We start our study with fluids at rest, such as water in a glass or a lake. Pressure in a fluid increases with depth, a fact that allows less dense objects to float—the pressure underneath is higher than on top. When fluids flow, such as water or air, interesting effects occur because the pressure in the fluid is lower where the fluid velocity is higher (Bernoulli's principle).

The great mass of a glacier's ice (photos here) moves slowly, like a viscous liquid. The dark lines are "moraines," made up of rock broken off mountain walls by the moving ice, and represent streamlines. The two photos, taken in 1929 and 2009 by Italian expeditions to the mountain K2 (on the right in the distance), show the same glacier has become less thick, presumably due to global warming.

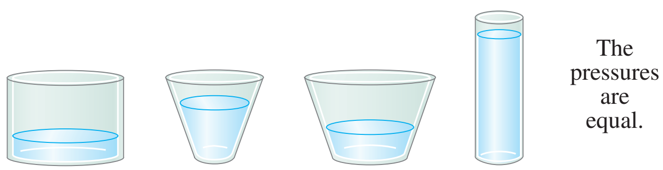
CHAPTER 10

Fluids

CONTENTS

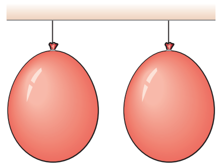
CHAPTER-OPENING QUESTIONS—Guess now!

1. Which container has the largest pressure at the bottom? Assume each container holds the same volume of water.



2. Two balloons are tied and hang with their nearest edges about 3 cm apart. If you blow between the balloons (not *at* the balloons, but at the opening between them), what will happen?

(a) Nothing.
(b) The balloons will move closer together.
(c) The balloons will move farther apart.



In previous Chapters we considered objects that were solid and assumed to maintain their shape except for a small amount of elastic deformation. We sometimes treated objects as point particles. Now we are going to shift our attention to materials that are very deformable and can flow. Such "fluids" include liquids and gases. We will examine fluids both at rest (fluid statics) and in motion (fluid dynamics).

10–1 Phases of Matter

The three common **phases**, or **states**, of matter are solid, liquid, and gas. A simple way to distinguish these three phases is as follows. A **solid** maintains a generally fixed size and shape; usually it requires a large force to change the volume or shape of a solid[†] (although a thin object might bend). A **liquid** does not maintain a fixed shape—it takes on the shape of its container, and it can flow; but like a solid it is not readily compressible, and its volume can be changed significantly only by a very large force. A **gas** has neither a fixed shape nor a fixed volume—it will expand to fill its container. For example, when air is pumped into an automobile tire, the air does not all run to the bottom of the tire as a liquid would; it spreads out to fill the whole volume of the tire.

Because liquids and gases do not maintain a fixed shape, they both have the ability to flow. They are thus referred to collectively as **fluids**.

The division of matter into three phases is not always simple. How, for example, should butter be classified? Furthermore, a fourth phase of matter can be distinguished, the **plasma** phase, which occurs only at very high temperatures and consists of ionized atoms (electrons separated from the nuclei). Some scientists believe that *colloids* (suspensions of tiny particles in a liquid) should also be considered a separate phase of matter. **Liquid crystals**, used in TV, cell phone, and computer screens, can be considered a phase of matter in between solids and liquids. For now, we will be interested in the three ordinary phases of matter.

10–2 Density and Specific Gravity

It is sometimes said that iron is "heavier" than wood. This cannot really be true since a large log clearly weighs more than an iron nail. What we should say is that iron is more *dense* than wood.

The **density**, ρ, of a substance (ρ is the lowercase Greek letter rho) is defined as its mass per unit volume:

$$\rho = \frac{m}{V}, \tag{10–1}$$

where m is the mass of a sample of the substance and V its volume. Density is a characteristic property of any pure substance. Objects made of a particular pure substance, such as pure gold, can have any size or mass, but the density will be the same for each.

We can use the concept of density, Eq. 10–1, to write the mass of an object as

$$m = \rho V,$$

and the weight of an object as

$$mg = \rho V g.$$

The SI unit for density is $\mathrm{kg/m^3}$. Sometimes densities are given in $\mathrm{g/cm^3}$. Note that a density given in $\mathrm{g/cm^3}$ must be multiplied by 1000 to give the result in $\mathrm{kg/m^3}$ $\left[1\,\mathrm{kg/m^3} = 1000\,\mathrm{g}/(100\,\mathrm{cm})^3 = 10^3\,\mathrm{g}/10^6\,\mathrm{cm^3} = 10^{-3}\,\mathrm{g/cm^3}\right]$. For example, the density of aluminum is $\rho = 2.70\,\mathrm{g/cm^3}$, which equals $2700\,\mathrm{kg/m^3}$. The densities of various substances are given in Table 10–1. The Table specifies temperature and atmospheric pressure because they affect density (the effect is slight for liquids and solids). Note that air is about 1000 times less dense than water.

EXAMPLE 10–1 Mass, given volume and density. What is the mass of a solid iron wrecking ball of radius 18 cm?

APPROACH First we use the standard formula $V = \frac{4}{3}\pi r^3$ (see inside rear cover) to obtain the sphere's volume. Then Eq. 10–1 and Table 10–1 give us the mass m.

SOLUTION The volume of the sphere is

$$V = \tfrac{4}{3}\pi r^3 = \tfrac{4}{3}(3.14)(0.18\,\mathrm{m})^3 = 0.024\,\mathrm{m^3}.$$

From Table 10–1, the density of iron is $\rho = 7800\,\mathrm{kg/m^3}$, so Eq. 10–1 gives

$$m = \rho V = \left(7800\,\mathrm{kg/m^3}\right)\left(0.024\,\mathrm{m^3}\right) = 190\,\mathrm{kg}.$$

TABLE 10–1 Densities of Substances[‡]

Substance	Density, ρ ($\mathrm{kg/m^3}$)
Solids	
Aluminum	2.70×10^3
Iron and steel	7.8×10^3
Copper	8.9×10^3
Lead	11.3×10^3
Gold	19.3×10^3
Concrete	2.3×10^3
Granite	2.7×10^3
Wood (typical)	$0.3–0.9 \times 10^3$
Glass, common	$2.4–2.8 \times 10^3$
Ice (H_2O)	0.917×10^3
Bone	$1.7–2.0 \times 10^3$
Liquids	
Water (4°C)	1.000×10^3
Sea water	1.025×10^3
Blood, plasma	1.03×10^3
Blood, whole	1.05×10^3
Mercury	13.6×10^3
Alcohol, ethyl	0.79×10^3
Gasoline	$0.7–0.8 \times 10^3$
Gases	
Air	1.29
Helium	0.179
Carbon dioxide	1.98
Water (steam) (100°C)	0.598

[‡]Densities are given at 0°C and 1 atm pressure unless otherwise specified.

[†]Section 9–5.

The **specific gravity** of a substance is defined as the ratio of the density of that substance to the density of water at 4.0°C. Because specific gravity (abbreviated SG) is a ratio, it is a simple number without dimensions or units. For example (see Table 10–1), the specific gravity of lead is 11.3 $[(11.3 \times 10^3\,\text{kg/m}^3)/(1.00 \times 10^3\,\text{kg/m}^3)]$. The SG of alcohol is 0.79.

The concepts of density and specific gravity are especially helpful in the study of fluids because we are not always dealing with a fixed volume or mass.

10–3 Pressure in Fluids

Pressure and force are related, but they are not the same thing. **Pressure** is defined as force per unit area, where the force F is understood to be the magnitude of the force acting perpendicular to the surface area A:

$$\text{pressure} = P = \frac{F}{A}. \tag{10–2}$$

CAUTION
Pressure is a scalar, not a vector

Although force is a vector, pressure is a scalar. Pressure has magnitude only. The SI unit of pressure is N/m^2. This unit has the official name **pascal** (Pa), in honor of Blaise Pascal (see Section 10–5); that is, $1\,\text{Pa} = 1\,\text{N/m}^2$. However, for simplicity, we will often use N/m^2. Other units sometimes used are dynes/cm^2, and lb/in.^2 (pounds per square inch, abbreviated "psi"). Several other units for pressure are discussed in Sections 10–4 and 10–6, along with conversions between them (see also the Table inside the front cover).

EXAMPLE 10–2 **Calculating pressure.** A 60-kg person's two feet cover an area of $500\,\text{cm}^2$. (*a*) Determine the pressure exerted by the two feet on the ground. (*b*) If the person stands on one foot, what will be the pressure under that foot?

APPROACH Assume the person is at rest. Then the ground pushes up on her with a force equal to her weight mg, and she exerts a force mg on the ground where her feet (or foot) contact it. Because $1\,\text{cm}^2 = (10^{-2}\,\text{m})^2 = 10^{-4}\,\text{m}^2$, then $500\,\text{cm}^2 = 0.050\,\text{m}^2$.

SOLUTION (*a*) The pressure on the ground exerted by the two feet is

$$P = \frac{F}{A} = \frac{mg}{A} = \frac{(60\,\text{kg})(9.8\,\text{m/s}^2)}{(0.050\,\text{m}^2)} = 12 \times 10^3\,\text{N/m}^2.$$

(*b*) If the person stands on one foot, the force is still equal to the person's weight, but the area will be half as much, so the pressure will be twice as much: $24 \times 10^3\,\text{N/m}^2$.

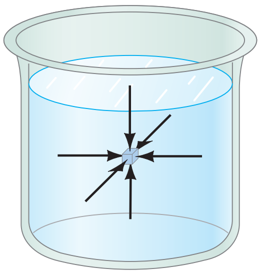
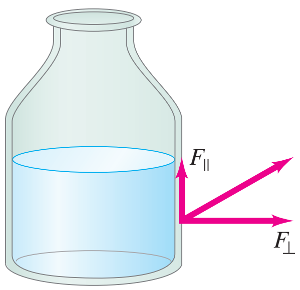


FIGURE 10–1 Pressure is the same in every direction in a nonmoving fluid at a given depth. If this weren't true, the fluid would be in motion.

FIGURE 10–2 If there were a component of force parallel to the solid surface of the container, the liquid would move in response to it. For a liquid at rest, $F_{\parallel} = 0$.



Pressure is particularly useful for dealing with fluids. It is an experimental observation that *a fluid exerts pressure in every direction*. This is well known to swimmers and divers who feel the water pressure on all parts of their bodies. At any depth in a fluid at rest, the pressure is the same in all directions at that given depth. To see why, consider a tiny cube of the fluid (Fig. 10–1) which is so small that we can consider it a point and can ignore the force of gravity on it. The pressure on one side of it must equal the pressure on the opposite side. If this weren't true, there would be a net force on the cube and it would start moving. If the fluid is not flowing, then the pressures must be equal.

For a fluid at rest, the force due to fluid pressure always acts *perpendicular* to any solid surface it touches. If there were a component of the force parallel to the surface, as shown in Fig. 10–2, then according to Newton's third law the solid surface would exert a force back on the fluid, which would cause the fluid to flow—in contradiction to our assumption that the fluid is at rest. Thus the force due to the pressure in a fluid at rest is always perpendicular to the surface.

We now calculate quantitatively how the pressure in a liquid of uniform density varies with depth. Let us look at a depth h below the surface of the liquid as shown in Fig. 10–3 (that is, the liquid's top surface is a height h above this level). The pressure due to the liquid at this depth h is due to the weight of the column of liquid above it. Thus the force due to the weight of liquid acting on the area A is $F = mg = (\rho V)g = \rho Ahg$, where Ah is the volume of the column of liquid, ρ is the density of the liquid (assumed to be constant), and g is the acceleration of gravity. The pressure P due to the weight of liquid is then

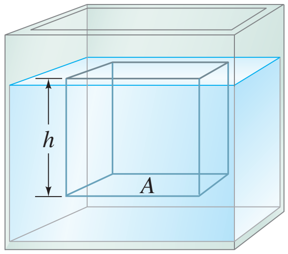


FIGURE 10–3 Calculating the pressure at a depth h in a liquid, due to the weight of the liquid above.

$$P = \frac{F}{A} = \frac{\rho Ahg}{A}$$

$$P = \rho gh. \qquad \text{[liquid]} \quad \textbf{(10–3a)}$$

Note that the area A doesn't affect the pressure at a given depth. The fluid pressure is directly proportional to the density of the liquid and to the depth within the liquid. In general, *the pressure at equal depths within a uniform liquid is the same.*

EXERCISE A Return to Chapter-Opening Question 1, page 260, and answer it again now. Try to explain why you may have answered differently the first time.

Equation 10–3a is extremely useful. It is valid for fluids whose density is constant and does not change with depth—that is, if the fluid is *incompressible*. This is usually a good approximation for liquids (although at great depths in the ocean, the density of water is increased some by compression due to the great weight of water above).

If the density of a fluid does vary, a useful relation can be found by considering a thin horizontal slab of the fluid of thickness $\Delta h = h_2 - h_1$. The pressure on the top of the slab, at depth h_1, is $P_1 = \rho gh_1$. The pressure on the bottom of the slab (pushing upward), at depth h_2, is $P_2 = \rho gh_2$. The difference in pressure is

$$\Delta P = P_2 - P_1 = \rho g(h_2 - h_1)$$

or

$$\Delta P = \rho g\, \Delta h. \qquad [\rho \approx \text{constant over } \Delta h] \quad \textbf{(10–3b)}$$

Equation 10–3b tells us how the pressure changes over a small change in depth (Δh) within a fluid, even if compressible.

Gases are very compressible, and density can vary significantly with depth. For this more general case, in which ρ may vary, we need to use Eq. 10–3b where Δh should be small if ρ varies significantly with depth (or height).

EXAMPLE 10–3 **Pressure at a faucet.** The surface of the water in a storage tank is 30 m above a water faucet in the kitchen of a house, Fig. 10–4. Calculate the difference in water pressure between the faucet and the surface of the water in the tank.

PHYSICS APPLIED
Water supply

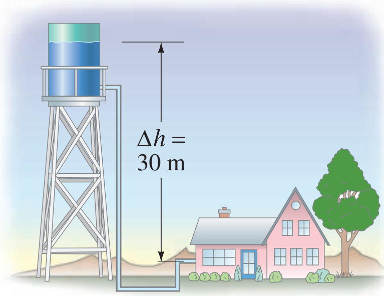
APPROACH Water is practically incompressible, so ρ is constant even for a $\Delta h = 30\,\text{m}$ when used in Eq. 10–3b. Only Δh matters; we can ignore the "route" of the pipe and its bends.

SOLUTION We assume the atmospheric pressure at the surface of the water in the storage tank is the same as at the faucet. So, the water pressure difference between the faucet and the surface of the water in the tank is

$$\Delta P = \rho g\, \Delta h = (1.0 \times 10^3\,\text{kg/m}^3)(9.8\,\text{m/s}^2)(30\,\text{m}) = 2.9 \times 10^5\,\text{N/m}^2.$$

NOTE The height Δh is sometimes called the **pressure head**. In this Example, the head of water is 30 m at the faucet. The very different diameters of the tank and faucet don't affect the result—only height does.

FIGURE 10–4 Example 10–3.



EXERCISE B A dam holds back a lake that is 85 m deep at the dam. If the lake is 20 km long, how much thicker should the dam be than if the lake were smaller, only 1.0 km long?

10–4 Atmospheric Pressure and Gauge Pressure

Atmospheric Pressure

The pressure of the Earth's atmosphere, as in any fluid, changes with depth. But the Earth's atmosphere is somewhat complicated: not only does the density of air vary greatly with altitude but there is no distinct top surface to the atmosphere from which h (in Eq. 10–3a) could be measured. We can, however, calculate the approximate difference in pressure between two altitudes above Earth's surface using Eq. 10–3b.

The pressure of the air at a given place varies slightly according to the weather. At sea level, the pressure of the atmosphere on average is $1.013 \times 10^5\,\mathrm{N/m^2}$ (or $14.7\,\mathrm{lb/in.^2}$). This value lets us define a commonly used unit of pressure, the **atmosphere** (abbreviated atm):

$$1\,\mathrm{atm} = 1.013 \times 10^5\,\mathrm{N/m^2} = 101.3\,\mathrm{kPa}.$$

Another unit of pressure sometimes used (in meteorology and on weather maps) is the **bar**, which is defined as

$$1\,\mathrm{bar} = 1.000 \times 10^5\,\mathrm{N/m^2}.$$

Thus standard atmospheric pressure is slightly more than 1 bar.

PHYSICS APPLIED
Pressure on living cells

The pressure due to the weight of the atmosphere is exerted on all objects immersed in this great sea of air, including our bodies. How does a human body withstand the enormous pressure on its surface? The answer is that living cells maintain an internal pressure that closely equals the external pressure, just as the pressure inside a balloon closely matches the outside pressure of the atmosphere. An automobile tire, because of its rigidity, can maintain internal pressures much greater than the external pressure.

CONCEPTUAL EXAMPLE 10–4 **Finger holds water in a straw.** You insert a straw of length ℓ into a tall glass of water. You place your finger over the top of the straw, capturing some air above the water but preventing any additional air from getting in or out, and then you lift the straw from the water. You find that the straw retains most of the water (Fig. 10–5a). Does the air in the space between your finger and the top of the water have a pressure P that is greater than, equal to, or less than, the atmospheric pressure P_0 outside the straw?

RESPONSE Consider the forces on the column of water (Fig. 10–5b). Atmospheric pressure outside the straw pushes upward on the water at the bottom of the straw, gravity pulls the water downward, and the air pressure inside the top of the straw pushes downward on the water. Since the water is in equilibrium, the upward force due to atmospheric pressure P_0 must balance the two downward forces. The only way this is possible is for the air pressure P inside the straw at the top to be *less than* the atmosphere pressure outside the straw. (When you initially remove the straw from the water glass, a little water may leave the bottom of the straw, thus increasing the volume of trapped air and reducing its density and pressure.)

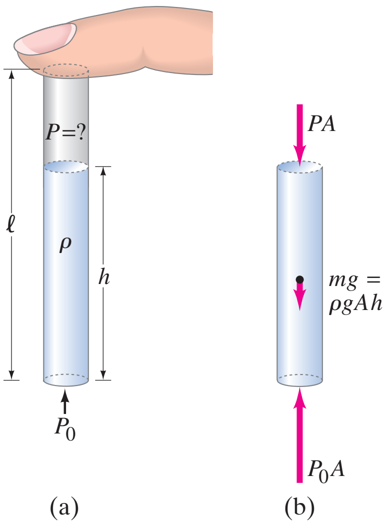


FIGURE 10–5 Example 10–4.

Gauge Pressure

It is important to note that tire gauges, and most other pressure gauges, register the pressure above and beyond atmospheric pressure. This is called **gauge pressure**. Thus, to get the **absolute pressure**, P, we must add the atmospheric pressure, P_0, to the gauge pressure, P_G:

$$P = P_G + P_0.$$

If a tire gauge registers 220 kPa, the absolute pressure within the tire is 220 kPa + 101 kPa = 321 kPa, equivalent to about 3.2 atm (2.2 atm gauge pressure).

10–5 Pascal's Principle

The Earth's atmosphere exerts a pressure on all objects with which it is in contact, including other fluids. External pressure acting on a fluid is transmitted throughout that fluid. For instance, according to Eq. 10–3a, the pressure due to the water at a depth of 100 m below the surface of a lake is $P = \rho g\,\Delta h = (1000\,\mathrm{kg/m^3})(9.8\,\mathrm{m/s^2})(100\,\mathrm{m}) = 9.8 \times 10^5\,\mathrm{N/m^2}$, or 9.7 atm. However, the total pressure at this point is due to the pressure of water plus the pressure of the air above it. Hence the total pressure (if the lake is near sea level) is 9.7 atm + 1.0 atm = 10.7 atm. This is just one example of a general principle attributed to the French philosopher and scientist Blaise Pascal (1623–1662). **Pascal's principle** states that *if an external pressure is applied to a confined fluid, the pressure at every point within the fluid increases by that amount.*

A number of practical devices make use of Pascal's principle. One example is the hydraulic lift, illustrated in Fig. 10–6a, in which a small input force is used to exert a large output force by making the area of the output piston larger than the area of the input piston. To see how this works, we assume the input and output pistons are at the same height (at least approximately). Then the external input force F_{in}, by Pascal's principle, increases the pressure equally throughout. Therefore, at the same level (see Fig. 10–6a),

PHYSICS APPLIED
Hydraulic lift

$$P_{\mathrm{out}} = P_{\mathrm{in}}$$

where the input quantities are represented by the subscript "in" and the output by "out." Since $P = F/A$, we write the above equality as

$$\frac{F_{\mathrm{out}}}{A_{\mathrm{out}}} = \frac{F_{\mathrm{in}}}{A_{\mathrm{in}}},$$

or

$$\frac{F_{\mathrm{out}}}{F_{\mathrm{in}}} = \frac{A_{\mathrm{out}}}{A_{\mathrm{in}}}.$$

The quantity $F_{\mathrm{out}}/F_{\mathrm{in}}$ is called the **mechanical advantage** of the hydraulic lift, and it is equal to the ratio of the areas. For example, if the area of the output piston is 20 times that of the input cylinder, the force is multiplied by a factor of 20. Thus a force of 200 lb could lift a 4000-lb car.

FIGURE 10–6 Applications of Pascal's principle: (a) hydraulic lift; (b) hydraulic brakes in a car.

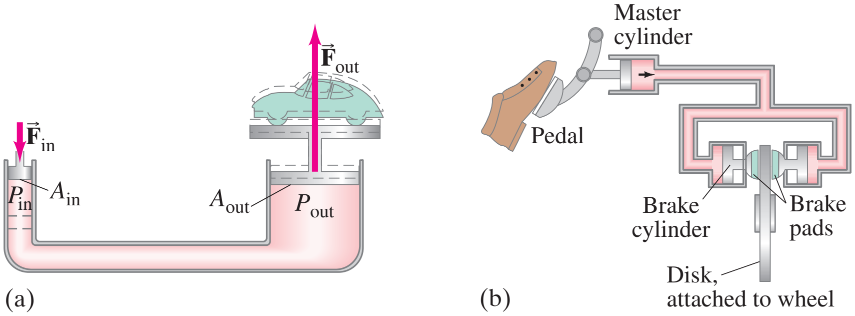


Figure 10–6b illustrates the brake system of a car. When the driver presses the brake pedal, the pressure in the master cylinder increases. This pressure increase occurs throughout the brake fluid, thus pushing the brake pads against the disk attached to the car's wheel.

