6–10 Power

Power is defined as the *rate at which work is done*. Average power equals the work done divided by the time to do it. Power can also be defined as the *rate at which energy is transformed*. Thus

$$\overline{P} = \text{average power} = \frac{\text{work}}{\text{time}} = \frac{\text{energy transformed}}{\text{time}}. \qquad \textbf{(6–17)}$$

The power rating of an engine refers to how much chemical or electrical energy can be transformed into mechanical energy per unit time. In SI units, power is measured in joules per second, and this unit is given a special name, the **watt** (W): $1\text{ W} = 1\text{ J/s}$. We are most familiar with the watt for electrical devices, such as the rate at which an electric lightbulb or heater changes electric energy into light or thermal energy. But the watt is used for other types of energy transformations as well.

In the British system, the unit of power is the foot-pound per second (ft·lb/s). For practical purposes, a larger unit is often used, the **horsepower**. One horsepower† (hp) is defined as 550 ft·lb/s, which equals 746 W. An engine's power is usually specified in hp or in kW $\left(1\text{ kW} \approx 1\tfrac{1}{3}\text{hp}\right)$‡.

To see the distinction between energy and power, consider the following example. A person is limited in the work he or she can do, not only by the total energy required, but also by how fast this energy is transformed: that is, by power. For example, a person may be able to walk a long distance or climb many flights of stairs before having to stop because so much energy has been expended. On the other hand, a person who runs very quickly up stairs may feel exhausted after only a flight or two. He or she is limited in this case by power, the rate at which his or her body can transform chemical energy into mechanical energy.

CAUTION
Distinguish between power and energy

EXAMPLE 6–13 **Stair-climbing power.** A 60-kg jogger runs up a long flight of stairs in 4.0 s (Fig. 6–28). The vertical height of the stairs is 4.5 m. (*a*) Estimate the jogger's power output in watts and horsepower. (*b*) How much energy did this require?

APPROACH The work done by the jogger is against gravity, and equals $W = mgy$. To get her average power output, we divide W by the time it took.

SOLUTION (*a*) The average power output was

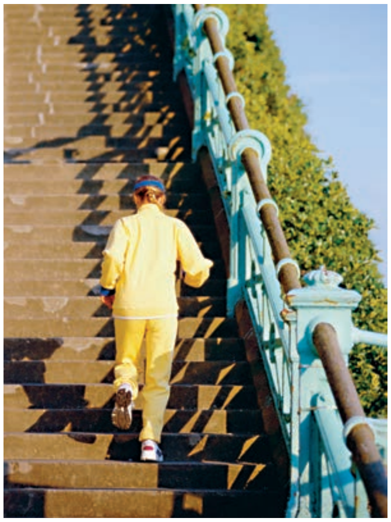
$$\overline{P} = \frac{W}{t} = \frac{mgy}{t} = \frac{(60\text{ kg})(9.8\text{ m/s}^2)(4.5\text{ m})}{4.0\text{ s}} = 660\text{ W}.$$

Since there are 746 W in 1 hp, the jogger is doing work at a rate of just under 1 hp. A human cannot do work at this rate for very long.

(*b*) The energy required is $E = \overline{P}t = (660\text{ J/s})(4.0\text{ s}) = 2600\text{ J}$. This result equals $W = mgy$.

NOTE The person had to transform more energy than this 2600 J. The total energy transformed by a person or an engine always includes some thermal energy (recall how hot you get running up stairs).

FIGURE 6–28 Example 6–13.



Automobiles do work to overcome the force of friction and air resistance, to climb hills, and to accelerate. A car is limited by the rate at which it can do work, which is why automobile engines are rated in horsepower or kilowatts.

PHYSICS APPLIED
Power needs of a car

†The unit was chosen by James Watt (1736–1819), who needed a way to specify the power of his newly developed steam engines. He found by experiment that a good horse can work all day at an average rate of about 360 ft·lb/s. So as not to be accused of exaggeration in the sale of his steam engines, he multiplied this by $1\tfrac{1}{2}$ when he defined the hp.

‡$1\text{ kW} = (1000\text{ W})/(746\text{ W/hp}) \approx 1\tfrac{1}{3}\text{hp}$.

