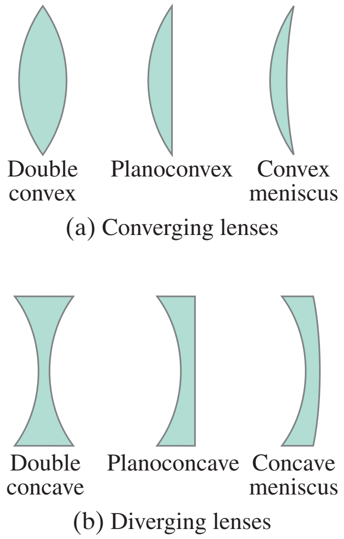
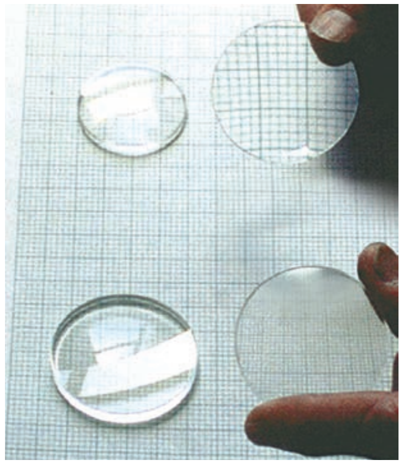


(c)



(d)

FIGURE 23–31 (a) Converging lenses and (b) diverging lenses, shown in cross section. Converging lenses are thicker at the center whereas diverging lenses are thicker at the edges. (c) Photo of a converging lens (on the left) and a diverging lens (right). (d) Converging lenses (above), and diverging lenses (below), lying flat, and raised off the paper to form images.

23–7 Thin Lenses; Ray Tracing

The most important simple optical device is the thin lens. The development of optical devices using lenses dates to the sixteenth and seventeenth centuries, although the earliest record of eyeglasses dates from the late thirteenth century. Today we find lenses in eyeglasses, cameras, magnifying glasses, telescopes, binoculars, microscopes, and medical instruments. A thin lens is usually circular, and its two faces are portions of a sphere. (Cylindrical faces are also possible, but we will concentrate on spherical.) The two faces can be concave, convex, or plane. Several types are shown in Figs. 23–31a and b in cross section. The importance of lenses is that they form images of objects—see Fig. 23–32.

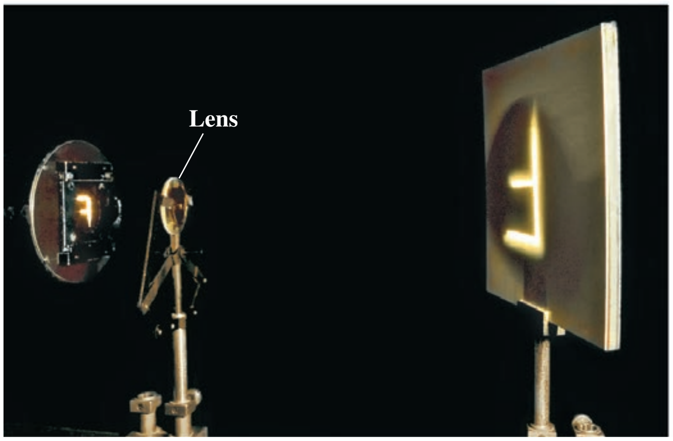


FIGURE 23–32 Converging lens (in holder) forms an image (large “F” on screen at right) of a bright object (illuminated “F” at the left).

Consider parallel rays striking the double convex lens shown in cross section in Fig. 23–33. We assume the lens is made of transparent material such as glass or transparent plastic with index of refraction greater than that of the air outside. The **axis** of a lens is a straight line passing through the center of the lens and perpendicular to its two surfaces (Fig. 23–33). From Snell’s law, we can see that each ray in Fig. 23–33 is bent toward the axis when the ray enters the lens and again when it leaves the lens at the back surface. (Note the dashed lines indicating the normals to each surface for the top ray.) If rays parallel to the axis fall on a thin lens, they will be focused to a point called the **focal point**, F. This will not be precisely true for a lens with spherical surfaces. But it will be very nearly true—that is, parallel rays will be focused to a tiny region that is nearly a point—if the diameter of the lens is small compared to the radii of curvature of the two lens surfaces. This criterion is satisfied by a **thin lens**, one that is very thin compared to its diameter, and we consider only thin lenses here.

