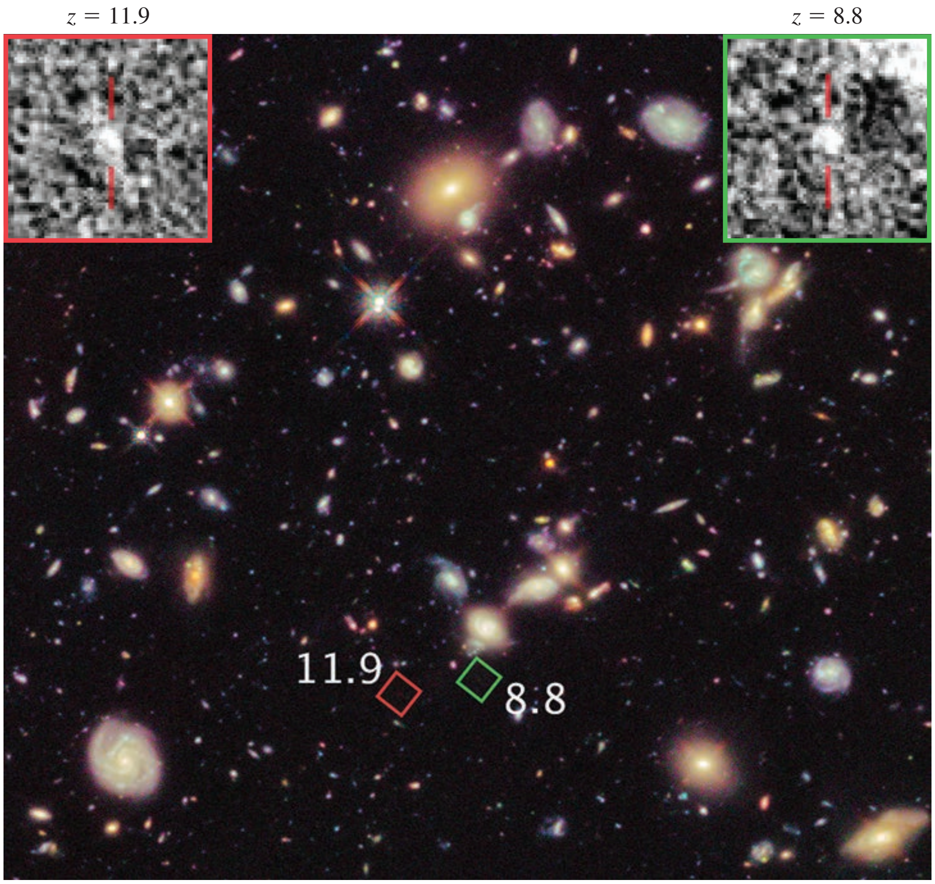


This Hubble eXtreme Deep Field (XDF) photograph is of a very small part of the sky. It includes what may be the most distant galaxies observable by us (small red and green squares, and shown enlarged in the corners), with $z \approx 8.8$ and 11.9, that already existed when the universe was about 0.4 billion years old. We see these galaxies as they appeared then, 13.4 billion years ago, which is when they emitted this light. The most distant galaxies were young and small and grew to become large galaxies by colliding and merging with other small galaxies.

We examine the latest theories on how stars and galaxies form and evolve, including the role of nucleosynthesis, as well as Einstein's general theory of relativity which deals with gravity and curvature of space. We take a thorough look at the evidence for the expansion of the universe, and the Standard Model of the universe evolving from an initial Big Bang. We point out some unsolved problems, including the nature of dark matter and dark energy that make up most of our universe.

CHAPTER 33

Astrophysics and Cosmology

CONTENTS

CHAPTER-OPENING QUESTIONS—Guess now!

1. Until recently, astronomers expected the expansion rate of the universe would be decreasing. Why?

(a) Friction.
(b) The second law of thermodynamics.
(c) Gravity.
(d) The electromagnetic force.

2. The universe began expanding right at the beginning. How long will it continue to expand?

(a) Until it runs out of room.
(b) Until friction slows it down and brings it to a stop.
(c) Until all galaxies are moving at the speed of light relative to the center.
(d) Possibly forever.

In the previous Chapter, we studied the tiniest objects in the universe—the elementary particles. Now we leap to the grandest objects in the universe—stars, galaxies, and clusters of galaxies—plus the history and structure of the universe itself. These two extreme realms, elementary particles and the cosmos, are among the most intriguing and exciting subjects in science. And, surprisingly, these two extreme realms are related in a fundamental way, as was already hinted in Chapter 32.

Use of the techniques and ideas of physics to study the night sky is often referred to as **astrophysics**. Central to our present theoretical understanding of the universe (or cosmos) is Einstein's *general theory of relativity* which represents our most complete understanding of gravitation. Many other aspects of physics are involved, from electromagnetism and thermodynamics to atomic and nuclear physics as well as elementary particles. General Relativity serves also as the foundation for modern **cosmology**, which is the study of the universe as a whole. Cosmology deals especially with the search for a theoretical framework to understand the observed universe, its origin, and its future. The questions posed by cosmology are profound and difficult; the possible answers stretch the imagination. They are questions like "Has the universe always existed, or did it have a beginning in time?" Either alternative is difficult to imagine: time going back indefinitely into the past, or an actual moment when the universe began (but, then, what was there before?). And what about the size of the universe? Is it infinite in size? It is hard to imagine infinity. Or is it finite in size? This is also hard to imagine, for if the universe is finite, it does not make sense to ask what is beyond it, because the universe is all there is.

In the last 10 to 20 years, so much progress has occurred in astrophysics and cosmology that many scientists are calling recent work a "Golden Age" for cosmology. Our survey will be qualitative, but we will nonetheless touch on the major ideas. We begin with a look at what can be seen beyond the Earth.

FIGURE 33–1 Sections of the Milky Way. In (a), the thin line is the trail of an artificial Earth satellite in this long time exposure. The dark diagonal area is due to dust absorption of visible light, blocking the view. In (b) the view is toward the center of the Galaxy (taken in summer from Arizona).

(a)

(b)

33–1 Stars and Galaxies

According to the ancients, the stars, except for the few that seemed to move relative to the others (the planets), were fixed on a sphere beyond the last planet. The universe was neatly self-contained, and we on Earth were at or near its center. But in the centuries following Galileo's first telescopic observations of the night sky in 1609, our view of the universe has changed dramatically. We no longer place ourselves at the center, and we view the universe as vastly larger. The distances involved are so great that we specify them in terms of the time it takes light to travel the given distance: for example,

$$1 \text{ light-second} = \left(3.0 \times 10^8 \text{ m/s}\right)(1.0 \text{ s}) = 3.0 \times 10^8 \text{ m} = 300{,}000 \text{ km};$$
$$1 \text{ light-minute} = \left(3.0 \times 10^8 \text{ m/s}\right)(60 \text{ s}) = 18 \times 10^6 \text{ km}.$$

The most common unit is the **light-year** (ly):

$$\begin{aligned} 1 \text{ ly} &= \left(2.998 \times 10^8 \text{ m/s}\right)\left(3.156 \times 10^7 \text{ s/yr}\right) \\ &= 9.46 \times 10^{15} \text{ m} \approx 10^{13} \text{ km} \approx 10^{16} \text{ m}. \end{aligned}$$

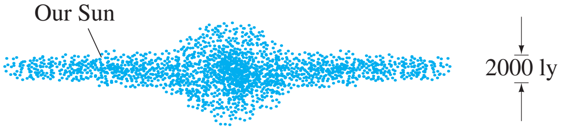
For specifying distances to the Sun and Moon, we usually use meters or kilometers, but we could specify them in terms of light seconds or minutes. The Earth–Moon distance is 384,000 km, which is 1.28 light-seconds. The Earth–Sun distance is 1.50×10^{11} m, or 150,000,000 km; this is equal to 8.3 light-minutes (it takes 8.3 min for light emitted by the Sun to reach us). Far out in our solar system, Pluto is about 6×10^9 km from the Sun, or 6×10^{-4} ly.† The nearest star to us, other than the Sun, is Proxima Centauri, about 4.2 ly away.

On a clear moonless night, thousands of stars of varying degrees of brightness can be seen, as well as the long cloudy stripe known as the Milky Way (Fig. 33–1). Galileo first observed, with his telescope, that the Milky Way is comprised of countless individual stars. A century and a half later (about 1750), Thomas Wright suggested that the Milky Way was a flat disk of stars extending to great distances in a plane, which we call the **Galaxy** (Greek for "milky way").

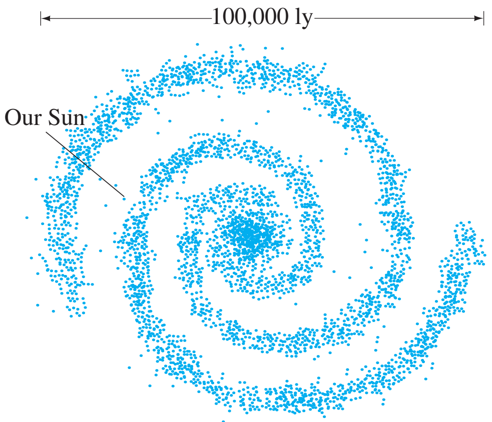
†We can also say this is about 5 light-hours.

Our Galaxy has a diameter of almost 100,000 light-years and a thickness of roughly 2000 ly. It has a central bulge and spiral arms (Fig. 33–2). Our Sun, which is a star like many others, is located about halfway from the galactic center to the edge, some 26,000 ly from the center. Our Galaxy contains roughly 400 billion (4×10^{11}) stars. The Sun orbits the galactic center approximately once every 250 million years, so its speed is roughly 200 km/s relative to the center of the Galaxy. The total mass of all the stars in our Galaxy is estimated to be about 4×10^{41} kg of ordinary matter. There is also strong evidence that our Galaxy is permeated and surrounded by a massive invisible "halo" of "dark matter" (Section 33–9).

FIGURE 33–2 Our Galaxy, as it would appear from the outside: (a) "edge view," in the plane of the disk; (b) "top view," looking down on the disk. (If only we could see it like this—from the outside!) (c) Infrared photograph of the inner reaches of the Milky Way, showing the central bulge and disk of our Galaxy. This very wide angle photo taken from the COBE satellite (Section 33–6) extends over 360° of sky. The white dots are nearby stars.



(a)



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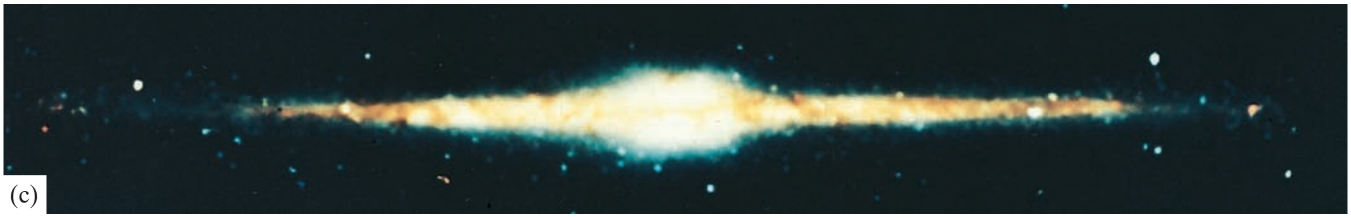


(c)

EXAMPLE 33–1 **ESTIMATE** **Our Galaxy's mass.** Estimate the total mass of our Galaxy using the orbital data above for the Sun about the center of the Galaxy. Assume the mass of the Galaxy is concentrated in the central bulge.

APPROACH We assume that the Sun (including our solar system) has total mass m and moves in a circular orbit about the center of the Galaxy (total mass M), and that the mass M can be considered as being located at the center of the Galaxy. We then apply Newton's second law, $F = ma$, with a being the centripetal acceleration, $a = v^2/r$, and for F we use the universal law of gravitation (Chapter 5).

SOLUTION Our Sun and solar system orbit the center of the Galaxy, according to the best measurements as mentioned above, with a speed of about $v = 200$ km/s at a distance from the Galaxy center of about $r = 26{,}000$ ly. We use Newton's second law:

$$F = ma$$

$$G\frac{Mm}{r^2} = m\frac{v^2}{r}$$

where M is the mass of the Galaxy and m is the mass of our Sun and solar system. Solving this, we find

$$M = \frac{rv^2}{G} \approx \frac{(26{,}000\ \text{ly})(10^{16}\ \text{m/ly})(2 \times 10^5\ \text{m/s})^2}{6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2} \approx 2 \times 10^{41}\ \text{kg}.$$

NOTE In terms of *numbers* of stars, if they are like our Sun $(m = 2.0 \times 10^{30}\ \text{kg})$, there would be about $(2 \times 10^{41}\ \text{kg})/(2 \times 10^{30}\ \text{kg}) \approx 10^{11}$ or very roughly on the order of 100 billion stars.

FIGURE 33–3 This globular star cluster is located in the constellation Hercules.

FIGURE 33–4 This gaseous nebula, found in the constellation Carina, is about 9000 light-years from us.

In addition to stars both within and outside the Milky Way, we can see by telescope many faint cloudy patches in the sky which were all referred to once as "nebulae" (Latin for "clouds"). A few of these, such as those in the constellations Andromeda and Orion, can actually be discerned with the naked eye on a clear night. Some are **star clusters** (Fig. 33–3), groups of stars that are so numerous they appear to be a cloud. Others are glowing clouds of gas or dust (Fig. 33–4), and it is for these that we now mainly reserve the word **nebula**.

Most fascinating are those that belong to a third category: they often have fairly regular elliptical shapes. Immanuel Kant (about 1755) guessed they are faint because they are a great distance beyond our Galaxy. At first it was not universally accepted that these objects were **extragalactic**—that is, outside our Galaxy. But the very large telescopes constructed in the twentieth century revealed that individual stars could be resolved within these extragalactic objects and that many contain spiral arms. Edwin Hubble (1889–1953) did much of this observational work in the 1920s using the 2.5-m (100-inch) telescope† on Mt. Wilson near Los Angeles, California, then the world's largest. Hubble demonstrated that these objects were indeed extragalactic because of their great distances. The distance to our nearest large galaxy,‡ Andromeda, is over 2 million light-years, a distance 20 times greater than the diameter of our Galaxy. It seemed logical that these nebulae must be **galaxies** similar to ours. (Note that it is usual to capitalize the word "galaxy" only when it refers to our own.) Today it is thought there are roughly 10^{11} galaxies in the observable universe—that is, roughly as many galaxies as there are stars in a galaxy. See Fig. 33–5.

Many galaxies tend to be grouped in **galaxy clusters** held together by their mutual gravitational attraction. There may be anywhere from a few dozen to many thousands of galaxies in each cluster. Furthermore, clusters themselves seem to be organized into even larger aggregates: clusters of clusters of galaxies, or **superclusters**. The farthest detectable galaxies are more than 10^{10} ly distant. See Table 33–1 (top of next page).

†2.5 m (= 100 inches) refers to the diameter of the curved objective mirror. The bigger the mirror, the more light it collects (greater brightness) and the less diffraction there is (better resolution), so more and fainter stars can be seen. See Chapter 25. Until recently, photographic films or plates were used to take long time exposures. Now large solid-state CCD or CMOS sensors (Section 25–1) are available containing hundreds of millions of pixels (compared to 10 million pixels in a good-quality digital camera).

‡The *Magellanic clouds* are much closer than Andromeda, but are small and are usually considered small satellite galaxies of our own Galaxy.

FIGURE 33–5 Photographs of galaxies. (a) Spiral galaxy in the constellation Hydra. (b) Two galaxies: the larger and more dramatic one is known as the Whirlpool galaxy. (c) An infrared image (given "false" colors) of the same galaxies as in (b), here showing the arms of the spiral as having more substance than in the visible light photo (b); the different colors correspond to different light intensities. Visible light is scattered and absorbed by interstellar dust much more than infrared is, so infrared gives us a clearer image.

(a) (b) (c)

CONCEPTUAL EXAMPLE 33–2 **Looking back in time.** Astronomers often think of their telescopes as time machines, looking back toward the origin of the universe. How far back do they look?

RESPONSE The distance in light-years measures how long in years the light has been traveling to reach us, so Table 33–1 tells us also how far back in time we are looking. For example, if we saw Proxima Centauri explode into a supernova today, then the event would have really occurred about 4.2 years ago. The most distant galaxies emitted the light we see now roughly 13×10^9 years ago. What we see was how they were then, 13×10^9 yr ago.

EXERCISE A Suppose we could place a huge mirror 1 light-year away from us. What would we see in this mirror if it is facing us on Earth? When did what we see in the mirror take place? (This might be called a "time machine.")

Table 33–1 Astronomical Distances

Object	Approx. Distance from Earth (ly)
Moon	4×10^{-8}
Sun	1.6×10^{-5}
Size of solar system (distance to Pluto)	6×10^{-4}
Nearest star (Proxima Centauri)	4.2
Center of our Galaxy	2.6×10^4
Nearest large galaxy	2.4×10^6
Farthest galaxies	13.4×10^9

Besides the usual stars, clusters of stars, galaxies, and clusters and superclusters of galaxies, the universe contains many other interesting objects. Among these are stars known as *red giants, white dwarfs*, *neutron stars*, exploding stars called *novae* and *supernovae*, and *black holes* whose gravity is so strong that even light cannot escape them. In addition, there is electromagnetic radiation that reaches the Earth but does not come from the bright pointlike objects we call stars: particularly important is the microwave background radiation that arrives nearly uniformly from all directions in the universe.

Finally, there are **active galactic nuclei** (**AGN**), which are very luminous pointlike sources of light in the centers of distant galaxies. The most dramatic examples of AGN are **quasars** ("quasistellar objects" or QSOs), which are so luminous that the surrounding starlight of the galaxy is drowned out. Their luminosity is thought to come from matter falling into a giant black hole at a galaxy's center.

33–2 Stellar Evolution: Birth and Death of Stars, Nucleosynthesis

The stars appear unchanging. Night after night the night sky reveals no significant variations. Indeed, on a human time scale, the vast majority of stars change very little (except for novae, supernovae, and certain variable stars). Although stars *seem* fixed in relation to each other, many move sufficiently for the motion to be detected. Speeds of stars relative to neighboring stars can be hundreds of km/s, but at their great distance from us, this motion is detectable only by careful measurement. There is also a great range of brightness among stars, due to differences in the rate stars emit energy and to their different distances from us.

Luminosity and Brightness of Stars

Any star or galaxy has an **intrinsic luminosity**, L (or simply **luminosity**), which is its total power radiated in watts. Also important is the **apparent brightness**, b, defined as the power crossing unit area at the Earth perpendicular to the path of the light. Given that energy is conserved, and ignoring any absorption in space, the total emitted power L when it reaches a distance d from the star will be spread over a sphere of surface area $4\pi d^2$. If d is the distance from the star to the Earth, then L must be equal to $4\pi d^2$ times b (power per unit area at Earth). That is,

$$b = \frac{L}{4\pi d^2}. \qquad \textbf{(33–1)}$$

EXAMPLE 33–3 **Apparent brightness.** Suppose a star has luminosity equal to that of our Sun. If it is 10 ly away from Earth, how much dimmer will it appear?

APPROACH We use the inverse square law in Eq. 33–1 to determine the relative brightness $(b \propto 1/d^2)$ since the luminosity L is the same for both stars.

