4–7 Solving Problems with Newton's Laws: Free-Body Diagrams

Newton's second law tells us that the acceleration of an object is proportional to the *net force* acting on the object. The **net force**, as mentioned earlier, is the *vector sum* of all forces acting on the object. Indeed, extensive experiments have shown that forces do add together as vectors precisely according to the rules we developed in Chapter 3. For example, in Fig. 4–18, two forces of equal magnitude (100 N each) are shown acting on an object at right angles to each other. Intuitively, we can see that the object will start moving at a 45° angle and thus the net force acts at a 45° angle. This is just what the rules of vector addition give. From the theorem of Pythagoras, the magnitude of the resultant force is $F_R = \sqrt{(100\text{ N})^2 + (100\text{ N})^2} = 141\text{ N}.$

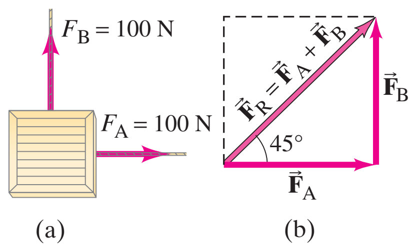


FIGURE 4–18 (a) Two horizontal forces, $\vec{F}_A$ and $\vec{F}_B$, exerted by workers A and B, act on a crate (we are looking down from above). (b) The sum, or resultant, of $\vec{F}_A$ and $\vec{F}_B$ is $\vec{F}_R$.

EXAMPLE 4–9 **Adding force vectors.** Calculate the sum of the two forces exerted on the boat by workers A and B in Fig. 4–19a.

APPROACH We add force vectors like any other vectors as described in Chapter 3. The first step is to choose an *xy* coordinate system (see Fig. 4–19a), and then resolve vectors into their components.

SOLUTION The two force vectors are shown resolved into components in Fig. 4–19b. We add the forces using the method of components. The components of $\vec{F}_A$ are

$$F_{Ax} = F_A \cos 45.0° = (40.0\text{ N})(0.707) = 28.3\text{ N},$$
$$F_{Ay} = F_A \sin 45.0° = (40.0\text{ N})(0.707) = 28.3\text{ N}.$$

The components of $\vec{F}_B$ are

$$F_{Bx} = +F_B \cos 37.0° = +(30.0\text{ N})(0.799) = +24.0\text{ N},$$
$$F_{By} = -F_B \sin 37.0° = -(30.0\text{ N})(0.602) = -18.1\text{ N}.$$

F_{By} is negative because it points along the negative *y* axis. The components of the resultant force are (see Fig. 4–19c)

$$F_{Rx} = F_{Ax} + F_{Bx} = 28.3\text{ N} + 24.0\text{ N} = 52.3\text{ N},$$
$$F_{Ry} = F_{Ay} + F_{By} = 28.3\text{ N} - 18.1\text{ N} = 10.2\text{ N}.$$

To find the magnitude of the resultant force, we use the Pythagorean theorem,

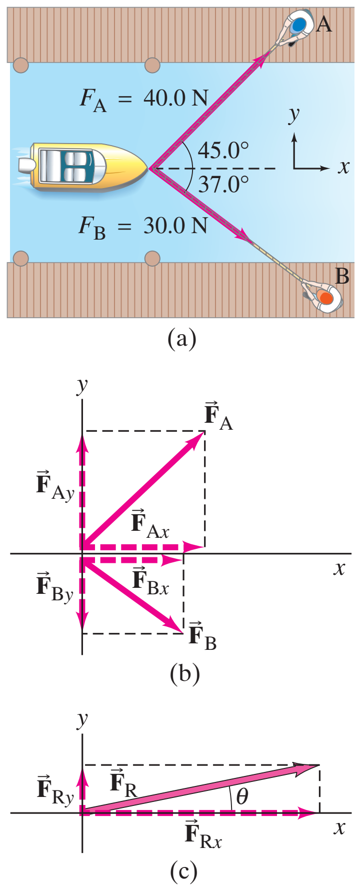
$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{(52.3)^2 + (10.2)^2}\text{ N} = 53.3\text{ N}.$$

The only remaining question is the angle θ that the net force $\vec{F}_R$ makes with the *x* axis. We use:

$$\tan\theta = \frac{F_{Ry}}{F_{Rx}} = \frac{10.2\text{ N}}{52.3\text{ N}} = 0.195,$$

and $\tan^{-1}(0.195) = 11.0°$. The net force on the boat has magnitude 53.3 N and acts at an 11.0° angle to the *x* axis.

FIGURE 4–19 Example 4–9: Two force vectors act on a boat.



When solving problems involving Newton's laws and force, it is very important to draw a diagram showing all the forces acting *on* each object involved. Such a diagram is called a **free-body diagram**, or **force diagram**: choose one object, and draw an arrow to represent each force acting on it. Include *every* force acting on that object. Do not show forces that the chosen object exerts on *other* objects. To help you identify each and every force that is exerted on your chosen object, ask yourself what other objects could exert a force on it. If your problem involves more than one object, a separate free-body diagram is needed for each object. For now, the likely forces that could be acting are *gravity* and *contact forces* (one object pushing or pulling another, normal force, friction). Later we will consider other types of force such as buoyancy, fluid pressure, and electric and magnetic forces.

PROBLEM SOLVING
Free-body diagram

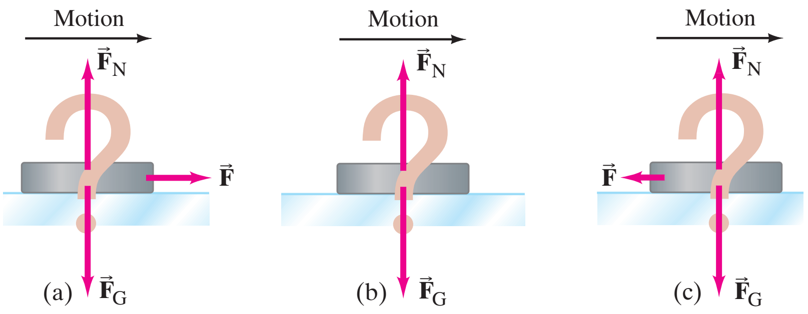


FIGURE 4–20 Example 4–10. Which is the correct free-body diagram for a hockey puck sliding across frictionless ice?

CONCEPTUAL EXAMPLE 4–10 **The hockey puck.** A hockey puck is sliding at constant velocity across a flat horizontal ice surface that is assumed to be frictionless. Which of the sketches in Fig. 4–20 is the correct free-body diagram for this puck? What would your answer be if the puck slowed down?

RESPONSE Did you choose (*a*)? If so, can you answer the question: what exerts the horizontal force labeled $\vec{\mathbf{F}}$ on the puck? If you say that it is the force needed to maintain the motion, ask yourself: what exerts this force? Remember that another object must exert any force—and there simply isn't any possibility here. Therefore, (*a*) is wrong. Besides, the force $\vec{\mathbf{F}}$ in Fig. 4–20a would give rise to an acceleration by Newton's second law. It is (*b*) that is correct. No net force acts on the puck, and the puck slides at constant velocity across the ice.