PHYSICS APPLIED
Hydraulic brakes

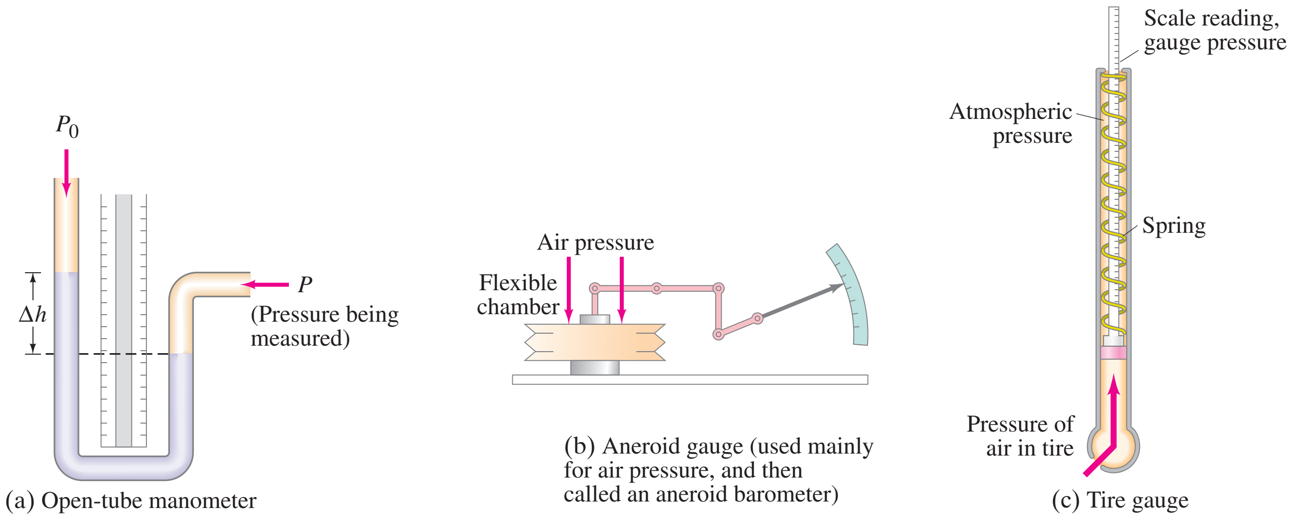


FIGURE 10–7 Pressure gauges: (a) open-tube manometer, (b) aneroid gauge, and (c) common tire pressure gauge.

10–6 Measurement of Pressure; Gauges and the Barometer

Many devices have been invented to measure pressure, some of which are shown in Fig. 10–7. The simplest is the *open-tube* **manometer** (Fig. 10–7a) which is a U-shaped tube partially filled with a liquid, usually mercury or water. The pressure P being measured is related (by Eq. 10–3b) to the difference in height Δh of the two levels of the liquid by the relation

$$P = P_0 + \rho g \, \Delta h, \qquad \text{[manometer]} \quad \textbf{(10–3c)}$$

where P_0 is atmospheric pressure (acting on the top of the liquid in the left-hand tube), and ρ is the density of the liquid. Note that the quantity $\rho g \, \Delta h$ is the gauge pressure—the amount by which P exceeds atmospheric pressure P_0. If the liquid in the left-hand column were lower than that in the right-hand column, P would have to be less than atmospheric pressure (and Δh would be negative).

Instead of calculating the product $\rho g \, \Delta h$, sometimes only the change in height Δh is specified. In fact, pressures are sometimes specified as so many "millimeters of mercury" (mm-Hg) or "mm of water" (mm-H_2O). The unit mm-Hg is equivalent to a pressure of 133 N/m^2, because $\rho g \, \Delta h$ for 1 mm $(= 1.0 \times 10^{-3}\,\text{m})$ of mercury gives

$$\begin{aligned} \rho g \, \Delta h &= \left(13.6 \times 10^3\,\text{kg/m}^3\right)\left(9.80\,\text{m/s}^2\right)\left(1.00 \times 10^{-3}\,\text{m}\right) \\ &= 1.33 \times 10^2\,\text{N/m}^2. \end{aligned}$$

PROBLEM SOLVING
Use SI unit in calculations:
$1\,\text{Pa} = 1\,\text{N/m}^2$

The unit mm-Hg is also called the **torr** in honor of Evangelista Torricelli (1608–1647), a student of Galileo's who invented the barometer (see top of next page). Conversion factors among the various units of pressure (an incredible nuisance!) are given in Table 10–2. It is important that only $\text{N/m}^2 = \text{Pa}$, the proper SI unit, be used in calculations involving other quantities specified in SI units.

Another type of pressure gauge is the **aneroid gauge** (Fig. 10–7b) in which the pointer is linked to the flexible ends of an evacuated thin metal chamber. In electronic gauges, the pressure may be applied to a thin metal diaphragm whose resulting deformation is translated into an electrical signal by a transducer. A common tire gauge uses a spring, as shown in Fig. 10–7c.

TABLE 10–2 Conversion Factors Between Different Units of Pressure

In Terms of $1\ \text{Pa} = 1\ \text{N/m}^2$	1 atm in Different Units
$1\ \text{atm} = 1.013 \times 10^5\ \text{N/m}^2$	$1\ \text{atm} = 1.013 \times 10^5\ \text{N/m}^2$
$= 1.013 \times 10^5\ \text{Pa} = 101.3\ \text{kPa}$	
$1\ \text{bar} = 1.000 \times 10^5\ \text{N/m}^2$	$1\ \text{atm} = 1.013\ \text{bar}$
$1\ \text{dyne/cm}^2 = 0.1\ \text{N/m}^2$	$1\ \text{atm} = 1.013 \times 10^6\ \text{dyne/cm}^2$
$1\ \text{lb/in.}^2 = 6.90 \times 10^3\ \text{N/m}^2$	$1\ \text{atm} = 14.7\ \text{lb/in.}^2$
$1\ \text{lb/ft}^2 = 47.9\ \text{N/m}^2$	$1\ \text{atm} = 2.12 \times 10^3\ \text{lb/ft}^2$
$1\ \text{cm-Hg} = 1.33 \times 10^3\ \text{N/m}^2$	$1\ \text{atm} = 76.0\ \text{cm-Hg}$
$1\ \text{mm-Hg} = 133\ \text{N/m}^2$	$1\ \text{atm} = 760\ \text{mm-Hg}$
$1\ \text{torr} = 133\ \text{N/m}^2$	$1\ \text{atm} = 760\ \text{torr}$
$1\ \text{mm-H}_2\text{O}\ (4°\text{C}) = 9.80\ \text{N/m}^2$	$1\ \text{atm} = 1.03 \times 10^4\ \text{mm-H}_2\text{O}\ (4°\text{C})$
	$\approx 10\ \text{m of water}$

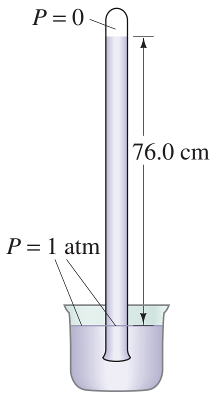


FIGURE 10–8 A mercury barometer, invented by Torricelli, is shown here when the air pressure is standard atmospheric, 76.0 cm-Hg.

Atmospheric pressure can be measured by a modified kind of mercury manometer with one end closed, called a mercury **barometer** (Fig. 10–8). The glass tube is completely filled with mercury and then inverted into the bowl of mercury. If the tube is long enough, the level of the mercury will drop, leaving a vacuum at the top of the tube, since atmospheric pressure can support a column of mercury only about 76 cm high (exactly 76.0 cm at standard atmospheric pressure). That is, a column of mercury 76 cm high exerts the same pressure as the atmosphere†:

$$\begin{aligned} P &= \rho g\, \Delta h \\ &= \left(13.6 \times 10^3\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)(0.760\ \text{m}) = 1.013 \times 10^5\ \text{N/m}^2 = 1.00\ \text{atm}. \end{aligned}$$

Household barometers are usually of the aneroid type (Fig. 10–7b), either mechanical (with dial) or electronic.

A calculation similar to that just done will show that atmospheric pressure can maintain a column of water 10.3 m high in a tube whose top is under vacuum (Fig. 10–9). No matter how good a vacuum pump is, water cannot be made to rise more than about 10 m under normal atmospheric pressure. To pump water out of deep mine shafts with a vacuum pump requires multiple stages for depths greater than 10 m. Galileo studied this problem, and his student Torricelli was the first to explain it. The point is that a pump does not really suck water up a tube—it merely reduces the pressure at the top of the tube. Atmospheric air pressure *pushes* the water up the tube if the top end is at low pressure (under a vacuum), just as it is air pressure that pushes (or maintains) the mercury 76 cm high in a barometer. [Force pumps, Section 10–14, can push higher.]

FIGURE 10–9 A water barometer: a full tube of water (longer than 10 m), closed at the top, is inserted into a tub of water. When the submerged bottom end of the tube is unplugged, some water flows out of the tube into the tub, leaving a vacuum at the top of the tube above the water's upper surface. Why? Because air pressure can support a column of water only 10 m high.

CONCEPTUAL EXAMPLE 10–5 **Suction.** A novice engineer proposes suction cup shoes for space shuttle astronauts working on the exterior of a spacecraft. Having just studied this Chapter, you gently remind him of the fallacy of this plan. What is it?

RESPONSE Suction cups work by pushing out the air underneath the cup. What holds the suction cup in place is the air pressure outside it. (This can be a substantial force when on Earth. For example, a 10-cm-diameter suction cup has an area of $7.9 \times 10^{-3}\ \text{m}^2$. The force of the atmosphere on it is $\left(7.9 \times 10^{-3}\ \text{m}^2\right)\left(1.0 \times 10^5\ \text{N/m}^2\right) \approx 800\ \text{N}$, about 180 lbs!) But in outer space, there is no air pressure to push the suction cup onto the spacecraft.

We sometimes mistakenly think of suction as something we actively do. For example, we intuitively think that we pull the soda up through a straw. Instead, what we do is lower the pressure at the top of the straw, and the atmosphere *pushes* the soda up the straw.

†This calculation confirms the entry in Table 10–2, $1\ \text{atm} = 76.0\ \text{cm-Hg}$.

10–7 Buoyancy and Archimedes' Principle

Objects submerged in a fluid appear to weigh less than they do when outside the fluid. For example, a large rock that you would have difficulty lifting off the ground can often be easily lifted from the bottom of a stream. When you lift the rock through the surface of the water, it suddenly seems to be much heavier. Many objects, such as wood, float on the surface of water. These are two examples of **buoyancy**. In each example, the force of gravity is acting downward. But in addition, an upward *buoyant force* is exerted by the liquid. The buoyant force on fish and underwater divers almost exactly balances the force of gravity downward, and allows them to "hover" in equilibrium.

The buoyant force occurs because the pressure in a fluid increases with depth. Thus the upward pressure on the bottom surface of a submerged object is greater than the downward pressure on its top surface. To see this effect, consider a cylinder of height Δh whose top and bottom ends have an area A and which is completely submerged in a fluid of density ρ_F, as shown in Fig. 10–10. The fluid exerts a pressure $P_1 = \rho_F g h_1$ at the top surface of the cylinder (Eq. 10–3a).

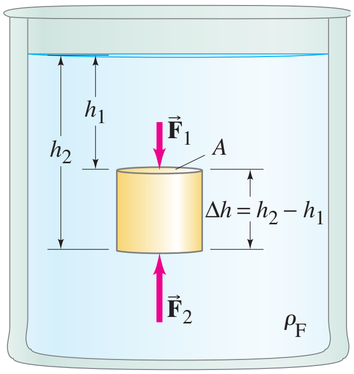


FIGURE 10–10 Determination of the buoyant force.

The force due to this pressure on top of the cylinder is $F_1 = P_1 A = \rho_F g h_1 A$, and it is directed downward. Similarly, the fluid exerts an upward force on the bottom of the cylinder equal to $F_2 = P_2 A = \rho_F g h_2 A$. The net force on the cylinder exerted by the fluid pressure, which is the **buoyant force**, $\vec{\mathbf{F}}_B$, acts upward and has the magnitude

$$\begin{aligned} F_B = F_2 - F_1 &= \rho_F g A (h_2 - h_1) \\ &= \rho_F g A \,\Delta h \\ &= \rho_F V g \\ &= m_F g, \end{aligned}$$

where $V = A\,\Delta h$ is the volume of the cylinder; the product $\rho_F V$ is the mass of the fluid displaced, and $\rho_F V g = m_F g$ is the weight of fluid which takes up a volume equal to the volume of the cylinder. Thus the buoyant force on the cylinder is equal to the weight of fluid displaced by the cylinder.

This result is valid no matter what the shape of the object. Its discovery is credited to Archimedes (287?–212 B.C.), and it is called **Archimedes' principle**:

the buoyant force on an object immersed in a fluid is equal to the weight of the fluid displaced by that object.

By "fluid displaced," we mean a volume of fluid equal to the submerged volume of the object (or that part of the object that is submerged). If the object is placed in a glass or tub initially filled to the brim with water, the water that flows over the top represents the water displaced by the object.

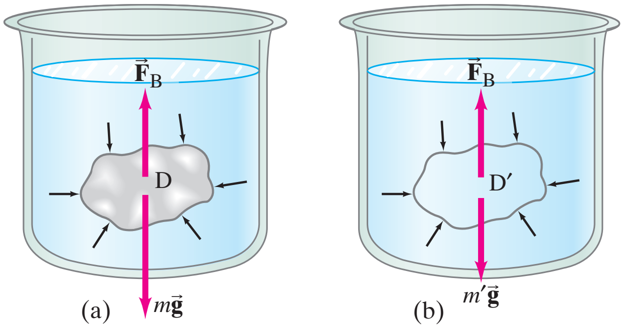


FIGURE 10–11 Archimedes' principle.

We can derive Archimedes' principle in general by the following simple but elegant argument. The irregularly shaped object D shown in Fig. 10–11a is acted on by the force of gravity (its weight, $m\vec{\mathbf{g}}$, downward) and the buoyant force, $\vec{\mathbf{F}}_B$, upward. We wish to determine F_B. To do so, we next consider a body (D′ in Fig. 10–11b), this time made of the fluid itself, with the same shape and size as the original object, and located at the same depth. You might think of this body of fluid as being separated from the rest of the fluid by an imaginary membrane. The buoyant force F_B on this body of fluid will be exactly the same as that on the original object since the surrounding fluid, which exerts F_B, is in exactly the same configuration. This body of fluid D′ is in equilibrium (the fluid as a whole is at rest). Therefore, $F_B = m'g$, where $m'g$ is the weight of the body of fluid D′. Hence the buoyant force F_B is equal to the weight of the body of fluid whose volume equals the volume of the original submerged object, which is Archimedes' principle.

Archimedes' discovery was made by experiment. What we have done is show that Archimedes' principle can be derived from Newton's laws.

CONCEPTUAL EXAMPLE 10–6 **Two pails of water.** Consider two identical pails of water filled to the brim. One pail contains only water, the other has a piece of wood floating in it. Which pail has the greater weight?

RESPONSE Both pails weigh the same. Recall Archimedes' principle: the wood displaces a volume of water with weight equal to the weight of the wood. Some water will overflow the pail, but Archimedes' principle tells us the spilled water has weight equal to the weight of the wood; so the pails have the same weight.

EXAMPLE 10–7 **Recovering a submerged statue.** A 70-kg ancient statue lies at the bottom of the sea. Its volume is $3.0 \times 10^4\,\mathrm{cm}^3$. How much force is needed to lift it (without acceleration)?

APPROACH The force F needed to lift the statue is equal to the statue's weight mg minus the buoyant force F_B. Figure 10–12 is the free-body diagram.

SOLUTION We apply Newton's second law, $\Sigma F = ma = 0$, which gives $F + F_B - mg = 0$ or

$$F = mg - F_B.$$

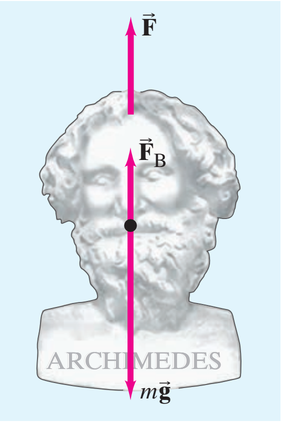
The buoyant force on the statue due to the water is equal to the weight of $3.0 \times 10^4\,\mathrm{cm}^3 = 3.0 \times 10^{-2}\,\mathrm{m}^3$ of water (for seawater, $\rho = 1.025 \times 10^3\,\mathrm{kg/m^3}$):

$$\begin{aligned} F_B = m_{H_2O}\,g = \rho_{H_2O} V g &= \left(1.025 \times 10^3\,\mathrm{kg/m^3}\right)\left(3.0 \times 10^{-2}\,\mathrm{m^3}\right)\left(9.8\,\mathrm{m/s^2}\right) \\ &= 3.0 \times 10^2\,\mathrm{N}, \end{aligned}$$

where we use the chemical symbol for water, H_2O, as a subscript. The weight of the statue is $mg = (70\,\mathrm{kg})\left(9.8\,\mathrm{m/s^2}\right) = 6.9 \times 10^2\,\mathrm{N}$. Hence the force F needed to lift it is $690\,\mathrm{N} - 300\,\mathrm{N} = 390\,\mathrm{N}$. It is as if the statue had a mass of only $(390\,\mathrm{N})/\left(9.8\,\mathrm{m/s^2}\right) = 40\,\mathrm{kg}$.

NOTE Here $F = 390\,\mathrm{N}$ is the force needed to lift the statue without acceleration when it is under water. As the statue comes *out* of the water, the force F increases, reaching 690 N when the statue is fully out of the water.

FIGURE 10–12 Example 10–7. The force needed to lift the statue is $\vec{\mathbf{F}}$.



Archimedes is said to have discovered his principle in his bath while thinking how he might determine whether the king's new crown was pure gold or a fake. Gold has a specific gravity of 19.3, somewhat higher than that of most metals, but a determination of specific gravity or density is not readily done directly because, even if the mass is known, the volume of an irregularly shaped object is not easily calculated. However, if the object is weighed in air ($= w$) and also "weighed" while it is under water ($= w'$), the density can be determined using Archimedes' principle, as the following Example shows. The quantity w' is called the **apparent weight** in water, and is what a scale reads when the object is submerged in water (see Fig. 10–13); w' equals the true weight ($w = mg$) minus the buoyant force.

EXAMPLE 10–8 **Archimedes: Is the crown gold?** When a crown of mass 14.7 kg is submerged in water, an accurate scale reads only 13.4 kg. Is the crown made of gold?

APPROACH If the crown is gold, its density and specific gravity must be very high, SG = 19.3 (see Section 10–2 and Table 10–1). We determine the specific gravity using Archimedes' principle and the two free-body diagrams shown in Fig. 10–13.

SOLUTION The *apparent weight* of the submerged object (the crown) is w' (what the scale reads), and is the force pulling down on the scale hook. By Newton's third law, w' equals the force F'_T that the scale exerts on the crown in Fig. 10–13b. The sum of the forces on the crown is zero, so w' equals the actual weight w ($= mg$) minus the buoyant force F_B:

$$w' = F'_T = w - F_B$$

so

$$w - w' = F_B.$$

Let V be the volume of the completely submerged object and ρ_O the object's density (so $\rho_O V$ is its mass), and let ρ_F be the density of the fluid (water). Then $(\rho_F V)g$ is the weight of fluid displaced ($= F_B$). Now we can write

$$w = mg = \rho_O V g$$
$$w - w' = F_B = \rho_F V g.$$

We divide these two equations and obtain

$$\frac{w}{w - w'} = \frac{\rho_O V g}{\rho_F V g} = \frac{\rho_O}{\rho_F}.$$

We see that $w/(w - w')$ is equal to the specific gravity of the object (the crown) if the fluid in which it is submerged is water $\left(\rho_F = 1.00 \times 10^3\ \mathrm{kg/m^3}\right)$. Thus

$$\frac{\rho_O}{\rho_{H_2O}} = \frac{w}{w - w'} = \frac{(14.7\ \mathrm{kg})g}{(14.7\ \mathrm{kg} - 13.4\ \mathrm{kg})g} = \frac{14.7\ \mathrm{kg}}{1.3\ \mathrm{kg}} = 11.3.$$

This corresponds to a density of 11,300 kg/m³. The crown is not gold, but seems to be made of lead (see Table 10–1).

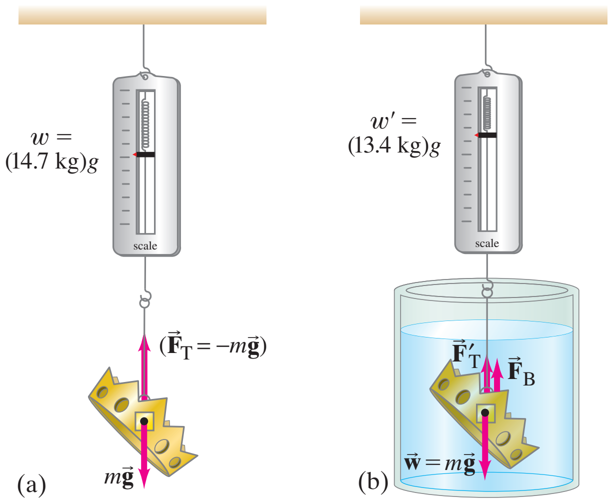


FIGURE 10–13 (a) A scale reads the mass of an object in air—in this case the crown of Example 10–8. All objects are at rest, so the tension F_T in the connecting cord equals the weight w of the object: $F_T = mg$. We show the free-body diagram of the crown, and F_T is what causes the scale reading (it is equal to the net downward force on the scale, by Newton's third law). (b) Submerged, the crown has an additional force on it, the buoyant force F_B. The net force is zero, so $F'_T + F_B = mg\ (= w)$. The scale now reads $m' = 13.4$ kg, where m' is related to the effective weight by $w' = m'g$. Thus $F'_T = w' = w - F_B$.

Archimedes' principle applies equally well to objects that float, such as wood. In general, *an object floats on a fluid if its density* (ρ_O) *is less than that of the fluid* (ρ_F). This is readily seen from Fig. 10–14a, where a submerged log of mass m_O will experience a net upward force and float to the surface if $F_B > m_O g$; that is, if $\rho_F V g > \rho_O V g$ or $\rho_F > \rho_O$. At equilibrium—that is, when floating—the buoyant force on an object has magnitude equal to the weight of the object. For example, a log whose specific gravity is 0.60 and whose volume is $2.0\ m^3$ has a mass

$$m_O = \rho_O V = (0.60 \times 10^3\ kg/m^3)(2.0\ m^3) = 1200\ kg.$$

If the log is fully submerged, it will displace a mass of water

$$m_F = \rho_F V = (1000\ kg/m^3)(2.0\ m^3) = 2000\ kg.$$

Hence the buoyant force on the log will be greater than its weight, and it will float upward to the surface (Fig. 10–14). The log will come to equilibrium when it displaces 1200 kg of water, which means that $1.2\ m^3$ of its volume will be submerged. This $1.2\ m^3$ corresponds to 60% of the volume of the log $(= 1.2/2.0 = 0.60)$, so 60% of the log is submerged.

In general when an object floats, we have $F_B = m_O g$, which we can write as (see Fig. 10–15)

$$\begin{aligned} F_B &= m_O g \\ \rho_F V_{displ} g &= \rho_O V_O g, \end{aligned}$$

where V_O is the full volume of the object and V_{displ} is the volume of fluid it displaces (= volume submerged). Thus

$$\frac{V_{displ}}{V_O} = \frac{\rho_O}{\rho_F}.$$

That is, the fraction of the object submerged is given by the ratio of the object's density to that of the fluid. If the fluid is water, this fraction equals the specific gravity of the object.

