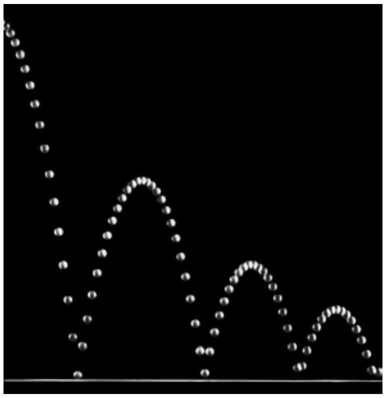


(a)

(b)

FIGURE 3–17 Photographs of (a) a bouncing ball and (b) a thrown basketball, each showing the characteristic "parabolic" path of projectile motion.

3–5 Projectile Motion

In Chapter 2, we studied the one-dimensional motion of an object in terms of displacement, velocity, and acceleration, including purely vertical motion of a falling object undergoing acceleration due to gravity. Now we examine the more general translational motion of objects moving through the air in two dimensions near the Earth's surface, such as a golf ball, a thrown or batted baseball, kicked footballs, and speeding bullets. These are all examples of **projectile motion** (see Fig. 3–17), which we can describe as taking place in two dimensions if there is no wind.

Although air resistance is often important, in many cases its effect can be ignored, and we will ignore it in the following analysis. We will not be concerned now with the process by which the object is thrown or projected. We consider only its motion *after* it has been projected, and *before* it lands or is caught—that is, we analyze our projected object only when it is moving freely through the air under the action of gravity alone. Then the acceleration of the object is that due to gravity, which acts downward with magnitude $g = 9.80\ \mathrm{m/s^2}$, and we assume it is constant.†

Galileo was the first to describe projectile motion accurately. He showed that it could be understood by analyzing the horizontal and vertical components of the motion separately. For convenience, we assume that the motion begins at time $t = 0$ at the origin of an xy coordinate system (so $x_0 = y_0 = 0$).

Let us look at a (tiny) ball rolling off the end of a horizontal table with an initial velocity in the horizontal (x) direction, $\vec{\mathbf{v}}_{x0}$. See Fig. 3–18, where an object falling vertically is also shown for comparison. The velocity vector $\vec{\mathbf{v}}$ at each instant points in the direction of the ball's motion at that instant and is thus always tangent to the path. Following Galileo's ideas, we treat the horizontal and vertical components of velocity and acceleration separately, and we can apply the kinematic equations (Eqs. 2–11a through 2–11c) to the x and y components of the motion.

First we examine the vertical (y) component of the motion. At the instant the ball leaves the table's top $(t = 0)$, it has only an x component of velocity. Once the ball leaves the table (at $t = 0$), it experiences a vertically downward acceleration g, the acceleration due to gravity. Thus v_y is initially zero $(v_{y0} = 0)$ but increases continually in the downward direction (until the ball hits the ground). Let us take y to be positive upward. Then the acceleration due to gravity is in the $-y$ direction, so $a_y = -g$. From Eq. 2–11a (using y in place of x) we can write $v_y = v_{y0} + a_y t = -gt$ since we set $v_{y0} = 0$. The vertical displacement is given by Eq. 2–11b written in terms of y: $y = y_0 + v_{y0} + \frac{1}{2} a_y t^2$. Given $y_0 = 0$, $v_{y0} = 0$, and $a_y = -g$, then $y = -\frac{1}{2} g t^2$.

†This restricts us to objects whose distance traveled and maximum height above the Earth are small compared to the Earth's radius (6400 km).

