Reflection from still water, as from a glass mirror, can be analyzed using the ray model of light.

Is this picture right side up, or upside down? How can you tell? What are the clues? Notice the people and position of the Sun. Ray diagrams, which we will learn to draw in this Chapter, can provide the answer. See Example 23–3.

In this first Chapter on light and optics, we use the ray model of light to understand the formation of images by mirrors, both plane and curved (spherical). We also study refraction—how light rays bend when they go from one medium to another—and how, via refraction, images are formed by lenses, which are the crucial part of so many optical instruments.

CHAPTER 23

Light: Geometric Optics

CONTENTS

CHAPTER-OPENING QUESTIONS—Guess now!

1. A 2.0-m-tall person is standing 2.0 m from a flat vertical mirror staring at her image. What minimum height must the mirror's reflecting glass have if the person is to see her entire body, from the top of her head to her feet?

(a) 0.50 m. **(b)** 1.0 m. **(c)** 1.5 m. **(d)** 2.0 m. **(e)** 2.5 m.

2. The focal length of a lens is

(a) the diameter of the lens.

(b) the thickness of the lens.

(c) the distance from the lens at which incoming parallel rays bend to intersect at a point.

(d) the distance from the lens at which all real images are formed.

The sense of sight is extremely important to us, for it provides us with a large part of our information about the world. How do we see? What is the something called *light* that enters our eyes and causes the sensation of sight? How does light behave so that we can see everything that we do? We saw in Chapter 22 that light can be considered a form of electromagnetic radiation. We now examine the subject of light in detail in the next three Chapters.

We see an object in one of two ways: (1) the object may be a *source* of light, such as a lightbulb, a flame, or a star, in which case we see the light emitted directly from the source; or, more commonly, (2) we see an object by light *reflected* from it.

In the latter case, the light may have originated from the Sun, artificial lights, or a campfire. An understanding of how objects *emit* light was not achieved until the 1920s, and will be discussed in Chapter 27. How light is *reflected* from objects was understood much earlier, and will be discussed in Section 23–2.

23–1 The Ray Model of Light

A great deal of evidence suggests that *light travels in straight lines* under a wide variety of circumstances.[†] For example, a source of light like the Sun (which at its great distance from us is nearly a "point source") casts distinct shadows, and the beam from a laser pointer appears to be a straight line. In fact, we infer the positions of objects in our environment by assuming that light moves from the object to our eyes in straight-line paths. Our orientation to the physical world is based on this assumption.

This reasonable assumption is the basis of the **ray model** of light. This model assumes that light travels in straight-line paths called light **rays**. Actually, a ray is an idealization; it is meant to represent an extremely narrow beam of light. When we see an object, according to the ray model, light reaches our eyes from each point on the object. Although light rays leave each point in many different directions, normally only a small bundle of these rays can enter the pupil of an observer's eye, as shown in Fig. 23–1. If the person's head moves to one side, a different bundle of rays will enter the eye from each point.

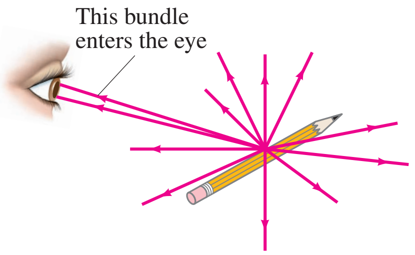

FIGURE 23–1 Light rays come from each single point on an object. A small bundle of rays leaving one point is shown entering a person's eye.

We saw in Chapter 22 that light can be considered as an electromagnetic wave. Although the ray model of light does not deal with this aspect of light (we discuss the wave nature of light in Chapter 24), the ray model has been very successful in describing many aspects of light such as reflection, refraction, and the formation of images by mirrors and lenses. Because these explanations involve straight-line rays at various angles, this subject is referred to as **geometric optics**.

23–2 Reflection; Image Formation by a Plane Mirror

When light strikes the surface of an object, some of the light is reflected. The rest can be absorbed by the object (and transformed to thermal energy) or, if the object is transparent like glass or water, part can be transmitted through. For a very smooth shiny object such as a silvered mirror, over 95% of the light may be reflected.