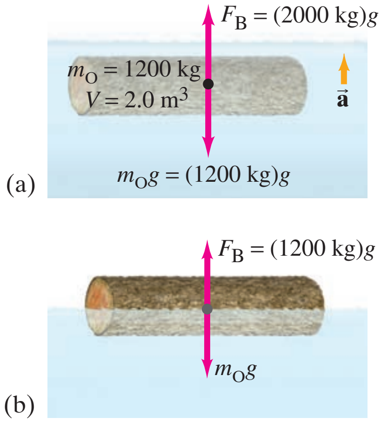
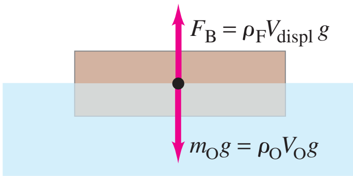


FIGURE 10–14 (a) The fully submerged log accelerates upward because $F_B > m_O g$. It comes to equilibrium (b) when $\Sigma F = 0$, so $F_B = m_O g = (1200\ kg)g$. Then 1200 kg, or $1.2\ m^3$, of water is displaced.

FIGURE 10–15 An object floating in equilibrium: $F_B = m_O g$.



EXAMPLE 10–9 **Hydrometer calibration.** A **hydrometer** is a simple instrument used to measure the specific gravity of a liquid by indicating how deeply the instrument sinks in the liquid. A particular hydrometer (Fig. 10–16) consists of a glass tube, weighted at the bottom, which is 25.0 cm long and $2.00\ cm^2$ in cross-sectional area, and has a mass of 45.0 g. How far from the weighted end should the 1.000 mark be placed?

APPROACH The hydrometer will float in water if its density ρ is less than $\rho_{H_2O} = 1.000\ g/cm^3$, the density of water. The fraction of the hydrometer submerged $(V_{displaced}/V_{total})$ is equal to the density ratio ρ/ρ_{H_2O}.

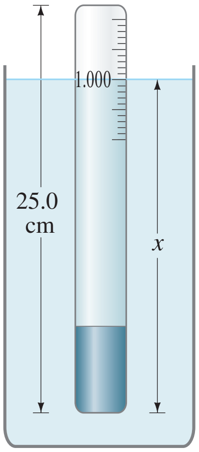
SOLUTION The hydrometer has an overall density

$$\rho = \frac{m}{V} = \frac{45.0\ g}{(2.00\ cm^2)(25.0\ cm)} = 0.900\ g/cm^3.$$

Thus, when placed in water, it will come to equilibrium when 0.900 of its volume is submerged. Since it is of uniform cross section, $(0.900)(25.0\ cm) = 22.5\ cm$ of its length will be submerged. The specific gravity of water is defined to be 1.000, so the mark should be placed 22.5 cm from the weighted end.

NOTE Hydrometers can be used to measure the density of liquids like car antifreeze coolant, car battery acid (a measure of its charge), wine fermenting in casks, and many others.

FIGURE 10–16 A hydrometer. Example 10–9.



EXERCISE C Which of the following objects, submerged in water, experiences the largest magnitude of the buoyant force? (*a*) A 1-kg helium balloon; (*b*) 1 kg of wood; (*c*) 1 kg of ice; (*d*) 1 kg of iron; (*e*) all the same.

EXERCISE D Which of the following objects, submerged in water, experiences the largest magnitude of the buoyant force? (*a*) A 1-m^3 helium balloon; (*b*) 1 m^3 of wood; (*c*) 1 m^3 of ice; (*d*) 1 m^3 of iron; (*e*) all the same.

PHYSICS APPLIED
Continental drift—plate tectonics

Archimedes' principle is also useful in geology. According to the theories of plate tectonics and continental drift, the continents float on a fluid "sea" of slightly deformable rock (mantle rock). Some interesting calculations can be done using very simple models, which we consider in the Problems at the end of the Chapter.

Air is a fluid, and it too exerts a buoyant force. Ordinary objects weigh less in air than they do in a vacuum. Because the density of air is so small, the effect for ordinary solids is slight. There are objects, however, that *float* in air—helium-filled balloons, for example, because the density of helium is less than the density of air.

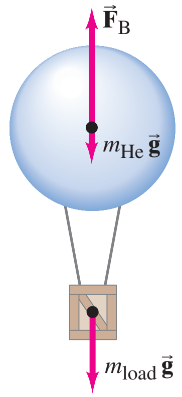


FIGURE 10–17 Example 10–10.

EXAMPLE 10–10 **Helium balloon.** What volume V of helium is needed if a balloon is to lift a load of 180 kg (including the weight of the empty balloon)?

APPROACH The buoyant force on the helium balloon, F_B, which is equal to the weight of displaced air, must be at least equal to the weight of the helium plus the weight of the balloon and load (Fig. 10–17). Table 10–1 gives the density of helium as $0.179\ \text{kg/m}^3$.

SOLUTION The buoyant force must have a minimum value of

$$F_B = (m_{He} + 180\ \text{kg})g.$$

This equation can be written in terms of density using Archimedes' principle:

$$\rho_{air} V g = (\rho_{He} V + 180\ \text{kg})g.$$

Solving now for V, we find

$$V = \frac{180\ \text{kg}}{\rho_{air} - \rho_{He}} = \frac{180\ \text{kg}}{(1.29\ \text{kg/m}^3 - 0.179\ \text{kg/m}^3)} = 160\ \text{m}^3.$$

NOTE This is the minimum volume needed near the Earth's surface, where $\rho_{air} = 1.29\ \text{kg/m}^3$. To reach a high altitude, a greater volume would be needed since the density of air decreases with altitude.

CONCEPTUAL EXAMPLE 10–11 **Throwing a rock overboard.** A rowboat carrying a large granite rock floats in a small lake. If the rock (SG $\approx$ 3, Table 10–1) is thrown overboard and sinks, does the lake level drop, rise, or stay the same?

RESPONSE Together the boat and rock float, so the buoyant force on them equals their total weight. The boat and rock displace a mass of water whose weight is equal to the weight of boat plus rock. When the rock is thrown into the lake, it displaces only its own volume, which is smaller than the volume of water the rock displaced when in the boat ($\approx \frac{1}{3}$ as much because the rock's density is ≈ 3 times greater than water). So less lake water is displaced and the water level of the lake *drops* when the rock is in the lake.

Maybe numbers can help. Suppose the boat and the rock each has a mass of 60 kg. Then the boat carrying the rock displaces 120 kg of water, which is a volume of $0.12\ \text{m}^3$ ($\rho = 1000\ \text{kg/m}^3$ for water, Table 10–1). When the rock is thrown into the lake, the boat alone now displaces $0.06\ \text{m}^3$. The rock displaces only its own volume of $0.02\ \text{m}^3$ ($\rho = m/V \approx 3$ so $V \approx 0.06\ \text{m}^3/3$). Thus a total of $0.08\ \text{m}^3$ of water is displaced. Less water is displaced so the water level of the lake drops.

EXERCISE E If you throw a flat 60-kg aluminum plate into water, the plate sinks. But if that aluminum is shaped into a rowboat, it floats. Explain.

10–8 Fluids in Motion; Flow Rate and the Equation of Continuity

We now turn to the subject of fluids in motion, which is called **fluid dynamics**, or (especially if the fluid is water) **hydrodynamics**.

We can distinguish two main types of fluid flow. If the flow is smooth, such that neighboring layers of the fluid slide by each other smoothly, the flow is said to be **streamline** or **laminar flow**.[†] In streamline flow, each particle of the fluid follows a smooth path, called a **streamline**, and these paths do not cross one another (Fig. 10–18a).

[†]The word laminar means "in layers."

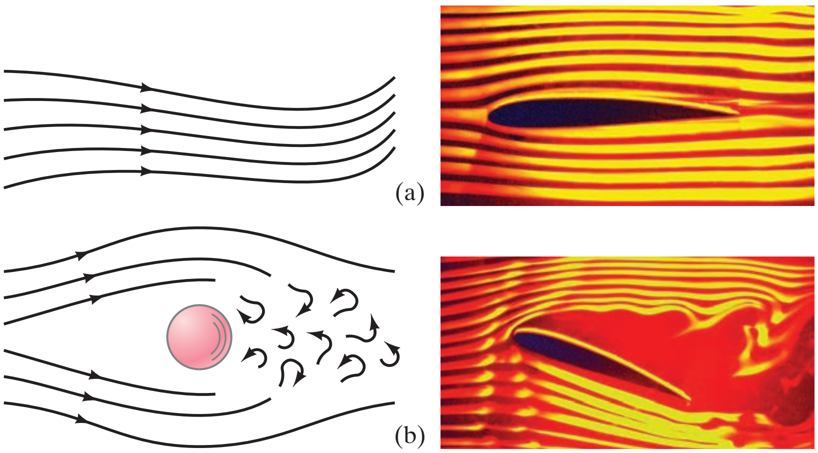


FIGURE 10–18 (a) Streamline, or laminar, flow; (b) turbulent flow. The photos show airflow around an airfoil or airplane wing (more in Section 10–10).

Above a certain speed, the flow becomes turbulent. **Turbulent flow** is characterized by erratic, small, whirlpool-like circles called *eddy currents* or *eddies* (Fig. 10–18b). Eddies absorb a great deal of energy, and although a certain amount of internal friction called **viscosity** is present even during streamline flow, it is much greater when the flow is turbulent. A few tiny drops of ink or food coloring dropped into a moving liquid can quickly reveal whether the flow is streamline or turbulent.

Let us consider the steady laminar flow of a fluid through an enclosed tube or pipe as shown in Fig. 10–19. First we determine how the speed of the fluid changes when the diameter of the tube changes. The mass **flow rate** is defined as the mass Δm of fluid that passes a given point per unit time Δt:

$$\text{mass flow rate} \;=\; \frac{\Delta m}{\Delta t}.$$

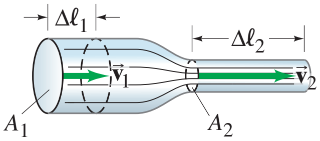


FIGURE 10–19 Fluid flow through a pipe of varying diameter.

In Fig. 10–19, the volume of fluid passing point 1 (through area A_1) in a time Δt is $A_1\,\Delta\ell_1$, where $\Delta\ell_1$ is the distance the fluid moves in time Δt. The velocity† of fluid (density ρ_1) passing point 1 is $v_1 = \Delta\ell_1/\Delta t$. Then the mass flow rate $\Delta m_1/\Delta t$ through area A_1 is

$$\frac{\Delta m_1}{\Delta t} \;=\; \frac{\rho_1\,\Delta V_1}{\Delta t} \;=\; \frac{\rho_1\,A_1\,\Delta\ell_1}{\Delta t} \;=\; \rho_1\,A_1\,v_1,$$

where $\Delta V_1 = A_1\,\Delta\ell_1$ is the volume of mass Δm_1. Similarly, at point 2 (through area A_2), the flow rate is $\rho_2\,A_2\,v_2$. Since no fluid flows in or out the sides of the tube, the flow rates through A_1 and A_2 must be equal. Thus

$$\frac{\Delta m_1}{\Delta t} \;=\; \frac{\Delta m_2}{\Delta t},$$

and

$$\rho_1\,A_1\,v_1 \;=\; \rho_2\,A_2\,v_2. \qquad \textbf{(10–4a)}$$

This is called the **equation of continuity**.

If the fluid is incompressible (ρ doesn't change with pressure), which is an excellent approximation for liquids under most circumstances (and sometimes for gases as well), then $\rho_1 = \rho_2$, and the equation of continuity becomes

$$A_1\,v_1 \;=\; A_2\,v_2. \qquad [\rho = \text{constant}] \quad \textbf{(10–4b)}$$

The product Av represents the *volume rate of flow* (volume of fluid passing a given point per second), since $\Delta V/\Delta t = A\,\Delta\ell/\Delta t = Av$, which in SI units is m^3/s. Equation 10–4b tells us that where the cross-sectional area is large, the velocity is small, and where the area is small, the velocity is large. That this is reasonable can be seen by looking at a river. A river flows slowly through a meadow where it is broad, but speeds up to torrential speed when passing through a narrow gorge.

†If there were no viscosity, the velocity would be the same across a cross section of the tube. Real fluids have viscosity, and this internal friction causes different layers of the fluid to flow at different speeds. In this case v_1 and v_2 represent the average speeds at each cross section.

PHYSICS APPLIED
Blood flow

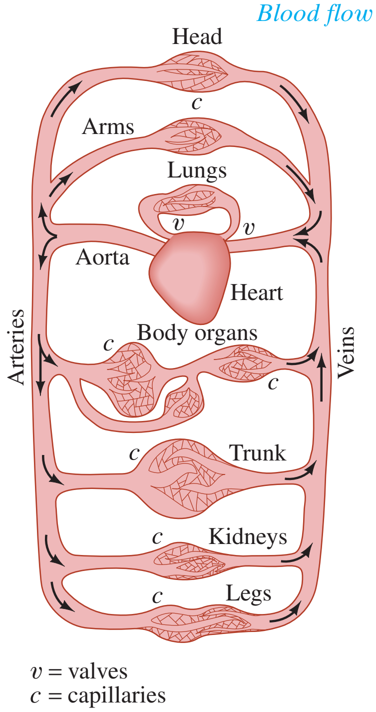


FIGURE 10–20 Human circulatory system.

EXAMPLE 10–12 ESTIMATE **Blood flow.** In humans, blood flows from the heart into the aorta, from which it passes into the major arteries, Fig. 10–20. These branch into the small arteries (arterioles), which in turn branch into myriads of tiny capillaries. The blood returns to the heart via the veins. The radius of the aorta is about 1.2 cm, and the blood passing through it has a speed of about 40 cm/s. A typical capillary has a radius of about 4×10^{-4} cm, and blood flows through it at a speed of about 5×10^{-4} m/s. Estimate the number of capillaries that are in the body.

APPROACH We assume the density of blood doesn't vary significantly from the aorta to the capillaries. By the equation of continuity, the volume flow rate in the aorta must equal the volume flow rate through *all* the capillaries. The total area of all the capillaries is given by the area of a typical capillary multiplied by the total number N of capillaries.

SOLUTION Let A_1 be the area of the aorta and A_2 be the area of *all* the capillaries through which blood flows. Then $A_2 = N\pi r_{\text{cap}}^2$, where $r_{\text{cap}} \approx 4 \times 10^{-4}$ cm is the estimated average radius of one capillary. From the equation of continuity (Eq. 10–4b), we have

$$\begin{aligned} v_2 A_2 &= v_1 A_1 \\ v_2 N\pi r_{\text{cap}}^2 &= v_1 \pi r_{\text{aorta}}^2 \end{aligned}$$

so

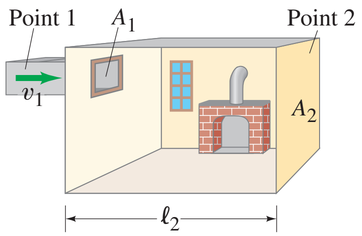
$$N = \frac{v_1}{v_2}\frac{r_{\text{aorta}}^2}{r_{\text{cap}}^2} = \left(\frac{0.40\text{ m/s}}{5 \times 10^{-4}\text{ m/s}}\right)\left(\frac{1.2 \times 10^{-2}\text{ m}}{4 \times 10^{-6}\text{ m}}\right)^2 \approx 7 \times 10^9,$$

or on the order of 10 billion capillaries.

PHYSICS APPLIED
Heating duct

EXAMPLE 10–13 **Heating duct to a room.** What area must a heating duct have if air moving 3.0 m/s along it can replenish the air every 15 minutes in a room of volume 300 m^3? Assume the air's density remains constant.

FIGURE 10–21 Example 10–13.



APPROACH We apply the equation of continuity at constant density, Eq. 10–4b, to the air that flows through the duct (point 1 in Fig. 10–21) and then into the room (point 2). The volume flow rate in the room equals the volume of the room divided by the 15-min replenishing time.

SOLUTION Consider the room as a large section of the duct, Fig. 10–21, and think of air equal to the volume of the room as passing by point 2 in $t = 15\text{ min} = 900\text{ s}$. Reasoning in the same way we did to obtain Eq. 10–4a (changing Δt to t), we write $v_2 = \ell_2/t$ so $A_2 v_2 = A_2\ell_2/t = V_2/t$, where V_2 is the volume of the room. Then the equation of continuity becomes $A_1 v_1 = A_2 v_2 = V_2/t$ and

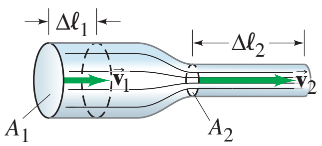
$$A_1 = \frac{V_2}{v_1 t} = \frac{300\text{ m}^3}{(3.0\text{ m/s})(900\text{ s})} = 0.11\text{ m}^2.$$

NOTE If the duct is square, then each side has length $\ell = \sqrt{A} = 0.33$ m, or 33 cm. A rectangular duct 20 cm × 55 cm will also do.

10–9 Bernoulli's Equation

Have you ever wondered why an airplane can fly, or how a sailboat can move against the wind? These are examples of a principle worked out by Daniel Bernoulli (1700–1782) concerning fluids in motion. In essence, **Bernoulli's principle** states that *where the velocity of a fluid is high, the pressure is low, and where the velocity is low, the pressure is high*. For example, if the pressure in the fluid is measured at points 1 and 2 of Fig. 10–19, it will be found that the pressure is lower at point 2, where the velocity is greater, than it is at point 1, where the velocity is smaller. At first glance, this might seem strange; you might expect that the greater speed at point 2 would imply a higher pressure. But this cannot be the case:

FIGURE 10–19 (Repeated.) Fluid flow through a pipe of varying diameter.



if the pressure in the fluid at point 2 were higher than at point 1, this higher pressure would slow the fluid down, whereas in fact it has sped up in going from point 1 to point 2. Thus the pressure at point 2 must be less than at point 1, to be consistent with the fact that the fluid accelerates.

To help clarify any misconceptions, a faster fluid might indeed exert a greater force bouncing off an obstacle placed in its path. But that is not what we mean by the pressure in a fluid. We are examining smooth streamline flow, with no obstacles that interrupt the flow. The fluid pressure is exerted on the walls of a tube or pipe, or on the surface of a material the fluid passes over.

Bernoulli developed an equation that expresses this principle quantitatively. To derive Bernoulli's equation, we assume the flow is steady and laminar, the fluid is incompressible, and the viscosity is small enough to be ignored. To be general, we assume the fluid is flowing in a tube of nonuniform cross section that varies in height above some reference level, Fig. 10–22. We will consider the volume of fluid shown in color and calculate the work done to move it from the position shown in Fig. 10–22a to that shown in Fig. 10–22b. In this process, fluid entering area A_1 flows a distance $\Delta\ell_1$ and forces the fluid at area A_2 to move a distance $\Delta\ell_2$. The fluid to the left of area A_1 exerts a pressure P_1 on our section of fluid and does an amount of work

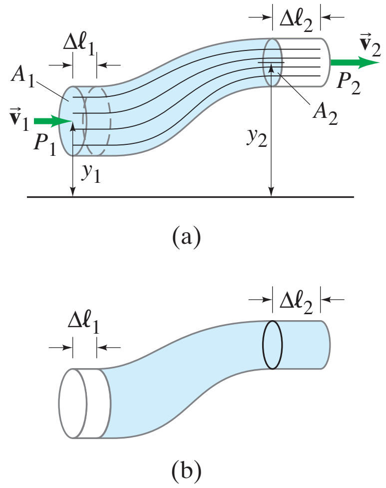


FIGURE 10–22 Fluid flow: for derivation of Bernoulli's equation.

$$W_1 = F_1\,\Delta\ell_1 = P_1 A_1\,\Delta\ell_1,$$

(since $P = F/A$). At point 2, the work done on our section of fluid is

$$W_2 = -P_2 A_2\,\Delta\ell_2.$$

The negative sign is present because the force exerted on the fluid is opposite to the displacement. Work is also done on the fluid by the force of gravity. The net effect of the process shown in Fig. 10–22 is to move a mass m of volume $A_1\,\Delta\ell_1$ ($= A_2\,\Delta\ell_2$, since the fluid is incompressible) from point 1 to point 2, so the work done by gravity is

$$W_3 = -mg(y_2 - y_1),$$

where y_1 and y_2 are heights of the center of the tube above some (arbitrary) reference level. In the case shown in Fig. 10–22, this term is negative since the motion is uphill against the force of gravity. The net work W done on the fluid is thus

$$W = W_1 + W_2 + W_3$$
$$W = P_1 A_1\,\Delta\ell_1 - P_2 A_2\,\Delta\ell_2 - mgy_2 + mgy_1.$$

According to the work-energy principle (Section 6–3), the net work done on a system is equal to its change in kinetic energy. Hence

$$\tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 = P_1 A_1\,\Delta\ell_1 - P_2 A_2\,\Delta\ell_2 - mgy_2 + mgy_1.$$

The mass m has volume $A_1\,\Delta\ell_1 = A_2\,\Delta\ell_2$ for an incompressible fluid. Thus we can substitute $m = \rho A_1\,\Delta\ell_1 = \rho A_2\,\Delta\ell_2$, and then divide through by $A_1\,\Delta\ell_1 = A_2\,\Delta\ell_2$, to obtain

$$\tfrac{1}{2}\rho v_2^2 - \tfrac{1}{2}\rho v_1^2 = P_1 - P_2 - \rho g y_2 + \rho g y_1,$$

which we rearrange to get

$$P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 = P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1. \qquad \textbf{(10–5)}$$

Bernoulli's equation

This is **Bernoulli's equation**. Since points 1 and 2 can be any two points along a tube of flow, Bernoulli's equation can be written as

$$P + \tfrac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

at every point in the fluid, where y is the height of the center of the tube above a fixed reference level. [Note that if there is no flow $(v_1 = v_2 = 0)$, then Eq. 10–5 reduces to the hydrostatic equation, Eq. 10–3b or c: $P_1 - P_2 = \rho g(y_2 - y_1)$.]

Bernoulli's equation is an expression of the law of energy conservation, since we derived it from the work-energy principle.

EXERCISE F As water in a level pipe passes from a narrow cross section of pipe to a wider cross section, how does the pressure against the walls change?

PHYSICS APPLIED
Hot-water heating system

EXAMPLE 10–14 Flow and pressure in a hot-water heating system. Water circulates throughout a house in a hot-water heating system. If the water is pumped at a speed of 0.50 m/s through a 4.0-cm-diameter pipe in the basement under a pressure of 3.0 atm, what will be the flow speed and pressure in a 2.6-cm-diameter pipe on the second floor 5.0 m above? Assume the pipes do not divide into branches.

APPROACH We use the equation of continuity at constant density to determine the flow speed on the second floor, and then Bernoulli's equation to find the pressure.

SOLUTION We take v_2 in the equation of continuity, Eq. 10–4, as the flow speed on the second floor, and v_1 as the flow speed in the basement. Noting that the areas are proportional to the radii squared $(A = \pi r^2)$, we obtain

$$v_2 = \frac{v_1 A_1}{A_2} = \frac{v_1 \pi r_1^2}{\pi r_2^2} = (0.50\,\mathrm{m/s})\frac{(0.020\,\mathrm{m})^2}{(0.013\,\mathrm{m})^2} = 1.2\,\mathrm{m/s}.$$

To find the pressure on the second floor, we use Bernoulli's equation (Eq. 10–5):

$$\begin{aligned} P_2 &= P_1 + \rho g(y_1 - y_2) + \tfrac{1}{2}\rho\left(v_1^2 - v_2^2\right) \\ &= \left(3.0\times10^5\,\mathrm{N/m^2}\right) + \left(1.0\times10^3\,\mathrm{kg/m^3}\right)\left(9.8\,\mathrm{m/s^2}\right)(-5.0\,\mathrm{m}) \\ &\quad + \tfrac{1}{2}\left(1.0\times10^3\,\mathrm{kg/m^3}\right)\left[(0.50\,\mathrm{m/s})^2 - (1.2\,\mathrm{m/s})^2\right] \\ &= \left(3.0\times10^5\,\mathrm{N/m^2}\right) - \left(4.9\times10^4\,\mathrm{N/m^2}\right) - \left(6.0\times10^2\,\mathrm{N/m^2}\right) \\ &= 2.5\times10^5\,\mathrm{N/m^2} = 2.5\,\mathrm{atm}. \end{aligned}$$

NOTE The velocity term contributes very little in this case.