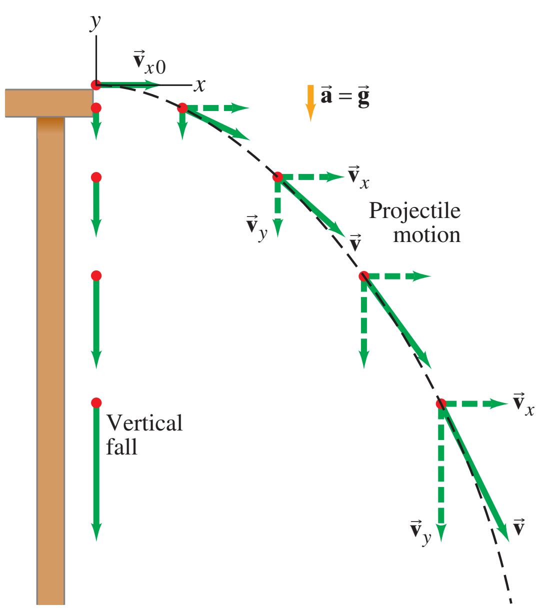


FIGURE 3–18 Projectile motion of a small ball projected horizontally with initial velocity $\vec{\mathbf{v}} = \vec{\mathbf{v}}_{x0}$. The dashed black line represents the path of the object. The velocity vector $\vec{\mathbf{v}}$ is in the direction of motion at each point, and thus is tangent to the path. The velocity vectors are green arrows, and velocity components are dashed. (A vertically falling object starting from rest at the same place and time is shown at the left for comparison; v_y is the same at each instant for the falling object and the projectile.)

In the horizontal direction, on the other hand, there is no acceleration (we are ignoring air resistance). With $a_x = 0$, the horizontal component of velocity, v_x, remains constant, equal to its initial value, v_{x0}, and thus has the same magnitude at each point on the path. The horizontal displacement (with $a_x = 0$) is given by $x = v_{x0}t + \frac{1}{2}a_x t^2 = v_{x0}t$. The two vector components, $\vec{\mathbf{v}}_x$ and $\vec{\mathbf{v}}_y$, can be added vectorially at any instant to obtain the velocity $\vec{\mathbf{v}}$ at that time (that is, for each point on the path), as shown in Fig. 3–18.

One result of this analysis, which Galileo himself predicted, is that *an object projected horizontally will reach the ground in the same time as an object dropped vertically*. This is because the vertical motions are the same in both cases, as shown in Fig. 3–18. Figure 3–19 is a multiple-exposure photograph of an experiment that confirms this.

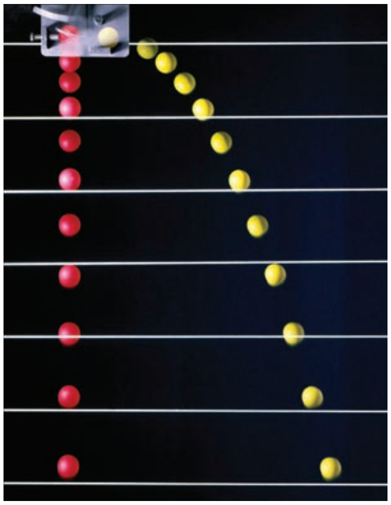


FIGURE 3–19 Multiple-exposure photograph showing positions of two balls at equal time intervals. One ball was dropped from rest at the same time the other ball was projected horizontally outward. The vertical position of each ball is seen to be the same at each instant.

EXERCISE C Two balls having different speeds roll off the edge of a horizontal table at the same time. Which hits the floor sooner, the faster ball or the slower one?

If an object is projected at an upward angle, as in Fig. 3–20, the analysis is similar, except that now there is an initial vertical component of velocity, v_{y0}. Because of the downward acceleration of gravity, the upward component of velocity v_y gradually decreases with time until the object reaches the highest point on its path, at which point $v_y = 0$. Subsequently the object moves downward (Fig. 3–20) and v_y increases in the downward direction, as shown (that is, becoming more negative). As before, v_x remains constant.

