Reflection from still water, as from a glass mirror, can be analyzed using the ray model of light.

Is this picture right side up, or upside down? How can you tell? What are the clues? Notice the people and position of the Sun. Ray diagrams, which we will learn to draw in this Chapter, can provide the answer. See Example 23–3.

In this first Chapter on light and optics, we use the ray model of light to understand the formation of images by mirrors, both plane and curved (spherical). We also study refraction—how light rays bend when they go from one medium to another—and how, via refraction, images are formed by lenses, which are the crucial part of so many optical instruments.

CHAPTER 23

Light: Geometric Optics

CONTENTS

23–1 The Ray Model of Light
23–2 Reflection; Image Formation by a Plane Mirror
23–3 Formation of Images by Spherical Mirrors
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23–7 Thin Lenses; Ray Tracing
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*23–9 Combinations of Lenses
*23–10 Lensmaker's Equation

CHAPTER-OPENING QUESTIONS—Guess now!

1. A 2.0-m-tall person is standing 2.0 m from a flat vertical mirror staring at her image. What minimum height must the mirror's reflecting glass have if the person is to see her entire body, from the top of her head to her feet?
(a) 0.50 m. **(b)** 1.0 m. **(c)** 1.5 m. **(d)** 2.0 m. **(e)** 2.5 m.

2. The focal length of a lens is
(a) the diameter of the lens.
(b) the thickness of the lens.
(c) the distance from the lens at which incoming parallel rays bend to intersect at a point.
(d) the distance from the lens at which all real images are formed.

The sense of sight is extremely important to us, for it provides us with a large part of our information about the world. How do we see? What is the something called *light* that enters our eyes and causes the sensation of sight? How does light behave so that we can see everything that we do? We saw in Chapter 22 that light can be considered a form of electromagnetic radiation. We now examine the subject of light in detail in the next three Chapters.

We see an object in one of two ways: (1) the object may be a *source* of light, such as a lightbulb, a flame, or a star, in which case we see the light emitted directly from the source; or, more commonly, (2) we see an object by light *reflected* from it.

In the latter case, the light may have originated from the Sun, artificial lights, or a campfire. An understanding of how objects *emit* light was not achieved until the 1920s, and will be discussed in Chapter 27. How light is *reflected* from objects was understood much earlier, and will be discussed in Section 23–2.

23–1 The Ray Model of Light

A great deal of evidence suggests that *light travels in straight lines* under a wide variety of circumstances.† For example, a source of light like the Sun (which at its great distance from us is nearly a "point source") casts distinct shadows, and the beam from a laser pointer appears to be a straight line. In fact, we infer the positions of objects in our environment by assuming that light moves from the object to our eyes in straight-line paths. Our orientation to the physical world is based on this assumption.

This reasonable assumption is the basis of the **ray model** of light. This model assumes that light travels in straight-line paths called light **rays**. Actually, a ray is an idealization; it is meant to represent an extremely narrow beam of light. When we see an object, according to the ray model, light reaches our eyes from each point on the object. Although light rays leave each point in many different directions, normally only a small bundle of these rays can enter the pupil of an observer's eye, as shown in Fig. 23–1. If the person's head moves to one side, a different bundle of rays will enter the eye from each point.

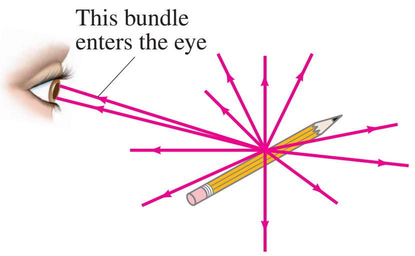


FIGURE 23–1 Light rays come from each single point on an object. A small bundle of rays leaving one point is shown entering a person's eye.

We saw in Chapter 22 that light can be considered as an electromagnetic wave. Although the ray model of light does not deal with this aspect of light (we discuss the wave nature of light in Chapter 24), the ray model has been very successful in describing many aspects of light such as reflection, refraction, and the formation of images by mirrors and lenses. Because these explanations involve straight-line rays at various angles, this subject is referred to as **geometric optics**.

23–2 Reflection; Image Formation by a Plane Mirror

When light strikes the surface of an object, some of the light is reflected. The rest can be absorbed by the object (and transformed to thermal energy) or, if the object is transparent like glass or water, part can be transmitted through. For a very smooth shiny object such as a silvered mirror, over 95% of the light may be reflected.

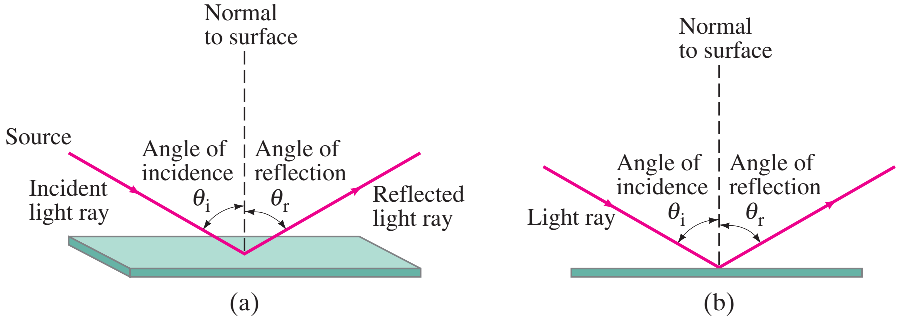


FIGURE 23–2 Law of reflection: (a) shows a 3-D view of an incident ray being reflected at the top of a flat surface; (b) shows a side or "end-on" view, which we will usually use because of its clarity.

When a narrow beam of light strikes a flat surface (Fig. 23–2), we define the **angle of incidence**, $\boldsymbol{\theta_i}$, to be the angle an incident ray makes with the normal (perpendicular) to the surface, and the **angle of reflection**, $\boldsymbol{\theta_r}$, to be the angle the reflected ray makes with the normal. It is found that the *incident and reflected rays lie in the same plane with the normal to the surface*, and that

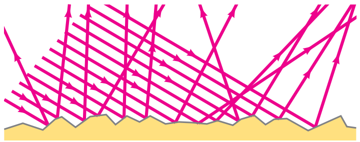
LAW OF REFLECTION

the angle of reflection equals the angle of incidence, $\boldsymbol{\theta_r = \theta_i}$.

This is the **law of reflection**, and it is depicted in Fig. 23–2. It was known to the ancient Greeks, and you can confirm it yourself by shining a narrow flashlight beam or a laser pointer at a mirror in a darkened room.

†In a uniform transparent medium such as air or glass: But not always, such as for nonuniform air that allows optical illusions and mirages which we discuss in Section 24–2 (Fig. 24–4).

FIGURE 23–3 Diffuse reflection from a rough surface.



When light is incident upon a rough surface, even microscopically rough such as this page, it is reflected in many directions, as shown in Fig. 23–3. This is called **diffuse reflection**. The law of reflection still holds, however, at each small section of the surface. Because of diffuse reflection in all directions, an ordinary object can be seen at many different angles by the light reflected from it. When you move your head to the side, different reflected rays reach your eye from each point on the object (such as this page), Fig. 23–4a. Let us compare diffuse reflection to reflection from a mirror, which is known as **specular reflection**. ("Speculum" is Latin for mirror.) When a narrow beam of light shines on a mirror, the light will not reach your eye unless your eye is positioned at just the right place where the law of reflection is satisfied, as shown in Fig. 23–4b. This is what gives rise to the special image-forming properties of mirrors.

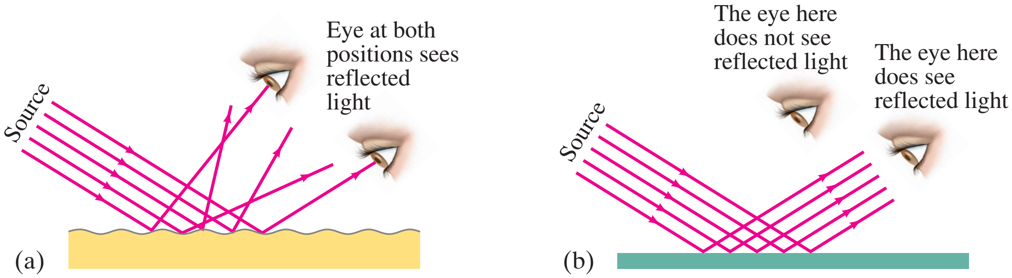


FIGURE 23–4 A narrow beam of light shines on (a) white paper, and (b) a mirror. In part (a), you can see with your eye the white light (and printed words) reflected at various positions because of diffuse reflection. But in part (b), you see the reflected light only when your eye is placed correctly $(\theta_r = \theta_i)$; mirror reflection is also known as specular reflection. (Galileo, using similar arguments, showed that the Moon must have a rough surface rather than a highly polished surface like a mirror, as some people thought.)

EXAMPLE 23–1 **Reflection from flat mirrors.** Two flat mirrors are perpendicular to each other. An incoming beam of light makes an angle of 15° with the first mirror as shown in Fig. 23–5a. What angle will the outgoing beam make with the second mirror?

APPROACH We sketch the path of the beam as it reflects off the two mirrors, and draw the two normals to the mirrors for the two reflections. We use geometry and the law of reflection to find the various angles.

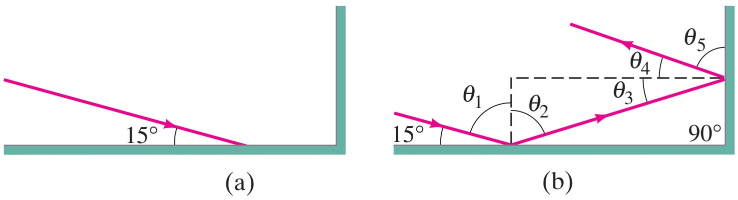


FIGURE 23–5 Example 23–1.

SOLUTION In Fig. 23–5b, $\theta_1 + 15° = 90°$, so $\theta_1 = 75°$; by the law of reflection $\theta_2 = \theta_1 = 75°$ too. Using the fact that the sum of the three angles of a triangle is always 180°, and noting that the two normals to the two mirrors are perpendicular to each other, we have $\theta_2 + \theta_3 + 90° = 180°$. Thus $\theta_3 = 180° - 90° - 75° = 15°$. By the law of reflection, $\theta_4 = \theta_3 = 15°$, so $\theta_5 = 75°$ is the angle the reflected ray makes with the second mirror surface.

NOTE The outgoing ray is parallel to the incoming ray. Reflectors on bicycles, cars, and other applications use this principle.

When you look straight into a mirror, you see what appears to be yourself as well as various objects around and behind you, Fig. 23–6. Your face and the other objects look as if they are in front of you, beyond the mirror. But what you see in the mirror is an **image** of the objects, including yourself, that are in front of the mirror. Also, you don't see yourself as others see you, because left and right appear reversed in the image.

A **plane mirror** is one with a smooth flat reflecting surface. Figure 23–7 shows how an image is formed by a plane mirror according to the ray model. We are viewing the mirror, on edge, in the diagram of Fig. 23–7, and the rays are shown reflecting from the front surface. (Good mirrors are generally made by putting a highly reflective metallic coating on one surface of a very flat piece of glass.) Rays from two different points on an object (the bottle on the left in Fig. 23–7) are shown: two rays are shown leaving from a point on the top of the bottle, and two more from a point on the bottom. Rays leave each point on the object going in many directions (as in Fig. 23–1), but only those that enclose the bundle of rays that enter the eye from each of the two points are shown. Each set of diverging rays that reflect from the mirror and enter the eye *appear to come from a single point* behind the mirror, called the **image point**, as shown by the dashed lines. That is, our eyes and brain interpret any rays that enter an eye as having traveled straight-line paths. The point from which each bundle of rays seems to come is one point on the image. For each point on the object, there is a corresponding image point. (This analysis of how a plane mirror forms an image was published by Kepler in 1604.)

FIGURE 23–6 When you look in a mirror, you see an image of yourself and objects around you. You don't see yourself as others see you, because left and right appear reversed in the image.

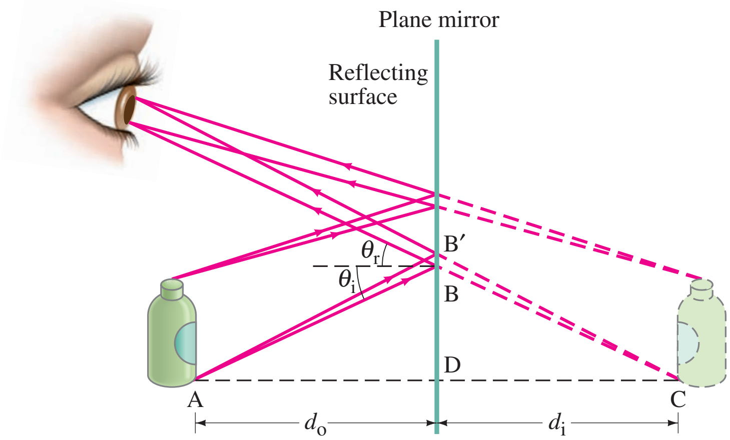


FIGURE 23–7 Formation of a virtual image by a plane mirror. Only the bundle of rays from the top and bottom of the object which reach the eye is shown.

Let us concentrate on the two rays that leave point A on the object in Fig. 23–7, and strike the mirror at points B and B′. We use geometry now, for the rays at B. The angles ADB and CDB are right angles; and because of the law of reflection, $\theta_i = \theta_r$ at point B. Therefore, by geometry, angles ABD and CBD are also equal. The two triangles ABD and CBD are thus congruent, and the length AD = CD. That is, the image appears as far behind the mirror as the object is in front. The **image distance**, d_i (perpendicular distance from mirror to image, Fig. 23–7), equals the **object distance**, d_o (perpendicular distance from object to mirror). From the geometry, we also can see that the height of the image is the same as that of the object.

The light rays do not actually pass through the image location itself in Fig. 23–7. (Note where the red lines are dashed to show they are our projections, not rays.) The image would not appear on paper or film placed at the location of the image. Therefore, it is called a **virtual image**. This is to distinguish it from a **real image** in which the light does pass through the image and which therefore could appear on a white surface, or on film or on an electronic sensor placed at the image position. Our eyes can see both real and virtual images, as long as the diverging rays enter our pupils. We will see that curved mirrors and lenses can form real images, as well as virtual. A movie projector lens, for example, produces a real image that is visible on the screen.

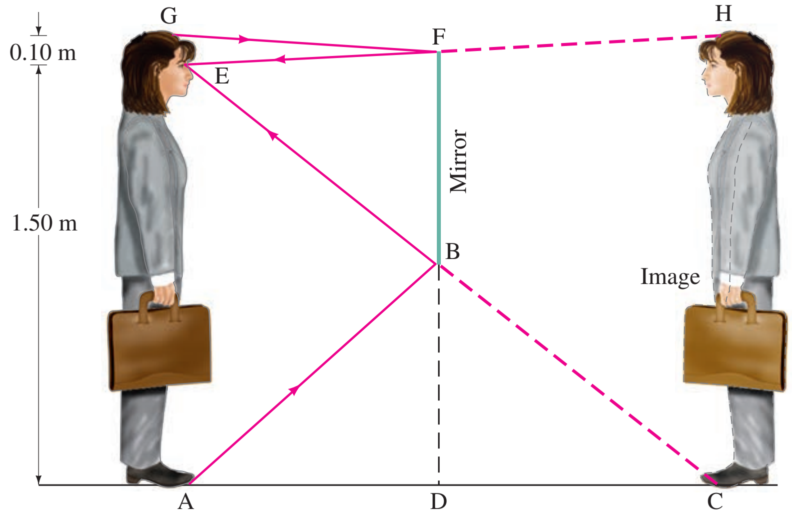


FIGURE 23–8 Seeing oneself in a mirror. Example 23–2.

EXAMPLE 23–2 **How tall must a full-length mirror be?** A woman 1.60 m tall stands in front of a vertical plane mirror. What is the minimum height of the mirror, and how high must its lower edge be above the floor, if she is to be able to see her whole body? Assume that her eyes are 10 cm below the top of her head.

APPROACH For her to see her whole body, light rays from the top of her head (point G) and from the bottom of her foot (A) must reflect from the mirror and enter her eye, Fig. 23–8. We don't show two rays diverging from each point as we did in Fig. 23–7, where we wanted to find where the image is. Now that we know the image is the same distance behind a plane mirror as the object is in front, we only need to show one ray leaving point G (top of head) and one ray leaving point A (her toe), and then use geometry.

PHYSICS APPLIED
How tall a mirror do you need to see a reflection of your entire self?

SOLUTION First consider the ray that leaves her foot at A, reflects at B, and enters the eye at E. The mirror needs to extend no lower than B. The angle of reflection equals the angle of incidence, so the height BD is half of the height AE. Because $AE = 1.60\,\text{m} - 0.10\,\text{m} = 1.50\,\text{m}$, then $BD = 0.75\,\text{m}$. Similarly, if the woman is to see the top of her head, the top edge of the mirror only needs to reach point F, which is 5 cm below the top of her head (half of $GE = 10\,\text{cm}$). Thus, $DF = 1.55\,\text{m}$, and the mirror needs to have a vertical height of only $(1.55\,\text{m} - 0.75\,\text{m}) = 0.80\,\text{m}$. And the mirror's bottom edge must be 0.75 m above the floor.

NOTE We see that a mirror, if positioned at the correct height (as in Fig. 23–8), need be only half as tall as a person for that person to be able to see all of himself or herself.

EXERCISE A Does the result of Example 23–2 depend on your distance from the mirror? (Try it and see, it's fun.)

EXERCISE B Return to Chapter-Opening Question 1, page 644, and answer it again now. Try to explain why you may have answered differently the first time.

CONCEPTUAL EXAMPLE 23–3 **Is the photo upside down?** Close examination of the photograph on the first page of this Chapter reveals that in the top portion, the image of the Sun is seen clearly, whereas in the lower portion, the image of the Sun is partially blocked by the tree branches. Show why the reflection is not the same as the real scene by drawing a sketch of this situation, showing the Sun, the camera, the branch, and two rays going from the Sun to the camera (one direct and one reflected). Is the photograph right side up?