10–10 Applications of Bernoulli's Principle: Torricelli, Airplanes, Baseballs, Blood Flow

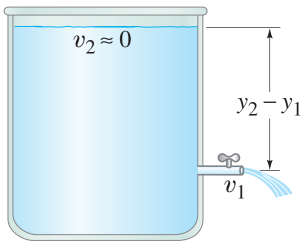


FIGURE 10–23 Torricelli's theorem: $v_1 = \sqrt{2g(y_2 - y_1)}$.

Bernoulli's equation can be applied to many situations. One example is to calculate the velocity, v_1, of a liquid flowing out of a spigot at the bottom of a reservoir, Fig. 10–23. We choose point 2 in Eq. 10–5 to be the top surface of the liquid. Assuming the diameter of the reservoir is large compared to that of the spigot, v_2 will be almost zero. Points 1 (the spigot) and 2 (top surface) are open to the atmosphere, so the pressure at both points is equal to atmospheric pressure: $P_1 = P_2$. Then Bernoulli's equation becomes

$$\tfrac{1}{2}\rho v_1^2 + \rho g y_1 = \rho g y_2$$

or

$$v_1 = \sqrt{2g(y_2 - y_1)}. \qquad \textbf{(10–6)}$$

This result is called **Torricelli's theorem**. Although it is seen to be a special case of Bernoulli's equation, it was discovered a century earlier by Evangelista Torricelli. Equation 10–6 tells us that the liquid leaves the spigot with the same speed that a freely falling object would attain if falling from the same height. This should not be too surprising since the derivation of Bernoulli's equation relies on the conservation of energy.

Another special case of Bernoulli's equation arises when a fluid is flowing horizontally with no appreciable change in height; that is, $y_1 = y_2$. Then Eq. 10–5 becomes

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2, \qquad \textbf{(10–7)}$$

which tells us quantitatively that the speed is high where the pressure is low, and vice versa. It explains many common phenomena, some of which are illustrated in Figs. 10–24 to 10–30. The pressure in the air blown at high speed across the top of the vertical tube of a perfume atomizer (Fig. 10–24a) is less than the normal air pressure acting on the surface of the liquid in the bowl. Thus atmospheric pressure in the bowl pushes the perfume up the tube because of the lower pressure at the top. A Ping-Pong ball can be made to float above a blowing jet of air (a hair dryer or a vacuum cleaner that can also blow air), Fig. 10–24b; if the ball begins to leave the jet of air, the higher pressure in the still air outside the jet pushes the ball back in.

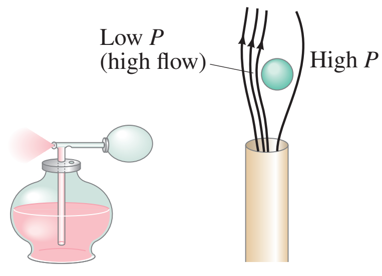


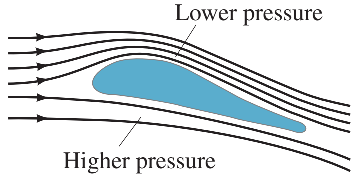
FIGURE 10–24 Examples of Bernoulli's principle: (a) atomizer, (b) Ping-Pong ball in jet of air.

EXERCISE G Return to Chapter-Opening Question 2, page 260, and answer it again now. Try to explain why you may have answered differently the first time. Try it and see.

Airplane Wings and Dynamic Lift

Airplanes experience a "lift" force on their wings, keeping them up in the air, if they are moving at a sufficiently high speed relative to the air and the wing is tilted upward at a small angle (the "attack angle"). See Fig. 10–25, where streamlines of air are shown rushing by the wing (we are in the reference frame of the wing, as if sitting on the wing). The upward tilt, as well as the rounded upper surface of the wing, causes the streamlines to be forced upward and to be crowded together above the wing. The area of air flowing between any two streamlines is smaller as the streamlines get closer together, so from the equation of continuity $(A_1 v_1 = A_2 v_2)$, the air speed increases above the wing where the streamlines are squished together. (Recall also how the crowded streamlines in a pipe constriction, Fig. 10–19, indicate the velocity is higher in the constriction.) Thus the air speed is greater above the wing than below it, so the pressure above the wing is less than the pressure below the wing (Bernoulli's principle). Hence there is a net upward force on the wing called **dynamic lift**. Experiments show that the speed of air above the wing can even be double the speed of the air below it. (Friction between the air and wing exerts a *drag force*, toward the rear, which must be overcome by the plane's engines.)

FIGURE 10–25 Lift on an airplane wing. We are in the reference frame of the wing, seeing the air flow by.



PHYSICS APPLIED
Airplanes and dynamic lift

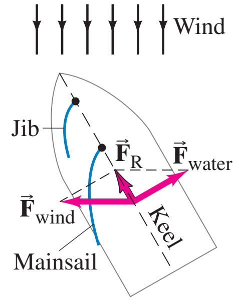
A flat wing, or one with symmetric cross section, will experience lift as long as the front of the wing is tilted upward (attack angle). The wing shown in Fig. 10–25 can experience lift even if the attack angle is zero, because the rounded upper surface deflects air up, squeezing the streamlines together. Airplanes can fly upside down, experiencing lift, if the attack angle is sufficient to deflect streamlines up and closer together.

Our picture considers streamlines; but if the attack angle is larger than about 15°, turbulence sets in (Fig. 10–18b) leading to greater drag and less lift, causing the plane to "stall" and then to drop.

From another point of view, the upward tilt of a wing means the air moving horizontally in front of the wing is deflected downward; the change in momentum of the rebounding air molecules results in an upward force on the wing (Newton's third law).

FIGURE 10–26 Sailboat (a) sailing against the wind with (b) analysis.

(a)



(b)

Sailboats

A sailboat can move "against" the wind, with the aid of the Bernoulli effect, by setting the sails at an angle, as shown in Fig. 10–26. The air traveling rapidly over the bulging front surface of the mainsail exerts a smaller pressure than the relatively still air behind the sail. The result is a net force on the sail, $\vec{\mathbf{F}}_{wind}$, as shown in Fig. 10–26b. This force would tend to make the boat move sideways if it weren't for the keel that extends vertically downward beneath the water: the water exerts a force $(\vec{\mathbf{F}}_{water})$ on the keel nearly perpendicular to the keel. The resultant of these two forces $(\vec{\mathbf{F}}_R)$ is almost directly forward as shown.

Baseball Curve

Why a spinning pitched baseball (or tennis ball) curves can also be explained using Bernoulli's principle. It is simplest if we put ourselves in the reference frame of the ball, with the air rushing by, just as we did for the airplane wing. Suppose the ball is rotating counterclockwise as seen from above, Fig. 10–27. A thin layer of air ("boundary layer") is being dragged around by the ball. We are looking down on the ball, and at point A in Fig. 10–27, this boundary layer tends to slow down the oncoming air. At point B, the air rotating with the ball adds its speed to that of the oncoming air, so the air speed is higher at B than at A. The higher speed at B means the pressure is lower at B than at A, resulting in a net force toward B. The ball's path curves toward the left (as seen by the pitcher).

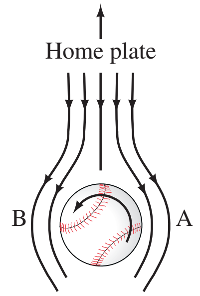
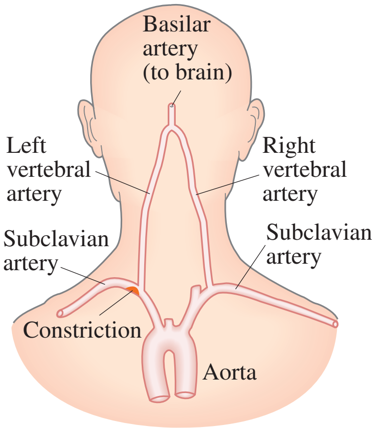


FIGURE 10–27 Looking down on a pitched baseball heading toward home plate. We are in the reference frame of the baseball, with the air flowing by.

Lack of Blood to the Brain—TIA

In medicine, one of many applications of Bernoulli's principle is to explain a TIA, a *transient ischemic attack* (meaning a temporary lack of blood supply to the brain). A person suffering a TIA may experience symptoms such as dizziness, double vision, headache, and weakness of the limbs. A TIA can occur as follows. Blood normally flows up to the brain at the back of the head via the two vertebral arteries—one going up each side of the neck—which meet to form the basilar artery just below the brain, as shown in Fig. 10–28. Each vertebral artery connects to the subclavian artery, as shown, before the blood passes to the arms. When an arm is exercised vigorously, blood flow increases to meet the needs of the arm's muscles. If the subclavian artery on one side of the body is partially blocked, however, as in arteriosclerosis (hardening of the arteries), the blood velocity will have to be higher on that side to supply the needed blood. (Recall the equation of continuity: smaller area means larger velocity for the same flow rate, Eqs. 10–4.) The increased blood velocity past the opening to the vertebral artery results in lower pressure (Bernoulli's principle). Thus, blood rising in the vertebral artery on the "good" side at normal pressure can be *diverted down* into the other vertebral artery because of the low pressure on that side, instead of passing upward to the brain. Hence the blood supply to the brain is reduced.

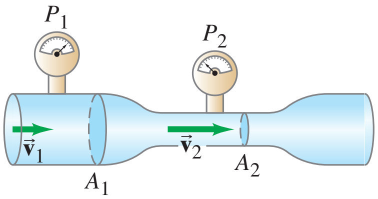
FIGURE 10–28 Rear of the head and shoulders showing arteries leading to the brain and to the arms. High blood velocity past the constriction in the left subclavian artery causes low pressure in the left vertebral artery, in which a reverse (downward) blood flow can then occur, resulting in a TIA, a loss of blood to the brain.



Other Applications

A **venturi tube** is essentially a pipe with a narrow constriction (the throat). The flowing fluid speeds up as it passes through this constriction, so the pressure is lower in the throat. A **venturi meter**, Fig. 10–29, is used to measure the flow speed of gases and liquids, including blood velocity in arteries. The velocity v_1 can be determined by measuring the pressure P_1 and P_2, the areas A_1 and A_2, as well as the density of the fluid. (The formula is given in Problem 50.)

FIGURE 10–29 Venturi meter.

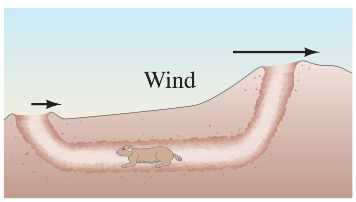


PHYSICS APPLIED
Smoke up a chimney
Underground air circulation

Why does smoke go up a chimney? It's partly because hot air rises (it's less dense and therefore buoyant). But Bernoulli's principle also plays a role. When wind blows across the top of a chimney, the pressure is less there than inside the house. Hence, air and smoke are pushed up the chimney by the higher indoor pressure. Even on an apparently still night there is usually enough ambient air flow at the top of a chimney to assist upward flow of smoke.

If gophers, prairie dogs, rabbits, and other animals that live underground are to avoid suffocation, the air must circulate in their burrows. The burrows always have at least two entrances (Fig. 10–30). The speed of air flow across different holes will usually be slightly different. This results in a slight pressure difference, which forces a flow of air through the burrow via Bernoulli's principle. The flow of air is enhanced if one hole is higher than the other (animals often build mounds) since wind speed tends to increase with height.

FIGURE 10–30 Bernoulli's principle explains air flow in underground burrows.



Bernoulli's equation ignores the effects of friction (viscosity) and the compressibility of the fluid. The energy that is transformed to internal (or potential) energy due to compression and to thermal energy by friction can be taken into account by adding terms to Eq. 10–5. These terms are difficult to calculate theoretically and are normally determined empirically for given situations. They do not significantly alter the explanations for the phenomena described above.

*10–11 Viscosity

Real fluids have a certain amount of internal friction called **viscosity**, as mentioned in Section 10–8. Viscosity exists in both liquids and gases, and is essentially a frictional force between adjacent layers of fluid as the layers move past one another. In liquids, viscosity is due to the electrical cohesive forces between the molecules. In gases, it arises from collisions between the molecules.

The viscosity of different fluids can be expressed quantitatively by a *coefficient of viscosity*, η (the Greek lowercase letter eta), which is defined in the following way. A thin layer of fluid is placed between two flat plates. One plate is stationary and the other is made to move, Fig. 10–31. The fluid directly in contact with each plate is held to the surface by the adhesive force between the molecules of the liquid and those of the plate. Thus the upper surface of the fluid moves with the same speed v as the upper plate, whereas the fluid in contact with the stationary plate remains stationary. The stationary layer of fluid retards the flow of the layer just above it, which in turn retards the flow of the next layer, and so on. Thus the velocity varies continuously from 0 to v, as shown. The increase in velocity divided by the distance over which this change is made—equal to v/ℓ—is called the *velocity gradient*. To move the upper plate requires a force, which you can verify by moving a flat plate across a puddle of syrup on a table. For a given fluid, it is found that the force required, F, is proportional to the area of fluid in contact with each plate, A, and to the speed, v, and is inversely proportional to the separation, ℓ, of the plates: $F \propto vA/\ell$. For different fluids, the more viscous the fluid, the greater is the required force. The proportionality constant for this equation is defined as the coefficient of viscosity, η:

$$F = \eta A \frac{v}{\ell}. \qquad \textbf{(10–8)}$$

Solving for η, we find $\eta = F\ell/vA$. The SI unit for η is $\mathrm{N \cdot s/m^2} = \mathrm{Pa \cdot s}$ (pascal·second). In the cgs system, the unit is $\mathrm{dyne \cdot s/cm^2}$, which is called a *poise* (P). Viscosities are often given in centipoise $(1\ \mathrm{cP} = 10^{-2}\ \mathrm{P} = 10^{-3}\ \mathrm{Pa \cdot s})$. Table 10–3 lists the coefficient of viscosity for various fluids. The temperature is also specified, since it has a strong effect; the viscosity of liquids such as motor oil, for example, decreases rapidly as temperature increases.‡

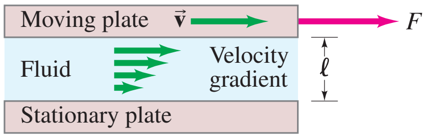


FIGURE 10–31 Determination of viscosity.

TABLE 10–3
Coefficients of Viscosity

Fluid (temperature in °C)	Coefficient of Viscosity, η (Pa·s)†
Water (0°)	1.8×10^{-3}
(20°)	1.0×10^{-3}
(100°)	0.3×10^{-3}
Whole blood (37°)	$\approx 4 \times 10^{-3}$
Blood plasma (37°)	$\approx 1.5 \times 10^{-3}$
Ethyl alcohol (20°)	1.2×10^{-3}
Engine oil (30°) (SAE 10)	200×10^{-3}
Glycerine (20°)	1500×10^{-3}
Air (20°)	0.018×10^{-3}
Hydrogen (0°)	0.009×10^{-3}
Water vapor (100°)	0.013×10^{-3}

†1 Pa·s = 10 poise (P) = 1000 cP.

*10–12 Flow in Tubes: Poiseuille's Equation, Blood Flow

If a fluid had no viscosity, it could flow through a level tube or pipe without a force being applied. Viscosity acts like a sort of friction (between fluid layers moving at slightly different speeds), so a pressure difference between the ends of a level tube is necessary for the steady flow of any real fluid, be it water or oil in a pipe, or blood in the circulatory system of a human.

‡The Society of Automotive Engineers assigns numbers to represent the viscosity of oils: 30-weight (SAE 30) is more viscous than 10-weight. Multigrade oils, such as 20–50, are designed to maintain viscosity as temperature increases; 20–50 means the oil acts like 20-weight when cool and is like 50-weight when it is hot (engine running temperature). In other words, the viscosity does not drop precipitously as the oil warms up, as a simple 20-weight oil would.

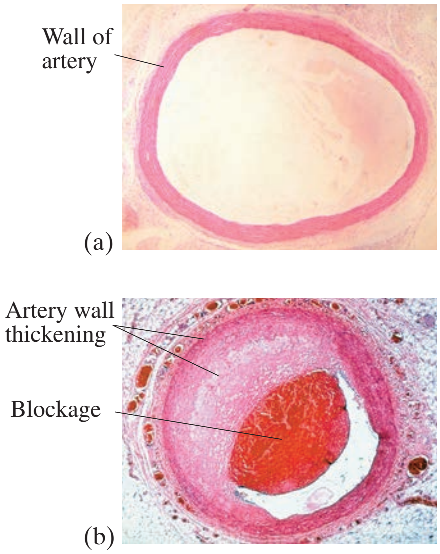


FIGURE 10–32 A cross section of a human artery that (a) is healthy, (b) is partly blocked as a result of arteriosclerosis.

PHYSICS APPLIED

Medicine—blood flow and heart disease

The French scientist J. L. Poiseuille (1799–1869), who was interested in the physics of blood circulation (and after whom the "poise" is named), determined how the variables affect the flow rate of an incompressible fluid undergoing laminar flow in a cylindrical tube. His result, known as **Poiseuille's equation**, is:

$$Q = \frac{\pi R^4 (P_1 - P_2)}{8\eta\ell}, \qquad \textbf{(10–9)}$$

where R is the inside radius of the tube, ℓ is the tube length, $P_1 - P_2$ is the pressure difference between the ends, η is the coefficient of viscosity, and Q is the volume rate of flow (volume of fluid flowing past a given point per unit time which in SI has units of m^3/s). Equation 10–9 applies only to laminar (streamline) flow.

Poiseuille's equation tells us that the flow rate Q is directly proportional to the "pressure gradient," $(P_1 - P_2)/\ell$, and it is inversely proportional to the viscosity of the fluid. This is just what we might expect. It may be surprising, however, that Q also depends on the *fourth* power of the tube's radius. This means that for the same pressure gradient, if the tube radius is halved, the flow rate is decreased by a factor of 16! Thus the rate of flow, or alternately the pressure required to maintain a given flow rate, is greatly affected by only a small change in tube radius.

An interesting example of this R^4 dependence is *blood flow* in the human body. Poiseuille's equation is valid only for the streamline flow of an incompressible fluid. So it cannot be precisely accurate for blood whose flow is not without turbulence and that contains blood cells (whose diameter is almost equal to that of a capillary). Nonetheless, Poiseuille's equation does give a reasonable first approximation. Because the radius of arteries is reduced as a result of arteriosclerosis (thickening and hardening of artery walls, Fig. 10–32) and by cholesterol buildup, the pressure gradient must be increased to maintain the same flow rate. If the radius is reduced by half, the heart would have to increase the pressure by a factor of about $2^4 = 16$ in order to maintain the same blood-flow rate. The heart must work much harder under these conditions, but usually cannot maintain the original flow rate. Thus, high blood pressure is an indication both that the heart is working harder and that the blood-flow rate is reduced.

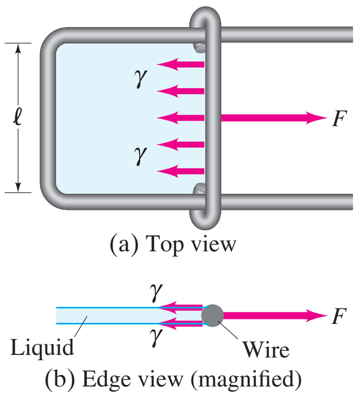
FIGURE 10–33 Spherical water droplets, dew on a blade of grass.

* 10–13 Surface Tension and Capillarity

The *surface* of a liquid at rest behaves in an interesting way, almost as if it were a stretched membrane under tension. For example, a drop of water on the end of a dripping faucet, or hanging from a thin branch in the early morning dew (Fig. 10–33), forms into a nearly spherical shape as if it were a tiny balloon filled with water. A steel needle can be made to float on the surface of water even though it is denser than the water. The surface of a liquid acts like it is under tension, and this tension, acting along the surface, arises from the attractive forces between the molecules. This effect is called **surface tension**. More specifically, a quantity called the *surface tension*, γ (the Greek letter gamma), is defined as the force F per unit length ℓ that acts perpendicular to any line or cut in a liquid surface, tending to pull the surface closed:

$$\gamma = \frac{F}{\ell}. \qquad \textbf{(10–10)}$$

FIGURE 10–34 U-shaped wire apparatus holding a film of liquid to measure surface tension ($\gamma = F/2\ell$).



To understand this, consider the U-shaped apparatus shown in Fig. 10–34 which encloses a thin film of liquid (such as a liquid soap film). Because of surface tension, a force F is required to pull the movable wire and thus increase the surface area of the liquid. The liquid contained by the wire apparatus is a thin film having both a top and a bottom surface. Hence the total length of the surface being increased is 2ℓ, and the surface tension is $\gamma = F/2\ell$. A delicate apparatus of this type can be used to measure the surface tension of various liquids. The surface tension of water is 0.072 N/m at 20°C. Table 10–4 (next page) gives the values for several substances. Note that temperature has a considerable effect on the surface tension.

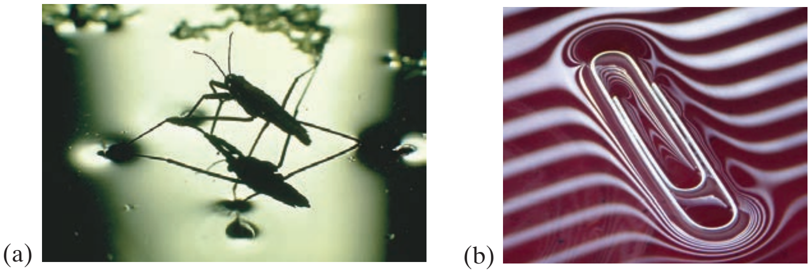


FIGURE 10–35 (a) Water strider. (b) Paper clip (light coming through window blinds).

Because of surface tension, some insects (Fig. 10–35a) can walk on water, and objects more dense than water, such as a paper clip (Fig. 10–35b), can float on the surface. Figure 10–36a shows how the surface tension can support the weight w of an object. Actually, the object sinks slightly into the fluid, so w is the "effective weight" of that object—its true weight less the buoyant force.

TABLE 10–4 Surface Tension of Some Substances

Substance	Surface Tension (N/m)
Mercury (20°C)	0.44
Blood, whole (37°C)	0.058
Blood, plasma (37°C)	0.073
Alcohol, ethyl (20°C)	0.023
Water (0°C)	0.076
(20°C)	0.072
(100°C)	0.059
Benzene (20°C)	0.029
Soap solution (20°C)	≈0.025
Oxygen (−193°C)	0.016

EXAMPLE 10–15 ESTIMATE **Insect walks on water.** The base of an insect's leg is approximately spherical in shape, with a radius of about 2.0×10^{-5} m. The 0.0030-g mass of the insect is supported equally by its six legs. Estimate the angle θ at which the surface tension force acts (see Fig. 10–36) for an insect on the surface of water. Assume the water temperature is 20°C.