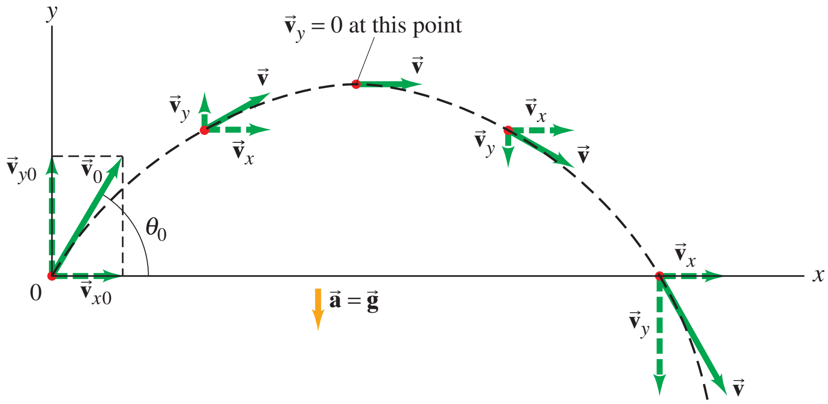


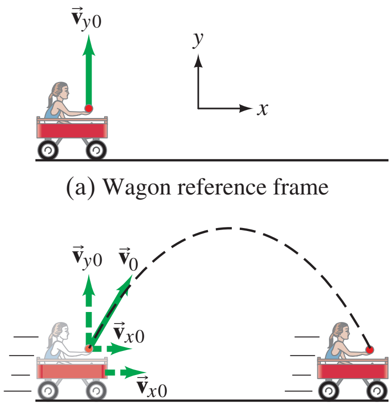
FIGURE 3–20 Path of a projectile launched with initial velocity $\vec{\mathbf{v}}_0$ at angle θ_0 to the horizontal. Path is shown dashed in black, the velocity vectors are green arrows, and velocity components are dashed. The figure does not show where the projectile hits the ground (at that point, projectile motion ceases).

EXERCISE D Where in Fig. 3–20 is (i) $\vec{\mathbf{v}} = 0$, (ii) $v_y = 0$, and (iii) $v_x = 0$?

CONCEPTUAL EXAMPLE 3–4 **Where does the apple land?** A child sits upright in a wagon which is moving to the right at constant speed as shown in Fig. 3–21. The child extends her hand and throws an apple straight upward (from her own point of view, Fig. 3–21a), while the wagon continues to travel forward at constant speed. If air resistance is neglected, will the apple land (*a*) behind the wagon, (*b*) in the wagon, or (*c*) in front of the wagon?

RESPONSE The child throws the apple straight up from her own reference frame with initial velocity $\vec{\mathbf{v}}_{y0}$ (Fig. 3–21a). But when viewed by someone on the ground, the apple also has an initial horizontal component of velocity equal to the speed of the wagon, $\vec{\mathbf{v}}_{x0}$. Thus, to a person on the ground, the apple will follow the path of a projectile as shown in Fig. 3–21b. The apple experiences no horizontal acceleration, so $\vec{\mathbf{v}}_{x0}$ will stay constant and equal to the speed of the wagon. As the apple follows its arc, the wagon will be directly under the apple at all times because they have the same horizontal velocity. When the apple comes down, it will drop right into the outstretched hand of the child. The answer is (*b*).

FIGURE 3–21 Example 3–4.



(a) Wagon reference frame

(b) Ground reference frame

EXERCISE E Return to the Chapter-Opening Question, page 49, and answer it again now. Try to explain why you may have answered differently the first time. Describe the role of the helicopter in this example of projectile motion.

3–6 Solving Projectile Motion Problems

We now work through several Examples of projectile motion quantitatively. We use the kinematic equations (2–11a through 2–11c) separately for the vertical and horizontal components of the motion. These equations are shown separately for the x and y components of the motion in Table 3–1, for the general case of two-dimensional motion at constant acceleration. Note that x and y are the respective displacements, that v_x and v_y are the components of the velocity, and that a_x and a_y are the components of the acceleration, each of which is constant. The subscript 0 means "at $t = 0$."

TABLE 3–1 General Kinematic Equations for Constant Acceleration in Two Dimensions

x component (horizontal)		y component (vertical)
$v_x = v_{x0} + a_x t$	(Eq. 2–11a)	$v_y = v_{y0} + a_y t$
$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$	(Eq. 2–11b)	$y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$
$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$	(Eq. 2–11c)	$v_y^2 = v_{y0}^2 + 2a_y(y - y_0)$

We can simplify Eqs. 2–11 to use for projectile motion because we can set $a_x = 0$. See Table 3–2, which assumes y is positive upward, so $a_y = -g = -9.80\,\mathrm{m/s^2}$.

TABLE 3–2 Kinematic Equations for Projectile Motion
(y positive upward; $a_x = 0,\ a_y = -g = -9.80\,\mathrm{m/s^2}$)

Horizontal Motion ($a_x = 0,\ v_x =$ constant)		Vertical Motion† ($a_y = -g =$ constant)
$v_x = v_{x0}$	(Eq. 2–11a)	$v_y = v_{y0} - gt$
$x = x_0 + v_{x0}t$	(Eq. 2–11b)	$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$
	(Eq. 2–11c)	$v_y^2 = v_{y0}^2 - 2g(y - y_0)$

† If y is taken positive downward, the minus ($-$) signs in front of g become $+$ signs.