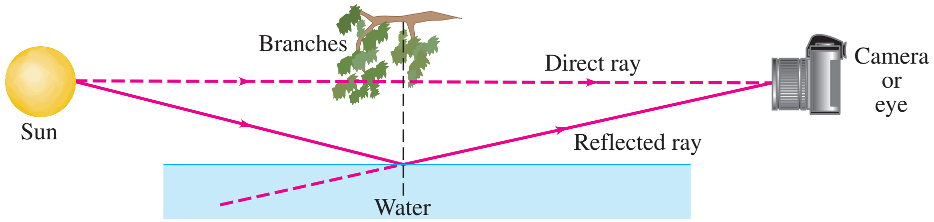


FIGURE 23–9 Example 23–3.

RESPONSE We need to draw two diagrams, one assuming the photo on p. 644 is right side up, and another assuming it is upside down. Figure 23–9 is drawn assuming the photo is upside down. In this case, the Sun blocked by the tree would be the direct view, and the full view of the Sun the reflection: the ray which reflects off the water and into the camera travels at an angle below the branch, whereas the ray that travels directly to the camera passes through the branches. This works. Try to draw a diagram assuming the photo is right side up (thus assuming that the image of the Sun in the reflection is higher above the horizon than it is as viewed directly). It won't work. The photo on p. 644 is upside down.

Also, what about the people in the photo? Try to draw a diagram showing why they don't appear in the reflection. [*Hint*: Assume they are not sitting at the edge of the pool, but back from the edge.] Then try to draw a diagram of the reverse (i.e., assume the photo is right side up so the people are visible only in the reflection). Reflected images are not perfect replicas when different planes (distances) are involved.

23–3 Formation of Images by Spherical Mirrors

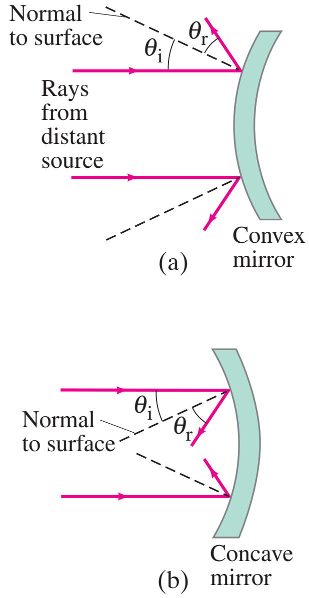


FIGURE 23–10 Mirrors with convex and concave spherical surfaces. Note that $\theta_r = \theta_i$ for each ray. (The dashed lines are perpendicular to the mirror surface at each point shown.)

Reflecting surfaces can also be *curved*, usually *spherical*, which means they form a section of a sphere. A **spherical mirror** is called **convex** if the reflection takes place on the outer surface of the spherical shape so that the center of the mirror surface bulges out toward the viewer, Fig. 23–10a. A mirror is called **concave** if the reflecting surface is on the inner surface of the sphere so that the mirror surface curves away from the viewer (like a "cave"), Fig. 23–10b. Concave mirrors are used as shaving or cosmetic mirrors (**magnifying mirrors**), Fig. 23–11a, because they magnify. Convex mirrors are sometimes used on cars and trucks (rearview mirrors) and in shops (to watch for theft), because they take in a wide field of view, Fig. 23–11b.

(a)

(b)


FIGURE 23–11 (a) A concave cosmetic mirror gives a magnified image. (b) A convex mirror in a store reduces image size and so includes a wide field of view. Note the extreme distortion—this mirror has a large curved surface and does not fit the "paraxial ray" approximation discussed on the next page.

Focal Point and Focal Length

To see how spherical mirrors form images, we first consider an object that is very far from a concave mirror. For a distant object, as shown in Fig. 23–12, the rays from each point on the object that strike the mirror will be nearly parallel. *For an object infinitely far away* (the Sun and stars approach this), *the rays would be precisely parallel.*

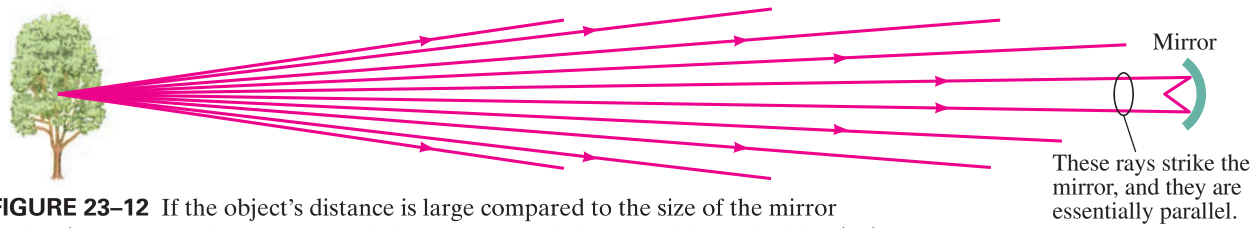


FIGURE 23–12 If the object's distance is large compared to the size of the mirror (or lens), the rays arrive nearly parallel. They are parallel for an object at infinity (∞).

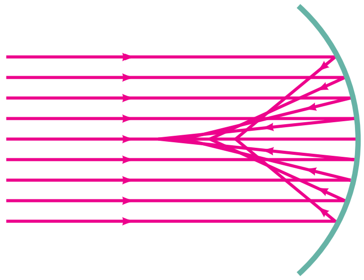


FIGURE 23–13 Parallel rays striking a concave spherical mirror do not intersect (or focus) at precisely a single point. (This "defect" is referred to as "spherical aberration.")

Now consider such parallel rays falling on a concave mirror as in Fig. 23–13. The law of reflection holds for each of these rays at the point each strikes the mirror. As can be seen, they are not all brought to a single point. In order to form a sharp image, the rays must come to a point. Thus a spherical mirror will not make as sharp an image as a plane mirror will. However, as we show below, if the mirror is small compared to its radius of curvature, so that a reflected ray makes only a *small angle* with the incident ray (2θ in Fig. 23–14), then the rays will cross each other at very nearly a single point, or **focus**. In the case shown in Fig. 23–14, the incoming rays are parallel to the **principal axis**, which is defined as the straight line perpendicular to the curved surface at its center (line CA in Fig. 23–14). The point F, where incident parallel rays come to a focus after reflection, is called the **focal point** of the mirror. The distance between F and the center of the mirror, length FA, is called the **focal length**, f, of the mirror. The focal point is also the *image point for an object infinitely far away* along the principal axis. The image of the Sun, for example, would be at F.

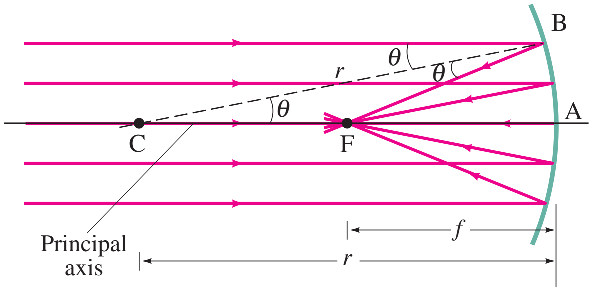


FIGURE 23–14 Rays parallel to the principal axis of a concave spherical mirror come to a focus at F, the focal point, as long as the mirror is small in width as compared to its radius of curvature, r, so that the rays are "paraxial"—that is, make only small angles with the horizontal axis.

Now we will show, for a mirror whose reflecting surface is small compared to its radius of curvature, that the rays very nearly meet at a common point, F, and we will also determine the focal length f. In this approximation, we consider only rays that make a small angle with the principal axis; such rays are called **paraxial rays**, and their angles are exaggerated in Fig. 23–14 to make the labels clear. First we consider a ray that strikes the mirror at B in Fig. 23–14. The point C is the center of curvature of the mirror (the center of the sphere of which the mirror is a part). So the dashed line CB is equal to r, the radius of curvature, and CB is normal to the mirror's surface at B. The incoming ray that hits the mirror at B makes an angle θ with this normal, and hence the reflected ray, BF, also makes an angle θ with the normal (law of reflection). The angle BCF is also θ, as shown. The triangle CBF is isosceles because two of its angles are equal. Thus length $\mathrm{CF} = \mathrm{FB}$. We assume the mirror surface is small compared to the mirror's radius of curvature, so the angles are small, and the length FB is nearly equal to length FA. In this approximation, $\mathrm{FA} = \mathrm{FC}$. But $\mathrm{FA} = f$, the focal length, and $\mathrm{CA} = 2 \times \mathrm{FA} = r$. Thus the focal length is half the radius of curvature:

$$f = \frac{r}{2}. \qquad \text{[spherical mirror]} \quad \textbf{(23–1)}$$

We assumed only that the angle θ was small, so this result applies for all other incident paraxial rays. Thus all paraxial rays pass through the same point F, the focal point.

Since it is only approximately true that the rays come to a perfect focus at F, the more curved the mirror, the worse the approximation (Fig. 23–13) and the more blurred the image. This "defect" of spherical mirrors is called **spherical aberration**; we will discuss it more with regard to lenses in Chapter 25. A **parabolic reflector**, on the other hand, will reflect the rays to a perfect focus. However, because parabolic shapes are much harder to make and thus much more expensive, spherical mirrors are used for most purposes. (Many astronomical telescopes use parabolic reflectors, as do TV satellite dish antennas which concentrate radio waves to nearly a point, Fig. 22–19.) We consider here only spherical mirrors and we will assume that they are small compared to their radius of curvature so that the image is sharp and Eq. 23–1 holds.

Image Formation—Ray Diagrams

We saw that for an object at infinity, the image is located at the focal point of a concave spherical mirror, where $f = r/2$. But where does the image lie for an object not at infinity? First consider the object shown as an arrow in Fig. 23–15a, which is placed between F and C at point O (O for object). Let us determine where the image will be for a given point O′ at the top of the object, by finding the point where rays drawn from the tip of the arrow converge after reflecting from the mirror. To do this we can draw several rays and make sure these reflect from the mirror such that the angle of reflection equals the angle of incidence.

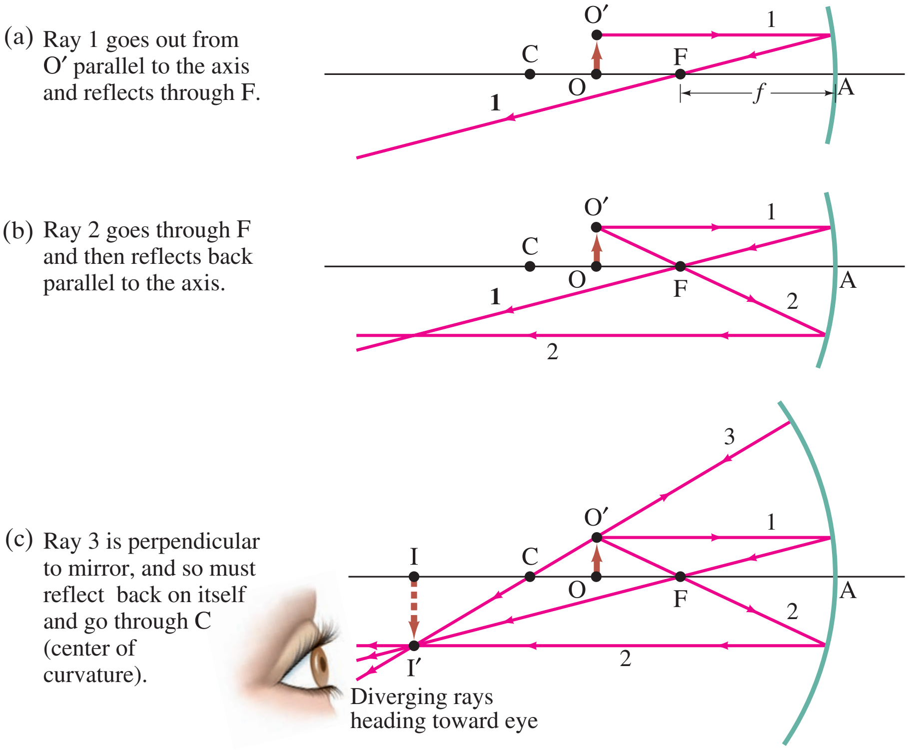


FIGURE 23–15 Rays leave point O′ on the object (an arrow). Shown are the three most useful rays for determining where the image I′ is formed. [Note that our mirror is not small compared to *f*, so our diagram will not give the precise position of the image.]

Many rays could be drawn leaving any point on an object, but determining the image position is faster if we deal with three particular rays. These are the rays labeled 1, 2, and 3 in Fig. 23–15 and we draw them leaving object point O′ as follows:

Ray 1 leaving O′ is drawn parallel to the axis; therefore after reflection it must pass along a line through F, Fig. 23–15a (just as parallel rays did in Fig. 23–14).

Ray 2 leaves O′ and is made to pass through F (Fig. 23–15b); therefore it must reflect so it is parallel to the axis. (In reverse, a parallel ray passes through F.)

Ray 3 is drawn along a radius of the spherical surface (Fig. 23–15c) and is perpendicular to the mirror, so it is reflected back on itself and passes through C, the center of curvature.

RAY DIAGRAM

Finding the image position for a curved mirror

All three rays leave a single point O′ on the object. After reflection from a (small) mirror, the point at which these rays cross is the image point I′. All other rays from the same object point will also pass through this image point. To find the image point for any object point, only these three types of rays need to be drawn. Only two of these rays are needed, but the third serves as a check.

PROBLEM SOLVING

Image point is where reflected rays intersect

We have shown the image point in Fig. 23–15 only for a single point on the object. Other points on the object are imaged nearby. For instance, the bottom of the arrow, on the principal axis at point O, is imaged on the axis at point I. So a complete image of the object is formed (dashed arrow in Fig. 23–15c). Because the light actually passes through the image, this is a **real image** that will appear on a white surface or film placed there. This can be compared to the virtual image formed by a plane mirror (the light does not pass through that image, Fig. 23–7).

The image in Fig. 23–15 can be seen by the eye only when the eye is placed to the left of the image, so that some of the rays *diverging* from each point on the image (as point I′) can enter the eye as shown in Fig. 23–15c (just as in Figs. 23–1 and 23–7).

Mirror Equation and Magnification

Image points can be determined, roughly, by drawing the three rays as just described, Fig. 23–15. But it is difficult to draw small angles for the "paraxial" rays as we assumed. For more accurate results, we now derive an equation that gives the image distance if the object distance and radius of curvature of the mirror are known. To do this, we refer to Fig. 23–16. The **object distance**, d_o, is the distance of the object (point O) from the center of the mirror. The **image distance**, d_i, is the distance of the image (point I) from the center of the mirror. The height of the object OO′ is called h_o and the height of the image, I′I, is h_i. Two rays leaving O′ are shown: O′FBI′ (same as ray 2 in Fig. 23–15) and O′AI′, which is a fourth type of ray that reflects at the center of the mirror and can also be used to find an image point.

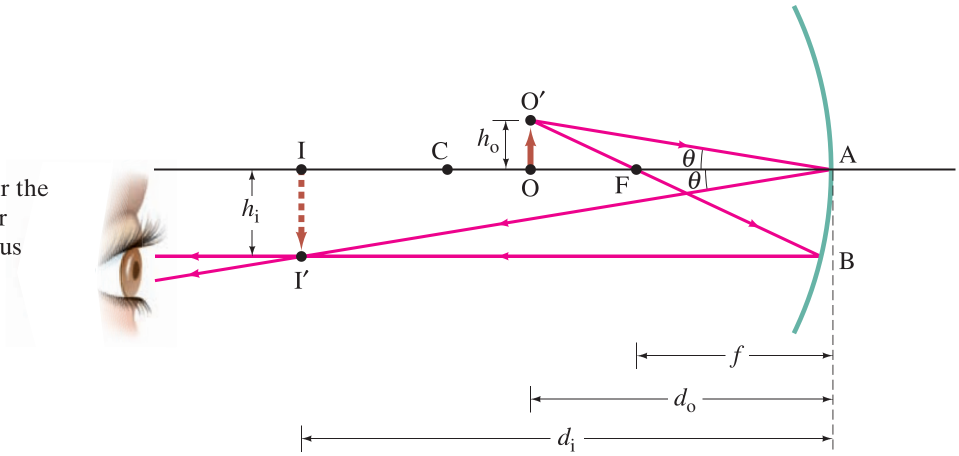


FIGURE 23–16 Diagram for deriving the mirror equation. For the derivation, we assume the mirror size is small compared to its radius of curvature.

The ray O′AI′ obeys the law of reflection, so the two right triangles O′AO and I′AI are similar. Therefore, we have

$$\frac{h_o}{h_i} = \frac{d_o}{d_i}.$$

For the other ray shown, O′FBI′, the triangles O′FO and AFB are also similar because the angles at F are equal and we use the approximation AB $= h_i$ (mirror small compared to its radius). Furthermore FA $= f$, the focal length of the mirror, so

$$\frac{h_o}{h_i} = \frac{\mathrm{OF}}{\mathrm{FA}} = \frac{d_o - f}{f}.$$

The left sides of the two preceding expressions are the same, so we can equate the right sides:

$$\frac{d_o}{d_i} = \frac{d_o - f}{f}.$$

We now divide both sides by d_o and rearrange to obtain

Mirror equation

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}. \qquad \textbf{(23–2)}$$

This is the equation we were seeking. It is called the **mirror equation** and relates the object and image distances to the focal length f (where $f = r/2$).

The mirror equation also holds for a plane mirror: the focal length is $f = r/2 = \infty$ (Eq. 23–1), and Eq. 23–2 gives $d_i = -d_o$.

