The space shuttle has released a parachute to reduce its speed quickly. The directions of the shuttle's velocity and acceleration are shown by the green ($\vec{\mathbf{v}}$) and gold ($\vec{\mathbf{a}}$) arrows.

Motion is described using the concepts of velocity and acceleration. In the case shown here, the velocity $\vec{\mathbf{v}}$ is to the right, in the direction of motion. The acceleration $\vec{\mathbf{a}}$ is in the opposite direction from the velocity $\vec{\mathbf{v}}$, which means the object is slowing down.

We examine in detail motion with constant acceleration, including the vertical motion of objects falling under gravity.

CHAPTER 2

Describing Motion: Kinematics in One Dimension

CHAPTER-OPENING QUESTION—Guess now!

[*Don't worry about getting the right answer now—you will get another chance later in the Chapter. See also p. 1 of Chapter 1 for more explanation.*]

Two small heavy balls have the same diameter but one weighs twice as much as the other. The balls are dropped from a second-story balcony at the exact same time. The time to reach the ground below will be:

(a) twice as long for the lighter ball as for the heavier one.
(b) longer for the lighter ball, but not twice as long.
(c) twice as long for the heavier ball as for the lighter one.
(d) longer for the heavier ball, but not twice as long.
(e) nearly the same for both balls.

CONTENTS

The motion of objects—baseballs, automobiles, joggers, and even the Sun and Moon—is an obvious part of everyday life. It was not until the sixteenth and seventeenth centuries that our modern understanding of motion was established. Many individuals contributed to this understanding, particularly Galileo Galilei (1564–1642) and Isaac Newton (1642–1727).

The study of the motion of objects, and the related concepts of force and energy, form the field called **mechanics**. Mechanics is customarily divided into two parts: **kinematics**, which is the description of how objects move, and **dynamics**, which deals with force and why objects move as they do. This Chapter and the next deal with kinematics.

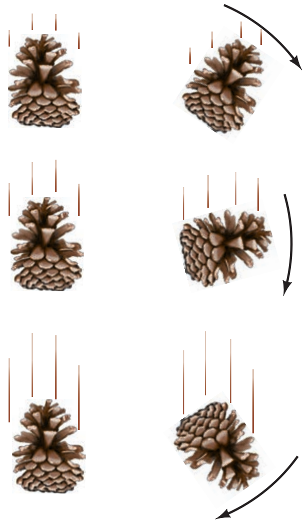


FIGURE 2–1 A falling pinecone undergoes (a) pure translation; (b) it is rotating as well as translating.

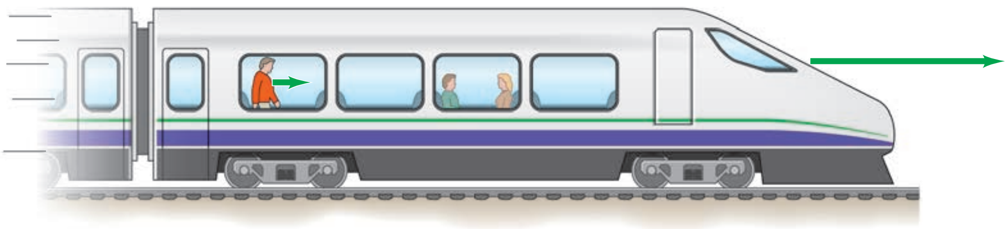
For now we only discuss objects that move without rotating (Fig. 2–1a). Such motion is called **translational motion**. In this Chapter we will be concerned with describing an object that moves along a straight-line path, which is one-dimensional translational motion. In Chapter 3 we will describe translational motion in two (or three) dimensions along paths that are not straight. (Rotation, shown in Fig. 2–1b, is discussed in Chapter 8.)

We will often use the concept, or *model*, of an idealized **particle** which is considered to be a mathematical **point** with no spatial extent (no size). A point particle can undergo only translational motion. The particle model is useful in many real situations where we are interested only in translational motion and the object's size is not significant. For example, we might consider a billiard ball, or even a spacecraft traveling toward the Moon, as a particle for many purposes.