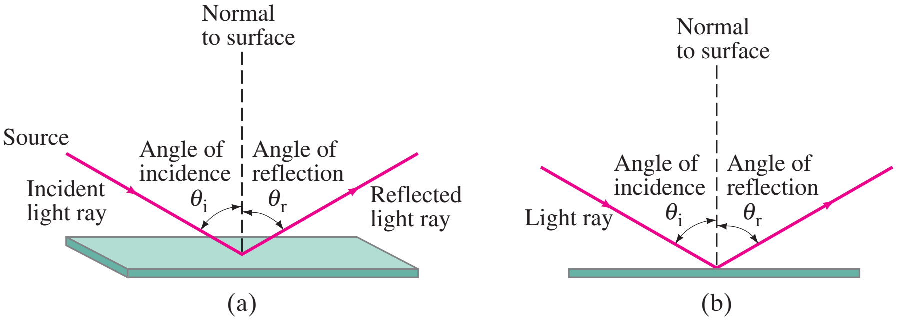

FIGURE 23–2 Law of reflection: (a) shows a 3-D view of an incident ray being reflected at the top of a flat surface; (b) shows a side or "end-on" view, which we will usually use because of its clarity.

When a narrow beam of light strikes a flat surface (Fig. 23–2), we define the **angle of incidence**, $\boldsymbol{\theta_i}$, to be the angle an incident ray makes with the normal (perpendicular) to the surface, and the **angle of reflection**, $\boldsymbol{\theta_r}$, to be the angle the reflected ray makes with the normal. It is found that the *incident and reflected rays lie in the same plane with the normal to the surface*, and that

the angle of reflection equals the angle of incidence, $\boldsymbol{\theta_r = \theta_i}$.

LAW OF REFLECTION

This is the **law of reflection**, and it is depicted in Fig. 23–2. It was known to the ancient Greeks, and you can confirm it yourself by shining a narrow flashlight beam or a laser pointer at a mirror in a darkened room.

[†]In a uniform transparent medium such as air or glass: But not always, such as for nonuniform air that allows optical illusions and mirages which we discuss in Section 24–2 (Fig. 24–4).

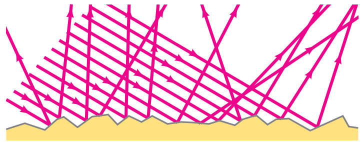

FIGURE 23–3 Diffuse reflection from a rough surface.

When light is incident upon a rough surface, even microscopically rough such as this page, it is reflected in many directions, as shown in Fig. 23–3. This is called **diffuse reflection**. The law of reflection still holds, however, at each small section of the surface. Because of diffuse reflection in all directions, an ordinary object can be seen at many different angles by the light reflected from it. When you move your head to the side, different reflected rays reach your eye from each point on the object (such as this page), Fig. 23–4a. Let us compare diffuse reflection to reflection from a mirror, which is known as **specular reflection**. ("Speculum" is Latin for mirror.) When a narrow beam of light shines on a mirror, the light will not reach your eye unless your eye is positioned at just the right place where the law of reflection is satisfied, as shown in Fig. 23–4b. This is what gives rise to the special image-forming properties of mirrors.

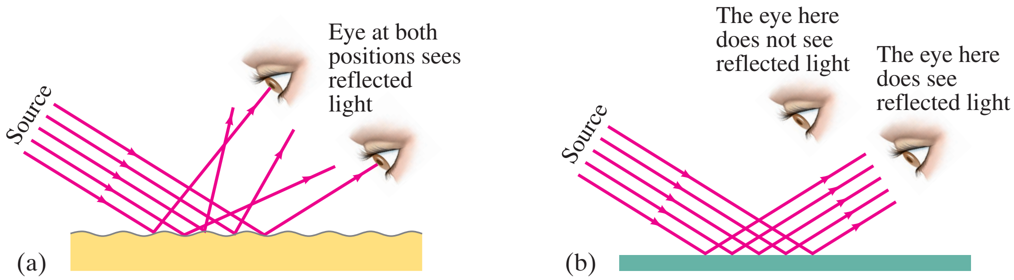

FIGURE 23–4 A narrow beam of light shines on (a) white paper, and (b) a mirror. In part (a), you can see with your eye the white light (and printed words) reflected at various positions because of diffuse reflection. But in part (b), you see the reflected light only when your eye is placed correctly $(\theta_r = \theta_i)$; mirror reflection is also known as specular reflection. (Galileo, using similar arguments, showed that the Moon must have a rough surface rather than a highly polished surface like a mirror, as some people thought.)