The **magnification**, m, of a mirror is defined as the height of the image divided by the height of the object. From our first set of similar triangles in Fig. 23–16, or the first equation just below Fig. 23–16, we can write:

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}. \qquad \textbf{(23–3)}$$

The minus sign in Eq. 23–3 is inserted as a convention. Indeed, we must be careful about the signs of all quantities in Eqs. 23–2 and 23–3. Sign conventions are chosen so as to give the correct locations and orientations of images, as predicted by ray diagrams. The **sign conventions** we use are:

PROBLEM SOLVING
Sign conventions for mirrors

1. the image height h_i is positive if the image is upright, and negative if inverted, relative to the object (assuming h_o is taken as positive);
2. d_i or d_o is positive if image or object is in front of the mirror (as in Fig. 23–16); if either image or object is behind the mirror, the corresponding distance is negative. [An example of $d_i < 0$ can be seen in Fig. 23–17, Example 23–6.]†

Thus the magnification (Eq. 23–3) is positive for an upright image and negative for an inverted image (upside down). We summarize sign conventions more fully in the Problem Solving Strategy following our discussion of convex mirrors later in this Section.

Concave Mirror Examples

EXAMPLE 23–4 Image in a concave mirror. A 1.50-cm-high object is placed 20.0 cm from a concave mirror with radius of curvature 30.0 cm. Determine (*a*) the position of the image, and (*b*) its size.

APPROACH We determine the focal length from the radius of curvature (Eq. 23–1), $f = r/2 = 15.0$ cm. The ray diagram is basically the same as Fig. 23–16, since the object is between F and C. The position and size of the image are found from Eqs. 23–2 and 23–3.

SOLUTION Referring to Fig. 23–16, we have $\mathrm{CA} = r = 30.0$ cm, $\mathrm{FA} = f = 15.0$ cm, and $\mathrm{OA} = d_o = 20.0$ cm.

(*a*) We start with the mirror equation, Eq. 23–2, rearranging it (subtracting $(1/d_o)$ from both sides):

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{15.0\text{ cm}} - \frac{1}{20.0\text{ cm}} = 0.0167\text{ cm}^{-1}.$$

CAUTION
Remember to take the reciprocal

So $d_i = 1/(0.0167\text{ cm}^{-1}) = 60.0$ cm. Because d_i is positive, the image is 60.0 cm in front of the mirror, on the same side as the object.

(*b*) From Eq. 23–3, the magnification is

$$m = -\frac{d_i}{d_o} = -\frac{60.0\text{ cm}}{20.0\text{ cm}} = -3.00.$$

The image is 3.0 times larger than the object, and its height is

$$h_i = mh_o = (-3.00)(1.5\text{ cm}) = -4.5\text{ cm}.$$

The minus sign reminds us that the image is inverted, as shown in Fig. 23–16.

NOTE When an object is further from a concave mirror than the focal point, we can see from Fig. 23–15 or 23–16 that the image is always inverted and real.

CONCEPTUAL EXAMPLE 23–5 Reversible rays. If the object in Example 23–4 is placed instead where the image is (see Fig. 23–16), where will the new image be?

RESPONSE The mirror equation is *symmetric* in d_o and d_i. Thus the new image will be where the old object was. Indeed, in Fig. 23–16 we need only reverse the direction of the rays to get our new situation.

†d_o is always positive for a real object; $d_o < 0$ can happen only if the object is an image formed by another mirror or lens—see Example 23–16.

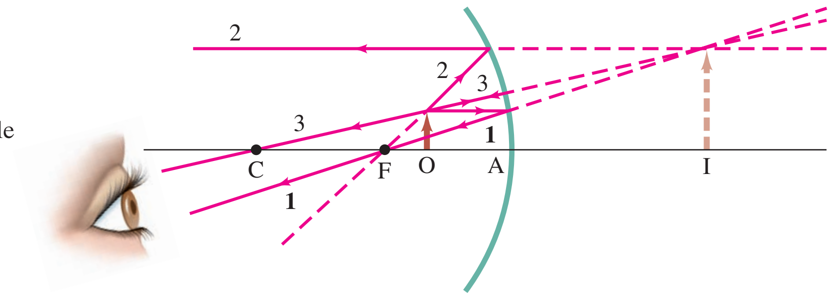


FIGURE 23–17 Object placed within the focal point F. The image is *behind* the mirror and is *virtual*, Example 23–6. [Note that the vertical scale (height of object $= 1.0$ cm) is different from the horizontal $(OA = 10.0\text{ cm})$ for ease of drawing, and reduces the precision of the drawing.]

EXAMPLE 23–6 Object closer to concave mirror than focal point. A 1.00-cm-high object is placed 10.0 cm from a concave mirror whose radius of curvature is 30.0 cm. (*a*) Draw a ray diagram to locate (approximately) the position of the image. (*b*) Determine the position of the image and the magnification analytically.

APPROACH We draw the ray diagram using the rays as in Fig. 23–15, page 651. An analytic solution uses Eqs. 23–1, 23–2, and 23–3.

SOLUTION (*a*) Since $f = r/2 = 15.0\text{ cm}$, the object is between the mirror and the focal point. We draw the three rays as described earlier (Fig. 23–15); they are shown leaving the tip of the object in Fig. 23–17. Ray 1 leaves the tip of our object heading toward the mirror parallel to the axis, and reflects through F. Ray 2 cannot head toward F because it would not strike the mirror; so ray 2 must point as if it started at F (dashed line in Fig. 23–17) and heads to the mirror, and then is reflected parallel to the principal axis. Ray 3 is perpendicular to the mirror and reflects back on itself. The rays reflected from the mirror diverge and so never meet at a point. They appear to be coming from a point behind the mirror (dashed lines). This point locates the image of the tip of the arrow. The image is thus behind the mirror and is *virtual*.

(*b*) We use Eq. 23–2 to find d_i when $d_o = 10.0\text{ cm}$:

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{15.0\text{ cm}} - \frac{1}{10.0\text{ cm}} = \frac{2-3}{30.0\text{ cm}} = -\frac{1}{30.0\text{ cm}}.$$

Therefore, $d_i = -30.0\text{ cm}$. The minus sign means the image is behind the mirror, which our diagram also showed us. The magnification is $m = -d_i/d_o = -(-30.0\text{ cm})/(10.0\text{ cm}) = +3.00$. So the image is 3.00 times larger than the object. The plus sign indicates that the image is upright (same as object), which is consistent with the ray diagram, Fig. 23–17.

NOTE The image distance cannot be obtained accurately by measuring on Fig. 23–17, because our diagram violates the paraxial ray assumption (we draw rays at steeper angles to make them clearly visible).

PHYSICS APPLIED
Magnifying mirror: Seeing yourself upright and magnified in a concave mirror

NOTE When the object is located inside the focal point of a concave mirror $(d_o < f)$, the image is always upright and virtual. If the object O in Fig. 23–17 is you, you see yourself clearly, because the reflected rays at point O (you) are diverging. Your image is upright and enlarged. This is how a shaving or cosmetic mirror is used—you must place your head closer to the mirror than the focal point if you are to see yourself right-side up (see the photograph, Fig. 23–11a). [If the object is *beyond* the focal point, as in Fig. 23–15, the image is real and inverted: upside down—and hard to use!]

Seeing the Image; Seeing Yourself

For a person's eye to see a sharp image, the eye must be at a place where it intercepts diverging rays from points on the image, as is the case for the eye's position in Figs. 23–15, 23–16, and 23–17. When we look at normal objects, we always detect rays diverging toward the eye as shown in Fig. 23–1. (Or, for very distant objects like stars, the rays become essentially parallel, as in Fig. 23–12.)

If you placed your eye between points O and I in Fig. 23–16, for example, *converging* rays from the object OO′ would enter your eye and the lens of your eye could not bring them to a focus; you would see a blurry image or no perceptible image at all. [We will discuss the eye more in Chapter 25.]

If *you* are the object OO′ in Fig. 23–16, situated between F and C, and are trying to see yourself in the mirror, you would see a blur; but the person whose eye is shown in Fig. 23–16 could see you clearly. If you are to the left of C in Fig. 23–16, where $d_o > 2f$, you can see yourself clearly, but upside down. Why? Because then the rays arriving from the image will be *diverging* at your position (Fig. 23–18), and your eye can then focus them. You can also see yourself clearly, and right side up, if you are closer to the mirror than its focal point $(d_o < f)$, as we saw in Example 23–6, Fig. 23–17.

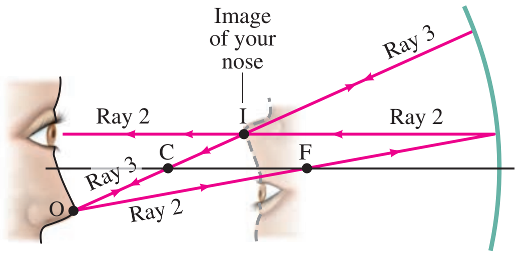


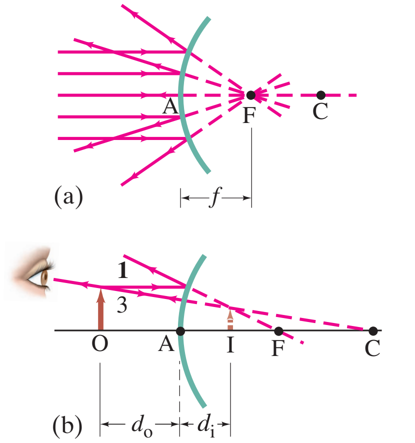
FIGURE 23–18 You can see a clear inverted image of your face in a concave mirror when you are beyond C $(d_o > 2f)$, because the rays that arrive at your eye are *diverging*. Standard rays 2 and 3 are shown leaving point O on your nose. Ray 2 (and other nearby rays) enters your eye. Notice that rays are diverging as they move to the left of image point I.

Convex Mirrors

The analysis used for concave mirrors can be applied to **convex** mirrors. Even the mirror equation (Eq. 23–2) holds for a convex mirror, although the quantities involved must be carefully defined. Figure 23–19a shows parallel rays falling on a convex mirror. Again spherical aberration is significant (Fig. 23–13), unless we assume the mirror is small compared to its radius of curvature. The reflected rays diverge, but seem to come from point F behind the mirror, Fig. 23–19a. This is the **focal point**, and its distance from the center of the mirror (point A) is the **focal length**, *f*. The equation $f = r/2$ is valid also for a convex mirror. We see that an object at infinity produces a virtual image in a convex mirror. Indeed, no matter where the object is placed on the reflecting side of a convex mirror, the image will be virtual and upright, as indicated in Fig. 23–19b. To find the image we draw rays 1 and 3 according to the rules used before on the concave mirror, as shown in Fig. 23–19b. Note that although rays 1 and 3 don't actually pass through points F and C, the line along which each is drawn does (shown dashed).

The mirror equation, Eq. 23–2, holds for convex mirrors but the focal length *f* and radius of curvature must be considered negative. The proof is left as a Problem. It is also left as a Problem to show that Eq. 23–3 for the magnification is also valid.

FIGURE 23–19 Convex mirror: (a) the focal point is at F, behind the mirror; (b) the image I of the object at O is virtual, upright, and smaller than the object. [Not to scale for Example 23–7.]



PROBLEM SOLVING

Spherical Mirrors

1. Always **draw a ray diagram** even though you are going to make an analytic calculation—the diagram serves as a check, even if not precise. From one point on the object, draw at least two, preferably three, of the easy-to-draw rays using the rules described in Fig. 23–15. The image point is where the reflected rays intersect (real image) or appear to intersect (virtual).
2. Apply the **mirror equation**, Eq. 23–2, and the **magnification equation**, Eq. 23–3. It is crucially important to follow the sign conventions—see the next point.
3. **Sign Conventions**
   (a) When the object, image, or focal point is on the reflecting side of the mirror (on the left in our drawings), the corresponding distance is positive. If any of these points is behind the mirror (on the right) the corresponding distance is negative.†
   (b) The image height h_i is positive if the image is upright, and negative if inverted, relative to the object (h_o is always taken as positive).
4. **Check** that the analytic solution is consistent with the ray diagram.

†Object distances are positive for material objects, but can be negative in systems with more than one mirror or lens—see Section 23–9.

PHYSICS APPLIED
Convex rearview mirror

FIGURE 23–20 Example 23–7.

EXAMPLE 23–7 **Convex rearview mirror.** An external rearview car mirror is convex with a radius of curvature of 16.0 m (Fig. 23–20). Determine the location of the image and its magnification for an object 10.0 m from the mirror.

APPROACH We follow the steps of the Problem Solving Strategy explicitly.

SOLUTION

1. **Draw a ray diagram.** The ray diagram will be like Fig. 23–19b, but the large object distance $(d_o = 10.0\,\text{m})$ makes a precise drawing difficult. We have a convex mirror, so r is negative by convention.
2. **Mirror and magnification equations.** The center of curvature of a convex mirror is behind the mirror, as is its focal point, so we set $r = -16.0\,\text{m}$ so that the focal length is $f = r/2 = -8.0\,\text{m}$. The object is in front of the mirror, $d_o = 10.0\,\text{m}$. Solving the mirror equation, Eq. 23–2, for $1/d_i$ gives
$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{-8.0\,\text{m}} - \frac{1}{10.0\,\text{m}} = \frac{-10.0 - 8.0}{80.0\,\text{m}} = -\frac{18}{80.0\,\text{m}}.$$
Thus $d_i = -80.0\,\text{m}/18 = -4.4\,\text{m}$. Equation 23–3 gives the magnification
$$m = -\frac{d_i}{d_o} = -\frac{(-4.4\,\text{m})}{(10.0\,\text{m})} = +0.44.$$
3. **Sign conventions.** The image distance is negative, $-4.4\,\text{m}$, so the image is *behind* the mirror. The magnification is $m = +0.44$, so the image is *upright* (same orientation as object, which is useful) and about half what it would be in a plane mirror.
4. **Check.** Our results are consistent with Fig. 23–19b.

Convex rearview mirrors on vehicles sometimes come with a warning that objects are closer than they appear in the mirror. The fact that d_i may be smaller than d_o (as in Example 23–7) seems to contradict this observation. The real reason the object seems farther away is that its image in the convex mirror is *smaller* than it would be in a plane mirror, and we judge distance of ordinary objects such as other cars mostly by their size.

23–4 Index of Refraction

We saw in Chapter 22 that the speed of light in vacuum (like other EM waves) is
$$c = 2.99792458 \times 10^8\,\text{m/s},$$
which is usually rounded off to
$$3.00 \times 10^8\,\text{m/s}$$
when extremely precise results are not required.

In air, the speed is only slightly less. In other transparent materials, such as glass and water, the speed is always less than that in vacuum. For example, in water light travels at about $\frac{3}{4}c$. The ratio of the speed of light in vacuum to the speed v in a given material is called the **index of refraction**, n, of that material:
$$n = \frac{c}{v}. \qquad \textbf{(23–4)}$$
The index of refraction is never less than 1, and values for various materials are given in Table 23–1. For example, since $n = 1.33$ for water, the speed of light in water is
$$v = \frac{c}{n} = \frac{(3.00 \times 10^8\,\text{m/s})}{1.33} = 2.26 \times 10^8\,\text{m/s}.$$
As we shall see later, n varies somewhat with the wavelength of the light—except in vacuum—so a particular wavelength is specified in Table 23–1, that of yellow light with wavelength $\lambda = 589\,\text{nm}$.

That light travels more slowly in matter than in vacuum can be explained at the atomic level as being due to the absorption and reemission of light by atoms and molecules of the material.

TABLE 23–1 Indices of Refraction†

Material	$n = \frac{c}{v}$
Vacuum	1.0000
Air (at STP)	1.0003
Water	1.33
Ethyl alcohol	1.36
Glass	
Fused quartz	1.46
Crown glass	1.52
Light flint	1.58
Plastic	
Acrylic, Lucite, CR-39	1.50
Polycarbonate	1.59
"High-index"	1.6–1.7
Sodium chloride	1.53
Diamond	2.42

† $\lambda = 589\,\text{nm}$.

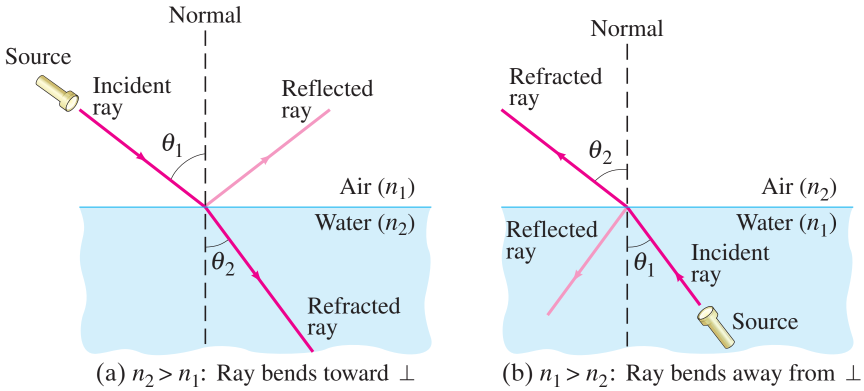


FIGURE 23–21 Refraction. (a) Light refracted when passing from air (n_1) into water (n_2): $n_2 > n_1$. (b) Light refracted when passing from water (n_1) into air (n_2): $n_1 > n_2$.

23–5 Refraction: Snell's Law

When light passes from one transparent medium into another with a different index of refraction, some or all of the incident light is reflected at the boundary. The rest passes into the new medium. If a ray of light is incident at an angle to the surface (other than perpendicular), the ray changes direction as it enters the new medium. This change in direction, or bending, of the light ray is called **refraction**.

Figure 23–21a shows a ray passing from air into water. Angle θ_1 is the angle the incident ray makes with the normal (perpendicular) to the surface and is called the **angle of incidence**. Angle θ_2 is the **angle of refraction**, the angle the refracted ray makes with the normal to the surface. Notice that the ray bends toward the normal when entering the water. This is always the case when the ray enters a medium where the speed of light is *less* (and the index of refraction is greater, Eq. 23–4). If light travels from one medium into a second where its speed is *greater*, the ray bends away from the normal; this is shown in Fig. 23–21b for a ray traveling from water to air.