2–1 Reference Frames and Displacement

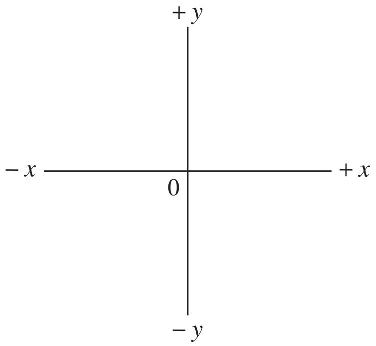
Any measurement of position, distance, or speed must be made with respect to a **reference frame**, or **frame of reference**. For example, while you are on a train traveling at 80 km/h, suppose a person walks past you toward the front of the train at a speed of, say, 5 km/h (Fig. 2–2). This 5 km/h is the person's speed with respect to the train as frame of reference. With respect to the ground, that person is moving at a speed of $80\,\mathrm{km/h} + 5\,\mathrm{km/h} = 85\,\mathrm{km/h}$. It is always important to specify the frame of reference when stating a speed. In everyday life, we usually mean "with respect to the Earth" without even thinking about it, but the reference frame must be specified whenever there might be confusion.

FIGURE 2–2 A person walks toward the front of a train at 5 km/h. The train is moving 80 km/h with respect to the ground, so the walking person's speed, relative to the ground, is 85 km/h.



When specifying the motion of an object, it is important to specify not only the speed but also the direction of motion. Often we can specify a direction by using north, east, south, and west, and by "up" and "down." In physics, we often draw a set of **coordinate axes**, as shown in Fig. 2–3, to represent a frame of reference. We can always place the origin 0, and the directions of the x and y axes, as we like for convenience. The x and y axes are always perpendicular to each other. The **origin** is where $x = 0$, $y = 0$. Objects positioned to the right of the origin of coordinates (0) on the x axis have an x coordinate which we almost always choose to be positive; then points to the left of 0 have a negative x coordinate. The position along the y axis is usually considered positive when above 0, and negative when below 0, although the reverse convention can be used if convenient. Any point on the plane can be specified by giving its x and y coordinates. In three dimensions, a z axis perpendicular to the x and y axes is added.

FIGURE 2–3 Standard set of xy coordinate axes, sometimes called "rectangular coordinates."



For one-dimensional motion, we often choose the x axis as the line along which the motion takes place. Then the **position** of an object at any moment is given by its x coordinate. If the motion is vertical, as for a dropped object, we usually use the y axis.

We need to make a distinction between the *distance* an object has traveled and its **displacement**, which is defined as the *change in position* of the object. That is, *displacement is how far the object is from its starting point*. To see the distinction between total distance and displacement, imagine a person walking 70 m to the east and then turning around and walking back (west) a distance of 30 m (see Fig. 2–4). The total *distance* traveled is 100 m, but the *displacement* is only 40 m since the person is now only 40 m from the starting point.

CAUTION

The displacement may not equal the total distance traveled

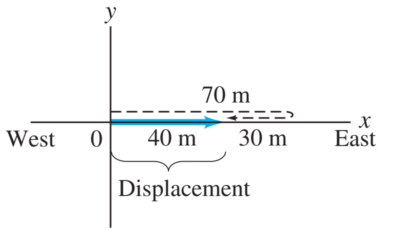


FIGURE 2–4 A person walks 70 m east, then 30 m west. The total distance traveled is 100 m (path is shown dashed in black); but the displacement, shown as a solid blue arrow, is 40 m to the east.

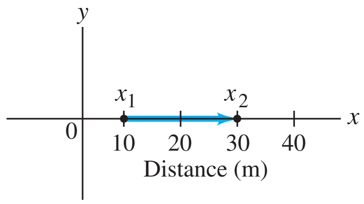
Displacement is a quantity that has both magnitude and direction. Such quantities are called **vectors**, and are represented by arrows in diagrams. For example, in Fig. 2–4, the blue arrow represents the displacement whose magnitude is 40 m and whose direction is to the right (east).

We will deal with vectors more fully in Chapter 3. For now, we deal only with motion in one dimension, along a line. In this case, vectors which point in one direction will be positive (typically to the right along the x axis). Vectors that point in the opposite direction will have a negative sign in front of their magnitude.

