce an electron–positron pair? (b) What is this photon’s wavelength?

**APPROACH** The minimum photon energy  $E$  equals the rest energy ( $mc^2$ ) of the two particles created, via Einstein’s famous equation  $E = mc^2$  (Eq. 26–7). There is no energy left over, so the particles produced will have zero kinetic energy. The wavelength is  $\lambda = c/f$  where  $E = hf$  for the original photon.

**SOLUTION** (a) Because  $E = mc^2$ , and the mass created is equal to two electron masses, the photon must have energy

$$E = 2(9.11 \times 10^{-31} \text{ kg})(3.00 \times 10^8 \text{ m/s})^2 = 1.64 \times 10^{-13} \text{ J} = 1.02 \text{ MeV}$$
 (1 MeV =  $10^6$  eV =  $1.60 \times 10^{-13}$  J). A photon with less energy cannot undergo pair production.

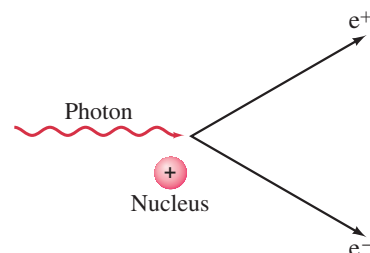
(b) Since  $E = hf = hc/\lambda$ , the wavelength of a 1.02-MeV photon is

$$\lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})(3.00 \times 10^8 \text{ m/s})}{(1.64 \times 10^{-13} \text{ J})} = 1.2 \times 10^{-12} \text{ m},$$

which is 0.0012 nm. Such photons are in the gamma-ray (or very short X-ray) region of the electromagnetic spectrum (Fig. 22–8).

**NOTE** Photons of higher energy (shorter wavelength) can also create an electron–positron pair, with the excess energy becoming kinetic energy of the particles.

Pair production cannot occur in empty space, for momentum could not be conserved. In Example 27–9, for instance, energy is conserved, but only enough energy was provided to create the electron–positron pair at rest and thus with zero momentum, which could not equal the initial momentum of the photon. Indeed, it can be shown that at any energy, an additional massive object, such as an atomic nucleus (Fig. 27–10), must take part in the interaction to carry off some of the momentum.



**FIGURE 27–10** Pair production: a photon disappears and produces an electron and a positron.

## 27–7 Wave–Particle Duality; the Principle of Complementarity



**FIGURE 27–11** Niels Bohr (right), walking with Enrico Fermi along the Appian Way outside Rome. This photo shows one important way physics is done.

### CAUTION

Not correct to say light is a wave and/or a particle. Light can **act** like a wave or like a particle

The photoelectric effect, the Compton effect, and other experiments have placed the particle theory of light on a firm experimental basis. But what about the classic experiments of Young and others (Chapter 24) on interference and diffraction which showed that the wave theory of light also rests on a firm experimental basis?

We seem to be in a dilemma. Some experiments indicate that light behaves like a wave; others indicate that it behaves like a stream of particles. These two theories seem to be incompatible, but both have been shown to have validity. Physicists finally came to the conclusion that this duality of light must be accepted as a fact of life. It is referred to as the **wave–particle duality**. Apparently, light is a more complex phenomenon than just a simple wave or a simple beam of particles.

To clarify the situation, the great Danish physicist Niels Bohr (1885–1962, Fig. 27–11) proposed his famous **principle of complementarity**. It states that to understand an experiment, sometimes we find an explanation using wave theory and sometimes using particle theory. Yet we must be aware of both the wave and particle aspects of light if we are to have a full understanding of light. Therefore these two aspects of light complement one another.

It is not easy to “visualize” this duality. We cannot readily picture a combination of wave and particle. Instead, we must recognize that the two aspects of light are different “faces” that light shows to experimenters.

Part of the difficulty stems from how we think. Visual pictures (or models) in our minds are based on what we see in the everyday world. We apply the concepts of waves and particles to light because in the macroscopic world we see that energy is transferred from place to place by these two methods. We cannot see directly whether light is a wave or particle, so we do indirect experiments. To explain the experiments, we apply the models of waves or of particles to the nature of light. But these are abstractions of the human mind. When we try to conceive of what light really “is,” we insist on a visual picture. Yet there is no reason why light should conform to these models (or visual images) taken from the macroscopic world. The “true” nature of light—if that means anything—is not possible to visualize. The best we can do is recognize that our knowledge is limited to the indirect experiments, and that in terms of everyday language and images, light reveals both wave and particle properties.

It is worth noting that Einstein’s equation  $E = hf$  itself links the particle and wave properties of a light beam. In this equation,  $E$  refers to the energy of a particle; and on the other side of the equation, we have the frequency  $f$  of the corresponding wave.

## 27–8 Wave Nature of Matter

In 1923, Louis de Broglie (1892–1987) extended the idea of the wave–particle duality. He appreciated the *symmetry* in nature, and argued that if light sometimes behaves like a wave and sometimes like a particle, then perhaps those things in nature thought to be particles—such as electrons and other material objects—might also have wave properties. De Broglie proposed that the wavelength of a material particle would be related to its momentum in the same way as for a photon, Eq. 27–6,  $p = h/\lambda$ . That is, for a particle having linear momentum  $p = mv$ , the wavelength  $\lambda$  is given by

$$\lambda = \frac{h}{p}, \quad (27-8)$$

and is valid classically ( $p = mv$  for  $v \ll c$ ) and relativistically ( $p = \gamma mv = mv/\sqrt{1 - v^2/c^2}$ ). This is sometimes called the **de Broglie wavelength** of a particle.

de Broglie wavelength



**EXAMPLE 27–10 Wavelength of a ball.** Calculate the de Broglie wavelength of a 0.20-kg ball moving with a speed of 15 m/s.

**APPROACH** We use Eq. 27–8.

**SOLUTION** 
$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{(6.6 \times 10^{-34} \text{ J}\cdot\text{s})}{(0.20 \text{ kg})(15 \text{ m/s})} = 2.2 \times 10^{-34} \text{ m}.$$

Ordinary objects, such as the ball of Example 27–10, have unimaginably small wavelengths. Even if the speed is extremely small, say  $10^{-4}$  m/s, the wavelength would be about  $10^{-29}$  m. Indeed, the wavelength of any ordinary object is much too small to be measured and detected. The problem is that the properties of waves, such as interference and diffraction, are significant only when the size of objects or slits is not much larger than the wavelength. And there are no known objects or slits to diffract waves only  $10^{-30}$  m long, so the wave properties of ordinary objects go undetected.

But tiny elementary particles, such as electrons, are another matter. Since the mass  $m$  appears in the denominator of Eq. 27–8, a very small mass should have a much larger wavelength.

**EXAMPLE 27–11 Wavelength of an electron.** Determine the wavelength of an electron that has been accelerated through a potential difference of 100 V.

**APPROACH** If the kinetic energy is much less than the rest energy, we can use the classical formula,  $\text{KE} = \frac{1}{2}mv^2$  (see end of Section 26–9). For an electron,  $mc^2 = 0.511 \text{ MeV}$ . We then apply conservation of energy: the kinetic energy acquired by the electron equals its loss in potential energy. After solving for  $v$ , we use Eq. 27–8 to find the de Broglie wavelength.

**SOLUTION** The gain in kinetic energy equals the loss in potential energy:  $\Delta \text{PE} = eV - 0$ . Thus  $\text{KE} = eV$ , so  $\text{KE} = 100 \text{ eV}$ . The ratio  $\text{KE}/mc^2 = 100 \text{ eV}/(0.511 \times 10^6 \text{ eV}) \approx 10^{-4}$ , so relativity is not needed. Thus

$$\frac{1}{2}mv^2 = eV$$

and

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{(2)(1.6 \times 10^{-19} \text{ C})(100 \text{ V})}{(9.1 \times 10^{-31} \text{ kg})}} = 5.9 \times 10^6 \text{ m/s}.$$

Then

$$\lambda = \frac{h}{mv} = \frac{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})}{(9.1 \times 10^{-31} \text{ kg})(5.9 \times 10^6 \text{ m/s})} = 1.2 \times 10^{-10} \text{ m},$$

or 0.12 nm.

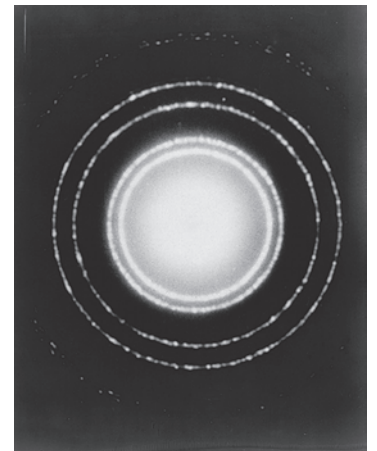
**EXERCISE E** As a particle travels faster, does its de Broglie wavelength decrease, increase, or remain the same?

**EXERCISE F** Return to the Chapter-Opening Question, page 771, and answer it again now. Try to explain why you may have answered differently the first time.

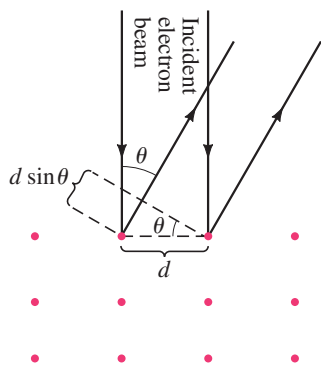
## Electron Diffraction

From Example 27–11, we see that electrons can have wavelengths on the order of  $10^{-10}$  m, and even smaller. Although small, this wavelength can be detected: the spacing of atoms in a crystal is on the order of  $10^{-10}$  m and the orderly array of atoms in a crystal could be used as a type of diffraction grating, as was done earlier for X-rays (see Section 25–11). C. J. Davisson and L. H. Germer performed the crucial experiment: they scattered electrons from the surface of a metal crystal and, in early 1927, observed that the electrons were scattered into a pattern of regular peaks. When they interpreted these peaks as a diffraction pattern, the wavelength of the diffracted electron wave was found to be just that predicted by de Broglie, Eq. 27–8. In the same year, G. P. Thomson (son of J. J. Thomson) used a different experimental arrangement and also detected diffraction of electrons. (See Fig. 27–12. Compare it to X-ray diffraction, Section 25–11.) Later experiments showed that protons, neutrons, and other particles also have wave properties.

**FIGURE 27–12** Diffraction pattern of electrons scattered from aluminum foil, as recorded on film.



**PHYSICS APPLIED**  
Electron diffraction



**FIGURE 27-13** Example 27-12. The red dots represent atoms in an orderly array in a solid.

**EXAMPLE 27-12 Electron diffraction.** The wave nature of electrons is manifested in experiments where an electron beam interacts with the atoms on the surface of a solid, especially crystals. By studying the angular distribution of the diffracted electrons, one can indirectly measure the geometrical arrangement of atoms. Assume that the electrons strike perpendicular to the surface of a solid (see Fig. 27-13), and that their energy is low,  $\text{KE} = 100 \text{ eV}$ , so that they interact only with the surface layer of atoms. If the smallest angle at which a diffraction maximum occurs is at  $24^\circ$ , what is the separation  $d$  between the atoms on the surface?

**SOLUTION** Treating the electrons as waves, we need to determine the condition where the difference in path traveled by the wave diffracted from adjacent atoms is an integer multiple of the de Broglie wavelength, so that constructive interference occurs. The path length difference is  $d \sin \theta$  (Fig. 27-13); so for the smallest value of  $\theta$  we must have

$$d \sin \theta = \lambda.$$

However,  $\lambda$  is related to the (non-relativistic) kinetic energy  $\text{KE}$  by

$$\text{KE} = \frac{p^2}{2m_e} = \frac{h^2}{2m_e \lambda^2}.$$

Thus

$$\begin{aligned} \lambda &= \frac{h}{\sqrt{2m_e \text{KE}}} \\ &= \frac{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})}{\sqrt{2(9.11 \times 10^{-31} \text{ kg})(100 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV})}} = 0.123 \text{ nm}. \end{aligned}$$

The surface inter-atomic spacing is

$$d = \frac{\lambda}{\sin \theta} = \frac{0.123 \text{ nm}}{\sin 24^\circ} = 0.30 \text{ nm}.$$

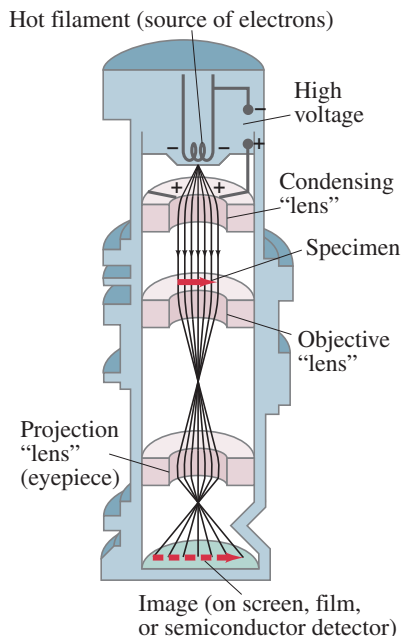
**NOTE** Experiments of this type verify both the wave nature of electrons and the orderly array of atoms in crystalline solids.