CAUTION
Angles of incidence and refraction are measured from the perpendicular, not from the surface

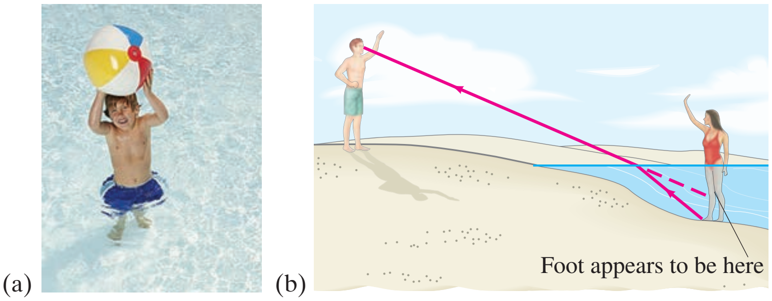


FIGURE 23–22 (a) Photograph, and (b) ray diagram showing why a person's legs look shorter standing in water: a ray from the bather's foot to the observer's eye bends at the water's surface, and our brain interprets the light as traveling in a straight line, from higher up (dashed line).

Refraction is responsible for a number of common optical illusions. For example, a person standing in waist-deep water appears to have shortened legs (Fig. 23–22). The rays leaving the person's foot are bent at the surface. The observer's brain assumes the rays to have traveled a straight-line path (dashed red line), and so the feet appear to be higher than they really are. Similarly, when you put a straw in water, it appears to be bent (Fig. 23–23). This also means that water is deeper than it appears.

FIGURE 23–23 A straw in water looks bent even when it isn't.

Snell's Law

The angle of refraction depends on the speed of light in the two media and on the incident angle. An analytic relation between θ_1 and θ_2 in Fig. 23–21 was arrived at experimentally about 1621 by Willebrord Snell (1591–1626). Known as **Snell's law**, it is written:

SNELL'S LAW (LAW OF REFRACTION)

$$n_1 \sin\theta_1 = n_2 \sin\theta_2. \qquad \textbf{(23–5)}$$

θ_1 is the angle of incidence and θ_2 is the angle of refraction; n_1 and n_2 are the respective indices of refraction in the materials. See Fig. 23–21. The incident and refracted rays lie in the same plane, which also includes the perpendicular to the surface. Snell's law is the **law of refraction**. (Snell's law was derived in Section 11–13 for water waves where Eq. 11–20 is just a combination of Eqs. 23–5 and 23–4, and we derive it again in Chapter 24 using the wave theory of light.)

Snell's law shows that if $n_2 > n_1$, then $\theta_2 < \theta_1$. Thus, if light enters a medium where n is greater (and its speed is less), the ray is bent toward the normal. And if $n_2 < n_1$, then $\theta_2 > \theta_1$, so the ray bends away from the normal. See Fig. 23–21.

EXERCISE C Light passes from a medium with $n = 1.3$ (water) into a medium with $n = 1.5$ (glass). Is the light bent toward or away from the perpendicular to the interface?

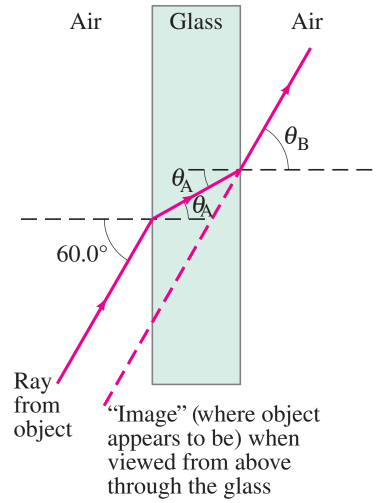


FIGURE 23–24 Light passing through a piece of glass (Example 23–8).

EXAMPLE 23–8 **Refraction through flat glass.** Light traveling in air strikes a flat piece of uniformly thick glass at an incident angle of 60.0°, as shown in Fig. 23–24. If the index of refraction of the glass is 1.50, (*a*) what is the angle of refraction θ_A in the glass; (*b*) what is the angle θ_B at which the ray emerges from the glass?

APPROACH We apply Snell's law twice: at the first surface, where the light enters the glass, and again at the second surface where it leaves the glass and enters the air.

SOLUTION (*a*) The incident ray is in air, so $n_1 = 1.00$ and $n_2 = 1.50$. Applying Snell's law where the light enters the glass $(\theta_1 = 60.0°,\ \theta_2 = \theta_A)$ gives

$$(1.00)\sin 60.0° = (1.50)\sin\theta_A$$

or

$$\sin\theta_A = \frac{1.00}{1.50}\sin 60.0° = 0.5774,$$

and $\theta_A = 35.3°$.

(*b*) Since the faces of the glass are parallel, the incident angle at the second surface is also θ_A (geometry), so $\sin\theta_A = 0.5774$. At this second interface, $n_1 = 1.50$ and $n_2 = 1.00$. Thus the ray re-enters the air at an angle θ_B given by

$$\sin\theta_B = \frac{1.50}{1.00}\sin\theta_A = 0.866,$$

and $\theta_B = 60.0°$. The direction of a light ray is thus unchanged by passing through a flat piece of glass of uniform thickness.

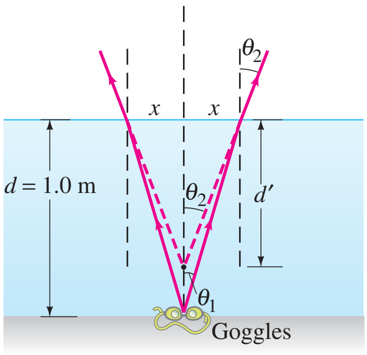
NOTE This result is valid for any angle of incidence. The ray is displaced slightly to one side, however. You can observe this by looking through a piece of glass (near its edge) at some object and then moving your head to the side slightly so that you see the object directly. It "jumps."

CAUTION (real life)
Water is deeper than it looks

EXAMPLE 23–9 **Apparent depth of a pool.** A swimmer has dropped her goggles to the bottom of a pool at the shallow end, marked as 1.0 m deep. But the goggles don't look that deep. Why? How deep do the goggles appear to be when you look straight down into the water?

APPROACH We draw a ray diagram showing two rays going upward from a point on the goggles at a small angle, and being refracted at the water's (flat) surface, Fig. 23–25. The two rays traveling upward from the goggles are refracted *away* from the normal as they exit the water, and so appear to be diverging from a point above the goggles (dashed lines), which is why the water seems less deep than it actually is. We are looking straight down, so all angles are small (but exaggerated in Fig. 23–25 for clarity).

FIGURE 23–25 Example 23–9.



SOLUTION To calculate the apparent depth d' (Fig. 23–25), given a real depth $d = 1.0$ m, we use Snell's law with $n_1 = 1.33$ for water and $n_2 = 1.0$ for air:

$$\sin\theta_2 = n_1 \sin\theta_1 .$$

We are considering only small angles, so $\sin\theta \approx \tan\theta \approx \theta$, with θ in radians. So Snell's law becomes

$$\theta_2 \approx n_1\theta_1 .$$

From Fig. 23–25, we see that $\theta_2 \approx \tan\theta_2 = x/d'$ and $\theta_1 \approx \tan\theta_1 = x/d$. Putting these into Snell's law, $\theta_2 \approx n_1\theta_1$, we get

$$\frac{x}{d'} \approx n_1\frac{x}{d}$$

or

$$d' \approx \frac{d}{n_1} = \frac{1.0\text{ m}}{1.33} = 0.75\text{ m}.$$

The pool seems only three-fourths as deep as it actually is.

NOTE Water in general is deeper than it looks—a useful safety guideline.

23–6 Total Internal Reflection; Fiber Optics

When light passes from one material into a second material where the index of refraction is less (say, from water into air), the refracted light ray bends away from the normal, as for rays I and J in Fig. 23–26. At a particular incident angle, the angle of refraction will be 90°, and the refracted ray would skim the surface (ray K).

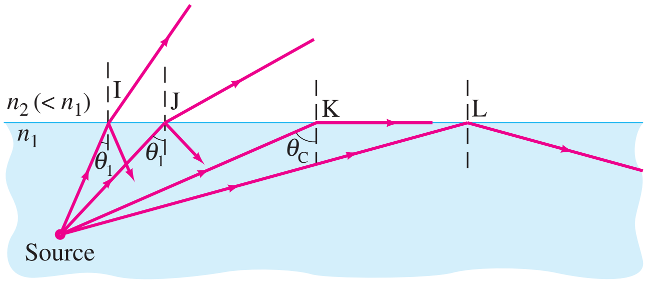


FIGURE 23–26 Since $n_2 < n_1$, light rays are totally internally reflected if the incident angle $\theta_1 > \theta_C$, as for ray L. If $\theta_1 < \theta_C$, as for rays I and J, only a part of the light is reflected, and the rest is refracted.

The incident angle at which this occurs is called the **critical angle**, θ_C. From Snell's law, θ_C is given by

$$\sin\theta_C = \frac{n_2}{n_1}\sin 90° = \frac{n_2}{n_1}. \qquad \textbf{(23–6)}$$

For any incident angle less than θ_C, there will be a refracted ray, although part of the light will also be reflected at the boundary. However, for incident angles θ_1 greater than θ_C, Snell's law would tell us that $\sin\theta_2$ $(= n_1 \sin\theta_1/n_2)$ would be greater than 1.00 when $n_2 < n_1$. Yet the sine of an angle can never be greater than 1.00. In this case there is no refracted ray at all, and *all of the light is reflected*, as for ray L in Fig. 23–26. This effect is called **total internal reflection**. Total internal reflection occurs only when light strikes a boundary where the medium beyond has a *lower* index of refraction.

CAUTION
Total internal reflection (occurs only if refractive index is smaller *beyond boundary)*

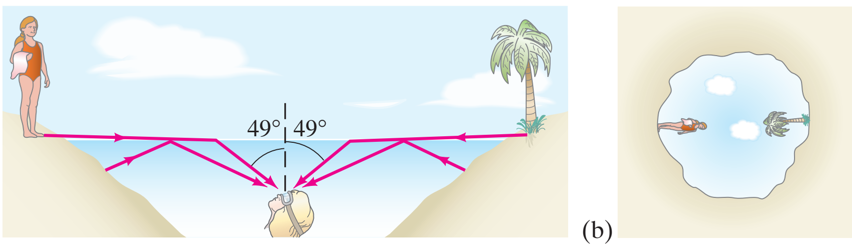
CONCEPTUAL EXAMPLE 23–10 **View up from under water.** Describe what a person would see who looked up at the world from beneath the perfectly smooth surface of a lake or swimming pool.

RESPONSE For an air–water interface, the critical angle is given by

$$\sin\theta_C = \frac{1.00}{1.33} = 0.750.$$

Therefore, $\theta_C = 49°$. Thus the person would see the outside world compressed into a circle whose edge makes a 49° angle with the vertical. Beyond this angle, the person would see reflections from the sides and bottom of the lake or pool (Fig. 23–27).

EXERCISE D Light traveling in air strikes a glass surface with $n = 1.48$. For what range of angles will total internal reflection occur?



(a) (b)

FIGURE 23–27 (a) Light rays entering submerged person's eye, and (b) view looking upward from beneath the water (the surface of the water must be very smooth). Example 23–10.

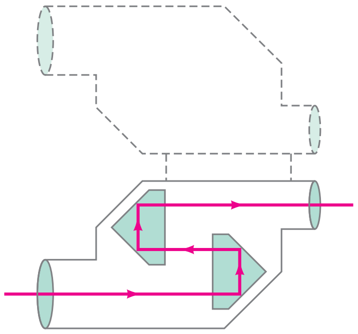


FIGURE 23–28 Total internal reflection of light by prisms in binoculars.

Many optical instruments, such as binoculars, use total internal reflection within a prism to reflect light. The advantage is that very nearly 100% of the light is reflected, whereas even the best mirrors reflect somewhat less than 100%. Thus the image is brighter, especially after several reflections. For glass with $n = 1.50$, $\theta_C = 41.8°$. Therefore, 45° prisms will reflect all the light internally, if oriented as shown in the binoculars of Fig. 23–28.

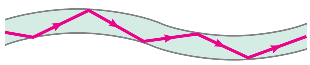
EXERCISE E What would happen if we immersed the 45° glass prisms in Fig. 23–28 in water?

Fiber Optics; Medical Instruments

PHYSICS APPLIED
Fiber optics in communications and medicine—bronchoscopes, colonoscopes, endoscopes

Total internal reflection is the principle behind **fiber optics**. Glass and plastic fibers as thin as a few micrometers in diameter are commonly used. A bundle of such slender transparent fibers is called a **light pipe** or **fiber-optic cable**. Light† can be transmitted along the fiber with almost no loss because of total internal reflection. Figure 23–29 shows how light traveling down a thin fiber makes only glancing collisions with the walls so that total internal reflection occurs. Even if the light pipe is bent gently into a complicated shape, the critical angle still won't be exceeded, so light is transmitted practically undiminished to the other end. Very small losses do occur, mainly by reflection at the ends and absorption within the fiber.

FIGURE 23–29 Light reflected totally at the interior surface of a glass or transparent plastic fiber.



Important applications of fiber-optic cables are in communications and medicine. They are used in place of wire to carry telephone calls, video signals, and computer data. The signal is a modulated light beam (a light beam whose intensity can be varied) and data is transmitted at a much higher rate and with less loss and less interference than an electrical signal in a copper wire. Fibers have been developed that can support over one hundred separate wavelengths, each modulated to carry more than 10 gigabits $\left(10^{10}\text{ bits}\right)$ of information per second. That amounts to a terabit $\left(10^{12}\text{ bits}\right)$ per second for one hundred wavelengths.

The use of fiber optics to transmit a clear picture is particularly useful in medicine, Fig. 23–30. For example, a patient's lungs can be examined by inserting a fiber-optic cable known as a bronchoscope through the mouth and down the bronchial tube. Light is sent down an outer set of fibers to illuminate the lungs. The reflected light returns up a central core set of fibers. Light directly in front of each fiber travels up that fiber. At the opposite end, a viewer sees a series of bright and dark spots, much like a TV screen—that is, a picture of what lies at the opposite end. Lenses are used at each end of the cable. The image may be viewed directly or on a monitor screen or film. The fibers must be optically insulated from one another, usually by a thin coating of material with index of refraction less than that of the fiber. The more fibers there are, and the smaller they are, the more detailed the picture. Such instruments, including bronchoscopes, colonoscopes (for viewing the colon), and endoscopes (stomach or other organs), are extremely useful for examining hard-to-reach places.

†Fiber-optic devices use not only visible light but also infrared light, ultraviolet light, and microwaves.

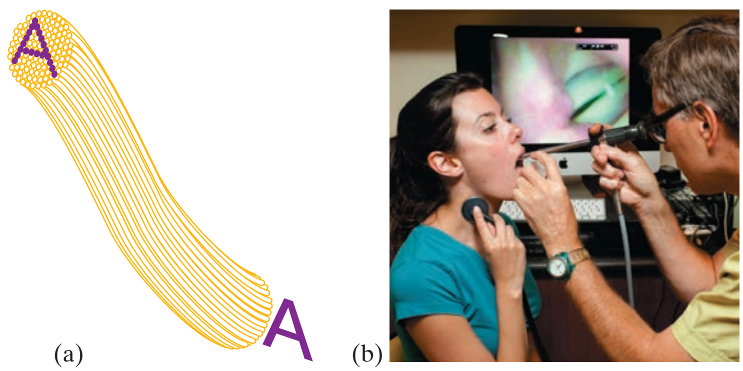
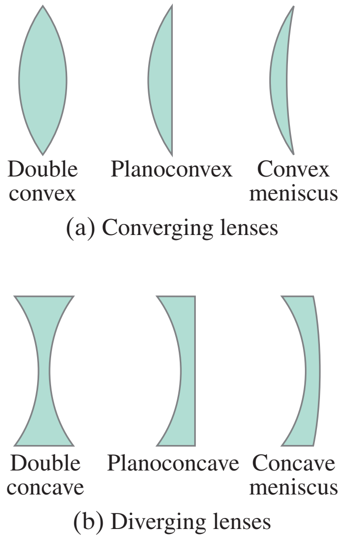


FIGURE 23–30 (a) How a fiber-optic image is made. (b) Example of a fiber-optic device inserted through the mouth to view the vocal cords, with the image on screen.




(c)

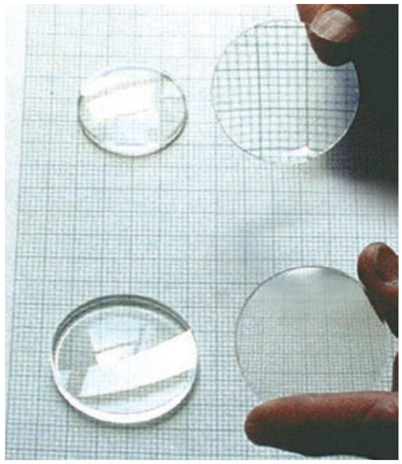

(d)

FIGURE 23–31 (a) Converging lenses and (b) diverging lenses, shown in cross section. Converging lenses are thicker at the center whereas diverging lenses are thicker at the edges. (c) Photo of a converging lens (on the left) and a diverging lens (right). (d) Converging lenses (above), and diverging lenses (below), lying flat, and raised off the paper to form images.

23–7 Thin Lenses; Ray Tracing

The most important simple optical device is the thin lens. The development of optical devices using lenses dates to the sixteenth and seventeenth centuries, although the earliest record of eyeglasses dates from the late thirteenth century. Today we find lenses in eyeglasses, cameras, magnifying glasses, telescopes, binoculars, microscopes, and medical instruments. A thin lens is usually circular, and its two faces are portions of a sphere. (Cylindrical faces are also possible, but we will concentrate on spherical.) The two faces can be concave, convex, or plane. Several types are shown in Figs. 23–31a and b in cross section. The importance of lenses is that they form images of objects—see Fig. 23–32.