APPROACH Since the insect is in equilibrium, the upward surface tension force is equal to the pull of gravity downward on each leg. We ignore buoyant forces for this estimate.

SOLUTION For each leg, we assume the surface tension force acts all around a circle of radius r, at an angle θ, as shown in Fig. 10–36a. Only the vertical component, $\gamma \cos\theta$, acts to balance the weight mg. We set the length ℓ in Eq. 10–10 equal to the circumference of the circle, $\ell \approx 2\pi r$. Then the net upward force due to surface tension is $F_y \approx (\gamma\cos\theta)\ell \approx 2\pi r\gamma\cos\theta$. We set this surface tension force equal to one-sixth the weight of the insect since it has six legs:

$$\begin{aligned} 2\pi r\gamma\cos\theta &\approx \tfrac{1}{6}mg \\ (6.28)(2.0\times10^{-5}\,\mathrm{m})(0.072\,\mathrm{N/m})\cos\theta &\approx \tfrac{1}{6}(3.0\times10^{-6}\,\mathrm{kg})(9.8\,\mathrm{m/s^2}) \\ \cos\theta &\approx 0.54. \end{aligned}$$

So $\theta \approx 57°$.

NOTE If $\cos\theta$ had come out greater than 1, the surface tension would not have been great enough to support the insect's weight. If the insect is very light, it will sink less into the water and θ (Fig. 10–36a) will be larger than calculated above.

NOTE Our estimate ignored the buoyant force and ignored any difference between the radius of the insect's "foot" and the radius of the surface depression.

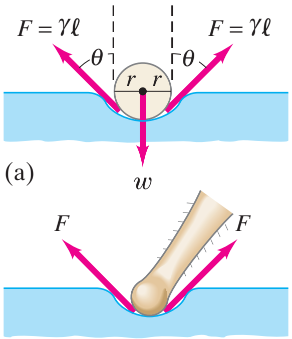


FIGURE 10–36 Surface tension acting on (a) a sphere, and (b) an insect leg. Example 10–15.

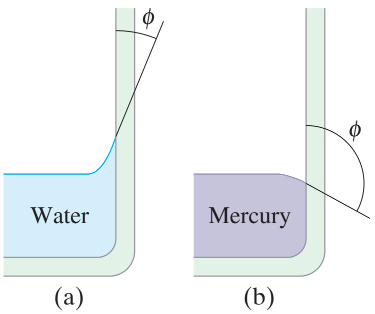
Soaps and detergents lower the surface tension of water. This is desirable for washing and cleaning since the high surface tension of pure water prevents it from penetrating easily between the fibers of material and into tiny crevices. Substances that reduce the surface tension of a liquid are called *surfactants*.

PHYSICS APPLIED
Soaps and detergents

*Capillarity

Surface tension plays a role in another interesting phenomenon, *capillarity*. It is a common observation that water in a glass container rises up slightly where it touches the glass, Fig. 10–37a. The water is said to "wet" the glass. Mercury, on the other hand, is depressed when it touches the glass, Fig. 10–37b; the mercury does not wet the glass. Whether a liquid wets a solid surface is determined by the relative strength of the cohesive forces between the molecules of the liquid compared to the adhesive forces between the molecules of the liquid and those of the container. **Cohesion** refers to the force between molecules of the same type, whereas **adhesion** refers to the force between molecules of different types. Water wets glass because the water molecules are more strongly attracted to the glass molecules than they are to other water molecules. The opposite is true for mercury: the cohesive forces are stronger than the adhesive forces.

FIGURE 10–37 (a) Water "wets" the surface of glass, whereas (b) mercury does not "wet" the glass.



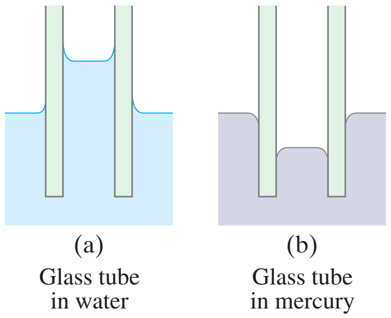


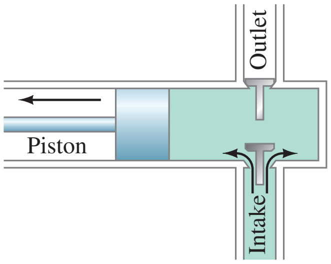
FIGURE 10–38 Capillarity.

In tubes having very small diameters, liquids are observed to rise or fall relative to the level of the surrounding liquid. This phenomenon is called **capillarity**, and such thin tubes are called **capillaries**. Whether the liquid rises or falls (Fig. 10–38) depends on the relative strengths of the adhesive and cohesive forces. Thus water rises in a glass tube, whereas mercury falls. The actual amount of rise (or fall) depends on the surface tension—which is what keeps the liquid surface from breaking apart.

*10–14 Pumps, and the Heart

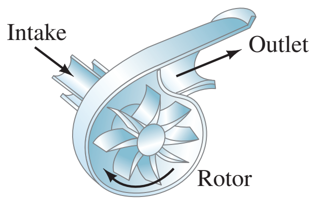
We conclude this Chapter with a brief discussion of pumps, including the heart. Pumps can be classified into categories according to their function. A *vacuum pump* is designed to reduce the pressure (usually of air) in a given vessel. A *force pump*, on the other hand, is a pump that is intended to increase the pressure—for example, to lift a liquid (such as water from a well) or to push a fluid through a pipe. Figure 10–39 illustrates the principle behind a simple reciprocating pump. It could be a vacuum pump, in which case the intake is connected to the vessel to be evacuated. A similar mechanism is used in some force pumps, and in this case the fluid is forced under increased pressure through the outlet.

FIGURE 10–39 One kind of pump (reciprocating type): the intake valve opens and air (or fluid that is being pumped) fills the empty space when the piston moves to the left. When the piston moves to the right (not shown), the outlet valve opens and fluid is forced out.



Another type of pump is the centrifugal pump, shown in Fig. 10–40. It, or any force pump, can be used as a *circulating pump*—that is, to circulate a fluid around a closed path, such as the cooling water or lubricating oil in an automobile.

FIGURE 10–40 Centrifugal pump: the rotating blades force fluid through the outlet pipe; this kind of pump is used in vacuum cleaners and as a water pump in automobiles.



PHYSICS APPLIED
Heart as a pump

The heart of a human (and of other animals as well) is essentially a circulating pump. The action of a human heart is shown in Fig. 10–41. There are actually two separate paths for blood flow. The longer path takes blood to the parts of the body, via the arteries, bringing oxygen to body tissues and picking up carbon dioxide, which it carries back to the heart via veins. This blood is then pumped to the lungs (the second path), where the carbon dioxide is released and oxygen is taken up. The oxygen-laden blood is returned to the heart, where it is again pumped to the tissues of the body.

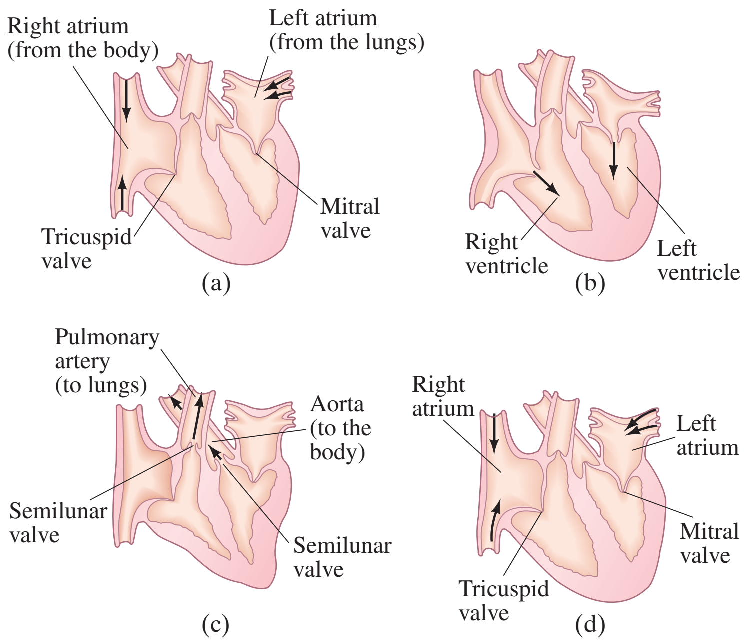


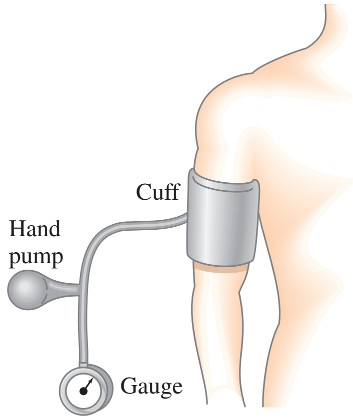
FIGURE 10–41 Pumping human heart. (a) In the diastole phase, the heart relaxes between beats. Blood moves into the heart; both atria fill rapidly. (b) When the atria contract, the systole or pumping phase begins. The contraction pushes the blood through the mitral and tricuspid valves into the ventricles. (c) The contraction of the ventricles forces the blood through the semilunar valves into the pulmonary artery, which leads to the lungs, and to the aorta (the body's largest artery), which leads to the arteries serving all the body. (d) When the heart relaxes, the semilunar valves close; blood fills the atria, beginning the cycle again.

Blood pressure is measured using one of the types of gauge mentioned earlier (Section 10–6), and it is usually calibrated in mm-Hg. The gauge is attached to a closed, air-filled cuff that is wrapped around the upper arm at the level of the heart, Fig. 10–42. Two values of blood pressure are measured: the maximum pressure when the heart is pumping, called *systolic pressure*; and the pressure when the heart is in the resting part of the cycle, called *diastolic pressure*. Initially, the air pressure in the cuff is increased high above the systolic pressure by a pump, compressing the main (brachial) artery in the arm and briefly cutting off the flow of blood. The air pressure is then reduced slowly until blood again begins to flow into the arm; it can be detected by listening with a stethoscope to the characteristic tapping sound† of the blood returning to the forearm. At this point, systolic pressure is just equal to the air pressure in the arm cuff which can be read off the gauge. The air pressure is subsequently reduced further, and the tapping sound disappears when blood at low pressure can enter the artery. At this point, the gauge indicates the diastolic pressure. Normal systolic pressure is around 120 mm-Hg, whereas normal diastolic pressure is around 70 or 80 mm-Hg. Blood pressure is reported in the form 120/70.

†When the blood starts flowing through the constriction caused by the tight cuff, its velocity is high and the flow is turbulent. It is the turbulence that causes the tapping sound.

PHYSICS APPLIED
Blood pressure

FIGURE 10–42 Device for measuring blood pressure.



Summary

The three common phases of matter are **solid**, **liquid**, and **gas**. Liquids and gases are collectively called **fluids**, meaning they have the ability to flow. The **density** of a material is defined as its mass per unit volume:

$$\rho = \frac{m}{V}. \tag{10–1}$$

Specific gravity (SG) is the ratio of the density of the material to the density of water (at 4°C).

Pressure is defined as force per unit area:

$$P = \frac{F}{A}. \tag{10–2}$$

The pressure P at a depth h in a liquid of constant density ρ, due to the weight of the liquid, is given by

$$P = \rho g h, \tag{10–3a}$$

where g is the acceleration due to gravity.

Pascal's principle says that an external pressure applied to a confined fluid is transmitted throughout the fluid.

Pressure is measured using a **manometer** or other type of gauge. A **barometer** is used to measure atmospheric pressure. Standard **atmospheric pressure** (average at sea level) is $1.013 \times 10^5\ \mathrm{N/m^2}$. **Gauge pressure** is the total (absolute) pressure minus atmospheric pressure.

Archimedes' principle states that an object submerged wholly or partially in a fluid is buoyed up by a force equal to the weight of fluid it displaces $(F_B = m_F g = \rho_F V_{displ}\, g)$.

Fluid flow can be characterized either as **streamline** (also called **laminar**), in which the layers of fluid move smoothly and regularly along paths called **streamlines**, or as **turbulent**, in which case the flow is not smooth and regular but is characterized by irregularly shaped whirlpools.

Fluid flow rate is the mass or volume of fluid that passes a given point per unit time. The **equation of continuity** states that for an incompressible fluid flowing in an enclosed tube, the product of the velocity of flow and the cross-sectional area of the tube remains constant:

$$Av = \text{constant}. \tag{10–4}$$

Bernoulli's principle tells us that where the velocity of a fluid is high, the pressure in it is low, and where the velocity is low, the pressure is high. For steady laminar flow of an incompressible and nonviscous fluid, **Bernoulli's equation**, which is based on the law of conservation of energy, is

$$P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 = P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1, \tag{10–5}$$

for two points along the flow.

[***Viscosity** refers to friction within a fluid and is essentially a frictional force between adjacent layers of fluid as they move past one another.]

[*Liquid surfaces hold together as if under tension (**surface tension**), allowing drops to form and objects like needles and insects to stay on the surface.]

Questions

1. If one material has a higher density than another, must the molecules of the first be heavier than those of the second? Explain.
2. Consider what happens when you push both a pin and the blunt end of a pen against your skin with the same force. Decide what determines whether your skin is cut—the net force applied to it or the pressure.
3. A small amount of water is boiled in a 1-gallon metal can. The can is removed from the heat and the lid put on. As the can cools, it collapses and looks crushed. Explain.
4. An ice cube floats in a glass of water filled to the brim. What can you say about the density of ice? As the ice melts, will the water overflow? Explain.
5. Will an ice cube float in a glass of alcohol? Why or why not?

6. A submerged can of Coke® will sink, but a can of Diet Coke® will float. (Try it!) Explain.
7. Why don't ships made of iron sink?
8. A barge filled high with sand approaches a low bridge over the river and cannot quite pass under it. Should sand be added to, or removed from, the barge? [*Hint*: Consider Archimedes' principle.]
9. Explain why helium weather balloons, which are used to measure atmospheric conditions at high altitudes, are normally released while filled to only 10–20% of their maximum volume.
10. Will an empty balloon have precisely the same apparent weight on a scale as a balloon filled with air? Explain.
11. Why do you float higher in salt water than in fresh water?
12. Why does the stream of water from a faucet become narrower as it falls (Fig. 10–43)?

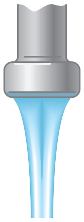


FIGURE 10–43 Question 12. Water coming from a faucet.

13. Children are told to avoid standing too close to a rapidly moving train because they might get sucked under it. Is this possible? Explain.
14. A tall Styrofoam cup is filled with water. Two holes are punched in the cup near the bottom, and water begins rushing out. If the cup is dropped so it falls freely, will the water continue to flow from the holes? Explain.
15. Why do airplanes normally take off into the wind?
16. Two ships moving in parallel paths close to one another risk colliding. Why?
17. If you dangle two pieces of paper vertically, a few inches apart (Fig. 10–44), and blow between them, how do you think the papers will move? Try it and see. Explain.

FIGURE 10–44 Question 17.

18. Why does the canvas top of a convertible bulge out when the car is traveling at high speed? [*Hint*: The windshield deflects air upward, pushing streamlines closer together.]
19. Roofs of houses are sometimes "blown" off (or are they pushed off?) during a tornado or hurricane. Explain using Bernoulli's principle.
20. Explain how the tube in Fig. 10–45, known as a **siphon**, can transfer liquid from one container to a lower one even though the liquid must flow uphill for part of its journey. (Note that the tube must be filled with liquid to start with.)

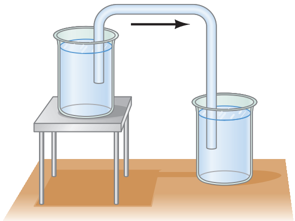


FIGURE 10–45 Question 20. A siphon.

*21. When blood pressure is measured, why must the arm cuff be held at the level of the heart?

MisConceptual Questions

1. You hold a piece of wood in one hand and a piece of iron in the other. Both pieces have the same volume, and you hold them fully under water at the same depth. At the moment you let go of them, which one experiences the greater buoyancy force?
(*a*) The piece of wood.
(*b*) The piece of iron.
(*c*) They experience the same buoyancy force.
(*d*) More information is needed.
2. Three containers are filled with water to the same height and have the same surface area at the base, but the total weight of water is different for each (Fig. 10–46). In which container does the water exert the greatest force on the bottom of the container?
(*a*) Container A.
(*b*) Container B.
(*c*) Container C.
(*d*) All three are equal.

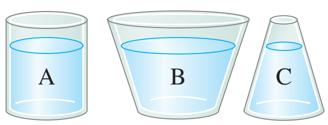


FIGURE 10–46 MisConceptual Question 2.

3. Beaker A is filled to the brim with water. Beaker B is the same size and contains a small block of wood which floats when the beaker is filled with water to the brim. Which beaker weighs more?
(*a*) Beaker A.
(*b*) Beaker B.
(*c*) The same for both.
4. Why does an ocean liner float?
(*a*) It is made of steel, which floats.
(*b*) Its very big size changes the way water supports it.
(*c*) It is held up in the water by large Styrofoam compartments.
(*d*) The average density of the ocean liner is less than that of seawater.
(*e*) Remember the *Titanic*—ocean liners do not float.
5. A rowboat floats in a swimming pool, and the level of the water at the edge of the pool is marked. Consider the following situations. (i) The boat is removed from the water. (ii) The boat in the water holds an iron anchor which is removed from the boat and placed on the shore. For each situation, the level of the water will
(*a*) rise. (*b*) fall. (*c*) stay the same.

6. You put two ice cubes in a glass and fill the glass to the rim with water. As the ice melts, the water level
   (*a*) drops below the rim.
   (*b*) rises and water spills out of the glass.
   (*c*) remains the same.
   (*d*) drops at first, then rises until a little water spills out.
7. Hot air is less dense than cold air. Could a hot-air balloon be flown on the Moon, where there is no atmosphere?
   (*a*) No, there is no cold air to displace, so no buoyancy force would exist.
   (*b*) Yes, warm air always rises, especially in a weak gravitational field like that of the Moon.
   (*c*) Yes, but the balloon would have to be filled with helium instead of hot air.
8. An object that can float in both water and in oil (whose density is less than that of water) experiences a buoyant force that is
   (*a*) greater when it is floating in oil than when floating in water.
   (*b*) greater when it is floating in water than when floating in oil.
   (*c*) the same when it is floating in water or in oil.
9. As water flows from a low elevation to a higher elevation through a pipe that changes in diameter,
   (*a*) the water pressure will increase.
   (*b*) the water pressure will decrease.
   (*c*) the water pressure will stay the same.
   (*d*) Need more information to determine how the water pressure changes.
10. Water flows in a horizontal pipe that is narrow but then widens and the speed of the water becomes less. The pressure in the water moving in the pipe is
    (*a*) greater in the wide part.
    (*b*) greater in the narrow part.
    (*c*) the same in both parts.
    (*d*) greater where the speed is higher.
    (*e*) greater where the speed is lower.
11. When a baseball curves to the right (a curveball), air is flowing
    (*a*) faster over the left side than over the right side.
    (*b*) faster over the right side than over the left side.
    (*c*) faster over the top than underneath.
    (*d*) at the same speed all around the baseball, but the ball curves as a result of the way the wind is blowing on the field.
12. How is the smoke drawn up a chimney affected when a wind is blowing outside?
    (*a*) Smoke rises more rapidly in the chimney.
    (*b*) Smoke rises more slowly in the chimney.
    (*c*) Smoke is forced back down the chimney.
    (*d*) Smoke is unaffected.

For assigned homework and other learning materials, go to the MasteringPhysics website.

Problems

10–2 Density and Specific Gravity

1. (I) The approximate volume of the granite monolith known as El Capitan in Yosemite National Park (Fig. 10–47) is about $10^8\ \text{m}^3$. What is its approximate mass?

FIGURE 10–47 Problem 1.

2. (I) What is the approximate mass of air in a living room $5.6\ \text{m} \times 3.6\ \text{m} \times 2.4\ \text{m}$?
3. (I) If you tried to smuggle gold bricks by filling your backpack, whose dimensions are $54\ \text{cm} \times 31\ \text{cm} \times 22\ \text{cm}$, what would its mass be?
4. (I) State your mass and then estimate your volume. [*Hint*: Because you can swim on or just under the surface of the water in a swimming pool, you have a pretty good idea of your density.]
5. (II) A bottle has a mass of 35.00 g when empty and 98.44 g when filled with water. When filled with another fluid, the mass is 89.22 g. What is the specific gravity of this other fluid?
6. (II) If 4.0 L of antifreeze solution (specific gravity = 0.80) is added to 5.0 L of water to make a 9.0-L mixture, what is the specific gravity of the mixture?
7. (III) The Earth is not a uniform sphere, but has regions of varying density. Consider a simple model of the Earth divided into three regions—inner core, outer core, and mantle. Each region is taken to have a unique constant density (the average density of that region in the real Earth):

Region	Radius (km)	Density (kg/m^3)
Inner Core	0–1220	13,000
Outer Core	1220–3480	11,100
Mantle	3480–6380	4400

(*a*) Use this model to predict the average density of the entire Earth. (*b*) If the radius of the Earth is 6380 km and its mass is 5.98×10^{24} kg, determine the actual average density of the Earth and compare it (as a percent difference) with the one you determined in (*a*).

10–3 to 10–6 Pressure; Pascal's Principle

8. (I) Estimate the pressure needed to raise a column of water to the same height as a 46-m-tall pine tree.

9. (I) Estimate the pressure exerted on a floor by (*a*) one pointed heel of area = 0.45 cm^2, and (*b*) one wide heel of area 16 cm^2, Fig. 10–48. The person wearing the shoes has a mass of 56 kg.

FIGURE 10–48 Problem 9.

10. (I) What is the difference in blood pressure (mm-Hg) between the top of the head and bottom of the feet of a 1.75-m-tall person standing vertically?

11. (I) (*a*) Calculate the total force of the atmosphere acting on the top of a table that measures 1.7 m $\times$ 2.6 m. (*b*) What is the total force acting upward on the underside of the table?

12. (II) How high would the level be in an alcohol barometer at normal atmospheric pressure?

13. (II) In a movie, Tarzan evades his captors by hiding under water for many minutes while breathing through a long, thin reed. Assuming the maximum pressure difference his lungs can manage and still breathe is −85 mm-Hg, calculate the deepest he could have been.

14. (II) The maximum gauge pressure in a hydraulic lift is 17.0 atm. What is the largest-size vehicle (kg) it can lift if the diameter of the output line is 25.5 cm?

15. (II) The gauge pressure in each of the four tires of an automobile is 240 kPa. If each tire has a "footprint" of 190 cm^2 (area touching the ground), estimate the mass of the car.

16. (II) (*a*) Determine the total force and the absolute pressure on the bottom of a swimming pool 28.0 m by 8.5 m whose uniform depth is 1.8 m. (*b*) What will be the pressure against the *side* of the pool near the bottom?

17. (II) A house at the bottom of a hill is fed by a full tank of water 6.0 m deep and connected to the house by a pipe that is 75 m long at an angle of 61° from the horizontal (Fig. 10–49). (*a*) Determine the water gauge pressure at the house. (*b*) How high could the water shoot if it came vertically out of a broken pipe in front of the house?

