FIGURE 2–18 Painting of Galileo demonstrating to the Grand Duke of Tuscany his argument for the action of gravity being uniform acceleration. He used an inclined plane to slow down the action. A ball rolling down the plane still accelerates. Tiny bells placed at equal distances along the inclined plane would ring at shorter time intervals as the ball "fell," indicating that the speed was increasing.

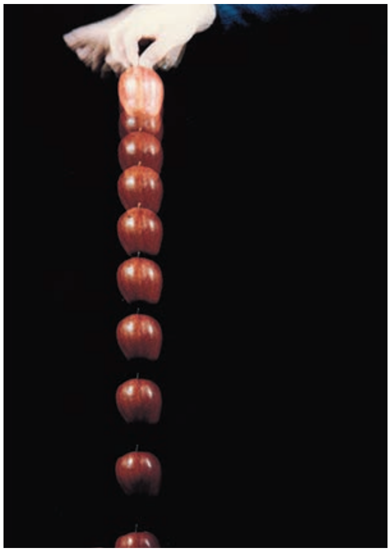


FIGURE 2–19 Multiflash photograph of a falling apple, at equal time intervals. The apple falls farther during each successive interval, which means it is accelerating.

2–7 Freely Falling Objects

One of the most common examples of uniformly accelerated motion is that of an object allowed to fall freely near the Earth's surface. That a falling object is accelerating may not be obvious at first. And beware of thinking, as was widely believed before the time of Galileo (Fig. 2–18), that heavier objects fall faster than lighter objects and that the speed of fall is proportional to how heavy the object is. *The speed of a falling object is* not *proportional to its mass.*

Galileo made use of his new technique of imagining what would happen in idealized (simplified) cases. For free fall, he postulated that *all objects would fall with the same constant acceleration in the absence of air or other resistance*. He showed that this postulate predicts that for an object falling from rest, the distance traveled will be proportional to the square of the time (Fig. 2–19); that is, $d \propto t^2$. We can see this from Eq. 2–11b for constant acceleration; but Galileo was the first to derive this mathematical relation.

To support his claim that falling objects increase in speed as they fall, Galileo made use of a clever argument: a heavy stone dropped from a height of 2 m will drive a stake into the ground much further than will the same stone dropped from a height of only 0.2 m. Clearly, the stone must be moving faster in the former case.

Galileo claimed that *all* objects, light or heavy, fall with the *same* acceleration, at least in the absence of air. If you hold a piece of paper flat and horizontal in one hand, and a heavier object like a baseball in the other, and release them at the same time as in Fig. 2–20a, the heavier object will reach the ground first. But if you repeat the experiment, this time crumpling the paper into a small wad, you will find (see Fig. 2–20b) that the two objects reach the floor at nearly the same time.

Galileo was sure that air acts as a resistance to very light objects that have a large surface area. But in many ordinary circumstances this air resistance is negligible. In a chamber from which the air has been removed, even light objects like a feather or a horizontally held piece of paper will fall with the same acceleration as any other object (see Fig. 2–21). Such a demonstration in vacuum was not possible in Galileo's time, which makes Galileo's achievement all the greater. Galileo is often called the "father of modern science," not only for the *content* of his science (astronomical discoveries, inertia, free fall) but also for his new methods of *doing* science (idealization and simplification, mathematization of theory, theories that have testable consequences, experiments to test theoretical predictions).

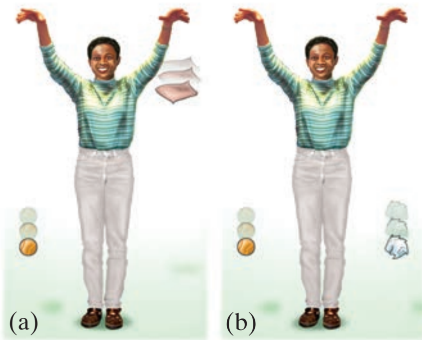
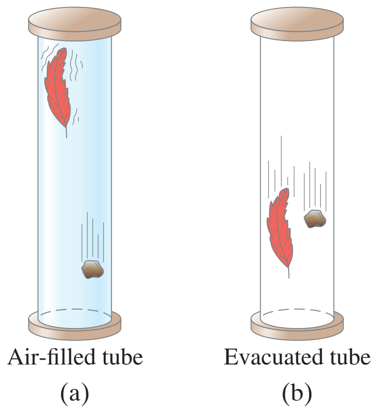


FIGURE 2–20 (a) A ball and a light piece of paper are dropped at the same time. (b) Repeated, with the paper wadded up.