EXAMPLE 23–1 **Reflection from flat mirrors.** Two flat mirrors are perpendicular to each other. An incoming beam of light makes an angle of 15° with the first mirror as shown in Fig. 23–5a. What angle will the outgoing beam make with the second mirror?

APPROACH We sketch the path of the beam as it reflects off the two mirrors, and draw the two normals to the mirrors for the two reflections. We use geometry and the law of reflection to find the various angles.

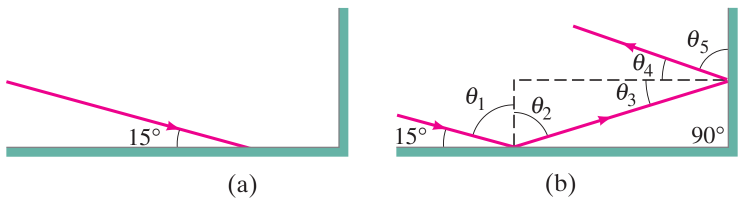

FIGURE 23–5 Example 23–1.

SOLUTION In Fig. 23–5b, $\theta_1 + 15° = 90°$, so $\theta_1 = 75°$; by the law of reflection $\theta_2 = \theta_1 = 75°$ too. Using the fact that the sum of the three angles of a triangle is always 180°, and noting that the two normals to the two mirrors are perpendicular to each other, we have $\theta_2 + \theta_3 + 90° = 180°$. Thus $\theta_3 = 180° - 90° - 75° = 15°$. By the law of reflection, $\theta_4 = \theta_3 = 15°$, so $\theta_5 = 75°$ is the angle the reflected ray makes with the second mirror surface.

NOTE The outgoing ray is parallel to the incoming ray. Reflectors on bicycles, cars, and other applications use this principle.

When you look straight into a mirror, you see what appears to be yourself as well as various objects around and behind you, Fig. 23–6. Your face and the other objects look as if they are in front of you, beyond the mirror. But what you see in the mirror is an **image** of the objects, including yourself, that are in front of the mirror. Also, you don't see yourself as others see you, because left and right appear reversed in the image.

A **plane mirror** is one with a smooth flat reflecting surface. Figure 23–7 shows how an image is formed by a plane mirror according to the ray model. We are viewing the mirror, on edge, in the diagram of Fig. 23–7, and the rays are shown reflecting from the front surface. (Good mirrors are generally made by putting a highly reflective metallic coating on one surface of a very flat piece of glass.) Rays from two different points on an object (the bottle on the left in Fig. 23–7) are shown: two rays are shown leaving from a point on the top of the bottle, and two more from a point on the bottom. Rays leave each point on the object going in many directions (as in Fig. 23–1), but only those that enclose the bundle of rays that enter the eye from each of the two points are shown. Each set of diverging rays that reflect from the mirror and enter the eye *appear to come from a single point* behind the mirror, called the **image point**, as shown by the dashed lines. That is, our eyes and brain interpret any rays that enter an eye as having traveled straight-line paths. The point from which each bundle of rays seems to come is one point on the image. For each point on the object, there is a corresponding image point. (This analysis of how a plane mirror forms an image was published by Kepler in 1604.)

FIGURE 23–6 When you look in a mirror, you see an image of yourself and objects around you. You don't see yourself as others see you, because left and right appear reversed in the image.