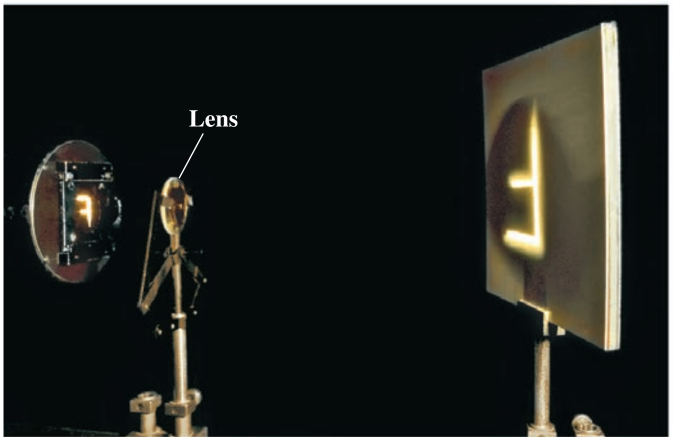


FIGURE 23–32 Converging lens (in holder) forms an image (large "F" on screen at right) of a bright object (illuminated "F" at the left).

Consider parallel rays striking the double convex lens shown in cross section in Fig. 23–33. We assume the lens is made of transparent material such as glass or transparent plastic with index of refraction greater than that of the air outside. The **axis** of a lens is a straight line passing through the center of the lens and perpendicular to its two surfaces (Fig. 23–33). From Snell's law, we can see that each ray in Fig. 23–33 is bent toward the axis when the ray enters the lens and again when it leaves the lens at the back surface. (Note the dashed lines indicating the normals to each surface for the top ray.) If rays parallel to the axis fall on a thin lens, they will be focused to a point called the **focal point**, F. This will not be precisely true for a lens with spherical surfaces. But it will be very nearly true—that is, parallel rays will be focused to a tiny region that is nearly a point—if the diameter of the lens is small compared to the radii of curvature of the two lens surfaces. This criterion is satisfied by a **thin lens**, one that is very thin compared to its diameter, and we consider only thin lenses here.

FIGURE 23–33 Parallel rays are brought to a focus by a converging thin lens.

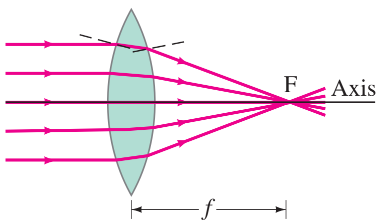


FIGURE 23–34 Image of the Sun burning wood.

FIGURE 23–35 Parallel rays at an angle are focused on the focal plane.

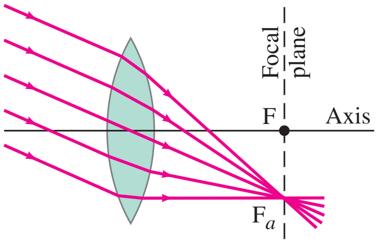
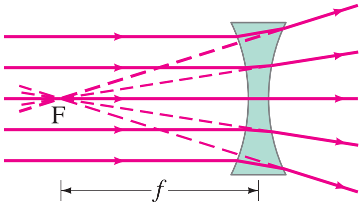


FIGURE 23–36 Diverging lens.



The rays from a point on a distant object are essentially parallel—see Fig. 23–12. Therefore we can say that *the focal point is the image point for an object at infinity on the lens axis*, Fig. 23–33. Thus, the focal point of a lens can be found by locating the point where the Sun's rays (or those from some other distant object) are brought to a sharp image, Fig. 23–34. The distance of the focal point from the center of the lens is called the **focal length**, f, Fig. 23–33. A lens can be turned around so that light can pass through it from the opposite side. The *focal length is the same on both sides*, as we shall see later, even if the curvatures of the two lens surfaces are different. If parallel rays fall on a lens at an angle, as in Fig. 23–35, they focus at a point F_a. The plane containing all focus points, such as F and F_a in Fig. 23–35, is called the **focal plane** of the lens.

Any lens (in air) that is thicker in the center than at the edges will make parallel rays converge to a point, and is called a **converging lens** (see Fig. 23–31a). Lenses that are thinner in the center than at the edges (Fig. 23–31b) are called **diverging lenses** because they make parallel light diverge, as shown in Fig. 23–36. The focal point, F, of a diverging lens is defined as that point from which refracted rays, originating from parallel incident rays, seem to emerge as shown in Fig. 23–36. And the distance from F to the center of the lens is called the **focal length**, f, just as for a converging lens.

EXERCISE F Return to Chapter-Opening Question 2, page 644, and answer it again now. Try to explain why you may have answered differently the first time.

Optometrists and ophthalmologists, instead of using the focal length, use the reciprocal of the focal length to specify the strength of eyeglass (or contact) lenses. This is called the **power**, P, of a lens:

$$P = \frac{1}{f}. \qquad \textbf{(23–7)}$$

The unit for lens power is the **diopter** (D), which is an inverse meter: $1\text{ D} = 1\text{ m}^{-1}$. For example, a 20-cm-focal-length lens has a power $P = 1/(0.20\text{ m}) = 5.0\text{ D}$. We will mainly use the focal length, but we will refer again to the power of a lens when we discuss eyeglass lenses in Chapter 25.

The most important parameter of a lens is its focal length f, which is the same on both sides of the lens. For a converging lens, f can be measured by finding the image point for the Sun or other distant objects. Once f is known, the image position can be determined for any object. To find the image point by drawing rays would be difficult if we had to determine the refractive angles at the front surface of the lens and again at the back surface where the ray exits. We can save ourselves a lot of effort by making use of certain facts we already know, such as that a ray parallel to the axis of the lens passes (after refraction) through the focal point. To determine an image point, we can consider only the three rays indicated in Fig. 23–37, which uses an arrow (on the left) as the object, and a converging lens forming an image (dashed arrow) to the right. These rays, emanating from a single point on the object, are drawn as if the lens were infinitely thin, and we show only a single sharp bend at the center line of the lens instead of the refractions at each surface. These three rays are drawn as follows:

RAY DIAGRAM

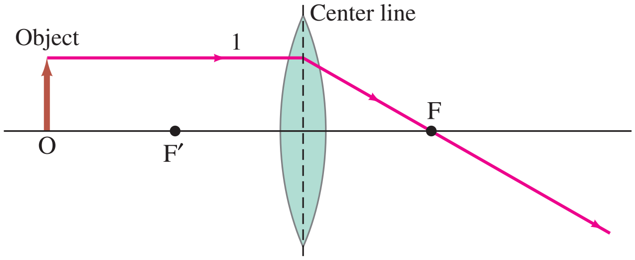
Finding the image position formed by a thin lens

Ray 1 is drawn parallel to the axis, Fig. 23–37a; therefore it is refracted by the lens so that it passes along a line through the focal point F behind the lens.

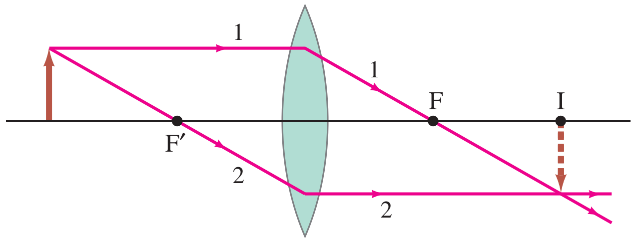
Ray 2 is drawn to pass through the other focal point F′ (front side of lens in Fig. 23–37) and emerge from the lens parallel to the axis, Fig. 23–37b. (In reverse it would be a parallel ray going left and passing through F′.)

Ray 3 is directed toward the very center of the lens, where the two surfaces are essentially parallel to each other, Fig. 23–37c. This ray therefore emerges from the lens at the same angle as it entered. The ray would be displaced slightly to one side, as we saw in Example 23–8; but since we assume the lens is thin, we draw ray 3 straight through as shown.

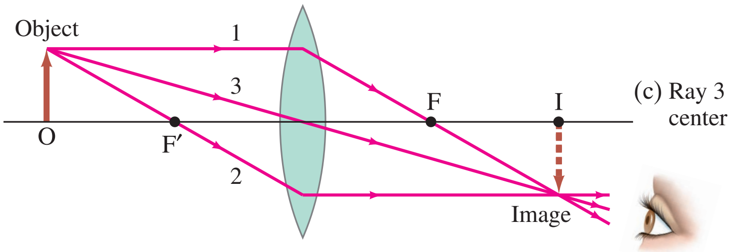
The point where these three rays cross is the image point for that object point. Actually, any two of these rays will suffice to locate the image point, but drawing the third ray can serve as a check.



(a) Ray 1 leaves one point on object going parallel to the axis, then refracts through focal point behind the lens.



(b) Ray 2 passes through F′ in front of the lens; therefore it is parallel to the axis behind the lens.



(c) Ray 3 passes straight through the center of the lens (assumed very thin).

FIGURE 23–37 Finding the image by ray tracing for a converging lens. Rays are shown leaving one point on the object (an arrow). Shown are the three most useful rays, leaving the tip of the object, for determining where the image of that point is formed. (Note that the focal points F and F′ on either side of the lens are the same distance *f* from the center of the lens.)

Using these three rays for one object point, we can find the image point for that point of the object (the top of the arrow in Fig. 23–37). The image points for all other points on the object can be found similarly to determine the complete image of the object. Because the rays actually pass through the image for the case shown in Fig. 23–37, it is a **real image** (see pages 647 and/or 651). The image could be detected by film or electronic sensor, and actually be seen on a white surface or screen placed at the position of the image (Fig. 23–38).

CONCEPTUAL EXAMPLE 23–11 **Half-blocked lens.** What happens to the image of an object if the top half of a lens is covered by a piece of cardboard?

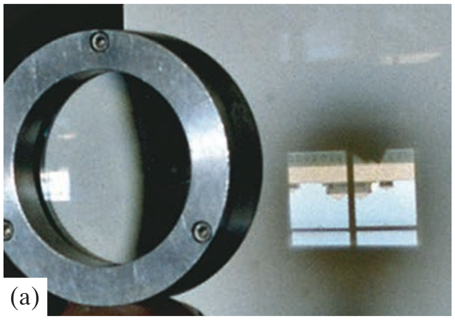
RESPONSE Let us look at the rays in Fig. 23–37. If the top half (or any half of the lens) is blocked, you might think that half the image is blocked. But in Fig. 23–37c, we see how the rays used to create the "top" of the image pass through both the top and the bottom of the lens. Only three of many rays are shown—many more rays pass through the lens, and they can form the image. You don't lose the image. But covering part of the lens cuts down on the total light received and reduces the brightness of the image.

NOTE If the lens is partially blocked by your thumb, you may notice an out of focus image of part of that thumb.

Seeing the Image

The image can also be seen directly by the eye when the eye is placed behind the image, as shown in Fig. 23–37c, so that some of the rays diverging from each point on the image can enter the eye. We can see a sharp image only for rays *diverging* from each point on the image, because we see normal objects when diverging rays from each point enter the eye as shown in Fig. 23–1. A normal eye cannot focus converging rays; if your eye was positioned between points F and I in Fig. 23–37c, it would not see a clear image. (More about our eyes in Section 25–2.) Figure 23–38 shows an image seen (a) on a white surface and (b) directly by the eye (and a camera) placed behind the image. The eye can see both real and virtual images (see next page) as long as the eye is positioned so rays diverging from the image enter it.

FIGURE 23–38 (a) A converging lens can form a real image (here of a distant building, upside down) on a white wall. (b) That same real image is also directly visible to the eye. [Figure 23–31d shows images (graph paper) seen by the eye made by both diverging and converging lenses.]



(a)

(b)

Diverging Lens

By drawing the same three rays emerging from a single object point, we can determine the image position formed by a diverging lens, as shown in Fig. 23–39. Note that ray 1 is drawn parallel to the axis, but does not pass through the focal point F′ behind the lens. Instead it seems to come (dashed line) from the focal point F in front of the lens. Ray 2 is directed toward F′ and is refracted parallel to the lens axis by the lens. Ray 3 passes directly through the center of the lens. The three refracted rays seem to emerge from a point on the left of the lens. This is the image point, I. Because the rays do not pass through the image, it is a **virtual image**. Note that the eye does not distinguish between real and virtual images—both are visible.

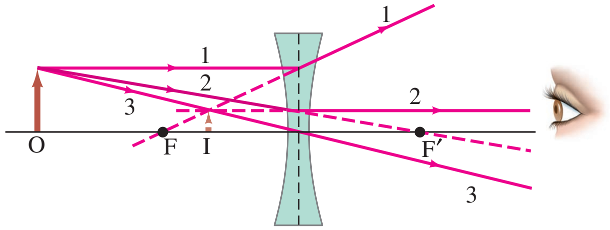


FIGURE 23–39 Finding the image by ray tracing for a diverging lens.

23–8 The Thin Lens Equation

We now derive an equation that relates the image distance to the object distance and the focal length of a thin lens. This equation will make the determination of image position quicker and more accurate than doing ray tracing. Let d_o be the object distance, the distance of the object from the center of the lens, and d_i be the image distance, the distance of the image from the center of the lens, Fig. 23–40.

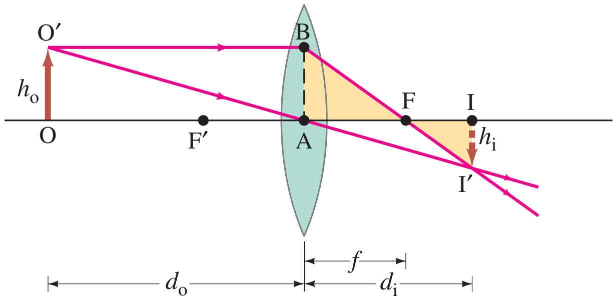


FIGURE 23–40 Deriving the lens equation for a converging lens.

Let h_o and h_i refer to the heights of the object and image. Consider the two rays shown in Fig. 23–40 for a converging lens, assumed to be very thin. The right triangles FI′I and FBA (highlighted in yellow) are similar because angle AFB equals angle IFI′; so

$$\frac{h_i}{h_o} = \frac{d_i - f}{f},$$

since length $AB = h_o$. Triangles OAO′ and IAI′ are similar as well. Therefore,

$$\frac{h_i}{h_o} = \frac{d_i}{d_o}.$$

We equate the right sides of these two equations (the left sides are the same), and divide by d_i to obtain

$$\frac{1}{f} - \frac{1}{d_i} = \frac{1}{d_o}$$

or

THIN LENS EQUATION

$$\boxed{\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}.} \qquad \text{(23–8)}$$

This is called the **thin lens equation**. It relates the image distance d_i to the object distance d_o and the focal length f. It is the most useful equation in geometric optics. (Interestingly, it is exactly the same as the mirror equation, Eq. 23–2.)

If the object is at infinity, then $1/d_o = 0$, so $d_i = f$. Thus the focal length is the image distance for an object at infinity, as mentioned earlier.

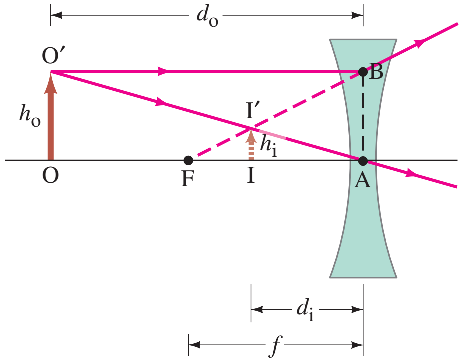


FIGURE 23–41 Deriving the lens equation for a diverging lens.

We can derive the lens equation for a diverging lens using Fig. 23–41. Triangles IAI′ and OAO′ are similar; and triangles IFI′ and AFB are similar. Thus (noting that length AB $= h_o$)

$$\frac{h_i}{h_o} = \frac{d_i}{d_o}$$

and

$$\frac{h_i}{h_o} = \frac{f - d_i}{f}.$$

When we equate the right sides of these two equations and simplify, we obtain

$$\frac{1}{d_o} - \frac{1}{d_i} = -\frac{1}{f}.$$

This equation becomes the same as Eq. 23–8 if we make f and d_i negative. That is, we take f to be *negative for a diverging lens*, and d_i negative when the image is on the same side of the lens as the light comes from. Thus Eq. 23–8 will be valid for both converging and diverging lenses, and for *all* situations, if we use the following **sign conventions**:

CAUTION
Focal length is negative for diverging lens

1. The focal length is positive for converging lenses and negative for diverging lenses.
2. The object distance is positive if the object is on the side of the lens from which the light is coming (this is always the case for real objects; but when lenses are used in combination, it might not be so: see Example 23–16); otherwise, it is negative.
3. The image distance is positive if the image is on the opposite side of the lens from where the light is coming; if it is on the same side, d_i is negative. Equivalently, the image distance is positive for a real image (Fig. 23–40) and negative for a virtual image (Fig. 23–41).
4. The height of the image, h_i, is positive if the image is upright, and negative if the image is inverted relative to the object. (h_o is always taken as upright and positive.)

PROBLEM SOLVING
SIGN CONVENTIONS for lenses

The **magnification**, m, of a lens is defined as the ratio of the image height to object height, $m = h_i/h_o$. From Figs. 23–40 and 23–41 and the conventions just stated (for which we will need a minus sign), we have

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}. \tag{23–9}$$

For an upright image the magnification is positive, and for an inverted image the magnification is negative.

From sign convention 1, it follows that the power (Eq. 23–7) of a converging lens, in diopters, is positive, whereas the power of a diverging lens is negative. A converging lens is sometimes referred to as a **positive lens**, and a diverging lens as a **negative lens**.

Diverging lenses (see Fig. 23–41) always produce an upright virtual image for any real object, no matter where that object is. Converging lenses can produce real (inverted) images as in Fig. 23–40, or virtual (upright) images, depending on object position, as we shall see.