FIGURE 2–21 A rock and a feather are dropped simultaneously (a) in air, (b) in a vacuum.



Galileo's specific contribution to our understanding of the motion of falling objects can be summarized as follows:

> **at a given location on the Earth and in the absence of air resistance, all objects fall with the same constant acceleration.**

We call this acceleration the **acceleration due to gravity** at the surface of the Earth, and we give it the symbol g. Its magnitude is approximately

$$g = 9.80\ \mathrm{m/s^2}. \qquad \left[\begin{array}{l}\text{acceleration due to gravity}\\ \text{at surface of Earth}\end{array}\right]$$

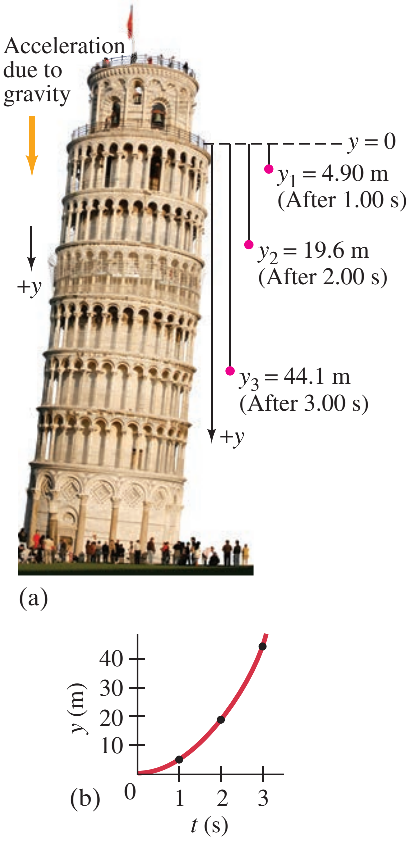
In British units g is about $32\ \mathrm{ft/s^2}$. Actually, g varies slightly according to latitude and elevation on the Earth's surface, but these variations are so small that we will ignore them for most purposes. (Acceleration of gravity in space beyond the Earth's surface is treated in Chapter 5.) The effects of air resistance are often small, and we will neglect them for the most part. However, air resistance will be noticeable even on a reasonably heavy object if the velocity becomes large.† Acceleration due to gravity is a vector, as is any acceleration, and its direction is downward toward the center of the Earth.

When dealing with freely falling objects we can make use of Eqs. 2–11, where for a we use the value of g given above. Also, since the motion is vertical we will substitute y in place of x, and y_0 in place of x_0. We take $y_0 = 0$ unless otherwise specified. *It is arbitrary whether we choose y to be positive in the upward direction or in the downward direction; but we must be consistent about it throughout a problem's solution.*

PROBLEM SOLVING
You can choose y to be positive either up or down

EXERCISE E Return to the Chapter-Opening Question, page 21, and answer it again now, assuming minimal air resistance. Try to explain why you may have answered differently the first time.

FIGURE 2–22 Example 2–10. (a) An object dropped from a tower falls with progressively greater speed and covers greater distance with each successive second. (See also Fig. 2–19.) (b) Graph of y vs. t.



EXAMPLE 2–10 Falling from a tower. Suppose that a ball is dropped ($v_0 = 0$) from a tower. How far will it have fallen after a time $t_1 = 1.00\ \mathrm{s}$, $t_2 = 2.00\ \mathrm{s}$, and $t_3 = 3.00\ \mathrm{s}$? Ignore air resistance.