SOLUTION Using the inverse square law, the star appears dimmer by a factor

$$\frac{b_{\text{star}}}{b_{\text{Sun}}} = \frac{d_{\text{Sun}}^2}{d_{\text{star}}^2} = \frac{(1.5 \times 10^8\,\text{km})^2}{(10\,\text{ly})^2(10^{13}\,\text{km/ly})^2} \approx 2 \times 10^{-12}.$$

Careful study of nearby stars has shown that the luminosity for most stars depends on the mass: *the more massive the star*, *the greater its luminosity*[†]. Another important parameter of a star is its surface temperature, which can be determined from the spectrum of electromagnetic frequencies it emits. As we saw in Chapter 27, as the temperature of a body increases, the spectrum shifts from predominantly lower frequencies (and longer wavelengths, such as red) to higher frequencies (and shorter wavelengths such as blue). Quantitatively, the relation is given by Wien's law (Eq. 27–2): the wavelength λ_P at the peak of the spectrum of light emitted by a blackbody (we often approximate stars as blackbodies) is inversely proportional to its Kelvin temperature T; that is, $\lambda_P T = 2.90 \times 10^{-3}\,\mathrm{m\cdot K}$. The surface temperatures of stars typically range from about 3000 K (reddish) to about 50,000 K (UV).

EXAMPLE 33–4 Determining star temperature and star size. Suppose that the distances from Earth to two nearby stars can be reasonably estimated, and that their measured apparent brightnesses suggest the two stars have about the same luminosity, L. The spectrum of one of the stars peaks at about 700 nm (so it is reddish). The spectrum of the other peaks at about 350 nm (bluish). Use Wien's law (Eq. 27–2) and the Stefan-Boltzmann equation (Section 14–8) to determine (*a*) the surface temperature of each star, and (*b*) how much larger one star is than the other.

APPROACH We determine the surface temperature T for each star using Wien's law and each star's peak wavelength. Then, using the Stefan-Boltzmann equation (power output or luminosity $\propto AT^4$ where A = surface area of emitter), we can find the surface area ratio and relative sizes of the two stars.

SOLUTION (*a*) Wien's law (Eq. 27–2) states that $\lambda_P T = 2.90 \times 10^{-3}\,\mathrm{m\cdot K}$. So the temperature of the reddish star is

$$T_r = \frac{2.90 \times 10^{-3}\,\mathrm{m\cdot K}}{\lambda_P} = \frac{2.90 \times 10^{-3}\,\mathrm{m\cdot K}}{700 \times 10^{-9}\,\mathrm{m}} = 4140\,\mathrm{K}.$$

The temperature of the bluish star will be double this because its peak wavelength is half (350 nm vs. 700 nm):

$$T_b = 8280\,\mathrm{K}.$$

(*b*) The Stefan-Boltzmann equation, Eq. 14–6, states that the power radiated *per unit area* of surface from a blackbody is proportional to the fourth power of the Kelvin temperature, T^4. The temperature of the bluish star is double that of the reddish star, so the bluish one must radiate $(T_b/T_r)^4 = 2^4 = 16$ times as much energy per unit area. But we are given that they have the same luminosity (the same total power output); so the surface area of the blue star must be $\frac{1}{16}$ that of the red one. The surface area of a sphere is $4\pi r^2$, so the radius of the reddish star is $\sqrt{16} = 4$ times larger than the radius of the bluish star (or $4^3 = 64$ times the volume).

H–R Diagram

An important astronomical discovery, made around 1900, was that for most stars, the color is related to the intrinsic luminosity and therefore to the mass. A useful way to present this relationship is by the so-called Hertzsprung–Russell (H–R) diagram. On the H–R diagram, the horizontal axis shows the surface temperature T and the vertical axis is the luminosity L; each star is represented by a point

[†]Applies to "main-sequence" stars (see next page). The mass of a star can be determined by observing its gravitational effects on other visible objects. Many stars are part of a cluster, the simplest being a binary star in which two stars orbit around each other, allowing their masses to be determined using rotational mechanics.

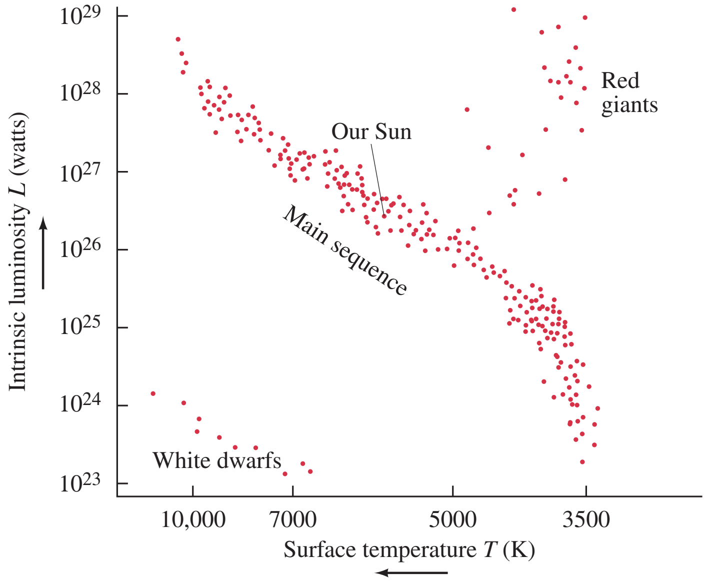


FIGURE 33–6 Hertzsprung–Russell (H–R) diagram is a logarithmic graph of luminosity vs. surface temperature T of stars (note that T increases to the left).

on the diagram, Fig. 33–6. Most stars fall along the diagonal band termed the **main sequence**. Starting at the lower right we find the coolest stars: by Wien's law, $\lambda_P T$ = constant, their light output peaks at long wavelengths, so they are reddish in color. They are also the least luminous and therefore of low mass. Farther up toward the left we find hotter and more luminous stars that are whitish, like our Sun. Still farther up we find even more luminous and more massive stars, bluish in color. Stars that fall on this diagonal band are called *main-sequence stars*. There are also stars that fall outside the main sequence. Above and to the right we find extremely large stars, with high luminosities but with low (reddish) color temperature: these are called **red giants**. At the lower left, there are a few stars of low luminosity but with high temperature: these are the **white dwarfs**.

EXAMPLE 33–5 ESTIMATE Distance to a star using the H–R diagram and color. Suppose that detailed study of a certain star suggests that it most likely fits on the main sequence of an H–R diagram. Its measured apparent brightness is $b = 1.0 \times 10^{-12}\,\mathrm{W/m^2}$, and the peak wavelength of its spectrum is $\lambda_P \approx 600\,\mathrm{nm}$. Estimate its distance from us.

APPROACH We find the temperature using Wien's law, Eq. 27–2. The luminosity is estimated for a main-sequence star on the H–R diagram of Fig. 33–6, and then the distance is found using the relation between brightness and luminosity, Eq. 33–1.

SOLUTION The star's temperature, from Wien's law (Eq. 27–2), is

$$T \approx \frac{2.90 \times 10^{-3}\,\mathrm{m \cdot K}}{600 \times 10^{-9}\,\mathrm{m}} \approx 4800\,\mathrm{K}.$$

A star on the main sequence of an H–R diagram at this temperature has luminosity of about $L \approx 1 \times 10^{26}\,\mathrm{W}$, read off of Fig. 33–6. Then, from Eq. 33–1,

$$d = \sqrt{\frac{L}{4\pi b}} \approx \sqrt{\frac{1 \times 10^{26}\,\mathrm{W}}{4(3.14)\left(1.0 \times 10^{-12}\,\mathrm{W/m^2}\right)}} \approx 3 \times 10^{18}\,\mathrm{m}.$$

Its distance from us in light-years is

$$d = \frac{3 \times 10^{18}\,\mathrm{m}}{10^{16}\,\mathrm{m/ly}} \approx 300\,\mathrm{ly}.$$

EXERCISE B Estimate the distance to a 6000-K main-sequence star with an apparent brightness of $2.0 \times 10^{-12}\,\mathrm{W/m^2}$.

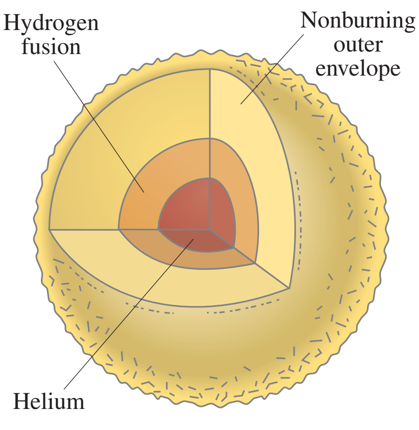
Stellar Evolution; Nucleosynthesis

Why are there different types of stars, such as red giants and white dwarfs, as well as main-sequence stars? Were they all born this way, in the beginning? Or might each different type represent a different age in the life cycle of a star? Astronomers and astrophysicists today believe the latter is the case. Note, however, that we cannot actually follow any but the tiniest part of the life cycle of any given star because they live for ages vastly greater than ours, on the order of millions or billions of years. Nonetheless, let us follow the process of **stellar evolution** from the birth to the death of a star, as astrophysicists have theoretically reconstructed it today.

Stars are born, it is believed, when gaseous clouds (mostly hydrogen) contract due to the pull of gravity. A huge gas cloud might fragment into numerous contracting masses, each mass centered in an area where the density is only slightly greater than that at nearby points. Once such "globules" form, gravity causes each to contract in toward its center of mass. As the particles of such a *protostar* accelerate inward, their kinetic energy increases. Eventually, when the kinetic energy is sufficiently high, the Coulomb repulsion between the positive charges is not strong enough to keep all the hydrogen nuclei apart, and nuclear fusion can take place.

In a star like our Sun, the fusion of hydrogen (sometimes referred to as "burning")† occurs via the *proton–proton chain* (Section 31–3, Eqs. 31–6), in which four protons fuse to form a ${}^4_2\mathrm{He}$ nucleus with the release of γ rays, positrons, and neutrinos: $4\,{}^1_1\mathrm{H} \rightarrow {}^4_2\mathrm{He} + 2\,\mathrm{e}^+ + 2\nu_\mathrm{e} + 2\gamma$. These reactions require a temperature of about 10^7 K, corresponding to an average kinetic energy ($\approx kT$) of about 1 keV (Eq. 13–8). In more massive stars, the carbon cycle produces the same net effect: four ${}^1_1\mathrm{H}$ produce a ${}^4_2\mathrm{He}$—see Section 31–3. The fusion reactions take place primarily in the core of a star, where T may be on the order of 10^7 to 10^8 K. (The surface temperature is much lower—on the order of a few thousand kelvins.) The tremendous release of energy in these fusion reactions produces an outward pressure sufficient to halt the inward gravitational contraction. Our protostar, now really a young *star*, stabilizes on the *main sequence*. Exactly where the star falls along the main sequence depends on its mass. The more massive the star, the farther up (and to the left) it falls on the H–R diagram of Fig. 33–6. Our Sun required perhaps 30 million years to reach the main sequence, and is expected to remain there about 10 billion years $\left(10^{10}\,\mathrm{yr}\right)$. Although most stars are billions of years old, evidence is strong that stars are actually being born at this moment. More massive stars have shorter lives, because they are hotter and the Coulomb repulsion is more easily overcome, so they use up their fuel faster. Our Sun may remain on the main sequence for 10^{10} years, but a star ten times more massive may reside there for only 10^7 years.

FIGURE 33–7 A shell of "burning" hydrogen (fusing to become helium) surrounds the core where the newly formed helium gravitates.



As hydrogen fuses to form helium, the helium that is formed is denser and tends to accumulate in the central core where it was formed. As the core of helium grows, hydrogen continues to fuse in a shell around it: see Fig. 33–7. When much of the hydrogen within the core has been consumed, the production of energy decreases at the center and is no longer sufficient to prevent the huge gravitational forces from once again causing the core to contract and heat up. The hydrogen in the shell around the core then fuses even more fiercely because of this rise in temperature, allowing the outer envelope of the star to expand and to cool. The surface temperature, thus reduced, produces a spectrum of light that peaks at longer wavelength (reddish).

This process marks a new step in the evolution of a star. The star has become redder, it has grown in size, and it has become more luminous, which means it has left the main sequence. It will have moved to the right and upward on the

†The word "burn," meaning fusion, is put in quotation marks because these high-temperature fusion reactions occur via a *nuclear* process, and must not be confused with ordinary burning (of, say, paper, wood, or coal) in air, which is a *chemical* reaction, occurring at the *atomic* level (and at a much lower temperature).

H–R diagram, as shown in Fig. 33–8. As it moves upward, it enters the **red giant** stage. Thus, theory explains the origin of red giants as a natural step in a star's evolution. Our Sun, for example, has been on the main sequence for about $4\frac{1}{2}$ billion years. It will probably remain there another 5 or 6 billion years. When our Sun leaves the main sequence, it is expected to grow in diameter (as it becomes a red giant) by a factor of 100 or more, possibly swallowing up inner planets such as Mercury and possibly Venus and even Earth.

If the star is like our Sun, or larger, further fusion can occur. As the star's outer envelope expands, its core continues to shrink and heat up. When the temperature reaches about 10^8 K, even helium nuclei, in spite of their greater charge and hence greater electrical repulsion, can come close enough to each other to undergo fusion. The reactions are

$$
\begin{aligned}
{}^4_2\text{He} + {}^4_2\text{He} &\rightarrow {}^8_4\text{Be} \\
{}^4_2\text{He} + {}^8_4\text{Be} &\rightarrow {}^{12}_6\text{C}
\end{aligned}
\qquad \textbf{(33–2)}
$$

with the emission of two γ rays. These two reactions must occur in quick succession (because ${}^8_4\text{Be}$ is very unstable), and the net effect is

$$3\,{}^4_2\text{He} \rightarrow {}^{12}_6\text{C} + 2\gamma. \qquad (Q = 7.3\,\text{MeV})$$

This fusion of helium causes a change in the star which moves rapidly to the "horizontal branch" on the H–R diagram (Fig. 33–8). Further fusion reactions are possible, with ${}^4_2\text{He}$ fusing with ${}^{12}_6\text{C}$ to form ${}^{16}_8\text{O}$. In more massive stars, higher Z elements like ${}^{20}_{10}\text{Ne}$ or ${}^{24}_{12}\text{Mg}$ can be made. This process of creating heavier nuclei from lighter ones (or by absorption of neutrons which tends to occur at higher Z) is called **nucleosynthesis**.

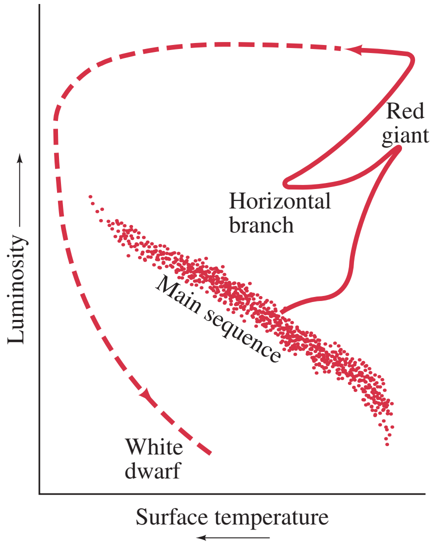


FIGURE 33–8 Evolutionary "track" of a star like our Sun represented on an H–R diagram.

Low Mass Stars—White Dwarfs

The final fate of a star depends on its mass. Stars can lose mass as parts of their outer envelope move off into space. Stars born with a mass less than about 8 solar masses (8× the mass of our Sun) eventually end up with a residual mass less than about 1.4 solar masses. A residual mass of 1.4 solar masses is known as the **Chandrasekhar limit**. For stars smaller than this, no further fusion energy can be obtained because of the large Coulomb repulsion between nuclei. The core of such a "low mass" star (original mass $\lesssim$ 8 solar masses) contracts under gravity. The outer envelope expands again and the star becomes an even brighter and larger red giant, Fig. 33–8. Eventually the outer layers escape into space, and the newly revealed surface is hotter than before. So the star moves to the left in the H–R diagram (horizontal dashed line in Fig. 33–8). Then, as the core shrinks the star cools, and typically follows the downward dashed route shown on the left in Fig. 33–8, becoming a **white dwarf**. A white dwarf with a residual mass equal to that of the Sun would be about the size of the Earth. A white dwarf contracts to the point at which the electrons start to overlap, but no further because, by the Pauli exclusion principle, no two electrons can be in the same quantum state. At this point the star is supported against further collapse by this **electron degeneracy** pressure. A white dwarf continues to lose internal energy by radiation, decreasing in temperature and becoming dimmer until it glows no more. It has then become a cold dark chunk of extremely dense material.

High Mass Stars—Supernovae, Neutron Stars, Black Holes

Stars whose original mass is greater than about 8 solar masses are thought to follow a very different scenario. A star with this great a mass can contract under gravity and heat up even further. At temperatures $T \approx 3$ or 4×10^9 K, nuclei as heavy as ${}^{56}_{26}\text{Fe}$ and ${}^{56}_{28}\text{Ni}$ can be made. But here the formation of heavy nuclei from lighter ones, by fusion, ends. As we saw in Fig. 30–1, the average binding energy per nucleon begins to decrease for A greater than about 60. Further fusions would *require* energy, rather than release it.

At these extremely high temperatures, well above 10^9 K, high-energy collisions can cause the breaking apart of iron and nickel nuclei into He nuclei, and eventually into protons and neutrons:

$$^{56}_{26}\text{Fe} \rightarrow 13\,^{4}_{2}\text{He} + 4\text{n}$$

$$^{4}_{2}\text{He} \rightarrow 2\text{p} + 2\text{n}.$$

These are energy-requiring (endothermic) reactions, which rob energy from the core, allowing gravitational contraction to begin. This then can force electrons and protons together to form neutrons in **inverse β decay**:

$$e^- + p \rightarrow n + \nu.$$

As a result of these reactions, the pressure in the core drops precipitously. As the core collapses under the huge gravitational forces, the tremendous mass becomes essentially an enormous nucleus made up almost exclusively of neutrons. The size of the star is no longer limited by the exclusion principle applied to electrons, but rather by **neutron degeneracy** pressure, and the star contracts rapidly to form an enormously dense **neutron star**. The core of a neutron star contracts to the point at which all neutrons are as close together as they are in an atomic nucleus. That is, the density of a neutron star is on the order of 10^{14} times greater than normal solids and liquids on Earth. A cupful of such dense matter would weigh billions of tons. A neutron star that has a mass 1.5 times that of our Sun would have a diameter of only about 20 km. (Compare this to a white dwarf with 1 solar mass whose diameter would be $\approx 10^4$ km, as mentioned on the previous page.)

The contraction of the core of a massive star would mean a great reduction in gravitational potential energy. Somehow this energy would have to be released. Indeed, it was suggested in the 1930s that the final core collapse to a neutron star could be accompanied by a catastrophic explosion known as a **supernova** (plural = supernovae). The tremendous energy release (Fig. 33–9) could form virtually all elements of the Periodic Table (see below) and blow away the entire outer envelope of the star, spreading its contents into interstellar space. The presence of heavy elements on Earth and in our solar system suggests that our solar system formed from the debris of many such supernova explosions.

(a) (b)

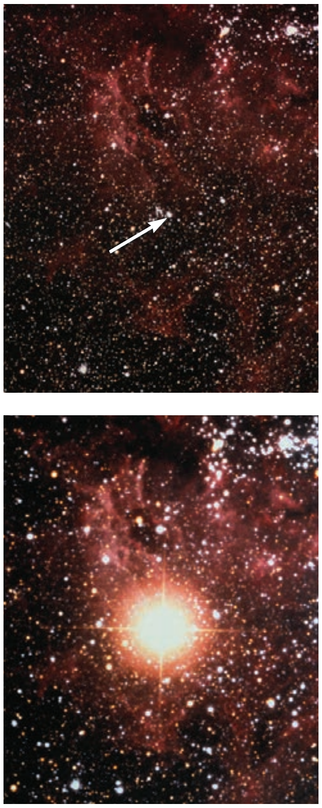
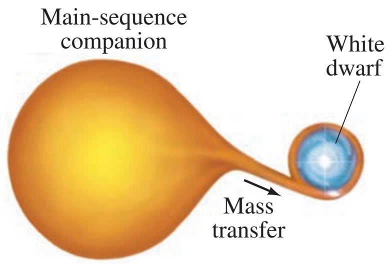


FIGURE 33–9 The star indicated by the arrow in (a) exploded in 1987 as a supernova (SN1987A), as shown in (b). The bright spot in (b) indicates a huge release of energy but does not represent the physical size.