If the projection angle θ_0 is chosen relative to the $+x$ axis (Fig. 3–20), then

$$v_{x0} = v_0 \cos\theta_0, \qquad \text{and} \qquad v_{y0} = v_0 \sin\theta_0 .$$

PROBLEM SOLVING
Choice of time interval

In doing Problems involving projectile motion, we must consider a time interval for which our chosen object is in the air, influenced only by gravity. We do not consider the throwing (or projecting) process, nor the time after the object lands or is caught, because then other influences act on the object, and we can no longer set $\vec{\mathbf{a}} = \vec{\mathbf{g}}$.

PROBLEM SOLVING

Projectile Motion

Our approach to solving Problems in Section 2–6 also applies here. Solving Problems involving projectile motion can require creativity, and cannot be done just by following some rules. Certainly you must avoid just plugging numbers into equations that seem to "work."

1. As always, **read** carefully; **choose** the **object** (or objects) you are going to analyze.
2. **Draw** a careful **diagram** showing what is happening to the object.
3. **Choose** an origin and an xy **coordinate system**.
4. Decide on the **time interval**, which for projectile motion can only include motion under the effect of gravity alone, not throwing or landing. The time interval must be the same for the x and y analyses. The x and y motions are connected by the common time, t.
5. **Examine** the horizontal (x) and vertical (y) **motions** separately. If you are given the initial velocity, you may want to resolve it into its x and y components.
6. List the **known** and **unknown** quantities, choosing $a_x = 0$ and $a_y = -g$ or $+g$, where $g = 9.80\,\mathrm{m/s^2}$, and using the $-$ or $+$ sign, depending on whether you choose y positive up or down. Remember that v_x never changes throughout the trajectory, and that $v_y = 0$ at the highest point of any trajectory that returns downward. The velocity just before landing is generally not zero.
7. Think for a minute before jumping into the equations. A little planning goes a long way. **Apply** the **relevant equations** (Table 3–2), combining equations if necessary. You may need to combine components of a vector to get magnitude and direction (Eqs. 3–4).

EXAMPLE 3–5 **Driving off a cliff.** A movie stunt driver on a motorcycle speeds horizontally off a 50.0-m-high cliff. How fast must the motorcycle leave the cliff top to land on level ground below, 90.0 m from the base of the cliff where the cameras are? Ignore air resistance.

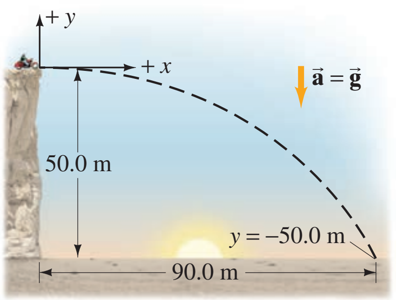


FIGURE 3–22 Example 3–5.

APPROACH We explicitly follow the steps of the Problem Solving Strategy on the previous page.

SOLUTION

1. and **2. Read, choose the object, and draw a diagram**. Our object is the motorcycle and driver, taken as a single unit. The diagram is shown in Fig. 3–22.
3. **Choose a coordinate system**. We choose the y direction to be positive upward, with the top of the cliff as $y_0 = 0$. The x direction is horizontal with $x_0 = 0$ at the point where the motorcycle leaves the cliff.
4. **Choose a time interval**. We choose our time interval to begin $(t = 0)$ just as the motorcycle leaves the cliff top at position $x_0 = 0,\ y_0 = 0$. Our time interval ends just before the motorcycle touches the ground below.
5. **Examine *x* and *y* motions**. In the horizontal (x) direction, the acceleration $a_x = 0$, so the velocity is constant. The value of x when the motorcycle reaches the ground is $x = +90.0\text{ m}$. In the vertical direction, the acceleration is the acceleration due to gravity, $a_y = -g = -9.80\text{ m/s}^2$. The value of y when the motorcycle reaches the ground is $y = -50.0\text{ m}$. The initial velocity is horizontal and is our unknown, v_{x0}; the initial vertical velocity is zero, $v_{y0} = 0$.
6. **List knowns and unknowns**. See the Table in the margin. Note that in addition to not knowing the initial horizontal velocity v_{x0} (which stays constant until landing), we also do not know the time t when the motorcycle reaches the ground.