APPROACH Let us take y as positive downward, so the acceleration is $a = g = +9.80\ \mathrm{m/s^2}$. We set $v_0 = 0$ and $y_0 = 0$. We want to find the position y of the ball after three different time intervals. Equation 2–11b, with x replaced by y, relates the given quantities (t, a, and v_0) to the unknown y.

SOLUTION We set $t = t_1 = 1.00\ \mathrm{s}$ in Eq. 2–11b:

$$\begin{aligned} y_1 &= v_0 t_1 + \tfrac{1}{2}at_1^2 \\ &= 0 + \tfrac{1}{2}at_1^2 = \tfrac{1}{2}(9.80\ \mathrm{m/s^2})(1.00\ \mathrm{s})^2 = 4.90\ \mathrm{m}. \end{aligned}$$

The ball has fallen a distance of 4.90 m during the time interval $t = 0$ to $t_1 = 1.00\ \mathrm{s}$. Similarly, after 2.00 s $(= t_2)$, the ball's position is

$$y_2 = \tfrac{1}{2}at_2^2 = \tfrac{1}{2}(9.80\ \mathrm{m/s^2})(2.00\ \mathrm{s})^2 = 19.6\ \mathrm{m}.$$

Finally, after 3.00 s $(= t_3)$, the ball's position is (see Fig. 2–22)

$$y_3 = \tfrac{1}{2}at_3^2 = \tfrac{1}{2}(9.80\ \mathrm{m/s^2})(3.00\ \mathrm{s})^2 = 44.1\ \mathrm{m}.$$

NOTE Whenever we say "dropped," it means $v_0 = 0$. Note also the graph of y vs. t (Fig. 2–22b): the curve is not straight but bends upward because y is proportional to t^2.

†The speed of an object falling in air (or other fluid) does not increase indefinitely. If the object falls far enough, it will reach a maximum velocity called the **terminal velocity** due to air resistance.

EXAMPLE 2–11 **Thrown down from a tower.** Suppose the ball in Example 2–10 is *thrown* downward with an initial velocity of 3.00 m/s, instead of being dropped. (*a*) What then would be its position after 1.00 s and 2.00 s? (*b*) What would its speed be after 1.00 s and 2.00 s? Compare with the speeds of a dropped ball.

APPROACH Again we use Eq. 2–11b, but now v_0 is not zero, it is $v_0 = 3.00\,\mathrm{m/s}$.

SOLUTION (*a*) At $t_1 = 1.00\,\mathrm{s}$, the position of the ball as given by Eq. 2–11b is

$$y = v_0 t + \tfrac{1}{2}at^2 = (3.00\,\mathrm{m/s})(1.00\,\mathrm{s}) + \tfrac{1}{2}(9.80\,\mathrm{m/s^2})(1.00\,\mathrm{s})^2 = 7.90\,\mathrm{m}.$$

At $t_2 = 2.00\,\mathrm{s}$ (time interval $t = 0$ to $t = 2.00\,\mathrm{s}$), the position is

$$y = v_0 t + \tfrac{1}{2}at^2 = (3.00\,\mathrm{m/s})(2.00\,\mathrm{s}) + \tfrac{1}{2}(9.80\,\mathrm{m/s^2})(2.00\,\mathrm{s})^2 = 25.6\,\mathrm{m}.$$

As expected, the ball falls farther each second than if it were dropped with $v_0 = 0$.

