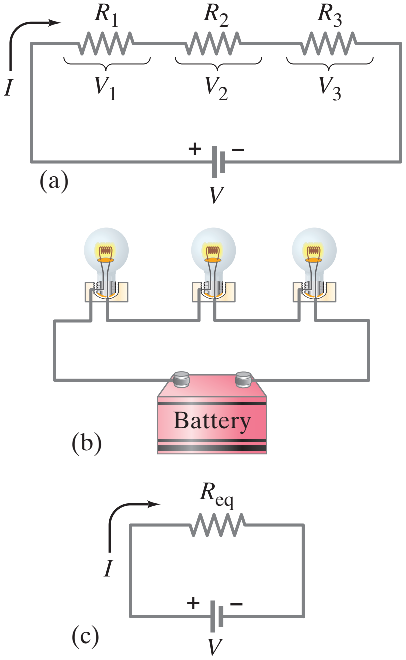
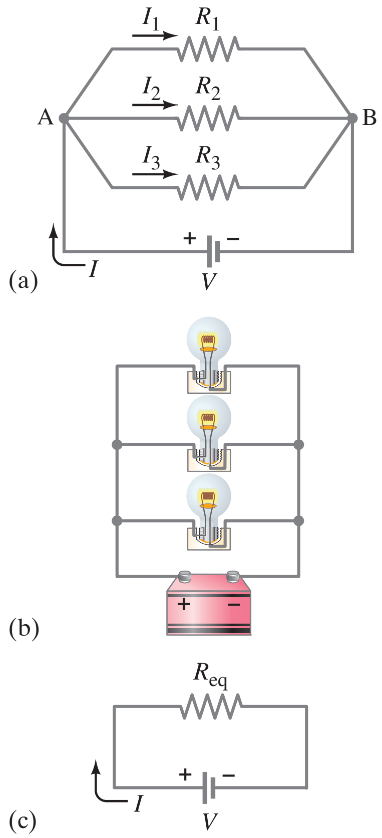


FIGURE 19–3 (a) Resistances connected in series. (b) Resistances could be lightbulbs, or any other type of resistance. (c) Equivalent single resistance R_{eq} that draws the same current: $R_{eq} = R_1 + R_2 + R_3$.

FIGURE 19–4 (a) Resistances connected in parallel. (b) Resistances could be lightbulbs. (c) The equivalent circuit with R_{eq} obtained from Eq. 19–4:
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.$$



Unless stated otherwise, we assume the battery's internal resistance is negligible, and the battery voltage given is its terminal voltage, which we will usually write as V rather than V_{ab}. Do not confuse V (italic) for voltage, with V (not italic) for the volt unit.

19–2 Resistors in Series and in Parallel

When two or more resistors are connected end to end along a single path as shown in Fig. 19–3a, they are said to be connected in **series**. The resistors could be simple resistors as were pictured in Fig. 18–11, or they could be lightbulbs (Fig. 19–3b), or heating elements, or other resistive devices. Any charge that passes through R_1 in Fig. 19–3a will also pass through R_2 and then R_3. Hence the same current I passes through each resistor. (If it did not, this would imply that either charge was not conserved, or that charge was accumulating at some point in the circuit, which does not happen in the steady state.)

We let V represent the potential difference (voltage) across all three resistors in Fig. 19–3a. We assume all other resistance in the circuit can be ignored, so V equals the terminal voltage supplied by the battery. We let V_1, V_2, and V_3 be the potential differences across each of the resistors, R_1, R_2, and R_3, respectively. From Ohm's law, $V = IR$, we can write $V_1 = IR_1$, $V_2 = IR_2$, and $V_3 = IR_3$. Because the resistors are connected end to end, energy conservation tells us that the total voltage V is equal to the sum of the voltages across each resistor:

$$V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3. \qquad \text{[series]} \quad \textbf{(19–2)}$$

Now let us determine the equivalent single resistance R_{eq} that would draw the same current I as our combination of three resistors in series; see Fig. 19–3c. Such a single resistance R_{eq} would be related to V by

$$V = IR_{eq}.$$

We equate this expression with Eq. 19–2, $V = I(R_1 + R_2 + R_3)$, and find

$$R_{eq} = R_1 + R_2 + R_3. \qquad \text{[series]} \quad \textbf{(19–3)}$$

When we put several resistances in series, the total or equivalent resistance is the sum of the separate resistances. (Sometimes we call it "net resistance.") This sum applies to any number of resistances in series. Note that when you add more resistance to the circuit, the current through the circuit will decrease. For example, if a 12-V battery is connected to a 4-Ω resistor, the current will be 3 A. But if the 12-V battery is connected to three 4-Ω resistors in series, the total resistance is 12 Ω and the current through the entire circuit will be only 1 A.