Known	Unknown
$x_0 = y_0 = 0$	v_{x0}
$x = 90.0\text{ m}$	t
$y = -50.0\text{ m}$	
$a_x = 0$	
$a_y = -g = -9.80\text{ m/s}^2$	
$v_{y0} = 0$	

7. **Apply relevant equations**. The motorcycle maintains constant v_x as long as it is in the air. The time it stays in the air is determined by the y motion—when it reaches the ground. So we first find the time using the y motion, and then use this time value in the x equations. To find out how long it takes the motorcycle to reach the ground below, we use Eq. 2–11b (Tables 3–1 and 3–2) for the vertical (y) direction with $y_0 = 0$ and $v_{y0} = 0$:

$$\begin{aligned} y &= y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \\ &= 0 + \ \ 0 \ \ + \tfrac{1}{2}(-g)t^2 \end{aligned}$$

or

$$y = -\tfrac{1}{2}gt^2.$$

We solve for t and set $y = -50.0\text{ m}$:

$$t = \sqrt{\frac{2y}{-g}} = \sqrt{\frac{2(-50.0\text{ m})}{-9.80\text{ m/s}^2}} = 3.19\text{ s}.$$

To calculate the initial velocity, v_{x0}, we again use Eq. 2–11b, but this time for the horizontal (x) direction, with $a_x = 0$ and $x_0 = 0$:

$$\begin{aligned} x &= x_0 + v_{x0}t + \tfrac{1}{2}a_x t^2 \\ &= 0 + v_{x0}t + 0 \end{aligned}$$

or

$$x = v_{x0}t.$$

Then

$$v_{x0} = \frac{x}{t} = \frac{90.0\text{ m}}{3.19\text{ s}} = 28.2\text{ m/s},$$

which is about 100 km/h (roughly 60 mi/h).

NOTE In the time interval of the projectile motion, the only acceleration is g in the negative y direction. The acceleration in the x direction is zero.

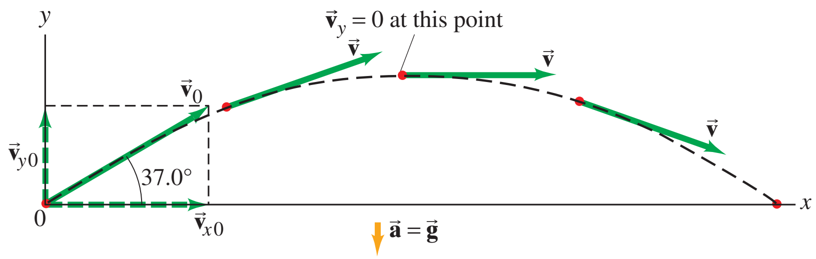


FIGURE 3–23 Example 3–6.

PHYSICS APPLIED
Sports

EXAMPLE 3–6 **A kicked football.** A kicked football leaves the ground at an angle $\theta_0 = 37.0°$ with a velocity of 20.0 m/s, as shown in Fig. 3–23. Calculate (*a*) the maximum height, (*b*) the time of travel before the football hits the ground, and (*c*) how far away it hits the ground. Assume the ball leaves the foot at ground level, and ignore air resistance and rotation of the ball.

APPROACH This may seem difficult at first because there are so many questions. But we can deal with them one at a time. We take the *y* direction as positive upward, and treat the *x* and *y* motions separately. The total time in the air is again determined by the *y* motion. The *x* motion occurs at constant velocity. The *y* component of velocity varies, being positive (upward) initially, decreasing to zero at the highest point, and then becoming negative as the football falls.