(*b*) The velocity is obtained from Eq. 2–11a:

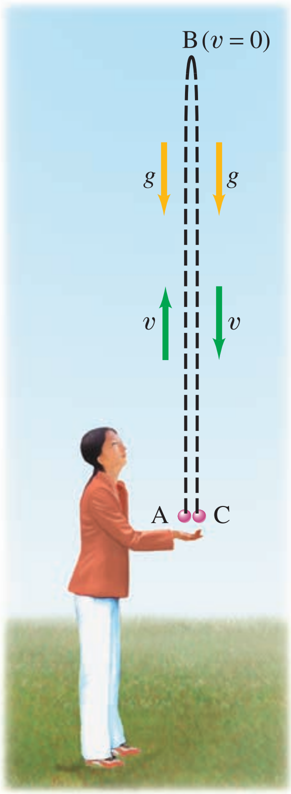
$$\begin{aligned} v &= v_0 + at \\ &= 3.00\,\mathrm{m/s} + (9.80\,\mathrm{m/s^2})(1.00\,\mathrm{s}) = 12.8\,\mathrm{m/s} \qquad [\text{at } t_1 = 1.00\,\mathrm{s}] \\ &= 3.00\,\mathrm{m/s} + (9.80\,\mathrm{m/s^2})(2.00\,\mathrm{s}) = 22.6\,\mathrm{m/s}. \qquad [\text{at } t_2 = 2.00\,\mathrm{s}] \end{aligned}$$

In Example 2–10, when the ball was dropped $(v_0 = 0)$, the first term (v_0) in these equations was zero, so

$$\begin{aligned} v &= 0 + at \\ &= (9.80\,\mathrm{m/s^2})(1.00\,\mathrm{s}) = 9.80\,\mathrm{m/s} \qquad [\text{at } t_1 = 1.00\,\mathrm{s}] \\ &= (9.80\,\mathrm{m/s^2})(2.00\,\mathrm{s}) = 19.6\,\mathrm{m/s}. \qquad [\text{at } t_2 = 2.00\,\mathrm{s}] \end{aligned}$$

NOTE For both Examples 2–10 and 2–11, the speed increases linearly in time by 9.80 m/s during each second. But the speed of the downwardly thrown ball at any instant is always 3.00 m/s (its initial speed) higher than that of a dropped ball.

FIGURE 2–23 An object thrown into the air leaves the thrower's hand at A, reaches its maximum height at B, and returns to the original position at C. Examples 2–12, 2–13, 2–14, and 2–15.



EXAMPLE 2–12 **Ball thrown upward.** A person throws a ball *upward* into the air with an initial velocity of 15.0 m/s. Calculate how high it goes. Ignore air resistance.

APPROACH We are not concerned here with the throwing action, but only with the motion of the ball *after* it leaves the thrower's hand (Fig. 2–23) and until it comes back to the hand again. Let us choose y to be positive in the upward direction and negative in the downward direction. (This is a different convention from that used in Examples 2–10 and 2–11, and so illustrates our options.) The acceleration due to gravity is downward and so will have a negative sign, $a = -g = -9.80\,\mathrm{m/s^2}$. As the ball rises, its speed decreases until it reaches the highest point (B in Fig. 2–23), where its speed is zero for an instant; then it descends, with increasing speed.

SOLUTION We consider the time interval from when the ball leaves the thrower's hand until the ball reaches the highest point. To determine the maximum height, we calculate the position of the ball when its velocity equals zero ($v = 0$ at the highest point). At $t = 0$ (point A in Fig. 2–23) we have $y_0 = 0$, $v_0 = 15.0\,\mathrm{m/s}$, and $a = -9.80\,\mathrm{m/s^2}$. At time t (maximum height), $v = 0$, $a = -9.80\,\mathrm{m/s^2}$, and we wish to find y. We use Eq. 2–11c, replacing x with y: $v^2 = v_0^2 + 2ay$. We solve this equation for y:

$$y = \frac{v^2 - v_0^2}{2a} = \frac{0 - (15.0\,\mathrm{m/s})^2}{2(-9.80\,\mathrm{m/s^2})} = 11.5\,\mathrm{m}.$$

The ball reaches a height of 11.5 m above the hand.