Another way to connect resistors is in **parallel**, so that the current from the source splits into separate branches or paths (Fig. 19–4a). Wiring in houses and buildings is arranged so all electric devices are in parallel, as we saw in Chapter 18, Fig. 18–20. With parallel wiring, if you disconnect one device (say, R_1 in Fig. 19–4a), the current to the other devices is not interrupted. Compare to a series circuit, where if one device (say, R_1 in Fig. 19–3a) is disconnected, the current *is* stopped to all others.

In a parallel circuit, Fig. 19–4a, the total current I that leaves the battery splits into three separate paths. We let I_1, I_2, and I_3 be the currents through each of the resistors, R_1, R_2, and R_3, respectively. Because *electric charge is conserved*, the current I flowing into junction A (where the different wires or conductors meet, Fig. 19–4a) must equal the current flowing out of the junction. Thus

$$I = I_1 + I_2 + I_3. \qquad \text{[parallel]}$$

When resistors are connected in parallel, each has the same voltage across it. (Indeed, any two points in a circuit connected by a wire of negligible resistance are at the same potential.) Hence the full voltage of the battery is applied to each resistor in Fig. 19–4a. Applying Ohm's law to each resistor, we have

$$I_1 = \frac{V}{R_1}, \quad I_2 = \frac{V}{R_2}, \quad \text{and} \quad I_3 = \frac{V}{R_3}.$$

Let us now determine what single resistor R_{eq} (Fig. 19–4c) will draw the same current I as these three resistances in parallel. This equivalent resistance R_{eq} must satisfy Ohm's law too:

$$I = \frac{V}{R_{eq}}.$$

We now combine the equations above:

$$I = I_1 + I_2 + I_3,$$

$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}.$$

When we divide out the V from each term, we have

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}. \qquad \text{[parallel]} \quad \textbf{(19–4)}$$

For example, suppose you connect two 4-Ω loudspeakers in parallel to a single set of output terminals of an amplifier. The equivalent resistance of the two 4-Ω "resistors" in parallel is

$$\frac{1}{R_{eq}} = \frac{1}{4\,\Omega} + \frac{1}{4\,\Omega} = \frac{2}{4\,\Omega} = \frac{1}{2\,\Omega},$$

and so $R_{eq} = 2\,\Omega$. Thus the net (or equivalent) resistance is *less* than each single resistance. This may at first seem surprising. But remember that when you connect resistors in parallel, you are giving the current additional paths to follow. Hence the net resistance will be less.†

Equations 19–3 and 19–4 make good sense. Recalling Eq. 18–3 for resistivity, $R = \rho\ell/A$, we see that placing resistors in series effectively increases the length and therefore the resistance; putting resistors in parallel effectively increases the area through which current flows, thus reducing the overall resistance.

Note that whenever a group of resistors is replaced by the equivalent resistance, current and voltage and power in the rest of the circuit are unaffected.

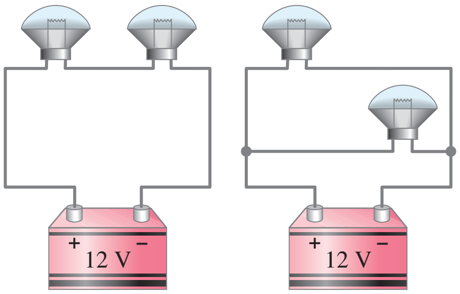
EXERCISE A You have a 10-Ω and a 15-Ω resistor. What is the smallest and largest equivalent resistance that you can make with these two resistors?

CONCEPTUAL EXAMPLE 19–2 **Series or parallel?** (*a*) The lightbulbs in Fig. 19–5 are identical. Which configuration produces more light? (*b*) Which way do you think the headlights of a car are wired? Ignore change of filament resistance R with current.