In the real world, where even smooth ice exerts at least a tiny friction force, then (*c*) is the correct answer. The tiny friction force is in the direction opposite to the motion, and the puck's velocity decreases, even if very slowly.

PROBLEM SOLVING

Newton's Laws; Free-Body Diagrams

1. **Draw a sketch** of the situation, after carefully reading the Problem at least twice.
2. Consider only one object (at a time), and draw a **free-body diagram** for that object, showing *all* the forces acting *on* that object. Include any unknown forces that you have to solve for. Do not show any forces that the chosen object exerts on other objects.

   Draw the arrow for each force vector reasonably accurately for direction and magnitude. Label each force acting on the object, including forces you must solve for, according to its source (gravity, person, friction, and so on).

   If several objects are involved, draw a free-body diagram for each object *separately*. For each object, show all the forces acting *on that object* (and *only* forces acting on that object). For each (and every) force, you must be clear about: *on* what object that force acts, and *by* what object that force is exerted. Only forces acting *on* a given object can be included in $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$ for that object.
3. Newton's second law involves vectors, and it is usually important to **resolve vectors** into components. **Choose** x and y **axes** in a way that simplifies the calculation. For example, it often saves work if you choose one coordinate axis to be in the direction of the acceleration (if known).
4. For each object, **apply Newton's second law** to the x and y components separately. That is, the x component of the net force on that object is related to the x component of that object's acceleration: $\Sigma F_x = ma_x$, and similarly for the y direction.
5. **Solve** the equation or equations for the unknown(s). Put in numerical values only at the end, and keep track of units.

This Problem Solving Strategy should not be considered a prescription. Rather it is a summary of things to do that will start you thinking and getting involved in the problem at hand.

CAUTION
Treating an object as a particle

When we are concerned only about translational motion, all the forces on a given object can be drawn as acting at the center of the object, thus treating the object as a *point particle*. However, for problems involving rotation or statics, the place *where* each force acts is also important, as we shall see in Chapters 8 and 9.

In the Examples in this Section, we assume that all surfaces are very smooth so that friction can be ignored. (Friction, and Examples using it, are discussed in Section 4–8.)

EXAMPLE 4–11 **Pulling the mystery box.** Suppose a friend asks to examine the 10.0-kg box you were given (Example 4–6, Fig. 4–15), hoping to guess what is inside; and you respond, "Sure, pull the box over to you." She then pulls the box by the attached cord, as shown in Fig. 4–21a, along the smooth surface of the table. The magnitude of the force exerted by the person is $F_P = 40.0\,\text{N}$, and it is exerted at a 30.0° angle as shown. Calculate (*a*) the acceleration of the box, and (*b*) the magnitude of the upward force F_N exerted by the table on the box. Assume that friction can be neglected.

APPROACH We follow the Problem Solving Strategy on the previous page.

SOLUTION

1. **Draw a sketch**: The situation is shown in Fig. 4–21a; it shows the box and the force applied by the person, F_P.
2. **Free-body diagram**: Figure 4–21b shows the free-body diagram of the box. To draw it correctly, we show *all* the forces acting on the box and *only* the forces acting on the box. They are: the force of gravity $m\vec{\mathbf{g}}$; the normal force exerted by the table $\vec{\mathbf{F}}_N$; and the force exerted by the person $\vec{\mathbf{F}}_P$. We are interested only in translational motion, so we can show the three forces acting at a point, Fig. 4–21c.
3. **Choose axes and resolve vectors**: We expect the motion to be horizontal, so we choose the x axis horizontal and the y axis vertical. The pull of 40.0 N has components

$$F_{Px} = (40.0\,\text{N})(\cos 30.0°) = (40.0\,\text{N})(0.866) = 34.6\,\text{N},$$
$$F_{Py} = (40.0\,\text{N})(\sin 30.0°) = (40.0\,\text{N})(0.500) = 20.0\,\text{N}.$$

In the horizontal (x) direction, $\vec{\mathbf{F}}_N$ and $m\vec{\mathbf{g}}$ have zero components. Thus the horizontal component of the net force is F_{Px}.

4. (*a*) **Apply Newton's second law** to get the x component of the acceleration:

$$F_{Px} = ma_x.$$

5. (*a*) **Solve**:

$$a_x = \frac{F_{Px}}{m} = \frac{(34.6\,\text{N})}{(10.0\,\text{kg})} = 3.46\,\text{m/s}^2.$$

The acceleration of the box is $3.46\,\text{m/s}^2$ to the right.

(*b*) Next we want to find F_N.

4′. (*b*) **Apply Newton's second law** to the vertical (y) direction, with upward as positive:

$$\Sigma F_y = ma_y$$
$$F_N - mg + F_{Py} = ma_y.$$

5′. (*b*) **Solve**: We have $mg = (10.0\,\text{kg})(9.80\,\text{m/s}^2) = 98.0\,\text{N}$ and, from point 3 above, $F_{Py} = 20.0\,\text{N}$. Furthermore, since $F_{Py} < mg$, the box does not move vertically, so $a_y = 0$. Thus

$$F_N - 98.0\,\text{N} + 20.0\,\text{N} = 0,$$

so

$$F_N = 78.0\,\text{N}.$$

NOTE F_N is less than mg: the table does not push against the full weight of the box because part of the pull exerted by the person is in the upward direction.

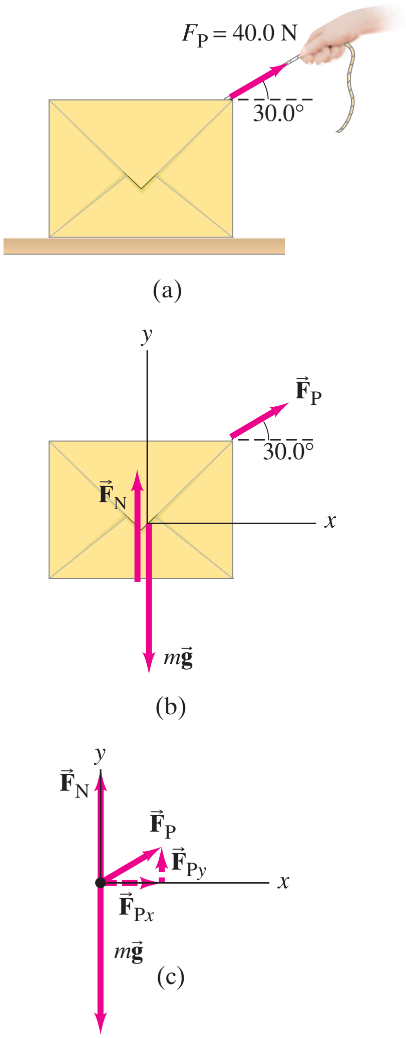


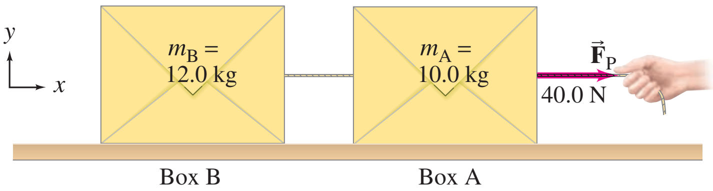
FIGURE 4–21 (a) Pulling the box, Example 4–11; (b) is the free-body diagram for the box, and (c) is the free-body diagram considering all the forces to act at a point (translational motion only, which is what we have here).