The elements heavier than Ni are thought to form mainly by **neutron capture** in these exploding supernovae (rather than by fusion, as for elements up to Ni). Large numbers of free neutrons, resulting from nuclear reactions, are present inside those highly evolved stars and they can readily combine with, say, a $^{56}_{26}\text{Fe}$ nucleus to form (if three are captured) $^{59}_{26}\text{Fe}$, which decays to $^{59}_{27}\text{Co}$. The $^{59}_{27}\text{Co}$ can capture neutrons, also becoming neutron rich and decaying by β^- to the next higher Z element, and so on to the highest Z elements.

The final state of a neutron star depends on its mass. If the final mass is less than about three solar masses, the subsequent evolution of the neutron star is thought to resemble that of a white dwarf. If the mass is greater than this (original mass $\gtrsim 40$ solar masses), the neutron star collapses under gravity, overcoming even neutron degeneracy. Gravity would then be so strong that emitted light could not escape—it would be pulled back in by the force of gravity. Since no radiation could escape from such a "star," we could not see it—it would be black. An object may pass by it and be deflected by its gravitational field, but if the object came too close it would be swallowed up, never to escape. This is a **black hole**.

FIGURE 33–10 Hypothetical model for novae and Type Ia supernovae, showing how a white dwarf could pull mass from its normal companion.



Novae and Supernovae

Novae (singular is *nova*, meaning "new" in Latin) are faint stars that have suddenly increased in brightness by as much as a factor of 10^6 and last for a month or two before fading. Novae are thought to be faint white dwarfs that have pulled mass from a nearby companion (they make up a *binary* system), as illustrated in Fig. 33–10. The captured mass of hydrogen suddenly fuses into helium at a high rate for a few weeks. Many novae (maybe all) are *recurrent*—they repeat their bright glow years later.

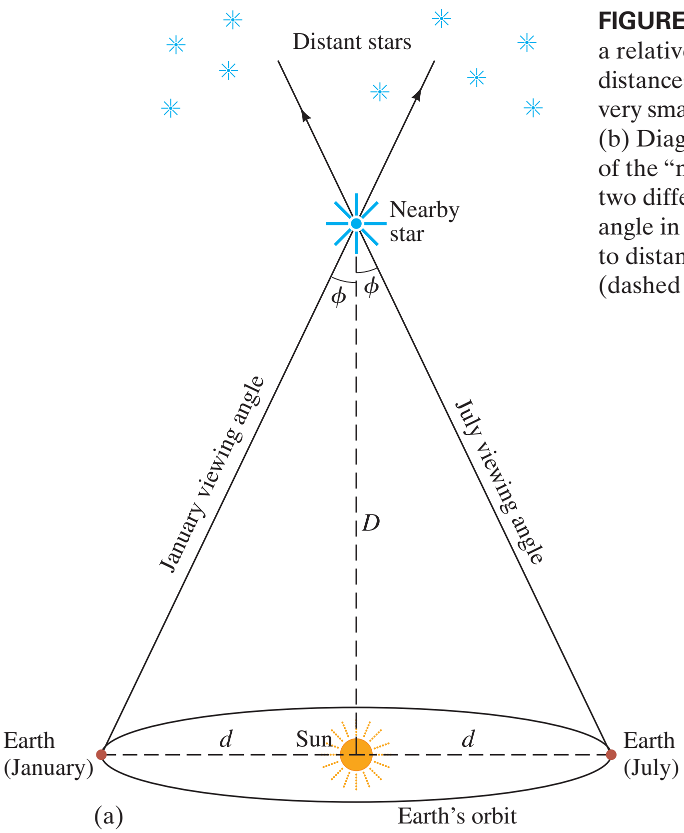
Supernovae are also brief explosive events, but release millions of times more energy than novae, up to 10^{10} times more luminous than our Sun. The peak of brightness may exceed that of the entire galaxy in which they are located, but lasts only a few days or weeks. They slowly fade over a few months. Many supernovae form by core collapse to a neutron star as described above. See Fig. 33–9.

Type Ia supernovae are different. They all seem to have very nearly the same luminosity. They are believed to be binary stars, one of which is a white dwarf that pulls mass from its companion, much like for a nova, Fig. 33–10. The mass is higher, and as mass is captured and the total mass approaches the Chandrasekhar limit of 1.4 solar masses, it explodes as a "white-dwarf" supernova by undergoing a "thermonuclear runaway"—an uncontrolled chain of nuclear reactions that entirely destroys the white dwarf. Type Ia supernovae are useful to us as "standard candles" in the night sky to help us determine distance—see next Section.

33–3 Distance Measurements

Parallax

We have talked about the vast distances of objects in the universe. But how do we measure these distances? One basic technique employs simple geometry to measure the **parallax** of a star. By parallax we mean the apparent motion of a star, against the background of much more distant stars, due to the Earth's motion around the Sun. As shown in Fig. 33–11, we can measure the angle 2ϕ that the star appears to shift, relative to very distant stars, when viewed 6 months apart. If we know the distance d from Earth to Sun, we can reconstruct the right triangles shown in Fig. 33–11 and can then determine the distance D to the star. This is essentially the way the heights of mountains are determined, by "triangulation": see Example 1–8.



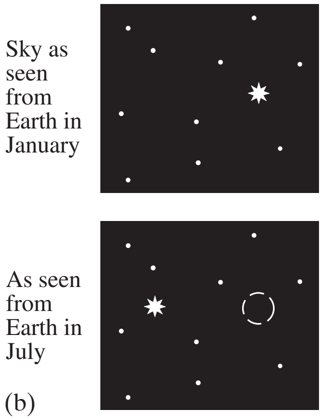


FIGURE 33–11 (a) Determining the distance D to a relatively nearby star using parallax. Horizontal distances are greatly exaggerated: in reality ϕ is a very small angle (less than $\frac{1}{3600}^{\circ} = 1'' = 1$ second of arc). (b) Diagram of the sky showing the apparent position of the "nearby" star relative to more distant stars, at two different times (January and July). The viewing angle in January puts the star more to the right relative to distant stars, whereas in July it is more to the left (dashed circle shows January location).

EXAMPLE 33–6 ESTIMATE **Distance to a star using parallax.** Estimate the distance D to a star if the angle 2ϕ in Fig. 33–11a is measured to be $2\phi = 0.00012°$.

APPROACH From trigonometry, $\tan\phi = d/D$ in Fig. 33–11a. The Sun–Earth distance is $d = 1.5 \times 10^8$ km (inside front cover).

SOLUTION The angle $\phi = 0.00006°$, or about $(0.00006°)(2\pi\ \text{rad}/360°) = 1.0 \times 10^{-6}$ radians. We can use $\tan\phi \approx \phi$ because ϕ is very small. We solve for D in $\tan\phi = d/D$. The distance D to the star is

$$D = \frac{d}{\tan\phi} \approx \frac{d}{\phi} = \frac{1.5 \times 10^8\ \text{km}}{1.0 \times 10^{-6}\ \text{rad}} = 1.5 \times 10^{14}\ \text{km},$$

or about 15 ly.

* Parsec

Distances to stars are often specified in terms of parallax angle (ϕ in Fig. 33–11a) given in seconds of arc: 1 second $(1'')$ is $\frac{1}{60}$ of one minute $(1')$ of arc, which is $\frac{1}{60}$ of a degree, so $1'' = \frac{1}{3600}$ of a degree. The distance is then specified in **parsecs** (pc) (meaning *par*allax angle in *sec*onds of arc): $D = 1/\phi$ with ϕ in seconds of arc. In Example 33–6, $\phi = (6 \times 10^{-5})°(3600) = 0.22''$ of arc, so we would say the star is at a distance of $1/0.22'' = 4.5$ pc. One parsec is given by (recall $D = d/\phi$, and we set the Sun–Earth distance (Fig. 33–11a) as $d = 1.496 \times 10^{11}$ m):

$$1\ \text{pc} = \frac{d}{1''} = \frac{1.496 \times 10^{11}\ \text{m}}{(1'')\left(\frac{1'}{60''}\right)\left(\frac{1°}{60'}\right)\left(\frac{2\pi\ \text{rad}}{360°}\right)} = 3.086 \times 10^{16}\ \text{m}$$

$$1\ \text{pc} = (3.086 \times 10^{16}\ \text{m})\left(\frac{1\ \text{ly}}{9.46 \times 10^{15}\ \text{m}}\right) = 3.26\ \text{ly}.$$

Distant Stars and Galaxies

Parallax can be used to determine the distance to stars as far away as about 100 light-years from Earth, and from an orbiting spacecraft perhaps 5 to 10 times farther. Beyond that distance, parallax angles are too small to measure. For greater distances, more subtle techniques must be employed. We might compare the apparent brightnesses of two stars, or two galaxies, and use the *inverse square law* (apparent brightness drops off as the square of the distance) to roughly estimate their relative distances. We can't expect this technique to be very precise because we don't expect any two stars, or two galaxies, to have the same intrinsic luminosity. When comparing galaxies, a perhaps better estimate assumes the brightest stars in all galaxies (or the brightest galaxies in galaxy clusters) are similar and have about the same intrinsic luminosity. Consequently, their *apparent brightness* would be a measure of how far away they were.

Another technique makes use of the H–R diagram. Measurement of a star's surface temperature (from its spectrum) places it at a certain point (within 20%) on the H–R diagram, assuming it is a main-sequence star, and then its luminosity can be estimated from the vertical axis (Fig. 33–6). Its apparent brightness and Eq. 33–1 give its approximate distance; see Example 33–5.

A better estimate comes from comparing *variable stars*, especially *Cepheid variables* whose luminosity varies over time with a period that is found to be related to their average luminosity. Thus, from their period and apparent brightness we get their distance.

Distance via SNIa, Redshift

The largest distances are estimated by comparing the apparent brightnesses of Type Ia supernovae ("SNIa"). Type Ia supernovae all have a similar origin (as described on the previous page and Fig. 33–10), and their brief explosive burst of light is expected to be of nearly the same luminosity. They are thus sometimes referred to as "standard candles."

Another important technique for estimating the distance of very distant galaxies is from the "redshift" in the line spectra of elements and compounds. The redshift is related to the expansion of the universe, as we shall discuss in Section 33–5. It is useful for objects farther than 10^7 to 10^8 ly away.

As we look farther and farther away, measurement techniques are less and less reliable, so there is more uncertainty in the measurements of large distances.

33–4 General Relativity: Gravity and the Curvature of Space

We have seen that the force of gravity plays an important role in the processes that occur in stars. Gravity too is important for the evolution of the universe as a whole. The reasons gravity plays a dominant role in the universe, and not one of the other of the four forces in nature, are (1) it is long-range and (2) it is always attractive. The strong and weak nuclear forces act over very short distances only, on the order of the size of a nucleus; hence they do not act over astronomical distances (they do act between nuclei and nucleons in stars to produce nuclear reactions). The electromagnetic force, like gravity, acts over great distances. But it can be either attractive or repulsive. And since the universe does not seem to contain large areas of net electric charge, a large net force does not occur. But gravity acts only as an *attractive* force between *all* masses, and there are large accumulations of mass in the universe. The force of gravity as Newton described it in his law of universal gravitation was modified by Einstein. In his general theory of relativity, Einstein developed a theory of gravity that now forms the basis of cosmological dynamics.

In the *special theory of relativity* (Chapter 26), Einstein concluded that there is no way for an observer to determine whether a given frame of reference is at rest or is moving at constant velocity in a straight line. Thus the laws of physics must be the same in different inertial reference frames. But what about the more general case of motion where reference frames can be *accelerating*?

Einstein tackled the problem of accelerating reference frames in his **general theory of relativity** and in it also developed a theory of gravity. The mathematics of General Relativity is complex, so our discussion will be mainly qualitative.

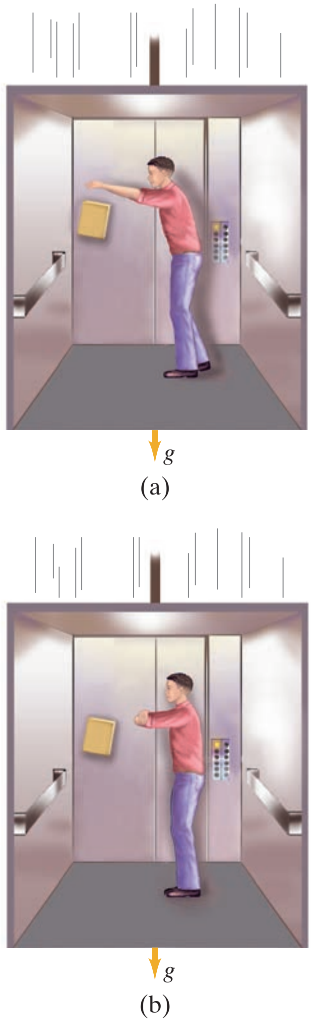
We begin with Einstein's **principle of equivalence**, which states that

> **no experiment can be performed that could distinguish between a uniform gravitational field and an equivalent uniform acceleration.**

If observers sensed that they were accelerating (as in a vehicle speeding around a sharp curve), they could not prove by any experiment that in fact they weren't simply experiencing the pull of a gravitational field. Conversely, we might think we are being pulled by gravity when in fact we are undergoing an acceleration having nothing to do with gravity.

As a thought experiment, consider a person in a freely falling elevator near the Earth's surface. If our observer held out a book and let go of it, what would happen? Gravity would pull it downward toward the Earth, but at the same rate $(g = 9.8\,\mathrm{m/s^2})$ at which the person and elevator were falling. So the book would hover right next to the person's hand (Fig. 33–12). The effect is exactly the same as if this reference frame was at rest and *no* forces were acting. On the other hand, if the elevator was out in space where the gravitational field is essentially zero, the released book would float, just as it does in Fig. 33–12. Next, if the elevator (out in space) is accelerated upward (using rockets) at an acceleration of $9.8\,\mathrm{m/s^2}$, the book as seen by our observer would fall to the floor with an acceleration of $9.8\,\mathrm{m/s^2}$, just as if it were falling due to gravity at the surface of the Earth. According to the principle of equivalence, the observer could not determine whether the book fell because the elevator was accelerating upward, or because a gravitational field was acting downward and the elevator was at rest. The two descriptions are equivalent.

FIGURE 33–12 In an elevator falling freely under gravity, (a) a person releases a book; (b) the released book hovers next to the owner's hand; (b) is a few moments after (a).



The principle of equivalence is related to the concept that there are two types of mass. Newton's second law, $F = ma$, uses **inertial mass**. We might say that inertial mass represents "resistance" to any type of force. The second type of mass is **gravitational mass**. When one object attracts another by the gravitational force (Newton's law of universal gravitation, $F = Gm_1m_2/r^2$, Chapter 5), the strength of the force is proportional to the product of the *gravitational masses* of the two objects. This is much like Coulomb's law for the electric force between two objects which is proportional to the product of their electric charges. The electric charge on an object is not related to its inertial mass; so why should we expect that an object's gravitational mass (call it gravitational charge if you like) be related to its inertial mass? All along we have assumed they were the same. Why? Because no experiment—not even of high precision—has been able to discern any measurable difference between inertial mass and gravitational mass. (For example, in the absence of air resistance, all objects fall at the same acceleration, g, on Earth.) This is another way to state the equivalence principle: *gravitational mass is equivalent to inertial mass*.

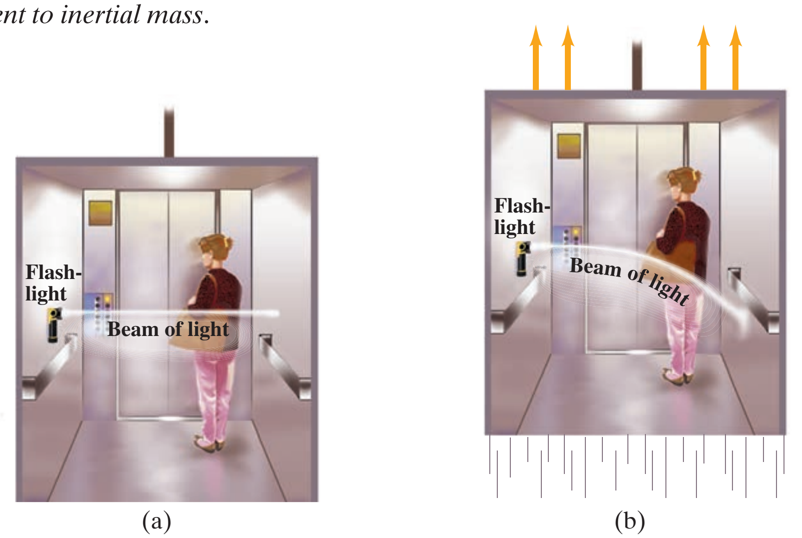
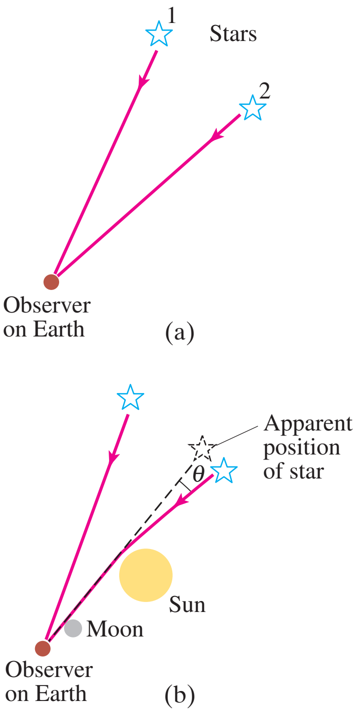


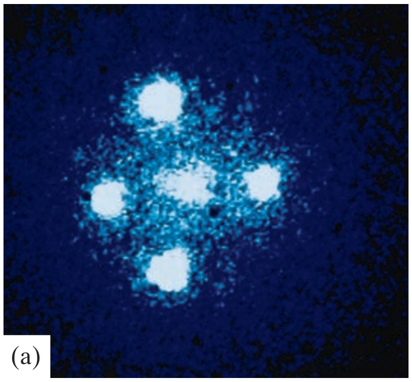
FIGURE 33–13 (a) Light beam goes straight across an elevator which is not accelerating. (b) The light beam bends (exaggerated) according to an observer in an accelerating elevator whose speed increases in the upward direction.

The principle of equivalence can be used to show that light ought to be deflected by the gravitational force due to a massive object. Consider another thought experiment, in which an elevator is in free space where virtually no gravity acts. If a light beam is emitted by a flashlight attached to the side of the elevator, the beam travels straight across the elevator and makes a spot on the opposite side if the elevator is at rest or moving at constant velocity (Fig. 33–13a). If instead the elevator is accelerating upward, as in Fig. 33–13b, the light beam still travels straight across in a reference frame at rest. In the upwardly accelerating elevator, however, the beam is observed to curve downward. Why? Because during the time the light travels from one side of the elevator to the other, the elevator is moving upward at a vertical speed that is increasing relative to the light. Next we note that according to the equivalence principle, an upwardly accelerating reference frame is equivalent to a downward gravitational field. Hence, we can picture the curved light path in Fig. 33–13b as being due to the effect of a gravitational field. Thus, from the principle of equivalence, we expect gravity to exert a force on a beam of light and to bend it out of a straight-line path!