Consider the motion of an object over a particular time interval. Suppose that at some initial time, call it t_1, the object is on the x axis at the position x_1 in the coordinate system shown in Fig. 2–5. At some later time, t_2, suppose the object has moved to position x_2. The displacement of our object is $x_2 - x_1$, and is represented by the arrow pointing to the right in Fig. 2–5. It is convenient to write

$$\Delta x = x_2 - x_1,$$

FIGURE 2–5 The arrow represents the displacement $x_2 - x_1$. Distances are in meters.



where the symbol Δ (Greek letter delta) means "change in." Then Δx means "the change in x," or "change in position," which is the displacement. The **change in** any quantity means *the final value of that quantity, minus the initial value*. Suppose $x_1 = 10.0\text{ m}$ and $x_2 = 30.0\text{ m}$, as in Fig. 2–5. Then

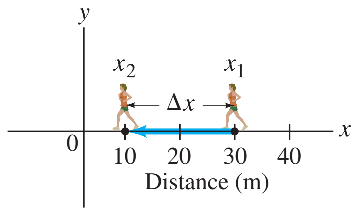
$$\Delta x = x_2 - x_1 = 30.0\text{ m} - 10.0\text{ m} = 20.0\text{ m},$$

so the displacement is 20.0 m in the positive direction, Fig. 2–5.

Now consider an object moving to the left as shown in Fig. 2–6. Here the object, a person, starts at $x_1 = 30.0\text{ m}$ and walks to the left to the point $x_2 = 10.0\text{ m}$. In this case her displacement is

$$\Delta x = x_2 - x_1 = 10.0\text{ m} - 30.0\text{ m} = -20.0\text{ m},$$

FIGURE 2–6 For the displacement $\Delta x = x_2 - x_1 = 10.0\text{ m} - 30.0\text{ m}$, the displacement vector points left.



and the blue arrow representing the vector displacement points to the left. For one-dimensional motion along the x axis, a vector pointing to the right is positive, whereas a vector pointing to the left has a negative sign.

EXERCISE A An ant starts at $x = 20\text{ cm}$ on a piece of graph paper and walks along the x axis to $x = -20\text{ cm}$. It then turns around and walks back to $x = -10\text{ cm}$. Determine (a) the ant's displacement and (b) the total distance traveled.

2–2 Average Velocity

An important aspect of the motion of a moving object is how *fast* it is moving—its speed or velocity.

The term "speed" refers to how far an object travels in a given time interval, regardless of direction. If a car travels 240 kilometers (km) in 3 hours (h), we say its average speed was 80 km/h. In general, the **average speed** of an object is defined as *the total distance traveled along its path divided by the time it takes to travel this distance*:

$$\text{average speed} = \frac{\text{distance traveled}}{\text{time elapsed}}. \qquad \textbf{(2–1)}$$

The terms "velocity" and "speed" are often used interchangeably in ordinary language. But in physics we make a distinction between the two. Speed is simply a positive number, with units. **Velocity**, on the other hand, is used to signify both the *magnitude* (numerical value) of how fast an object is moving and also the *direction* in which it is moving. Velocity is therefore a *vector*.

There is a second difference between speed and velocity: namely, the *average velocity* is defined in terms of *displacement*, rather than total distance traveled:

$$\text{average velocity} = \frac{\text{displacement}}{\text{time elapsed}} = \frac{\text{final position} - \text{initial position}}{\text{time elapsed}}.$$

CAUTION
Average speed is not necessarily equal to the magnitude of the average velocity

Average speed and average velocity have the same magnitude when the motion is all in one direction. In other cases, they may differ: recall the walk we described earlier, in Fig. 2–4, where a person walked 70 m east and then 30 m west. The total distance traveled was $70\,\text{m} + 30\,\text{m} = 100\,\text{m}$, but the displacement was 40 m. Suppose this walk took 70 s to complete. Then the average speed was:

$$\frac{\text{distance}}{\text{time elapsed}} = \frac{100\,\text{m}}{70\,\text{s}} = 1.4\,\text{m/s}.$$

The magnitude of the average velocity, on the other hand, was:

$$\frac{\text{displacement}}{\text{time elapsed}} = \frac{40\,\text{m}}{70\,\text{s}} = 0.57\,\text{m/s}.$$

To discuss one-dimensional motion of an object in general, suppose that at some moment in time, call it t_1, the object is on the x axis at position x_1 in a coordinate system, and at some later time, t_2, suppose it is at position x_2. The **elapsed time** (= change in time) is $\Delta t = t_2 - t_1$; during this time interval the displacement of our object is $\Delta x = x_2 - x_1$. Then the **average velocity**, defined as *the displacement divided by the elapsed time*, can be written

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}, \qquad \text{[average velocity]} \quad \textbf{(2–2)}$$

where v stands for velocity and the bar ($^-$) over the v is a standard symbol meaning "average."