EXERCISE F A 10.0-kg box is dragged on a horizontal frictionless surface by a horizontal force of 10.0 N. If the applied force is doubled, the normal force on the box will (*a*) increase; (*b*) remain the same; (*c*) decrease.

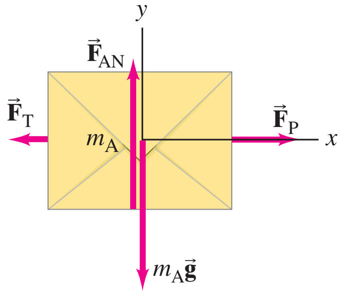
Tension in a Flexible Cord

When a flexible cord pulls on an object, the cord is said to be under **tension**, and the force it exerts on the object is the tension F_T. If the cord has negligible mass, the force exerted at one end is transmitted undiminished to each adjacent piece of cord along the entire length to the other end. Why? Because $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}} = 0$ for the cord if the cord's mass m is zero (or negligible) no matter what $\vec{\mathbf{a}}$ is. Hence the forces pulling on the cord at its two ends must add up to zero (F_T and $-F_T$). Note that flexible cords and strings can only pull. They can't push because they bend.

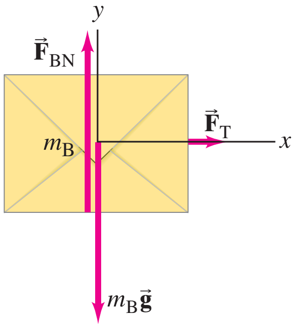
PROBLEM SOLVING
Cords can pull but can't push; tension exists throughout a taut cord



(a)



(b)



(c)

FIGURE 4–22 Example 4–12. (a) Two boxes, A and B, are connected by a cord. A person pulls horizontally on box A with force $F_P = 40.0$ N. (b) Free-body diagram for box A. (c) Free-body diagram for box B.

Our next Example involves two boxes connected by a cord. We can refer to this group of objects as a system. A **system** is any group of one or more objects we choose to consider and study.

EXAMPLE 4–12 **Two boxes connected by a cord.** Two boxes, A and B, are connected by a lightweight cord and are resting on a smooth (frictionless) table. The boxes have masses of 12.0 kg and 10.0 kg. A horizontal force F_P of 40.0 N is applied to the 10.0-kg box, as shown in Fig. 4–22a. Find (*a*) the acceleration of each box, and (*b*) the tension in the cord connecting the boxes.

APPROACH We streamline our approach by not listing each step. We have two boxes so we draw a free-body diagram for each. To draw them correctly, we must consider the forces on *each* box by itself, so that Newton's second law can be applied to each. The person exerts a force F_P on box A. Box A exerts a force F_T on the connecting cord, and the cord exerts an opposite but equal magnitude force F_T back on box A (Newton's third law). The two horizontal forces on box A are shown in Fig. 4–22b, along with the force of gravity $m_A\vec{g}$ downward and the normal force $\vec{F}_{AN}$ exerted upward by the table. The cord is light, so we neglect its mass. The tension at each end of the cord is thus the same. Hence the cord exerts a force F_T on the second box. Figure 4–22c shows the forces on box B, which are $\vec{F}_T$, $m_B\vec{g}$, and the normal force $\vec{F}_{BN}$. There will be only horizontal motion. We take the positive x axis to the right.

SOLUTION (*a*) We apply $\Sigma F_x = ma_x$ to box A:

$$\Sigma F_x = F_P - F_T = m_A a_A. \qquad \text{[box A]}$$

For box B, the only horizontal force is F_T, so

$$\Sigma F_x = F_T = m_B a_B. \qquad \text{[box B]}$$

The boxes are connected, and if the cord remains taut and doesn't stretch, then the two boxes will have the same acceleration a. Thus $a_A = a_B = a$. We are given $m_A = 10.0$ kg and $m_B = 12.0$ kg. We can add the two equations above to eliminate an unknown (F_T) and obtain

$$(m_A + m_B)a = F_P - F_T + F_T = F_P$$

or

$$a = \frac{F_P}{m_A + m_B} = \frac{40.0\text{ N}}{22.0\text{ kg}} = 1.82\text{ m/s}^2.$$

This is what we sought.

(*b*) From the equation for box B above $(F_T = m_B a_B)$, the tension in the cord is

$$F_T = m_B a = (12.0\text{ kg})(1.82\text{ m/s}^2) = 21.8\text{ N}.$$

Thus, $F_T < F_P$ (= 40.0 N), as we expect, since F_T acts to accelerate only m_B.

Alternate Solution to (a) We would have obtained the same result had we considered a single system, of mass $m_A + m_B$, acted on by a net horizontal force equal to F_P. (The tension forces F_T would then be considered internal to the system as a whole, and summed together would make zero contribution to the net force on the *whole* system.)

NOTE It might be tempting to say that the force the person exerts, F_P, acts not only on box A but also on box B. It doesn't. F_P acts only on box A. It affects box B via the tension in the cord, F_T, which acts on box B and accelerates it. (You could look at it this way: $F_T < F_P$ because F_P accelerates *both* boxes whereas F_T only accelerates box B.)

CAUTION

For any object, use only the forces on that object *in calculating* $\Sigma F = ma$

EXAMPLE 4–13 **Elevator and counterweight (Atwood machine).** A system of two objects suspended over a pulley by a flexible cable, as shown in Fig. 4–23a, is sometimes referred to as an *Atwood machine*. Consider the real-life application of an elevator (m_E) and its counterweight (m_C). To minimize the work done by the motor to raise and lower the elevator safely, m_E and m_C are made similar in mass. We leave the motor out of the system for this calculation, and assume that the cable's mass is negligible and that the mass of the pulley, as well as any friction, is small and ignorable. These assumptions ensure that the tension F_T in the cable has the same magnitude on both sides of the pulley. Let the mass of the counterweight be $m_C = 1000\,\text{kg}$. Assume the mass of the empty elevator is 850 kg, and its mass when carrying four passengers is $m_E = 1150\,\text{kg}$. For the latter case $(m_E = 1150\,\text{kg})$, calculate (*a*) the acceleration of the elevator and (*b*) the tension in the cable.

PHYSICS APPLIED
Elevator (as Atwood machine)

APPROACH Again we have two objects, and we will need to apply Newton's second law to each of them separately. Each mass has two forces acting on it: gravity downward and the cable tension pulling upward, $\vec{\mathbf{F}}_T$. Figures 4–23b and c show the free-body diagrams for the elevator (m_E) and for the counterweight (m_C). The elevator, being the heavier, will accelerate downward, whereas the counterweight will accelerate upward. The magnitudes of their accelerations will be equal (we assume the cable is massless and doesn't stretch). For the counterweight, $m_C g = (1000\,\text{kg})(9.80\,\text{m/s}^2) = 9800\,\text{N}$, so F_T must be greater than 9800 N (in order that m_C will accelerate upward). For the elevator, $m_E g = (1150\,\text{kg})(9.80\,\text{m/s}^2) = 11{,}300\,\text{N}$, which must have greater magnitude than F_T so that m_E accelerates downward. Thus our calculation must give F_T between 9800 N and 11,300 N.