PROBLEM SOLVING

Thin Lenses

1. Draw a **ray diagram**, as precise as possible, but even a rough one can serve as confirmation of analytic results. Choose one point on the object and draw at least two, or preferably three, of the easy-to-draw rays described in Figs. 23–37 and 23–39. The image point is where the rays intersect.
2. For analytic solutions, solve for unknowns in the **thin lens equation** (Eq. 23–8) and the **magnification equation** (Eq. 23–9). The thin lens equation involves reciprocals—don't forget to take the reciprocal.
3. Follow the **sign conventions** listed just above.
4. Check that your analytic answers are **consistent** with your ray diagram.

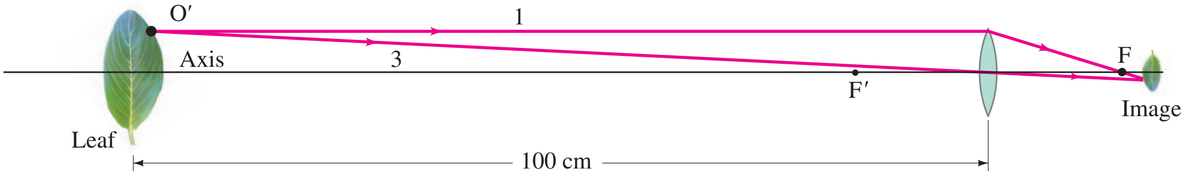


FIGURE 23–42 Example 23–12. (Not to scale.)

EXAMPLE 23–12 **Image formed by converging lens.** What is (*a*) the position, and (*b*) the size, of the image of a 7.6-cm-high leaf placed 1.00 m from a +50.0-mm-focal-length camera lens?

APPROACH We follow the steps of the Problem Solving Strategy explicitly.

SOLUTION

1. **Ray diagram.** Figure 23–42 is an approximate ray diagram, showing only rays 1 and 3 for a single point on the leaf. We see that the image ought to be a little behind the focal point F, to the right of the lens.
2. **Thin lens and magnification equations.** (*a*) We find the image position analytically using the thin lens equation, Eq. 23–8. The camera lens is converging, with $f = +5.00\text{ cm}$, and $d_o = 100\text{ cm}$, and so the thin lens equation gives

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{5.00\text{ cm}} - \frac{1}{100\text{ cm}} = \frac{20.0 - 1.0}{100\text{ cm}} = \frac{19.0}{100\text{ cm}}.$$

Then, taking the reciprocal,

$$d_i = \frac{100\text{ cm}}{19.0} = 5.26\text{ cm},$$

or 52.6 mm behind the lens.

(*b*) The magnification is

$$m = -\frac{d_i}{d_o} = -\frac{5.26\text{ cm}}{100\text{ cm}} = -0.0526,$$

so

$$h_i = mh_o = (-0.0526)(7.6\text{ cm}) = -0.40\text{ cm}.$$

The image is 4.0 mm high.

3. **Sign conventions.** The image distance d_i came out positive, so the image is behind the lens. The image height is $h_i = -0.40\text{ cm}$; the minus sign means the image is inverted.
4. **Consistency.** The analytic results of steps 2 and 3 are consistent with the ray diagram, Fig. 23–42: the image is behind the lens and inverted.

NOTE Part (*a*) tells us that the image is 2.6 mm farther from the lens than the image for an object at infinity, which would equal the focal length, 50.0 mm. Indeed, when focusing a camera lens, the closer the object is to the camera, the farther the lens must be from the sensor or film.

EXERCISE G If the leaf (object) of Example 23–12 is moved farther from the lens, does the image move closer to or farther from the lens? (Don't calculate!)

EXAMPLE 23–13 **Object close to converging lens.** An object is placed 10 cm from a 15-cm-focal-length converging lens. Determine the image position and size (*a*) analytically, and (*b*) using a ray diagram.

APPROACH The object is within the focal point—closer to the lens than the focal point F as $d_o < f$. We first use Eqs. 23–8 and 23–9 to obtain an analytic solution, and then confirm with a ray diagram using the special rays 1, 2, and 3 for a single object point.

SOLUTION (*a*) Given $f = 15\text{ cm}$ and $d_o = 10\text{ cm}$, then

$$\frac{1}{d_i} = \frac{1}{15\text{ cm}} - \frac{1}{10\text{ cm}} = \frac{2-3}{30\text{ cm}} = -\frac{1}{30\text{ cm}},$$

and $d_i = -30\text{ cm}$. (Remember to take the reciprocal!) Because d_i is negative, the image must be virtual and on the same side of the lens as the object (sign convention 3, page 665). The magnification

$$m = -\frac{d_i}{d_o} = -\frac{-30\text{ cm}}{10\text{ cm}} = 3.0.$$

The image is three times as large as the object and is upright. This lens is being used as a magnifying glass, which we discuss in more detail in Section 25–3.

(*b*) The ray diagram is shown in Fig. 23–43 and confirms the result in part (*a*). We choose point O′ on the top of the object and draw ray 1. Ray 2, however, may take some thought: if we draw it heading toward F′, it is going the wrong way—so we have to draw it as if coming from F′ (and so dashed), striking the lens, and then going out parallel to the lens axis. We project it backward, with a dashed line, as we must do also for ray 1, in order to find where they cross. Ray 3 is drawn through the lens center, and it crosses the other two rays at the image point, I′.

NOTE From Fig. 23–43 we can see that, when an object is placed between a converging lens and its focal point, the image is virtual.

CAUTION

Don't forget to take the reciprocal

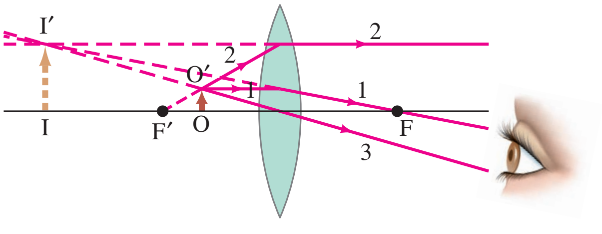


FIGURE 23–43 An object placed within the focal point of a converging lens produces a virtual image. Example 23–13.

EXAMPLE 23–14 **Diverging lens.** Where must a small insect be placed if a 25-cm-focal-length diverging lens is to form a virtual image 20 cm from the lens, on the same side as the object?

APPROACH The ray diagram is basically that of Fig. 23–41 because our lens here is diverging and our image is given as in front of the lens within the focal distance. (It would be a valuable exercise to draw the ray diagram to scale, precisely, now.) The insect's distance, d_o, can be calculated using the thin lens equation.

SOLUTION The lens is diverging, so f is negative: $f = -25\text{ cm}$. The image distance must be negative too because the image is in front of the lens (sign conventions), so $d_i = -20\text{ cm}$. The lens equation, Eq. 23–8, gives

$$\frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i} = -\frac{1}{25\text{ cm}} + \frac{1}{20\text{ cm}} = \frac{-4+5}{100\text{ cm}} = \frac{1}{100\text{ cm}}.$$

So the object must be 100 cm in front of the lens.

*23–9 Combinations of Lenses

Many optical instruments use lenses in combination. When light passes through more than one lens, we find the image formed by the first lens as if it were alone. Then this image becomes the *object* for the second lens. Next we find the image formed by this second lens using the first image as object. This second image is the final image if there are only two lenses. The total magnification will be the product of the separate magnifications of each lens. Even if the second lens intercepts the light from the first lens before it forms an image, this technique still works.

EXAMPLE 23–15 **A two-lens system.** Two converging lenses, A and B, with focal lengths $f_A = 20.0\text{ cm}$ and $f_B = 25.0\text{ cm}$, are placed 80.0 cm apart, as shown in Fig. 23–44a. An object is placed 60.0 cm in front of the first lens as shown in Fig. 23–44b. Determine (*a*) the position, and (*b*) the magnification, of the final image formed by the combination of the two lenses.

APPROACH Starting at the tip of our object O, we draw rays 1, 2, and 3 for the first lens, A, and also a ray 4 which, after passing through lens A, acts for the second lens, B, as ray 3′ (through the center). We use primes now for the standard rays relative to lens B. Ray 2 for lens A exits parallel, and so is ray 1′ for lens B. To determine the position of the image I_A formed by lens A, we use Eq. 23–8 with $f_A = 20.0\text{ cm}$ and $d_{oA} = 60.0\text{ cm}$. The distance of I_A (lens A's image) from lens B is the object distance d_{oB} for lens B. The final image is found using the thin lens equation, this time with all distances relative to lens B. For (*b*) the magnifications are found from Eq. 23–9 for each lens in turn.

SOLUTION (*a*) The object is a distance $d_{oA} = +60.0\text{ cm}$ from the first lens, A, and this lens forms an image whose position can be calculated using the thin lens equation:

$$\frac{1}{d_{iA}} = \frac{1}{f_A} - \frac{1}{d_{oA}} = \frac{1}{20.0\text{ cm}} - \frac{1}{60.0\text{ cm}} = \frac{3-1}{60.0\text{ cm}} = \frac{1}{30.0\text{ cm}}.$$

⚠ **CAUTION**
Object distance for second lens is **not** *equal to the image distance for first lens*

So the first image I_A is at $d_{iA} = 30.0\text{ cm}$ behind the first lens. This image becomes the object for the second lens, B. It is a distance $d_{oB} = 80.0\text{ cm} - 30.0\text{ cm} = 50.0\text{ cm}$ in front of lens B (Fig. 23–44b). The image formed by lens B, again using the thin lens equation, is at a distance d_{iB} from the lens B:

$$\frac{1}{d_{iB}} = \frac{1}{f_B} - \frac{1}{d_{oB}} = \frac{1}{25.0\text{ cm}} - \frac{1}{50.0\text{ cm}} = \frac{2-1}{50.0\text{ cm}} = \frac{1}{50.0\text{ cm}}.$$

Hence $d_{iB} = 50.0\text{ cm}$ behind lens B. This is the final image—see Fig. 23–44b.

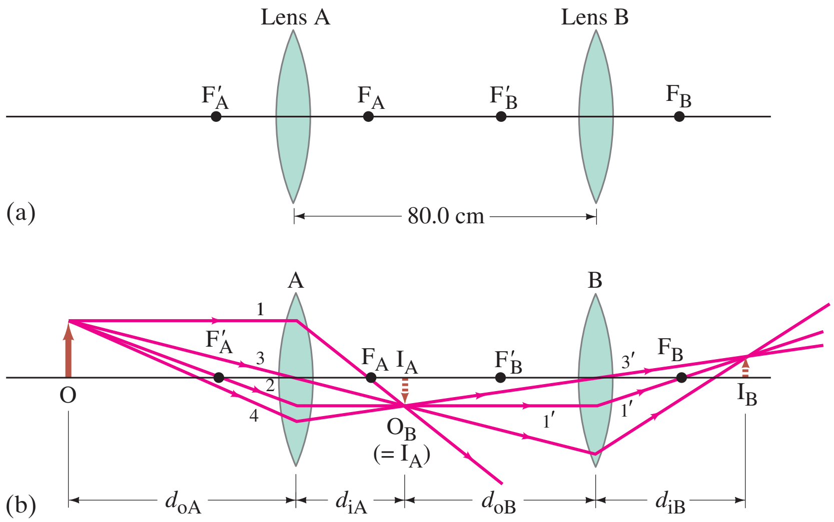


FIGURE 23–44 Two lenses, A and B, used in combination, Example 23–15. The small numbers refer to the easily drawn rays.

(b) Lens A has a magnification (Eq. 23–9)

$$m_A = -\frac{d_{iA}}{d_{oA}} = -\frac{30.0\text{ cm}}{60.0\text{ cm}} = -0.500.$$

Thus, the first image is inverted and is half as high as the object (again Eq. 23–9):

$$h_{iA} = m_A h_{oA} = -0.500 h_{oA}.$$

Lens B takes this first image as object and changes its height by a factor

$$m_B = -\frac{d_{iB}}{d_{oB}} = -\frac{50.0\text{ cm}}{50.0\text{ cm}} = -1.000.$$

The second lens reinverts the image (the minus sign) but doesn't change its size. The final image height is (remember h_{oB} is the same as h_{iA})

$$h_{iB} = m_B h_{oB} = m_B h_{iA} = m_B m_A h_{oA} = (m_{total})h_{oA}.$$

The total magnification is the product of m_A and m_B, which here equals $m_{total} = m_A m_B = (-1.000)(-0.500) = +0.500$, or half the original height, and the final image is upright.

PROBLEM SOLVING
Total magnification is $m_{total} = m_A m_B$

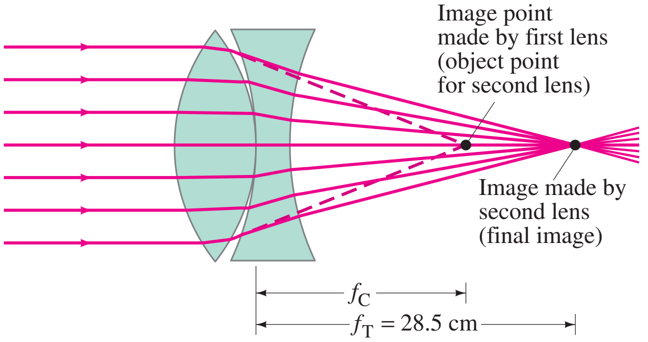


FIGURE 23–45 Determining the focal length of a diverging lens. Example 23–16.

EXAMPLE 23–16 **Measuring *f* for a diverging lens.** To measure the focal length of a diverging lens, a converging lens is placed in contact with it, as shown in Fig. 23–45. The Sun's rays are focused by this combination at a point 28.5 cm behind the lenses as shown. If the converging lens has a focal length f_C of 16.0 cm, what is the focal length f_D of the diverging lens? Assume both lenses are thin and the space between them is negligible.

APPROACH The image distance for the first lens equals its focal length (16.0 cm) since the object distance is infinity (∞). The position of this image, even though it is never actually formed, acts as the object for the second (diverging) lens. We apply the thin lens equation to the diverging lens to find its focal length, given that the final image is at $d_i = 28.5$ cm.

SOLUTION Rays from the Sun are focused 28.5 cm behind the combination, so the focal length of the total combination is $f_T = 28.5$ cm. If the diverging lens was absent, the converging lens would form the image at its focal point—that is, at a distance $f_C = 16.0$ cm behind it (dashed lines in Fig. 23–45). When the diverging lens is placed next to the converging lens, we treat the image formed by the first lens as the *object* for the second lens. Since this object lies to the right of the diverging lens, this is a situation where d_o is negative (see the sign conventions, page 665). Thus, for the diverging lens, the object is virtual and $d_o = -16.0$ cm. The diverging lens forms the image of this virtual object at a distance $d_i = 28.5$ cm away (given). Thus,

CAUTION
$d_o < 0$

$$\frac{1}{f_D} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{-16.0\text{ cm}} + \frac{1}{28.5\text{ cm}} = -0.0274\text{ cm}^{-1}.$$

We take the reciprocal to find $f_D = -1/(0.0274\text{ cm}^{-1}) = -36.5$ cm.

NOTE If this technique is to work, the converging lens must be "stronger" than the diverging lens—that is, it must have a focal length whose magnitude is less than that of the diverging lens.

*23–10 Lensmaker's Equation

A useful equation, called the **lensmaker's equation**, relates the focal length of a lens to the radii of curvature R_1 and R_2 of its two surfaces and its index of refraction n:

Lensmaker's equation

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right). \qquad \textbf{(23–10)}$$

If both surfaces are convex, R_1 and R_2 are considered positive.† For a concave surface, the radius must be considered *negative*.

Notice that Eq. 23–10 is *symmetrical* in R_1 and R_2. Thus, if a lens is turned around so that light impinges on the other surface, the focal length is the same even if the two lens surfaces are different. This confirms what we said earlier: a lens' focal length is the same on both sides of the lens.

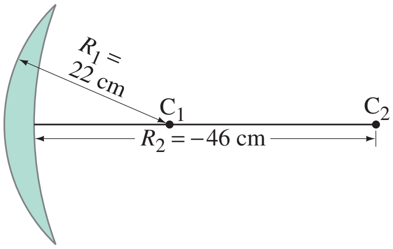


FIGURE 23–46 Example 23–17. The left surface is convex (center bulges outward); the right surface is concave.

EXAMPLE 23–17 **Calculating *f* for a converging lens.** A convex meniscus lens (Figs. 23–31a and 23–46) is made from glass with $n = 1.50$. The radius of curvature of the convex surface (left in Fig. 23–46) is 22 cm. The surface on the right is concave with radius of curvature 46 cm. What is the focal length?

APPROACH We use the lensmaker's equation, Eq. 23–10, to find f.

SOLUTION $R_1 = 0.22\,\text{m}$ and $R_2 = -0.46\,\text{m}$ (concave surface). Then

$$\frac{1}{f} = (1.50 - 1.00)\left(\frac{1}{0.22\,\text{m}} - \frac{1}{0.46\,\text{m}}\right) = 1.19\,\text{m}^{-1}.$$

So

$$f = \frac{1}{1.19\,\text{m}^{-1}} = 0.84\,\text{m},$$

and the lens is converging since $f > 0$.

NOTE If we turn the lens around so that $R_1 = -46\,\text{cm}$ and $R_2 = +22\,\text{cm}$, we get the same result.

NOTE Because Eq. 23–10 gives $1/f$, it gives directly the power of a lens in diopters, Eq. 23–7. The power of this lens is about 1.2 D.

†Some books use a different convention: R_1 and R_2 may be considered positive if their centers of curvature are to the right of the lens; then a minus sign replaces the + sign in their version of Eq. 23–10.

Summary

Light appears to travel along straight-line paths, called **rays**, through uniform transparent materials including air and glass. When light reflects from a flat surface, the *angle of reflection equals the angle of incidence*. This **law of reflection** explains why mirrors can form **images**.

In a **plane mirror**, the image is virtual, upright, the same size as the object, and as far behind the mirror as the object is in front.

A **spherical mirror** can be concave or convex. A **concave** spherical mirror focuses parallel rays of light (light from a very distant object) to a point called the **focal point**. The distance of this point from the mirror is the **focal length** f of the mirror and

$$f = \frac{r}{2} \qquad \textbf{(23–1)}$$

where r is the radius of curvature of the mirror.