FIGURE 23–33 Parallel rays are brought to a focus by a converging thin lens.

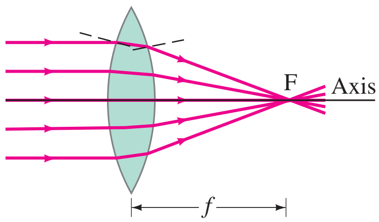


FIGURE 23–34 Image of the Sun burning wood.

FIGURE 23–35 Parallel rays at an angle are focused on the focal plane.

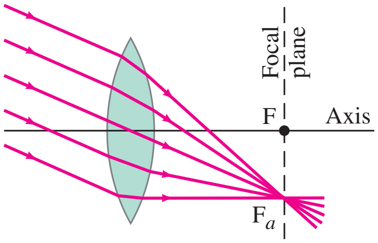
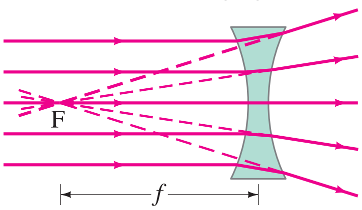


FIGURE 23–36 Diverging lens.



The rays from a point on a distant object are essentially parallel—see Fig. 23–12. Therefore we can say that *the focal point is the image point for an object at infinity on the lens axis*, Fig. 23–33. Thus, the focal point of a lens can be found by locating the point where the Sun's rays (or those from some other distant object) are brought to a sharp image, Fig. 23–34. The distance of the focal point from the center of the lens is called the **focal length**, f, Fig. 23–33. A lens can be turned around so that light can pass through it from the opposite side. The *focal length is the same on both sides*, as we shall see later, even if the curvatures of the two lens surfaces are different. If parallel rays fall on a lens at an angle, as in Fig. 23–35, they focus at a point F_a. The plane containing all focus points, such as F and F_a in Fig. 23–35, is called the **focal plane** of the lens.

Any lens (in air) that is thicker in the center than at the edges will make parallel rays converge to a point, and is called a **converging lens** (see Fig. 23–31a). Lenses that are thinner in the center than at the edges (Fig. 23–31b) are called **diverging lenses** because they make parallel light diverge, as shown in Fig. 23–36. The focal point, F, of a diverging lens is defined as that point from which refracted rays, originating from parallel incident rays, seem to emerge as shown in Fig. 23–36. And the distance from F to the center of the lens is called the **focal length**, f, just as for a converging lens.

EXERCISE F Return to Chapter-Opening Question 2, page 644, and answer it again now. Try to explain why you may have answered differently the first time.

Optometrists and ophthalmologists, instead of using the focal length, use the reciprocal of the focal length to specify the strength of eyeglass (or contact) lenses. This is called the **power**, P, of a lens:

$$P = \frac{1}{f}. \tag{23–7}$$

The unit for lens power is the **diopter** (D), which is an inverse meter: $1\text{ D} = 1\text{ m}^{-1}$. For example, a 20-cm-focal-length lens has a power $P = 1/(0.20\text{ m}) = 5.0\text{ D}$. We will mainly use the focal length, but we will refer again to the power of a lens when we discuss eyeglass lenses in Chapter 25.