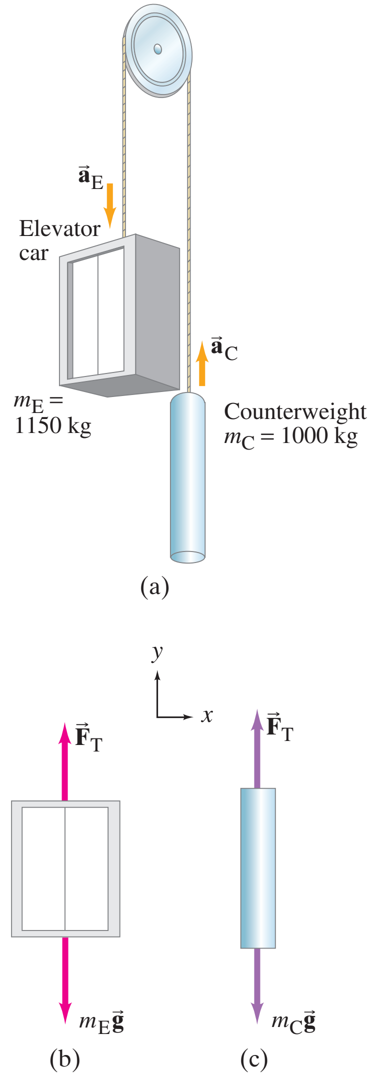


FIGURE 4–23 Example 4–13. (a) Atwood machine in the form of an elevator–counterweight system. (b) and (c) Free-body diagrams for the two objects.

SOLUTION (*a*) To find F_T as well as the acceleration a, we apply Newton's second law, $\Sigma F = ma$, to each object. We take upward as the positive y direction for both objects. With this choice of axes, $a_C = a$ because m_C accelerates upward, and $a_E = -a$ because m_E accelerates downward. Thus

$$F_T - m_E g = m_E a_E = -m_E a$$
$$F_T - m_C g = m_C a_C = +m_C a.$$

We can subtract the first equation from the second to get

$$(m_E - m_C)g = (m_E + m_C)a,$$

where a is now the only unknown. We solve this for a:

$$a = \frac{m_E - m_C}{m_E + m_C}g = \frac{1150\,\text{kg} - 1000\,\text{kg}}{1150\,\text{kg} + 1000\,\text{kg}}g = 0.070g = 0.68\,\text{m/s}^2.$$

The elevator (m_E) accelerates downward (and the counterweight m_C upward) at $a = 0.070g = 0.68\,\text{m/s}^2$.

(*b*) The tension in the cable F_T can be obtained from either of the two $\Sigma F = ma$ equations at the start of our solution, setting $a = 0.070g = 0.68\,\text{m/s}^2$:

$$\begin{aligned} F_T = m_E g - m_E a &= m_E(g - a) \\ &= 1150\,\text{kg}\,(9.80\,\text{m/s}^2 - 0.68\,\text{m/s}^2) = 10{,}500\,\text{N}, \end{aligned}$$

or

$$\begin{aligned} F_T = m_C g + m_C a &= m_C(g + a) \\ &= 1000\,\text{kg}\,(9.80\,\text{m/s}^2 + 0.68\,\text{m/s}^2) = 10{,}500\,\text{N}, \end{aligned}$$

which are consistent. As predicted, our result lies between 9800 N and 11,300 N.

NOTE We can check our equation for the acceleration a in this Example by noting that if the masses were equal ($m_E = m_C$), then our equation above for a would give $a = 0$, as we should expect. Also, if one of the masses is zero (say, $m_C = 0$), then the other mass ($m_E \neq 0$) would be predicted by our equation to accelerate at $a = g$, again as expected.

PROBLEM SOLVING
Check your result by seeing if it works in situations where the answer is easily guessed

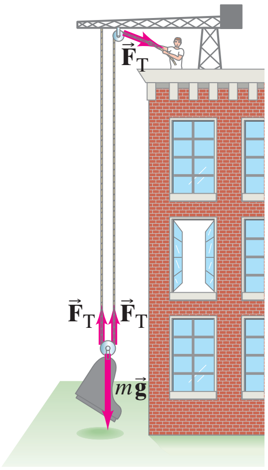


FIGURE 4–24 Example 4–14.

CONCEPTUAL EXAMPLE 4–14 **The advantage of a pulley.** A mover is trying to lift a piano (slowly) up to a second-story apartment (Fig. 4–24). He is using a rope looped over two pulleys as shown. What force must he exert on the rope to slowly lift the piano's 1600-N weight?

RESPONSE The magnitude of the tension force F_T within the rope is the same at any point along the rope if we assume we can ignore its mass. First notice the forces acting on the lower pulley at the piano. The weight of the piano $(= mg)$ pulls down on the pulley. The tension in the rope, looped through this pulley, pulls up *twice*, once on each side of the pulley. Let us apply Newton's second law to the pulley–piano combination (of mass m), choosing the upward direction as positive:

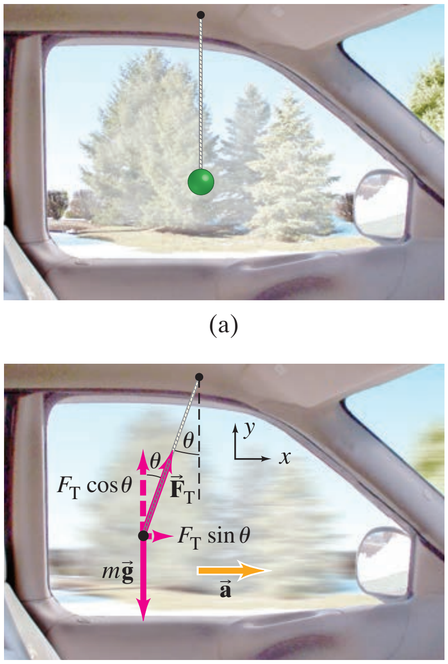
$$2F_T - mg = ma.$$

To move the piano with constant speed (set $a = 0$ in this equation) thus requires a tension in the rope, and hence a pull on the rope, of $F_T = mg/2$. The piano mover can exert a force equal to half the piano's weight.

NOTE We say the pulley has given a **mechanical advantage** of 2, since without the pulley the mover would have to exert twice the force.

PHYSICS APPLIED
Accelerometer

FIGURE 4–25 Example 4–15.



(a)

(b)

EXAMPLE 4–15 **Accelerometer.** A small mass m hangs from a thin string and can swing like a pendulum. You attach it above the window of your car as shown in Fig. 4–25a. When the car is at rest, the string hangs vertically. What angle θ does the string make (a) when the car accelerates at a constant $a = 1.20\,\mathrm{m/s^2}$, and (b) when the car moves at constant velocity, $v = 90\,\mathrm{km/h}$?