Parallel rays falling on a **convex mirror** reflect from the mirror as if they diverged from a common point behind the mirror. The distance of this point from the mirror is the focal length and is considered negative for a convex mirror.

For a given object, the approximate position and size of the image formed by a mirror can be found by ray tracing. Algebraically, the relation between image and object distances, d_i and d_o, and the focal length f, is given by the **mirror equation**:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}. \qquad \textbf{(23–2)}$$

The ratio of image height h_i to object height h_o, which equals the magnification m of a mirror, is

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}. \qquad \textbf{(23–3)}$$

If the rays that converge to form an image actually pass through the image, so the image would appear on a screen or film placed there, the image is said to be a **real image**. If the light rays do not actually pass through the image, the image is a **virtual image**.

The speed of light v depends on the **index of refraction**, n, of the material:

$$n = \frac{c}{v}, \qquad \textbf{(23–4)}$$

where c is the speed of light in vacuum.

When light passes from one transparent medium into another, the rays bend or refract. The **law of refraction** (**Snell's law**) states that

$$n_1 \sin\theta_1 = n_2 \sin\theta_2, \qquad \textbf{(23–5)}$$

where n_1 and θ_1 are the index of refraction and angle with the normal (perpendicular) to the surface for the incident ray, and n_2 and θ_2 are for the refracted ray.

When light rays reach the boundary of a material where the index of refraction decreases, the rays will be **totally internally reflected** if the incident angle, θ_1, is such that Snell's law would predict $\sin\theta_2 > 1$. This occurs if θ_1 exceeds the critical angle θ_C given by

$$\sin\theta_C = \frac{n_2}{n_1}. \qquad \textbf{(23–6)}$$

A lens uses refraction to produce a real or virtual image. Parallel rays of light are focused to a point, the **focal point**, by a **converging** lens. The distance of the focal point from the lens is the **focal length** f of the lens. It is the same on both sides of the lens.

After parallel rays pass through a **diverging** lens, they appear to diverge from a point in front of the lens, which is its focal point; and the corresponding focal length is considered negative.

The **power** P of a lens, which is $P = 1/f$ (Eq. 23–7), is given in diopters, which are units of inverse meters (m^{-1}).

For a given object, the position and size of the image formed by a lens can be found approximately by ray tracing. Algebraically, the relation between image and object distances, d_i and d_o, and the focal length f, is given by the **thin lens equation**:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}. \qquad \textbf{(23–8)}$$

The ratio of image height to object height, which equals the **magnification** m for a lens, is

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}. \qquad \textbf{(23–9)}$$

When using the various equations of geometric optics, you must remember the **sign conventions** for all quantities involved: carefully review them (pages 655 and 665) when doing Problems.

[*When two (or more) thin lenses are used in combination to produce an image, the thin lens equation can be used for each lens in sequence. The image produced by the first lens acts as the object for the second lens.]

[*The **lensmaker's equation** relates the radii of curvature of the lens surfaces and the lens' index of refraction to the focal length of the lens.]

Questions

1. Archimedes is said to have burned the whole Roman fleet in the harbor of Syracuse, Italy, by focusing the rays of the Sun with a huge spherical mirror. Is this† reasonable?
2. What is the focal length of a plane mirror? What is the magnification of a plane mirror?
3. Although a plane mirror appears to reverse left and right, it doesn't reverse up and down. Discuss why this happens, noting that front to back is also reversed. Also discuss what happens if, while standing, you look up vertically at a horizontal mirror on the ceiling.
4. An object is placed along the principal axis of a spherical mirror. The magnification of the object is -2.0. Is the image real or virtual, inverted or upright? Is the mirror concave or convex? On which side of the mirror is the image located?
5. If a concave mirror produces a real image, is the image necessarily inverted? Explain.
6. How might you determine the speed of light in a solid, rectangular, transparent object?
7. When you look at the Moon's reflection from a rippy sea, it appears elongated (Fig. 23–47). Explain.

FIGURE 23–47 Question 7.

8. What is the angle of refraction when a light ray is incident perpendicular to the boundary between two transparent materials?

†Students at MIT did a feasibility study. See www.mit.edu/2.009/www/experiments/deathray/10_ArchimedesResult.html.

9. When you look down into a swimming pool or a lake, are you likely to overestimate or underestimate its depth? Explain. How does the apparent depth vary with the viewing angle? (Use ray diagrams.)
10. Draw a ray diagram to show why a stick or straw looks bent when part of it is under water (Fig. 23–23).
11. When a wide beam of parallel light enters water at an angle, the beam broadens. Explain.
12. You look into an aquarium and view a fish inside. One ray of light from the fish is shown emerging from the tank in Fig. 23–48. The apparent position of the fish is also shown (dashed ray). In the drawing, indicate the approximate position of the actual fish. Briefly justify your answer.

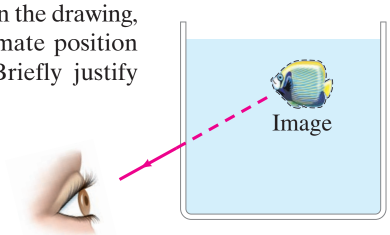


FIGURE 23–48 Question 12.

13. How can you "see" a round drop of water on a table even though the water is transparent and colorless?
14. A ray of light is refracted through three different materials (Fig. 23–49). Which material has (a) the largest index of refraction, (b) the smallest?

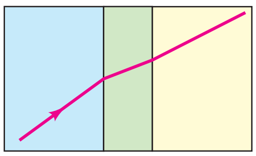


FIGURE 23–49 Question 14.

15. A child looks into a pool to see how deep it is. She then drops a small toy into the pool to help decide how deep the pool is. After this careful investigation, she decides it is safe to jump in—only to discover the water is over her head. What went wrong with her interpretation of her experiment?
16. Can a light ray traveling in air be totally reflected when it strikes a smooth water surface if the incident angle is chosen correctly? Explain.

17. What type of mirror is shown in Fig. 23–50? Explain.

FIGURE 23–50 Question 17 and Problem 15.

18. Light rays from stars (including our Sun) always bend toward the vertical direction as they pass through the Earth's atmosphere. (*a*) Why does this make sense? (*b*) What can you conclude about the apparent positions of stars as viewed from Earth? Draw a circle for Earth, a dot for you, and 3 or 4 stars at different angles.

19. Where must the film be placed if a camera lens is to make a sharp image of an object far away? Explain.

20. A photographer moves closer to his subject and then refocuses. Does the camera lens move farther away from or closer to the camera film or sensor? Explain.

21. Can a diverging lens form a real image under any circumstances? Explain.

22. Light rays are said to be "reversible." Is this consistent with the thin lens equation? Explain.

23. Can real images be projected on a screen? Can virtual images? Can either be photographed? Discuss carefully.

24. A thin converging lens is moved closer to a nearby object. Does the real image formed change (*a*) in position, (*b*) in size? If yes, describe how.

25. If a glass converging lens is placed in water, its focal length in water will be (*a*) longer, (*b*) shorter, or (*c*) the same as in air. Explain.

26. Compare the mirror equation with the thin lens equation. Discuss similarities and differences, especially the sign conventions for the quantities involved.

27. A lens is made of a material with an index of refraction $n = 1.25$. In air, it is a converging lens. Will it still be a converging lens if placed in water? Explain, using a ray diagram.

28. (*a*) Does the focal length of a lens depend on the fluid in which it is immersed? (*b*) What about the focal length of a spherical mirror? Explain.

29. An underwater lens consists of a carefully shaped thin-walled plastic container filled with air. What shape should it have in order to be (*a*) converging, (*b*) diverging? Use ray diagrams to support your answer.

30. The thicker a double convex lens is in the center as compared to its edges, the shorter its focal length for a given lens diameter. Explain.

*31. A non-symmetrical lens (say, planoconvex) forms an image of a nearby object. Use the lensmaker's equation to explain if the image point changes when the lens is turned around.

*32. Example 23–16 shows how to use a converging lens to measure the focal length of a diverging lens. (*a*) Why can't you measure the focal length of a diverging lens directly? (*b*) It is said that for this to work, the converging lens must be stronger than the diverging lens. What is meant by "stronger," and why is this statement true?

MisConceptual Questions

1. Suppose you are standing about 3 m in front of a mirror. You can see yourself just from the top of your head to your waist, where the bottom of the mirror cuts off the rest of your image. If you walk one step closer to the mirror
(*a*) you will not be able to see any more of your image.
(*b*) you will be able to see more of your image, below your waist.
(*c*) you will see less of your image, with the cutoff rising to be above your waist.

2. When the reflection of an object is seen in a flat mirror, the image is
(*a*) real and upright.
(*b*) real and inverted.
(*c*) virtual and upright.
(*d*) virtual and inverted.

3. You want to create a spotlight that will shine a bright beam of light with all of the light rays parallel to each other. You have a large concave spherical mirror and a small lightbulb. Where should you place the lightbulb?
(*a*) At the focal point of the mirror.
(*b*) At the radius of curvature of the mirror.
(*c*) At any point, because all rays bouncing off the mirror will be parallel.
(*d*) None of the above; you can't make parallel rays with a concave mirror.

4. When you look at a fish in a still stream from the bank, the fish appears shallower than it really is due to refraction. From directly above, it appears
(*a*) deeper than it really is.
(*b*) at its actual depth.
(*c*) shallower than its real depth.
(*d*) It depends on your height above the water.

5. Parallel light rays cross interfaces from medium 1 into medium 2 and then into medium 3 as shown in Fig. 23–51. What can we say about the relative sizes of the indices of refraction of these media?
(*a*) $n_1 > n_2 > n_3$.
(*b*) $n_3 > n_2 > n_1$.
(*c*) $n_2 > n_3 > n_1$.
(*d*) $n_1 > n_3 > n_2$.
(*e*) $n_2 > n_1 > n_3$.
(*f*) None of the above.

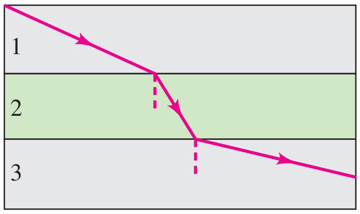


FIGURE 23–51 MisConceptual Question 5.

6. To shoot a swimming fish with an intense light beam from a *laser gun*, you should aim
(*a*) directly at the image.
(*b*) slightly above the image.
(*c*) slightly below the image.

7. When moonlight strikes the surface of a calm lake, what happens to this light?
(*a*) All of it reflects from the water surface back to the air.
(*b*) Some of it reflects back to the air; some enters the water.
(*c*) All of it enters the water.
(*d*) All of it disappears via absorption by water molecules.

8. If you shine a light through an optical fiber, why does it come out the end but not out the sides?
(*a*) It does come out the sides, but this effect is not obvious because the sides are so much longer than the ends.
(*b*) The sides are mirrored, so the light reflects.
(*c*) Total internal reflection makes the light reflect from the sides.
(*d*) The light flows along the length of the fiber, never touching the sides.

9. A converging lens, such as a typical magnifying glass,
(*a*) always produces a magnified image (taller than object).
(*b*) always produces an image smaller than the object.
(*c*) always produces an upright image.
(*d*) always produces an inverted image (upside down).
(*e*) None of these statements are true.

10. Virtual images can be formed by
(*a*) only mirrors.
(*b*) only lenses.
(*c*) only plane mirrors.
(*d*) only curved mirrors or lenses.
(*e*) plane and curved mirrors, and lenses.

11. A lens can be characterized by its *power*, which
(*a*) is the same as the magnification.
(*b*) tells how much light the lens can focus.
(*c*) depends on where the object is located.
(*d*) is the reciprocal of the focal length.

12. You cover half of a lens that is forming an image on a screen. Compare what happens when you cover the top half of the lens versus the bottom half.
(*a*) When you cover the top half of the lens, the top half of the image disappears; when you cover the bottom half of the lens, the bottom half of the image disappears.
(*b*) When you cover the top half of the lens, the bottom half of the image disappears; when you cover the bottom half of the lens, the top half of the image disappears.
(*c*) The image becomes half as bright in both cases.
(*d*) Nothing happens in either case.
(*e*) The image disappears in both cases.

13. Which of the following can form an image?
(*a*) A plane mirror.
(*b*) A curved mirror.
(*c*) A lens curved on both sides.
(*d*) A lens curved on only one side.
(*e*) All of the above.

14. As an object moves from just outside the focal point of a converging lens to just inside it, the image goes from _____ and _____ to _____ and _____.
(*a*) large; inverted; large; upright.
(*b*) large; upright; large; inverted.
(*c*) small; inverted; small; upright.
(*d*) small; upright; small; inverted.

For assigned homework and other learning materials, go to the MasteringPhysics website.

Problems

23–2 Reflection; Plane Mirrors

1. (I) When you look at yourself in a 60-cm-tall plane mirror, you see the same amount of your body whether you are close to the mirror or far away. (Try it and see.) Use ray diagrams to show why this should be true.

2. (I) Suppose that you want to take a photograph of yourself as you look at your image in a mirror 3.1 m away. For what distance should the camera lens be focused?

3. (II) Two plane mirrors meet at a 135° angle, Fig. 23–52. If light rays strike one mirror at 34° as shown, at what angle ϕ do they leave the second mirror?

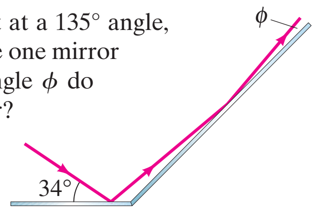


FIGURE 23–52 Problem 3.

4. (II) A person whose eyes are 1.72 m above the floor stands 2.20 m in front of a vertical plane mirror whose bottom edge is 38 cm above the floor, Fig. 23–53. What is the horizontal distance x to the base of the wall supporting the mirror of the nearest point on the floor that can be seen reflected in the mirror?

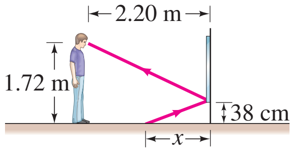


FIGURE 23–53 Problem 4.

5. (II) Stand up two plane mirrors so they form a 90.0° angle as in Fig. 23–54. When you look into this double mirror, you see yourself as others see you, instead of reversed as in a single mirror. Make a ray diagram to show how this occurs.

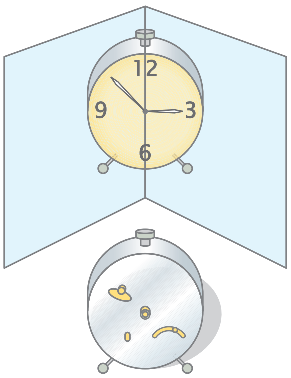


FIGURE 23–54 Problem 5.

6. (II) Two plane mirrors, nearly parallel, are facing each other 2.3 m apart as in Fig. 23–55. You stand 1.6 m away from one of these mirrors and look into it. You will see multiple images of yourself. (*a*) How far away from you are the first three images of yourself in the mirror in front of you? (*b*) Are these first three images facing toward you or away from you?

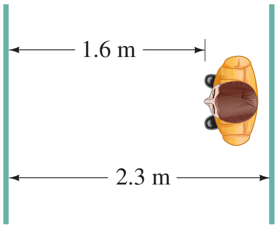


FIGURE 23–55 Problem 6.

23–3 Spherical Mirrors

7. (I) A solar cooker, really a concave mirror pointed at the Sun, focuses the Sun's rays 18.8 cm in front of the mirror. What is the radius of the spherical surface from which the mirror was made?

8. (I) How far from a concave mirror (radius 21.0 cm) must an object be placed if its image is to be at infinity?

9. (II) A small candle is 38 cm from a concave mirror having a radius of curvature of 24 cm. (*a*) What is the focal length of the mirror? (*b*) Where will the image of the candle be located? (*c*) Will the image be upright or inverted?

10. (II) An object 3.0 mm high is placed 16 cm from a convex mirror of radius of curvature 16 cm. (*a*) Show by ray tracing that the image is virtual, and estimate the image distance. (*b*) Show that the (negative) image distance can be computed from Eq. 23–2 using a focal length of −8.0 cm. (*c*) Compute the image size, using Eq. 23–3.

11. (II) A dentist wants a small mirror that, when 2.00 cm from a tooth, will produce a 4.0× upright image. What kind of mirror must be used and what must its radius of curvature be?

12. (II) You are standing 3.4 m from a convex security mirror in a store. You estimate the height of your image to be half of your actual height. Estimate the radius of curvature of the mirror.

13. (II) The image of a distant tree is virtual and very small when viewed in a curved mirror. The image appears to be 19.0 cm behind the mirror. What kind of mirror is it, and what is its radius of curvature?

14. (II) A mirror at an amusement park shows an upright image of any person who stands 1.9 m in front of it. If the image is three times the person's height, what is the radius of curvature of the mirror? (See Fig. 23–50.)

15. (II) In Example 23–4, show that if the object is moved 10.0 cm farther from the concave mirror, the object's image size will equal the object's actual size. Stated as a multiple of the focal length, what is the object distance for this "actual-sized image" situation?

16. (II) You look at yourself in a shiny 8.8-cm-diameter Christmas tree ball. If your face is 25.0 cm away from the ball's front surface, where is your image? Is it real or virtual? Is it upright or inverted?

17. (II) Some rearview mirrors produce images of cars to your rear that are smaller than they would be if the mirror were flat. Are the mirrors concave or convex? What is a mirror's radius of curvature if cars 16.0 m away appear 0.33 their normal size?

18. (II) When walking toward a concave mirror you notice that the image flips at a distance of 0.50 m. What is the radius of curvature of the mirror?

19. (II) (*a*) Where should an object be placed in front of a concave mirror so that it produces an image at the same location as the object? (*b*) Is the image real or virtual? (*c*) Is the image inverted or upright? (*d*) What is the magnification of the image?

20. (II) A shaving or makeup mirror is designed to magnify your face by a factor of 1.40 when your face is placed 20.0 cm in front of it. (*a*) What type of mirror is it? (*b*) Describe the type of image that it makes of your face. (*c*) Calculate the required radius of curvature for the mirror.