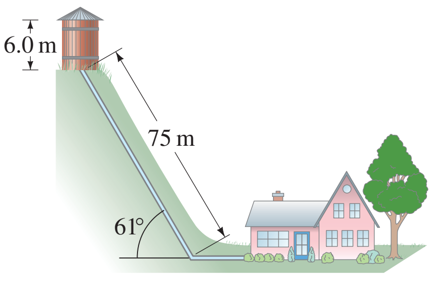


FIGURE 10–49 Problem 17.

18. (II) Water and then oil (which don't mix) are poured into a U-shaped tube, open at both ends. They come to equilibrium as shown in Fig. 10–50. What is the density of the oil? [*Hint*: Pressures at points a and b are equal. Why?]

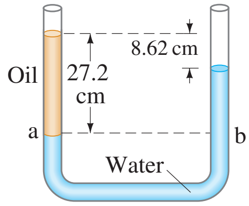


FIGURE 10–50 Problem 18.

19. (II) How high would the atmosphere extend if it were of uniform density throughout, equal to half the present density at sea level?

20. (II) Determine the minimum gauge pressure needed in the water pipe leading into a building if water is to come out of a faucet on the fourteenth floor, 44 m above that pipe.

21. (II) A **hydraulic press** for compacting powdered samples has a large cylinder which is 10.0 cm in diameter, and a small cylinder with a diameter of 2.0 cm (Fig. 10–51). A lever is attached to the small cylinder as shown. The sample, which is placed on the large cylinder, has an area of 4.0 cm^2. What is the pressure on the sample if 320 N is applied to the lever?

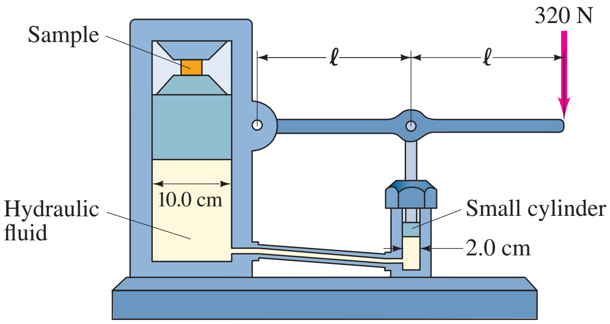


FIGURE 10–51 Problem 21.

22. (II) An open-tube mercury manometer is used to measure the pressure in an oxygen tank. When the atmospheric pressure is 1040 mbar, what is the absolute pressure (in Pa) in the tank if the height of the mercury in the open tube is (*a*) 18.5 cm higher, (*b*) 5.6 cm lower, than the mercury in the tube connected to the tank? See Fig. 10–7a.

10–7 Buoyancy and Archimedes' Principle

23. (II) What fraction of a piece of iron will be submerged when it floats in mercury?

24. (II) A geologist finds that a Moon rock whose mass is 9.28 kg has an apparent mass of 6.18 kg when submerged in water. What is the density of the rock?

25. (II) A crane lifts the 18,000-kg steel hull of a sunken ship out of the water. Determine (*a*) the tension in the crane's cable when the hull is fully submerged in the water, and (*b*) the tension when the hull is completely out of the water.

26. (II) A spherical balloon has a radius of 7.15 m and is filled with helium. How large a cargo can it lift, assuming that the skin and structure of the balloon have a mass of 930 kg? Neglect the buoyant force on the cargo volume itself.

27. (II) What is the likely identity of a metal (see Table 10–1) if a sample has a mass of 63.5 g when measured in air and an apparent mass of 55.4 g when submerged in water?

28. (II) Calculate the true mass (in vacuum) of a piece of aluminum whose apparent mass is 4.0000 kg when weighed in air.

29. (II) Because gasoline is less dense than water, drums containing gasoline will float in water. Suppose a 210-L steel drum is completely full of gasoline. What total volume of steel can be used in making the drum if the gasoline-filled drum is to float in fresh water?

30. (II) A scuba diver and her gear displace a volume of 69.6 L and have a total mass of 72.8 kg. (*a*) What is the buoyant force on the diver in seawater? (*b*) Will the diver sink or float?

31. (II) The specific gravity of ice is 0.917, whereas that of seawater is 1.025. What percent of an iceberg is above the surface of the water?

32. (II) Archimedes' principle can be used to determine the specific gravity of a solid using a known liquid (Example 10–8). The reverse can be done as well. (*a*) As an example, a 3.80-kg aluminum ball has an apparent mass of 2.10 kg when submerged in a particular liquid: calculate the density of the liquid. (*b*) Determine a formula for finding the density of a liquid using this procedure.

33. (II) A 32-kg child decides to make a raft out of empty 1.0-L soda bottles and duct tape. Neglecting the mass of the duct tape and plastic in the bottles, what minimum number of soda bottles will the child need to be able stay dry on the raft?

34. (II) A two-component model used to determine percent body fat in a human body assumes that a fraction $f\ (< 1)$ of the body's total mass m is composed of fat with a density of $0.90\ \mathrm{g/cm^3}$, and that the remaining mass of the body is composed of fat-free tissue with a density of $1.10\ \mathrm{g/cm^3}$. If the specific gravity of the entire body's density is X, show that the percent body fat $(= f \times 100)$ is given by

$$\%\ \text{Body fat} = \frac{495}{X} - 450.$$

35. (II) On dry land, an athlete weighs 70.2 kg. The same athlete, when submerged in a swimming pool and hanging from a scale, has an "apparent weight" of 3.4 kg. Using Example 10–8 as a guide, (*a*) find the total volume V of the submerged athlete. (*b*) Assume that when submerged, the athlete's body contains a residual volume $V_R = 1.3 \times 10^{-3}\ \mathrm{m^3}$ of air (mainly in the lungs). Taking $V - V_R$ to be the actual volume of the athlete's body, find the body's specific gravity, SG. (*c*) What is the athlete's percent body fat assuming it is given by the formula $(495/\mathrm{SG}) - 450$?

36. (III) A 3.65-kg block of wood $(\mathrm{SG} = 0.50)$ floats on water. What minimum mass of lead, hung from the wood by a string, will cause the block to sink?

10–8 to 10–10 Fluid Flow, Bernoulli's Equation

37. (I) A 12-cm-radius air duct is used to replenish the air of a room 8.2 m × 5.0 m × 3.5 m every 12 min. How fast does the air flow in the duct?

38. (I) Calculate the average speed of blood flow in the major arteries of the body, which have a total cross-sectional area of about $2.0\ \mathrm{cm^2}$. Use the data of Example 10–12.

39. (I) How fast does water flow from a hole at the bottom of a very wide, 4.7-m-deep storage tank filled with water? Ignore viscosity.

40. (I) Show that Bernoulli's equation reduces to the hydrostatic variation of pressure with depth (Eq. 10–3b) when there is no flow $(v_1 = v_2 = 0)$.

41. (II) What is the volume rate of flow of water from a 1.85-cm-diameter faucet if the pressure head is 12.0 m?

42. (II) A fish tank has dimensions 36 cm wide by 1.0 m long by 0.60 m high. If the filter should process all the water in the tank once every 3.0 h, what should the flow speed be in the 3.0-cm-diameter input tube for the filter?

43. (II) What gauge pressure in the water pipes is necessary if a fire hose is to spray water to a height of 16 m?

44. (II) A 180-km/h wind blowing over the flat roof of a house causes the roof to lift off the house. If the house is 6.2 m × 12.4 m in size, estimate the weight of the roof. Assume the roof is not nailed down.

45. (II) A 6.0-cm-diameter horizontal pipe gradually narrows to 4.5 cm. When water flows through this pipe at a certain rate, the gauge pressure in these two sections is 33.5 kPa and 22.6 kPa, respectively. What is the volume rate of flow?

46. (II) Estimate the air pressure inside a category 5 hurricane, where the wind speed is 300 km/h (Fig. 10–52).

FIGURE 10–52 Problem 46.

47. (II) What is the lift (in newtons) due to Bernoulli's principle on a wing of area 88 m^2 if the air passes over the top and bottom surfaces at speeds of 280 m/s and 150 m/s, respectively?

48. (II) Water at a gauge pressure of 3.8 atm at street level flows into an office building at a speed of 0.78 m/s through a pipe 5.0 cm in diameter. The pipe tapers down to 2.8 cm in diameter by the top floor, 16 m above (Fig. 10–53), where the faucet has been left open. Calculate the flow velocity and the gauge pressure in the pipe on the top floor. Assume no branch pipes and ignore viscosity.

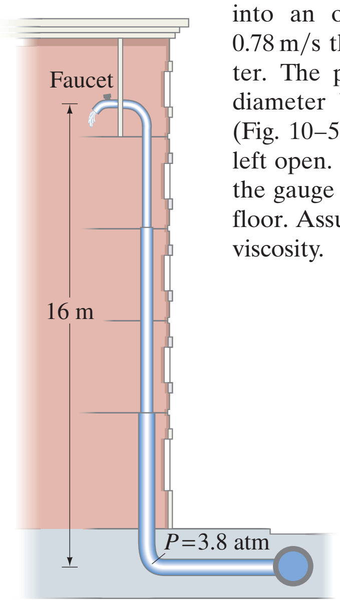


FIGURE 10–53 Problem 48.

49. (II) Show that the power needed to drive a fluid through a pipe with uniform cross-section is equal to the volume rate of flow, Q, times the pressure difference, $P_1 - P_2$. Ignore viscosity.

50. (III) (*a*) Show that the flow speed measured by a venturi meter (see Fig. 10–29) is given by the relation

$$v_1 = A_2\sqrt{\frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}}.$$

(*b*) A venturi meter is measuring the flow of water; it has a main diameter of 3.5 cm tapering down to a throat diameter of 1.0 cm. If the pressure difference is measured to be 18 mm-Hg, what is the speed of the water entering the venturi throat?

51. (III) A fire hose exerts a force on the person holding it. This is because the water accelerates as it goes from the hose through the nozzle. How much force is required to hold a 7.0-cm-diameter hose delivering 420 L/min through a 0.75-cm-diameter nozzle?

*10–11 Viscosity

***52.** (II) A viscometer consists of two concentric cylinders, 10.20 cm and 10.60 cm in diameter. A liquid fills the space between them to a depth of 12.0 cm. The outer cylinder is fixed, and a torque of 0.024 m·N keeps the inner cylinder turning at a steady rotational speed of 57 rev/min. What is the viscosity of the liquid?

*10–12 Flow in Tubes; Poiseuille's Equation

***53.** (I) Engine oil (assume SAE 10, Table 10–3) passes through a fine 1.80-mm-diameter tube that is 10.2 cm long. What pressure difference is needed to maintain a flow rate of 6.2 mL/min?

***54.** (I) A gardener feels it is taking too long to water a garden with a $\frac{3}{8}$-in.-diameter hose. By what factor will the time be cut using a $\frac{5}{8}$-in.-diameter hose instead? Assume nothing else is changed.

***55.** (II) What diameter must a 15.5-m-long air duct have if the ventilation and heating system is to replenish the air in a room 8.0 m × 14.0 m × 4.0 m every 15.0 min? Assume the pump can exert a gauge pressure of 0.710×10^{-3} atm.

***56.** (II) What must be the pressure difference between the two ends of a 1.6-km section of pipe, 29 cm in diameter, if it is to transport oil ($\rho = 950\ \mathrm{kg/m^3}$, $\eta = 0.20\ \mathrm{Pa\cdot s}$) at a rate of 650 cm^3/s?

***57.** (II) Poiseuille's equation does not hold if the flow velocity is high enough that turbulence sets in. The onset of turbulence occurs when the **Reynolds number**, *Re*, exceeds approximately 2000. *Re* is defined as

$$Re = \frac{2\bar{v}r\rho}{\eta},$$

where $\bar{v}$ is the average speed of the fluid, ρ is its density, η is its viscosity, and r is the radius of the tube in which the fluid is flowing. (*a*) Determine if blood flow through the aorta is laminar or turbulent when the average speed of blood in the aorta ($r = 0.80$ cm) during the resting part of the heart's cycle is about 35 cm/s. (*b*) During exercise, the blood-flow speed approximately doubles. Calculate the Reynolds number in this case, and determine if the flow is laminar or turbulent.

***58.** (II) Assuming a constant pressure gradient, if blood flow is reduced by 65%, by what factor is the radius of a blood vessel decreased?

***59.** (II) Calculate the pressure drop per cm along the aorta using the data of Example 10–12 and Table 10–3.

***60.** (III) A patient is to be given a blood transfusion. The blood is to flow through a tube from a raised bottle to a needle inserted in the vein (Fig. 10–54). The inside diameter of the 25-mm-long needle is 0.80 mm, and the required flow rate is 2.0 cm^3 of blood per minute. How high h should the bottle be placed above the needle? Obtain ρ and η from the Tables. Assume the blood pressure is 78 torr above atmospheric pressure.

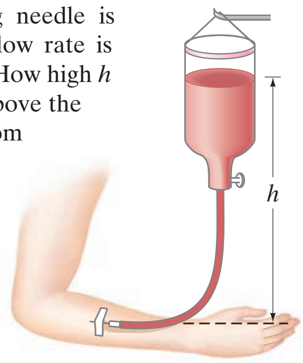


FIGURE 10–54 Problem 60.

*10–13 Surface Tension and Capillarity

*61. (I) If the force F needed to move the wire in Fig. 10–34 is 3.4×10^{-3} N, calculate the surface tension γ of the enclosed fluid. Assume $\ell = 0.070$ m.

*62. (I) Calculate the force needed to move the wire in Fig. 10–34 if it holds a soapy solution (Table 10–4) and the wire is 21.5 cm long.

*63. (II) The surface tension of a liquid can be determined by measuring the force F needed to just lift a circular platinum ring of radius r from the surface of the liquid. (*a*) Find a formula for γ in terms of F and r. (*b*) At 30°C, if $F = 6.20 \times 10^{-3}$ N and $r = 2.9$ cm, calculate γ for the tested liquid.

*64. (II) If the base of an insect's leg has a radius of about 3.0×10^{-5} m and the insect's mass is 0.016 g, would you expect the six-legged insect to remain on top of the water? Why or why not?

*65. (III) Estimate the diameter of a steel needle that can just barely remain on top of water due to surface tension.

*10–14 Pumps; the Heart

*66. (II) A physician judges the health of a heart by measuring the pressure with which it pumps blood. If the physician mistakenly attaches the pressurized cuff around a standing patient's calf (about 1 m below the heart) instead of the arm (Fig. 10–42), what error (in Pa) would be introduced in the heart's blood pressure measurement?

General Problems

67. A 3.2-N force is applied to the plunger of a hypodermic needle. If the diameter of the plunger is 1.3 cm and that of the needle is 0.20 mm, (*a*) with what force does the fluid leave the needle? (*b*) What force on the plunger would be needed to push fluid into a vein where the gauge pressure is 75 mm-Hg? Answer for the instant just before the fluid starts to move.

68. A beaker of water rests on an electronic balance that reads 975.0 g. A 2.6-cm-diameter solid copper ball attached to a string is submerged in the water, but does not touch the bottom. What are the tension in the string and the new balance reading?

69. Estimate the difference in air pressure between the top and the bottom of the Empire State Building in New York City. It is 380 m tall and is located at sea level. Express as a fraction of atmospheric pressure at sea level.

70. A hydraulic lift is used to jack a 960-kg car 42 cm off the floor. The diameter of the output piston is 18 cm, and the input force is 380 N. (*a*) What is the area of the input piston? (*b*) What is the work done in lifting the car 42 cm? (*c*) If the input piston moves 13 cm in each stroke, how high does the car move up for each stroke? (*d*) How many strokes are required to jack the car up 42 cm? (*e*) Show that energy is conserved.

71. When you ascend or descend a great deal when driving in a car, your ears "pop," which means that the pressure behind the eardrum is being equalized to that outside. If this did not happen, what would be the approximate force on an eardrum of area $0.20\ \text{cm}^2$ if a change in altitude of 1250 m takes place?

72. Giraffes are a wonder of cardiovascular engineering. Calculate the difference in pressure (in atmospheres) that the blood vessels in a giraffe's head must accommodate as the head is lowered from a full upright position to ground level for a drink. The height of an average giraffe is about 6 m.

73. How high should the pressure head be if water is to come from a faucet at a speed of 9.2 m/s? Ignore viscosity.

74. Suppose a person can reduce the pressure in his lungs to −75 mm-Hg gauge pressure. How high can water then be "sucked" up a straw?

75. A bicycle pump is used to inflate a tire. The initial tire (gauge) pressure is 210 kPa (30 psi). At the end of the pumping process, the final pressure is 310 kPa (45 psi). If the diameter of the plunger in the cylinder of the pump is 2.5 cm, what is the range of the force that needs to be applied to the pump handle from beginning to end?

76. Estimate the pressure on the mountains underneath the Antarctic ice sheet, which is typically 2 km thick.

77. A simple model (Fig. 10–55) considers a continent as a block $(\text{density} \approx 2800\ \text{kg/m}^3)$ floating in the mantle rock around it $(\text{density} \approx 3300\ \text{kg/m}^3)$. Assuming the continent is 35 km thick (the average thickness of the Earth's continental crust), estimate the height of the continent above the surrounding mantle rock.

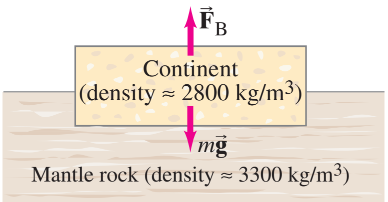


FIGURE 10–55 Problem 77.

78. A ship, carrying fresh water to a desert island in the Caribbean, has a horizontal cross-sectional area of $2240\ \text{m}^2$ at the waterline. When unloaded, the ship rises 8.25 m higher in the sea. How much water (m^3) was delivered?

79. During each heartbeat, approximately 70 cm^3 of blood is pushed from the heart at an average pressure of 105 mm-Hg. Calculate the power output of the heart, in watts, assuming 70 beats per minute.

80. Four lawn sprinkler heads are fed by a 1.9-cm-diameter pipe. The water comes out of the heads at an angle of 35° above the horizontal and covers a radius of 6.0 m. (*a*) What is the velocity of the water coming out of each sprinkler head? (Assume zero air resistance.) (*b*) If the output diameter of each head is 3.0 mm, how many liters of water do the four heads deliver per second? (*c*) How fast is the water flowing inside the 1.9-cm-diameter pipe?

81. The contraction of the left ventricle (chamber) of the heart pumps blood to the body. Assuming that the inner surface of the left ventricle has an area of 82 cm^2 and the maximum pressure in the blood is 120 mm-Hg, estimate the force exerted by that ventricle at maximum pressure.

82. An airplane has a mass of 1.7×10^6 kg, and the air flows past the lower surface of the wings at 95 m/s. If the wings have a surface area of 1200 m^2, how fast must the air flow over the upper surface of the wing if the plane is to stay in the air?

83. A hurricane-force wind of 180 km/h blows across the face of a storefront window. Estimate the force on the 2.0 m × 3.0 m window due to the difference in air pressure inside and outside the window. Assume the store is airtight so the inside pressure remains at 1.0 atm. (This is why you should not tightly seal a building in preparation for a hurricane.)

84. One arm of a U-shaped tube (open at both ends) contains water, and the other alcohol. If the two fluids meet at exactly the bottom of the U, and the alcohol is at a height of 16.0 cm, at what height will the water be?

85. Blood is placed in a bottle 1.40 m above a 3.8-cm-long needle, of inside diameter 0.40 mm, from which it flows at a rate of 4.1 cm^3/min. What is the viscosity of this blood?

86. You are watering your lawn with a hose when you put your finger over the hose opening to increase the distance the water reaches. If you are holding the hose horizontally, and the distance the water reaches increases by a factor of 4, what fraction of the hose opening did you block?

87. A copper (Cu) weight is placed on top of a 0.40-kg block of wood (density $= 0.60 \times 10^3$ kg/m^3) floating in water, as shown in Fig. 10–56. What is the mass of the copper if the top of the wood block is exactly at the water's surface?

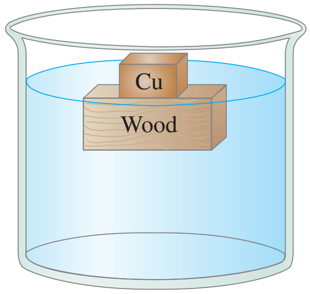


FIGURE 10–56 Problem 87.

*88. If cholesterol buildup reduces the diameter of an artery by 25%, by what % will the blood flow rate be reduced, assuming the same pressure difference?

Search and Learn

1. A 5.0-kg block and 4.0 kg of water in a 0.50-kg container are placed symmetrically on a board that can balance at the center (Fig. 10–57). A solid aluminum cube of sides 10.0 cm is lowered into the water. How much of the aluminum must be under water to make this system balance? How would your answer change for a lead cube of the same size? Explain. (See Sections 10–7 and 9–1.)

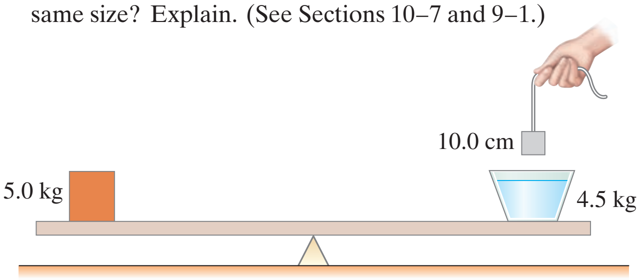


FIGURE 10–57 Search and Learn 1.

2. (*a*) Show that the buoyant force F_B on a partially submerged object such as a ship acts at the center of gravity of the fluid before it is displaced, Fig. 10–58. This point is called the **center of buoyancy**. (*b*) To ensure that a ship is in stable equilibrium, would it be better if its center of buoyancy was above, below, or at the same point as its center of gravity? Explain. (See Section 10–7 and Chapter 9.)

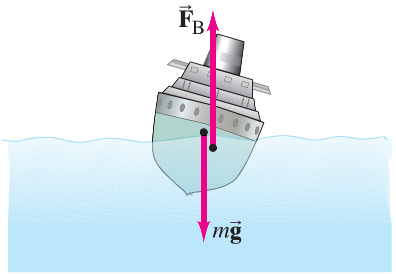


FIGURE 10–58 Search and Learn 2.

3. (*a*) When submerged in water, two objects with different volumes have the same *apparent* weight. When taken out of water, compare their weights in air. (*b*) Which object has the greater density?

4. What approximations are made in the derivation of Bernoulli's equation? Qualitatively, how do you think Bernoulli's equation would change if each of these approximations was not made? (See Sections 10–8, 10–9, 10–11, and 10–12.)