APPROACH The free-body diagram of Fig. 4–25b shows the pendulum at some angle θ relative to the vertical, and the forces on it: $m\vec{\mathbf{g}}$ downward, and the tension $\vec{\mathbf{F}}_T$ in the cord (including its components). These forces do not add up to zero if $\theta \neq 0$; and since we have an acceleration a, we expect $\theta \neq 0$.

SOLUTION (a) The acceleration $a = 1.20\,\mathrm{m/s^2}$ is horizontal $(= a_x)$, and the only horizontal force is the x component of $\vec{\mathbf{F}}_T$, $F_T \sin\theta$ (Fig. 4–25b). Then from Newton's second law,

$$ma = F_T \sin\theta.$$

The vertical component of Newton's second law gives, since $a_y = 0$,

$$0 = F_T \cos\theta - mg.$$

So

$$mg = F_T \cos\theta.$$

Dividing these two equations, we obtain

$$\tan\theta = \frac{F_T \sin\theta}{F_T \cos\theta} = \frac{ma}{mg} = \frac{a}{g}$$

or

$$\tan\theta = \frac{1.20\,\mathrm{m/s^2}}{9.80\,\mathrm{m/s^2}} = 0.122,$$

so

$$\theta = 7.0°.$$

(b) The velocity is constant, so $a = 0$ and $\tan\theta = 0$. Hence the pendulum hangs vertically $(\theta = 0°)$.

NOTE This simple device is an **accelerometer**—it can be used to determine acceleration, by measuring the angle θ.

4–8 Problems Involving Friction, Inclines

Friction

Until now we have ignored friction, but it must be taken into account in most practical situations. Friction exists between two solid surfaces because even the smoothest looking surface is quite rough on a microscopic scale, Fig. 4–26. When we try to slide an object across a surface, these microscopic bumps impede the motion. Exactly what is happening at the microscopic level is not yet fully understood. It is thought that the atoms on a bump of one surface may come so close to the atoms of the other surface that attractive electric forces between the atoms could "bond" as a tiny weld between the two surfaces. Sliding an object across a surface is often jerky, perhaps due to the making and breaking of these bonds. Even when a round object rolls across a surface, there is still some friction, called *rolling friction*, although it is generally much less than when an object slides across a surface. We focus now on sliding friction, which is usually called **kinetic friction** (*kinetic* is from the Greek for "moving").

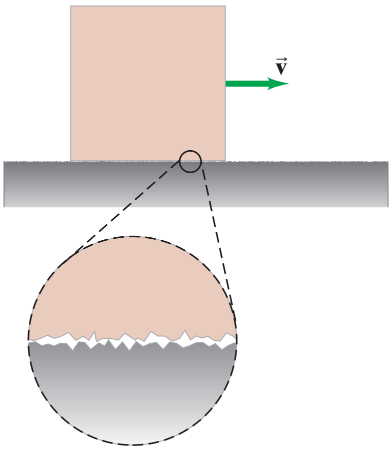
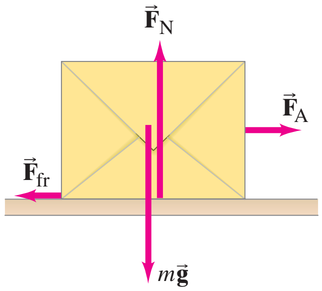


FIGURE 4–26 An object moving to the right on a table. The two surfaces in contact are assumed smooth, but are rough on a microscopic scale.

When an object slides along a rough surface, the force of kinetic friction acts opposite to the direction of the object's velocity. The magnitude of the force of kinetic friction depends on the nature of the two sliding surfaces. For given surfaces, experiment shows that the friction force is approximately proportional to the *normal force* between the two surfaces, which is the force that either object exerts on the other and is perpendicular to their common surface of contact (see Fig. 4–27). The force of friction between hard surfaces in many cases depends very little on the total surface area of contact; that is, the friction force on this book is roughly the same whether it is being slid across a table on its wide face or on its spine, assuming the surfaces have the same smoothness. We consider a simple model of friction in which we make this assumption that the friction force is independent of area. Then we write the proportionality between the magnitudes of the friction force F_{fr} and the normal force F_N as an equation by inserting a constant of proportionality, μ_k:

$$F_{fr} = \mu_k F_N. \qquad \text{[kinetic friction]}$$

FIGURE 4–27 When an object is pulled along a surface by an applied force ($\vec{\mathbf{F}}_A$), the force of friction $\vec{\mathbf{F}}_{fr}$ opposes the motion. The magnitude of $\vec{\mathbf{F}}_{fr}$ is proportional to the magnitude of the normal force (F_N).



CAUTION

$\vec{\mathbf{F}}_{fr} \perp \vec{\mathbf{F}}_N$

This relation is not a fundamental law; it is an experimental relation between the magnitude of the friction force F_{fr}, which acts parallel to the two surfaces, and the magnitude of the normal force F_N, which acts perpendicular to the surfaces. It is *not* a vector equation since the two forces have different directions, perpendicular to one another. The term μ_k is called the *coefficient of kinetic friction*, and its value depends on the nature of the two surfaces. Measured values for a variety of surfaces are given in Table 4–2. These are only approximate, however, since μ depends on whether the surfaces are wet or dry, on how much they have been sanded or rubbed, if any burrs remain, and other such factors. But μ_k (which has no units) is roughly independent of the sliding speed, as well as the area in contact.

TABLE 4–2 Coefficients of Friction[†]

Surfaces	Coefficient of Static Friction, μ_s	Coefficient of Kinetic Friction, μ_k
Wood on wood	0.4	0.2
Ice on ice	0.1	0.03
Metal on metal (lubricated)	0.15	0.07
Steel on steel (unlubricated)	0.7	0.6
Rubber on dry concrete	1.0	0.8
Rubber on wet concrete	0.7	0.5
Rubber on other solid surfaces	1–4	1
Teflon® on Teflon in air	0.04	0.04
Teflon on steel in air	0.04	0.04
Lubricated ball bearings	<0.01	<0.01
Synovial joints (in human limbs)	0.01	0.01

[†] Values are approximate and intended only as a guide.

What we have been discussing up to now is *kinetic friction*, when one object slides over another. There is also **static friction**, which refers to a force parallel to the two surfaces that can arise even when they are not sliding. Suppose an object such as a desk is resting on a horizontal floor. If no horizontal force is exerted on the desk, there also is no friction force. But now suppose you try to push the desk, and it doesn't move. You are exerting a horizontal force, but the desk isn't moving, so there must be another force on the desk keeping it from moving (the net force is zero on an object at rest). This is the force of *static friction* exerted by the floor on the desk. If you push with a greater force without moving the desk, the force of static friction also has increased. If you push hard enough, the desk will eventually start to move, and kinetic friction takes over. At this point, you have exceeded the maximum force of static friction, which is given by $(F_{\text{fr}})_{\text{max}} = \mu_s F_N$, where μ_s is the *coefficient of static friction* (Table 4–2). Because the force of static friction can vary from zero to this maximum value, we write

$$F_{\text{fr}} \leq \mu_s F_N. \qquad \text{[static friction]}$$

You may have noticed that it is often easier to keep a heavy object sliding than it is to start it sliding in the first place. This is consistent with μ_s generally being greater than μ_k (see Table 4–2).