21. (II) Use two techniques, (*a*) a ray diagram, and (*b*) the mirror equation, to show that the magnitude of the magnification of a concave mirror is less than 1 if the object is beyond the center of curvature C $(d_o > r)$, and is greater than 1 if the object is within C $(d_o < r)$.

22. (III) Show, using a ray diagram, that the magnification m of a convex mirror is $m = -d_i/d_o$, just as for a concave mirror. [*Hint*: Consider a ray from the top of the object that reflects at the center of the mirror.]

23. (III) An object is placed a distance r in front of a wall, where r exactly equals the radius of curvature of a certain concave mirror. At what distance from the wall should this mirror be placed so that a real image of the object is formed on the wall? What is the magnification of the image?

23–4 Index of Refraction

24. (I) The speed of light in ice is 2.29×10^8 m/s. What is the index of refraction of ice?

25. (I) What is the speed of light in (*a*) ethyl alcohol, (*b*) lucite, (*c*) crown glass?

26. (II) The speed of light in a certain substance is 82% of its value in water. What is the index of refraction of that substance?

23–5 Refraction; Snell's Law

27. (I) A flashlight beam strikes the surface of a pane of glass $(n = 1.56)$ at a 67° angle to the normal. What is the angle of refraction?

28. (I) A diver shines a flashlight upward from beneath the water at a 35.2° angle to the vertical. At what angle does the light leave the water?

29. (I) A light beam coming from an underwater spotlight exits the water at an angle of 56.0°. At what angle of incidence did it hit the air–water interface from below the surface?

30. (I) Rays of the Sun are seen to make a 36.0° angle to the vertical beneath the water. At what angle above the horizon is the Sun?

31. (II) An aquarium filled with water has flat glass sides whose index of refraction is 1.54. A beam of light from outside the aquarium strikes the glass at a 43.5° angle to the perpendicular (Fig. 23–56). What is the angle of this light ray when it enters (*a*) the glass, and then (*b*) the water? (*c*) What would be the refracted angle if the ray entered the water directly?

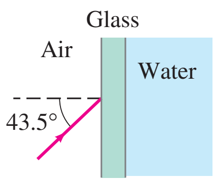


FIGURE 23–56 Problem 31.

32. (II) In searching the bottom of a pool at night, a watchman shines a narrow beam of light from his flashlight, 1.3 m above the water level, onto the surface of the water at a point 2.5 m from his foot at the edge of the pool (Fig. 23–57). Where does the spot of light hit the bottom of the 2.1-m-deep pool? Measure from the bottom of the wall beneath his foot.

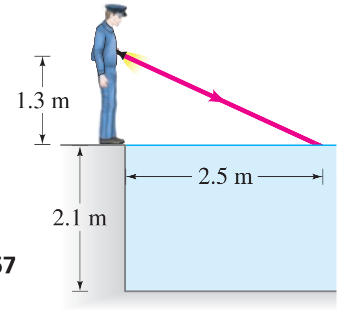


FIGURE 23–57 Problem 32.

23–6 Total Internal Reflection

33. (I) What is the critical angle for the interface between water and crown glass? To be internally reflected, the light must start in which material?

34. (I) The critical angle for a certain liquid–air surface is 47.2°. What is the index of refraction of the liquid?

35. (II) A beam of light is emitted in a pool of water from a depth of 82.0 cm. Where must it strike the air–water interface, relative to the spot directly above it, in order that the light does *not* exit the water?

36. (II) A beam of light is emitted 8.0 cm beneath the surface of a liquid and strikes the air surface 7.6 cm from the point directly above the source. If total internal reflection occurs, what can you say about the index of refraction of the liquid?

37. (III) (*a*) What is the minimum index of refraction for a glass or plastic prism to be used in binoculars (Fig. 23–28) so that total internal reflection occurs at 45°? (*b*) Will binoculars work if their prisms (assume $n = 1.58$) are immersed in water? (*c*) What minimum n is needed if the prisms are immersed in water?

38. (III) A beam of light enters the end of an optic fiber as shown in Fig. 23–58. (*a*) Show that we can guarantee total internal reflection at the side surface of the material (at point A), if the index of refraction is greater than about 1.42. In other words, regardless of the angle α, the light beam reflects back into the material at point A, assuming air outside. (*b*) What if the fiber were immersed in water?

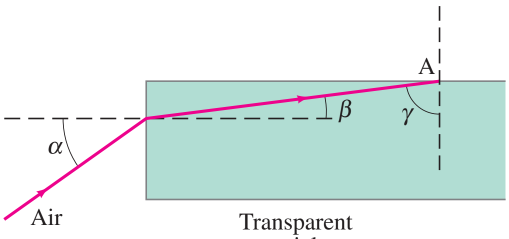


FIGURE 23–58 Problem 38.

23–7 and 23–8 Thin Lenses

39. (I) A sharp image is located 391 mm behind a 215-mm-focal-length converging lens. Find the object distance (*a*) using a ray diagram, (*b*) by calculation.

40. (I) Sunlight is observed to focus at a point 16.5 cm behind a lens. (*a*) What kind of lens is it? (*b*) What is its power in diopters?

41. (I) (*a*) What is the power of a 32.5-cm-focal-length lens? (*b*) What is the focal length of a −6.75-D lens? Are these lenses converging or diverging?

42. (II) A certain lens focuses light from an object 1.55 m away as an image 48.3 cm on the other side of the lens. What type of lens is it and what is its focal length? Is the image real or virtual?

43. (II) A 105-mm-focal-length lens is used to focus an image on the sensor of a camera. The maximum distance allowed between the lens and the sensor plane is 132 mm. (*a*) How far in front of the sensor should the lens (assumed thin) be positioned if the object to be photographed is 10.0 m away? (*b*) 3.0 m away? (*c*) 1.0 m away? (*d*) What is the closest object this lens could photograph sharply?

44. (II) Use ray diagrams to show that a real image formed by a thin lens is always inverted, whereas a virtual image is always upright if the object is real.

45. (II) A stamp collector uses a converging lens with focal length 28 cm to view a stamp 16 cm in front of the lens. (*a*) Where is the image located? (*b*) What is the magnification?

46. (II) A −7.00-D lens is held 12.5 cm from an ant 1.00 mm high. Describe the position, type, and height of the image.

47. (II) An object is located 1.50 m from a 6.5-D lens. By how much does the image move if the object is moved (*a*) 0.90 m closer to the lens, and (*b*) 0.90 m farther from the lens?

48. (II) (*a*) How far from a 50.0-mm-focal-length lens must an object be placed if its image is to be magnified 2.50× and be real? (*b*) What if the image is to be virtual and magnified 2.50×?

49. (II) Repeat Problem 48 for a −50.0-mm-focal-length lens. [*Hint*: Consider objects real or virtual (formed by some other piece of optics).]

50. (II) How far from a converging lens with a focal length of 32 cm should an object be placed to produce a real image which is the same size as the object?

51. (II) (*a*) A 2.40-cm-high insect is 1.30 m from a 135-mm-focal-length lens. Where is the image, how high is it, and what type is it? (*b*) What if $f = -135$ mm?

52. (III) A bright object and a viewing screen are separated by a distance of 86.0 cm. At what location(s) between the object and the screen should a lens of focal length 16.0 cm be placed in order to produce a sharp image on the screen? [*Hint*: First draw a diagram.]

53. (III) How far apart are an object and an image formed by an 85-cm-focal-length converging lens if the image is 3.25× larger than the object and is real?

54. (III) In a film projector, the film acts as the object whose image is projected on a screen (Fig. 23–59). If a 105-mm-focal-length lens is to project an image on a screen 25.5 m away, how far from the lens should the film be? If the film is 24 mm wide, how wide will the picture be on the screen?

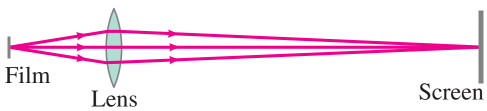


FIGURE 23–59 Film projector, Problem 54.

*23–9 Lens Combinations

*55. (II) A diverging lens with $f = -36.5$ cm is placed 14.0 cm behind a converging lens with $f = 20.0$ cm. Where will an object at infinity be focused?

*56. (II) Two 25.0-cm-focal-length converging lenses are placed 16.5 cm apart. An object is placed 35.0 cm in front of one lens. Where will the final image formed by the second lens be located? What is the total magnification?

*57. (II) A 38.0-cm-focal-length converging lens is 28.0 cm behind a diverging lens. Parallel light strikes the diverging lens. After passing through the converging lens, the light is again parallel. What is the focal length of the diverging lens? [*Hint*: First draw a ray diagram.]

*58. (II) A lighted candle is placed 36 cm in front of a converging lens of focal length $f_1 = 13$ cm, which in turn is 56 cm in front of another converging lens of focal length $f_2 = 16$ cm (see Fig. 23–60). (*a*) Draw a ray diagram and estimate the location and the relative size of the final image. (*b*) Calculate the position and relative size of the final image.

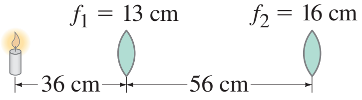


FIGURE 23–60 Problem 58.

*23–10 Lensmaker's Equation

*59. (I) A double concave lens has surface radii of 33.4 cm and 28.8 cm. What is the focal length if $n = 1.52$?

*60. (I) Both surfaces of a double convex lens have radii of 34.1 cm. If the focal length is 28.9 cm, what is the index of refraction of the lens material?

*61. (I) A planoconvex lens (Fig. 23–31a) with $n = 1.55$ is to have a focal length of 16.3 cm. What is the radius of curvature of the convex surface?

*62. (II) A prescription for an eyeglass lens calls for +3.50 diopters. The lensmaker grinds the lens from a "blank" with $n = 1.56$ and convex front surface of radius of curvature of 30.0 cm. What should be the radius of curvature of the other surface?

*63. (III) An object is placed 96.5 cm from a glass lens ($n = 1.52$) with one concave surface of radius 22.0 cm and one convex surface of radius 18.5 cm. Where is the final image? What is the magnification?

General Problems

64. Sunlight is reflected off the Moon. How long does it take that light to reach us from the Moon?

65. You hold a small flat mirror 0.50 m in front of you and can see your reflection twice in that mirror because there is a full-length mirror 1.0 m behind you (Fig. 23–61). Determine the distance of each image from you.

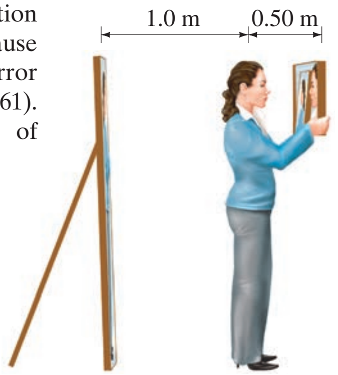


FIGURE 23–61 Problem 65.

66. We wish to determine the depth of a swimming pool filled with water by measuring the width ($x = 6.50$ m) and then noting that the far bottom edge of the pool is just visible at an angle of 13.0° above the horizontal as shown in Fig. 23–62. Calculate the depth of the pool.

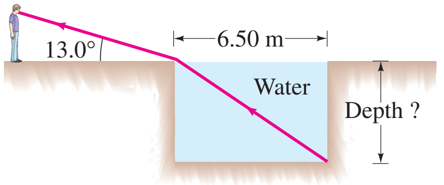


FIGURE 23–62 Problem 66.

67. The critical angle of a certain piece of plastic in air is $\theta_C = 37.8°$. What is the critical angle of the same plastic if it is immersed in water?

68. A pulse of light takes 2.63 ns (see Table 1–4) to travel 0.500 m in a certain material. Determine the material's index of refraction, and identify this material.

69. When an object is placed 60.0 cm from a certain converging lens, it forms a real image. When the object is moved to 40.0 cm from the lens, the image moves 10.0 cm farther from the lens. Find the focal length of this lens.

70. A 4.5-cm-tall object is placed 32 cm in front of a spherical mirror. It is desired to produce a virtual image that is upright and 3.5 cm tall. (*a*) What type of mirror should be used? (*b*) Where is the image located? (*c*) What is the focal length of the mirror? (*d*) What is the radius of curvature of the mirror?

71. Light is emitted from an ordinary lightbulb filament in wave-train bursts of about 10^{-8} s in duration. What is the length in space of such wave trains?

72. If the apex angle of a prism is $\phi = 75°$ (see Fig. 23–63), what is the minimum incident angle for a ray if it is to emerge from the opposite side (i.e., not be totally internally reflected), given $n = 1.58$?

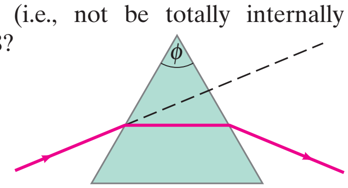


FIGURE 23–63 Problem 72.

73. An object is placed 18 cm from a certain mirror. The image is half the height of the object, inverted, and real. How far is the image from the mirror, and what is the radius of curvature of the mirror?

74. Light is incident on an equilateral glass prism at a 45.0° angle to one face, Fig. 23–64. Calculate the angle at which light emerges from the opposite face. Assume that $n = 1.54$.

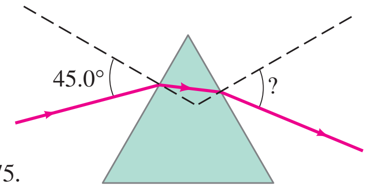


FIGURE 23–64
Problems 74 and 75.

75. Suppose a ray strikes the left face of the prism in Fig. 23–64 at 45.0° as shown, but is totally internally reflected at the opposite side. If the apex angle (at the top) is $\theta = 65.0°$, what can you say about the index of refraction of the prism?

76. (*a*) An object 37.5 cm in front of a certain lens is imaged 8.20 cm in front of that lens (on the same side as the object). What type of lens is this, and what is its focal length? Is the image real or virtual? (*b*) If the image were located, instead, 44.5 cm in front of the lens, what type of lens would it be and what focal length would it have?

77. How large is the image of the Sun on a camera sensor with (*a*) a 35-mm-focal-length lens, (*b*) a 50-mm-focal-length lens, and (*c*) a 105-mm-focal-length lens? The Sun has diameter 1.4×10^6 km, and it is 1.5×10^8 km away.

78. Figure 23–65 is a photograph of an eyeball with the image of a boy in a doorway. (*a*) Is the eye here acting as a lens or as a mirror? (*b*) Is the eye being viewed right side up or is the camera taking this photo upside down? (*c*) Explain, based on all possible images made by a convex mirror or lens.

FIGURE 23–65
Problem 78.

79. Which of the two lenses shown in Fig. 23–66 is converging, and which is diverging? Explain using ray diagrams and show how each image is formed.

FIGURE 23–66 Problem 79.

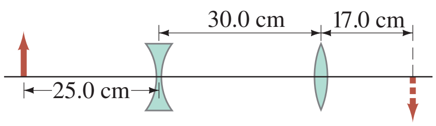
***80.** (*a*) Show that if two thin lenses of focal lengths f_1 and f_2 are placed in contact with each other, the focal length of the combination is given by $f_T = f_1 f_2/(f_1 + f_2)$. (*b*) Show that the power P of the combination of two lenses is the sum of their separate powers, $P = P_1 + P_2$.

***81.** Two converging lenses are placed 30.0 cm apart. The focal length of the lens on the right is 20.0 cm, and the focal length of the lens on the left is 15.0 cm. An object is placed to the left of the 15.0-cm-focal-length lens. A final image from both lenses is inverted and located halfway between the two lenses. How far to the left of the 15.0-cm-focal-length lens is the original object?

***82.** An object is placed 30.0 cm from a +5.0-D lens. A spherical mirror with focal length 25 cm is placed 75 cm behind the lens. Where is the final image? (Note that the mirror reflects light back through the lens.) Be sure to draw a diagram.

***83.** A small object is 25.0 cm from a diverging lens as shown in Fig. 23–67. A converging lens with a focal length of 12.0 cm is 30.0 cm to the right of the diverging lens. The two-lens system forms a real inverted image 17.0 cm to the right of the converging lens. What is the focal length of the diverging lens?

FIGURE 23–67
Problem 83.



Search and Learn

1. Both a converging lens and a concave mirror can produce virtual images that are larger than the object. Concave mirrors can be used as makeup mirrors, but converging lenses cannot be. (*a*) Draw ray diagrams to explain why not. (*b*) If a concave mirror has the same focal length as a converging lens, and an object is placed first at a distance of $\frac{1}{2}f$ from the lens and then at a distance of $\frac{1}{2}f$ from the mirror, how will the magnification of the object compare in the two cases?

2. (*a*) Did the person we see in Fig. 23–68 shoot the picture we are looking at? We see her in three different mirrors. Describe (*b*) what type of mirror each is, and (*c*) her position relative to the focal point and center of curvature.

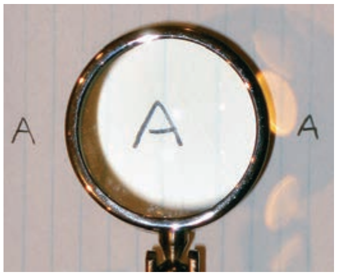
FIGURE 23–68 Search and Learn 2.

3. Justify the second part of sign convention 3, page 665, starting "Equivalently." Use ray diagrams for all possible situations. Cite Figures already in the text and draw any others needed.

4. Make a table showing the sign conventions for mirrors and lenses. Include the sign convention for the mirrors and lenses themselves and for the image and object heights and distances for each.

5. Figure 23–69 shows a converging lens held above three equal-sized letters A. In (a) the lens is 5 cm from the paper, and in (b) the lens is 15 cm from the paper. Estimate the focal length of the lens. What is the image position for each case?

(a)



(b)

FIGURE 23–69
Search and Learn 5.

ANSWERS TO EXERCISES

A: No.
B: (*b*).
C: Toward.
D: None.
E: No total internal reflection, $\theta_C > 45°$.
F: (*c*).
G: Closer to it.