SOLUTION We resolve the initial velocity into its components (Fig. 3–23):

$$v_{x0} = v_0 \cos 37.0° = (20.0\,\text{m/s})(0.799) = 16.0\,\text{m/s}$$
$$v_{y0} = v_0 \sin 37.0° = (20.0\,\text{m/s})(0.602) = 12.0\,\text{m/s}.$$

(*a*) To find the maximum height, we consider a time interval that begins just after the football loses contact with the foot until the ball reaches its maximum height. During this time interval, the acceleration is g downward. At the maximum height, the velocity is horizontal (Fig. 3–23), so $v_y = 0$. This occurs at a time given by $v_y = v_{y0} - gt$ with $v_y = 0$ (see Eq. 2–11a in Table 3–2), so $v_{y0} = gt$ and

$$t = \frac{v_{y0}}{g} = \frac{(12.0\,\text{m/s})}{(9.80\,\text{m/s}^2)} = 1.224\,\text{s} \approx 1.22\,\text{s}.$$

From Eq. 2–11b, with $y_0 = 0$, we can solve for y at this time $(t = v_{y0}/g)$:

$$y = v_{y0}t - \tfrac{1}{2}gt^2 = \frac{v_{y0}^2}{g} - \frac{1}{2}\frac{v_{y0}^2}{g} = \frac{v_{y0}^2}{2g} = \frac{(12.0\,\text{m/s})^2}{2(9.80\,\text{m/s}^2)} = 7.35\,\text{m}.$$

The maximum height is 7.35 m. [Solving Eq. 2–11c for y gives the same result.]

(*b*) To find the time it takes for the ball to return to the ground, we consider a different time interval, starting at the moment the ball leaves the foot $(t = 0, y_0 = 0)$ and ending just before the ball touches the ground $(y = 0$ again). We can use Eq. 2–11b with $y_0 = 0$ and also set $y = 0$ (ground level):

$$y = y_0 + v_{y0}t - \tfrac{1}{2}gt^2$$
$$0 = 0 + v_{y0}t - \tfrac{1}{2}gt^2.$$

This equation can be factored:

$$t\left(\tfrac{1}{2}gt - v_{y0}\right) = 0.$$

There are two solutions, $t = 0$ (which corresponds to the initial point, y_0), and

$$t = \frac{2v_{y0}}{g} = \frac{2(12.0\,\text{m/s})}{(9.80\,\text{m/s}^2)} = 2.45\,\text{s},$$

which is the total travel time of the football.

(*c*) The total distance traveled in the *x* direction is found by applying Eq. 2–11b with $x_0 = 0$, $a_x = 0$, $v_{x0} = 16.0$ m/s, and $t = 2.45$ s:

$$x = v_{x0}t = (16.0\,\text{m/s})(2.45\,\text{s}) = 39.2\,\text{m}.$$

PROBLEM SOLVING
Symmetry

NOTE In (*b*), the time needed for the whole trip, $t = 2v_{y0}/g = 2.45$ s, is double the time to reach the highest point, calculated in (*a*). That is, the time to go up equals the time to come back down to the same level (ignoring air resistance).

EXERCISE F In Example 3–6, what is (*a*) the velocity vector at the maximum height, and (*b*) the acceleration vector at maximum height?

In Example 3–6, we treated the football as if it were a particle, ignoring its rotation. We also ignored air resistance. Because air resistance is significant on a football, our results are only estimates (mainly overestimates).

CONCEPTUAL EXAMPLE 3–7 **The wrong strategy.** A boy on a small hill aims his water-balloon slingshot horizontally, straight at a second boy hanging from a tree branch a distance d away, Fig. 3–24. At the instant the water balloon is released, the second boy lets go and falls from the tree, hoping to avoid being hit. Show that he made the wrong move. (He hadn't studied physics yet.) Ignore air resistance.

RESPONSE Both the water balloon and the boy in the tree start falling at the same instant, and in a time t they each fall the same vertical distance $y = \frac{1}{2}gt^2$, much like Fig. 3–19. In the time it takes the water balloon to travel the horizontal distance d, the balloon will have the same y position as the falling boy. Splat. If the boy had stayed in the tree, he would have avoided the humiliation.

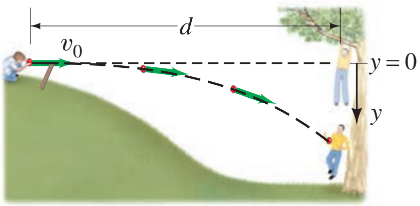


FIGURE 3–24 Example 3–7.