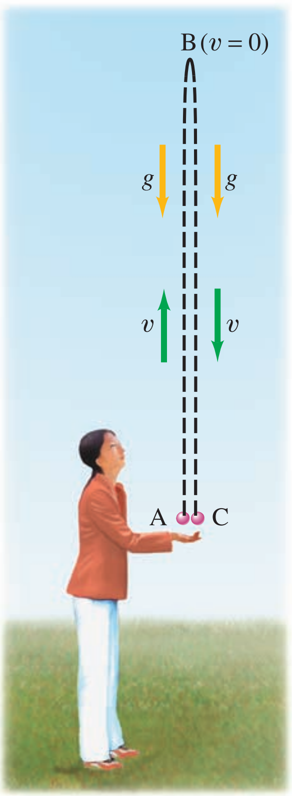


FIGURE 2–23 (Repeated.) An object thrown into the air leaves the thrower's hand at A, reaches its maximum height at B, and returns to the original position at C. Examples 2–12, 2–13, 2–14, and 2–15.

EXAMPLE 2–13 **Ball thrown upward, II.** In Fig. 2–23, Example 2–12, how long is the ball in the air before it comes back to the hand?

APPROACH We need to choose a time interval to calculate how long the ball is in the air before it returns to the hand. We could do this calculation in two parts by first determining the time required for the ball to reach its highest point, and then determining the time it takes to fall back down. However, it is simpler to consider the time interval for the entire motion from A to B to C (Fig. 2–23) in one step and use Eq. 2–11b. We can do this because y is position or displacement, and not the total distance traveled. Thus, at both points A and C, $y = 0$.

SOLUTION We use Eq. 2–11b with $a = -9.80\,\mathrm{m/s^2}$ and find

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2$$
$$0 = 0 + (15.0\,\mathrm{m/s})t + \tfrac{1}{2}(-9.80\,\mathrm{m/s^2})t^2.$$

This equation can be factored (we factor out one t):

$$(15.0\,\mathrm{m/s} - 4.90\,\mathrm{m/s^2}\,t)t = 0.$$

There are two solutions:

$$t = 0 \quad \text{and} \quad t = \frac{15.0\,\mathrm{m/s}}{4.90\,\mathrm{m/s^2}} = 3.06\,\mathrm{s}.$$

The first solution $(t = 0)$ corresponds to the initial point (A) in Fig. 2–23, when the ball was first thrown from $y = 0$. The second solution, $t = 3.06\,\mathrm{s}$, corresponds to point C, when the ball has returned to $y = 0$. Thus the ball is in the air 3.06 s.

NOTE We have ignored air resistance in these last two Examples, which could be significant, so our result is only an approximation to a real, practical situation.

We did not consider the throwing action in these Examples. Why? Because during the throw, the thrower's hand is touching the ball and accelerating the ball at a rate unknown to us—the acceleration is *not* g. We consider only the time when the ball is in the air and the acceleration is equal to g.

CAUTION

Quadratic equations have two solutions. Sometimes only one corresponds to reality, sometimes both

Every quadratic equation (where the variable is squared) mathematically produces two solutions. In physics, sometimes only one solution corresponds to the real situation, as in Example 2–8, in which case we ignore the "unphysical" solution. But in Example 2–13, both solutions to our equation in t^2 are physically meaningful: $t = 0$ and $t = 3.06\,\mathrm{s}$.

CONCEPTUAL EXAMPLE 2–14 **Two possible misconceptions.** Give examples to show the error in these two common misconceptions: (1) that acceleration and velocity are always in the same direction, and (2) that an object thrown upward has zero acceleration at the highest point (B in Fig. 2–23).

CAUTION

(1) Velocity and acceleration are not always in the same direction; the acceleration (of gravity) always points down

(2) $a \neq 0$ even at the highest point of a trajectory

RESPONSE Both are wrong. (1) Velocity and acceleration are *not* necessarily in the same direction. When the ball in Fig. 2–23 is moving upward, its velocity is positive (upward), whereas the acceleration is negative (downward). (2) At the highest point (B in Fig. 2–23), the ball has zero velocity for an instant. Is the acceleration also zero at this point? No. The velocity near the top of the arc points upward, then becomes zero for an instant (zero time) at the highest point, and then points downward. Gravity does not stop acting, so $a = -g = -9.80\,\mathrm{m/s^2}$ even there. Thinking that $a = 0$ at point B would lead to the conclusion that upon reaching point B, the ball would stay there: if the acceleration ($=$ rate of change of velocity) were zero, the velocity would stay zero at the highest point, and the ball would stay up there without falling. Remember: the acceleration of gravity always points down toward the Earth, even when the object is moving up.