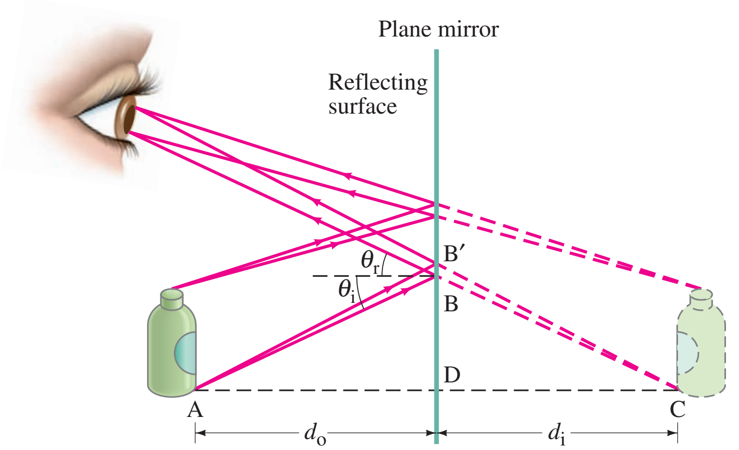

FIGURE 23–7 Formation of a virtual image by a plane mirror. Only the bundle of rays from the top and bottom of the object which reach the eye is shown.

Let us concentrate on the two rays that leave point A on the object in Fig. 23–7, and strike the mirror at points B and B′. We use geometry now, for the rays at B. The angles ADB and CDB are right angles; and because of the law of reflection, $\theta_i = \theta_r$ at point B. Therefore, by geometry, angles ABD and CBD are also equal. The two triangles ABD and CBD are thus congruent, and the length AD = CD. That is, the image appears as far behind the mirror as the object is in front. The **image distance**, d_i (perpendicular distance from mirror to image, Fig. 23–7), equals the **object distance**, d_o (perpendicular distance from object to mirror). From the geometry, we also can see that the height of the image is the same as that of the object.

The light rays do not actually pass through the image location itself in Fig. 23–7. (Note where the red lines are dashed to show they are our projections, not rays.) The image would not appear on paper or film placed at the location of the image. Therefore, it is called a **virtual image**. This is to distinguish it from a **real image** in which the light does pass through the image and which therefore could appear on a white surface, or on film or on an electronic sensor placed at the image position. Our eyes can see both real and virtual images, as long as the diverging rays enter our pupils. We will see that curved mirrors and lenses can form real images, as well as virtual. A movie projector lens, for example, produces a real image that is visible on the screen.

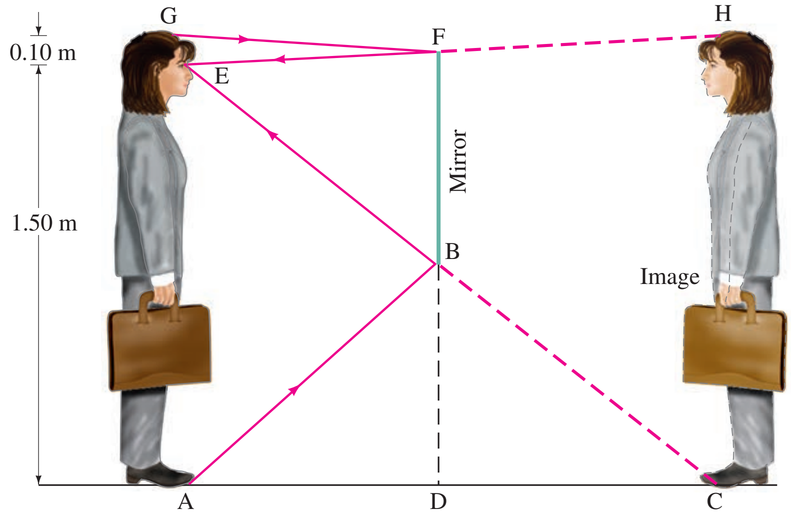

FIGURE 23–8 Seeing oneself in a mirror. Example 23–2.

EXAMPLE 23–2 **How tall must a full-length mirror be?** A woman 1.60 m tall stands in front of a vertical plane mirror. What is the minimum height of the mirror, and how high must its lower edge be above the floor, if she is to be able to see her whole body? Assume that her eyes are 10 cm below the top of her head.