## What Is an Electron?

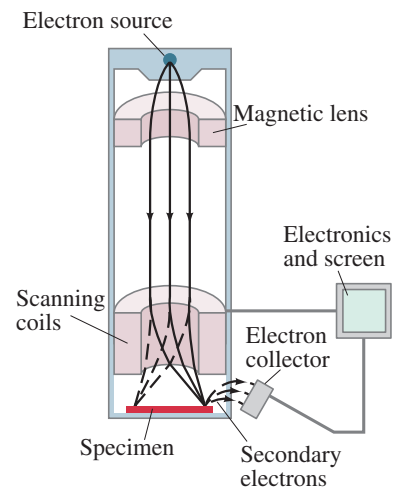
We might ask ourselves: “What is an electron?” The early experiments of J. J. Thomson (Section 27-1) indicated a glow in a tube, and that glow moved when a magnetic field was applied. The results of these and other experiments were best interpreted as being caused by tiny negatively charged particles which we now call electrons. No one, however, has actually seen an electron directly. The drawings we sometimes make of electrons as tiny spheres with a negative charge on them are merely convenient pictures (now recognized to be inaccurate). Again we must rely on experimental results, some of which are best interpreted using the particle model and others using the wave model. These models are mere pictures that we use to extrapolate from the macroscopic world to the tiny microscopic world of the atom. And there is no reason to expect that these models somehow reflect the reality of an electron. We thus use a wave or a particle model (whichever works best in a situation) so that we can talk about what is happening. But we should not be led to believe that an electron is a wave or a particle. Instead we could say that an electron is the set of its properties that we can measure. Bertrand Russell said it well when he wrote that an electron is “a logical construction.”

## 27-9 Electron Microscopes

The idea that electrons have wave properties led to the development of the **electron microscope (EM)**, which can produce images of much greater magnification than a light microscope. Figures 27-14 and 27-15 are diagrams of two types, developed around the middle of the twentieth century: the **transmission electron microscope (TEM)**, which produces a two-dimensional image, and the **scanning electron microscope (SEM)**, which produces images with a three-dimensional quality.

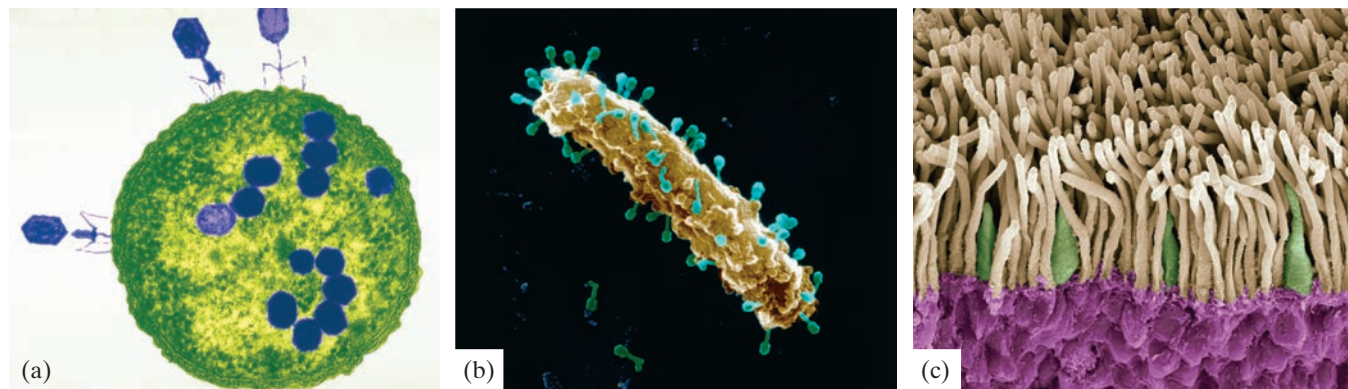


**FIGURE 27-14** Transmission electron microscope. The magnetic field coils are designed to be “magnetic lenses,” which bend the electron paths and bring them to a focus, as shown. The sensors of the image measure electron intensity only, no color.



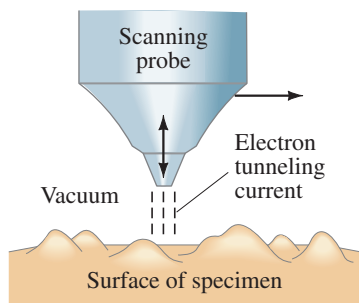
**FIGURE 27-15** Scanning electron microscope. Scanning coils move an electron beam back and forth across the specimen. Secondary electrons produced when the beam strikes the specimen are collected and their intensity affects the brightness of pixels in a monitor to produce a picture.

In both types, the objective and eyepiece lenses are actually magnetic fields that exert forces on the electrons to bring them to a focus. The fields are produced by carefully designed current-carrying coils of wire. Photographs using each type are shown in Fig. 27-16. EMs measure the intensity of electrons, producing monochromatic photos. Color is often added artificially to highlight.



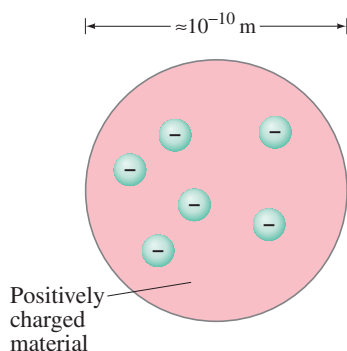
**FIGURE 27-16** Electron micrographs, in false color, of (a) viruses attacking a cell of the bacterium *Escherichia coli* (TEM,  $\approx 50,000\times$ ). (b) Same subject by an SEM ( $\approx 35,000\times$ ). (c) SEM image of an eye’s retina (Section 25-2); the rods and cones have been colored beige and green, respectively. Part (c) is also on the cover of this book.

As discussed in Sections 25-7 and 25-8, the maximum resolution of details on an object is about the size of the wavelength of the radiation used to view it. Electrons accelerated by voltages on the order of  $10^5$  V have wavelengths of about 0.004 nm. The maximum resolution obtainable would be on this order, but in practice, aberrations in the magnetic lenses limit the resolution in transmission electron microscopes to about 0.1 to 0.5 nm. This is still 1000 times better than a visible-light microscope, and corresponds to a useful magnification of about a million. Such magnifications are difficult to achieve, and more common magnifications are  $10^4$  to  $10^5$ . The maximum resolution of a scanning electron microscope is less, typically 5 to 10 nm although new high-resolution SEMs approach 1 nm.



**FIGURE 27-17** The probe tip of a scanning tunneling electron microscope, as it is moved horizontally, automatically moves up and down to maintain a constant tunneling current, and this motion is translated into an image of the surface.

**FIGURE 27-18** Plum-pudding model of the atom.



The **scanning tunneling electron microscope (STM)**, developed in the 1980s, contains a tiny probe, whose tip may be only one (or a few) atoms wide, that is moved across the specimen to be examined in a series of linear passes. The tip, as it scans, remains very close to the surface of the specimen, about 1 nm above it, Fig. 27-17. A small voltage applied between the probe and the surface causes electrons to leave the surface and pass through the vacuum to the probe, by a process known as *tunneling* (discussed in Section 30-12). This “tunneling” current is very sensitive to the gap width, so a feedback mechanism can be used to raise and lower the probe to maintain a constant electron current. The probe’s vertical motion, following the surface of the specimen, is then plotted as a function of position, scan after scan, producing a three-dimensional image of the surface. Surface features as fine as the size of an atom can be resolved: a resolution better than 50 pm (0.05 nm) laterally and 0.01 to 0.001 nm vertically. This kind of resolution has given a great impetus to the study of the surface structure of materials. The “topographic” image of a surface actually represents the distribution of electron charge.

The **atomic force microscope (AFM)**, developed in the 1980s, is in many ways similar to an STM, but can be used on a wider range of sample materials. Instead of detecting an electric current, the AFM measures the force between a cantilevered tip and the sample, a force which depends strongly on the tip-sample separation at each point. The tip is moved as for the STM.

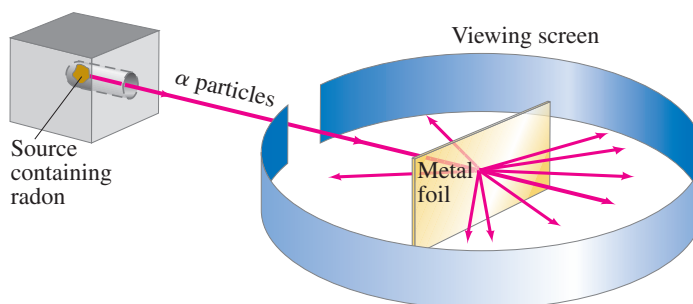
## 27-10 Early Models of the Atom

The idea that matter is made up of atoms was accepted by most scientists by 1900. With the discovery of the electron in the 1890s, scientists began to think of the atom itself as having a structure with electrons as part of that structure. We now discuss how our modern view of the atom developed, and the quantum theory with which it is intertwined.<sup>†</sup>

A typical model of the atom in the 1890s visualized the atom as a homogeneous sphere of positive charge inside of which there were tiny negatively charged electrons, a little like plums in a pudding, Fig. 27-18.

Around 1911, Ernest Rutherford (1871–1937) and his colleagues performed experiments whose results contradicted the plum-pudding model of the atom. In these experiments a beam of positively charged alpha ( $\alpha$ ) particles was directed at a thin sheet of metal foil such as gold, Fig. 27-19. (These newly discovered  $\alpha$  particles were emitted by certain radioactive materials and were soon shown to be doubly ionized helium atoms—that is, having a charge of  $+2e$ .) It was

**FIGURE 27-19** Experimental setup for Rutherford’s experiment:  $\alpha$  particles emitted by radon are deflected by the atoms of a thin metal foil and a few rebound backward.



expected from the plum-pudding model that the alpha particles would not be deflected significantly because electrons are so much lighter than alpha particles, and the alpha particles should not have encountered any massive concentration of positive charge to strongly repel them. The experimental results completely contradicted these predictions. It was found that most of the alpha particles passed through the foil unaffected, as if the foil were mostly empty space.

<sup>†</sup>Some readers may say: “Tell us the facts as we know them today, and don’t bother us with the historical background and its outmoded theories.” Such an approach would ignore the creative aspect of science and thus give a false impression of how science develops. Moreover, it is not really possible to understand today’s view of the atom without insight into the concepts that led to it.



And of those deflected, a few were deflected at very large angles—some even backward, nearly in the direction from which they had come. This could happen, Rutherford reasoned, only if the positively charged alpha particles were being repelled by a massive positive charge concentrated in a very small region of space (see Fig. 27–20). He hypothesized that the atom must consist of a tiny but massive positively charged nucleus, containing over 99.9% of the mass of the atom, surrounded by much lighter electrons some distance away. The electrons would be moving in orbits about the nucleus—much as the planets move around the Sun—because if they were at rest, they would fall into the nucleus due to electrical attraction. See Fig. 27–21. Rutherford’s experiments suggested that the nucleus must have a radius of about  $10^{-15}$  to  $10^{-14}$  m. From kinetic theory, and especially Einstein’s analysis of Brownian motion (see Section 13–1), the radius of atoms was estimated to be about  $10^{-10}$  m. Thus the electrons would seem to be at a distance from the nucleus of about 10,000 to 100,000 times the radius of the nucleus itself. (If the nucleus were the size of a baseball, the atom would have the diameter of a big city several kilometers across.) So an atom would be mostly empty space.