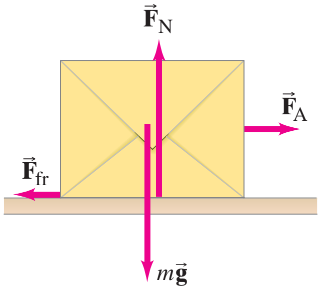


FIGURE 4–27 Repeated for Example 4–16.

EXAMPLE 4–16 **Friction: static and kinetic.** Our 10.0-kg mystery box rests on a horizontal floor. The coefficient of static friction is $\mu_s = 0.40$ and the coefficient of kinetic friction is $\mu_k = 0.30$. Determine the force of friction, F_{fr}, acting on the box if a horizontal applied force F_A is exerted on it of magnitude: (*a*) 0, (*b*) 10 N, (*c*) 20 N, (*d*) 38 N, and (*e*) 40 N.

APPROACH We don't know, right off, if we are dealing with static friction or kinetic friction, nor if the box remains at rest or accelerates. We need to draw a free-body diagram, and then determine in each case whether or not the box will move: the box starts moving if F_A is greater than the maximum static friction force (Newton's second law). The forces on the box are gravity $m\vec{\mathbf{g}}$, the normal force exerted by the floor $\vec{\mathbf{F}}_N$, the horizontal applied force $\vec{\mathbf{F}}_A$, and the friction force $\vec{\mathbf{F}}_{\text{fr}}$, as shown in Fig. 4–27.

SOLUTION The free-body diagram of the box is shown in Fig. 4–27. In the vertical direction there is no motion, so Newton's second law in the vertical direction gives $\Sigma F_y = ma_y = 0$, which tells us $F_N - mg = 0$. Hence the normal force is

$$F_N = mg = (10.0\,\text{kg})(9.80\,\text{m/s}^2) = 98.0\,\text{N}.$$

(*a*) Because $F_A = 0$ in this first case, the box doesn't move, and $F_{\text{fr}} = 0$.

(*b*) The force of static friction will oppose any applied force up to a maximum of

$$\mu_s F_N = (0.40)(98.0\,\text{N}) = 39\,\text{N}.$$

When the applied force is $F_A = 10\,\text{N}$, the box will not move. Newton's second law gives $\Sigma F_x = F_A - F_{\text{fr}} = 0$, so $F_{\text{fr}} = 10\,\text{N}$.

(*c*) An applied force of 20 N is also not sufficient to move the box. Thus $F_{\text{fr}} = 20\,\text{N}$ to balance the applied force.

(*d*) The applied force of 38 N is still not quite large enough to move the box; so the friction force has now increased to 38 N to keep the box at rest.

(*e*) A force of 40 N will start the box moving since it exceeds the maximum force of static friction, $\mu_s F_N = (0.40)(98\,\text{N}) = 39\,\text{N}$. Instead of static friction, we now have kinetic friction, and its magnitude is

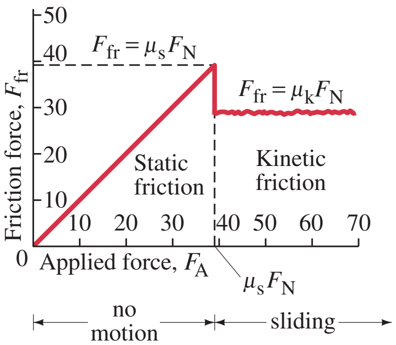
$$F_{\text{fr}} = \mu_k F_N = (0.30)(98.0\,\text{N}) = 29\,\text{N}.$$

There is now a net (horizontal) force on the box of magnitude $F = 40\,\text{N} - 29\,\text{N} = 11\,\text{N}$, so the box will accelerate at a rate

$$a_x = \frac{\Sigma F}{m} = \frac{11\,\text{N}}{10.0\,\text{kg}} = 1.1\,\text{m/s}^2$$

as long as the applied force is 40 N. Figure 4–28 shows a graph that summarizes this Example.

FIGURE 4–28 Example 4–16. Magnitude of the force of friction as a function of the external force applied to an object initially at rest. As the applied force is increased in magnitude, the force of static friction increases in proportion until the applied force equals $\mu_s F_N$. If the applied force increases further, the object will begin to move, and the friction force drops to a roughly constant value characteristic of kinetic friction.



Friction can be a hindrance. It slows down moving objects and causes heating and binding of moving parts in machinery. Friction can be reduced by using lubricants such as oil. More effective in reducing friction between two surfaces is to maintain a layer of air or other gas between them. Devices using this concept, which is not practical for most situations, include air tracks and air tables in which the layer of air is maintained by forcing air through many tiny holes. Another technique to maintain the air layer is to suspend objects in air using magnetic fields ("magnetic levitation").

On the other hand, friction can be helpful. Our ability to walk depends on friction between the soles of our shoes (or feet) and the ground. (Walking involves static friction, not kinetic friction. Why?) The movement of a car, and also its stability, depend on friction. When friction is low, such as on ice, safe walking or driving becomes difficult.

CONCEPTUAL EXAMPLE 4–17 **A box against a wall.** You can hold a box against a rough wall (Fig. 4–29) and prevent it from slipping down by pressing hard horizontally. How does the application of a horizontal force keep an object from moving vertically?

RESPONSE This won't work well if the wall is slippery. You need friction. Even then, if you don't press hard enough, the box will slip. The horizontal force you apply produces a normal force on the box exerted by the wall (the net force horizontally is zero since the box doesn't move horizontally). The force of gravity mg, acting downward on the box, can now be balanced by an upward static friction force whose maximum magnitude is proportional to the normal force. The harder you push, the greater F_N is and the greater F_{fr} can be. If you don't press hard enough, then $mg > \mu_s F_N$ and the box begins to slide down.

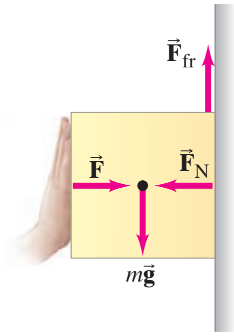


FIGURE 4–29 Example 4–17.

EXERCISE G If $\mu_s = 0.40$ and $mg = 20\,\text{N}$, what minimum force F will keep the box from falling: (*a*) 100 N; (*b*) 80 N; (*c*) 50 N; (*d*) 20 N; (*e*) 8 N?

EXAMPLE 4–18 **Pulling against friction.** A 10.0-kg box is pulled along a horizontal surface by a force F_P of 40.0 N applied at a 30.0° angle above horizontal. This is like Example 4–11 except now there is friction, and we assume a coefficient of kinetic friction of 0.30. Calculate the acceleration.

APPROACH The free-body diagram is shown in Fig. 4–30. It is much like that in Fig. 4–21b, but with one more force, friction.

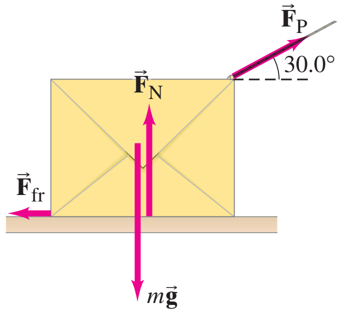


FIGURE 4–30 Example 4–18.

