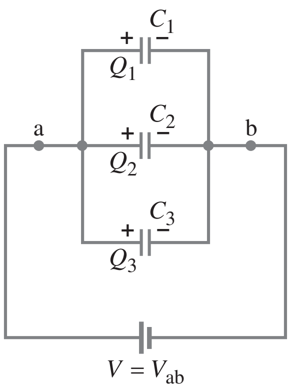


FIGURE 19–17 Capacitors in parallel: $C_{eq} = C_1 + C_2 + C_3$.

19–5 Circuits Containing Capacitors in Series and in Parallel

Just as resistors can be placed in series or in parallel in a circuit, so can capacitors (Chapter 17). We first consider a **parallel** connection as shown in Fig. 19–17. If a battery supplies a potential difference V to points a and b, this same potential difference $V = V_{ab}$ exists across each of the capacitors. That is, since the left-hand plates of all the capacitors are connected by conductors, they all reach the same potential V_a when connected to the battery; and the right-hand plates each reach potential V_b. Each capacitor plate acquires a charge given by $Q_1 = C_1 V$, $Q_2 = C_2 V$, and $Q_3 = C_3 V$. The total charge Q that must leave the battery is then

$$Q = Q_1 + Q_2 + Q_3 = C_1 V + C_2 V + C_3 V.$$

Let us try to find a single equivalent capacitor that will hold the same charge Q at the same voltage $V = V_{ab}$. It will have a capacitance C_{eq} given by

$$Q = C_{eq} V.$$

Combining the two previous equations, we have

$$C_{eq} V = C_1 V + C_2 V + C_3 V = (C_1 + C_2 + C_3)V$$

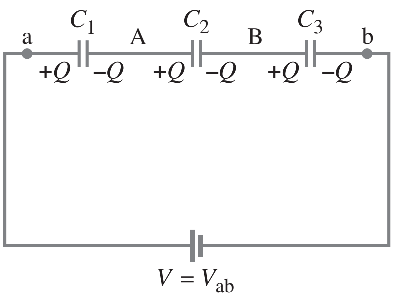
or

$$C_{eq} = C_1 + C_2 + C_3. \qquad \text{[parallel]} \quad \textbf{(19–5)}$$

The net effect of connecting capacitors in parallel is thus to *increase* the capacitance. Connecting capacitors in parallel is essentially increasing the area of the plates where charge can accumulate (see, for example, Eq. 17–8).

Capacitors can also be connected in **series**: that is, end to end as shown in Fig. 19–18. A charge $+Q$ flows from the battery to one plate of C_1, and $-Q$ flows to one plate of C_3. The regions A and B between the capacitors were originally neutral, so the net charge there must still be zero. The $+Q$ on the left plate of C_1 attracts a charge of $-Q$ on the opposite plate. Because region A must have a zero net charge, there is $+Q$ on the left plate of C_2. The same considerations apply to the other capacitors, so we see that the charge on each capacitor plate has the same magnitude Q. A single capacitor that could replace these three in series without affecting the circuit (that is, Q and V the same) would have a capacitance C_{eq} where

FIGURE 19–18 Capacitors in series: $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3}$.



$$Q = C_{eq} V.$$

The total voltage V across the three capacitors in series must equal the sum of the voltages across each capacitor:

$$V = V_1 + V_2 + V_3.$$

We also have for each capacitor $Q = C_1 V_1$, $Q = C_2 V_2$, and $Q = C_3 V_3$, so we substitute for V_1, V_2, V_3, and V into the last equation and get

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

or

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}. \qquad \text{[series]} \quad \textbf{(19–6)}$$

Notice that the equivalent capacitance C_{eq} is *smaller* than the smallest contributing capacitance. Notice also that the forms of the equations for capacitors in series or in parallel are the reverse of their counterparts for resistance. That is, the formula for capacitors in series resembles the formula for resistors in parallel.

CAUTION
Formula for capacitors in series resembles formula for resistors in parallel

EXAMPLE 19–10 **Equivalent capacitance.** Determine the capacitance of a single capacitor that will have the same effect as the combination shown in Fig. 19–19a. Take $C_1 = C_2 = C_3 = C$.

APPROACH First we find the equivalent capacitance of C_2 and C_3 in parallel, and then consider that capacitance in series with C_1.