The most important parameter of a lens is its focal length f, which is the same on both sides of the lens. For a converging lens, f can be measured by finding the image point for the Sun or other distant objects. Once f is known, the image position can be determined for any object. To find the image point by drawing rays would be difficult if we had to determine the refractive angles at the front surface of the lens and again at the back surface where the ray exits. We can save ourselves a lot of effort by making use of certain facts we already know, such as that a ray parallel to the axis of the lens passes (after refraction) through the focal point. To determine an image point, we can consider only the three rays indicated in Fig. 23–37, which uses an arrow (on the left) as the object, and a converging lens forming an image (dashed arrow) to the right. These rays, emanating from a single point on the object, are drawn as if the lens were infinitely thin, and we show only a single sharp bend at the center line of the lens instead of the refractions at each surface. These three rays are drawn as follows:

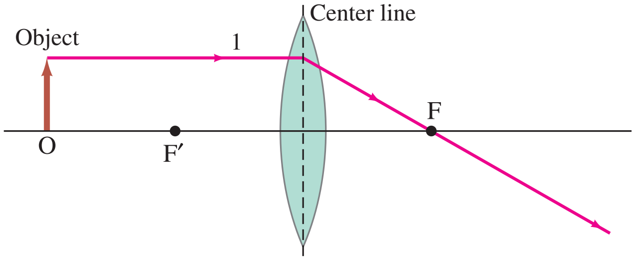
RAY DIAGRAM
Finding the image position formed by a thin lens

Ray 1 is drawn parallel to the axis, Fig. 23–37a; therefore it is refracted by the lens so that it passes along a line through the focal point F behind the lens.

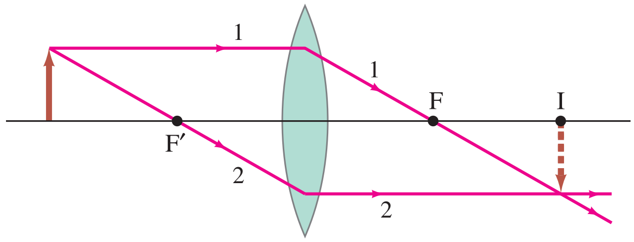
Ray 2 is drawn to pass through the other focal point F′ (front side of lens in Fig. 23–37) and emerge from the lens parallel to the axis, Fig. 23–37b. (In reverse it would be a parallel ray going left and passing through F′.)

Ray 3 is directed toward the very center of the lens, where the two surfaces are essentially parallel to each other, Fig. 23–37c. This ray therefore emerges from the lens at the same angle as it entered. The ray would be displaced slightly to one side, as we saw in Example 23–8; but since we assume the lens is thin, we draw ray 3 straight through as shown.

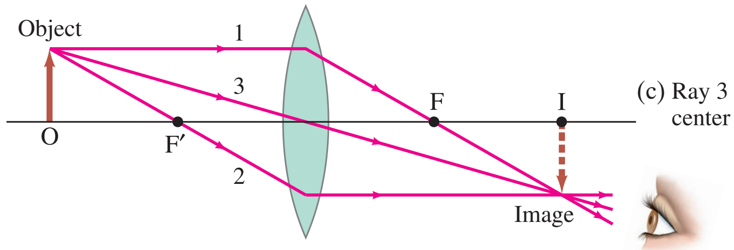
The point where these three rays cross is the image point for that object point. Actually, any two of these rays will suffice to locate the image point, but drawing the third ray can serve as a check.



(a) Ray 1 leaves one point on object going parallel to the axis, then refracts through focal point behind the lens.



(b) Ray 2 passes through F′ in front of the lens; therefore it is parallel to the axis behind the lens.



(c) Ray 3 passes straight through the center of the lens (assumed very thin).

FIGURE 23–37 Finding the image by ray tracing for a converging lens. Rays are shown leaving one point on the object (an arrow). Shown are the three most useful rays, leaving the tip of the object, for determining where the image of that point is formed. (Note that the focal points F and F′ on either side of the lens are the same distance f from the center of the lens.)

Using these three rays for one object point, we can find the image point for that point of the object (the top of the arrow in Fig. 23–37). The image points for all other points on the object can be found similarly to determine the complete image of the object. Because the rays actually pass through the image for the case shown in Fig. 23–37, it is a **real image** (see pages 647 and/or 651). The image could be detected by film or electronic sensor, and actually be seen on a white surface or screen placed at the position of the image (Fig. 23–38).