APPROACH For her to see her whole body, light rays from the top of her head (point G) and from the bottom of her foot (A) must reflect from the mirror and enter her eye, Fig. 23–8. We don't show two rays diverging from each point as we did in Fig. 23–7, where we wanted to find where the image is. Now that we know the image is the same distance behind a plane mirror as the object is in front, we only need to show one ray leaving point G (top of head) and one ray leaving point A (her toe), and then use geometry.

PHYSICS APPLIED
How tall a mirror do you need to see a reflection of your entire self?

SOLUTION First consider the ray that leaves her foot at A, reflects at B, and enters the eye at E. The mirror needs to extend no lower than B. The angle of reflection equals the angle of incidence, so the height BD is half of the height AE. Because $AE = 1.60\text{ m} - 0.10\text{ m} = 1.50\text{ m}$, then $BD = 0.75\text{ m}$. Similarly, if the woman is to see the top of her head, the top edge of the mirror only needs to reach point F, which is 5 cm below the top of her head (half of $GE = 10\text{ cm}$). Thus, $DF = 1.55\text{ m}$, and the mirror needs to have a vertical height of only $(1.55\text{ m} - 0.75\text{ m}) = 0.80\text{ m}$. And the mirror's bottom edge must be 0.75 m above the floor.

NOTE We see that a mirror, if positioned at the correct height (as in Fig. 23–8), need be only half as tall as a person for that person to be able to see all of himself or herself.

EXERCISE A Does the result of Example 23–2 depend on your distance from the mirror? (Try it and see, it's fun.)

EXERCISE B Return to Chapter-Opening Question 1, page 644, and answer it again now. Try to explain why you may have answered differently the first time.

CONCEPTUAL EXAMPLE 23–3 **Is the photo upside down?** Close examination of the photograph on the first page of this Chapter reveals that in the top portion, the image of the Sun is seen clearly, whereas in the lower portion, the image of the Sun is partially blocked by the tree branches. Show why the reflection is not the same as the real scene by drawing a sketch of this situation, showing the Sun, the camera, the branch, and two rays going from the Sun to the camera (one direct and one reflected). Is the photograph right side up?

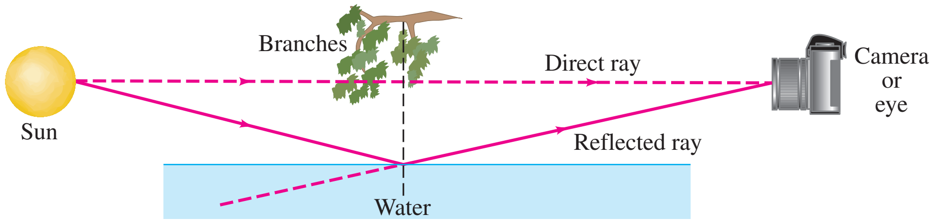

FIGURE 23–9 Example 23–3.

RESPONSE We need to draw two diagrams, one assuming the photo on p. 644 is right side up, and another assuming it is upside down. Figure 23–9 is drawn assuming the photo is upside down. In this case, the Sun blocked by the tree would be the direct view, and the full view of the Sun the reflection: the ray which reflects off the water and into the camera travels at an angle below the branch, whereas the ray that travels directly to the camera passes through the branches. This works. Try to draw a diagram assuming the photo is right side up (thus assuming that the image of the Sun in the reflection is higher above the horizon than it is as viewed directly). It won't work. The photo on p. 644 is upside down.

Also, what about the people in the photo? Try to draw a diagram showing why they don't appear in the reflection. [*Hint*: Assume they are not sitting at the edge of the pool, but back from the edge.] Then try to draw a diagram of the reverse (i.e., assume the photo is right side up so the people are visible only in the reflection). Reflected images are not perfect replicas when different planes (distances) are involved.