SOLUTION The calculation for the vertical (y) direction is just the same as in Example 4–11b, $mg = (10.0\,\text{kg})(9.80\,\text{m/s}^2) = 98.0\,\text{N}$ and $F_{Py} = (40.0\,\text{N})(\sin 30.0°) = 20.0\,\text{N}$. With y positive upward and $a_y = 0$, we have

$$F_N - mg + F_{Py} = ma_y$$
$$F_N - 98.0\,\text{N} + 20.0\,\text{N} = 0,$$

so the normal force is $F_N = 78.0\,\text{N}$. Now we apply Newton's second law for the horizontal (x) direction (positive to the right), and include the friction force:

$$F_{Px} - F_{fr} = ma_x.$$

The friction force is kinetic friction as long as $F_{fr} = \mu_k F_N$ is less than $F_{Px} = (40.0\,\text{N})\cos 30.0° = 34.6\,\text{N}$, which it is:

$$F_{fr} = \mu_k F_N = (0.30)(78.0\,\text{N}) = 23.4\,\text{N}.$$

Hence the box does accelerate:

$$a_x = \frac{F_{Px} - F_{fr}}{m} = \frac{34.6\,\text{N} - 23.4\,\text{N}}{10.0\,\text{kg}} = 1.1\,\text{m/s}^2.$$

In the absence of friction, as we saw in Example 4–11, the acceleration would be much greater than this.

NOTE Our final answer has only two significant figures because our least significant input value $(\mu_k = 0.30)$ has two.

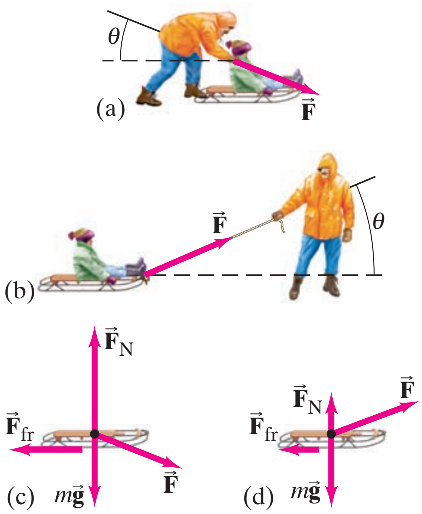
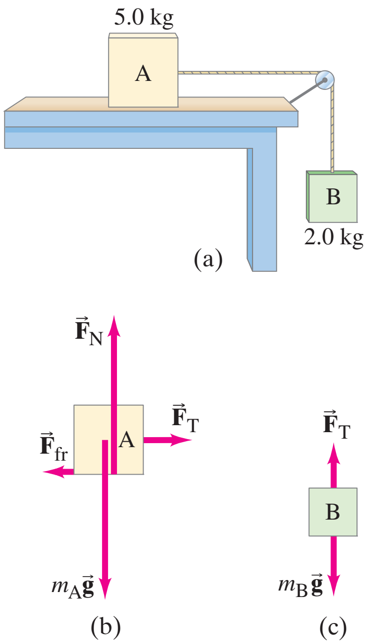


FIGURE 4–31 Example 4–19.

FIGURE 4–32 Example 4–20.



CAUTION

Tension in a cord supporting a falling object may not equal object's weight

CONCEPTUAL EXAMPLE 4–19 **To push or to pull a sled?** Your little sister wants a ride on her sled. If you are on flat ground, will you exert less force if you push her or pull her? See Figs. 4–31a and b. Assume the same angle θ in each case.

RESPONSE Let us draw free-body diagrams for the sled–sister combination, as shown in Figs. 4–31c and d. They show, for the two cases, the forces exerted by you, $\vec{\mathbf{F}}$ (an unknown), by the snow, $\vec{\mathbf{F}}_{\mathrm{N}}$ and $\vec{\mathbf{F}}_{\mathrm{fr}}$, and gravity $m\vec{\mathbf{g}}$. (*a*) If you push her, and $\theta > 0$, there is a vertically downward component to your force. Hence the normal force upward exerted by the ground (Fig. 4–31c) will be larger than mg (where m is the mass of sister plus sled). (*b*) If you pull her, your force has a vertically upward component, so the normal force F_{N} will be less than mg, Fig. 4–31d. Because the friction force is proportional to the normal force, F_{fr} will be less if you pull her. So you exert less force if you pull her.

EXAMPLE 4–20 **Two boxes and a pulley.** In Fig. 4–32a, two boxes are connected by a cord running over a pulley. The coefficient of kinetic friction between box A and the table is 0.20. We ignore the mass of the cord and pulley and any friction in the pulley, which means we can assume that a force applied to one end of the cord will have the same magnitude at the other end. We wish to find the acceleration, a, of the system, which will have the same magnitude for both boxes assuming the cord doesn't stretch. As box B moves down, box A moves to the right.

APPROACH The free-body diagrams for each box are shown in Figs. 4–32b and c. The forces on box A are the pulling force of the cord F_{T}, gravity $m_{\mathrm{A}}g$, the normal force exerted by the table F_{N}, and a friction force exerted by the table F_{fr}; the forces on box B are gravity $m_{\mathrm{B}}g$, and the cord pulling up, F_{T}.

SOLUTION Box A does not move vertically, so Newton's second law tells us the normal force just balances the weight,

$$F_{\mathrm{N}} = m_{\mathrm{A}}g = (5.0\,\mathrm{kg})(9.8\,\mathrm{m/s^2}) = 49\,\mathrm{N}.$$

In the horizontal direction, there are two forces on box A (Fig. 4–32b): F_{T}, the tension in the cord (whose value we don't know), and the force of friction

$$F_{\mathrm{fr}} = \mu_{\mathrm{k}}F_{\mathrm{N}} = (0.20)(49\,\mathrm{N}) = 9.8\,\mathrm{N}.$$

The horizontal acceleration (box A) is what we wish to find; we use Newton's second law in the x direction, $\Sigma F_{\mathrm{A}x} = m_{\mathrm{A}}a_x$, which becomes (taking the positive direction to the right and setting $a_{\mathrm{A}x} = a$):

$$\Sigma F_{\mathrm{A}x} = F_{\mathrm{T}} - F_{\mathrm{fr}} = m_{\mathrm{A}}a. \qquad \text{[box A]}$$

Next consider box B. The force of gravity $m_{\mathrm{B}}g = (2.0\,\mathrm{kg})(9.8\,\mathrm{m/s^2}) = 19.6\,\mathrm{N}$ pulls downward; and the cord pulls upward with a force F_{T}. So we can write Newton's second law for box B (taking the downward direction as positive):

$$\Sigma F_{\mathrm{B}y} = m_{\mathrm{B}}g - F_{\mathrm{T}} = m_{\mathrm{B}}a. \qquad \text{[box B]}$$

[Notice that if $a \neq 0$, then F_{T} is not equal to $m_{\mathrm{B}}g$.]