Level Horizontal Range

The total distance the football traveled in Example 3–6 is called the horizontal **range** R. We now derive a formula for the range, which applies to a projectile that lands at the same level it started ($= y_0$): that is, y (final) $= y_0$ (see Fig. 3–25a). Looking back at Example 3–6 part (*c*), we see that $x = R = v_{x0}t$ where (from part *b*) $t = 2v_{y0}/g$. Thus

$$R = v_{x0}t = v_{x0}\left(\frac{2v_{y0}}{g}\right) = \frac{2v_{x0}v_{y0}}{g} = \frac{2v_0^2 \sin\theta_0 \cos\theta_0}{g}, \qquad [y = y_0]$$

where $v_{x0} = v_0 \cos\theta_0$ and $v_{y0} = v_0 \sin\theta_0$. This can be rewritten, using the trigonometric identity $2\sin\theta\cos\theta = \sin 2\theta$ (Appendix A or inside the rear cover):

$$R = \frac{v_0^2 \sin 2\theta_0}{g}. \qquad [\text{only if } y \text{ (final)} = y_0]$$

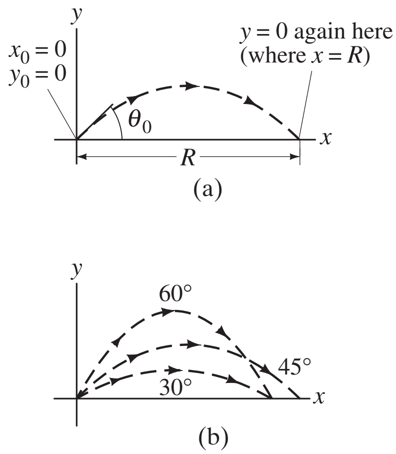
Note that the *maximum* range, for a given initial velocity v_0, is obtained when $\sin 2\theta$ takes on its maximum value of 1.0, which occurs for $2\theta_0 = 90°$; so

$$\theta_0 = 45° \text{ for maximum range, and } R_{max} = v_0^2/g.$$

The maximum range increases by the square of v_0, so doubling the muzzle velocity of a cannon increases its maximum range by a factor of 4.

When air resistance is important, the range is less for a given v_0, and the maximum range is obtained at an angle smaller than 45°.

FIGURE 3–25 (a) The range R of a projectile. (b) There are generally two angles θ_0 that will give the same range. If one angle is θ_{01}, the other is $\theta_{02} = 90° - \theta_{01}$. Example 3–8.



EXAMPLE 3–8 **Range of a cannon ball.** Suppose one of Napoleon's cannons had a muzzle speed, v_0, of 60.0 m/s. At what angle should it have been aimed (ignore air resistance) to strike a target 320 m away?

APPROACH We use the equation just derived for the range, $R = v_0^2 \sin 2\theta_0/g$, with $R = 320$ m.

SOLUTION We solve for $\sin 2\theta_0$ in the range formula:

$$\sin 2\theta_0 = \frac{Rg}{v_0^2} = \frac{(320\text{ m})(9.80\text{ m/s}^2)}{(60.0\text{ m/s})^2} = 0.871.$$

We want to solve for an angle θ_0 that is between 0° and 90°, which means $2\theta_0$ in this equation can be as large as 180°. Thus, $2\theta_0 = 60.6°$ is a solution, so $\theta_0 = 30.3°$. But $2\theta_0 = 180° - 60.6° = 119.4°$ is also a solution (see Appendix A–7), so θ_0 can also be $\theta_0 = 59.7°$. In general we have two solutions (see Fig. 3–25b), which in the present case are given by

$$\theta_0 = 30.3° \quad \text{or} \quad 59.7°.$$

Either angle gives the same range. Only when $\sin 2\theta_0 = 1$ (so $\theta_0 = 45°$) is there a single solution (that is, both solutions are the same).