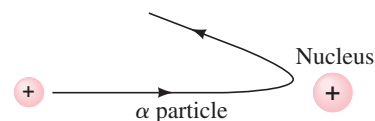
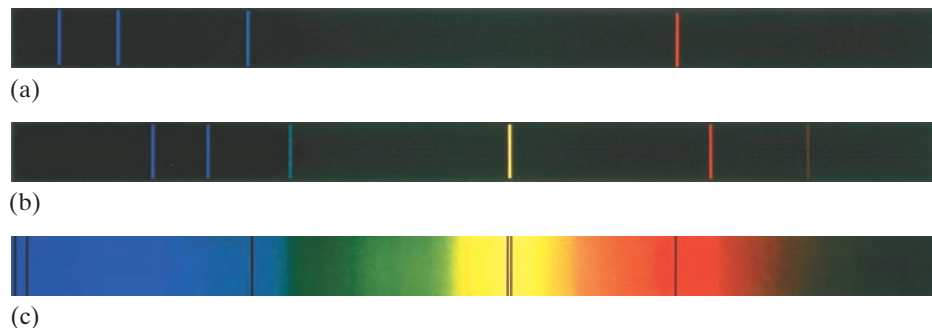
Rutherford’s **planetary model** of the atom (also called the **nuclear model** of the atom) was a major step toward how we view the atom today. It was not, however, a complete model and presented some major problems, as we shall see.

## 27–11 Atomic Spectra: Key to the Structure of the Atom

Earlier in this Chapter we saw that heated solids (as well as liquids and dense gases) emit light with a continuous spectrum of wavelengths. This radiation is assumed to be due to oscillations of atoms and molecules, which are largely governed by the interaction of each atom or molecule with its neighbors.

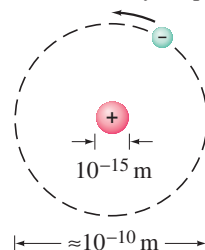
Rarefied gases can also be excited to emit light. This is done by intense heating, or more commonly by applying a high voltage to a “discharge tube” containing the gas at low pressure, Fig. 27–22. The radiation from excited gases had been observed early in the nineteenth century, and it was found that the spectrum was not continuous. Rather, excited gases emit light of only certain wavelengths, and when this light is analyzed through the slit of a spectroscope or spectrometer, a **line spectrum** is seen rather than a continuous spectrum. The line spectra emitted by a number of elements in the visible region are shown below in Fig. 27–23, and in Chapter 24, Fig. 24–28. The **emission spectrum** is characteristic of the material and can serve as a type of “fingerprint” for identification of the gas.

We also saw (Chapter 24) that if a continuous spectrum passes through a rarefied gas, dark lines are observed in the emerging spectrum, at wavelengths corresponding to lines normally emitted by the gas. This is called an **absorption spectrum** (Fig. 27–23c), and it became clear that gases can absorb light at the same frequencies at which they emit. Using film sensitive to ultraviolet and to infrared light, it was found that gases emit and absorb discrete frequencies in these regions as well as in the visible.

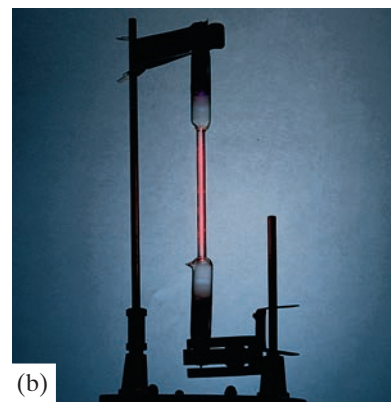
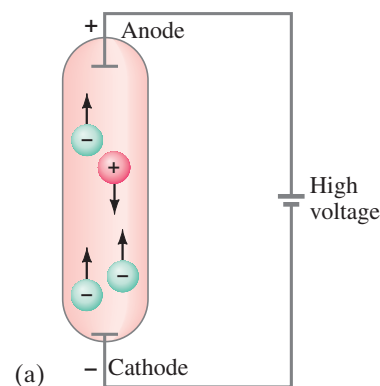


**FIGURE 27–20** Backward rebound of  $\alpha$  particles in Fig. 27–19 explained as the repulsion from a heavy positively charged nucleus.

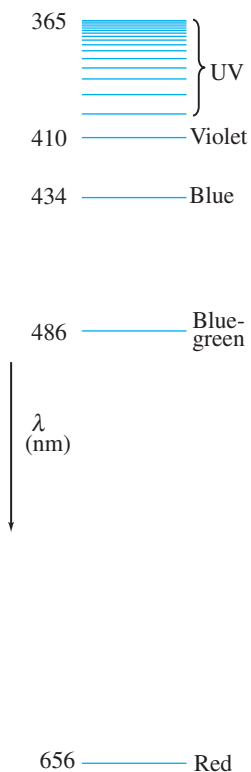
**FIGURE 27–21** Rutherford’s model of the atom: electrons orbit a tiny positive nucleus (not to scale). The atom is visualized as mostly empty space.



**FIGURE 27–22** Gas-discharge tube: (a) diagram; (b) photo of an actual discharge tube for hydrogen.

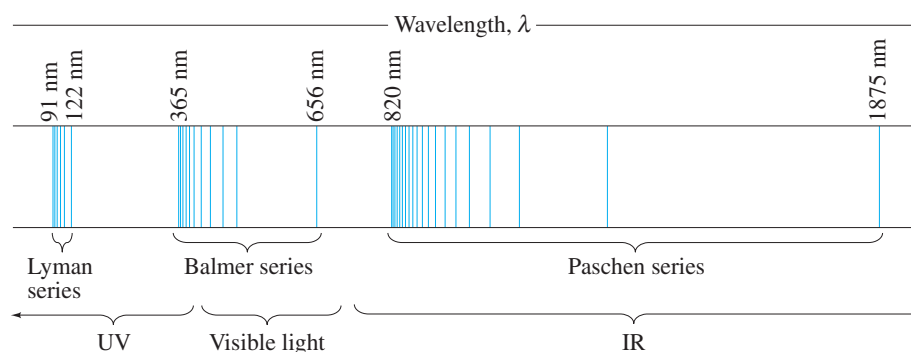


**FIGURE 27–23** Emission spectra of the gases (a) atomic hydrogen, (b) helium, and (c) the *solar absorption* spectrum.



**FIGURE 27-24** Balmer series of lines for hydrogen.

**FIGURE 27-25** Line spectrum of atomic hydrogen. Each series fits the formula  $\frac{1}{\lambda} = R\left(\frac{1}{n'^2} - \frac{1}{n^2}\right)$  where  $n' = 1$  for the Lyman series,  $n' = 2$  for the Balmer series,  $n' = 3$  for the Paschen series, and so on;  $n$  can take on all integer values from  $n = n' + 1$  up to infinity. The only lines in the visible region of the electromagnetic spectrum are part of the Balmer series.



fit a formula with the same form as Eq. 27-9 but with the  $1/2^2$  replaced by  $1/1^2$ ,  $1/3^2$ ,  $1/4^2$ , and so on. For example, the **Lyman series** contains lines with wavelengths from 91 nm to 122 nm (in the UV region) and fits the formula

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right), \quad n = 2, 3, \dots$$

The wavelengths of the **Paschen series** (in the IR region) fit

$$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right), \quad n = 4, 5, \dots$$

The Rutherford model was unable to explain why atoms emit line spectra. It had other difficulties as well. According to the Rutherford model, electrons orbit the nucleus, and since their paths are curved the electrons are accelerating. Hence they should give off light like any other accelerating electric charge (Chapter 22).

In low-density gases, the atoms are far apart on average and hence the light emitted or absorbed is assumed to be by *individual atoms* rather than through interactions between atoms, as in a solid, liquid, or dense gas. Thus the line spectra serve as a key to the structure of the atom: any theory of atomic structure must be able to explain why atoms emit light only of discrete wavelengths, and it should be able to predict what these wavelengths are.

Hydrogen is the simplest atom—it has only one electron orbiting its nucleus. It also has the simplest spectrum. The spectrum of most atoms shows little apparent regularity. But the spacing between lines in the hydrogen spectrum decreases in a regular way, Fig. 27-24. Indeed, in 1885, J. J. Balmer (1825–1898) showed that the four lines in the visible portion of the hydrogen spectrum (with measured wavelengths 656 nm, 486 nm, 434 nm, and 410 nm) have wavelengths that fit the formula

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right), \quad n = 3, 4, \dots \quad (27-9)$$

Here  $n$  takes on the values 3, 4, 5, 6 for the four visible lines, and  $R$ , called the **Rydberg constant**, has the value  $R = 1.0974 \times 10^7 \text{ m}^{-1}$ . Later it was found that this **Balmer series** of lines extended into the UV region, ending at  $\lambda = 365 \text{ nm}$ , as shown in Fig. 27-24. Balmer's formula, Eq. 27-9, also worked for these lines with higher integer values of  $n$ . The lines near 365 nm become too close together to distinguish, but the limit of the series at 365 nm corresponds to  $n = \infty$  (so  $1/n^2 = 0$  in Eq. 27-9).

Later experiments on hydrogen showed that there were similar series of lines in the UV and IR regions, and each series had a pattern just like the Balmer series, but at different wavelengths, Fig. 27-25. Each of these series was found to

Since light carries off energy and energy is conserved, the electron's own energy must decrease to compensate. Hence electrons would be expected to spiral into the nucleus. As they spiraled inward, their frequency would increase in a short time and so too would the frequency of the light emitted. Thus the two main difficulties of the Rutherford model are these: (1) it predicts that light of a continuous range of frequencies will be emitted, whereas experiment shows line spectra; (2) it predicts that atoms are unstable—electrons would quickly spiral into the nucleus—but we know that atoms in general are stable, because there is stable matter all around us.

Clearly Rutherford's model was not sufficient. Some sort of modification was needed, and Niels Bohr provided it in a model that included the quantum hypothesis. Although the Bohr model has been superseded, it did provide a crucial stepping stone to our present understanding. And some aspects of the Bohr model are still useful today, so we examine it in detail in the next Section.

## 27–12 The Bohr Model

Bohr had studied in Rutherford's laboratory for several months in 1912 and was convinced that Rutherford's planetary model of the atom had validity. But in order to make it work, he felt that the newly developing quantum theory would somehow have to be incorporated in it. The work of Planck and Einstein had shown that in heated solids, the energy of oscillating electric charges must change discontinuously—from one discrete energy state to another, with the emission of a quantum of light. Perhaps, Bohr argued, the electrons in an atom also cannot lose energy continuously, but must do so in quantum “jumps.” In working out his model during the next year, Bohr postulated that electrons move about the nucleus in circular orbits, but that only certain orbits are allowed. He further postulated that an electron in each orbit would have a definite energy and would move in the orbit *without radiating energy* (even though this violated classical ideas since accelerating electric charges are supposed to emit EM waves; see Chapter 22). He thus called the possible orbits **stationary states**. In this **Bohr model**, light is emitted only when an electron jumps from a higher (upper) stationary state to another of lower energy, Fig. 27–26. When such a transition occurs, a single photon of light is emitted whose energy, by energy conservation, is given by

$$hf = E_u - E_\ell, \quad (27-10)$$

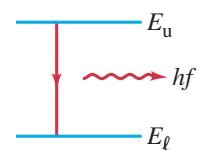
where  $E_u$  refers to the energy of the upper state and  $E_\ell$  the energy of the lower state.