A car needs power most when climbing hills and when accelerating. In the next Example, we will calculate how much power is needed in these situations for a car of reasonable size. Even when a car travels on a level road at constant speed, it needs some power just to do work to overcome the retarding forces of internal friction and air resistance. These forces depend on the conditions and speed of the car, but are typically in the range 400 N to 1000 N.

It is often convenient to write power in terms of the net force F applied to an object and its speed v. This is readily done because $\overline{P} = W/t$ and $W = Fd$, where d is the distance traveled. Then

$$\overline{P} = \frac{W}{t} = \frac{Fd}{t} = F\bar{v}, \tag{6–18}$$

where $\bar{v} = d/t$ is the average speed of the object.

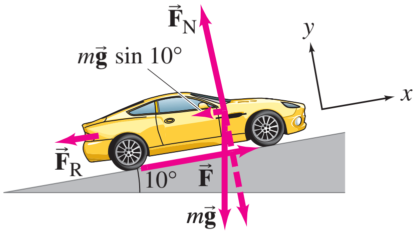


FIGURE 6–29 Example 6–14. Calculation of power needed for a car to climb a hill.

EXAMPLE 6–14 **Power needs of a car.** Calculate the power required of a 1400-kg car under the following circumstances: (a) the car climbs a 10° hill (a fairly steep hill) at a steady 80 km/h; and (b) the car accelerates along a level road from 90 to 110 km/h in 6.0 s to pass another car. Assume the average retarding force on the car is $F_R = 700$ N throughout. See Fig. 6–29.

APPROACH First we must be careful not to confuse $\vec{\mathbf{F}}_R$, which is due to air resistance and friction that retards the motion, with the force $\vec{\mathbf{F}}$ needed to accelerate the car, which is the frictional force exerted by the road on the tires—the reaction to the motor-driven tires pushing against the road. We must determine the magnitude of the force F before calculating the power.

SOLUTION (a) To move at a steady speed up the hill, the car must, by Newton's second law, exert a force F equal to the sum of the retarding force, 700 N, and the component of gravity parallel to the hill, mg sin 10°, Fig. 6–29. Thus

$$\begin{aligned} F &= 700\,\text{N} + mg \sin 10^\circ \\ &= 700\,\text{N} + (1400\,\text{kg})(9.80\,\text{m/s}^2)(0.174) = 3100\,\text{N}. \end{aligned}$$

Since $\bar{v} = 80\,\text{km/h} = 22\,\text{m/s}^\dagger$ and is parallel to $\vec{\mathbf{F}}$, then (Eq. 6–18) the power is

$$\overline{P} = F\bar{v} = (3100\,\text{N})(22\,\text{m/s}) = 6.8 \times 10^4\,\text{W} = 68\,\text{kW} = 91\,\text{hp}.$$

(b) The car accelerates from 25.0 m/s to 30.6 m/s (90 to 110 km/h) on the flat. The car must exert a force that overcomes the 700-N retarding force plus that required to give it the acceleration

$$\bar{a}_x = \frac{(30.6\,\text{m/s} - 25.0\,\text{m/s})}{6.0\,\text{s}} = 0.93\,\text{m/s}^2.$$

We apply Newton's second law with x being the horizontal direction of motion (no component of gravity):

$$ma_x = \Sigma F_x = F - F_R.$$

We solve for the force required, F:

$$\begin{aligned} F &= ma_x + F_R \\ &= (1400\,\text{kg})(0.93\,\text{m/s}^2) + 700\,\text{N} = 1300\,\text{N} + 700\,\text{N} = 2000\,\text{N}. \end{aligned}$$

Since $\overline{P} = F\bar{v}$, the required power increases with speed and the motor must be able to provide a maximum power output in this case of

$$\overline{P} = (2000\,\text{N})(30.6\,\text{m/s}) = 6.1 \times 10^4\,\text{W} = 61\,\text{kW} = 82\,\text{hp}.$$

NOTE Even taking into account the fact that only 60 to 80% of the engine's power output reaches the wheels, it is clear from these calculations that an engine of 75 to 100 kW (100 to 130 hp) is adequate from a practical point of view.

† Recall $1\,\text{km/h} = 1000\,\text{m}/3600\,\text{s} = 0.278\,\text{m/s}$.