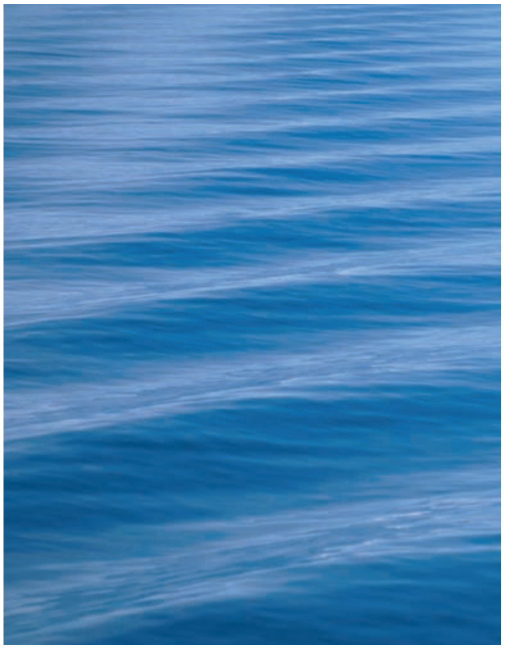
*5. Estimate the density of the water 5.4 km deep in the sea. (See Table 9–1 and Section 9–5 regarding bulk modulus.) By what fraction does it differ from the density at the surface?

ANSWERS TO EXERCISES

A: (*d*).
B: The same. Pressure depends on depth, not on length.
C: (*a*).
D: (*e*).
E: The rowboat is shaped to have a lot of empty, air-filled space, so its "average" density is much lower than that of water (unless the boat becomes full of water, in which case it sinks). Steel ships float for the same reason.
F: Increases.
G: (*b*).

An object attached to a coil spring can exhibit oscillatory motion. Many kinds of oscillatory motion are sinusoidal in time, or nearly so, and are referred to as simple harmonic motion. Real systems generally have at least some friction, causing the motion to be damped. The automobile spring shown here has a shock absorber (yellow) that purposefully dampens the oscillation to make for a smooth ride. When an external sinusoidal force is exerted on a system able to oscillate, resonance occurs if the driving force is at or near the natural frequency of oscillation.

Vibrations can give rise to waves—such as water waves or waves traveling along a cord—which travel outward from their source.

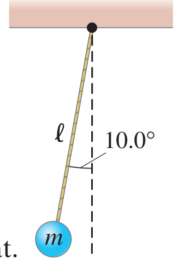


CHAPTER 11

Oscillations and Waves

CONTENTS

CHAPTER-OPENING QUESTIONS—Guess now!



1. A simple pendulum consists of a mass m (the "bob") hanging on the end of a thin string of length ℓ and negligible mass. The bob is pulled sideways so the string makes a 5.0° angle to the vertical; when released, it oscillates back and forth at a frequency f. If the pendulum is started at a 10.0° angle instead, its frequency would be

(a) twice as great.
(b) half as great.
(c) the same, or very close to it.
(d) not quite twice as great.
(e) a bit more than half as great.

2. You drop a rock into a pond, and water waves spread out in circles.

(a) The waves carry water outward, away from where the rock hit. That moving water carries energy outward.
(b) The waves only make the water move up and down. No energy is carried outward from where the rock hit.
(c) The waves only make the water move up and down, but the waves do carry energy outward, away from where the rock hit.

Many objects vibrate or oscillate—an object on the end of a spring, a tuning fork, the balance wheel of an old watch, a pendulum, a plastic ruler held firmly over the edge of a table and gently struck, the strings of a guitar or piano. Spiders detect prey by the vibrations of their webs; cars oscillate up and down when they hit a bump; buildings and bridges vibrate when heavy trucks pass or the wind is fierce. Indeed, because most solids are elastic (see Section 9–5), they vibrate (at least briefly) when given an impulse. Electrical oscillations occur in radio and television sets. At the atomic level, atoms oscillate within a molecule, and the atoms of a solid oscillate about their relatively fixed positions.

Because it is so common in everyday life and occurs in so many areas of physics, oscillatory (or vibrational) motion is of great importance. Mechanical oscillations or vibrations are fully described on the basis of Newtonian mechanics.

Vibrations and wave motion are intimately related. Waves—whether ocean waves, waves on a string, earthquake waves, or sound waves in air—have as their source a vibration. In the case of sound, not only is the source a vibrating object, but so is the detector—the eardrum or the membrane of a microphone. Indeed, when a wave travels through a medium, the medium oscillates (such as air for sound waves). In the second half of this Chapter, after we discuss oscillations, we will discuss simple waves such as those on water or on a string. In Chapter 12 we will study sound waves, and in later Chapters we will encounter other forms of wave motion, including electromagnetic waves and light.

11–1 Simple Harmonic Motion—Spring Oscillations

When an object **vibrates** or **oscillates** back and forth, over the same path, each oscillation taking the same amount of time, the motion is **periodic**. The simplest form of periodic motion is represented by an object oscillating on the end of a uniform coil spring. Because many other types of oscillatory motion closely resemble this system, we will look at it in detail. We assume that the mass of the spring can be ignored, and that the spring is mounted horizontally, as shown in Fig. 11–1a, so that the object of mass m slides without friction on the horizontal surface. Any spring has a natural length at which it exerts no force on the mass m. The position of the mass at this point is called the **equilibrium position**. If the mass is moved either to the left, which compresses the spring, or to the right, which stretches it, the spring exerts a force on the mass that acts in the direction of returning the mass to the equilibrium position; hence it is called a *restoring force*. We consider the common situation where we can assume the restoring force F is directly proportional to the displacement x the spring has been stretched (Fig. 11–1b) or compressed (Fig. 11–1c) from the equilibrium position:

$$F = -kx. \qquad \text{[force exerted by spring]} \qquad \textbf{(11–1)}$$

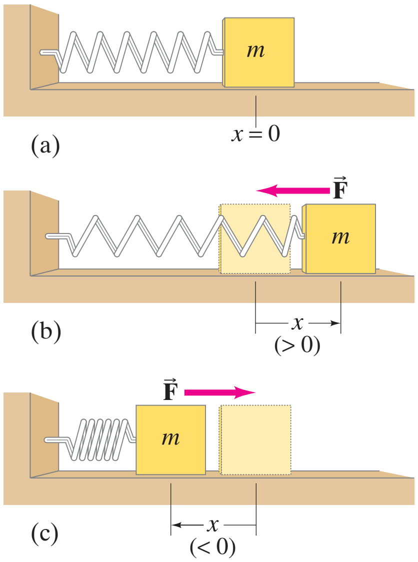


FIGURE 11–1 An object of mass m oscillating at the end of a uniform spring. The force $\vec{\mathbf{F}}$ on the object at the different positions is shown *above* the object.

Note that the equilibrium position has been chosen at $x = 0$ and the minus sign in Eq. 11–1 indicates that the restoring force is always in the direction opposite to the displacement x. For example, if we choose the positive direction to the right in Fig. 11–1, x is positive when the spring is stretched (Fig. 11–1b), but the direction of the restoring force is to the left (negative direction). If the spring is compressed, x is negative (to the left) but the force F acts toward the right (Fig. 11–1c).

Equation 11–1 is often referred to as Hooke's law (Sections 6–4 and 9–5), and is accurate only if the spring is not compressed to where the coils are close to touching, or stretched beyond the elastic region (see Fig. 9–19). Hooke's law works not only for springs but for other oscillating solids as well; it thus has wide applicability, even though it is valid only over a certain range of F and x values.

The proportionality constant k in Eq. 11–1 is called the *spring constant* for that particular spring, or its *spring stiffness constant* (units = N/m). To stretch the spring a distance x, an (external) force must be exerted on the free end of the spring with a magnitude at least equal to

$$F_{\text{ext}} = +kx. \qquad \text{[external force on spring]}$$

The greater the value of k, the greater the force needed to stretch a spring a given distance. That is, the stiffer the spring, the greater the spring constant k.

Note that the force F in Eq. 11–1 is *not* a constant, but varies with position. Therefore the acceleration of the mass m is not constant, so we *cannot* use the equations for constant acceleration developed in Chapter 2.

CAUTION
Eqs. 2–11 for constant acceleration do not apply to a spring

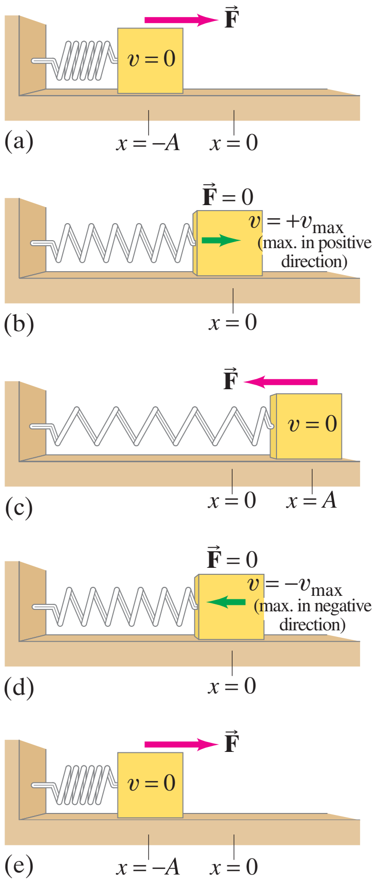


FIGURE 11–2 An object oscillating on a frictionless surface, indicating the force on the object and its velocity at different positions of its oscillation cycle.

Let us examine what happens when our uniform spring is initially compressed a distance $x = -A$, as shown in Fig. 11–2a, and then our object of mass m is released on the frictionless surface. The spring exerts a force on the mass that accelerates it toward the equilibrium position. Because the mass has inertia, it passes the equilibrium position with considerable speed. Indeed, as the mass reaches the equilibrium position, the force on it decreases to zero, but its speed at this point is a maximum, v_{max} (Fig. 11–2b). As the mass moves farther to the right, the force on it acts to slow it down, and it stops for an instant at $x = A$ (Fig. 11–2c). It then begins moving back in the opposite direction, accelerating until it passes the equilibrium point (Fig. 11–2d), and then slows down until it reaches zero speed at the original starting point, $x = -A$ (Fig. 11–2e). It then repeats the motion, moving back and forth symmetrically between $x = A$ and $x = -A$.

EXERCISE A A mass is oscillating on a frictionless surface at the end of a horizontal spring. Where, if anywhere, is the acceleration of the mass zero (see Fig. 11–2)? (*a*) At $x = -A$; (*b*) at $x = 0$; (*c*) at $x = +A$; (*d*) at both $x = -A$ and $x = +A$; (*e*) nowhere.

To discuss oscillatory motion, we need to define a few terms. The distance x of the mass from the equilibrium point at any moment is the **displacement** (with a + or − sign). The maximum displacement—the greatest distance from the equilibrium point—is called the **amplitude**, A. One **cycle** refers to the complete to-and-fro motion from some initial point back to that same point—say, from $x = -A$ to $x = +A$ and back to $x = -A$. The **period**, T, is defined as the time required to complete one cycle. Finally, the **frequency**, f, is the number of complete cycles per second. Frequency is generally specified in hertz (Hz), where $1\,\text{Hz} = 1$ cycle per second (s^{-1}). Given their definitions, frequency and period are inversely related, as we saw earlier (Eqs. 5–2 and 8–8):

$$f = \frac{1}{T} \quad \text{and} \quad T = \frac{1}{f}. \qquad \textbf{(11–2)}$$

For example, if the frequency is 2 cycles per second, then each cycle takes $\frac{1}{2}$ s.

EXERCISE B If an oscillating mass has a frequency of 1.25 Hz, it makes 100 oscillations in (*a*) 12.5 s, (*b*) 125 s, (*c*) 80 s, (*d*) 8.0 s.

CAUTION

For vertical spring, measure displacement (x or y) from the vertical equilibrium position

The oscillation of a spring hung vertically is similar to that of a horizontal spring; but because of gravity, the length of a vertical spring with a mass m on the end will be longer at equilibrium than when that same spring is horizontal. See Fig. 11–3. The spring is in equilibrium when $\Sigma F = 0 = mg - kx_0$, so the spring stretches an extra amount $x_0 = mg/k$ to be in equilibrium. If x is measured from this new equilibrium position, Eq. 11–1 can be used directly with the same value of k.

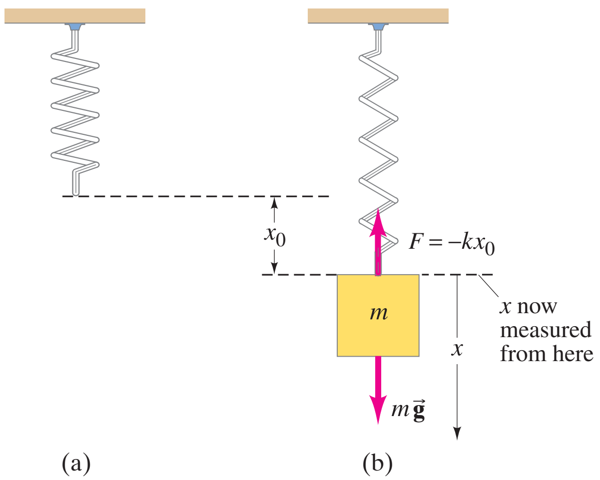


FIGURE 11–3
(a) Free spring, hung vertically.
(b) Mass m attached to spring in new equilibrium position, which occurs when $\Sigma F = 0 = mg - kx_0$.

EXAMPLE 11–1 **Car springs.** When a family of four with a total mass of 200 kg step into their 1200-kg car, the car's springs compress 3.0 cm. (*a*) What is the spring constant of the car's springs (Fig. 11–4), assuming they act as a single spring? (*b*) How far will the car lower if loaded with 300 kg rather than 200 kg?

APPROACH We use Hooke's law: the weight of the people, mg, causes a 3.0-cm displacement.

SOLUTION (*a*) The added force of $(200\,\mathrm{kg})(9.8\,\mathrm{m/s^2}) = 1960\,\mathrm{N}$ causes the springs to compress $3.0 \times 10^{-2}\,\mathrm{m}$. Therefore (Eq. 11–1), the spring constant is

$$k = \frac{F}{x} = \frac{1960\,\mathrm{N}}{3.0 \times 10^{-2}\,\mathrm{m}} = 6.5 \times 10^{4}\,\mathrm{N/m}.$$

(*b*) If the car is loaded with 300 kg, Hooke's law gives

$$x = \frac{F}{k} = \frac{(300\,\mathrm{kg})(9.8\,\mathrm{m/s^2})}{(6.5 \times 10^{4}\,\mathrm{N/m})} = 4.5 \times 10^{-2}\,\mathrm{m},$$

or 4.5 cm.

NOTE In (*b*), we could have obtained x without solving for k: since x is proportional to F, if 200 kg compresses the spring 3.0 cm, then 1.5 times the force will compress the spring 1.5 times as much, or 4.5 cm.

PHYSICS APPLIED
Car springs

FIGURE 11–4 Photo of a car's spring. (Also visible is the shock absorber, in blue—see Section 11–5.)

Any oscillating system for which the net restoring force is directly proportional to the negative of the displacement (as in Eq. 11–1, $F = -kx$) is said to exhibit **simple harmonic motion** (SHM).† Such a system is often called a **simple harmonic oscillator** (SHO). We saw in Section 9–5 that most solid materials stretch or compress according to Eq. 11–1 as long as the displacement is not too great. Because of this, many natural oscillations are simple harmonic, or sufficiently close to it that they can be treated using this SHM model.

CONCEPTUAL EXAMPLE 11–2 **Is the motion simple harmonic?** Which of the following forces would cause an object to move in simple harmonic motion? (*a*) $F = -0.5x^2$, (*b*) $F = -2.3y$, (*c*) $F = 8.6x$, (*d*) $F = -4\theta$?

RESPONSE Both (*b*) and (*d*) will give simple harmonic motion because they give the force as minus a constant times a displacement. The displacement need not be x, but the minus sign is required to restore the system to equilibrium, which is why (*c*) does not produce SHM.

11–2 Energy in Simple Harmonic Motion

With forces that are not constant, such as here with simple harmonic motion, it is often convenient and useful to use the energy approach, as we saw in Chapter 6.

To stretch or compress a spring, work has to be done. Hence potential energy is stored in a stretched or compressed spring. We have already seen in Section 6–4 that elastic potential energy is given by

$$\mathrm{PE} = \tfrac{1}{2}kx^2.$$

The total mechanical energy E is the sum of the kinetic and potential energies,

$$E = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2, \qquad \textbf{(11–3)}$$

where v is the speed of the mass m at a distance x from the equilibrium position.

†The word "harmonic" refers to the motion being sinusoidal, which we discuss in Section 11–3. It is "simple" when the motion is sinusoidal of a single frequency. This can happen only if friction or other forces are not acting.

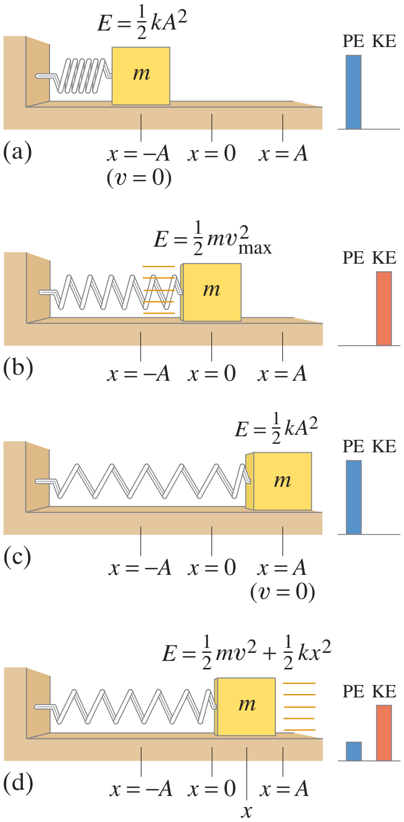


FIGURE 11–5 Energy changes from potential energy to kinetic energy and back again as the spring oscillates. Energy bar graphs (on the right) were used in Section 6–7.

SHM can occur only if friction is negligible so that the total mechanical energy E remains constant. As the mass oscillates back and forth, the energy continuously changes from potential energy to kinetic energy, and back again (Fig. 11–5). At the extreme points, $x = -A$ and $x = A$ (Fig. 11–5a, c), all the energy is stored in the spring as potential energy (and is the same whether the spring is compressed or stretched to the full amplitude). At these extreme points, the mass stops for an instant as it changes direction, so $v = 0$ and

$$E = \tfrac{1}{2}m(0)^2 + \tfrac{1}{2}kA^2 = \tfrac{1}{2}kA^2. \tag{11–4a}$$

Thus, the **total mechanical energy of a simple harmonic oscillator is proportional to the square of the amplitude**. At the equilibrium point, $x = 0$ (Fig. 11–5b), all the energy is kinetic:

$$E = \tfrac{1}{2}mv^2_{max} + \tfrac{1}{2}k(0)^2 = \tfrac{1}{2}mv^2_{max}, \tag{11–4b}$$

where v_{max} is the maximum speed during the motion (which occurs at $x = 0$). At intermediate points (Fig. 11–5d), the energy is part kinetic and part potential; because energy is conserved (we use Eqs. 11–3 and 11–4a),

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}kA^2. \tag{11–4c}$$

From this conservation of energy equation, we can obtain the velocity as a function of position. Solving for v^2, we have

$$v^2 = \frac{k}{m}\left(A^2 - x^2\right) = \frac{k}{m}A^2\left(1 - \frac{x^2}{A^2}\right).$$

From Eqs. 11–4a and 11–4b, we have $\frac{1}{2}mv^2_{max} = \frac{1}{2}kA^2$, so $v^2_{max} = (k/m)A^2$ or

$$v_{max} = \sqrt{\frac{k}{m}}\,A. \tag{11–5a}$$

Inserting this equation into the equation just above it and taking the square root, we have

$$v = \pm\, v_{max}\sqrt{1 - \frac{x^2}{A^2}}. \tag{11–5b}$$

This gives the velocity of the object at any position x. The object moves back and forth, so its velocity can be either in the + or − direction, but its magnitude depends only on its position x.

CONCEPTUAL EXAMPLE 11–3 **Doubling the amplitude.** Suppose the spring in Fig. 11–5 is stretched twice as far (to $x = 2A$). What happens to (*a*) the energy of the system, (*b*) the maximum velocity of the oscillating mass, (*c*) the maximum acceleration of the mass?

RESPONSE (*a*) From Eq. 11–4a, the total energy is proportional to the square of the amplitude A, so stretching it twice as far quadruples the energy $(2^2 = 4)$. You may protest, "I did work stretching the spring from $x = 0$ to $x = A$. Don't I do the same work stretching it from A to $2A$?" No. The force you exert is proportional to the displacement x, so for the second displacement, from $x = A$ to $2A$, you do more work than for the first displacement $(x = 0$ to $A)$.

(*b*) From Eq. 11–5a, we can see that when the amplitude is doubled, the maximum velocity must be doubled.

(*c*) Since the force is twice as great when we stretch the spring twice as far $(F = kx)$, the acceleration is also twice as great: $a \propto F \propto x$.

EXERCISE C Suppose the spring in Fig. 11–5 is compressed to $x = -A$, but is given a push to the right so that the initial speed of the mass m is v_0. What effect does this push have on (*a*) the energy of the system, (*b*) the maximum velocity, (*c*) the maximum acceleration?

EXAMPLE 11–4 **Spring calculations.** A spring stretches 0.150 m when a 0.300-kg mass is gently suspended from it as in Fig. 11–3b. The spring is then set up horizontally with the 0.300-kg mass resting on a frictionless table as in Fig. 11–5. The mass is pulled so that the spring is stretched 0.100 m from the equilibrium point, and released from rest. Determine: (*a*) the spring stiffness constant k; (*b*) the amplitude of the horizontal oscillation A; (*c*) the magnitude of the maximum velocity v_{max}; (*d*) the magnitude of the velocity v when the mass is 0.050 m from equilibrium; and (*e*) the magnitude of the maximum acceleration a_{max} of the mass.

APPROACH Wow, a lot of questions, but we can take them one by one. When the 0.300-kg mass hangs at rest from the spring as in Fig. 11–3b, we apply Newton's second law for the vertical forces: $\Sigma F = 0 = mg - kx_0$, so $k = mg/x_0$. For the horizontal oscillations, the amplitude is given, the velocities are found using conservation of energy, and the acceleration is found from $F = ma$.

SOLUTION (*a*) The spring stretches 0.150 m due to the 0.300-kg load, so

$$k = \frac{F}{x_0} = \frac{mg}{x_0} = \frac{(0.300\ \text{kg})(9.80\ \text{m/s}^2)}{0.150\ \text{m}} = 19.6\ \text{N/m}.$$

(*b*) The spring is now horizontal (on a table). It is stretched 0.100 m from equilibrium and is given no initial speed, so $A = 0.100$ m.

(*c*) The maximum velocity v_{max} is attained as the mass passes through the equilibrium point where all the energy is kinetic. By comparing the total energy (see Eq. 11–3) at equilibrium with that at full extension, conservation of energy tells us that

$$\tfrac{1}{2}mv_{max}^2 + 0 = 0 + \tfrac{1}{2}kA^2,$$

where $A = 0.100$ m. Solving for v_{max} (or using Eq. 11–5a), we have

$$v_{max} = A\sqrt{\frac{k}{m}} = (0.100\ \text{m})\sqrt{\frac{19.6\ \text{N/m}}{0.300\ \text{kg}}} = 0.808\ \text{m/s}.$$

(*d*) We use conservation of energy, or Eq. 11–5b derived from it, and find that

$$v = v_{max}\sqrt{1 - \frac{x^2}{A^2}} = (0.808\ \text{m/s})\sqrt{1 - \frac{(0.050\ \text{m})^2}{(0.100\ \text{m})^2}} = 0.700\ \text{m/s}.$$

(*e*) By Newton's second law, $F = ma$. So the maximum acceleration occurs where the force is greatest—that is, when $x = A = 0.100$ m. Thus

$$a_{max} = \frac{F_{max}}{m} = \frac{kA}{m} = \frac{(19.6\ \text{N/m})(0.100\ \text{m})}{0.300\ \text{kg}} = 6.53\ \text{m/s}^2.$$

NOTE We cannot use the kinematic equations, Eqs. 2–11, because the acceleration is not constant in SHM.