In 1912–13, Bohr set out to determine what energies these orbits would have in the simplest atom, hydrogen; the spectrum of light emitted could then be predicted from Eq. 27–10. In the Balmer formula he had the key he was looking for. Bohr quickly found that his theory would agree with the Balmer formula if he assumed that the electron's angular momentum  $L$  is quantized and equal to an integer  $n$  times  $h/2\pi$ . As we saw in Chapter 8 angular momentum is given by  $L = I\omega$ , where  $I$  is the moment of inertia and  $\omega$  is the angular velocity. For a single particle of mass  $m$  moving in a circle of radius  $r$  with speed  $v$ ,  $I = mr^2$  and  $\omega = v/r$ ; hence,  $L = I\omega = (mr^2)(v/r) = mvr$ . Bohr's **quantum condition** is

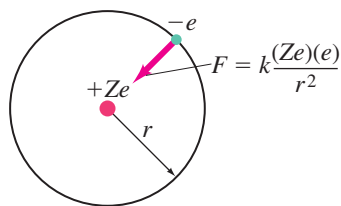
$$L = mvr_n = n \frac{h}{2\pi}, \quad n = 1, 2, 3, \dots, \quad (27-11)$$

where  $n$  is an integer and  $r_n$  is the radius of the  $n^{\text{th}}$  possible orbit. The allowed orbits are numbered 1, 2, 3,  $\dots$ , according to the value of  $n$ , which is called the **principal quantum number** of the orbit.

Equation 27–11 did not have a firm theoretical foundation. Bohr had searched for some “quantum condition,” and such tries as  $E = hf$  (where  $E$  represents the energy of the electron in an orbit) did not give results in accord with experiment. Bohr's reason for using Eq. 27–11 was simply that it worked; and we now look at how. In particular, let us determine what the Bohr theory predicts for the measurable wavelengths of emitted light.



**FIGURE 27–26** An atom emits a photon (energy =  $hf$ ) when its energy changes from  $E_u$  to a lower energy  $E_\ell$ .



**FIGURE 27-27** Electric force (Coulomb's law) keeps the negative electron in orbit around the positively charged nucleus.

An electron in a circular orbit of radius  $r$  (Fig. 27-27) would have a centripetal acceleration  $v^2/r$  produced by the electrical force of attraction between the negative electron and the positive nucleus. This force is given by Coulomb's law,

$$F = k \frac{(Ze)(e)}{r^2},$$

where  $k = 1/4\pi\epsilon_0 = 8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2$ . The charge on the electron is  $q_1 = -e$ , and that on the nucleus is  $q_2 = +Ze$ , where  $Ze$  is the charge on the nucleus:  $+e$  is the charge on a proton,  $Z$  is the number of protons in the nucleus (called "atomic number," Section 28-7).<sup>†</sup> For the hydrogen atom,  $Z = +1$ .

In Newton's second law,  $F = ma$ , we substitute Coulomb's law for  $F$  and  $a = v^2/r_n$  for a particular allowed orbit of radius  $r_n$ , and obtain

$$\begin{aligned} F &= ma \\ k \frac{Ze^2}{r_n^2} &= \frac{mv^2}{r_n}. \end{aligned}$$

We solve this for  $r_n$ ,

$$r_n = \frac{kZe^2}{mv^2},$$

and then substitute for  $v$  from Eq. 27-11 (which says  $v = nh/2\pi mr_n$ ):

$$r_n = \frac{kZe^2 4\pi^2 m r_n^2}{n^2 h^2}.$$

We solve for  $r_n$  (it appears on both sides, so we cancel one of them) and find

$$r_n = \frac{n^2 h^2}{4\pi^2 m k Z e^2} = \frac{n^2}{Z} r_1 \quad n = 1, 2, 3, \dots, \quad (27-12)$$

where  $n$  is an integer (Eq. 27-11), and

$$r_1 = \frac{h^2}{4\pi^2 m k e^2}.$$

Equation 27-12 gives the radii of all possible orbits. The smallest orbit is for  $n = 1$ , and for hydrogen ( $Z = 1$ ) has the value

$$\begin{aligned} r_1 &= \frac{(1)^2 (6.626 \times 10^{-34} \text{ J}\cdot\text{s})^2}{4\pi^2 (9.11 \times 10^{-31} \text{ kg}) (8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2) (1.602 \times 10^{-19} \text{ C})^2} \\ r_1 &= 0.529 \times 10^{-10} \text{ m}. \end{aligned} \quad (27-13)$$

The radius of the smallest orbit in hydrogen,  $r_1$ , is sometimes called the **Bohr radius**. From Eq. 27-12, we see that the radii of the larger orbits<sup>‡</sup> increase as  $n^2$ , so

$$\begin{aligned} r_2 &= 4r_1 = 2.12 \times 10^{-10} \text{ m}, \\ r_3 &= 9r_1 = 4.76 \times 10^{-10} \text{ m}, \\ &\vdots \\ r_n &= n^2 r_1, \quad n = 1, 2, 3, \dots. \end{aligned}$$

The first four orbits are shown in Fig. 27-28. Notice that, according to Bohr's model, an electron can exist only in the orbits given by Eq. 27-12. There are no allowable orbits in between.

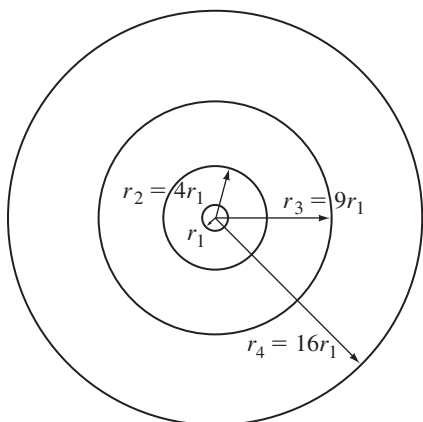
For an atom with  $Z \neq 1$ , we can write the orbital radii,  $r_n$ , using Eq. 27-12:

$$r_n = \frac{n^2}{Z} (0.529 \times 10^{-10} \text{ m}), \quad n = 1, 2, 3, \dots. \quad (27-14)$$

<sup>†</sup>We include  $Z$  in our derivation so that we can treat other single-electron ("hydrogenlike") atoms such as the ions  $\text{He}^+$  ( $Z = 2$ ) and  $\text{Li}^{2+}$  ( $Z = 3$ ). Helium in the neutral state has two electrons; if one electron is missing, the remaining  $\text{He}^+$  ion consists of one electron revolving around a nucleus of charge  $+2e$ . Similarly, doubly ionized lithium,  $\text{Li}^{2+}$ , also has a single electron, and in this case  $Z = 3$ .

<sup>‡</sup>Be careful not to believe that these well-defined orbits actually exist. Today electrons are better thought of as forming "clouds," as discussed in Chapter 28.

**FIGURE 27-28** The four smallest orbits in the Bohr model of hydrogen;  $r_1 = 0.529 \times 10^{-10} \text{ m}$ .





In each of its possible orbits, the electron in a Bohr model atom would have a definite energy, as the following calculation shows. The total energy equals the sum of the kinetic and potential energies. The potential energy of the electron is given by  $PE = qV = -eV$ , where  $V$  is the potential due to a point charge  $+Ze$  as given by Eq. 17-5:  $V = kQ/r = kZe/r$ . So

$$PE = -eV = -k \frac{Ze^2}{r}.$$

The total energy  $E_n$  for an electron in the  $n^{\text{th}}$  orbit of radius  $r_n$  is the sum of the kinetic and potential energies:

$$E_n = \frac{1}{2}mv^2 - \frac{kZe^2}{r_n}.$$

When we substitute  $v$  from Eq. 27-11 and  $r_n$  from Eq. 27-12 into this equation, we obtain

$$E_n = -\frac{2\pi^2 Z^2 e^4 m k^2}{h^2} \frac{1}{n^2} \quad n = 1, 2, 3, \dots \quad (27-15a)$$

If we evaluate the constant term in Eq. 27-15a and convert it to electron volts, as is customary in atomic physics, we obtain

$$E_n = -(13.6 \text{ eV}) \frac{Z^2}{n^2}, \quad n = 1, 2, 3, \dots \quad (27-15b)$$

The lowest energy level ( $n = 1$ ) for hydrogen ( $Z = 1$ ) is

$$E_1 = -13.6 \text{ eV}.$$

Since  $n^2$  appears in the denominator of Eq. 27-15b, the energies of the larger orbits in hydrogen ( $Z = 1$ ) are given by

$$E_n = \frac{-13.6 \text{ eV}}{n^2}.$$

For example,

$$E_2 = \frac{-13.6 \text{ eV}}{4} = -3.40 \text{ eV},$$

$$E_3 = \frac{-13.6 \text{ eV}}{9} = -1.51 \text{ eV}.$$

We see that not only are the orbit radii quantized, but from Eqs. 27-15, so is the energy. The quantum number  $n$  that labels the orbit radii also labels the energy levels. The lowest **energy level** or **energy state** has energy  $E_1$ , and is called the **ground state**. The higher states,  $E_2$ ,  $E_3$ , and so on, are called **excited states**. The fixed energy levels are also called **stationary states**.

Notice that although the energy for the larger orbits has a smaller numerical value, all the energies are less than zero. Thus,  $-3.4 \text{ eV}$  is a higher energy than  $-13.6 \text{ eV}$ . Hence the orbit closest to the nucleus ( $r_1$ ) has the lowest energy (the most negative). The reason the energies have negative values has to do with the way we defined the zero for potential energy. For two point charges,  $PE = kq_1 q_2 / r$  corresponds to zero potential energy when the two charges are infinitely far apart (Section 17-5). Thus, an electron that can just barely be free from the atom by reaching  $r = \infty$  (or, at least, far from the nucleus) with zero kinetic energy will have  $E = KE + PE = 0 + 0 = 0$ , corresponding to  $n = \infty$  in Eqs. 27-15. If an electron is free and has kinetic energy, then  $E > 0$ . To remove an electron that is part of an atom requires an energy input (otherwise atoms would not be stable). Since  $E \geq 0$  for a free electron, then an electron bound to an atom needs to have  $E < 0$ . That is, energy must be added to bring its energy up, from a negative value to at least zero in order to free it.

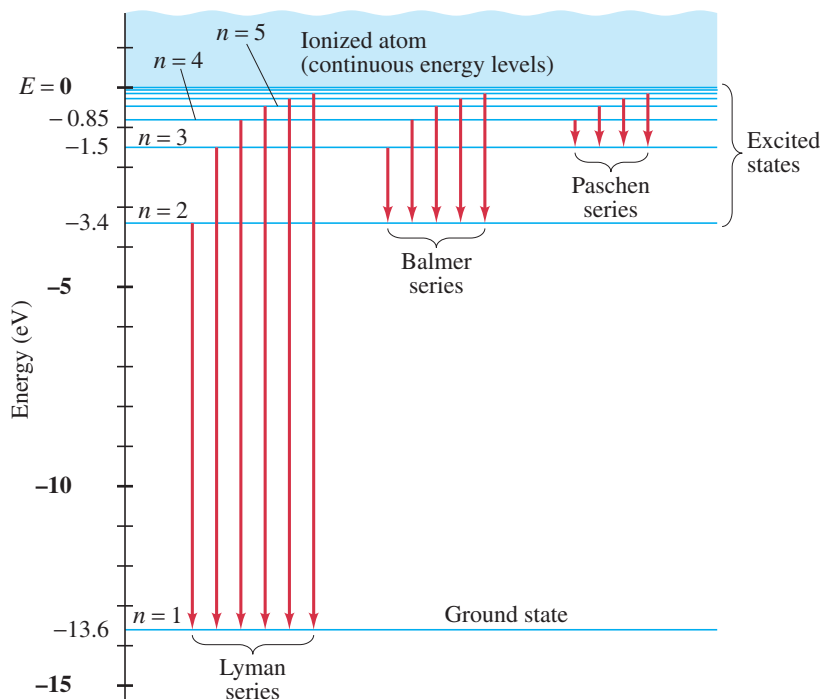
The minimum energy required to remove an electron from an atom initially in the ground state is called the **binding energy** or **ionization energy**. The ionization energy for hydrogen has been measured to be  $13.6 \text{ eV}$ , and this corresponds precisely to removing an electron from the lowest state,  $E_1 = -13.6 \text{ eV}$ , up to  $E = 0$  where it can be free.

## Spectra Lines Explained

It is useful to show the various possible energy values as horizontal lines on an energy-level diagram. This is shown for hydrogen in Fig. 27–29. The electron in a hydrogen atom can be in any one of these levels according to Bohr theory. But it could never be in between, say at  $-9.0$  eV. At room temperature, nearly all H atoms will be in the ground state ( $n = 1$ ). At higher temperatures, or during an electric discharge when there are many collisions between free electrons and atoms, many atoms can be in excited states ( $n > 1$ ). Once in an excited state, an atom's electron can jump down to a lower state, and give off a photon in the process. This is, according to the Bohr model, the origin of the emission spectra of excited gases.