That light is affected by gravity is an important prediction of Einstein's general theory of relativity. And it can be tested. The amount a light beam would be deflected from a straight-line path must be small even when passing a massive object. (For example, light near the Earth's surface after traveling 1 km is predicted to drop only about 10^{-10} m, which is equal to the diameter of a small atom and not detectable.) The most massive object near us is the Sun, and it was calculated that light from a distant star would be deflected by 1.75″ of arc (tiny but detectable) as it passed by the edge of the Sun (Fig. 33–14). However, such a measurement could be made only during a total eclipse of the Sun, so that the Sun's tremendous brightness would not obscure the starlight passing near its edge.

FIGURE 33–14 (a) Two stars in the sky observed from Earth. (b) If the light from one of these stars passes very near the Sun, whose gravity bends the rays, the star will appear higher than it actually is (follow the ray backwards). [Not to scale.]





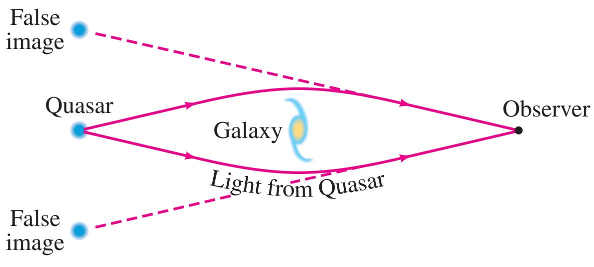


FIGURE 33–15 (a) Hubble Space Telescope photograph of the so-called "Einstein cross," thought to represent "gravitational lensing": the central spot is a relatively nearby galaxy, whereas the four other spots are thought to be images of a single quasar *behind* the galaxy. (b) Diagram showing how the galaxy could bend the light coming from the quasar behind it to produce the four images. See also Fig. 33–14. [If the shape of the nearby galaxy and distant quasar were perfect spheres and perfectly aligned, we would expect the "image" of the distant quasar to be a circular ring or halo instead of the four separate images seen here. Such a ring is called an "Einstein ring."]

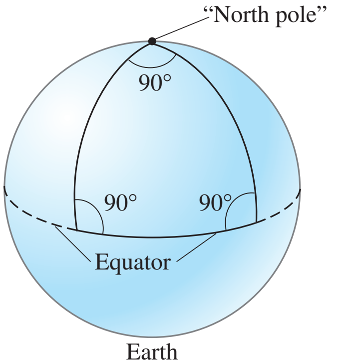
An opportune eclipse occurred in 1919, and scientists journeyed to the South Atlantic to observe it. Their photos of stars just behind the Sun revealed shifts in accordance with Einstein's prediction. Another example of gravitational deflection of light is **gravitational lensing**, as described in Fig. 33–15. The very distant galaxies shown in the XDF photo at the start of this Chapter, page 947, are thought to be visible only because of gravitational lensing (and magnification of their emitted light) by nearer galaxies—as if the nearby galaxies acted as a magnifying glass.

The mathematician Fermat showed in the 1600s that optical phenomena, including reflection, refraction, and effects of lenses, can be derived from a simple principle: that light traveling between two points follows the shortest path in space. Thus if gravity curves the path of light, then gravity must be able to curve space itself. That is, *space itself can be curved*, and it is gravitational mass that causes the curvature. Indeed, the curvature of space—or rather, of four-dimensional space-time—is a basic aspect of Einstein's General Relativity.

What is meant by **curved space**? To understand, recall that our normal method of viewing the world is via Euclidean plane geometry. In Euclidean geometry, there are many axioms and theorems we take for granted, such as that the sum of the angles of any triangle is 180°. Non-Euclidean geometries, which involve curved space, have also been imagined by mathematicians. It is hard enough to imagine three-dimensional curved space, much less curved four-dimensional space-time. So let us try to understand the idea of curved space by using two-dimensional surfaces.

Consider, for example, the two-dimensional surface of a sphere. It is clearly curved, Fig. 33–16, at least to us who view it from the outside—from our three-dimensional world. But how would hypothetical two-dimensional creatures determine whether their two-dimensional space was flat (a plane) or curved? One way would be to measure the sum of the angles of a triangle. If the surface is a plane, the sum of the angles is 180°, as we learn in plane geometry. But if the space is curved, and a sufficiently large triangle is constructed, the sum of the angles will *not* be 180°. To construct a triangle on a curved surface, say the sphere of Fig. 33–16, we must use the equivalent of a straight line: that is, the shortest distance between two points, which is called a **geodesic**. On a sphere, a geodesic is an arc of a great circle (an arc in a plane passing through the center of the sphere) such as the Earth's equator and the Earth's longitude lines. Consider, for example, the large triangle of Fig. 33–16: its sides are two longitude lines passing from the north pole to the equator, and the third side is a section of the equator as shown. The two longitude lines make 90° angles with the equator (look at a world globe to see this more clearly). They make an angle with each other at the north pole, which could be, say, 90° as shown; the sum of these angles is $90° + 90° + 90° = 270°$. This is clearly *not* a Euclidean space. Note, however, that if the triangle is small in comparison to the radius of the sphere, the angles will add up to nearly 180°, and the triangle (and space) will seem flat.

FIGURE 33–16 On a two-dimensional curved surface, the sum of the angles of a triangle may not be 180°.



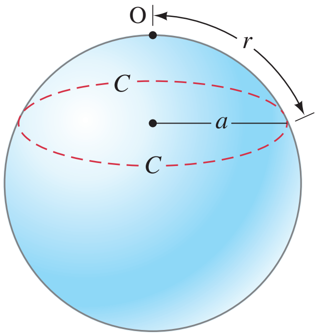


FIGURE 33–17 On a spherical surface (a two-dimensional world) a circle of circumference C is drawn (red) about point O as the center. The radius of the circle (not the sphere) is the distance r along the surface. (Note that in our three-dimensional view, we can tell that $C = 2\pi a$. Since $r > a$, then $C < 2\pi r$.)

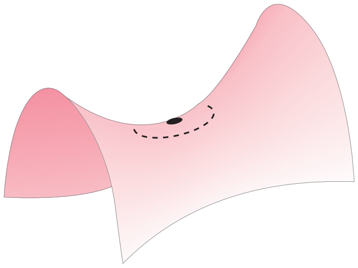


FIGURE 33–18 Example of a two-dimensional surface with negative curvature.

Another way to test the curvature of space is to measure the radius r and circumference C of a large circle. On a plane surface, $C = 2\pi r$. But on a two-dimensional spherical surface, C is *less* than $2\pi r$, as can be seen in Fig. 33–17. The proportionality between C and r is *less* than 2π. Such a surface is said to have *positive curvature*. On the saddlelike surface of Fig. 33–18, the circumference of a circle is greater than $2\pi r$, and the sum of the angles of a triangle is less than 180°. Such a surface is said to have a *negative curvature*.

Curvature of the Universe

What about our universe? On a large scale (not just near a large mass), what is the overall curvature of the universe? Does it have positive curvature, negative curvature, or is it flat (zero curvature)? We perceive our world as Euclidean (flat), but we can not exclude the possibility that space could have a curvature so slight that we don't normally notice it. This is a crucial question in cosmology, and it can be answered only by precise experimentation.

If the universe had a positive curvature, the universe would be *closed*, or *finite* in volume. This would *not* mean that the stars and galaxies extended out to a certain boundary, beyond which there is empty space. There is no boundary or edge in such a universe. The universe is all there is. If a particle were to move in a straight line in a particular direction, it would eventually return to the starting point—perhaps eons of time later.

On the other hand, if the curvature of space was zero or negative, the universe would be *open*. It could just go on forever. An open universe could be *infinite*; but according to recent research, even that may not necessarily be so.

Today the evidence is very strong that the universe on a large scale is very close to being flat. Indeed, it is so close to being flat that we can't tell if it might have very slightly positive or very slightly negative curvature.

Black Holes

According to Einstein's theory of general relativity (sometimes abbreviated GR), space-time is curved near massive objects. We might think of space as being like a thin rubber sheet: if a heavy weight is placed on the sheet, it sags as shown in Fig. 33–19a (top of next page). The weight corresponds to a huge mass that causes space (space itself!) to curve. Thus, in the context of general relativity† we do not speak of the “force” of gravity acting on objects. Instead we say that objects and light rays move as they do because space-time is curved. An object starting at rest or moving slowly near the great mass of Fig. 33–19a would follow a geodesic (the equivalent of a straight line in plane geometry) toward that great mass.

The extreme curvature of space-time shown in Fig. 33–19b could be produced by a **black hole**. A black hole, as we mentioned in Section 33–2, has such strong gravity that even light cannot escape from it. To become a black hole, an object of mass M must undergo **gravitational collapse**, contracting by gravitational self-attraction to within a radius called the **Schwarzschild radius**,

$$R = \frac{2GM}{c^2},$$

where G is the gravitational constant and c the speed of light. If an object collapses to within this radius, it is predicted by general relativity to collapse to a point at $r = 0$, forming an infinitely dense singularity. This prediction is uncertain, however, because in this realm we need to combine quantum mechanics with gravity, a unification of theories not yet achieved (Section 32–12).

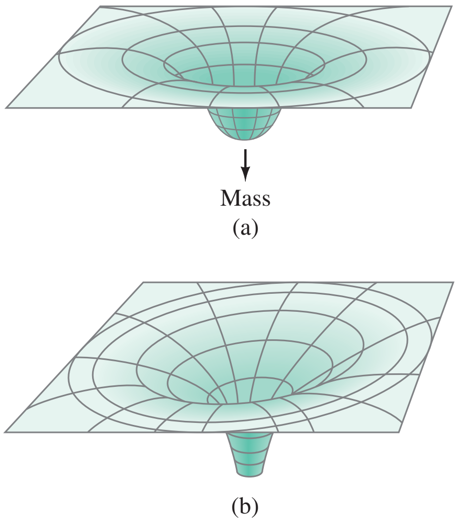


FIGURE 33–19 (a) Rubber-sheet analogy for space-time curved by matter. (b) Same analogy for a black hole, which can “swallow up” objects that pass near.

EXERCISE C What is the Schwarzschild radius for an object with 10 solar masses?

The Schwarzschild radius also represents the event horizon of a black hole. By **event horizon** we mean the surface beyond which no emitted signals can ever reach us, and thus inform us of events that happen beyond that surface. As a star collapses toward a black hole, the light it emits is pulled harder and harder by gravity, but we can still see it. Once the matter passes within the event horizon, the emitted light cannot escape but is pulled back in by gravity (= curvature of space-time).

All we can know about a black hole is its mass, its angular momentum (rotating black holes), and its electric charge. No other information, no details of its structure or the kind of matter it was formed of, can be known because no information can escape.

How might we observe black holes? We cannot see them because no light can escape from them. They would be black objects against a black sky. But they do exert a gravitational force on nearby objects, and also on light rays (or photons) that pass nearby (just like in Fig. 33–15). The black hole believed to be at the center of our Galaxy ($M \approx 4 \times 10^6\, M_{\mathrm{Sun}}$) was discovered by examining the motion of matter in its vicinity. Another technique is to examine stars which appear to move as if they were one member of a *binary system* (two stars rotating about their common center of mass), but without a visible companion. If the unseen star is a black hole, it might be expected to pull off gaseous material from its visible companion (as in Fig. 33–10). As this matter approached the black hole, it would be highly accelerated and should emit X-rays of a characteristic type before plunging inside the event horizon. Such X-rays, plus a sufficiently high mass estimate from the rotational motion, can provide evidence for a black hole. One of the many candidates for a black hole is in the binary-star system Cygnus X-1. It is widely believed that the center of most galaxies is occupied by a black hole with a mass 10^6 to 10^9 times the mass of a typical star like our Sun.

EXERCISE D A black hole has radius R. Its mass is proportional to (a) R, (b) R^2, (c) R^3. Justify your answer.

†Alexander Pope (1688–1744) wrote an epitaph for Newton:
“Nature, and Nature’s laws lay hid in night:
God said, *Let Newton be!* and all was light.”

Sir John Squire (1884–1958), perhaps uncomfortable with Einstein’s profound thoughts, added:
“It did not last: the Devil howling ‘*Ho!*
Let Einstein be!’ restored the status quo.”

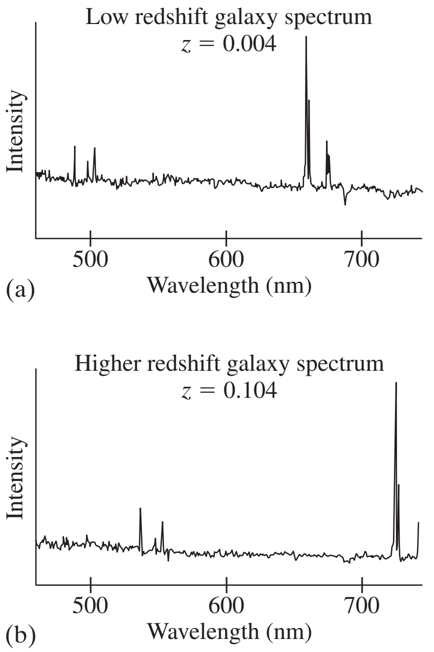


FIGURE 33–20 Atoms and molecules emit and absorb light of particular frequencies depending on the spacing of their energy levels, as we saw in Chapters 27 to 29. (a) The spectrum of light received from a relatively slow-moving galaxy. (b) Spectrum of a galaxy moving away from us at a much higher speed. Note how the peaks (or lines) in the spectrum have moved to longer wavelengths. The redshift is $z = (\lambda_{\text{obs}} - \lambda_{\text{rest}})/\lambda_{\text{rest}}$.

33–5 The Expanding Universe: Redshift and Hubble's Law

We discussed in Section 33–2 how individual stars evolve from their birth to their death as white dwarfs, neutron stars, or black holes. But what about the universe as a whole: is it static, or does it change? One of the most important scientific discoveries of the twentieth century was that distant galaxies are racing away from us, and that the farther they are from us at a given time, the faster they are moving away. How astronomers arrived at this astonishing idea, and what it means for the past history of the universe as well as its future, will occupy us for the remainder of the book.

Observational evidence that the universe is expanding was first put forth by Edwin Hubble in 1929. This idea was based on distance measurements of galaxies (Section 33–3), and determination of their velocities by the Doppler shift of spectral lines in the light received from them (Fig. 33–20). In Chapter 12 we saw how the frequency of sound is higher and the wavelength shorter if the source and observer move toward each other. If the source moves away from the observer, the frequency is lower and the wavelength longer. The **Doppler effect** occurs also for light, but the formula for light is slightly different than for sound and is given by†

$$\lambda_{\text{obs}} = \lambda_{\text{rest}}\sqrt{\frac{1 + v/c}{1 - v/c}}, \qquad \left[\begin{array}{l}\text{source and observer moving}\\ \text{away from each other}\end{array}\right] \qquad \textbf{(33–3)}$$

where λ_{rest} is the emitted wavelength as seen in a reference frame at rest with respect to the source, and λ_{obs} is the wavelength observed in a frame moving with velocity v away from the source along the line of sight. (For relative motion *toward* each other, $v < 0$ in this formula.) When a distant source emits light of a particular wavelength, and the source is moving away from us, the wavelength appears longer to us: the color of the light (if it is visible) is shifted toward the red end of the visible spectrum, an effect known as a **redshift**. (If the source moves toward us, the color shifts toward the blue or shorter wavelength.)

In the spectra of stars in other galaxies, lines are observed that correspond to lines in the known spectra of particular atoms (see Section 27–11 and Figs. 24–28 and 27–23). What Hubble found was that the lines seen in the spectra from distant galaxies were generally *redshifted*, and that the amount of shift seemed to be approximately proportional to the distance of the galaxy from us. That is, the velocity v of a galaxy moving away from us is proportional to its distance d from us:

HUBBLE'S LAW

$$v = H_0 d. \qquad \textbf{(33–4)}$$

This is **Hubble's law**, one of the most fundamental astronomical ideas. It was first suggested, in 1927, by Georges Lemaître, a Belgian physics professor and priest, who also first proposed what later came to be called the Big Bang. The constant H_0 is called the **Hubble parameter**.

The value of H_0 until recently was uncertain by over 20%, and thought to be between 15 and 25 km/s/Mly. But recent measurements now put its value more precisely at

$$H_0 = 21\ \text{km/s/Mly}$$

(that is, 21 km/s per million light-years of distance). The current uncertainty is about 2%, or ± 0.5 km/s/Mly. [H_0 can be written in terms of parsecs (Section 33–3) as $H_0 = 67$ km/s/Mpc (that is, 67 km/s per megaparsec of distance) with an uncertainty of about ± 1.2 km/s/Mpc.]

†For light there is no medium and we can make no distinction between motion of the source and motion of the observer (special relativity), as we did for sound which travels in a medium.

Redshift Origins

Galaxies very near us seem to be moving randomly relative to us: some move towards us (blueshifted), others away from us (redshifted); their speeds are on the order of $0.001c$. But for more distant galaxies, the velocity of recession is much greater than the velocity of local random motion, and so is dominant and Hubble's law (Eq. 33–4) holds very well. More distant galaxies have higher recession velocity and a larger redshift, and we call their redshift a **cosmological redshift**. We interpret this redshift today as due to the *expansion of space* itself. We can think of the originally emitted wavelength λ_{rest} as being stretched out (becoming longer) along with the expanding space around it, as suggested in Fig. 33–21. Although Hubble thought of the redshift as a Doppler shift, now we prefer to understand it in this sense of expanding space. (But note that atoms in galaxies do not expand as space expands; they keep their regular size.)

There is a third way to produce a redshift, which we mention for completeness: a **gravitational redshift**. Light leaving a massive star is gaining in gravitational potential energy (just like a stone thrown upward from Earth). So the kinetic energy of each photon, hf, must be getting smaller (to conserve energy). A smaller frequency f means a larger (longer) wavelength $\lambda\ (= c/f)$, which is a redshift.

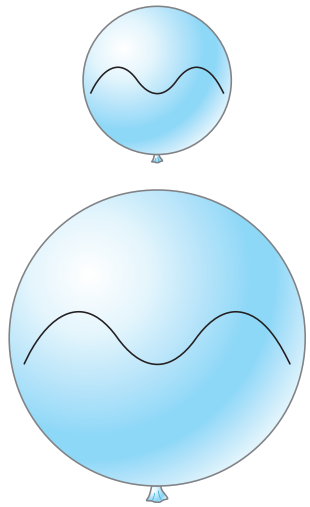


FIGURE 33–21 Simplified model of a 2-dimensional universe, imagined as a balloon. As you blow up the balloon (= expanding universe), the wavelength of a wave on its surface gets longer (redshifted).

The amount of a redshift is specified by the **redshift parameter**, z, defined as

$$z = \frac{\lambda_{\text{obs}} - \lambda_{\text{rest}}}{\lambda_{\text{rest}}} = \frac{\Delta\lambda}{\lambda_{\text{rest}}}, \qquad \textbf{(33–5a)}$$

where λ_{rest} is a wavelength as seen by an observer at rest relative to the source, and λ_{obs} is the wavelength measured by a moving observer. Equation 33–5a can be written as

$$z = \frac{\lambda_{\text{obs}}}{\lambda_{\text{rest}}} - 1 \qquad \textbf{(33–5b)}$$

and

$$z + 1 = \frac{\lambda_{\text{obs}}}{\lambda_{\text{rest}}}. \qquad \textbf{(33–5c)}$$

For low speeds not close to the speed of light ($v \lesssim 0.1\,c$), the Doppler formula (Eq. 33–3) can be used to show (Problem 31) that z is proportional to the speed of the source toward or away from us:

$$z = \frac{\lambda_{\text{obs}} - \lambda_{\text{rest}}}{\lambda_{\text{rest}}} = \frac{\Delta\lambda}{\lambda_{\text{rest}}} \approx \frac{v}{c}. \qquad [v \ll c] \quad \textbf{(33–6)}$$

But redshifts are not always small, in which case the approximation of Eq. 33–6 is not valid. For high z galaxies, not even Eq. 33–3 applies because the redshift is due to the expansion of space (cosmological redshift), not the Doppler effect. Our Chapter-Opening Photograph, page 947, shows two very distant high z galaxies, $z = 8.8$ and 11.9, which are also shown enlarged.