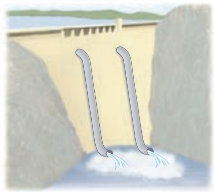
RESPONSE (*a*) The equivalent resistance of the parallel circuit is found from Eq. 19–4, $1/R_{eq} = 1/R + 1/R = 2/R$. Thus $R_{eq} = R/2$. The parallel combination then has lower resistance $(= R/2)$ than the series combination $(R_{eq} = R + R = 2R)$. There will be more total current in the parallel configuration (2), since $I = V/R_{eq}$ and V is the same for both circuits. The total power transformed, which is related to the light produced, is $P = IV$, so the greater current in (2) means more light is produced.

(*b*) Headlights are wired in parallel (2), because if one bulb goes out, the other bulb can stay lit. If they were in series (1), when one bulb burned out (the filament broke), the circuit would be open and no current would flow, so neither bulb would light.

FIGURE 19–5 Example 19–2.



EXERCISE B Return to the Chapter-Opening Question, page 526, and answer it again now. Try to explain why you may have answered differently the first time.



†An analogy may help. Consider two identical pipes taking in water near the top of a dam and releasing it at the bottom as shown in the figure to the right. If both pipes are open, rather than only one, twice as much water will flow through. That is, the net resistance to the flow of water will be reduced by half with two equal pipes open, just as for electrical resistors in parallel.

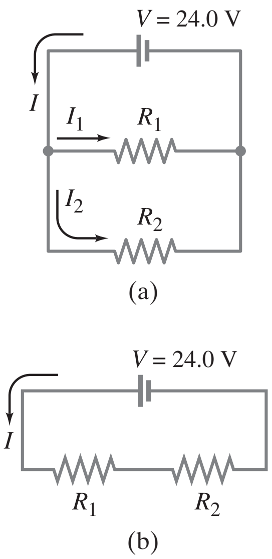


FIGURE 19–6 Example 19–3.

EXAMPLE 19–3 **Series and parallel resistors.** Two 100-Ω resistors are connected (*a*) in parallel, and (*b*) in series, to a 24.0-V battery (Fig. 19–6). What is the current through each resistor and what is the equivalent resistance of each circuit?

APPROACH We use Ohm's law and the ideas just discussed for series and parallel connections to get the current in each case. We can also use Eqs. 19–3 and 19–4.

SOLUTION (*a*) Any given charge (or electron) can flow through only one or the other of the two resistors in Fig. 19–6a. Just as a river may break into two streams when going around an island, here too the total current I from the battery (Fig. 19–6a) splits to flow through each resistor, so I equals the sum of the separate currents through the two resistors:

$$I = I_1 + I_2.$$

The potential difference across each resistor is the battery voltage $V = 24.0\text{ V}$. Applying Ohm's law to each resistor gives

$$I = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2} = \frac{24.0\text{ V}}{100\,\Omega} + \frac{24.0\text{ V}}{100\,\Omega} = 0.24\text{ A} + 0.24\text{ A} = 0.48\text{ A}.$$

The equivalent resistance is

$$R_{\text{eq}} = \frac{V}{I} = \frac{24.0\text{ V}}{0.48\text{ A}} = 50\,\Omega.$$

We could also have obtained this result from Eq. 19–4:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{100\,\Omega} + \frac{1}{100\,\Omega} = \frac{2}{100\,\Omega} = \frac{1}{50\,\Omega},$$

so $R_{\text{eq}} = 50\,\Omega$.

(*b*) All the current that flows out of the battery passes first through R_1 and then through R_2 because they lie along a single path, Fig. 19–6b. So the current I is the same in both resistors; the potential difference V across the battery equals the total change in potential across the two resistors:

$$V = V_1 + V_2.$$

Ohm's law gives $V = IR_1 + IR_2 = I(R_1 + R_2)$. Hence

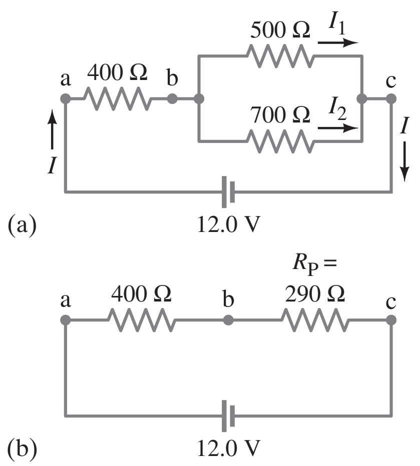
$$I = \frac{V}{R_1 + R_2} = \frac{24.0\text{ V}}{100\,\Omega + 100\,\Omega} = 0.120\text{ A}.$$