CONCEPTUAL EXAMPLE 23–11 **Half-blocked lens.** What happens to the image of an object if the top half of a lens is covered by a piece of cardboard?

RESPONSE Let us look at the rays in Fig. 23–37. If the top half (or any half of the lens) is blocked, you might think that half the image is blocked. But in Fig. 23–37c, we see how the rays used to create the "top" of the image pass through both the top and the bottom of the lens. Only three of many rays are shown—many more rays pass through the lens, and they can form the image. You don't lose the image. But covering part of the lens cuts down on the total light received and reduces the brightness of the image.

NOTE If the lens is partially blocked by your thumb, you may notice an out of focus image of part of that thumb.

Seeing the Image

The image can also be seen directly by the eye when the eye is placed behind the image, as shown in Fig. 23–37c, so that some of the rays diverging from each point on the image can enter the eye. We can see a sharp image only for rays *diverging* from each point on the image, because we see normal objects when diverging rays from each point enter the eye as shown in Fig. 23–1. A normal eye cannot focus converging rays; if your eye was positioned between points F and I in Fig. 23–37c, it would not see a clear image. (More about our eyes in Section 25–2.) Figure 23–38 shows an image seen (a) on a white surface and (b) directly by the eye (and a camera) placed behind the image. The eye can see both real and virtual images (see next page) as long as the eye is positioned so rays diverging from the image enter it.

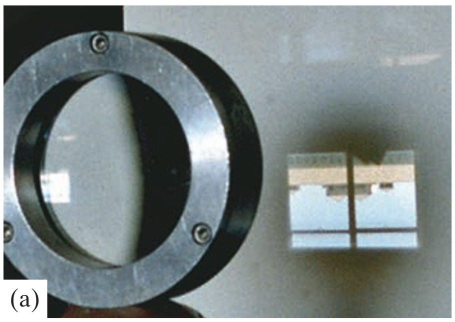


FIGURE 23–38 (a) A converging lens can form a real image (here of a distant building, upside down) on a white wall. (b) That same real image is also directly visible to the eye. [Figure 23–31d shows images (graph paper) seen by the eye made by both diverging and converging lenses.]

Diverging Lens

By drawing the same three rays emerging from a single object point, we can determine the image position formed by a diverging lens, as shown in Fig. 23–39. Note that ray 1 is drawn parallel to the axis, but does not pass through the focal point F′ behind the lens. Instead it seems to come (dashed line) from the focal point F in front of the lens. Ray 2 is directed toward F′ and is refracted parallel to the lens axis by the lens. Ray 3 passes directly through the center of the lens. The three refracted rays seem to emerge from a point on the left of the lens. This is the image point, I. Because the rays do not pass through the image, it is a **virtual image**. Note that the eye does not distinguish between real and virtual images—both are visible.

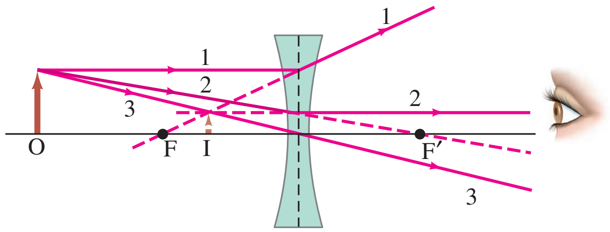


FIGURE 23–39 Finding the image by ray tracing for a diverging lens.

23–8 The Thin Lens Equation

We now derive an equation that relates the image distance to the object distance and the focal length of a thin lens. This equation will make the determination of image position quicker and more accurate than doing ray tracing. Let d_o be the object distance, the distance of the object from the center of the lens, and d_i be the image distance, the distance of the image from the center of the lens, Fig. 23–40.