PROBLEM SOLVING
+ or − sign can signify the direction for linear motion

For one-dimensional motion in the usual case of the $+x$ axis to the right, note that if x_2 is less than x_1, the object is moving to the left, and then $\Delta x = x_2 - x_1$ is less than zero. The sign of the displacement, and thus of the average velocity, indicates the direction: the average velocity is positive for an object moving to the right along the x axis and negative when the object moves to the left. The direction of the average velocity is always the same as the direction of the displacement.

CAUTION
Time interval = elapsed time

It is always important to choose (and state) the *elapsed time*, or **time interval**, $t_2 - t_1$, the time that passes during our chosen period of observation.

EXAMPLE 2–1 **Runner's average velocity.** The position of a runner is plotted as moving along the x axis of a coordinate system. During a 3.00-s time interval, the runner's position changes from $x_1 = 50.0\,\text{m}$ to $x_2 = 30.5\,\text{m}$, as shown in Fig. 2–7. What is the runner's average velocity?

APPROACH We want to find the average velocity, which is the displacement divided by the elapsed time.

SOLUTION The displacement is

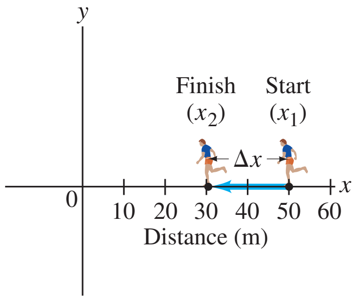
$$\begin{aligned}\Delta x &= x_2 - x_1\\ &= 30.5\,\text{m} - 50.0\,\text{m} = -19.5\,\text{m}.\end{aligned}$$

The elapsed time, or time interval, is given as $\Delta t = 3.00\,\text{s}$. The average velocity (Eq. 2–2) is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{-19.5\,\text{m}}{3.00\,\text{s}} = -6.50\,\text{m/s}.$$

The displacement and average velocity are negative, which tells us that the runner is moving to the left along the x axis, as indicated by the arrow in Fig. 2–7. The runner's average velocity is 6.50 m/s to the left.

FIGURE 2–7 Example 2–1. A person runs from $x_1 = 50.0\,\text{m}$ to $x_2 = 30.5\,\text{m}$. The displacement is $-19.5\,\text{m}$.



EXAMPLE 2–2 **Distance a cyclist travels.** How far can a cyclist travel in 2.5 h along a straight road if her average velocity is 18 km/h?

APPROACH We want to find the distance traveled, so we solve Eq. 2–2 for Δx.

SOLUTION In Eq. 2–2, $\bar{v} = \Delta x/\Delta t$, we multiply both sides by Δt and obtain

$$\Delta x = \bar{v}\,\Delta t = (18\,\text{km/h})(2.5\,\text{h}) = 45\,\text{km}.$$

EXAMPLE 2–3 **Car changes speed.** A car travels at a constant 50 km/h for 100 km. It then speeds up to 100 km/h and is driven another 100 km. What is the car's average speed for the 200-km trip?

APPROACH At 50 km/h, the car takes 2.0 h to travel 100 km. At 100 km/h it takes only 1.0 h to travel 100 km. We use the defintion of average velocity, Eq. 2–2.

SOLUTION Average velocity (Eq. 2–2) is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{100\,\text{km} + 100\,\text{km}}{2.0\,\text{h} + 1.0\,\text{h}} = 67\,\text{km/h}.$$

NOTE Averaging the two speeds, $(50\,\text{km/h} + 100\,\text{km/h})/2 = 75\,\text{km/h}$, gives a wrong answer. Can you see why? You must use the definition of $\bar{v}$, Eq. 2–2.