The equivalent resistance, using Eq. 19–3, is $R_{\text{eq}} = R_1 + R_2 = 200\,\Omega$. We can also get R_{eq} by thinking from the point of view of the battery: the total resistance R_{eq} must equal the battery voltage divided by the current it delivers:

$$R_{\text{eq}} = \frac{V}{I} = \frac{24.0\text{ V}}{0.120\text{ A}} = 200\,\Omega.$$

NOTE The voltage across R_1 is $V_1 = IR_1 = (0.120\text{ A})(100\,\Omega) = 12.0\text{ V}$, and that across R_2 is $V_2 = IR_2 = 12.0\text{ V}$, each being half of the battery voltage. A simple circuit like Fig. 19–6b is thus often called a simple **voltage divider**.

FIGURE 19–7 (a) Circuit for Examples 19–4 and 19–5. (b) Equivalent circuit, showing the equivalent resistance of 290 Ω for the two parallel resistors in (a).



EXAMPLE 19–4 **Circuit with series and parallel resistors.** How much current is drawn from the battery shown in Fig. 19–7a?

APPROACH The current I that flows out of the battery all passes through the 400-Ω resistor, but then it splits into I_1 and I_2 passing through the 500-Ω and 700-Ω resistors. The latter two resistors are in parallel with each other. We look for something that we already know how to treat. So let's start by finding the equivalent resistance, R_{P}, of the parallel resistors, 500 Ω and 700 Ω. Then we can consider this R_{P} to be in series with the 400-Ω resistor.

SOLUTION The equivalent resistance, R_{P}, of the 500-Ω and 700-Ω resistors in parallel is

$$\frac{1}{R_{\text{P}}} = \frac{1}{500\,\Omega} + \frac{1}{700\,\Omega} = 0.0020\,\Omega^{-1} + 0.0014\,\Omega^{-1} = 0.0034\,\Omega^{-1}.$$

This is $1/R_{\text{P}}$, so we take the reciprocal to find R_{P}.

It is a common mistake to forget to take this reciprocal. The units of reciprocal ohms, Ω^{-1}, are a reminder. Thus

$$R_P = \frac{1}{0.0034\ \Omega^{-1}} = 290\ \Omega.$$

This 290 Ω is the equivalent resistance of the two parallel resistors, and is in series with the 400-Ω resistor (see equivalent circuit, Fig. 19–7b). To find the total equivalent resistance R_{eq}, we add the 400-Ω and 290-Ω resistances, since they are in series:

$$R_{eq} = 400\ \Omega + 290\ \Omega = 690\ \Omega.$$

The total current flowing from the battery is then

$$I = \frac{V}{R_{eq}} = \frac{12.0\ \text{V}}{690\ \Omega} = 0.0174\ \text{A} \approx 17\ \text{mA}.$$

NOTE This I is also the current flowing through the 400-Ω resistor, but not through the 500-Ω and 700-Ω resistors (both currents are less—see the next Example).

CAUTION

Remember to take the reciprocal

EXAMPLE 19–5 Current in one branch. What is the current I_1 through the 500-Ω resistor in Fig. 19–7a?

APPROACH We need the voltage across the 500-Ω resistor, which is the voltage between points b and c in Fig. 19–7a, and we call it V_{bc}. Once V_{bc} is known, we can apply Ohm's law, $V = IR$, to get the current. First we find the voltage across the 400-Ω resistor, V_{ab}, since we know that 17.4 mA passes through it (Example 19–4).