Note that above  $E = 0$ , an electron is free and can have any energy ( $E$  is not quantized). Thus there is a continuum of energy states above  $E = 0$ , as indicated in the energy-level diagram of Fig. 27–29.

**FIGURE 27–29** Energy-level diagram for the hydrogen atom, showing the transitions for the spectral lines of the Lyman, Balmer, and Paschen series (Fig. 27–25). Each vertical arrow represents an atomic transition that gives rise to the photons of one spectral line (a single wavelength or frequency).



The vertical arrows in Fig. 27–29 represent the transitions or jumps that correspond to the various observed spectral lines. For example, an electron jumping from the level  $n = 3$  to  $n = 2$  would give rise to the 656-nm line in the Balmer series, and the jump from  $n = 4$  to  $n = 2$  would give rise to the 486-nm line (see Fig. 27–24). We can predict wavelengths of the spectral lines emitted according to Bohr theory by combining Eq. 27–10 with Eq. 27–15. Since  $hf = hc/\lambda$ , we have from Eq. 27–10

$$\frac{1}{\lambda} = \frac{hf}{hc} = \frac{1}{hc} (E_n - E_{n'}),$$

where  $n$  refers to the upper state and  $n'$  to the lower state. Then using Eq. 27–15,

$$\frac{1}{\lambda} = \frac{2\pi^2 Z^2 e^4 m k^2}{h^3 c} \left( \frac{1}{n'^2} - \frac{1}{n^2} \right). \quad (27-16)$$

This theoretical formula has the same form as the experimental Balmer formula, Eq. 27–9, with  $n' = 2$ . Thus we see that the Balmer series of lines corresponds to transitions or “jumps” that bring the electron down to the second energy level. Similarly,  $n' = 1$  corresponds to the Lyman series and  $n' = 3$  to the Paschen series (see Fig. 27–29).

When the constant in Eq. 27–16 is evaluated with  $Z = 1$ , it is found to have the measured value of the Rydberg constant,  $R = 1.0974 \times 10^7 \text{ m}^{-1}$  in Eq. 27–9, in accord with experiment (see Problem 47).

The great success of Bohr's model is that it gives an explanation for why atoms emit line spectra, and accurately predicts the wavelengths of emitted light for hydrogen. The Bohr model also explains absorption spectra: photons of just the right wavelength can knock an electron from one energy level to a higher one. To conserve energy, only photons that have just the right energy will be absorbed. This explains why a continuous spectrum of light entering a gas will emerge with dark (absorption) lines at frequencies that correspond to emission lines (Fig. 27–23c).

The Bohr theory also ensures the stability of atoms. It establishes stability by decree: the ground state is the lowest state for an electron and there is no lower energy level to which it can go and emit more energy. Finally, as we saw above, the Bohr theory accurately predicts the ionization energy of 13.6 eV for hydrogen. However, the Bohr model was not so successful for other atoms, and has been superseded as we shall discuss in the next Chapter. We discuss the Bohr model because it *was* an important start and because we still use the concept of stationary states, the ground state, and transitions between states. Also, the terminology used in the Bohr model is still used by chemists and spectroscopists.

**EXAMPLE 27–13 Wavelength of a Lyman line.** Use Fig. 27–29 to determine the wavelength of the first Lyman line, the transition from  $n = 2$  to  $n = 1$ . In what region of the electromagnetic spectrum does this lie?

**APPROACH** We use Eq. 27–10,  $hf = E_u - E_\ell$ , with the energies obtained from Fig. 27–29 to find the energy and the wavelength of the transition. The region of the electromagnetic spectrum is found using the EM spectrum in Fig. 22–8.

**SOLUTION** In this case,  $hf = E_2 - E_1 = \{-3.4 \text{ eV} - (-13.6 \text{ eV})\} = 10.2 \text{ eV} = (10.2 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV}) = 1.63 \times 10^{-18} \text{ J}$ . Since  $\lambda = c/f$ , we have

$$\lambda = \frac{c}{f} = \frac{hc}{E_2 - E_1} = \frac{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})(3.00 \times 10^8 \text{ m/s})}{1.63 \times 10^{-18} \text{ J}} = 1.22 \times 10^{-7} \text{ m},$$

or 122 nm, which is in the UV region of the EM spectrum, Fig. 22–8. See also Fig. 27–25, where this value is confirmed experimentally.

**NOTE** An alternate approach: use Eq. 27–16 to find  $\lambda$ , and get the same result.

**EXAMPLE 27–14 Wavelength of a Balmer line.** Use the Bohr model to determine the wavelength of light emitted when a hydrogen atom makes a transition from the  $n = 6$  to the  $n = 2$  energy level.

**APPROACH** We can use Eq. 27–16 or its equivalent, Eq. 27–9, with  $R = 1.097 \times 10^7 \text{ m}^{-1}$ .

**SOLUTION** We find

$$\frac{1}{\lambda} = (1.097 \times 10^7 \text{ m}^{-1}) \left( \frac{1}{4} - \frac{1}{36} \right) = 2.44 \times 10^6 \text{ m}^{-1}.$$

So  $\lambda = 1/(2.44 \times 10^6 \text{ m}^{-1}) = 4.10 \times 10^{-7} \text{ m}$  or 410 nm. This is the fourth line in the Balmer series, Fig. 27–24, and is violet in color.

**EXAMPLE 27–15 Absorption wavelength.** Use Fig. 27–29 to determine the maximum wavelength that hydrogen in its ground state can absorb. What would be the next smaller wavelength that would work?

**APPROACH** Maximum wavelength corresponds to minimum energy, and this would be the jump from the ground state up to the first excited state (Fig. 27–29). The next smaller wavelength occurs for the jump from the ground state to the second excited state.

**SOLUTION** The energy needed to jump from the ground state to the first excited state is  $13.6 \text{ eV} - 3.4 \text{ eV} = 10.2 \text{ eV}$ ; the required wavelength, as we saw in Example 27–13, is 122 nm. The energy to jump from the ground state to the second excited state is  $13.6 \text{ eV} - 1.5 \text{ eV} = 12.1 \text{ eV}$ , which corresponds to a wavelength

$$\lambda = \frac{c}{f} = \frac{hc}{hf} = \frac{hc}{E_3 - E_1} = \frac{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})(3.00 \times 10^8 \text{ m/s})}{(12.1 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})} = 103 \text{ nm}.$$

**EXAMPLE 27-16** **He<sup>+</sup> ionization energy.** (a) Use the Bohr model to determine the ionization energy of the He<sup>+</sup> ion, which has a single electron. (b) Also calculate the maximum wavelength a photon can have to cause ionization. The helium atom is the second atom, after hydrogen, in the Periodic Table (next Chapter); its nucleus contains 2 protons and normally has 2 electrons circulating around it, so  $Z = 2$ .

**APPROACH** We want to determine the minimum energy required to lift the electron from its ground state and to barely reach the free state at  $E = 0$ . The ground state energy of He<sup>+</sup> is given by Eq. 27-15b with  $n = 1$  and  $Z = 2$ .

**SOLUTION** (a) Since all the symbols in Eq. 27-15b are the same as for the calculation for hydrogen, except that  $Z$  is 2 instead of 1, we see that  $E_1$  will be  $Z^2 = 2^2 = 4$  times the  $E_1$  for hydrogen:

$$E_1 = 4(-13.6 \text{ eV}) = -54.4 \text{ eV}.$$

Thus, to ionize the He<sup>+</sup> ion should require 54.4 eV, and this value agrees with experiment.

(b) The maximum wavelength photon that can cause ionization will have energy  $hf = 54.4 \text{ eV}$  and wavelength

$$\lambda = \frac{c}{f} = \frac{hc}{hf} = \frac{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})(3.00 \times 10^8 \text{ m/s})}{(54.4 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})} = 22.8 \text{ nm}.$$

If  $\lambda > 22.8 \text{ nm}$ , ionization can not occur.

**NOTE** If the atom absorbed a photon of greater energy (wavelength shorter than 22.8 nm), the atom could still be ionized and the freed electron would have kinetic energy of its own.

In this Example 27-16, we saw that  $E_1$  for the He<sup>+</sup> ion is four times more negative than that for hydrogen. Indeed, the energy-level diagram for He<sup>+</sup> looks just like that for hydrogen, Fig. 27-29, except that the numerical values for each energy level are four times larger. Note, however, that we are talking here about the He<sup>+</sup> ion. Normal (neutral) helium has two electrons and its energy level diagram is entirely different.

**CONCEPTUAL EXAMPLE 27-17** **Hydrogen at 20°C.** (a) Estimate the average kinetic energy of whole hydrogen atoms (not just the electrons) at room temperature. (b) Use the result to explain why, at room temperature, very few H atoms are in excited states and nearly all are in the ground state, and hence emit no light.

**RESPONSE** According to kinetic theory (Chapter 13), the average kinetic energy of atoms or molecules in a gas is given by Eq. 13-8:

$$\overline{\text{KE}} = \frac{3}{2}kT,$$

where  $k = 1.38 \times 10^{-23} \text{ J/K}$  is Boltzmann's constant, and  $T$  is the kelvin (absolute) temperature. Room temperature is about  $T = 300 \text{ K}$ , so

$$\overline{\text{KE}} = \frac{3}{2}(1.38 \times 10^{-23} \text{ J/K})(300 \text{ K}) = 6.2 \times 10^{-21} \text{ J},$$

or, in electron volts:

$$\overline{\text{KE}} = \frac{6.2 \times 10^{-21} \text{ J}}{1.6 \times 10^{-19} \text{ J/eV}} = 0.04 \text{ eV}.$$

The average KE of an atom as a whole is thus very small compared to the energy between the ground state and the next higher energy state ( $13.6 \text{ eV} - 3.4 \text{ eV} = 10.2 \text{ eV}$ ). Any atoms in excited states quickly fall to the ground state and emit light. Once in the ground state, collisions with other atoms can transfer energy of only 0.04 eV on the average. A small fraction of atoms can have much more energy (see Section 13-10 on the distribution of molecular speeds), but even a kinetic energy that is 10 times the average is not nearly enough to excite atoms into states above the ground state. Thus, at room temperature, practically all atoms are in the ground state. Atoms can be excited to upper states by very high temperatures, or by applying a high voltage so a current of high energy electrons passes through the gas as in a discharge tube (Fig. 27-22).



## Correspondence Principle

We should note that Bohr made some radical assumptions that were at variance with classical ideas. He assumed that electrons in fixed orbits do not radiate light even though they are accelerating (moving in a circle), and he assumed that angular momentum is quantized. Furthermore, he was not able to say how an electron moved when it made a transition from one energy level to another. On the other hand, there is no real reason to expect that in the tiny world of the atom electrons would behave as ordinary-sized objects do. Nonetheless, he felt that where quantum theory overlaps with the macroscopic world, it should predict classical results. This is the **correspondence principle**, already mentioned in regard to relativity (Section 26–11). This principle does work for Bohr's theory of the hydrogen atom. The orbit sizes and energies are quite different for  $n = 1$  and  $n = 2$ , say. But orbits with  $n = 100,000,000$  and  $100,000,001$  would be very close in radius and energy (see Fig. 27–29). Indeed, transitions between such large orbits, which would approach macroscopic sizes, would be imperceptible. Such orbits would thus appear to be continuously spaced, which is what we expect in the everyday world.

Finally, it must be emphasized that the well-defined orbits of the Bohr model do not actually exist. The Bohr model is only a model, not reality. The idea of electron orbits was rejected a few years later, and today electrons are thought of (Chapter 28) as forming “probability clouds.”

## 27–13 de Broglie's Hypothesis Applied to Atoms

Bohr's theory was largely of an *ad hoc* nature. Assumptions were made so that theory would agree with experiment. But Bohr could give no reason why the orbits were quantized, nor why there should be a stable ground state. Finally, ten years later, a reason was proposed by Louis de Broglie. We saw in Section 27–8 that in 1923, de Broglie proposed that material particles, such as electrons, have a wave nature; and that this hypothesis was confirmed by experiment several years later.