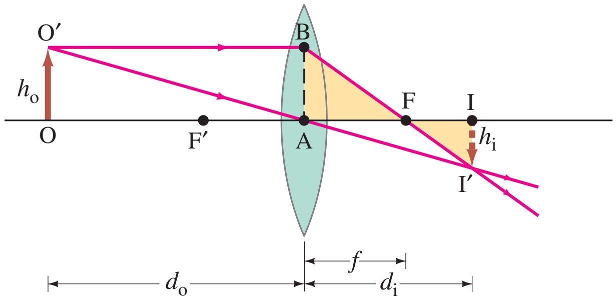


FIGURE 23–40 Deriving the lens equation for a converging lens.

Let h_o and h_i refer to the heights of the object and image. Consider the two rays shown in Fig. 23–40 for a converging lens, assumed to be very thin. The right triangles FI′I and FBA (highlighted in yellow) are similar because angle AFB equals angle IFI′; so

$$\frac{h_i}{h_o} = \frac{d_i - f}{f},$$

since length $AB = h_o$. Triangles OAO′ and IAI′ are similar as well. Therefore,

$$\frac{h_i}{h_o} = \frac{d_i}{d_o}.$$

We equate the right sides of these two equations (the left sides are the same), and divide by d_i to obtain

$$\frac{1}{f} - \frac{1}{d_i} = \frac{1}{d_o}$$

or

THIN LENS EQUATION

$$\boxed{\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}.} \qquad \textbf{(23–8)}$$

This is called the **thin lens equation**. It relates the image distance d_i to the object distance d_o and the focal length f. It is the most useful equation in geometric optics. (Interestingly, it is exactly the same as the mirror equation, Eq. 23–2.)

If the object is at infinity, then $1/d_o = 0$, so $d_i = f$. Thus the focal length is the image distance for an object at infinity, as mentioned earlier.

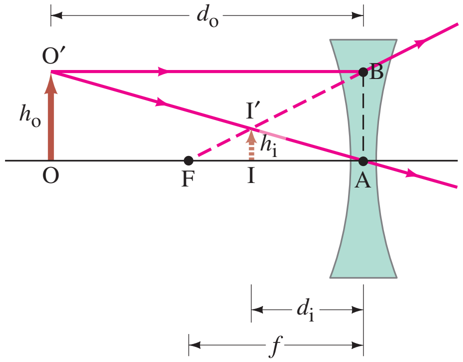


FIGURE 23–41 Deriving the lens equation for a diverging lens.

We can derive the lens equation for a diverging lens using Fig. 23–41. Triangles IAI′ and OAO′ are similar; and triangles IFI′ and AFB are similar. Thus (noting that length AB = h_o)

$$\frac{h_i}{h_o} = \frac{d_i}{d_o}$$

and

$$\frac{h_i}{h_o} = \frac{f - d_i}{f}.$$

When we equate the right sides of these two equations and simplify, we obtain

$$\frac{1}{d_o} - \frac{1}{d_i} = -\frac{1}{f}.$$

This equation becomes the same as Eq. 23–8 if we make f and d_i negative. That is, we take f to be *negative for a diverging lens*, and d_i negative when the image is on the same side of the lens as the light comes from. Thus Eq. 23–8 will be valid for both converging and diverging lenses, and for *all* situations, if we use the following **sign conventions**:

CAUTION
Focal length is negative for diverging lens

1. The focal length is positive for converging lenses and negative for diverging lenses.
2. The object distance is positive if the object is on the side of the lens from which the light is coming (this is always the case for real objects; but when lenses are used in combination, it might not be so: see Example 23–16); otherwise, it is negative.
3. The image distance is positive if the image is on the opposite side of the lens from where the light is coming; if it is on the same side, d_i is negative. Equivalently, the image distance is positive for a real image (Fig. 23–40) and negative for a virtual image (Fig. 23–41).
4. The height of the image, h_i, is positive if the image is upright, and negative if the image is inverted relative to the object. (h_o is always taken as upright and positive.)