SOLUTION V_{ab} can be found using $V = IR$:

$$V_{ab} = (0.0174\ \text{A})(400\ \Omega) = 7.0\ \text{V}.$$

The total voltage across the network of resistors is $V_{ac} = 12.0\ \text{V}$, so V_{bc} must be $12.0\ \text{V} - 7.0\ \text{V} = 5.0\ \text{V}$. Ohm's law gives the current I_1 through the 500-Ω resistor:

$$I_1 = \frac{5.0\ \text{V}}{500\ \Omega} = 1.0 \times 10^{-2}\ \text{A} = 10\ \text{mA}.$$

This is the answer we wanted. We can also calculate the current I_2 through the 700-Ω resistor since the voltage across it is also 5.0 V:

$$I_2 = \frac{5.0\ \text{V}}{700\ \Omega} = 7\ \text{mA}.$$

NOTE When I_1 combines with I_2 to form the total current I (at point c in Fig. 19–7a), their sum is $10\ \text{mA} + 7\ \text{mA} = 17\ \text{mA}$. This equals the total current I as calculated in Example 19–4, as it should.

CONCEPTUAL EXAMPLE 19–6 Bulb brightness in a circuit. The circuit in Fig. 19–8 has three identical lightbulbs, each of resistance R. (*a*) When switch S is closed, how will the brightness of bulbs A and B compare with that of bulb C? (*b*) What happens when switch S is opened? Use a minimum of mathematics.

RESPONSE (*a*) With switch S closed, the current that passes through bulb C must split into two equal parts when it reaches the junction leading to bulbs A and B because the resistance of bulb A equals that of B. Thus, A and B each receive half of C's current; A and B will be equally bright, but less bright than C ($P = I^2R$).
(*b*) When the switch S is open, no current can flow through bulb A, so it will be dark. Now, the same current passes through bulbs B and C, so B and C will be equally bright. The equivalent resistance of this circuit ($= R + R$) is greater than that of the circuit with the switch closed, so the current leaving the battery is reduced. Thus, bulb C will be dimmer when we open the switch, but bulb B will be brighter because it gets more current when the switch is open (you may want to use some mathematics here).

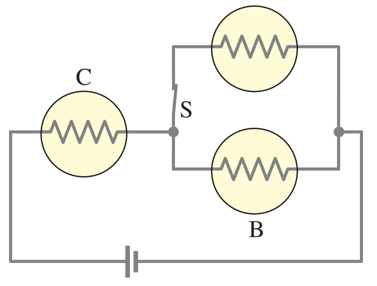
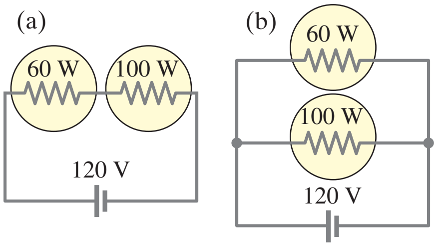


FIGURE 19–8 Example 19–6, three identical lightbulbs. Each yellow circle with —WW— inside represents a lightbulb and its resistance.

FIGURE 19–9 Exercise C.



EXERCISE C A 100-W, 120-V lightbulb and a 60-W, 120-V lightbulb are connected in two different ways as shown in Fig. 19–9. In each case, which bulb glows more brightly? Ignore change of filament resistance with current (and temperature).

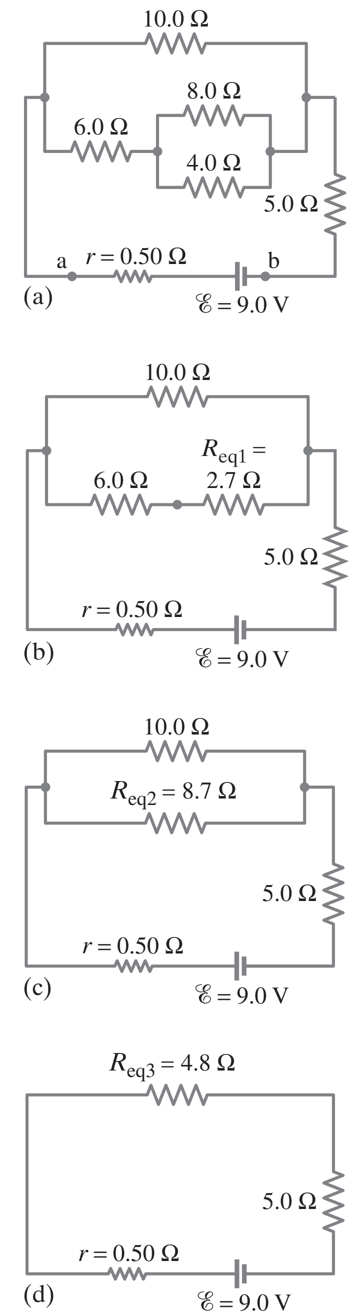


FIGURE 19–10 Circuit for Example 19–7, where r is the internal resistance of the battery.