EXAMPLE 2–15 **Ball thrown upward, III.** Let us consider again the ball thrown upward of Examples 2–12 and 2–13, and make more calculations. Calculate (a) how much time it takes for the ball to reach the maximum height (point B in Fig. 2–23), and (b) the velocity of the ball when it returns to the thrower's hand (point C).

APPROACH Again we assume the acceleration is constant, so we can use Eqs. 2–11. We have the maximum height of 11.5 m and initial speed of 15.0 m/s from Example 2–12. Again we take y as positive upward.

SOLUTION (a) We consider the time interval between the throw $(t = 0, v_0 = 15.0\,\mathrm{m/s})$ and the top of the path $(y = +11.5\,\mathrm{m},\ v = 0)$, and we want to find t. The acceleration is constant at $a = -g = -9.80\,\mathrm{m/s^2}$. Both Eqs. 2–11a and 2–11b contain the time t with other quantities known. Let us use Eq. 2–11a with $a = -9.80\,\mathrm{m/s^2}$, $v_0 = 15.0\,\mathrm{m/s}$, and $v = 0$:

$$v = v_0 + at;$$

setting $v = 0$ gives $0 = v_0 + at$, which we rearrange to solve for t: $at = -v_0$ or

$$\begin{aligned} t &= -\frac{v_0}{a} \\ &= -\frac{15.0\,\mathrm{m/s}}{-9.80\,\mathrm{m/s^2}} = 1.53\,\mathrm{s}. \end{aligned}$$

This is just half the time it takes the ball to go up and fall back to its original position [3.06 s, calculated in Example 2–13]. Thus it takes the same time to reach the maximum height as to fall back to the starting point.

(b) Now we consider the time interval from the throw $(t = 0,\ v_0 = 15.0\,\mathrm{m/s})$ until the ball's return to the hand, which occurs at $t = 3.06\,\mathrm{s}$ (as calculated in Example 2–13), and we want to find v when $t = 3.06\,\mathrm{s}$:

$$\begin{aligned} v &= v_0 + at \\ &= 15.0\,\mathrm{m/s} - (9.80\,\mathrm{m/s^2})(3.06\,\mathrm{s}) = -15.0\,\mathrm{m/s}. \end{aligned}$$

NOTE The ball has the same speed (magnitude of velocity) when it returns to the starting point as it did initially, but in the opposite direction (this is the meaning of the negative sign). And, as we saw in part (a), the time is the same up as down. Thus the motion is *symmetrical* about the maximum height.

The acceleration of objects such as rockets and fast airplanes is often given as a multiple of $g = 9.80\,\mathrm{m/s^2}$. For example, a plane pulling out of a dive (see Fig. 2–24) and undergoing $3.00\,g$'s would have an acceleration of $(3.00)(9.80\,\mathrm{m/s^2}) = 29.4\,\mathrm{m/s^2}$.

PROBLEM SOLVING
Acceleration in g's

FIGURE 2–24 Several planes, in formation, are just coming out of a downward dive.

EXERCISE F Two balls are thrown from a cliff. One is thrown directly up, the other directly down. Both balls have the same initial speed, and both hit the ground below the cliff but at different times. Which ball hits the ground at the greater speed: (*a*) the ball thrown upward, (*b*) the ball thrown downward, or (*c*) both the same? Ignore air resistance.