One of de Broglie's original arguments in favor of the wave nature of electrons was that it provided an explanation for Bohr's theory of the hydrogen atom. According to de Broglie, a particle of mass  $m$  moving with a nonrelativistic speed  $v$  would have a wavelength (Eq. 27–8) of

$$\lambda = \frac{h}{mv}.$$

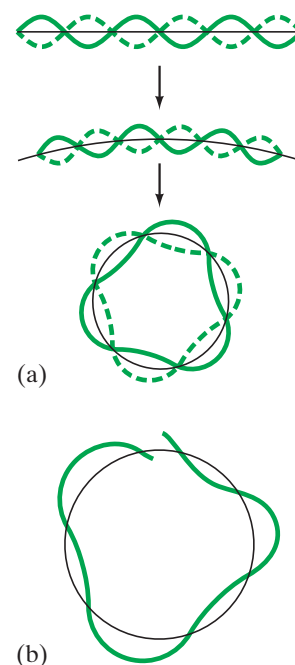
Each electron orbit in an atom, he proposed, is actually a standing wave. As we saw in Chapter 11, when a violin or guitar string is plucked, a vast number of wavelengths are excited. But only certain ones—those that have nodes at the ends—are sustained. These are the *resonant* modes of the string. Waves with other wavelengths interfere with themselves upon reflection and their amplitudes quickly drop to zero. With electrons moving in circles, according to Bohr's theory, de Broglie argued that the electron wave was a *circular* standing wave that closes on itself, Fig. 27–30a. If the wavelength of a wave does not close on itself, as in Fig. 27–30b, destructive interference takes place as the wave travels around the loop, and the wave quickly dies out. Thus, the only waves that persist are those for which the circumference of the circular orbit contains a whole number of wavelengths, Fig. 27–31. The circumference of a Bohr orbit of radius  $r_n$  is  $2\pi r_n$ , so to have constructive interference, we need

$$2\pi r_n = n\lambda, \quad n = 1, 2, 3, \dots$$

When we substitute  $\lambda = h/mv$ , we get  $2\pi r_n = nh/mv$ , or

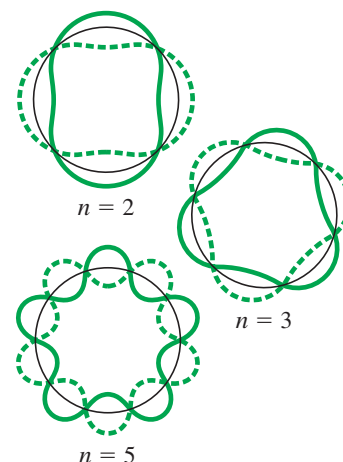
$$mvr_n = \frac{nh}{2\pi}.$$

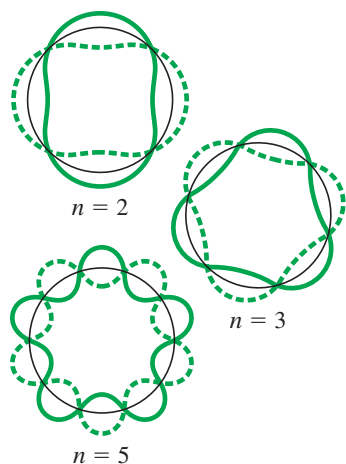
This is just the *quantum condition* proposed by Bohr on an *ad hoc* basis, Eq. 27–11. It is from this equation that the discrete orbits and energy levels were derived.



**FIGURE 27–30** (a) An ordinary standing wave compared to a circular standing wave. (b) When a wave does not close (and hence interferes destructively with itself), it rapidly dies out.

**FIGURE 27–31** Standing circular waves for two, three, and five wavelengths on the circumference;  $n$ , the number of wavelengths, is also the quantum number.





**FIGURE 27-31** (Repeated.) Standing circular waves for two, three, and five wavelengths on the circumference;  $n$ , the number of wavelengths, is also the quantum number.

Now we have a first explanation for the quantized orbits and energy states in the Bohr model: they are due to the wave nature of the electron, and only resonant “standing” waves can persist.<sup>†</sup> This implies that the *wave-particle duality* is at the root of atomic structure.

In viewing the circular electron waves of Fig. 27–31, the electron is not to be thought of as following the oscillating wave pattern. In the Bohr model of hydrogen, the electron moves in a circle. The circular wave, on the other hand, represents the *amplitude* of the electron “matter wave,” and in Fig. 27–31 the wave amplitude is shown superimposed on the circular path of the particle orbit for convenience.

Bohr’s theory worked well for hydrogen and for one-electron ions. But it did not prove successful for multi-electron atoms. Bohr theory could not predict line spectra even for the next simplest atom, helium. It could not explain why some emission lines are brighter than others, nor why some lines are split into two or more closely spaced lines (“fine structure”). A new theory was needed and was indeed developed in the 1920s. This new and radical theory is called *quantum mechanics*. It finally solved the problem of atomic structure, but it gives us a very different view of the atom: the idea of electrons in well-defined orbits was replaced with the idea of electron “clouds.” This new theory of quantum mechanics has given us a wholly different view of the basic mechanisms underlying physical processes.

<sup>†</sup>We note, however, that Eq. 27–11 is no longer considered valid, as discussed in the next Chapter.

## Summary

The electron was discovered using an evacuated cathode ray tube. The measurement of the charge-to-mass ratio ( $e/m$ ) of the electron was done using magnetic and electric fields. The charge  $e$  on the electron was first measured in the Millikan oil-drop experiment and then its mass was obtained from the measured value of the  $e/m$  ratio.

Quantum theory has its origins in **Planck’s quantum hypothesis** that molecular oscillations are **quantized**: their energy  $E$  can only be integer ( $n$ ) multiples of  $hf$ , where  $h$  is Planck’s constant and  $f$  is the natural frequency of oscillation:

$$E = nhf. \quad (27-3)$$

This hypothesis explained the spectrum of radiation emitted by a **blackbody** at high temperature.

Einstein proposed that for some experiments, light could be pictured as being emitted and absorbed as **quanta** (particles), which we now call **photons**, each with energy

$$E = hf \quad (27-4)$$

and momentum

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}. \quad (27-6)$$

He proposed the photoelectric effect as a test for the photon theory of light. In the **photoelectric effect**, the photon theory says that each incident photon can strike an electron in a material and eject it if the photon has sufficient energy. The maximum energy of ejected electrons is then linearly related to the frequency of the incident light.

The photon theory is also supported by the **Compton effect** and the observation of electron–positron **pair production**.

The **wave-particle duality** refers to the idea that light and matter (such as electrons) have both wave and particle properties. The wavelength of an object is given by

$$\lambda = \frac{h}{p}, \quad (27-8)$$

where  $p$  is the momentum of the object ( $p = mv$  for a particle of mass  $m$  and speed  $v$ ).

The **principle of complementarity** states that we must be aware of both the particle and wave properties of light and of matter for a complete understanding of them.

Electron microscopes (EM) make use of the wave properties of electrons to form an image: their “lenses” are magnetic. Various types of EM exist: some can magnify 100,000 $\times$  (1000 $\times$  better than a light microscope); others can give a 3-D image.

Early models of the atom include Rutherford’s planetary (or nuclear) model of an atom which consists of a tiny but massive positively charged nucleus surrounded (at a relatively great distance) by electrons.

To explain the **line spectra** emitted by atoms, as well as the stability of atoms, the **Bohr model** postulated that: (1) electrons bound in an atom can only occupy orbits for which the angular momentum is quantized, which results in discrete values for the radius and energy; (2) an electron in such a **stationary state** emits no radiation; (3) if an electron jumps to a lower state, it emits a photon whose energy equals the difference in energy between the two states; (4) the angular momentum  $L$  of atomic electrons is quantized by the rule  $L = nh/2\pi$ , where  $n$  is an integer called the **quantum number**. The  $n = 1$  state is the **ground state**, which in hydrogen has an energy  $E_1 = -13.6$  eV. Higher values of  $n$  correspond to **excited states**, and their energies are

$$E_n = -(13.6 \text{ eV}) \frac{Z^2}{n^2}, \quad (27-15b)$$

where  $Ze$  is the charge on the nucleus. Atoms are excited to these higher states by collisions with other atoms or electrons, or by absorption of a photon of just the right frequency.

De Broglie’s hypothesis that electrons (and other matter) have a wavelength  $\lambda = h/mv$  gave an explanation for Bohr’s quantized orbits by bringing in the wave-particle duality: the orbits correspond to circular standing waves in which the circumference of the orbit equals a whole number of wavelengths.

## Questions

- Does a lightbulb at a temperature of 2500 K produce as white a light as the Sun at 6000 K? Explain.
- If energy is radiated by all objects, why can we not see them in the dark? (See also Section 14–8.)
- What can be said about the relative temperatures of whitish-yellow, reddish, and bluish stars? Explain.
- Darkrooms for developing black-and-white film were sometimes lit by a red bulb. Why red? Explain if such a bulb would work in a darkroom for developing color film.
- If the threshold wavelength in the photoelectric effect increases when the emitting metal is changed to a different metal, what can you say about the work functions of the two metals?
- Explain why the existence of a cutoff frequency in the photoelectric effect more strongly favors a particle theory rather than a wave theory of light.
- UV light causes sunburn, whereas visible light does not. Suggest a reason.
- The work functions for sodium and cesium are 2.28 eV and 2.14 eV, respectively. For incident photons of a given frequency, which metal will give a higher maximum kinetic energy for the electrons? Explain.
- Explain how the photoelectric circuit of Fig. 27–6 could be used in (a) a burglar alarm, (b) a smoke detector, (c) a photographic light meter.
- (a) Does a beam of infrared photons always have less energy than a beam of ultraviolet photons? Explain. (b) Does a single photon of infrared light always have less energy than a single photon of ultraviolet light? Why?
- Light of 450-nm wavelength strikes a metal surface, and a stream of electrons emerges from the metal. If light of the same intensity but of wavelength 400 nm strikes the surface, are more electrons emitted? Does the energy of the emitted electrons change? Explain.
- If an X-ray photon is scattered by an electron, does the photon's wavelength change? If so, does it increase or decrease? Explain.
- In both the photoelectric effect and in the Compton effect, a photon collides with an electron causing the electron to fly off. What is the difference between the two processes?
- Why do we say that light has wave properties? Why do we say that light has particle properties?
- Why do we say that electrons have wave properties? Why do we say that electrons have particle properties?
- What are the differences between a photon and an electron? Be specific: make a list.
- If an electron and a proton travel at the same speed, which has the shorter wavelength? Explain.
- An electron and a proton are accelerated through the same voltage. Which has the longer wavelength? Explain why.
- In Rutherford's planetary model of the atom, what keeps the electrons from flying off into space?
- When a wide spectrum of light passes through hydrogen gas at room temperature, absorption lines are observed that correspond only to the Lyman series. Why don't we observe the other series?
- How can you tell if there is oxygen near the surface of the Sun?
- (a) List at least three successes of the Bohr model of the atom, according to Section 27–12. (b) List at least two observations that the Bohr model could not explain, according to Section 27–13.
- According to Section 27–11, what were the two main difficulties of the Rutherford model of the atom?
- Is it possible for the de Broglie wavelength of a "particle" to be greater than the dimensions of the particle? To be smaller? Is there any direct connection? Explain.
- How can the spectrum of hydrogen contain so many lines when hydrogen contains only one electron?
- Explain how the closely spaced energy levels for hydrogen near the top of Fig. 27–29 correspond to the closely spaced spectral lines at the top of Fig. 27–24.
- In a helium atom, which contains two electrons, do you think that on average the electrons are closer to the nucleus or farther away than in a hydrogen atom? Why?
- The Lyman series is brighter than the Balmer series, because this series of transitions ends up in the most common state for hydrogen, the ground state. Why then was the Balmer series discovered first?
- Use conservation of momentum to explain why photons emitted by hydrogen atoms have slightly less energy than that predicted by Eq. 27–10.
- State if a continuous or a line spectrum is produced by each of the following: (a) a hot solid object; (b) an excited, rarefied gas; (c) a hot liquid; (d) light from a hot solid that passes through a cooler rarefied gas; (e) a hot dense gas. For each, if a line spectrum is produced, is it an emission or an absorption spectrum?
- Suppose we obtain an emission spectrum for hydrogen at very high temperature (when some of the atoms are in excited states), and an absorption spectrum at room temperature, when all atoms are in the ground state. Will the two spectra contain identical lines?