2–3 Instantaneous Velocity

If you drive a car along a straight road for 150 km in 2.0 h, the magnitude of your average velocity is 75 km/h. It is unlikely, though, that you were moving at precisely 75 km/h at every instant. To describe this situation we need the concept of *instantaneous velocity*, which is the velocity at any instant of time. (Its magnitude is the number, with units, indicated by a speedometer, Fig. 2–8.) More precisely, the **instantaneous velocity** at any moment is defined as *the average velocity over an infinitesimally short time interval*. That is, Eq. 2–2 is to be evaluated in the limit of Δt becoming extremely small, approaching zero. We can write the definition of instantaneous velocity, v, for one-dimensional motion as

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}. \qquad \text{[instantaneous velocity]} \quad \textbf{(2–3)}$$

The notation $\lim_{\Delta t \to 0}$ means the ratio $\Delta x/\Delta t$ is to be evaluated in the limit of Δt approaching zero.†

For instantaneous velocity we use the symbol v, whereas for average velocity we use $\bar{v}$, with a bar above. In the rest of this book, when we use the term "velocity" it will refer to instantaneous velocity. When we want to speak of the average velocity, we will make this clear by including the word "average."

Note that the *instantaneous speed* always equals the magnitude of the instantaneous velocity. Why? Because distance traveled and the magnitude of the displacement become the same when they become infinitesimally small.

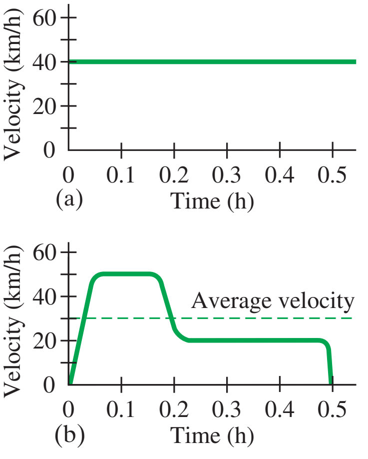
If an object moves at a uniform (that is, constant) velocity during a particular time interval, then its instantaneous velocity at any instant is the same as its average velocity (see Fig. 2–9a). But in many situations this is not the case. For example, a car may start from rest, speed up to 50 km/h, remain at that velocity for a time, then slow down to 20 km/h in a traffic jam, and finally stop at its destination after traveling a total of 15 km in 30 min. This trip is plotted on the graph of Fig. 2–9b. Also shown on the graph is the average velocity (dashed line), which is $\bar{v} = \Delta x/\Delta t = 15\,\text{km}/0.50\,\text{h} = 30\,\text{km/h}$.

Graphs are often useful for analysis of motion; we discuss additional insights graphs can provide as we go along, especially in Section 2–8.

EXERCISE B What is your instantaneous speed at the instant you turn around to move in the opposite direction? (*a*) Depends on how quickly you turn around; (*b*) always zero; (*c*) always negative; (*d*) none of the above.

FIGURE 2–8 Car speedometer showing mi/h in white, and km/h in orange.

FIGURE 2–9 Velocity of a car as a function of time: (a) at constant velocity; (b) with velocity varying in time.



†We do not simply set $\Delta t = 0$ in this definition, for then Δx would also be zero, and we would have an undetermined number. Rather, we consider the *ratio* $\Delta x/\Delta t$, as a whole. As we let Δt approach zero, Δx approaches zero as well. But the ratio $\Delta x/\Delta t$ approaches some definite value, which is the instantaneous velocity at a given instant.

2–4 Acceleration

An object whose velocity is changing is said to be accelerating. For instance, a car whose velocity increases in magnitude from zero to 80 km/h is accelerating. Acceleration specifies how *rapidly* the velocity of an object is changing.

Average acceleration is defined as the change in velocity divided by the time taken to make this change:

$$\text{average acceleration} = \frac{\text{change of velocity}}{\text{time elapsed}}.$$

In symbols, the **average acceleration**, $\bar{a}$, over a time interval $\Delta t = t_2 - t_1$, during which the velocity changes by $\Delta v = v_2 - v_1$, is defined as

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}. \quad \text{[average acceleration]} \quad \textbf{(2–4)}$$

We saw that velocity is a vector (it has magnitude and direction), so acceleration is a vector too. But for one dimensional motion, we need only use a plus or minus sign to indicate acceleration direction relative to a chosen coordinate axis. (Usually, right is +, left is −.)

The **instantaneous acceleration**, a, can be defined in analogy to instantaneous velocity as the average acceleration over an infinitesimally short time interval at a given instant:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}. \quad \text{[instantaneous acceleration]} \quad \textbf{(2–5)}$$

Here Δv is the very small change in velocity during the very short time interval Δt.