SOLUTION Capacitors C_2 and C_3 are connected in parallel, so they are equivalent to a single capacitor having capacitance

$$C_{23} = C_2 + C_3 = C + C = 2C.$$

This C_{23} is in series with C_1, Fig. 19–19b, so the equivalent capacitance of the entire circuit, C_{eq}, is given by

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_{23}} = \frac{1}{C} + \frac{1}{2C} = \frac{3}{2C}.$$

Hence the equivalent capacitance of the entire combination is $C_{\text{eq}} = \frac{2}{3}C$, and it is smaller than any of the contributing capacitances, $C_1 = C_2 = C_3 = C$.

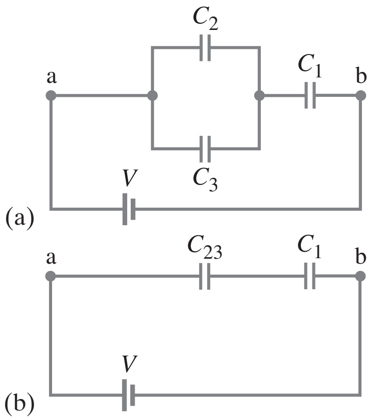


FIGURE 19–19 Examples 19–10 and 19–11.

PROBLEM SOLVING
Remember to take the reciprocal

EXERCISE F Consider two identical capacitors $C_1 = C_2 = 10\,\mu\text{F}$. What are the smallest and largest capacitances that can be obtained by connecting these in series or parallel combinations? (*a*) $0.2\,\mu$F, $5\,\mu$F; (*b*) $0.2\,\mu$F, $10\,\mu$F; (*c*) $0.2\,\mu$F, $20\,\mu$F; (*d*) $5\,\mu$F, $10\,\mu$F; (*e*) $5\,\mu$F, $20\,\mu$F; (*f*) $10\,\mu$F, $20\,\mu$F.

EXAMPLE 19–11 **Charge and voltage on capacitors.** Determine the charge on each capacitor in Fig. 19–19a of Example 19–10 and the voltage across each, assuming $C = 3.0\,\mu\text{F}$ and the battery voltage is $V = 4.0$ V.

APPROACH We have to work "backward" through Example 19–10. That is, we find the charge Q that leaves the battery, using the equivalent capacitance. Then we find the charge on each separate capacitor and the voltage across each. Each step uses Eq. 17–7, $Q = CV$.

SOLUTION The 4.0-V battery behaves as if it is connected to a capacitance $C_{\text{eq}} = \frac{2}{3}C = \frac{2}{3}(3.0\,\mu\text{F}) = 2.0\,\mu\text{F}$. Therefore the charge Q that leaves the battery, by Eq. 17–7, is

$$Q = CV = (2.0\,\mu\text{F})(4.0\text{ V}) = 8.0\,\mu\text{C}.$$

From Fig. 19–19a, this charge arrives at the negative plate of C_1, so $Q_1 = 8.0\,\mu\text{C}$. The charge Q that leaves the positive plate of the battery is split evenly between C_2 and C_3 (*symmetry*: $C_2 = C_3$) and is $Q_2 = Q_3 = \frac{1}{2}Q = 4.0\,\mu\text{C}$. Next, the voltages across C_2 and C_3 have to be the same. The voltage across each capacitor is obtained using $V = Q/C$. So

$$\begin{aligned} V_1 &= Q_1/C_1 = (8.0\,\mu\text{C})/(3.0\,\mu\text{F}) = 2.7\text{ V} \\ V_2 &= Q_2/C_2 = (4.0\,\mu\text{C})/(3.0\,\mu\text{F}) = 1.3\text{ V} \\ V_3 &= Q_3/C_3 = (4.0\,\mu\text{C})/(3.0\,\mu\text{F}) = 1.3\text{ V}. \end{aligned}$$

19–6 *RC* Circuits—Resistor and Capacitor in Series

Capacitor Charging

Capacitors and resistors are often found together in a circuit. Such ***RC* circuits** are common in everyday life. They are used to control the speed of a car's windshield wipers and the timing of traffic lights; they are used in camera flashes, in heart pacemakers, and in many other electronic devices. In *RC* circuits, we are not so interested in the final "steady state" voltage and charge on the capacitor, but rather in how these variables change in time.