PROBLEM SOLVING
SIGN CONVENTIONS for lenses

The **magnification**, m, of a lens is defined as the ratio of the image height to object height, $m = h_i/h_o$. From Figs. 23–40 and 23–41 and the conventions just stated (for which we will need a minus sign), we have

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}. \qquad \textbf{(23–9)}$$

For an upright image the magnification is positive, and for an inverted image the magnification is negative.

From sign convention 1, it follows that the power (Eq. 23–7) of a converging lens, in diopters, is positive, whereas the power of a diverging lens is negative. A converging lens is sometimes referred to as a **positive lens**, and a diverging lens as a **negative lens**.

Diverging lenses (see Fig. 23–41) always produce an upright virtual image for any real object, no matter where that object is. Converging lenses can produce real (inverted) images as in Fig. 23–40, or virtual (upright) images, depending on object position, as we shall see.

PROBLEM SOLVING

Thin Lenses

1. Draw a **ray diagram**, as precise as possible, but even a rough one can serve as confirmation of analytic results. Choose one point on the object and draw at least two, or preferably three, of the easy-to-draw rays described in Figs. 23–37 and 23–39. The image point is where the rays intersect.
2. For analytic solutions, solve for unknowns in the **thin lens equation** (Eq. 23–8) and the **magnification equation** (Eq. 23–9). The thin lens equation involves reciprocals—don't forget to take the reciprocal.
3. Follow the **sign conventions** listed just above.
4. Check that your analytic answers are **consistent** with your ray diagram.

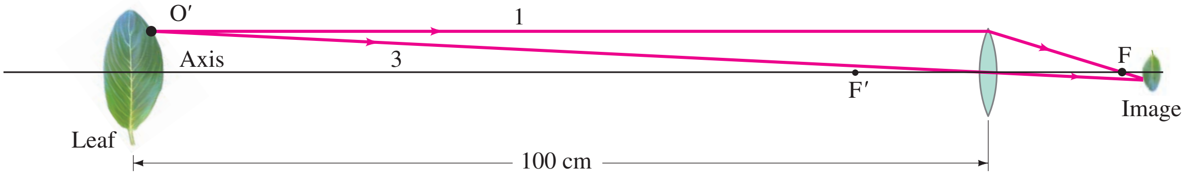


FIGURE 23–42 Example 23–12. (Not to scale.)

EXAMPLE 23–12 Image formed by converging lens. What is (*a*) the position, and (*b*) the size, of the image of a 7.6-cm-high leaf placed 1.00 m from a +50.0-mm-focal-length camera lens?

APPROACH We follow the steps of the Problem Solving Strategy explicitly.

SOLUTION

1. **Ray diagram.** Figure 23–42 is an approximate ray diagram, showing only rays 1 and 3 for a single point on the leaf. We see that the image ought to be a little behind the focal point F, to the right of the lens.
2. **Thin lens and magnification equations.** (*a*) We find the image position analytically using the thin lens equation, Eq. 23–8. The camera lens is converging, with $f = +5.00\text{ cm}$, and $d_o = 100\text{ cm}$, and so the thin lens equation gives

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{5.00\text{ cm}} - \frac{1}{100\text{ cm}} = \frac{20.0 - 1.0}{100\text{ cm}} = \frac{19.0}{100\text{ cm}}.$$

Then, taking the reciprocal,

$$d_i = \frac{100\text{ cm}}{19.0} = 5.26\text{ cm},$$

or 52.6 mm behind the lens.

(*b*) The magnification is

$$m = -\frac{d_i}{d_o} = -\frac{5.26\text{ cm}}{100\text{ cm}} = -0.0526,$$

so

$$h_i = mh_o = (-0.0526)(7.6\text{ cm}) = -0.40\text{ cm}.$$

The image is 4.0 mm high.
3. **Sign conventions.** The image distance d_i came out positive, so the image is behind the lens. The image height is $h_i = -0.40\text{ cm}$; the minus sign means the image is inverted.
4. **Consistency.** The analytic results of steps 2 and 3 are consistent with the ray diagram, Fig. 23–42: the image is behind the lens and inverted.