EXAMPLE 2–4 **Average acceleration.** A car accelerates on a straight road from rest to 75 km/h in 5.0 s, Fig. 2–10. What is the magnitude of its average acceleration?

APPROACH Average acceleration is the change in velocity divided by the elapsed time, 5.0 s. The car starts from rest, so $v_1 = 0$. The final velocity is $v_2 = 75$ km/h.

SOLUTION From Eq. 2–4, the average acceleration is

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{75\text{ km/h} - 0\text{ km/h}}{5.0\text{ s}} = 15\,\frac{\text{km/h}}{\text{s}}.$$

This is read as "fifteen kilometers per hour per second" and means that, on average, the velocity changed by 15 km/h during each second. That is, assuming the acceleration was constant, during the first second the car's velocity increased from zero to 15 km/h. During the next second its velocity increased by another 15 km/h, reaching a velocity of 30 km/h at $t = 2.0$ s, and so on. See Fig. 2–10.

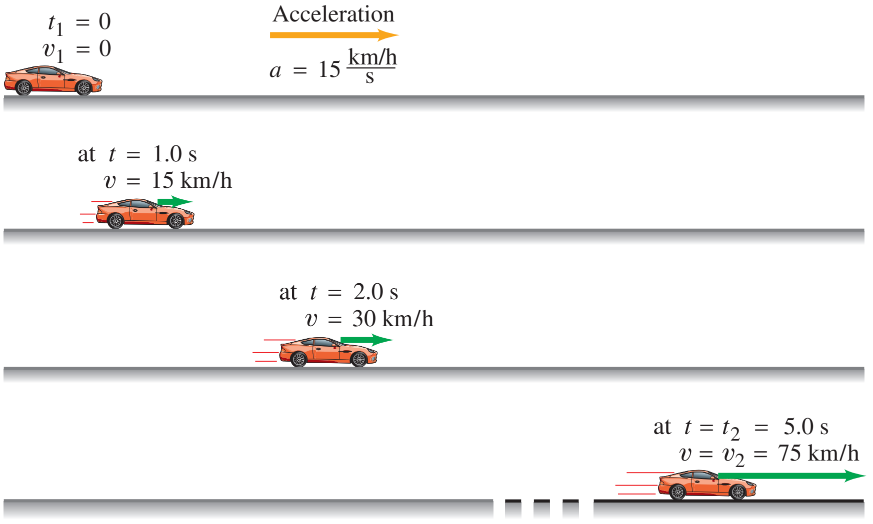


FIGURE 2–10 Example 2–4. The car is shown at the start with $v_1 = 0$ at $t_1 = 0$. The car is shown three more times, at $t = 1.0$ s, $t = 2.0$ s, and at the end of our time interval, $t_2 = 5.0$ s. The green arrows represent the velocity vectors, whose length represents the magnitude of the velocity at that moment. The acceleration vector is the orange arrow, whose magnitude is constant and equals 15 km/h/s or 4.2 m/s^2 (see top of next page). Distances are not to scale.

Our result in Example 2–4 contains two different time units: hours and seconds. We usually prefer to use only seconds. To do so we can change km/h to m/s (see Section 1–6, and Example 1–5):

$$75\ \mathrm{km/h} = \left(75\,\frac{\cancel{\mathrm{km}}}{\cancel{\mathrm{h}}}\right)\left(\frac{1000\ \mathrm{m}}{1\ \cancel{\mathrm{km}}}\right)\left(\frac{1\ \cancel{\mathrm{h}}}{3600\ \mathrm{s}}\right) = 21\ \mathrm{m/s}.$$

Then

$$\bar{a} = \frac{21\ \mathrm{m/s} - 0.0\ \mathrm{m/s}}{5.0\ \mathrm{s}} = 4.2\,\frac{\mathrm{m/s}}{\mathrm{s}} = 4.2\,\frac{\mathrm{m}}{\mathrm{s}^2}.$$

We almost always write the units for acceleration as $\mathrm{m/s^2}$ (meters per second squared) instead of m/s/s. This is possible because:

$$\frac{\mathrm{m/s}}{\mathrm{s}} = \frac{\mathrm{m}}{\mathrm{s\cdot s}} = \frac{\mathrm{m}}{\mathrm{s}^2}.$$

CAUTION
Distinguish velocity from acceleration

Note that *acceleration tells us how quickly the velocity changes*, whereas *velocity tells us how quickly the position changes*.