EXAMPLE 19–7 **Analyzing a circuit.** A 9.0-V battery whose internal resistance r is 0.50 Ω is connected in the circuit shown in Fig. 19–10a. (*a*) How much current is drawn from the battery? (*b*) What is the terminal voltage of the battery? (*c*) What is the current in the 6.0-Ω resistor?

APPROACH To find the current out of the battery, we first need to determine the equivalent resistance R_{eq} of the entire circuit, including r, which we do by identifying and isolating simple series or parallel combinations of resistors. Once we find I from Ohm's law, $I = \mathscr{E}/R_{eq}$, we get the terminal voltage using $V_{ab} = \mathscr{E} - Ir$. For (*c*) we apply Ohm's law to the 6.0-Ω resistor.

SOLUTION (*a*) We want to determine the equivalent resistance of the circuit. But where do we start? We note that the 4.0-Ω and 8.0-Ω resistors are in parallel, and so have an equivalent resistance R_{eq1} given by

$$\frac{1}{R_{eq1}} = \frac{1}{8.0\,\Omega} + \frac{1}{4.0\,\Omega} = \frac{3}{8.0\,\Omega};$$

so $R_{eq1} = 2.7\,\Omega$. This 2.7 Ω is in series with the 6.0-Ω resistor, as shown in the equivalent circuit of Fig. 19–10b. The net resistance of the lower arm of the circuit is then

$$R_{eq2} = 6.0\,\Omega + 2.7\,\Omega = 8.7\,\Omega,$$

as shown in Fig. 19–10c. The equivalent resistance R_{eq3} of the 8.7-Ω and 10.0-Ω resistances in parallel is given by

$$\frac{1}{R_{eq3}} = \frac{1}{10.0\,\Omega} + \frac{1}{8.7\,\Omega} = 0.21\,\Omega^{-1},$$

so $R_{eq3} = \left(1/0.21\,\Omega^{-1}\right) = 4.8\,\Omega$. This 4.8 Ω is in series with the 5.0-Ω resistor and the 0.50-Ω internal resistance of the battery (Fig. 19–10d), so the total equivalent resistance R_{eq} of the circuit is $R_{eq} = 4.8\,\Omega + 5.0\,\Omega + 0.50\,\Omega = 10.3\,\Omega$. Hence the current drawn is

$$I = \frac{\mathscr{E}}{R_{eq}} = \frac{9.0\,\text{V}}{10.3\,\Omega} = 0.87\,\text{A}.$$

(*b*) The terminal voltage of the battery is

$$V_{ab} = \mathscr{E} - Ir = 9.0\,\text{V} - (0.87\,\text{A})(0.50\,\Omega) = 8.6\,\text{V}.$$

(*c*) Now we can work back and get the current in the 6.0-Ω resistor. It must be the same as the current through the 8.7 Ω shown in Fig. 19–10c (why?). The voltage across that 8.7 Ω will be the emf of the battery minus the voltage drops across r and the 5.0-Ω resistor: $V_{8.7} = 9.0\,\text{V} - (0.87\,\text{A})(0.50\,\Omega + 5.0\,\Omega)$. Applying Ohm's law, we get the current (call it I')

$$I' = \frac{9.0\,\text{V} - (0.87\,\text{A})(0.50\,\Omega + 5.0\,\Omega)}{8.7\,\Omega} = 0.48\,\text{A}.$$

This is the current through the 6.0-Ω resistor.

19–3 Kirchhoff's Rules

In the last few Examples we have been able to find the currents in circuits by combining resistances in series and parallel, and using Ohm's law. This technique can be used for many circuits. However, some circuits are too complicated for that analysis. For example, we cannot find the currents in each part of the circuit shown in Fig. 19–11 simply by combining resistances as we did before.

To deal with complicated circuits, we use Kirchhoff's rules, devised by G. R. Kirchhoff (1824–1887) in the mid-nineteenth century. There are two rules, and they are simply convenient applications of the laws of conservation of charge and energy.

FIGURE 19–11 Currents can be calculated using Kirchhoff's rules.