*Scale Factor (advanced)

The expansion of space can be described as a scaling of the typical distance between two points or objects in the universe. If two distant galaxies are a distance d_0 apart at some initial time, then a time t later they will be separated by a greater distance $d(t)$. The **scale factor** is the same as for light, expressed in Eq. 33–5a:

$$\frac{d(t) - d_0}{d_0} = \frac{\Delta\lambda}{\lambda} = z$$

or

$$\frac{d(t)}{d_0} = 1 + z.$$

Thus, for example, if a galaxy has $z = 3$, then the scale factor is now $(1 + 3) = 4$ times larger than when the light was emitted from that galaxy. That is, the average distance between galaxies has become 4 times larger. Thus the factor by which the wavelength has increased since it was emitted tells us by what factor the universe (or the typical distance between objects) has increased.

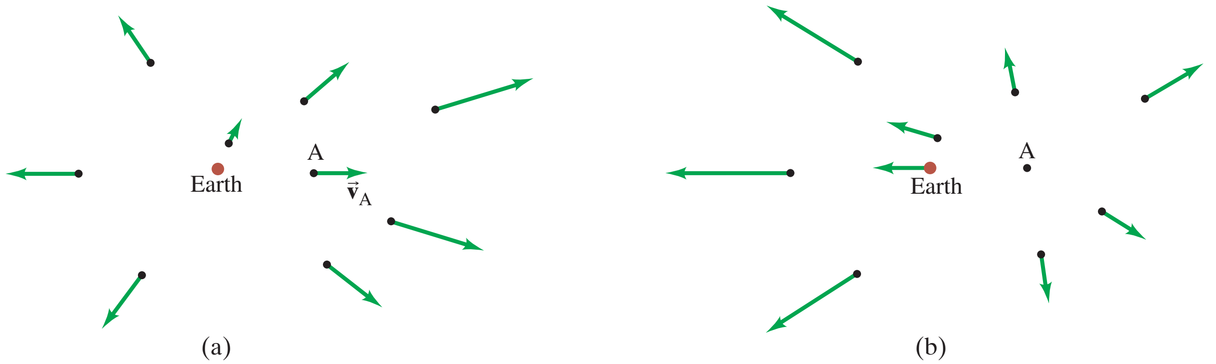


FIGURE 33–22 Expansion of the universe looks the same from any point in the universe. If you are on Earth as shown in part (a), or you are instead at galaxy A (which is at rest in the reference frame shown in (b)), all other galaxies appear to be racing away from you.

Expansion, and the Cosmological Principle

What does it mean that distant galaxies are all moving away from us, and with ever greater speed the farther they are from us? It seems to suggest some kind of explosive expansion that started at some very distant time in the past. And at first sight we seem to be in the middle of it all. But we aren't. The expansion appears the same from any other point in the universe. To understand why, see Fig. 33–22. In Fig. 33–22a we have the view from Earth (or from our Galaxy). The velocities of surrounding galaxies are indicated by arrows, pointing away from us, and the arrows are longer (faster speeds) for galaxies more distant from us. Now, what if we were on the galaxy labeled A in Fig. 33–22a? From Earth, galaxy A appears to be moving to the right at a velocity, call it $\vec{\mathbf{v}}_A$, represented by the arrow pointing to the right. If we were *on* galaxy A, Earth would appear to be moving to the left at velocity $-\vec{\mathbf{v}}_A$. To determine the velocities of other galaxies relative to A, we vectorially add the velocity vector, $-\vec{\mathbf{v}}_A$, to all the velocity arrows shown in Fig. 33–22a. This yields Fig. 33–22b, where we see that the universe is expanding away from galaxy A as well; and the velocities of galaxies receding from A are proportional to their current distance from A. *The universe looks pretty much the same from different points*.

Thus the expansion of the universe can be stated as follows: all galaxies are racing away from *each other* at an average rate of about 21 km/s per million light-years of distance between them. The ramifications of this idea are profound, and we discuss them in a moment.

A basic assumption in cosmology has been that on a large scale, the universe would look the same to observers at different places at the same time. In other words, the universe is both *isotropic* (looks the same in all directions) and *homogeneous* (would look the same if we were located elsewhere, say in another galaxy). This assumption is called the **cosmological principle**. On a local scale, say in our solar system or within our Galaxy, it clearly does not apply (the sky looks different in different directions). But it has long been thought to be valid if we look on a large enough scale, so that the average population density of galaxies and clusters of galaxies ought to be the same in different areas of the sky. This seems to be valid on distances greater than about 700 Mly. The expansion of the universe (Fig. 33–22) is consistent with the cosmological principle; and the near uniformity of the cosmic microwave background radiation (discussed in Section 33–6) supports it. Another way to state the cosmological principle is that *our place in the universe is not special*.

The expansion of the universe, as described by Hubble's law, strongly suggests that galaxies must have been closer together in the past than they are now. This is, in fact, the basis of the *Big Bang* theory of the origin of the universe, which pictures the universe as a relentless expansion starting from a very hot and compressed beginning. We discuss the Big Bang in detail shortly, but first let us see what can be said about the age of the universe.

One way to estimate the age of the universe uses the Hubble parameter. With $H_0 \approx 21\,\mathrm{km/s}$ per 10^6 light-years, the time required for the galaxies to arrive at their present separations would be approximately (starting with $v = d/t$ and using Hubble's law, Eq. 33–4),

$$t = \frac{d}{v} = \frac{d}{H_0 d} = \frac{1}{H_0} \approx \frac{(10^6\,\mathrm{ly})(0.95 \times 10^{13}\,\mathrm{km/ly})}{(21\,\mathrm{km/s})(3.16 \times 10^7\,\mathrm{s/yr})} \approx 14 \times 10^9\,\mathrm{yr},$$

or 14 billion years. The age of the universe calculated in this way is called the *characteristic expansion time* or "Hubble age." It is a very rough estimate and assumes the rate of expansion of the universe was constant (which today we are quite sure is not true). Today's best measurements give the age of the universe as about $13.8 \times 10^9\,\mathrm{yr}$, in remarkable agreement with the rough Hubble age estimate.

* Steady-State Model

Before discussing the Big Bang in detail, we mention one alternative to the Big Bang—the **steady-state model**—which assumed that the universe is infinitely old and on average looks the same now as it always has. (This assumed uniformity in time as well as space was called the *perfect cosmological principle*.) According to the steady-state model, no large-scale changes have taken place in the universe as a whole, particularly no Big Bang. To maintain this view in the face of the recession of galaxies away from each other, matter would need to be created continuously to maintain the assumption of uniformity. The rate of mass creation required is very small—about one nucleon per cubic meter every 10^9 years.

The steady-state model provided the Big Bang model with healthy competition in the mid-twentieth century. But the discovery of the cosmic microwave background radiation (next Section), as well as other observations of the universe, has made the Big Bang model universally accepted.

33–6 The Big Bang and the Cosmic Microwave Background

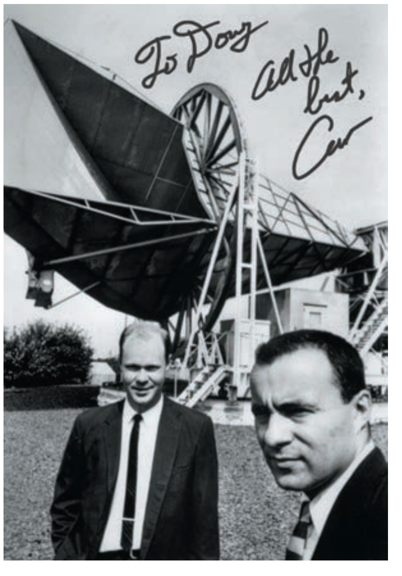
The expansion of the universe suggests that typical objects in the universe were once much closer together than they are now. This is the basis for the idea that the universe began about 14 billion years ago as an expansion from a state of very high density and temperature known affectionately as the **Big Bang**.

The birth of the universe was not an explosion, because an explosion blows pieces out into the surrounding space. Instead, the Big Bang was the start of an expansion of space itself. The observable universe was relatively very small at the start and has been expanding, getting ever larger, ever since. The initial tiny universe of extremely dense matter is not to be thought of as a concentrated mass in the midst of a much larger space around it. The initial tiny but dense universe was the *entire universe*. There wouldn't have been anything else. When we say that the universe was once smaller than it is now, we mean that the average separation between objects (such as electrons or galaxies) was less. The universe may have been infinite in extent even then, and it may still be now (only bigger). The **observable universe** (that which we have the possibility of observing because light has had time to reach us) is, however, finite.

A major piece of evidence supporting the Big Bang is the **cosmic microwave background** radiation (or CMB) whose discovery came about as follows.

In 1964, Arno Penzias and Robert Wilson pointed their horn antenna for detecting radio waves (Fig. 33–23) into the sky. With it they detected widespread emission, and became convinced that it was coming from outside our Galaxy. They made precise measurements at a wavelength $\lambda = 7.35\,\mathrm{cm}$, in the microwave region of the electromagnetic spectrum (Fig. 22–8). The intensity of this radiation was found initially not to vary by day or night or time of year, nor to depend on direction. It came from all directions in the universe with equal intensity, to a precision of better than 1%. It could only be concluded that this radiation came from the universe as a whole.

FIGURE 33–23 Photo of Arno Penzias (right, who signed it "Arno") and Robert Wilson. Behind them their "horn antenna."



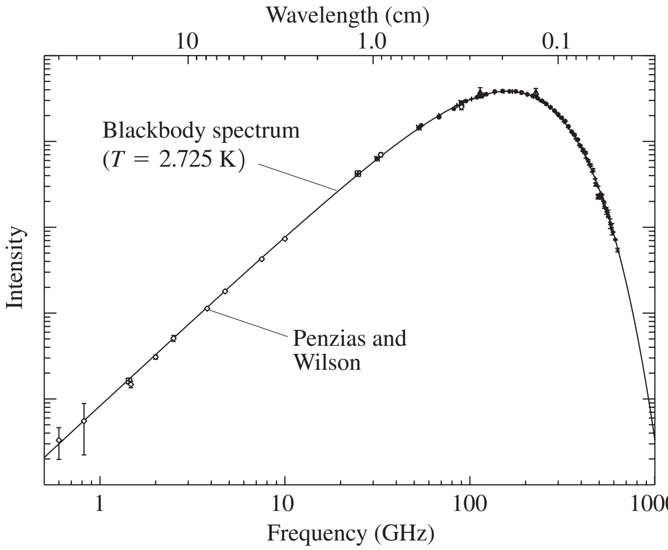
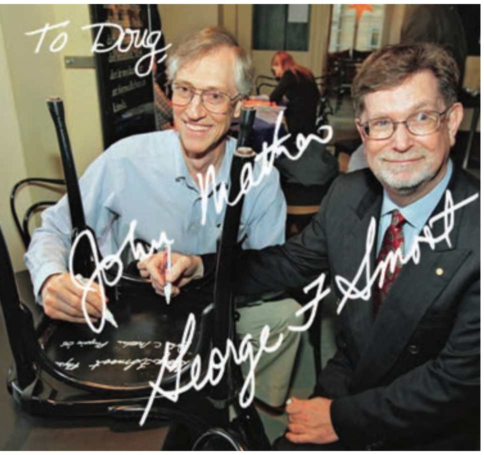


FIGURE 33–24 Spectrum of cosmic microwave background radiation, showing blackbody curve and experimental measurements including at the frequency detected by Penzias and Wilson. (Thanks to G. F. Smoot and D. Scott. The vertical bars represent the most recent experimental uncertainty in a measurement.)

The intensity of this CMB measured at $\lambda = 7.35\,\text{cm}$ corresponds to blackbody radiation (see Section 27–2) at a temperature of about 3 K. When radiation at other wavelengths was measured by the COBE satellite (COsmic Background Explorer), the intensities were found to fall on a nearly perfect blackbody curve as shown in Fig. 33–24, corresponding to a temperature of 2.725 K (± 0.002 K).

FIGURE 33–25 COBE scientists John Mather (chief scientist and responsible for measuring the blackbody form of the spectrum) and George Smoot (chief investigator for anisotropy experiment) shown here during celebrations for their Dec. 2006 Nobel Prize, given for their discovery of the spectrum and anisotropy of the CMB using the COBE instrument.



The remarkable uniformity of the CMB was in accordance with the cosmological principle. But theorists felt that there needed to be some small inhomogeneities, or "anisotropies," in the CMB that would have provided "seeds" at which galaxy formation could have started. Small areas of slightly higher density, which could have contracted under gravity to form clusters of galaxies, were indeed found. These tiny inhomogeneities in density and temperature were detected first by the COBE satellite experiment in 1992, led by George Smoot and John Mather (Fig. 33–25).

This discovery of the **anisotropy** of the CMB ranks with the discovery of the CMB itself in the history of cosmology. The blackbody fit and the anisotropy were the culmination of decades of research by pioneers such as Richard Muller, Paul Richards, and David Wilkinson. Subsequent experiments gave us greater detail in 2003, 2006, and 2012 with the WMAP (Wilkinson Microwave Anisotropy Probe) results, Fig. 33–26, and even more recently with the European Planck satellite results in 2013.

The CMB provides strong evidence in support of the Big Bang, and gives us information about conditions in the very early universe. In fact, in the late 1940s, George Gamow and his collaborators calculated that a Big Bang origin of the universe should have generated just such a microwave background radiation.

To understand why, let us look at what a Big Bang might have been like. (Today we usually use the term "Big Bang" to refer to the *process*, starting from a moment after the birth of the universe through the subsequent expansion.) The temperature must have been extremely high at the start, so high that there could not have been any atoms in the very early stages of the universe (high energy collisions would have broken atoms apart into nuclei and free electrons). Instead, the universe would have consisted solely of radiation (photons) and a plasma of charged electrons and other elementary particles. The universe would have been

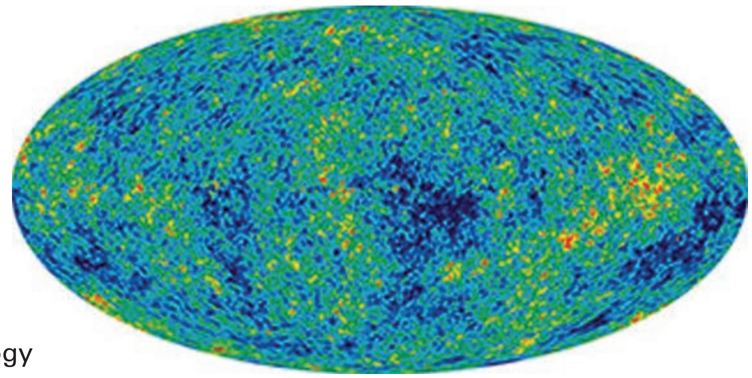


FIGURE 33–26 Measurements of the cosmic microwave background radiation over the entire sky, color-coded to represent differences in temperature from the average 2.725 K: the color scale ranges from $+200\,\mu$K (red) to $-200\,\mu$K (dark blue), representing slightly hotter and colder spots (associated with variations in density). Results are from the WMAP satellite in 2012: the angular resolution is 0.2°.

opaque—the photons in a sense "trapped," traveling very short distances before being scattered again, primarily by electrons. Indeed, the details of the microwave background radiation provide strong evidence that matter and radiation were once in equilibrium at a very high temperature. As the universe expanded, the energy spread out over an increasingly larger volume and the temperature dropped. Not long before the temperature had fallen to ~ 3000 K, some 380,000 years later, could nuclei and electrons combine together as stable atoms. With the disappearance of free electrons, as they combined with nuclei to form atoms, the radiation would have been freed—**decoupled** from matter, we say. The universe became *transparent* because photons were now free to travel nearly unimpeded straight through the universe.

It is this radiation, from 380,000 years after the birth of the universe, that we now see as the CMB. As the universe expanded, so too the wavelengths of the radiation lengthened, thus redshifting to longer wavelengths that correspond to lower temperature (recall Wien's law, $\lambda_P T = \text{constant}$, Section 27–2), until they would have reached the 2.7-K background radiation we observe today.

Looking Back toward the Big Bang—Lookback Time

Figure 33–27 shows our Earth point of view, looking out in all directions back toward the Big Bang and the brief (380,000-year-long) period when radiation was trapped in the early plasma (yellow band). The time it takes light to reach us from an event is called its **lookback time**. The "close-up" insert in Fig. 33–27 shows a photon scattering repeatedly inside that early plasma and then exiting the plasma in a straight line. No matter what direction we look, our view of the very early universe is blocked by this wall of plasma. It is like trying to look into a very thick fog or into the surface of the Sun—we can see only as far as its surface, called the **surface of last scattering**, but not into it. Wavelengths from there are redshifted by $z \approx 1100$. Time $\Delta t'$ in Fig. 33–27 is the lookback time (not real time that goes forward).

Recall that when we view an object far away, we are seeing it as it was then, when the light was emitted, not as it would appear today.

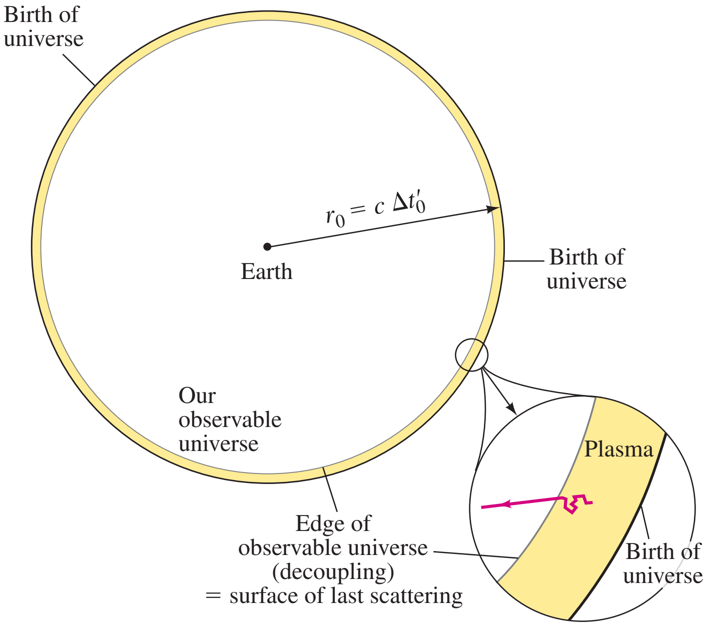


FIGURE 33–27 When we look out from the Earth, we look back in time. Any other observer in the universe would see more or less the same thing. The farther an object is from us, the longer ago the light we see had to have left it. We cannot see quite as far as the Big Bang; we can see only as far as the "surface of last scattering," which radiated the CMB. The insert on the lower right shows the earliest 380,000 years of the universe when it was opaque: a photon is shown scattering many times and then (at decoupling, 380,000 yr after the birth of the universe) becoming free to travel in a straight line. If this photon wasn't heading our way when "liberated," many others were. Galaxies are not shown, but would be concentrated close to Earth in this diagram because they were created relatively recently. *Note:* This diagram is not a normal map. Maps show a section of the world as might be seen all *at a given time*. This diagram shows space (like a map), but each point is *not* at the same time. The light coming from a point a distance r from Earth took a time $\Delta t' = r/c$ to reach Earth, and thus shows an event that took place long ago, a time $\Delta t' = r/c$ in the past, which we call its "lookback time." The universe began $\Delta t'_0 = 13.8\ \text{Gyr}$ ago.