A simple RC circuit is shown in Fig. 19–20a. When the switch S is closed, current immediately begins to flow through the circuit. Electrons will flow out from the negative terminal of the battery, through the resistor R, and accumulate on the upper plate of the capacitor. And electrons will flow into the positive terminal of the battery, leaving a positive charge on the other plate of the capacitor. As charge accumulates on the capacitor, the potential difference across it increases $(V_C = Q/C)$, and the current is reduced until eventually the voltage across the capacitor equals the emf of the battery, $\mathscr{E}$. There is then no further current flow, and no potential difference across the resistor. The potential difference V_C across the capacitor, which is proportional to the charge on it ($V_C = Q/C$, Eq. 17–7), thus increases in time as shown in Fig. 19–20b. The shape of this curve is a type of exponential, and is given by the formula†

$$V_C = \mathscr{E}\left(1 - e^{-t/RC}\right), \tag{19–7a}$$

where we use the subscript C to remind us that V_C is the voltage across the capacitor and is given here as a function of time t. [The constant e, known as the base for natural logarithms, has the value $e = 2.718\cdots$. Do not confuse this e with e for the charge on the electron.]

CAUTION

Don't confuse e for exponential with e for electron charge

We can write a similar formula for the charge $Q\ (= CV_C)$ on the capacitor:

$$Q = Q_0\left(1 - e^{-t/RC}\right), \tag{19–7b}$$

where Q_0 represents the maximum charge.

The product of the resistance R times the capacitance C, which appears in the exponent, is called the **time constant** τ of the circuit:

$$\tau = RC. \tag{19–7c}$$

The time constant is a measure of how quickly the capacitor becomes charged. [The units of RC are $\Omega\cdot\mathrm{F} = (\mathrm{V/A})(\mathrm{C/V}) = \mathrm{C/(C/s)} = \mathrm{s}$.] Specifically, it can be shown that the product RC gives the time required for the capacitor's voltage (and charge) to reach 63% of the maximum. This can be checked‡ using any calculator with an e^x key: $e^{-1} = 0.37$, so for $t = RC$, then $\left(1 - e^{-t/RC}\right) = \left(1 - e^{-1}\right) = (1 - 0.37) = 0.63$. In a circuit, for example, where $R = 200\ \mathrm{k\Omega}$ and $C = 3.0\ \mu\mathrm{F}$, the time constant is $\left(2.0 \times 10^5\ \Omega\right)\left(3.0 \times 10^{-6}\ \mathrm{F}\right) = 0.60\ \mathrm{s}$. If the resistance is much smaller, the time constant is much smaller and the capacitor becomes charged much more quickly. This makes sense, because a lower resistance will retard the flow of charge less. All circuits contain some resistance (if only in the connecting wires), so a capacitor can never be charged instantaneously when connected to a battery.

Finally, what is the voltage V_R across the resistor in Fig. 19–20a? The imposed battery voltage is $\mathscr{E}$, so

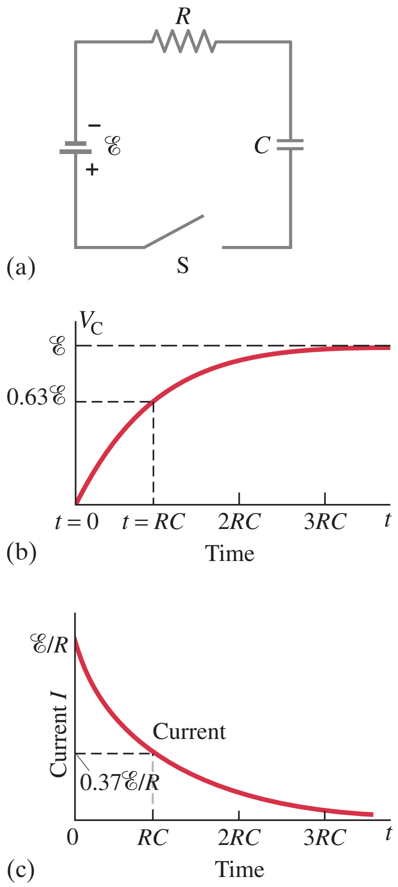
$$V_R = \mathscr{E} - V_C = \mathscr{E}\left(1 - 1 + e^{-t/RC}\right) = \mathscr{E}e^{-t/RC}.$$

This is called an **exponential decay**. The current I flowing in the circuit is that flowing through the resistor and is also an exponential decay:

$$I = \frac{V_R}{R} = \frac{\mathscr{E}}{R}e^{-t/RC}. \tag{19–7d}$$

When the switch of the circuit in Fig. 19–20a is closed, the current is largest at first because there is no charge on the capacitor to impede it. As charge builds on the capacitor, the current decreases in time. That is exactly what Eq. 19–7d and Fig. 19–20c tell us.

FIGURE 19–20 After the switch S closes in the RC circuit shown in (a), the voltage V_C across the capacitor increases with time as shown in (b), and the current through the resistor decreases with time as shown in (c).