NOTE Part (*a*) tells us that the image is 2.6 mm farther from the lens than the image for an object at infinity, which would equal the focal length, 50.0 mm. Indeed, when focusing a camera lens, the closer the object is to the camera, the farther the lens must be from the sensor or film.

EXERCISE G If the leaf (object) of Example 23–12 is moved farther from the lens, does the image move closer to or farther from the lens? (Don't calculate!)

EXAMPLE 23–13 **Object close to converging lens.** An object is placed 10 cm from a 15-cm-focal-length converging lens. Determine the image position and size (*a*) analytically, and (*b*) using a ray diagram.

APPROACH The object is within the focal point—closer to the lens than the focal point F as $d_o < f$. We first use Eqs. 23–8 and 23–9 to obtain an analytic solution, and then confirm with a ray diagram using the special rays 1, 2, and 3 for a single object point.

SOLUTION (*a*) Given $f = 15\text{ cm}$ and $d_o = 10\text{ cm}$, then

$$\frac{1}{d_i} = \frac{1}{15\text{ cm}} - \frac{1}{10\text{ cm}} = \frac{2-3}{30\text{ cm}} = -\frac{1}{30\text{ cm}},$$

and $d_i = -30\text{ cm}$. (Remember to take the reciprocal!) Because d_i is negative, the image must be virtual and on the same side of the lens as the object (sign convention 3, page 665). The magnification

$$m = -\frac{d_i}{d_o} = -\frac{-30\text{ cm}}{10\text{ cm}} = 3.0.$$

The image is three times as large as the object and is upright. This lens is being used as a magnifying glass, which we discuss in more detail in Section 25–3.

(*b*) The ray diagram is shown in Fig. 23–43 and confirms the result in part (*a*). We choose point O′ on the top of the object and draw ray 1. Ray 2, however, may take some thought: if we draw it heading toward F′, it is going the wrong way—so we have to draw it as if coming from F′ (and so dashed), striking the lens, and then going out parallel to the lens axis. We project it backward, with a dashed line, as we must do also for ray 1, in order to find where they cross. Ray 3 is drawn through the lens center, and it crosses the other two rays at the image point, I′.

NOTE From Fig. 23–43 we can see that, when an object is placed between a converging lens and its focal point, the image is virtual.

CAUTION
Don't forget to take the reciprocal

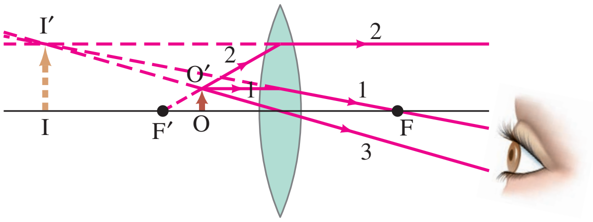


FIGURE 23–43 An object placed within the focal point of a converging lens produces a virtual image. Example 23–13.

EXAMPLE 23–14 **Diverging lens.** Where must a small insect be placed if a 25-cm-focal-length diverging lens is to form a virtual image 20 cm from the lens, on the same side as the object?

APPROACH The ray diagram is basically that of Fig. 23–41 because our lens here is diverging and our image is given as in front of the lens within the focal distance. (It would be a valuable exercise to draw the ray diagram to scale, precisely, now.) The insect's distance, d_o, can be calculated using the thin lens equation.

SOLUTION The lens is diverging, so f is negative: $f = -25\text{ cm}$. The image distance must be negative too because the image is in front of the lens (sign conventions), so $d_i = -20\text{ cm}$. The lens equation, Eq. 23–8, gives

$$\frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i} = -\frac{1}{25\text{ cm}} + \frac{1}{20\text{ cm}} = \frac{-4+5}{100\text{ cm}} = \frac{1}{100\text{ cm}}.$$

So the object must be 100 cm in front of the lens.