The Observable Universe

Figure 33–27 can easily be misinterpreted: it is not a picture of the universe at a given instant, but is intended to suggest how we look out in all directions from our observation point (the Earth, or near it). Be careful not to think that the birth of the universe took place in a circle or a sphere surrounding us as if Fig. 33–27 were a photo taken at a given moment. What Fig. 33–27 does show is what we can see, the *observable universe*. Better yet, it shows the *most* we could see.

We would undoubtedly be arrogant to think that we could see the entire universe. Indeed, theories assume that we cannot see everything, that the **entire universe** is greater than the **observable universe**, which is a sphere of radius $r_0 = ct_0$ centered on the observer, with t_0 being the age of the universe. We can never see further back than the time it takes light to reach us.

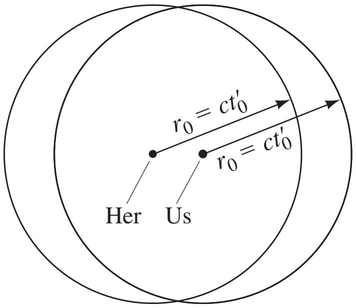


FIGURE 33–28 Two observers, on widely separated galaxies, have different horizons, different observable universes.

Consider, for example, an observer in another galaxy, very far from us, located to the left of our observation point in Fig. 33–27. That observer would not yet have seen light coming from the far right of the large circle in Fig. 33–27 that we see—it will take some time for that light to reach her. But she will have already, some time ago, seen the light coming from the left that we are seeing now. In fact, her observable universe, superimposed on ours, is suggested by Fig. 33–28.

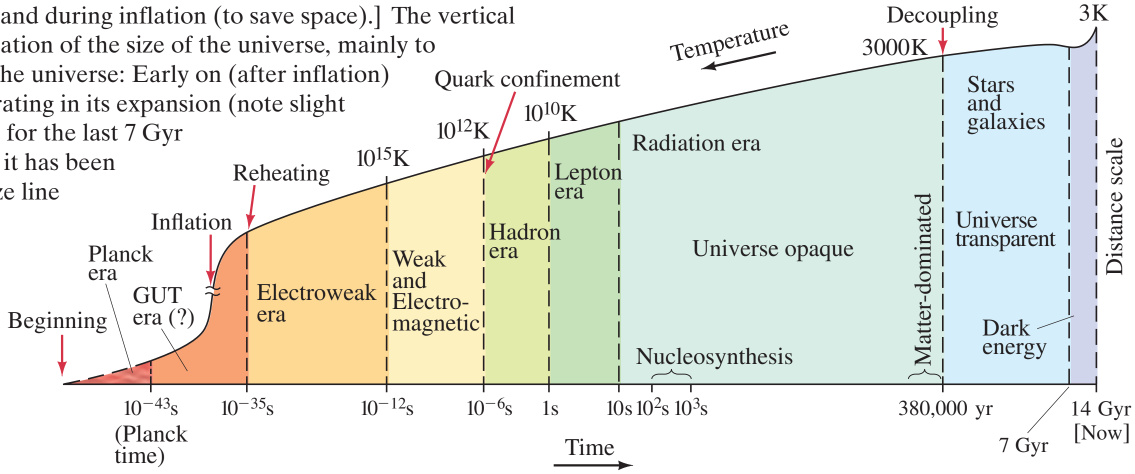
The edge of our observable universe is called the **horizon**. We could, in principle, see as far as the horizon, but not beyond it. An observer in another galaxy, far from us, will have a different horizon.

33–7 The Standard Cosmological Model: Early History of the Universe

In the last decade or two, a convincing theory of the origin and evolution of the universe has been developed, now called the **Standard Cosmological Model**. Part of this theory is based on recent theoretical and experimental advances in elementary particle physics, and part from observations of the universe including COBE, WMAP, and Planck. Indeed, cosmology and elementary particle physics have cross-fertilized to a surprising extent.

Let us go back to the earliest of times—as close as possible to the Big Bang—and follow a Standard Model theoretical scenario of events as the universe expanded and cooled after the Big Bang. Initially we talk of extremely small time intervals as well as extremely high temperatures, far higher than any temperature in the universe today. Figure 33–29 is a compressed graphical representation of the events, and it may be helpful to consult it as we go along.

FIGURE 33–29 Compressed graphical representation of the development of the universe after the Big Bang, according to modern cosmology. [The time scale is mostly logarithmic (each factor of 10 in time gets equal treatment), except at the start (there can be no $t = 0$ on a log scale), and during inflation (to save space).] The vertical height is a rough indication of the size of the universe, mainly to suggest expansion of the universe: Early on (after inflation) the universe is decelerating in its expansion (note slight downward curve); but for the last 7 Gyr (= thin strip on right) it has been accelerating, so the size line on the top curves upward at upper right.



The History

We begin at a time only a minuscule fraction of a second after the "beginning" of the universe, 10^{-43} s. This time (sometimes referred to as the **Planck time**) is an unimaginably short time, and predictions can be only speculative. Earlier, we can say nothing because we do not have a theory of quantum gravity which would be needed for the incredibly high densities and temperatures during this "Planck era."

The first theories of the Big Bang assumed the universe was extremely hot in the beginning, maybe 10^{32} K, and then gradually cooled down while expanding. In those first moments after 10^{-43} s, the four forces of nature were thought to be united—there was only one force (Chapter 32, Fig. 32–22). Then a kind of “phase transition” would have occurred during which the gravitational force would have “condensed out” as a separate force. This and subsequent phase transitions, as shown in Fig. 32–22, are analogous to phase transitions water undergoes as it cools from a gas condensing into a liquid, and with further cooling freezes into ice.† The *symmetry* of the four forces would have been broken leaving the strong, weak, and electromagnetic forces still unified, and the universe would have entered the **grand unified era** (GUT—see Section 32–11).

This scenario of a *hot* Big Bang is now doubted by some important theorists, such as Andreí Linde, whose theories suggest the universe was much cooler at the Planck time. But what happened next to the universe, though very strange, is accepted by most cosmologists: a brilliant idea, suggested by Linde and Alan Guth in the early 1980s, proposed that the universe underwent an incredible exponential expansion, increasing in size by a factor of 10^{30} or maybe much more, in a tiny fraction of a second, perhaps 10^{-35} s or 10^{-32} s. The usefulness of this **inflationary scenario** is that it solved major problems with earlier Big Bang models, such as explaining why the universe is flat, as well as the thermal equilibrium to provide the nearly uniform CMB, as discussed below.

When inflation ended, whatever energy caused it then ended up being transformed into elementary particles with very high kinetic energy, corresponding to very high temperature (Eq. 13–8, $\overline{\text{KE}} = \frac{3}{2}kT$). That process is referred to as **reheating**, and the universe was now a “soup” of leptons, quarks, and other particles. We can think of this “soup” as a plasma of particles and antiparticles, as well as photons—all in roughly equal numbers—colliding with one another frequently and exchanging energy.

The temperature of the universe at the end of inflation was much lower than that expected by the hot Big Bang theory. But it would have been high enough so that the weak and electromagnetic forces were unified into a single force, and this stage of the universe is sometimes called the **electroweak era**. Approximately 10^{-12} s after the Big Bang, the temperature dropped to about 10^{15} K corresponding to randomly moving particles with an average kinetic energy KE of about 100 GeV (see Eq. 13–8):

$$\text{KE} \approx kT \approx \frac{(1.4\times10^{-23}\,\text{J/K})(10^{15}\,\text{K})}{1.6\times10^{-19}\,\text{J/eV}} \approx 10^{11}\,\text{eV} = 100\,\text{GeV}.$$

(As an estimate, we usually ignore the factor $\frac{3}{2}$ in Eq. 13–8.) At that time, symmetry between weak and electromagnetic forces would have broken down, and the weak force separated from the electromagnetic.

As the universe cooled down to about 10^{12} K (KE $\approx$ 100 MeV), approximately 10^{-6} s after the Big Bang, quarks stop moving freely and begin to “condense” into more normal particles: nucleons and the other hadrons and their antiparticles. With this **confinement of quarks**, the universe entered the **hadron era**. But it did not last long. Very soon the vast majority of hadrons disappeared. To see why, let us focus on the most familiar hadrons: nucleons and their antiparticles. When the average kinetic energy of particles was somewhat higher than 1 GeV, protons, neutrons, and their antiparticles were continually being created out of the energies of collisions involving photons and other particles, such as

$$\begin{aligned}\text{photons} &\rightarrow \text{p} + \overline{\text{p}}\\ &\rightarrow \text{n} + \overline{\text{n}}.\end{aligned}$$

But just as quickly, particles and antiparticles would annihilate: for example

$$\text{p} + \overline{\text{p}} \rightarrow \text{photons or leptons.}$$

So the processes of creation and annihilation of nucleons were in equilibrium. The numbers of nucleons and antinucleons were high—roughly as many as there were electrons, positrons, or photons. But as the universe expanded and cooled, and the average kinetic energy of particles dropped below about 1 GeV, which is the minimum energy needed in a typical collision to create nucleons and antinucleons (about 940 MeV each), the process of nucleon creation could not continue.

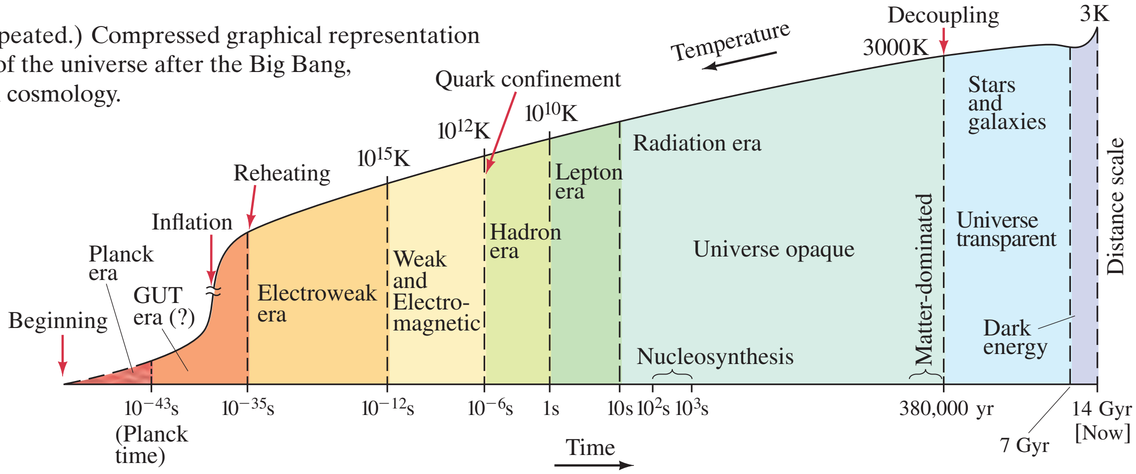
†It may be interesting to point out that this story of origins here bears some resemblance to ancient accounts (nonscientific) that mention the “void,” “formless wasteland” (or “darkness over the deep”), “abyss,” “divide the waters” (= a phase transition?), not to mention the sudden appearance of light.

Annihilation could continue, however, with antinucleons annihilating nucleons, until almost no nucleons were left. But not quite zero. Somehow we need to explain our present world of matter (nucleons and electrons) with very little antimatter in sight.

To explain our world of matter, we might suppose that earlier in the universe, after the inflationary period, a slight excess of quarks over antiquarks was formed.† This would have resulted in a slight excess of nucleons over antinucleons. And it is these "leftover" nucleons that we are made of today. The excess of nucleons over antinucleons was probably about one part in 10^9. During the hadron era, there should have been about as many nucleons as photons. After it ended, the "leftover" nucleons thus numbered only about one nucleon per 10^9 photons, and this ratio has persisted to this day. Protons, neutrons, and all other heavier particles were thus tremendously reduced in number by about 10^{-6} s after the Big Bang. The lightest hadrons, the pions, soon disappeared, about 10^{-4} s after the Big Bang; because they are the lightest mass hadrons (140 MeV), pions were the last hadrons able to be created as the temperature (and average kinetic energy) dropped. Lighter particles, including electrons and neutrinos, were the dominant form of matter, and the universe entered the **lepton era**.

By the time the first full second had passed (clearly the most eventful second in history!), the universe had cooled to about 10 billion degrees, 10^{10} K. The average kinetic energy was about 1 MeV. This was still sufficient energy to create electrons and positrons and balance their annihilation reactions, since their masses correspond to about 0.5 MeV. So there were about as many e^+ and e^- as there were photons. But within a few more seconds, the temperature had dropped sufficiently so that e^+ and e^- could no longer be formed. Annihilation $\left(e^+ + e^- \rightarrow \text{photons}\right)$ continued. And, like nucleons before them, electrons and positrons all but disappeared from the universe—except for a slight excess of electrons over positrons (later to join with nuclei to form atoms). Thus, about $t = 10\,\text{s}$ after the Big Bang, the universe entered the **radiation era** (Fig. 33–29). Its major constituents were photons and neutrinos. But the neutrinos, partaking only in the weak force, rarely interacted. So the universe, until then experiencing significant amounts of energy in matter and in radiation, now became **radiation-dominated**: much more energy was contained in radiation than in matter, a situation that would last more than 50,000 years.

FIGURE 33–29 (Repeated.) Compressed graphical representation of the development of the universe after the Big Bang, according to modern cosmology.



Meanwhile, during the next few minutes, crucial events were taking place. Beginning about 2 or 3 minutes after the Big Bang, nuclear fusion began to occur. The temperature had dropped to about 10^9 K, corresponding to an average kinetic energy $\overline{\text{KE}} \approx 100\,\text{keV}$, where nucleons could strike each other and be able to fuse (Section 31–3), but now cool enough so newly formed nuclei would not be immediately broken apart by subsequent collisions. Deuterium, helium, and very tiny amounts of lithium nuclei were made. But the universe was cooling too quickly, and larger nuclei were not made. After only a few minutes, probably not even a quarter of an hour after the Big Bang, the temperature dropped far enough that nucleosynthesis stopped, not to start again for millions of years (in stars).

†Why this could have happened is a question for which we are seeking an answer today.

Thus, after the first quarter hour or so of the universe, matter consisted mainly of bare nuclei of hydrogen (about 75%) and helium (about 25%)† as well as electrons. But radiation (photons) continued to dominate.

Our story is almost complete. The next important event is thought to have occurred 380,000 years later. The universe had expanded to about $\frac{1}{1000}$ of its present scale, and the temperature had cooled to about 3000 K. The average kinetic energy of nuclei, electrons, and photons was less than an electron volt. Since ionization energies of atoms are on the order of eV, then as the temperature dropped below this point, electrons could orbit the bare nuclei and remain there (without being ejected by collisions), thus forming atoms. This period is often called the **recombination** epoch (a misnomer since electrons had never before been combined with nuclei to form atoms). With the disappearance of free electrons and the birth of atoms, the photons—which had been continually scattering from the free electrons—now became free to spread throughout the universe. As mentioned in the previous Section, we say that the photons became **decoupled** from matter. Thus *decoupling* occurred at *recombination*. The energy contained in radiation had been decreasing (lengthening in wavelength as the universe expanded); and at about $t = 56{,}000\,\mathrm{yr}$ (even before decoupling) the energy contained in matter became dominant over radiation. The universe was said to have become **matter-dominated** (marked on Fig. 33–29). As the universe continued to expand, the electromagnetic radiation cooled further, to 2.7 K today, forming the cosmic microwave background radiation we detect from everywhere in the universe.

After the birth of atoms, then stars and galaxies could begin to form: by self-gravitation around mass concentrations (inhomogeneities). Stars began to form about 200 million years after the Big Bang, galaxies after almost 10^9 years. The universe continued to evolve until today, some 14 billion years after it started.

* * *

This scenario, like other scientific models, cannot be said to be "proven." Yet this model is remarkably effective in explaining the evolution of the universe we live in, and makes predictions which can be tested against the next generation of observations.

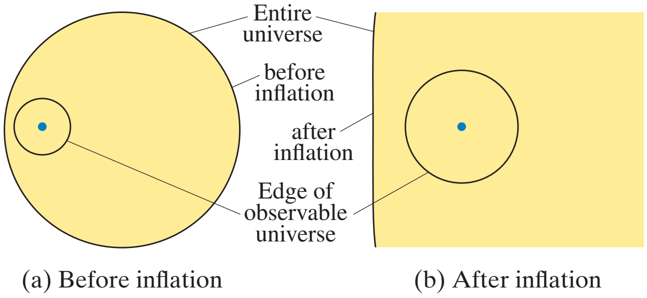
A major event, and something only discovered recently, is that when the universe was about half as old as it is now (about 7 Gyr ago), its expansion began to accelerate. This was a big surprise because it was assumed the expansion of the universe would slow down due to gravitational attraction of all objects toward each other. This acceleration in the expansion of the universe is said to be due to "dark energy," as we discuss in Section 33–9. On the right in Fig. 33–29 is a narrow vertical strip that represents the most recent 7 billion years of the universe, during which *dark energy* seems to have dominated.

33–8 Inflation: Explaining Flatness, Uniformity, and Structure

The idea that the universe underwent a period of exponential inflation early in its life, expanding by a factor of 10^{30} or more (previous Section), was first put forth by Alan Guth and Andreí Linde. Many sophisticated models based on this general idea have since been proposed. The energy required for this wild expansion may have been due to fields somewhat like the Higgs field (Section 32–10). So far, the evidence for inflation is indirect; yet it is a feature of most viable cosmological models because it alone is able to provide natural explanations for several remarkable features of our universe.

†This Standard Model prediction of a 25% primordial production of helium agrees with what we observe today—the universe *does* contain about 25% He—and it is strong evidence in support of the Standard Big Bang Model. Furthermore, the theory says that 25% He abundance is fully consistent with there being three neutrino types, which is the number we observe. And it sets an upper limit of four to the maximum number of possible neutrino types. This is a striking example of the powerful connection between particle physics and cosmology.

FIGURE 33–30 (a) Simple 2-D model of the entire universe; the observable universe is suggested by the small circle centered on us (blue dot). (b) Edge of entire universe is essentially flat after the 10^{30}-fold expansion during inflation.



Flatness

First of all, our best measurements suggest that the universe is flat, that it has zero curvature. As scientists, we would like some reason for this remarkable result. To see how inflation explains flatness, consider a simple 2-dimensional model of the universe as we did earlier in Figs. 33–16 and 33–21. A circle in this 2-dimensional universe (= surface of a sphere, Fig. 33–30a) represents the *observable* universe as seen by an observer at the blue dot. A possible hypothesis is that inflation occurred over a time interval that very roughly doubled the age of the universe from, let us say, $t = 1 \times 10^{-35}$ s to $t = 2 \times 10^{-35}$ s. The size of the *observable* universe $(r = ct)$ would have increased by a factor of two during inflation, while the radius of curvature of the *entire* universe increased by an enormous factor of 10^{30} or more. Thus the edge of our 2-D sphere representing the entire universe would have seemed flat to a high degree of precision, as shown in Fig. 33–30b. Even if the time of inflation was a factor of 10 or 100 (instead of 2), the expansion factor of 10^{30} or more would have blotted out any possibility of observing anything but a flat universe.