†The derivation uses calculus.

‡More simply, since $e = 2.718\cdots$, then $e^{-1} = 1/e = 1/2.718 = 0.37$. Note that e is the inverse operation to the natural logarithm ln: $\ln(e) = 1$, and $\ln(e^x) = x$.

EXAMPLE 19–12 ***RC* circuit, with emf.** The capacitance in the circuit of Fig. 19–20a is $C = 0.30\,\mu\text{F}$, the total resistance is $R = 20\,\text{k}\Omega$, and the battery emf is 12 V. Determine (*a*) the time constant, (*b*) the maximum charge the capacitor could acquire, (*c*) the time it takes for the charge to reach 99% of this value, and (*d*) the maximum current.

APPROACH We use Fig. 19–20 and Eqs. 19–7a, b, c, and d.

SOLUTION (*a*) The time constant is $RC = (2.0 \times 10^4\,\Omega)(3.0 \times 10^{-7}\,\text{F}) = 6.0 \times 10^{-3}\,\text{s} = 6.0\,\text{ms}$.

(*b*) The maximum charge would occur when no further current flows, so $Q_0 = C\mathscr{E} = (3.0 \times 10^{-7}\,\text{F})(12\,\text{V}) = 3.6\,\mu\text{C}$.

(*c*) In Eq. 19–7b, we set $Q = 0.99C\mathscr{E}$:

$$0.99C\mathscr{E} = C\mathscr{E}\left(1 - e^{-t/RC}\right),$$

or

$$e^{-t/RC} = 1 - 0.99 = 0.01.$$

We take the natural logarithm of both sides (Appendix A–8), recalling that $\ln e^x = x$:

$$\frac{t}{RC} = -\ln(0.01) = 4.6$$

so

$$t = 4.6RC = (4.6)(6.0 \times 10^{-3}\,\text{s}) = 28 \times 10^{-3}\,\text{s}$$

or 28 ms (less than $\frac{1}{30}$ s).

(*d*) The current is a maximum at $t = 0$ (the moment when the switch is closed) and there is no charge yet on the capacitor $(Q = 0)$:

$$I_{\text{max}} = \frac{\mathscr{E}}{R} = \frac{12\,\text{V}}{2.0 \times 10^4\,\Omega} = 600\,\mu\text{A}.$$

Capacitor Discharging

The circuit just discussed involved the *charging* of a capacitor by a battery through a resistance. Now let us look at another situation: a capacitor is already charged to a voltage V_0 and charge Q_0, and it is then allowed to *discharge* through a resistance R as shown in Fig. 19–21a. In this case there is no battery. When the switch S is closed, charge begins to flow through resistor R from one side of the capacitor toward the other side, until the capacitor is fully discharged. The voltage across the capacitor decreases, as shown in Fig. 19–21b. This "exponential decay" curve is given by

$$V_{\text{C}} = V_0 e^{-t/RC},$$

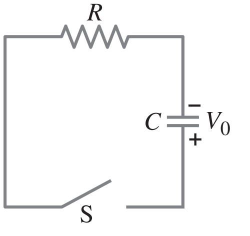
where V_0 is the initial voltage across the capacitor. The voltage falls 63% of the way to zero (to $0.37V_0$) in a time $\tau = RC$. Because the charge Q on the capacitor is $Q = CV$ (and $Q_0 = CV_0$), we can write

$$Q = Q_0 e^{-t/RC}$$

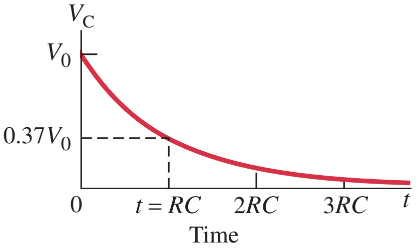
for a discharging capacitor, where Q_0 is the initial charge.

The voltage across the resistor will have the same magnitude as that across the capacitor at any instant, but the opposite sign, because there is zero applied emf: $V_{\text{C}} + V_{\text{R}} = 0$ so $V_{\text{R}} = -V_{\text{C}} = -V_0 e^{-t/RC}$. A graph of V_{R} vs. time would just be Fig. 19–21b upside down. The current $I = V_{\text{R}}/R = -(V_0/R)e^{-t/RC} = -I_0 e^{-t/RC}$. The current has its greatest magnitude at $t = 0$ and decreases exponentially in time. (The current has a minus sign because in Fig. 19–21a it flows in the opposite direction as compared to the current in Fig. 19–20a.)