We mentioned in Example 6–14 that only part of the energy output of a car engine reaches the wheels. Not only is some energy wasted in getting from the engine to the wheels, in the engine itself most of the input energy (from the burning of gasoline or other fuel) does not do useful work. An important characteristic of all engines is their overall **efficiency** e, defined as the ratio of the useful power output of the engine, P_{out}, to the power input, P_{in} (provided by burning of gasoline, for example):

$$e = \frac{P_{\text{out}}}{P_{\text{in}}}.$$

The efficiency is always less than 1.0 because no engine can create energy, and no engine can even transform energy from one form to another without some energy going to friction, thermal energy, and other nonuseful forms of energy. For example, an automobile engine converts chemical energy released in the burning of gasoline into mechanical energy that moves the pistons and eventually the wheels. But nearly 85% of the input energy is "wasted" as thermal energy that goes into the cooling system or out the exhaust pipe, plus friction in the moving parts. Thus car engines are roughly only about 15% efficient. We will discuss efficiency in more detail in Chapter 15.

Summary

Work is done on an object by a force when the object moves through a distance d. If the direction of a constant force $\vec{\mathbf{F}}$ makes an angle θ with the direction of motion, the work done by this force is

$$W = Fd\cos\theta. \qquad \textbf{(6–1)}$$

Energy can be defined as the ability to do work. In SI units, work and energy are measured in **joules** $(1\text{ J} = 1\text{ N}\cdot\text{m})$.

Kinetic energy (KE) is energy of motion. An object of mass m and speed v has **translational kinetic energy**

$$\text{KE} = \tfrac{1}{2}mv^2. \qquad \textbf{(6–3)}$$

The **work-energy principle** states that the *net* work done on an object (by the *net* force) equals the change in kinetic energy of that object:

$$W_{\text{net}} = \Delta\text{KE} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2. \qquad \textbf{(6–4)}$$

Potential energy (PE) is energy associated with forces that depend on the position or configuration of objects. Gravitational potential energy is

$$\text{PE}_{\text{G}} = mgy, \qquad \textbf{(6–6)}$$

where y is the height of the object of mass m above an arbitrary reference point. Elastic potential energy is given by

$$\text{PE}_{\text{el}} = \tfrac{1}{2}kx^2 \qquad \textbf{(6–9)}$$

for a stretched or compressed spring, where x is the displacement from the unstretched position and k is the **spring stiffness constant**. Other potential energies include chemical, electrical, and nuclear energy. The *change in potential energy* when an object changes position *is equal to the external work* needed to take the object from one position to the other.

Potential energy is associated only with **conservative forces**, for which the work done by the force in moving an object from one position to another depends only on the two positions and not on the path taken. **Nonconservative forces** like friction are different—work done by them does depend on the path taken and potential energy cannot be defined for them.

The **law of conservation of energy** states that energy can be transformed from one type to another, but the total energy remains constant. It is valid even when friction is present, because the heat generated can be considered a form of energy transfer. When only *conservative forces* act, the **total mechanical energy** is conserved:

$$\text{KE} + \text{PE} = \text{constant}. \qquad \textbf{(6–12)}$$

When nonconservative forces such as friction act, then

$$W_{\text{NC}} = \Delta\text{KE} + \Delta\text{PE}, \qquad \textbf{(6–10, 6–15)}$$

where W_{NC} is the work done by nonconservative forces.

Power is defined as the rate at which work is done, or the rate at which energy is transformed. The SI unit of power is the **watt** $(1\text{ W} = 1\text{ J/s})$.

Questions

1. In what ways is the word "work" as used in everyday language the same as it is defined in physics? In what ways is it different? Give examples of both.
2. Can a centripetal force ever do work on an object? Explain.
3. Why is it tiring to push hard against a solid wall even though you are doing no work?
4. Can the normal force on an object ever do work? Explain.
5. You have two springs that are identical except that spring 1 is stiffer than spring 2 $(k_1 > k_2)$. On which spring is more work done: (*a*) if they are stretched using the same force; (*b*) if they are stretched the same distance?
6. If the speed of a particle triples, by what factor does its kinetic energy increase?
7. List some everyday forces that are not conservative, and explain why they aren't.