CAUTION
If v or a is zero, is the other zero too?

CONCEPTUAL EXAMPLE 2–5 **Velocity and acceleration.** (*a*) If the velocity of an object is zero, does it mean that the acceleration is zero? (*b*) If the acceleration is zero, does it mean that the velocity is zero? Think of some examples.

RESPONSE A zero velocity does not necessarily mean that the acceleration is zero, nor does a zero acceleration mean that the velocity is zero. (*a*) For example, when you put your foot on the gas pedal of your car which is at rest, the velocity starts from zero but the acceleration is not zero since the velocity of the car changes. (How else could your car start forward if its velocity weren't changing—that is, accelerating?) (*b*) As you cruise along a straight highway at a constant velocity of 100 km/h, your acceleration is zero: $a = 0,\ v \neq 0$.

EXAMPLE 2–6 **Car slowing down.** An automobile is moving to the right along a straight highway, which we choose to be the positive x axis (Fig. 2–11). Then the driver steps on the brakes. If the initial velocity (when the driver hits the brakes) is $v_1 = 15.0\ \mathrm{m/s}$, and it takes 5.0 s to slow down to $v_2 = 5.0\ \mathrm{m/s}$, what was the car's average acceleration?

APPROACH We put the given initial and final velocities, and the elapsed time, into Eq. 2–4 for $\bar{a}$.

SOLUTION In Eq. 2–4, we call the initial time $t_1 = 0$, and set $t_2 = 5.0\ \mathrm{s}$:

$$\bar{a} = \frac{5.0\ \mathrm{m/s} - 15.0\ \mathrm{m/s}}{5.0\ \mathrm{s}} = -2.0\ \mathrm{m/s^2}.$$

The negative sign appears because the final velocity is less than the initial velocity. In this case the direction of the acceleration is to the left (in the negative x direction)—even though the velocity is always pointing to the right. We say that the acceleration is $2.0\ \mathrm{m/s^2}$ to the left, and it is shown in Fig. 2–11 as an orange arrow.

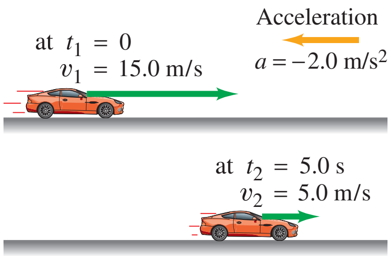


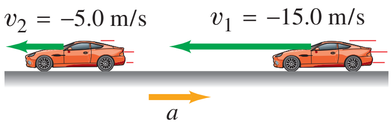
FIGURE 2–11 Example 2–6, showing the position of the car at times t_1 and t_2, as well as the car's velocity represented by the green arrows. The acceleration vector (orange) points to the left because the car slows down as it moves to the right.

Deceleration

When an object is slowing down, we can say it is **decelerating**. But be careful: deceleration does *not* mean that the acceleration is necessarily negative. The velocity of an object moving to the right along the positive x axis is positive; if the object is slowing down (as in Fig. 2–11), the acceleration *is* negative. But the same car moving to the left (decreasing x), and slowing down, has positive acceleration that points to the right, as shown in Fig. 2–12. We have a deceleration whenever the magnitude of the velocity is decreasing; thus the *velocity and acceleration point in opposite directions* when there is deceleration.

FIGURE 2–12 The car of Example 2–6, now moving to the *left* and decelerating. The acceleration is $a = (v_2 - v_1)/\Delta t$, or

$$a = \frac{(-5.0\ \mathrm{m/s}) - (-15.0\ \mathrm{m/s})}{5.0\ \mathrm{s}} = \frac{-5.0\ \mathrm{m/s} + 15.0\ \mathrm{m/s}}{5.0\ \mathrm{s}} = +2.0\ \mathrm{m/s^2}.$$



EXERCISE C A car moves along the x axis. What is the sign of the car's acceleration if it is moving in the positive x direction with (*a*) increasing speed or (*b*) decreasing speed? What is the sign of the acceleration if the car moves in the negative x direction with (*c*) increasing speed or (*d*) decreasing speed?