CMB Uniformity

Inflation also explains why the CMB is so uniform. Without inflation, the tiny universe at 10^{-35} s would not have been small enough for all parts of it to have been in contact and so reach the same temperature (information cannot travel faster than c). To see this, suppose that the currently observable universe came from a region of space about 1 cm in diameter at $t = 10^{-36}$ s, as per original Big Bang theory. In that 10^{-36} s, light could have traveled $d = ct = (3 \times 10^8\,\mathrm{m/s})(10^{-36}\,\mathrm{s}) = 10^{-27}\,\mathrm{m}$, way too small for the opposite sides of a 1-cm-wide "universe" to have been in communication. But if that region had been 10^{30} times smaller $(= 10^{-32}\,\mathrm{m})$, as proposed by the inflation model, there could have been contact and thermal equilibrium to produce the observed nearly uniform CMB. Inflation, by making the very early universe extremely small, assures that all parts of that region which is today's observable universe could have been in thermal equilibrium. And after inflation the universe could be large enough to give us today's observable universe.

Galaxy Seeds, Fluctuations, Magnetic Monopoles

Inflation also gives us a clue as to how the present structure of the universe (galaxies and clusters of galaxies) came about. We saw earlier that, according to the uncertainty principle, energy might be not conserved by an amount ΔE for a time $\Delta t \approx \hbar/\Delta E$. Forces, whether electromagnetic or other types, can undergo such tiny **quantum fluctuations** according to quantum theory, but they are so tiny they are not detectable unless magnified in some way. That is what inflation might have done: it could have magnified those fluctuations perhaps 10^{30} times in size, which would give us the density irregularities seen in the cosmic microwave background (WMAP, Fig. 33–26). That would be very nice, because the density variations we see in the CMB are what we believe were the seeds that later coalesced under gravity into galaxies and galaxy clusters, and our models fit the data extremely well.

Sometimes it is said that the quantum fluctuations occurred in the **vacuum state** or vacuum energy. This could be possible because the vacuum is no longer considered to be empty, as we discussed in Section 32–3 relative to positrons as holes in a negative energy sea of electrons. Indeed, the vacuum is thought to be filled with fields and particles occupying all the possible negative energy states.

Also, the virtual exchange particles that carry the forces, as discussed in Chapter 32, could leave their brief virtual states and actually become real as a result of the 10^{30} magnification of space (according to inflation) and the very short time over which it occurred ($\Delta t = \hbar/\Delta E$).

Inflation helps us too with the puzzle of why **magnetic monopoles** (Section 20–1) have never been observed, yet isolated magnetic poles may well have been copiously produced at the start. After inflation, they would have been so far apart that we have never stumbled on one.

Inflation may solve outstanding problems, but we may need new physics to understand how inflation occurred. Many predictions of inflationary theory have been confirmed by recent cosmological observations.

33–9 Dark Matter and Dark Energy

According to the Standard Big Bang Model, the universe is evolving and changing. Individual stars are being created, evolving, and then dying to become white dwarfs, neutron stars, or black holes. At the same time, the universe as a whole is expanding. One important question is whether the universe will continue to expand forever. Until the late 1990s, the universe was thought to be dominated by matter which interacts by gravity, and the fate of the universe was connected to the curvature of space-time (Section 33–4). If the universe had *negative* curvature, the expansion of the universe would never stop, although the rate of expansion would decrease due to the gravitational attraction of its parts. Such a universe would be *open* and infinite. If the universe is *flat* (no curvature), it would still be open and infinite but its expansion would slowly approach a zero rate. If the universe had *positive* curvature, it would be *closed* and finite; the effect of gravity would be strong enough that the expansion would eventually stop and the universe would begin to contract, collapsing back onto itself in a **big crunch**.

Critical Density

According to the above scenario (which does not include inflation or the recently discovered acceleration of the universe), the fate of the universe would depend on the average mass–energy density in the universe. For an average mass density greater than a critical value known as the **critical density**, estimated to be about

$$\rho_c \approx 10^{-26}\,\mathrm{kg/m^3}$$

(i.e., a few nucleons/m^3 on average throughout the universe), space-time would have a positive curvature and gravity would prevent expansion from continuing forever. Eventually (if $\rho > \rho_c$) gravity would pull the universe back into a big crunch. If instead the actual density was equal to the critical density, $\rho = \rho_c$, the universe would be flat and open, just barely expanding forever. If the actual density was less than the critical density, $\rho < \rho_c$, the universe would have negative curvature and would easily expand forever. See Fig. 33–31. Today we believe the universe is very close to flat. But recent evidence suggests the universe is expanding at an *accelerating* rate, as discussed below.

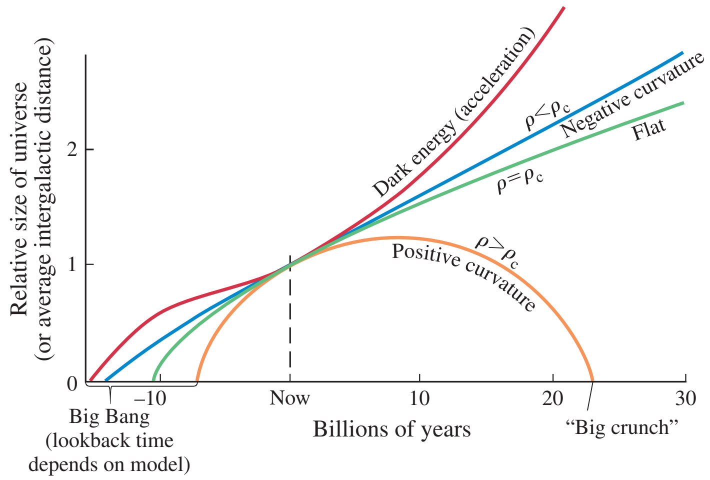


FIGURE 33–31 Three future possibilities for the universe, depending on the density ρ of ordinary matter, plus a fourth possibility that includes dark energy. Note that all curves have been chosen to have the same slope ($= H_0$, the Hubble parameter) right now. Looking back in time, the Big Bang occurs where each curve touches the horizontal (time) axis.

EXERCISE E Return to the Chapter-Opening Questions, page 947, and answer them again. Try to explain why you may have answered differently the first time.

Dark Matter

WMAP and other experiments have convinced scientists that the universe is flat and $\rho = \rho_c$. But this ρ cannot be only normal baryonic matter (atoms are 99.9% baryons—protons and neutrons—by weight). These recent experiments put the amount of normal baryonic matter in the universe at only about 5% of the critical density. What is the other 95%? There is strong evidence for a significant amount of nonluminous matter in the universe referred to as **dark matter**, which acts normally under gravity, but does not absorb or radiate light sufficiently to be visible. For example, observations of the rotation of galaxies suggest that they rotate as if they had considerably more mass than we can see. Recall from Chapter 5, Example 5–12, that for a satellite of mass m revolving around Earth (mass M)

$$m\frac{v^2}{r} = G\frac{mM}{r^2}$$

and hence $v = \sqrt{GM/r}$. If we apply this equation to stars in a galaxy, we see that their speed depends on galactic mass. Observations show that stars farther from the galactic center revolve much faster than expected if there is only the pull of visible matter, suggesting a great deal of invisible matter. Similarly, observations of the motion of galaxies within clusters also suggest that they have considerably more mass than can be seen. Furthermore, theory suggests that without dark matter, galaxies and stars probably would not have formed and would not exist. Dark matter seems to hold the universe together.

What might this nonluminous matter in the universe be? We don't know yet. But we hope to find out soon. It cannot be made of ordinary (baryonic) matter, so it must consist of some other sort of elementary particle, perhaps created at a very early time. Perhaps it is made up of previously undetected *weakly interacting massive particles* (**WIMP**s), possibly supersymmetric particles (Section 32–12) such as neutralinos. We are anxiously awaiting the results of intense searches for such particles, looking both at what arrives from far out in the cosmos with underground detectors†, and by producing them in particle colliders (the LHC, Section 32–1).

Dark matter makes up roughly 25% of the mass–energy of the universe, according to the latest observations and models. Thus the total mass–energy is 25% dark matter plus 5% baryons for a total of about 30%, which does not bring ρ up to ρ_c. What is the other 70%? We are not sure about that either, but we have given it a name: "dark energy."

Dark Energy—Cosmic Acceleration

FIGURE 33–32 Saul Perlmutter, center, flanked by Adam G. Riess (left) and Brian P. Schmidt, at the Nobel Prize celebrations, December 2011.

In 1998, just before the turn of the millennium, two groups, one led by Saul Perlmutter and the other by Brian Schmidt and Adam Riess (Fig. 33–32), reported a huge surprise. Gravity was assumed to be the predominant force on a large scale in the universe, and it was thought that the expansion of the universe ought to be slowing down in time because gravity acts as an attractive force between objects. But measurements of Type Ia supernovae (our best standard candles—see Section 33–3) unexpectedly showed that very distant (high z) supernovae were dimmer than expected. That is, given their great distance d as determined from their low brightness, their speed v as determined from the measured z was less than expected according to Hubble's law. This result suggests that nearer galaxies are moving away from us relatively faster than those very distant ones, meaning the expansion of the universe in more recent epochs has sped up.

†In deep mines and under mountains to block out most other particles.

This **acceleration** in the expansion of the universe (in place of the expected deceleration due to gravitational attraction between masses) seems to have begun roughly 7 billion years ago (7 Gyr, which would be about halfway back to what we call the Big Bang).

What could be causing the universe to accelerate in its expansion, against the attractive force of gravity? Does our understanding of gravity need to be revised? We don't yet know the answers to these questions. There are several speculations. Somehow there seems to be a long-range *repulsive* effect on space, like a negative gravity, causing objects to speed away from each other ever faster. Whatever it is, it has been given the name **dark energy**. Many scientists say dark energy is the biggest mystery facing physical science today.

One idea is a sort of quantum field given the name **quintessence**. Another possibility suggests an energy latent in space itself (**vacuum energy**) and relates to an aspect of General Relativity known as the **cosmological constant** (symbol Λ). When Einstein developed his equations, he found that they offered no solutions for a static universe. In those days (1917) it was thought the universe was static—unchanging and everlasting. Einstein added an arbitrary constant (Λ) to his equations to provide solutions for a static universe.† A decade later, when Hubble showed us an expanding universe, Einstein discarded his cosmological constant as no longer needed ($\Lambda = 0$). But today, measurements are consistent with dark energy being due to a nonzero cosmological constant, although further measurements are needed to see subtle differences among theories.

There is increasing evidence that the effects of some form of dark energy are very real. Observations of the CMB, supernovae, and large-scale structure (Section 33–10) agree well with theories and computer models when they input dark energy as providing about 70% of the mass–energy in the universe, and when the total mass–energy density equals the critical density ρ_c.

Today's best estimate of how the mass–energy in the universe is distributed is approximately (see also Fig. 33–33):

70% dark energy

30% matter, subject to the known gravitational force.

Of this 30%, about

25% is dark matter

5% is baryons (what atoms are made of); of this 5% only $\frac{1}{10}$ is readily visible matter—stars and galaxies (that is, 0.5% of the total); the other $\frac{9}{10}$ of ordinary matter, which is not visible, is mainly gaseous plasma.

It is remarkable that only 0.5% of all the mass–energy in the universe is visible as stars and galaxies.

The idea that the universe is dominated by completely unknown forms of matter and energy seems bizarre. Nonetheless, the ability of our present model to precisely explain observations of the CMB anisotropy, cosmic expansion, and large-scale structure (next Section) presents a compelling case.

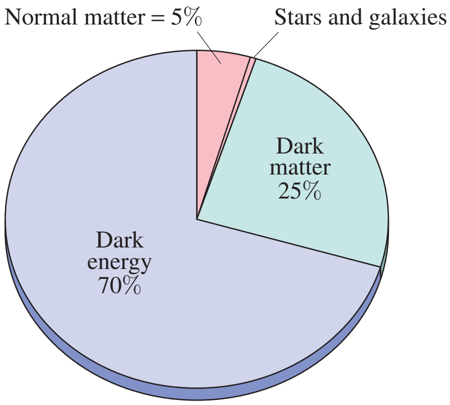


FIGURE 33–33 Portions of total mass–energy in the universe (approximate).

33–10 Large-Scale Structure of the Universe

The beautiful WMAP pictures of the sky (Fig. 33–26) show small but significant inhomogeneities in the temperature of the cosmic microwave background (CMB). These anisotropies reflect compressions and expansions in the primordial plasma just before decoupling (Fig. 33–29), from which galaxies and clusters of galaxies formed. Analyses of the irregularities in the CMB using mammoth computer

†It seems strange that Einstein and other scientists believed in a static universe. The ancients, including the Roman Lucretius argued against it: *If there was no birth-time of earth and heaven and they have been from everlasting, why before the* [Trojan] *war .... have not other poets as well sung other themes?* [The reference is to Homer being the oldest known writings.] The ancient Hebrews also argued for a beginning (like our Big Bang): see Genesis.

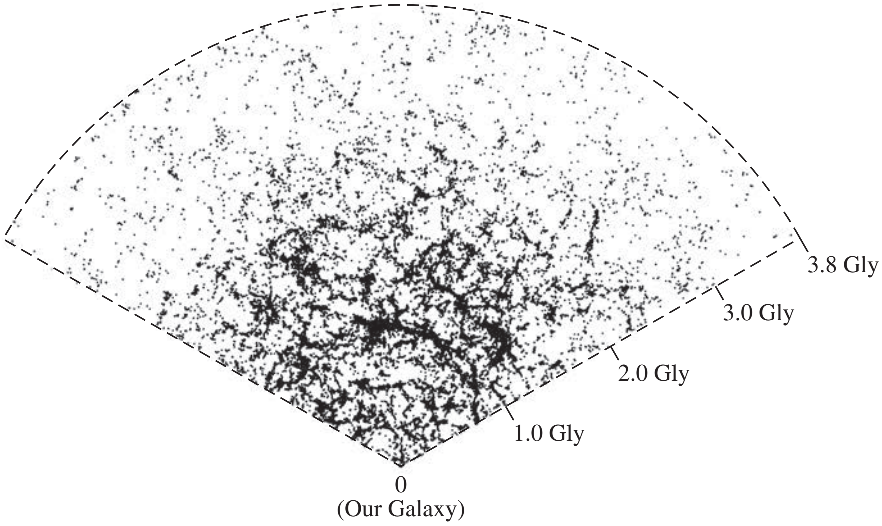


FIGURE 33–34 Distribution of some 50,000 galaxies in a 2.5° slice through almost half of the sky above the equator, as measured by the Sloan Digital Sky Survey (SDSS). Each dot represents a galaxy. The distance from us is obtained from the redshift and Hubble's law, and is given in units of 10^9 light-years (Gly). The point 0 represents us, our observation point. This diagram may seem to put us at the center, but remember that at greater distances, fewer galaxies are bright enough to be detected, thus resulting in an apparent thinning out of galaxies. Note the "walls" and "voids" of galaxies.

simulations predict a large-scale distribution of galaxies very similar to what is seen today (Fig. 33–34). These simulations are very successful if they contain dark energy and dark matter; and the dark matter needs to be *cold* (slow speed—think of Eq. 13–8, $\frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$ where T is temperature), rather than "hot" dark matter such as neutrinos which move at or very near the speed of light. Indeed, the modern **cosmological model** is called the **ΛCDM** model, where lambda (Λ) stands for the cosmological constant, and CDM is **cold dark matter**.

Cosmologists have gained substantial confidence in this cosmological model from such a precise fit between observations and theory. They can also extract very precise values for cosmological parameters which previously were only known with low accuracy. The CMB is such an important cosmological observable that every effort is being made to extract all of the information it contains. A new generation of ground, balloon, and satellite experiments is observing the CMB with greater resolution and sensitivity. They may detect interaction of **gravity waves** (produced in the inflationary epoch) with the CMB and thereby provide direct evidence for cosmic inflation, and also provide information about elementary particle physics at energies far beyond the reach of man-made accelerators and colliders.

33–11 Finally . . .

When we look up into the night sky, we see stars; and with the best telescopes, we see galaxies and the exotic objects we discussed earlier, including rare supernovae. But even with our best instruments we do not see the processes going on inside stars and supernovae that we hypothesized (and believe). We are dependent on brilliant theorists who come up with viable theories and verifiable models. We depend on complicated computer models whose parameters are varied until the outputs compare favorably with our observations and analyses of WMAP and other experiments. And we now have a surprisingly precise idea about some aspects of our universe: it is flat, it is about 14 billion years old, it contains only 5% "normal" baryonic matter (for atoms), and so on.

The questions raised by cosmology are difficult and profound, and may seem removed from everyday "reality." We can always say, "the Sun is shining, it's going to shine on for an unimaginably long time, all is well." Nonetheless, the questions of cosmology are deep ones that fascinate the human intellect. One aspect that is especially intriguing is this: calculations on the formation and evolution of the universe have been performed that deliberately varied the values—just slightly—of certain fundamental physical constants. The result?

A universe in which life as we know it could not exist. [For example, if the difference in mass between a proton and a neutron were zero, or less than the mass of the electron, $0.511\ \mathrm{MeV}/c^2$, there would be no atoms: electrons would be captured by protons to make neutrons.] Such results have contributed to a philosophical idea called the **anthropic principle**, which says that if the universe were even a little different than it is, we could not be here. We physicists are trying to find out if there are some undiscovered fundamental laws that determined those conditions that allowed us to exist. A poet might say that the universe is exquisitely tuned, almost as if to accommodate us.

Summary

The night sky contains myriads of stars including those in the Milky Way, which is a "side view" of our **Galaxy** looking along the plane of the disk. Our Galaxy includes over 10^{11} stars. Beyond our Galaxy are billions of other galaxies.

Astronomical distances are measured in **light-years** $(1\ \mathrm{ly} \approx 10^{13}\ \mathrm{km})$. The nearest star is about 4 ly away and the nearest large galaxy is 2 million ly away. Our Galactic disk has a diameter of about 100,000 ly. [Distances are sometimes specified in **parsecs**, where $1\ \mathrm{parsec} = 3.26\ \mathrm{ly}$.]

Stars are believed to begin life as collapsing masses of gas (protostars), largely hydrogen. As they contract, they heat up (potential energy is transformed to kinetic energy). When the temperature reaches about 10 million degrees, nuclear fusion begins and forms heavier elements (**nucleosynthesis**), mainly helium at first. The energy released during these reactions heats the gas so its outward pressure balances the inward gravitational force, and the young star stabilizes as a **main-sequence** star. The tremendous luminosity of stars comes from the energy released during these thermonuclear reactions. After billions of years, as helium is collected in the core and hydrogen is used up, the core contracts and heats further. The outer envelope expands and cools, and the star becomes a **red giant** (larger diameter, redder color).

The next stage of stellar evolution depends on the mass of the star, which may have lost much of its original mass as its outer envelope escaped into space. Stars of residual mass less than about 1.4 solar masses cool further and become **white dwarfs**, eventually fading and going out altogether. Heavier stars contract further due to their greater gravity: the density approaches nuclear density, the huge pressure forces electrons to combine with protons to form neutrons, and the star becomes essentially a huge nucleus of neutrons. This is a **neutron star**, and the energy released during its final core collapse is believed to produce **supernova** explosions. If the star is very massive, it may contract even further and form a **black hole**, which is so dense that no matter or light can escape from it.

In the **general theory of relativity**, the **equivalence principle** states that an observer cannot distinguish acceleration from a gravitational field. Said another way, gravitational and inertial masses are the same. The theory predicts gravitational bending of light rays to a degree consistent with experiment. Gravity is treated as a curvature in space and time, the curvature being greater near massive objects. The universe as a whole may be curved. With sufficient mass, the curvature of the universe would be positive, and the universe is *closed* and *finite*; otherwise, it would be *open* and *infinite*. Today we believe the universe is **flat**.