## MisConceptual Questions

- Which of the following statements is true regarding how blackbody radiation changes as the temperature of the radiating object increases?
  - Both the maximum intensity and the peak wavelength increase.
  - The maximum intensity increases, and the peak wavelength decreases.
  - Both the maximum intensity and the peak wavelength decrease.
  - The maximum intensity decreases, and the peak wavelength increases.
- As red light shines on a piece of metal, no electrons are released. When the red light is slowly changed to shorter-wavelength light (basically progressing through the rainbow), nothing happens until yellow light shines on the metal, at which point electrons are released from the metal. If this metal is replaced with a metal having a higher work function, which light would have the best chance of releasing electrons from the metal?
  - Blue.
  - Red.
  - Yellow would still work fine.
  - We need to know more about the metals involved.
- A beam of red light and a beam of blue light have equal intensities. Which statement is true?
  - There are more photons in the blue beam.
  - There are more photons in the red beam.
  - Both beams contain the same number of photons.
  - The number of photons is not related to intensity.
- Which of the following is necessarily true?
  - Red light has more energy than violet light.
  - Violet light has more energy than red light.
  - A single photon of red light has more energy than a single photon of violet light.
  - A single photon of violet light has more energy than a single photon of red light.
  - None of the above.
  - A combination of the above (specify).
- If a photon of energy  $E$  ejects electrons from a metal with kinetic energy  $KE$ , then a photon with energy  $E/2$ 
  - will eject electrons with kinetic energy  $KE/2$ .
  - will eject electrons with an energy greater than  $KE/2$ .
  - will eject electrons with an energy less than  $KE/2$ .
  - might not eject any electrons.
- If the momentum of an electron were doubled, how would its wavelength change?
  - No change.
  - It would be halved.
  - It would double.
  - It would be quadrupled.
  - It would be reduced to one-fourth.
- Which of the following can be thought of as either a wave or a particle?
  - Light.
  - An electron.
  - A proton.
  - All of the above.
- When you throw a baseball, its de Broglie wavelength is
  - the same size as the ball.
  - about the same size as an atom.
  - about the same size as an atom's nucleus.
  - much smaller than the size of an atom's nucleus.
- Electrons and photons of light are similar in that
  - both have momentum given by  $h/\lambda$ .
  - both exhibit wave-particle duality.
  - both are used in diffraction experiments to explore structure.
  - All of the above.
  - None of the above.
- In Rutherford's famous set of experiments described in Section 27-10, the fact that some alpha particles were deflected at large angles indicated that (choose all that apply)
  - the nucleus was positive.
  - charge was quantized.
  - the nucleus was concentrated in a small region of space.
  - most of the atom is empty space.
  - None of the above.
- Which of the following electron transitions between two energy states ( $n$ ) in the hydrogen atom corresponds to the emission of a photon with the longest wavelength?
  - $2 \rightarrow 5$ .
  - $5 \rightarrow 2$ .
  - $5 \rightarrow 8$ .
  - $8 \rightarrow 5$ .
- If we set the potential energy of an electron and a proton to be zero when they are an infinite distance apart, then the lowest energy a bound electron in a hydrogen atom can have is
  - 0.
  - $-13.6 \text{ eV}$ .
  - any possible value.
  - any value between  $-13.6 \text{ eV}$  and 0.
- Which of the following is the currently accepted model of the atom?
  - The plum-pudding model.
  - The Rutherford atom.
  - The Bohr atom.
  - None of the above.
- Light has all of the following except:
  - mass.
  - momentum.
  - kinetic energy.
  - frequency.
  - wavelength.





# Problems

## 27-1 Discovery of the Electron

- (I) What is the value of  $e/m$  for a particle that moves in a circle of radius 14 mm in a 0.86-T magnetic field if a perpendicular 640-V/m electric field will make the path straight?
- (II) (a) What is the velocity of a beam of electrons that go undeflected when passing through crossed (perpendicular) electric and magnetic fields of magnitude  $1.88 \times 10^4$  V/m and  $2.60 \times 10^{-3}$  T, respectively? (b) What is the radius of the electron orbit if the electric field is turned off?
- (II) An oil drop whose mass is  $2.8 \times 10^{-15}$  kg is held at rest between two large plates separated by 1.0 cm (Fig. 27-3), when the potential difference between the plates is 340 V. How many excess electrons does this drop have?

## 27-2 Blackbodies; Planck's Quantum Hypothesis

- (I) How hot is a metal being welded if it radiates most strongly at 520 nm?
- (I) Estimate the peak wavelength for radiation emitted from (a) ice at  $0^\circ\text{C}$ , (b) a floodlamp at 3100 K, (c) helium at 4 K, assuming blackbody emission. In what region of the EM spectrum is each?
- (I) (a) What is the temperature if the peak of a blackbody spectrum is at 18.0 nm? (b) What is the wavelength at the peak of a blackbody spectrum if the body is at a temperature of 2200 K?
- (I) An HCl molecule vibrates with a natural frequency of  $8.1 \times 10^{13}$  Hz. What is the difference in energy (in joules and electron volts) between successive values of the oscillation energy?
- (II) The steps of a flight of stairs are 20.0 cm high (vertically). If a 62.0-kg person stands with both feet on the same step, what is the gravitational potential energy of this person, relative to the ground, on (a) the first step, (b) the second step, (c) the third step, (d) the  $n^{\text{th}}$  step? (e) What is the change in energy as the person descends from step 6 to step 2?
- (II) Estimate the peak wavelength of light emitted from the pupil of the human eye (which approximates a blackbody) assuming normal body temperature.

## 27-3 and 27-4 Photons and the Photoelectric Effect

- (I) What is the energy of photons (joules) emitted by a 91.7-MHz FM radio station?
- (I) What is the energy range (in joules and eV) of photons in the visible spectrum, of wavelength 400 nm to 750 nm?
- (I) A typical gamma ray emitted from a nucleus during radioactive decay may have an energy of 320 keV. What is its wavelength? Would we expect significant diffraction of this type of light when it passes through an everyday opening, such as a door?
- (I) Calculate the momentum of a photon of yellow light of wavelength  $5.80 \times 10^{-7}$  m.
- (I) What is the momentum of a  $\lambda = 0.014$  nm X-ray photon?
- (II) About 0.1 eV is required to break a "hydrogen bond" in a protein molecule. Calculate the minimum frequency and maximum wavelength of a photon that can accomplish this.
- (II) What minimum frequency of light is needed to eject electrons from a metal whose work function is  $4.8 \times 10^{-19}$  J?

- (II) The human eye can respond to as little as  $10^{-18}$  J of light energy. For a wavelength at the peak of visual sensitivity, 550 nm, how many photons lead to an observable flash?
- (II) What is the longest wavelength of light that will emit electrons from a metal whose work function is 2.90 eV?
- (II) The work functions for sodium, cesium, copper, and iron are 2.3, 2.1, 4.7, and 4.5 eV, respectively. Which of these metals will not emit electrons when visible light shines on it?
- (II) In a photoelectric-effect experiment it is observed that no current flows unless the wavelength is less than 550 nm. (a) What is the work function of this material? (b) What stopping voltage is required if light of wavelength 400 nm is used?
- (II) What is the maximum kinetic energy of electrons ejected from barium ( $W_0 = 2.48$  eV) when illuminated by white light,  $\lambda = 400$  to 750 nm?
- (II) Barium has a work function of 2.48 eV. What is the maximum kinetic energy of electrons if the metal is illuminated by UV light of wavelength 365 nm? What is their speed?
- (II) When UV light of wavelength 255 nm falls on a metal surface, the maximum kinetic energy of emitted electrons is 1.40 eV. What is the work function of the metal?
- (II) The threshold wavelength for emission of electrons from a given surface is 340 nm. What will be the maximum kinetic energy of ejected electrons when the wavelength is changed to (a) 280 nm, (b) 360 nm?
- (II) When 250-nm light falls on a metal, the current through a photoelectric circuit (Fig. 27-6) is brought to zero at a stopping voltage of 1.64 V. What is the work function of the metal?
- (II) In a photoelectric experiment using a clean sodium surface, the maximum energy of the emitted electrons was measured for a number of different incident frequencies, with the following results.

Frequency ( $\times 10^{14}$ Hz)	Energy (eV)
11.8	2.60
10.6	2.11
9.9	1.81
9.1	1.47
8.2	1.10
6.9	0.57

Plot the graph of these results and find: (a) Planck's constant; (b) the cutoff frequency of sodium; (c) the work function.

- (II) Show that the energy  $E$  (in electron volts) of a photon whose wavelength is  $\lambda$  (nm) is given by

$$E = \frac{1.240 \times 10^3 \text{ eV} \cdot \text{nm}}{\lambda \text{ (nm)}}$$

Use at least 4 significant figures for values of  $h$ ,  $c$ ,  $e$  (see inside front cover).

## \*27-5 Compton Effect

- (I) A high-frequency photon is scattered off of an electron and experiences a change of wavelength of  $1.7 \times 10^{-4}$  nm. At what angle must a detector be placed to detect the scattered photon (relative to the direction of the incoming photon)?

- \*29. (II) The quantity  $h/mc$ , which has the dimensions of length, is called the *Compton wavelength*. Determine the Compton wavelength for (a) an electron, (b) a proton. (c) Show that if a photon has wavelength equal to the Compton wavelength of a particle, the photon's energy is equal to the rest energy of the particle,  $mc^2$ .
- \*30. (II) X-rays of wavelength  $\lambda = 0.140$  nm are scattered from carbon. What is the expected Compton wavelength shift for photons detected at angles (relative to the incident beam) of exactly (a)  $45^\circ$ , (b)  $90^\circ$ , (c)  $180^\circ$ ?

## 27-6 Pair Production

31. (I) How much total kinetic energy will an electron-positron pair have if produced by a 3.64-MeV photon?
32. (II) What is the longest wavelength photon that could produce a proton-antiproton pair? (Each has a mass of  $1.67 \times 10^{-27}$  kg.)
33. (II) What is the minimum photon energy needed to produce a  $\mu^+\mu^-$  pair? The mass of each  $\mu$  (muon) is 207 times the mass of an electron. What is the wavelength of such a photon?
34. (II) An electron and a positron, each moving at  $3.0 \times 10^5$  m/s, collide head on, disappear, and produce two photons, each with the same energy and momentum moving in opposite directions. Determine the energy and momentum of each photon.
35. (II) A gamma-ray photon produces an electron and a positron, each with a kinetic energy of 285 keV. Determine the energy and wavelength of the photon.

## 27-8 Wave Nature of Matter

36. (I) Calculate the wavelength of a 0.21-kg ball traveling at 0.10 m/s.
37. (I) What is the wavelength of a neutron ( $m = 1.67 \times 10^{-27}$  kg) traveling at  $8.5 \times 10^4$  m/s?
38. (II) Through how many volts of potential difference must an electron, initially at rest, be accelerated to achieve a wavelength of 0.27 nm?
39. (II) Calculate the ratio of the kinetic energy of an electron to that of a proton if their wavelengths are equal. Assume that the speeds are nonrelativistic.
40. (II) An electron has a de Broglie wavelength  $\lambda = 4.5 \times 10^{-10}$  m. (a) What is its momentum? (b) What is its speed? (c) What voltage was needed to accelerate it from rest to this speed?
41. (II) What is the wavelength of an electron of energy (a) 10 eV, (b) 100 eV, (c) 1.0 keV?
42. (II) Show that if an electron and a proton have the same nonrelativistic kinetic energy, the proton has the shorter wavelength.
43. (II) Calculate the de Broglie wavelength of an electron if it is accelerated from rest by 35,000 V as in Fig. 27-2. Is it relativistic? How does its wavelength compare to the size of the "neck" of the tube, typically 5 cm? Do we have to worry about diffraction problems blurring the picture on the CRT screen?

## 27-9 Electron Microscope

44. (II) What voltage is needed to produce electron wavelengths of 0.26 nm? (Assume that the electrons are nonrelativistic.)
45. (II) Electrons are accelerated by 2850 V in an electron microscope. Estimate the maximum possible resolution of the microscope.