FIGURE 19–21 For the *RC* circuit shown in (a), the voltage V_{C} across the capacitor decreases with time *t*, as shown in (b), after the switch S is closed at $t = 0$. The charge on the capacitor follows the same curve since $Q \propto V_{\text{C}}$.



(a)



(b)

EXAMPLE 19–13 **A discharging *RC* circuit.** If a charged capacitor, $C = 35\ \mu\text{F}$, is connected to a resistance $R = 120\ \Omega$ as in Fig. 19–21a, how much time will elapse until the voltage falls to 10% of its original (maximum) value?

APPROACH The voltage across the capacitor decreases according to $V_C = V_0 e^{-t/RC}$. We set $V_C = 0.10V_0$ (10% of V_0), but first we need to calculate $\tau = RC$.

SOLUTION The time constant for this circuit is given by

$$\tau = RC = (120\ \Omega)(35 \times 10^{-6}\ \text{F}) = 4.2 \times 10^{-3}\ \text{s}.$$

After a time t the voltage across the capacitor will be

$$V_C = V_0 e^{-t/RC}.$$

We want to know the time t for which $V_C = 0.10V_0$. We substitute into the above equation

$$0.10V_0 = V_0 e^{-t/RC}$$

so

$$e^{-t/RC} = 0.10.$$

The inverse operation to the exponential e is the natural log, ln. Thus

$$\ln\left(e^{-t/RC}\right) = -\frac{t}{RC} = \ln 0.10 = -2.3.$$

Solving for t, we find the elapsed time is

$$t = 2.3(RC) = (2.3)\left(4.2 \times 10^{-3}\ \text{s}\right) = 9.7 \times 10^{-3}\ \text{s} = 9.7\ \text{ms}.$$

NOTE We can find the time for any specified voltage across a capacitor by using $t = RC \ln\left(V_0/V_C\right)$.

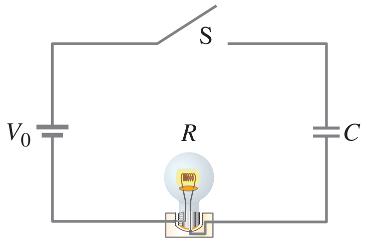


FIGURE 19–22 Example 19–14.

CONCEPTUAL EXAMPLE 19–14 **Bulb in *RC* circuit.** In the circuit of Fig. 19–22, the capacitor is originally uncharged. Describe the behavior of the lightbulb from the instant switch S is closed until a long time later.

RESPONSE When the switch is first closed, the current in the circuit is high and the lightbulb burns brightly. As the capacitor charges, the voltage across the capacitor increases, causing the current to be reduced, and the lightbulb dims. As the potential difference across the capacitor approaches the same voltage as the battery, the current decreases toward zero and the lightbulb goes out.

Medical and Other Applications of *RC* Circuits

PHYSICS APPLIED
Sawtooth voltage

PHYSICS APPLIED
Blinking flashers

The charging and discharging in an *RC* circuit can be used to produce voltage pulses at a regular frequency. The charge on the capacitor increases to a particular voltage, and then discharges. One way of initiating the discharge of the capacitor is by the use of a gas-filled tube which has an electrical breakdown when the voltage across it reaches a certain value V_0. After the discharge is finished, the tube no longer conducts current and the recharging process repeats itself, starting at a lower voltage V_0'. Figure 19–23 shows a possible circuit, and the **sawtooth voltage** it produces.

A simple blinking light can be an application of a sawtooth oscillator circuit. Here the emf is supplied by a battery; the neon bulb flashes on at a rate of perhaps 1 cycle per second. The main component of a "flasher unit" is a moderately large capacitor.

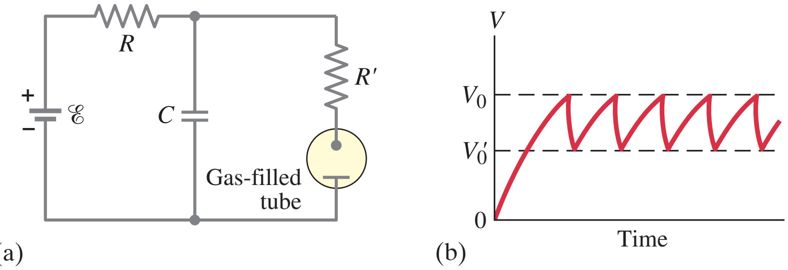


FIGURE 19–23 (a) An *RC* circuit, coupled with a gas-filled tube as a switch, can produce (b) a repeating "sawtooth" voltage.