23–3 Formation of Images by Spherical Mirrors

Reflecting surfaces can also be *curved*, usually *spherical*, which means they form a section of a sphere. A **spherical mirror** is called **convex** if the reflection takes place on the outer surface of the spherical shape so that the center of the mirror surface bulges out toward the viewer, Fig. 23–10a. A mirror is called **concave** if the reflecting surface is on the inner surface of the sphere so that the mirror surface curves away from the viewer (like a "cave"), Fig. 23–10b. Concave mirrors are used as shaving or cosmetic mirrors (**magnifying mirrors**), Fig. 23–11a, because they magnify. Convex mirrors are sometimes used on cars and trucks (rearview mirrors) and in shops (to watch for theft), because they take in a wide field of view, Fig. 23–11b.

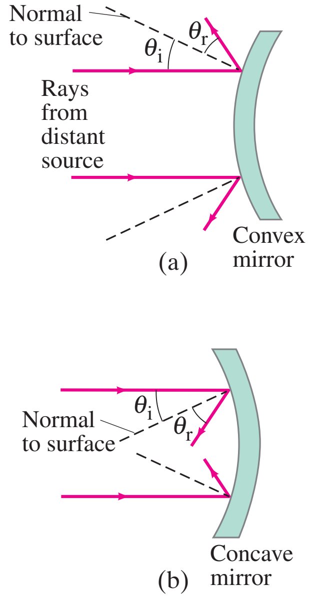

FIGURE 23–10 Mirrors with convex and concave spherical surfaces. Note that $\theta_r = \theta_i$ for each ray. (The dashed lines are perpendicular to the mirror surface at each point shown.)

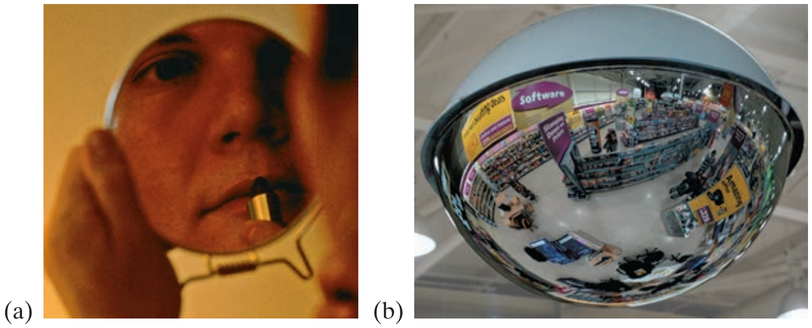

FIGURE 23–11 (a) A concave cosmetic mirror gives a magnified image. (b) A convex mirror in a store reduces image size and so includes a wide field of view. Note the extreme distortion—this mirror has a large curved surface and does not fit the "paraxial ray" approximation discussed on the next page.

Focal Point and Focal Length

To see how spherical mirrors form images, we first consider an object that is very far from a concave mirror. For a distant object, as shown in Fig. 23–12, the rays from each point on the object that strike the mirror will be nearly parallel. *For an object infinitely far away* (the Sun and stars approach this), *the rays would be precisely parallel*.

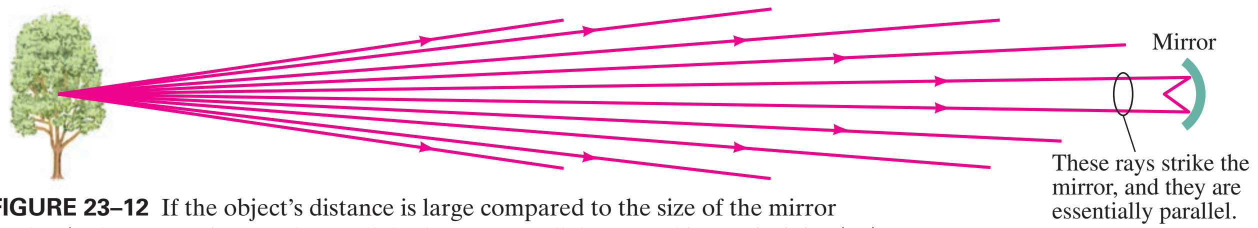

FIGURE 23–12 If the object's distance is large compared to the size of the mirror (or lens), the rays arrive nearly parallel. They are parallel for an object at infinity (∞).