## 27-11 and 27-12 Spectra and the Bohr Model

46. (I) For the three hydrogen transitions indicated below, with  $n$  being the initial state and  $n'$  being the final state, is the transition an absorption or an emission? Which is higher, the initial state energy or the final state energy of the atom? Finally, which of these transitions involves the largest energy photon? (a)  $n = 1, n' = 3$ ; (b)  $n = 6, n' = 2$ ; (c)  $n = 4, n' = 5$ .
47. (I) How much energy is needed to ionize a hydrogen atom in the  $n = 3$  state?
48. (I) The second longest wavelength in the Paschen series in hydrogen (Fig. 27-29) corresponds to what transition?
49. (I) Calculate the ionization energy of doubly ionized lithium,  $\text{Li}^{2+}$ , which has  $Z = 3$  (and is in the ground state).
50. (I) Evaluate the Rydberg constant  $R$  using the Bohr model (compare Eqs. 27-9 and 27-16) and show that its value is  $R = 1.0974 \times 10^7 \text{ m}^{-1}$ . (Use values inside front cover to 5 or 6 significant figures.)
51. (II) What is the longest wavelength light capable of ionizing a hydrogen atom in the ground state?
52. (II) What wavelength photon would be required to ionize a hydrogen atom in the ground state and give the ejected electron a kinetic energy of 11.5 eV?
53. (II) In the Sun, an ionized helium ( $\text{He}^+$ ) atom makes a transition from the  $n = 6$  state to the  $n = 2$  state, emitting a photon. Can that photon be absorbed by hydrogen atoms present in the Sun? If so, between what energy states will the hydrogen atom transition occur?
54. (II) Construct the energy-level diagram for the  $\text{He}^+$  ion (like Fig. 27-29).
55. (II) Construct the energy-level diagram for doubly ionized lithium,  $\text{Li}^{2+}$ .
56. (II) Determine the electrostatic potential energy and the kinetic energy of an electron in the ground state of the hydrogen atom.
57. (II) An excited hydrogen atom could, in principle, have a radius of 1.00 cm. What would be the value of  $n$  for a Bohr orbit of this size? What would its energy be?
58. (II) Is the use of nonrelativistic formulas justified in the Bohr atom? To check, calculate the electron's velocity,  $v$ , in terms of  $c$ , for the ground state of hydrogen, and then calculate  $\sqrt{1 - v^2/c^2}$ .
59. (III) Show that the magnitude of the electrostatic potential energy of an electron in any Bohr orbit of a hydrogen atom is twice the magnitude of its kinetic energy in that orbit.
60. (III) Suppose an electron was bound to a proton, as in the hydrogen atom, but by the gravitational force rather than by the electric force. What would be the radius, and energy, of the first Bohr orbit?

## General Problems

61. The Big Bang theory (Chapter 33) states that the beginning of the universe was accompanied by a huge burst of photons. Those photons are still present today and make up the so-called cosmic microwave background radiation. The universe radiates like a blackbody with a temperature today of about 2.7 K. Calculate the peak wavelength of this radiation.
62. At low temperatures, nearly all the atoms in hydrogen gas will be in the ground state. What minimum frequency photon is needed if the photoelectric effect is to be observed?
63. A beam of 72-eV electrons is scattered from a crystal, as in X-ray diffraction, and a first-order peak is observed at  $\theta = 38^\circ$ . What is the spacing between planes in the diffracting crystal? (See Section 25–11.)
64. A microwave oven produces electromagnetic radiation at  $\lambda = 12.2$  cm and produces a power of 720 W. Calculate the number of microwave photons produced by the microwave oven each second.
65. Sunlight reaching the Earth's atmosphere has an intensity of about  $1300 \text{ W/m}^2$ . Estimate how many photons per square meter per second this represents. Take the average wavelength to be 550 nm.
66. A beam of red laser light ( $\lambda = 633$  nm) hits a black wall and is fully absorbed. If this light exerts a total force  $F = 5.8$  nN on the wall, how many photons per second are hitting the wall?
67. A flashlight emits 2.5 W of light. As the light leaves the flashlight in one direction, a reaction force is exerted on the flashlight in the opposite direction. Estimate the size of this reaction force.
68. A **photomultiplier tube** (a very sensitive light sensor), is based on the photoelectric effect: incident photons strike a metal surface and the resulting ejected electrons are collected. By counting the number of collected electrons, the number of incident photons (i.e., the incident light intensity) can be determined. (a) If a photomultiplier tube is to respond properly for incident wavelengths throughout the visible range (410 nm to 750 nm), what is the maximum value for the work function  $W_0$  (eV) of its metal surface? (b) If  $W_0$  for its metal surface is above a certain threshold value, the photomultiplier will only function for incident ultraviolet wavelengths and be unresponsive to visible light. Determine this threshold value (eV).
69. An electron and a positron collide head on, annihilate, and create two 0.85-MeV photons traveling in opposite directions. What were the initial kinetic energies of electron and positron?
70. By what potential difference must (a) a proton ( $m = 1.67 \times 10^{-27}$  kg), and (b) an electron ( $m = 9.11 \times 10^{-31}$  kg), be accelerated from rest to have a wavelength  $\lambda = 4.0 \times 10^{-12}$  m?
71. In some of Rutherford's experiments (Fig. 27–19) the  $\alpha$  particles (mass  $= 6.64 \times 10^{-27}$  kg) had a kinetic energy of 4.8 MeV. How close could they get to the surface of a gold nucleus (radius  $\approx 7.0 \times 10^{-15}$  m, charge  $= +79e$ )? Ignore the recoil motion of the nucleus.
72. By what fraction does the mass of an H atom decrease when it makes an  $n = 3$  to  $n = 1$  transition?
73. Calculate the ratio of the gravitational force to the electric force for the electron in the ground state of a hydrogen atom. Can the gravitational force be reasonably ignored?
74. Electrons accelerated from rest by a potential difference of 12.3 V pass through a gas of hydrogen atoms at room temperature. What wavelengths of light will be emitted?
75. In a particular photoelectric experiment, a stopping potential of 2.10 V is measured when ultraviolet light of wavelength 270 nm is incident on the metal. Using the same setup, what will the new stopping potential be if blue light of wavelength 440 nm is used, instead?
76. Neutrons can be used in diffraction experiments to probe the lattice structure of crystalline solids. Since the neutron's wavelength needs to be on the order of the spacing between atoms in the lattice, about 0.3 nm, what should the speed of the neutrons be?
77. In Chapter 22, the intensity of light striking a surface was related to the electric field of the associated electromagnetic wave. For photons, the intensity is the number of photons striking a  $1\text{-m}^2$  area per second. Suppose  $1.0 \times 10^{12}$  photons of 497-nm light are incident on a  $1\text{-m}^2$  surface every second. What is the intensity of the light? Using the wave model of light, what is the maximum electric field of the electromagnetic wave?
78. The intensity of the Sun's light in the vicinity of the Earth is about  $1350 \text{ W/m}^2$ . Imagine a spacecraft with a mirrored square sail of dimension 1.0 km. Estimate how much thrust (in newtons) this craft will experience due to collisions with the Sun's photons. [Hint: Assume the photons bounce off the sail with no change in the magnitude of their momentum.]
79. Light of wavelength 280 nm strikes a metal whose work function is 2.2 eV. What is the shortest de Broglie wavelength for the electrons that are produced as photoelectrons?
80. What would be the theoretical limit of resolution for an electron microscope whose electrons are accelerated through 110 kV? (Relativistic formulas should be used.)
81. Assume hydrogen atoms in a gas are initially in their ground state. If free electrons with kinetic energy 12.75 eV collide with these atoms, what photon wavelengths will be emitted by the gas?
82. Visible light incident on a diffraction grating with slit spacing of 0.010 mm has the first maximum at an angle of  $3.6^\circ$  from the central peak. If electrons could be diffracted by the same grating, what electron velocity would produce the same diffraction pattern as the visible light?

83. (a) Suppose an unknown element has an absorption spectrum with lines corresponding to 2.5, 4.7, and 5.1 eV above its ground state and an ionization energy of 11.5 eV. Draw an energy level diagram for this element. (b) If a 5.1-eV photon is absorbed by an atom of this substance, in which state was the atom before absorbing the photon? What will be the energies of the photons that can subsequently be emitted by this atom?
84. A photon of momentum  $3.53 \times 10^{-28} \text{ kg} \cdot \text{m/s}$  is emitted from a hydrogen atom. To what spectrum series does this photon belong, and from what energy level was it ejected?
85. Light of wavelength 464 nm falls on a metal which has a work function of 2.28 eV. (a) How much voltage should be applied to bring the current to zero? (b) What is the maximum speed of the emitted electrons? (c) What is the de Broglie wavelength of these electrons?
86. An electron accelerated from rest by a 96-V potential difference is injected into a  $3.67 \times 10^{-4} \text{ T}$  magnetic field where it travels in an 18-cm-diameter circle. Calculate  $e/m$  from this information.
87. Estimate the number of photons emitted by the Sun in a year. (Take the average wavelength to be 550 nm and the intensity of sunlight reaching the Earth (outer atmosphere) as  $1350 \text{ W/m}^2$ .)
88. Apply Bohr's assumptions to the Earth–Moon system to calculate the allowed energies and radii of motion. Given the known distance between the Earth and Moon, is the quantization of the energy and radius apparent?
89. At what temperature would the average kinetic energy (Chapter 13) of a molecule of hydrogen gas ( $\text{H}_2$ ) be sufficient to excite a hydrogen atom out of the ground state?

## Search and Learn

- Imagine the following Young's double-slit experiment using matter rather than light: electrons are accelerated through a potential difference of 12 V, pass through two closely spaced slits separated by a distance  $d$ , and create an interference pattern. (a) Using Example 27–11 and Section 24–3 as guides, find the required value for  $d$  if the first-order interference fringe is to be produced at an angle of  $10^\circ$ . (b) Given the approximate size of atoms, would it be possible to construct the required two-slit set-up for this experiment?
- Does each of the following support the wave nature or the particle nature of light? (a) The existence of the cutoff frequency in the photoelectric effect; (b) Young's double-slit experiment; (c) the shift in the photon frequency in Compton scattering; (d) the diffraction of light.
- (a) From Sections 22–3, 24–4, and 27–3, estimate the minimum energy (eV) that initiates the chemical process on the retina responsible for vision. (b) Estimate the threshold photon energy above which the eye registers no sensation of sight.
- (a) A rubidium atom ( $m = 85 \text{ u}$ ) is at rest with one electron in an excited energy level. When the electron jumps to the ground state, the atom emits a photon of wavelength  $\lambda = 780 \text{ nm}$ . Determine the resulting (nonrelativistic) recoil speed  $v$  of the atom. (b) The recoil speed sets the lower limit on the temperature to which an ideal gas of rubidium atoms can be cooled in a laser-based **atom trap**. Using the kinetic theory of gases (Chapter 13), estimate this “lowest achievable” temperature.
- Suppose a particle of mass  $m$  is confined to a one-dimensional box of width  $L$ . According to quantum theory, the particle's wave (with  $\lambda = h/mv$ ) is a standing wave with nodes at the edges of the box. (a) Show the possible modes of vibration on a diagram. (b) Show that the kinetic energy of the particle has quantized energies given by  $\text{KE} = n^2 h^2 / 8mL^2$ , where  $n$  is an integer. (c) Calculate the ground-state energy ( $n = 1$ ) for an electron confined to a box of width  $0.50 \times 10^{-10} \text{ m}$ . (d) What is the ground-state energy, and speed, of a baseball ( $m = 140 \text{ g}$ ) in a box 0.65 m wide? (e) An electron confined to a box has a ground-state energy of 22 eV. What is the width of the box? [Hint: See Sections 27–8, 27–13, and 11–12.]

## ANSWERS TO EXERCISES

- A:  $\lambda_p = 725 \text{ nm}$ , so red.  
 B: More 1000-nm photons (each has lower energy).  
 C:  $5.50 \times 10^{14} \text{ Hz}$ , 545 nm.  
 D: Only  $\lambda$ .  
 E: Decrease.  
 F: (e).