The intermittent windshield wipers of a car can also use an RC circuit. The RC time constant, which can be changed using a multi-positioned switch for different values of R with fixed C, determines the rate at which the wipers come on.

PHYSICS APPLIED
Windshield wipers on "intermittent"

EXERCISE G A typical turn signal flashes perhaps twice per second, so its time constant is on the order of 0.5 s. Estimate the resistance in the circuit, assuming a moderate capacitor of $C = 1\,\mu\text{F}$.

PHYSICS APPLIED
Heart pacemaker

An important medical use of an RC circuit is the electronic heart pacemaker, which can make a stopped heart start beating again by applying an electric stimulus through electrodes attached to the chest. The stimulus can be repeated at the normal heartbeat rate if necessary. The heart itself contains *pacemaker* cells, which send out tiny electric pulses at a rate of 60 to 80 per minute. These signals induce the start of each heartbeat. In some forms of heart disease, the natural pacemaker fails to function properly, and the heart loses its beat. Such patients use *electronic pacemakers* which produce a regular voltage pulse that starts and controls the frequency of the heartbeat. The electrodes are implanted in or near the heart (Fig. 19–24), and the circuit contains a capacitor and a resistor. The charge on the capacitor increases to a certain point and then discharges a pulse to the heart. Then it starts charging again. The pulsing rate depends on the time constant RC.

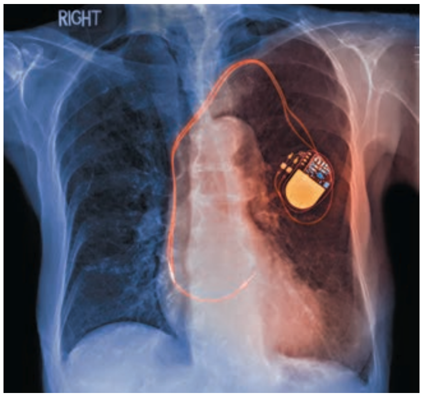


FIGURE 19–24 Electronic battery-powered pacemaker can be seen on the rib cage in this X-ray (color added).

19–7 Electric Hazards

PHYSICS APPLIED
Dangers of electricity

Excess electric current can overheat wires in buildings and cause fires, as discussed in Section 18–6. Electric current can also damage the human body or even be fatal. Electric current through the human body can cause damage in two ways: (1) heating tissue and causing burns; (2) stimulating nerves and muscles, and we feel a "shock." The severity of a shock depends on the magnitude of the current, how long it acts, and through what part of the body it passes. A current passing through vital organs such as the heart or brain is especially damaging.

A current of about 1 mA or more can be felt and may cause pain. Currents above 10 mA cause severe contraction of the muscles, and a person may not be able to let go of the source of the current (say, a faulty appliance or wire). Death from paralysis of the respiratory system can occur. Artificial respiration can sometimes revive a victim. If a current above about 80 to 100 mA passes across the torso, so that a portion passes through the heart for more than a second or two, the heart muscles will begin to contract irregularly and blood will not be properly pumped. This condition is called **ventricular fibrillation**. If it lasts for long, death results. Strangely enough, if the current is much larger, on the order of 1 A, death by heart failure may be less likely,[†] but such currents can cause serious burns if concentrated through a small area of the body.

It is current that harms, but it is voltage that drives the current. The seriousness of an electric shock depends on the current and thus on the applied voltage and the effective resistance of the body. Living tissue has low resistance because the fluid of cells contains ions that can conduct quite well. However, the outer layer of skin, when dry, offers high resistance and is thus protective. The effective resistance between two points on opposite sides of the body when the skin is dry is on the order of 10^4 to $10^6\,\Omega$. But when the skin is wet, the resistance may be $10^3\,\Omega$ or less. A person who is barefoot or wearing thin-soled shoes will be in good contact with the ground, and touching a 120-V line with a wet hand can result in a current

$$I = \frac{120\text{ V}}{1000\,\Omega} = 120\text{ mA}.$$

As we saw, this could be lethal.

[†]Larger currents apparently bring the entire heart to a standstill. Upon release of the current, the heart returns to its normal rhythm. This may not happen when fibrillation occurs because, once started, it can be hard to stop. Fibrillation may also occur as a result of a heart attack or during heart surgery. A device known as a *defibrillator* (described in Section 17–9) can apply a brief high current to the heart, causing complete heart stoppage which is often followed by resumption of normal beating.