We have two unknowns, a and F_{T}, and we also have two equations. We solve the box A equation for F_{T}:

$$F_{\mathrm{T}} = F_{\mathrm{fr}} + m_{\mathrm{A}}a,$$

and substitute this into the box B equation:

$$m_{\mathrm{B}}g - F_{\mathrm{fr}} - m_{\mathrm{A}}a = m_{\mathrm{B}}a.$$

Now we solve for a and put in numerical values:

$$a = \frac{m_{\mathrm{B}}g - F_{\mathrm{fr}}}{m_{\mathrm{A}} + m_{\mathrm{B}}} = \frac{19.6\,\mathrm{N} - 9.8\,\mathrm{N}}{5.0\,\mathrm{kg} + 2.0\,\mathrm{kg}} = 1.4\,\mathrm{m/s^2},$$

which is the acceleration of box A to the right, and of box B down.

If we wish, we can calculate F_{T} using the third equation up from here:

$$F_{\mathrm{T}} = F_{\mathrm{fr}} + m_{\mathrm{A}}a = 9.8\,\mathrm{N} + (5.0\,\mathrm{kg})(1.4\,\mathrm{m/s^2}) = 17\,\mathrm{N}.$$

NOTE Box B is not in free fall. It does not fall at $a = g$ because an additional force, F_{T}, is acting upward on it.

Inclines

Now we consider what happens when an object slides down an incline, such as a hill or ramp. Such problems are interesting because gravity is the accelerating force, yet the acceleration is not vertical. Solving problems is usually easier if we choose the xy coordinate system so the x axis points along the incline (the direction of motion) and the y axis is perpendicular to the incline, as shown in Fig. 4–33. Note also that the normal force is not vertical, but is perpendicular to the sloping surface of the plane, along the y axis in Fig. 4–33.

PROBLEM SOLVING
Good choice of coordinate system simplifies the calculation

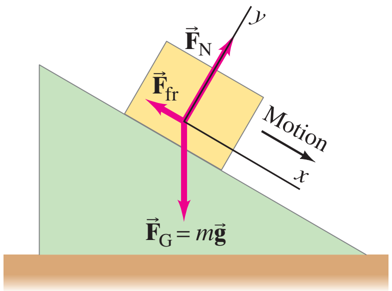


FIGURE 4–33 Forces on an object sliding down an incline.

EXERCISE H Is the normal force always perpendicular to an inclined plane? Is it always vertical?

EXAMPLE 4–21 The skier. The skier in Fig. 4–34a has begun descending the 30° slope. If the coefficient of kinetic friction is 0.10, what is her acceleration?

PHYSICS APPLIED
Skiing

APPROACH We choose the x axis along the slope, positive downslope in the direction of the skier's motion. The y axis is perpendicular to the surface. The forces acting on the skier are gravity, $\vec{\mathbf{F}}_G = m\vec{\mathbf{g}}$, which points vertically downward (*not* perpendicular to the slope), and the two forces exerted on her skis by the snow—the normal force perpendicular to the snowy slope (*not* vertical), and the friction force parallel to the surface. These three forces are shown acting at one point in Fig. 4–34b, which is our free-body diagram for the skier.

SOLUTION We have to resolve only one vector into components, the weight $\vec{\mathbf{F}}_G$, and its components are shown as dashed lines in Fig. 4–34c. To be general, we use θ rather than 30° for now. We use the definitions of sine ("side opposite") and cosine ("side adjacent") to obtain the components:

$$F_{Gx} = mg\sin\theta,$$
$$F_{Gy} = -mg\cos\theta$$

where F_{Gy} is in the negative y direction. To calculate the skier's acceleration down the hill, a_x, we apply Newton's second law to the x direction:

$$\Sigma F_x = ma_x$$
$$mg\sin\theta - \mu_k F_N = ma_x$$

where the two forces are the x component of the gravity force ($+x$ direction) and the friction force ($-x$ direction). We want to find the value of a_x, but we don't yet know F_N in the last equation. Let's see if we can get F_N from the y component of Newton's second law:

$$\Sigma F_y = ma_y$$
$$F_N - mg\cos\theta = ma_y = 0$$

where we set $a_y = 0$ because there is no motion in the y direction (perpendicular to the slope). Thus we can solve for F_N:

$$F_N = mg\cos\theta$$

and we can substitute this into our equation above for ma_x:

$$mg\sin\theta - \mu_k(mg\cos\theta) = ma_x.$$

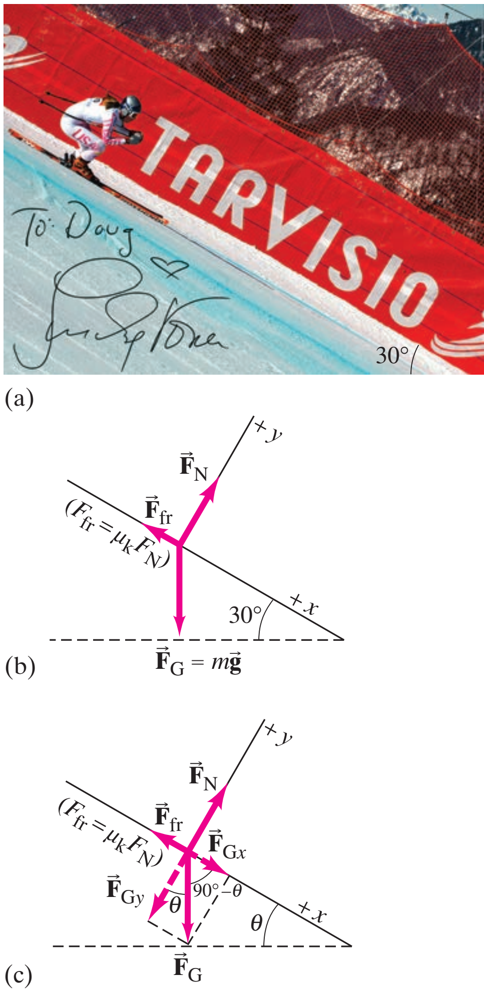
There is an m in each term which can be canceled out. Thus (setting $\theta = 30°$ and $\mu_k = 0.10$):

$$a_x = g\sin 30° - \mu_k g\cos 30°$$
$$= 0.50g - (0.10)(0.866)g = 0.41g.$$

The skier's acceleration is 0.41 times the acceleration of gravity, which in numbers[†] is $a = (0.41)(9.8\,\mathrm{m/s^2}) = 4.0\,\mathrm{m/s^2}$.

NOTE The mass canceled out, so we have the useful conclusion that *the acceleration doesn't depend on the mass*. That such a cancellation sometimes occurs, and thus may give a useful conclusion as well as saving calculation, is a big advantage of working with the algebraic equations and putting in the numbers only at the end.

FIGURE 4–34 Example 4–21. Skier descending a slope; $\vec{\mathbf{F}}_G = m\vec{\mathbf{g}}$ is the force of gravity (weight) on the skier.



PROBLEM SOLVING
It is often helpful to put in numbers only at the end

[†]We used values rounded off to 2 significant figures to obtain $a = 4.0\,\mathrm{m/s^2}$. If we kept all the extra digits in our calculator, we would find $a = 0.4134g \approx 4.1\,\mathrm{m/s^2}$. This difference is within the expected precision (number of significant figures, Section 1–4).