Additional Example—Using the Quadratic Formula

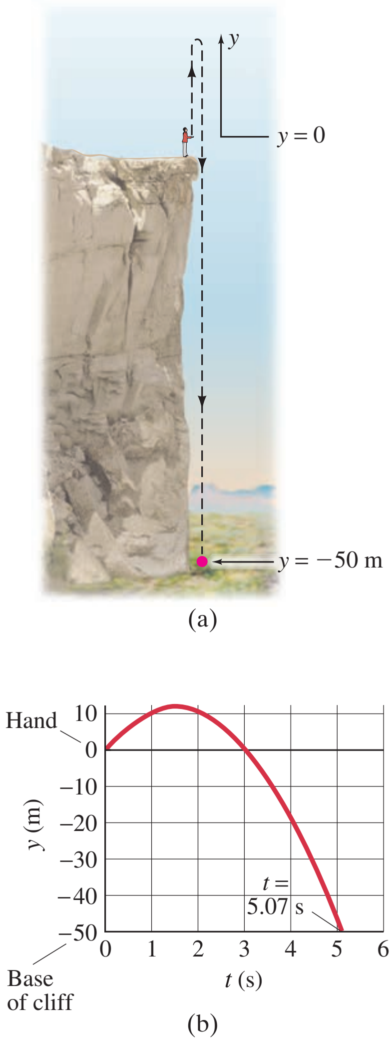


FIGURE 2–25 Example 2–16. (a) A person stands on the edge of a cliff. A ball is thrown upward, then falls back down past the thrower to the base of the cliff, 50.0 m below. (b) The y vs. t graph.

EXAMPLE 2–16 **Ball thrown upward at edge of cliff.** Suppose that the person of Examples 2–12, 2–13, and 2–15 throws the ball upward at 15.0 m/s while standing on the edge of a cliff, so that the ball can fall to the base of the cliff 50.0 m below, as shown in Fig. 2–25a. (*a*) How long does it take the ball to reach the base of the cliff? (*b*) What is the total distance traveled by the ball? Ignore air resistance (likely to be significant, so our result is an approximation).

APPROACH We again use Eq. 2–11b, with y as + upward, but this time we set $y = -50.0\,\mathrm{m}$, the bottom of the cliff, which is 50.0 m below the initial position $(y_0 = 0)$; hence the minus sign.

SOLUTION (*a*) We use Eq. 2–11b with $a = -9.80\,\mathrm{m/s^2}$, $v_0 = 15.0\,\mathrm{m/s}$, $y_0 = 0$, and $y = -50.0\,\mathrm{m}$:

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2$$
$$-50.0\,\mathrm{m} = 0 + (15.0\,\mathrm{m/s})t - \tfrac{1}{2}(9.80\,\mathrm{m/s^2})t^2.$$

To solve any quadratic equation of the form

$$at^2 + bt + c = 0,$$

where a, b, and c are constants (a is *not* acceleration here), we use the **quadratic formula** (see Appendix A–4):

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

We rewrite our y equation just above in standard form, $at^2 + bt + c = 0$:

$$(4.90\,\mathrm{m/s^2})t^2 - (15.0\,\mathrm{m/s})t - (50.0\,\mathrm{m}) = 0.$$

Using the quadratic formula, we find as solutions

$$t = 5.07\,\mathrm{s}$$

and

$$t = -2.01\,\mathrm{s}.$$

The first solution, $t = 5.07\,\mathrm{s}$, is the answer we are seeking: the time it takes the ball to rise to its highest point and then fall to the base of the cliff. To rise and fall back to the top of the cliff took 3.06 s (Example 2–13); so it took an additional 2.01 s to fall to the base. But what is the meaning of the other solution, $t = -2.01\,\mathrm{s}$? This is a time before the throw, when our calculation begins, so it isn't relevant here. It is outside our chosen time interval, and so is an *unphysical* solution (also in Example 2–8).

(*b*) From Example 2–12, the ball moves up 11.5 m, falls 11.5 m back down to the top of the cliff, and then down another 50.0 m to the base of the cliff, for a total distance traveled of 73.0 m. [Note that the *displacement*, however, was −50.0 m.] Figure 2–25b shows the y vs. t graph for this situation.

CAUTION

Sometimes a solution to a quadratic equation does not apply to the actual physical conditions of the Problem