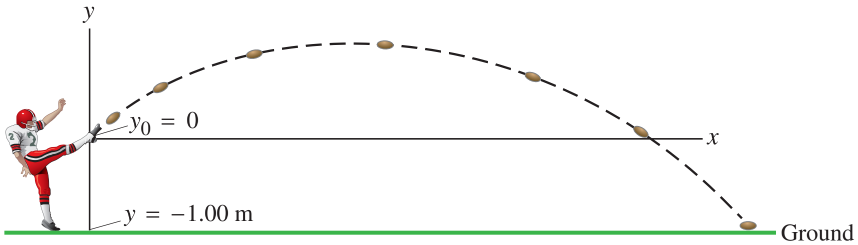


FIGURE 3–26 Example 3–9: the football leaves the punter's foot at $y = 0$, and reaches the ground where $y = -1.00$ m.

PHYSICS APPLIED
Sports

PROBLEM SOLVING
Do not use any formula unless you are sure its range of validity fits the problem; the range formula does not apply here because $y \neq y_0$

EXAMPLE 3–9 **A punt.** Suppose the football in Example 3–6 was punted, and left the punter's foot at a height of 1.00 m above the ground. How far did the football travel before hitting the ground? Set $x_0 = 0$, $y_0 = 0$.

APPROACH The only difference here from Example 3–6 is that the football hits the ground *below* its starting point of $y_0 = 0$. That is, the ball hits the ground at $y = -1.00$ m. See Fig. 3–26. Thus we cannot use the range formula which is valid only if $y\,(\text{final}) = y_0$. As in Example 3–6, $v_0 = 20.0$ m/s, $\theta_0 = 37.0°$.

SOLUTION With $y = -1.00$ m and $v_{y0} = 12.0$ m/s (see Example 3–6), we use the y version of Eq. 2–11b with $a_y = -g$,

$$y = y_0 + v_{y0}t - \tfrac{1}{2}gt^2,$$

and obtain

$$-1.00\text{ m} = 0 + (12.0\text{ m/s})t - \left(4.90\text{ m/s}^2\right)t^2.$$

We rearrange this equation into standard form $\left(ax^2 + bx + c = 0\right)$ so we can use the quadratic formula:

$$\left(4.90\text{ m/s}^2\right)t^2 - (12.0\text{ m/s})t - (1.00\text{ m}) = 0.$$

The quadratic formula (Appendix A–4) gives

$$t = \frac{12.0\text{ m/s} \pm \sqrt{(-12.0\text{ m/s})^2 - 4\left(4.90\text{ m/s}^2\right)(-1.00\text{ m})}}{2\left(4.90\text{ m/s}^2\right)}$$
$$= 2.53\text{ s} \quad \text{or} \quad -0.081\text{ s}.$$

The second solution would correspond to a time prior to the kick, so it doesn't apply. With $t = 2.53$ s for the time at which the ball touches the ground, the horizontal distance the ball traveled is (using $v_{x0} = 16.0$ m/s from Example 3–6):

$$x = v_{x0}t = (16.0\text{ m/s})(2.53\text{ s}) = 40.5\text{ m}.$$

Our assumption in Example 3–6 that the ball leaves the foot at ground level would result in an underestimate of about 1.3 m in the distance our punt traveled.

FIGURE 3–27 Examples of projectile motion: a boy jumping, and glowing lava from the volcano Stromboli.

*3–7 Projectile Motion Is Parabolic

We now show that the path followed by any projectile is a *parabola*, if we can ignore air resistance and can assume that $\vec{\mathbf{g}}$ is constant. To do so, we need to find y as a function of x by eliminating t between the two equations for horizontal and vertical motion (Eq. 2–11b in Table 3–2), and for simplicity we set $x_0 = y_0 = 0$:

$$x = v_{x0}t$$
$$y = v_{y0}t - \tfrac{1}{2}gt^2.$$

From the first equation, we have $t = x/v_{x0}$, and we substitute this into the second one to obtain

$$y = \left(\frac{v_{y0}}{v_{x0}}\right)x - \left(\frac{g}{2v_{x0}^2}\right)x^2. \qquad \textbf{(3–6)}$$

We see that y as a function of x has the form

$$y = Ax - Bx^2,$$

where A and B are constants for any specific projectile motion. This is the standard equation for a parabola. See Figs. 3–17 and 3–27.

The idea that projectile motion is parabolic was, in Galileo's day, at the forefront of physics research. Today we discuss it in Chapter 3 of introductory physics!

*Some Sections of this book, such as this one, may be considered *optional* at the discretion of the instructor. See the Preface for more details.