Distant galaxies display a **redshift** in their spectral lines, originally interpreted as a Doppler shift. The universe is observed to be **expanding**, its galaxies racing away from each other at speeds (v) proportional to the distance (d) between them:

$$v = H_0 d, \tag{33–4}$$

which is known as **Hubble's law** (H_0 is the **Hubble parameter**). This expansion of the universe suggests an explosive origin, the **Big Bang**, which occurred about 13.8 billion years ago. It is not like an ordinary explosion, but rather an expansion of space itself.

The **cosmological principle** assumes that the universe, on a large scale, is homogeneous and isotropic.

Important evidence for the Big Bang model of the universe was the discovery of the **cosmic microwave background** radiation (CMB), which conforms to a blackbody radiation curve at a temperature of 2.725 K.

The **Standard Model** of the Big Bang provides a possible scenario as to how the universe developed as it expanded and cooled after the Big Bang. Starting at 10^{-43} seconds after the Big Bang, according to this model, the universe underwent a brief but rapid exponential expansion, referred to as **inflation**. Shortly thereafter, quarks were **confined** into hadrons (the **hadron era**). About 10^{-4} s after the Big Bang, the majority of hadrons disappeared, having combined with anti-hadrons, producing photons, leptons, and energy, leaving mainly photons and leptons to freely move, thus introducing the **lepton era**. By the time the universe was about 10 s old, the electrons too had mostly disappeared, having combined with their antiparticles; the universe was **radiation-dominated**. A couple of minutes later, nucleosynthesis began, but lasted only a few minutes. It then took almost four hundred thousand years before the universe was cool enough for electrons to combine with nuclei to form atoms (**recombination**). Photons, up to then continually being scattered off of free electrons, could now move freely—they were **decoupled** from matter and the universe became transparent. The background radiation had expanded and cooled so much that its total energy became less than the energy in matter, and **matter dominated** increasingly over radiation. Then stars and galaxies formed, producing a universe not much different than it is today—some 14 billion years later.

Recent observations indicate that the universe is essentially flat, that it contains an as-yet unknown type of **dark matter**, and that it is dominated by a mysterious **dark energy** which exerts a sort of negative gravity causing the expansion of the universe to accelerate. The total contributions of baryonic (normal) matter, dark matter, and dark energy sum up to the **critical density**.

Questions

1. The Milky Way was once thought to be "murky" or "milky" but is now considered to be made up of point sources. Explain.
2. A star is in equilibrium when it radiates at its surface all the energy generated in its core. What happens when it begins to generate more energy than it radiates? Less energy? Explain.
3. Describe a red giant star. List some of its properties.
4. Does the H–R diagram directly reveal anything about the core of a star?
5. Why do some stars end up as white dwarfs, and others as neutron stars or black holes?
6. If you were measuring star parallaxes from the Moon instead of Earth, what corrections would you have to make? What changes would occur if you were measuring parallaxes from Mars?
7. *Cepheid variable* stars change in luminosity with a typical period of several days. The period has been found to have a definite relationship with the average intrinsic luminosity of the star. How could these stars be used to measure the distance to galaxies?
8. What is a geodesic? What is its role in General Relativity?
9. If it were discovered that the redshift of spectral lines of galaxies was due to something other than expansion, how might our view of the universe change? Would there be conflicting evidence? Discuss.
10. Almost all galaxies appear to be moving away from us. Are we therefore at the center of the universe? Explain.
11. If you were located in a galaxy near the boundary of our observable universe, would galaxies in the direction of the Milky Way appear to be approaching you or receding from you? Explain.
12. Compare an explosion on Earth to the Big Bang. Consider such questions as: Would the debris spread at a higher speed for more distant particles, as in the Big Bang? Would the debris come to rest? What type of universe would this correspond to, open or closed?
13. If nothing, not even light, escapes from a black hole, then how can we tell if one is there?
14. The Earth's age is often given as about 4.6 billion years. Find that time on Fig. 33–29. Modern humans have lived on Earth on the order of 200,000 years. Where is that on Fig. 33–29?
15. Why were atoms, as opposed to bare nuclei, unable to exist until hundreds of thousands of years after the Big Bang?
16. (*a*) Why are Type Ia supernovae so useful for determining the distances of galaxies? (*b*) How are their distances actually measured?
17. Under what circumstances would the universe eventually collapse in on itself?
18. (*a*) Why did astronomers expect that the expansion rate of the universe would be decreasing (decelerating) with time? (*b*) How, in principle, could astronomers hope to determine whether the universe used to expand faster than it does now?

MisConceptual Questions

1. Which one of the following is *not* expected to occur on an H–R diagram during the lifetime of a single star?
   (*a*) The star will move off the main sequence toward the upper right of the diagram.
   (*b*) Low-mass stars will become white dwarfs and end up toward the lower left of the diagram.
   (*c*) The star will move along the main sequence from one place to another.
   (*d*) All of the above.
2. When can parallax be used to determine the approximate distance from the Earth to a star?
   (*a*) Only during January and July.
   (*b*) Only when the star's distance is relatively small.
   (*c*) Only when the star's distance is relatively large.
   (*d*) Only when the star appears to move directly toward or away from the Earth.
   (*e*) Only when the star is the Sun.
   (*f*) Always.
   (*g*) Never.
3. Observations show that all galaxies tend to move away from Earth, and that more distant galaxies move away from Earth at faster velocities than do galaxies closer to the Earth. These observations imply that
   (*a*) the Earth is the center of the universe.
   (*b*) the universe is expanding.
   (*c*) the expansion of the universe will eventually stop.
   (*d*) All of the above.
4. Which process results in a tremendous amount of energy being emitted by the Sun?
   (*a*) Hydrogen atoms burn in the presence of oxygen—that is, hydrogen atoms oxidize.
   (*b*) The Sun contracts, decreasing its gravitational potential energy.
   (*c*) Protons in hydrogen atoms fuse, forming helium nuclei.
   (*d*) Radioactive atoms such as uranium, plutonium, and cesium emit gamma rays with high energy.
   (*e*) None of the above.
5. Which of the following methods can be used to find the distance from us to a star outside our galaxy? Choose all that apply.
   (*a*) Parallax.
   (*b*) Using luminosity and temperature from the H–R diagram and measuring the apparent brightness.
   (*c*) Using supernova explosions as a "standard candle."
   (*d*) Redshift in the line spectra of elements and compounds.
6. The history of the universe can be determined by observing astronomical objects at various (large) distances from the Earth. This method of discovery works because
   (*a*) time proceeds at different rates in different regions of the universe.
   (*b*) light travels at a finite speed.
   (*c*) matter warps space.
   (*d*) older galaxies are farther from the Earth than are younger galaxies.

7. Where did the Big Bang occur?
(*a*) Near the Earth.
(*b*) Near the center of the Milky Way Galaxy.
(*c*) Several billion light-years away.
(*d*) Throughout all space.
(*e*) Near the Andromeda Galaxy.

8. When and how were virtually all of the elements of the Periodic Table formed?
(*a*) In the very early universe a few seconds after the Big Bang.
(*b*) At the centers of stars during their main-sequence phases.
(*c*) At the centers of stars during novae.
(*d*) At the centers of stars during supernovae.
(*e*) On the surfaces of planets as they cooled and hardened.

9. We know that there must be dark matter in the universe because
(*a*) we see dark dust clouds.
(*b*) we see that the universe is expanding.
(*c*) we see that stars far from the galactic center are moving faster than can be explained by visible matter.
(*d*) we see that the expansion of the universe is accelerating.

10. Acceleration of the universe's expansion rate is due to
(*a*) the repulsive effect of dark energy.
(*b*) the attractive effect of dark matter.
(*c*) the attractive effect of gravity.
(*d*) the thermal expansion of stellar cores.

For assigned homework and other learning materials, go to the MasteringPhysics website.

Problems

33–1 to 33–3 Stars, Galaxies, Stellar Evolution, Distances

1. (I) The parallax angle of a star is 0.00029°. How far away is the star?

2. (I) A star exhibits a parallax of 0.27 seconds of arc. How far away is it?

3. (I) If one star is twice as far away from us as a second star, will the parallax angle of the farther star be greater or less than that of the nearer star? By what factor?

4. (II) What is the relative brightness of the Sun as seen from Jupiter, as compared to its brightness from Earth? (Jupiter is 5.2 times farther from the Sun than the Earth is.)

5. (II) When our Sun becomes a red giant, what will be its average density if it expands out to the orbit of Mercury (6×10^{10} m from the Sun)?

6. (II) We saw earlier (Chapter 14) that the rate energy reaches the Earth from the Sun (the "solar constant") is about $1.3 \times 10^3\ \mathrm{W/m^2}$. What is (*a*) the apparent brightness b of the Sun, and (*b*) the intrinsic luminosity L of the Sun?

7. (II) Estimate the angular width that our Galaxy would subtend if observed from the nearest galaxy to us (Table 33–1). Compare to the angular width of the Moon from Earth.

8. (II) Assuming our Galaxy represents a good average for all other galaxies, how many stars are in the observable universe?

9. (II) Calculate the density of a white dwarf whose mass is equal to the Sun's and whose radius is equal to the Earth's. How many times larger than Earth's density is this?

10. (II) A neutron star whose mass is 1.5 solar masses has a radius of about 11 km. Calculate its average density and compare to that for a white dwarf (Problem 9) and to that of nuclear matter.

***11.** (II) A star is 56 pc away. What is its parallax angle? State (*a*) in seconds of arc, and (*b*) in degrees.

***12.** (II) What is the parallax angle for a star that is 65 ly away? How many parsecs is this?

***13.** (II) A star is 85 pc away. How long does it take for its light to reach us?

14. (III) Suppose two stars of the same apparent brightness b are also believed to be the same size. The spectrum of one star peaks at 750 nm whereas that of the other peaks at 450 nm. Use Wien's law and the Stefan-Boltzmann equation (Eq. 14–6) to estimate their relative distances from us. [*Hint*: See Examples 33–4 and 33–5.]

15. (III) Stars located in a certain cluster are assumed to be about the same distance from us. Two such stars have spectra that peak at $\lambda_1 = 470$ nm and $\lambda_2 = 720$ nm, and the ratio of their apparent brightness is $b_1/b_2 = 0.091$. Estimate their relative sizes (give ratio of their diameters) using Wien's law and the Stefan-Boltzmann equation, Eq. 14–6.

33–4 General Relativity, Gravity and Curved Space

16. (I) Show that the Schwarzschild radius for Earth is 8.9 mm.

17. (II) What is the Schwarzschild radius for a typical galaxy (like ours)?

18. (II) What mass will give a Schwarzschild radius equal to that of the hydrogen atom in its ground state?

19. (II) What is the maximum sum-of-the-angles for a triangle on a sphere?

20. (II) Describe a triangle, drawn on the surface of a sphere, for which the sum of the angles is (*a*) 359°, and (*b*) 179°.

21. (III) What is the apparent deflection of a light beam in an elevator (Fig. 33–13) which is 2.4 m wide if the elevator is accelerating downward at $9.8\ \mathrm{m/s^2}$?

33–5 Redshift, Hubble's Law

22. (I) The redshift of a galaxy indicates a recession velocity of 1850 km/s. How far away is it?

23. (I) If a galaxy is traveling away from us at 1.5% of the speed of light, roughly how far away is it?

24. (II) A galaxy is moving away from Earth. The "blue" hydrogen line at 434 nm emitted from the galaxy is measured on Earth to be 455 nm. (*a*) How fast is the galaxy moving? (*b*) How far is it from Earth based on Hubble's law?

25. (II) Estimate the wavelength shift for the 656.3-nm line in the Balmer series of hydrogen emitted from a galaxy whose distance from us is (*a*) 7.0×10^6 ly, (*b*) 7.0×10^7 ly.

26. (II) If an absorption line of calcium is normally found at a wavelength of 393.4 nm in a laboratory gas, and you measure it to be at 423.4 nm in the spectrum of a galaxy, what is the approximate distance to the galaxy?

27. (II) What is the speed of a galaxy with $z = 0.060$?

28. (II) What would be the redshift parameter z for a galaxy traveling away from us at $v = 0.075c$?

29. (II) Estimate the distance d from the Earth to a galaxy whose redshift parameter $z = 1$.

30. (II) Estimate the speed of a galaxy, and its distance from us, if the wavelength for the hydrogen line at 434 nm is measured on Earth as being 610 nm.

31. (III) Starting from Eq. 33–3, show that the Doppler shift in wavelength is $\Delta\lambda/\lambda_{\text{rest}} \approx v/c$ (Eq. 33–6) for $v \ll c$. [*Hint*: Use the binomial expansion.]

33–6 to 33–8 The Big Bang, CMB, Universe Expansion

32. (I) Calculate the wavelength at the peak of the blackbody radiation distribution at 2.7 K using Wien's law.

33. (II) Calculate the peak wavelength of the CMB at 1.0 s after the birth of the universe. In what part of the EM spectrum is this radiation?

34. (II) The critical density for closure of the universe is $\rho_c \approx 10^{-26}\,\text{kg/m}^3$. State ρ_c in terms of the average number of nucleons per cubic meter.

35. (II) The scale factor of the universe (average distance between galaxies) at any given time is believed to have been inversely proportional to the absolute temperature. Estimate the size of the universe, compared to today, at (*a*) $t = 10^6\,\text{yr}$, (*b*) $t = 1\,\text{s}$, (*c*) $t = 10^{-6}\,\text{s}$, and (*d*) $t = 10^{-35}\,\text{s}$.

36. (II) At approximately what time had the universe cooled below the threshold temperature for producing (*a*) kaons $(M \approx 500\,\text{MeV}/c^2)$, (*b*) $\Upsilon\,(M \approx 9500\,\text{MeV}/c^2)$, and (*c*) muons $(M \approx 100\,\text{MeV}/c^2)$?

33–9 Dark Matter, Dark Energy

37. (II) Only about 5% of the energy in the universe is composed of baryonic matter. (*a*) Estimate the average density of baryonic matter in the observable universe with a radius of 14 billion light-years that contains 10^{11} galaxies, each with about 10^{11} stars like our Sun. (*b*) Estimate the density of dark matter in the universe.

General Problems

38. Use conservation of angular momentum to estimate the angular velocity of a neutron star which has collapsed to a diameter of 16 km, from a star whose core radius was equal to that of Earth $(6 \times 10^6\,\text{m})$. Assume its mass is 1.5 times that of the Sun, and that it rotated (like our Sun) about once a month.

39. By what factor does the rotational kinetic energy change when the star in Problem 38 collapses to a neutron star?

40. Suppose that three main-sequence stars could undergo the three changes represented by the three arrows, A, B, and C, in the H–R diagram of Fig. 33–35. For each case, describe the changes in temperature, intrinsic luminosity, and size.

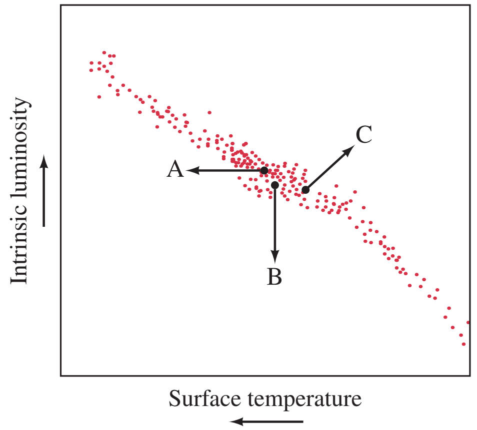


FIGURE 33–35 Problem 40.

41. Assume that the nearest stars to us have an intrinsic luminosity about the same as the Sun's. Their apparent brightness, however, is about 10^{11} times fainter than the Sun. From this, estimate the distance to the nearest stars.

42. A certain pulsar, believed to be a neutron star of mass 1.5 times that of the Sun, with diameter 16 km, is observed to have a rotation speed of 1.0 rev/s. If it loses rotational kinetic energy at the rate of 1 part in 10^9 per day, which is all transformed into radiation, what is the power output of the star?

43. The nearest large galaxy to our Galaxy is about 2×10^6 ly away. If both galaxies have a mass of 4×10^{41} kg, with what gravitational force does each galaxy attract the other? Ignore dark matter.

44. How large would the Sun be if its density equaled the critical density of the universe, $\rho_c \approx 10^{-26}\,\text{kg/m}^3$? Express your answer in light-years and compare with the Earth–Sun distance and the diameter of our Galaxy.

45. Two stars, whose spectra peak at 660 nm and 480 nm, respectively, both lie on the main sequence. Use Wien's law, the Stefan-Boltzmann equation, and the H–R diagram (Fig. 33–6) to estimate the ratio of their diameters.

46. (*a*) In order to measure distances with parallax at 100 ly, what minimum angular resolution (in degrees) is needed? (*b*) What diameter mirror or lens would be needed?

47. In the later stages of stellar evolution, a star (if massive enough) will begin fusing carbon nuclei to form, for example, magnesium:

$$^{12}_{6}\mathrm{C} + ^{12}_{6}\mathrm{C} \rightarrow ^{24}_{12}\mathrm{Mg} + \gamma.$$

(*a*) How much energy is released in this reaction (see Appendix B)? (*b*) How much kinetic energy must each carbon nucleus have (assume equal) in a head-on collision if they are just to "touch" (use Eq. 30–1) so that the strong force can come into play? (*c*) What temperature does this kinetic energy correspond to?

48. Use *dimensional analysis* with the fundamental constants c, G, and $\hbar$ to estimate the value of the so-called *Planck time*. It is thought that physics as we know it can say nothing about the universe before this time.

49. Estimate the mass of our observable universe using the following assumptions: Our universe is spherical in shape, it has been expanding at the speed of light since the Big Bang, and its density is the critical density.

Search and Learn

1. Estimate what neutrino mass (in eV/c^2) would provide the critical density to close the universe. Assume the neutrino density is, like photons, about 10^9 times that of nucleons, and that nucleons make up only (*a*) 2% of the mass needed, or (*b*) 5% of the mass needed.

2. Describe how we can estimate the distance from us to other stars. Which methods can we use for nearby stars, and which can we use for very distant stars? Which method gives the most accurate distance measurements for the most distant stars?

3. The evolution of stars, as discussed in Section 33–2, can lead to a white dwarf, a neutron star, or even a black hole, depending on the mass. (*a*) Referring to Sections 33–2 and 33–4, give the radius of (i) a white dwarf of 1 solar mass, (ii) a neutron star of 1.5 solar masses, and (iii) a black hole of 3 solar masses. (*b*) Express these three radii as ratios ($r_{\mathrm{i}} : r_{\mathrm{ii}} : r_{\mathrm{iii}}$).

4. When stable nuclei first formed, about 3 minutes after the Big Bang, there were about 7 times more protons than neutrons. Explain how this leads to a ratio of the mass of hydrogen to the mass of helium of 3:1. This is about the actual ratio observed in the universe.

5. We cannot use Hubble's law to measure the distances to nearby galaxies, because their random motions are larger than the overall expansion. Indeed, the closest galaxy to us, the Andromeda Galaxy, 2.5 million light-years away, is approaching us at a speed of about 130 km/s. (*a*) What is the shift in wavelength of the 656-nm line of hydrogen emitted from the Andromeda Galaxy, as seen by us? (*b*) Is this a redshift or a blueshift? (*c*) Ignoring the expansion, how soon will it and the Milky Way Galaxy collide?

ANSWERS TO EXERCISES

A: Our Earth and ourselves, 2 years ago.
B: 600 ly (estimating L from Fig. 33–6 as $L \approx 8 \times 10^{26}$ W; note that on a log scale, 6000 K is closer to 7000 K than it is to 5000 K).
C: 30 km.
D: (*a*); not the usual R^3, but R: see formula for the Schwarzschild radius.
E: (*c*); (*d*).