EXAMPLE 11–5 **Energy calculations.** For the simple harmonic oscillator of Example 11–4, determine (*a*) the total energy, and (*b*) the kinetic and potential energies at half amplitude ($x = \pm A/2$).

APPROACH We use conservation of energy for a mass–spring system, Eqs. 11–3 and 11–4.

SOLUTION (*a*) With $k = 19.6$ N/m and $A = 0.100$ m, the total energy E from Eq. 11–4a is

$$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}(19.6\ \text{N/m})(0.100\ \text{m})^2 = 9.80 \times 10^{-2}\ \text{J}.$$

(*b*) At $x = A/2 = 0.050$ m, we have

$$\text{PE} = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(19.6\ \text{N/m})(0.050\ \text{m})^2 = 2.45 \times 10^{-2}\ \text{J}.$$

By conservation of energy, the kinetic energy must be

$$\text{KE} = E - \text{PE} = 7.35 \times 10^{-2}\ \text{J}.$$

11–3 The Period and Sinusoidal Nature of SHM

The period of a simple harmonic oscillator is found to depend on the stiffness of the spring and also on the mass m that is oscillating. But—strange as it may seem—the *period does not depend on the amplitude*. You can find this out for yourself by using a watch and timing 10 or 20 cycles of an oscillating spring for a small amplitude and then for a large amplitude.

The period T is given by (see derivation on next page):

$$T = 2\pi\sqrt{\frac{m}{k}}. \tag{11–6a}$$

We see that the larger the mass, the longer the period; and the stiffer the spring (larger k), the shorter the period. This makes sense since a larger mass means more inertia and therefore slower response (smaller acceleration). And larger k means greater force and therefore quicker response (larger acceleration). Notice that Eq. 11–6a is not a direct proportion: the period varies as the *square root* of m/k. For example, the mass must be quadrupled to double the period. Equation 11–6a is fully in accord with experiment and is valid not only for a spring, but for all kinds of simple harmonic motion—that is, for motion subject to a restoring force proportional to displacement, Eq. 11–1.

We can write the frequency using $f = 1/T$ (Eq. 11–2):

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}. \tag{11–6b}$$

EXERCISE D By how much should the mass on the end of a spring be changed to halve the frequency of its oscillations? (*a*) No change; (*b*) doubled; (*c*) quadrupled; (*d*) halved; (*e*) quartered.

FIGURE 11–6 Example 11–6. A spider waits for its prey (on the left).

EXAMPLE 11–6 ESTIMATE **Spider web.** A spider of mass 0.30 g waits in its web of negligible mass (Fig. 11–6). A slight movement causes the web to vibrate with a frequency of about 15 Hz. (*a*) Estimate the value of the spring stiffness constant k for the web. (*b*) At what frequency would you expect the web to vibrate if an insect of mass 0.10 g were trapped in addition to the spider?

APPROACH We can only make a rough estimate because a spider's web is fairly complicated and may vibrate with a mixture of frequencies. We use SHM as an approximate model.

SOLUTION (*a*) The frequency of SHM is given by Eq. 11–6b,

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}.$$

We solve for k:

$$\begin{aligned} k &= (2\pi f)^2 m \\ &= (2\pi)^2\left(15\,\mathrm{s}^{-1}\right)^2\left(3.0\times10^{-4}\,\mathrm{kg}\right) = 2.7\,\mathrm{N/m}. \end{aligned}$$

(*b*) The total mass is now $0.10\,\mathrm{g} + 0.30\,\mathrm{g} = 4.0\times10^{-4}\,\mathrm{kg}$. We could substitute $m = 4.0\times10^{-4}\,\mathrm{kg}$ into Eq. 11–6b. Instead, we notice that the frequency decreases with the square root of the mass. Since the new mass is 4/3 times the first mass, the frequency changes by a factor of $1/\sqrt{4/3} = \sqrt{3/4}$. Thus $f = (15\,\mathrm{Hz})\left(\sqrt{3/4}\right) = 13\,\mathrm{Hz}$.

NOTE Check this result by direct substitution of k, found in part (*a*), and the new mass m into Eq. 11–6b.

EXAMPLE 11–7 **ESTIMATE** **A vibrating floor.** A large motor in a factory causes the floor to vibrate up and down at a frequency of 10 Hz. The amplitude of the floor's motion near the motor is about 3.0 mm. Estimate the maximum acceleration of the floor near the motor.

APPROACH Assuming the motion of the floor is roughly SHM, we can make an estimate for the maximum acceleration using $F = ma$ and Eq. 11–6b.

SOLUTION The maximum acceleration occurs when the force $(F = kx)$ is largest, which is when $x = A$. Thus, $a_{max} = F_{max}/m = kA/m = (k/m)A$. From Eq. 11–6b, $(k/m) = (2\pi f)^2$, so

$$a_{max} = \frac{F_{max}}{m} = \left(\frac{k}{m}\right)A = (2\pi f)^2 A = (2\pi)^2\left(10\,\mathrm{s}^{-1}\right)^2\left(3.0 \times 10^{-3}\,\mathrm{m}\right) = 12\,\mathrm{m/s^2}.$$

NOTE The maximum acceleration is a little over g, so when the floor accelerates down, objects sitting on the floor will actually lose contact with the floor momentarily, which will cause noise and serious wear.

PHYSICS APPLIED
Unwanted floor vibrations

Period and Frequency—Derivation

We can derive a formula for the period of simple harmonic motion (SHM) by comparing SHM to an object rotating uniformly in a circle. From this same "reference circle" we can obtain a second useful result—a formula for the position of an oscillating mass as a function of time. There is nothing actually rotating in a circle when a spring oscillates linearly, but it is the mathematical similarity that we find useful.

Consider a small object of mass m revolving counterclockwise in a circle of radius A, with constant speed v_{max}, on top of a table as shown in Fig. 11–7. As viewed from above, the motion is a circle in the xy plane. But a person who looks at the motion from the edge of the table sees an oscillatory motion back and forth, and this one-dimensional motion corresponds precisely to simple harmonic motion, as we shall now see.

What the person sees, and what we are interested in, is the projection of the circular motion onto the x axis (Fig. 11–7b). To see that this x motion is analogous to SHM, let us calculate the magnitude of the x component of the velocity v_{max}, which is labeled v in Fig. 11–7. The two triangles involving θ in Fig. 11–7a are similar, so

$$\frac{v}{v_{max}} = \frac{\sqrt{A^2 - x^2}}{A}$$

or

$$v = v_{max}\sqrt{1 - \frac{x^2}{A^2}}.$$

This is exactly the equation for the speed of a mass oscillating with SHM, as we saw in Eq. 11–5b. Thus the projection on the x axis of an object revolving in a circle has the same motion as a mass undergoing SHM.

We can now determine the period of SHM because it is equal to the time for our object revolving in a circle to make one complete revolution. First we note that the velocity v_{max} is equal to the circumference of the circle (distance) divided by the period T:

$$v_{max} = \frac{2\pi A}{T} = 2\pi A f. \qquad \textbf{(11–7)}$$

We solve for the period T in terms of A:

$$T = \frac{2\pi A}{v_{max}}.$$

From Eq. 11–5a, $A/v_{max} = \sqrt{m/k}$. Thus

$$T = 2\pi\sqrt{\frac{m}{k}},$$

which is Eq. 11–6a, the formula we were looking for. The period depends on the mass m and the spring stiffness constant k, but not on the amplitude A.

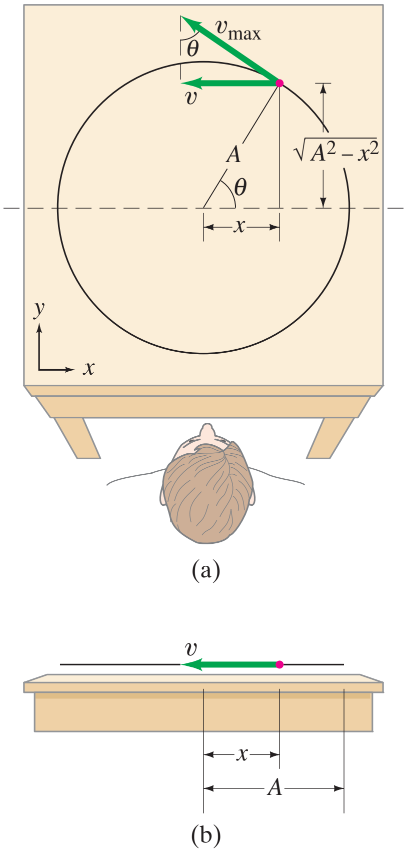


FIGURE 11–7 (a) Circular motion of a small (red) object. (b) Side view of circular motion (x component) is simple harmonic motion.

Position as a Function of Time

We now use the reference circle to find the position of a mass undergoing simple harmonic motion as a function of time. From Fig. 11–7, we see that $\cos\theta = x/A$, so the projection of the object's position on the x axis is

$$x = A\cos\theta.$$

The mass in the reference circle (Fig. 11–7) is rotating with uniform angular velocity ω. We then can write $\theta = \omega t$, where θ is in radians (Section 8–1). Thus

$$x = A\cos\omega t. \tag{11–8a}$$

Furthermore, since the angular velocity ω (specified in radians per second) can be written as $\omega = 2\pi f$, where f is the frequency (Eq. 8–7), we then write

$$x = A\cos(2\pi f t), \tag{11–8b}$$

or in terms of the period T,

$$x = A\cos(2\pi t/T). \tag{11–8c}$$

CAUTION
t is a variable (time); T is a constant for a given situation

Notice in Eq. 11–8c that when $t = T$ (that is, after a time equal to one period), we have the cosine of 2π (or 360°), which is the same as the cosine of zero. This makes sense since the motion repeats itself after a time $t = T$.

Because the cosine function varies between 1 and -1, Eqs. 11–8 tell us that x varies between A and $-A$, as it must. If a pen is attached to a vibrating mass as a sheet of paper is moved at a steady rate beneath it (Fig. 11–8), a sinusoidal curve will be drawn that accurately follows Eqs. 11–8.

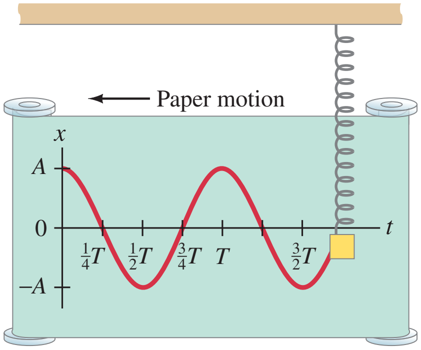


FIGURE 11–8 Position as a function of time for a simple harmonic oscillator, $x = A\cos(2\pi t/T)$.

EXAMPLE 11–8 Starting with $x = A\cos\omega t$. The displacement of an object is described by the following equation, where x is in meters and t is in seconds:

$$x = (0.30\text{ m})\cos(8.0\,t).$$

Determine the oscillating object's (*a*) amplitude, (*b*) frequency, (*c*) period, (*d*) maximum speed, and (*e*) maximum acceleration.

APPROACH We start by comparing the given equation for x with Eq. 11–8b, $x = A\cos(2\pi f t)$.

SOLUTION From $x = A\cos(2\pi f t)$, we see by inspection that (*a*) the amplitude $A = 0.30$ m, and (*b*) $2\pi f = 8.0\text{ s}^{-1}$; so $f = (8.0\text{ s}^{-1}/2\pi) = 1.27$ Hz. (*c*) Then $T = 1/f = 0.79$ s. (*d*) The maximum speed (see Eq. 11–7) is

$$\begin{aligned} v_{\max} &= 2\pi A f \\ &= (2\pi)(0.30\text{ m})(1.27\text{ s}^{-1}) = 2.4\text{ m/s}. \end{aligned}$$

(*e*) The maximum acceleration, by Newton's second law, is $a_{\max} = F_{\max}/m = kA/m$, because $F\,(= kx)$ is greatest when x is greatest. From Eq. 11–6b we see that $k/m = (2\pi f)^2$. Hence

$$\begin{aligned} a_{\max} &= \frac{k}{m}A = (2\pi f)^2 A \\ &= (2\pi)^2(1.27\text{ s}^{-1})^2(0.30\text{ m}) = 19\text{ m/s}^2. \end{aligned}$$

Sinusoidal Motion

Equation 11–8a, $x = A\cos\omega t$, assumes that the oscillating object starts from rest $(v = 0)$ at its maximum displacement $(x = A)$ at $t = 0$. Other equations for SHM are also possible, depending on the initial conditions (when you choose t to be zero).

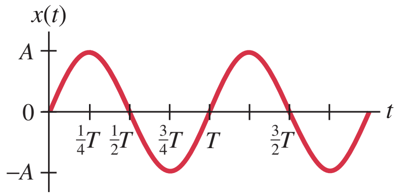


FIGURE 11–9 Sinusoidal nature of SHM, position as a function of time. In this case, $x = A\sin(2\pi t/T)$ because at $t = 0$ the mass is at the equilibrium position $x = 0$ and has (or is given) an initial speed at $t = 0$ that carries it to $x = A$ at $t = \frac{1}{4}T$.

For example, if at $t = 0$ the object is at the equilibrium position and the oscillations are begun by giving the object a push to the right $(+x)$, the equation would be

$$x = A\sin\omega t = A\sin(2\pi t/T).$$

This curve, shown in Fig. 11–9, has the same shape as the cosine curve shown in Fig. 11–8, except it is shifted to the right by a quarter cycle. Hence at $t = 0$ it starts out at $x = 0$ instead of at $x = A$.

Both sine and cosine curves are referred to as being **sinusoidal** (having the shape of a sine function). Thus simple harmonic motion† is said to be sinusoidal because the position varies as a sinusoidal function of time.

*Velocity and Acceleration as Functions of Time

Figure 11–10a, like Fig. 11–8, shows a graph of displacement x vs. time t, as given by Eqs. 11–8. We can also find the velocity v as a function of time from Fig. 11–7a. For the position shown (red dot in Fig. 11–7a), the magnitude of v is $v_{max}\sin\theta$, but $\vec{\mathbf{v}}$ points to the left, so $v = -v_{max}\sin\theta$. Again setting $\theta = \omega t = 2\pi ft = 2\pi t/T$, we have

$$v = -v_{max}\sin\omega t = -v_{max}\sin(2\pi ft) = -v_{max}\sin(2\pi t/T). \quad \textbf{(11–9)}$$

Just after $t = 0$, the velocity is negative (points to the left) and remains so until $t = \frac{1}{2}T$ (corresponding to $\theta = 180° = \pi$ radians). After $t = \frac{1}{2}T$ until $t = T$ the velocity is positive. The velocity as a function of time (Eq. 11–9) is plotted in Fig. 11–10b. From Eqs. 11–6b and 11–7,

$$v_{max} = 2\pi Af = A\sqrt{\frac{k}{m}}.$$

For a given spring–mass system, the maximum speed v_{max} is higher if the amplitude is larger, and always occurs as the mass passes the equilibrium point.

Newton's second law and Eqs. 11–8 give us the acceleration as a function of time:

$$a = \frac{F}{m} = \frac{-kx}{m} = -\left(\frac{kA}{m}\right)\cos\omega t = -a_{max}\cos(2\pi t/T) \quad \textbf{(11–10)}$$

where the maximum acceleration is

$$a_{max} = kA/m.$$

Equation 11–10 is plotted in Fig. 11–10c. Because the acceleration of a SHO is *not* constant, the equations for uniformly accelerated motion do *not* apply to SHM.

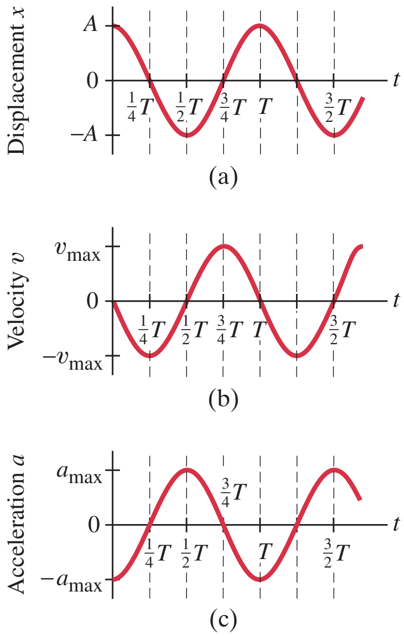
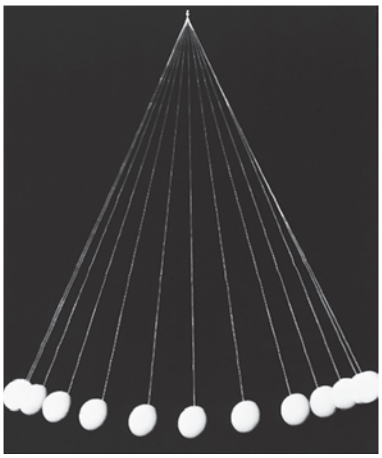


FIGURE 11–10 Graphs showing (a) displacement x as a function of time t: $x = A\cos(2\pi t/T)$; (b) velocity as a function of time: $v = -v_{max}\sin(2\pi t/T)$, where $v_{max} = A\sqrt{k/m}$; (c) acceleration as a function of time: $a = -a_{max}\cos(2\pi t/T)$, where $a_{max} = Ak/m$.

11–4 The Simple Pendulum

A **simple pendulum** consists of a small object (the pendulum bob) suspended from the end of a lightweight cord, Fig. 11–11. We assume that the cord does not stretch and that its mass can be ignored relative to that of the bob. The motion of a simple pendulum moving back and forth with negligible friction resembles simple harmonic motion: the pendulum bob oscillates along the arc of a circle with equal amplitude on either side of its equilibrium point, and as it passes through the equilibrium point (where it would hang vertically) it has its maximum speed. But is it really undergoing SHM? That is, is the restoring force proportional to its displacement? Let us find out.

FIGURE 11–11 Strobe-light photo of an oscillating pendulum, at equal time intervals.



†Simple harmonic motion can be *defined* as motion that is sinusoidal. This definition is fully consistent with our earlier definition in Section 11–1.

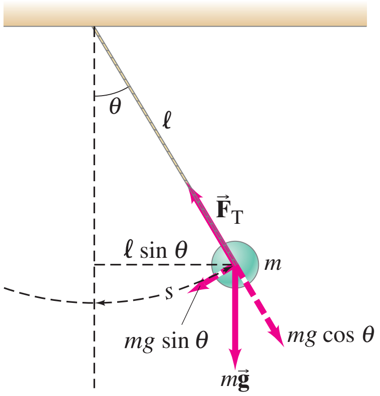


FIGURE 11–12 Simple pendulum, and a free-body diagram.

The displacement s of the pendulum along the arc is given by $s = \ell\theta$, where θ is the angle (in radians) that the cord makes with the vertical and ℓ is the length of the cord (Fig. 11–12). If the restoring force is proportional to s or to θ, the motion will be simple harmonic. The restoring force is the net force on the bob, which equals the component of the weight (mg) tangent to the arc:

$$F = -mg\sin\theta,$$

where g is the acceleration due to gravity. The minus sign here, as in Eq. 11–1, means the force is in the direction opposite to the angular displacement θ. Since F is proportional to the sine of θ and not to θ itself, the motion is *not* SHM. However, if θ is small, then $\sin\theta$ is very nearly equal to θ when the angle is specified in radians. This can be seen by noting in Fig. 11–12 that the arc length s $(= \ell\theta)$ is nearly the same length as the chord $(= \ell\sin\theta)$ indicated by the horizontal straight dashed line, *if θ is small*. For angles less than 15°, the difference between θ (in radians) and $\sin\theta$ is less than 1%—see Table 11–1. Thus, to a very good approximation for small angles,

$$F = -mg\sin\theta \approx -mg\theta.$$

Substituting $s = \ell\theta$, or $\theta = s/\ell$, we have

$$F \approx -\frac{mg}{\ell}s.$$

TABLE 11–1
Sin θ at Small Angles

θ (degrees)	θ (radians)	sin θ	% Difference
0	0	0	0
1°	0.01745	0.01745	0.005%
5°	0.08727	0.08716	0.1%
10°	0.17453	0.17365	0.5%
15°	0.26180	0.25882	1.1%
20°	0.34907	0.34202	2.0%
30°	0.52360	0.50000	4.5%

Thus, for small displacements, the motion can be modeled as being approximately simple harmonic, because this approximate equation fits Hooke's law, $F = -kx$, where in place of x we have arc length s. The effective force constant is $k = mg/\ell$. If we substitute $k = mg/\ell$ into Eq. 11–6a, we obtain the period of a simple pendulum:

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{mg/\ell}}$$

or

$$T = 2\pi\sqrt{\frac{\ell}{g}}. \qquad [\theta \text{ small}] \quad \textbf{(11–11a)}$$

The frequency is $f = 1/T$, so

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}. \qquad [\theta \text{ small}] \quad \textbf{(11–11b)}$$

The mass m of the pendulum bob does not appear in these formulas for T and f. Thus we have the surprising result that the period and frequency of a simple pendulum do not depend on the mass of the pendulum bob. You may have noticed this if you pushed a small child and then a large one on the same swing.

We also see from Eq. 11–11a that the period of a pendulum does not depend on the amplitude (like any SHM, Section 11–3), as long as the amplitude θ is small. Galileo is said to have first noted this fact while watching a swinging lamp in the cathedral at Pisa (Fig. 11–13). This discovery led to the invention of the pendulum clock, the first really precise timepiece, which became the standard for centuries.

FIGURE 11–13 The swinging motion of this elaborate lamp, hanging by a very long cord from the ceiling of the cathedral at Pisa, is said to have been observed by Galileo and to have inspired him to the conclusion that the period of a pendulum does not depend on amplitude.

EXERCISE E Return to Chapter-Opening Question 1, page 292, and answer it again now. Try to explain why you may have answered differently the first time.

EXERCISE F If a simple pendulum is taken from sea level to the top of a high mountain and started at the same angle of 5°, it would oscillate at the top of the mountain (*a*) slightly slower; (*b*) slightly faster; (*c*) at exactly the same frequency; (*d*) not at all—it would stop; (*e*) none of these.

PHYSICS APPLIED
Pendulum clock

Because a pendulum does not undergo *precisely* SHM, the period does depend slightly on the amplitude—the more so for large amplitudes. The accuracy of a pendulum clock would be affected, after many swings, by the decrease in amplitude due to friction. But the mainspring in a pendulum clock (or the falling weight in a grandfather clock) supplies energy to compensate for the friction and to maintain the amplitude constant, so that the timing remains precise.

EXAMPLE 11–9 **Measuring g.** A geologist uses a simple pendulum that has a length of 37.10 cm and a frequency of 0.8190 Hz at a particular location on the Earth. What is the acceleration due to gravity at this location?

APPROACH We can use the length ℓ and frequency f of the pendulum in Eq. 11–11b, which contains our unknown, g.

SOLUTION We solve Eq. 11–11b for g and obtain

$$g = (2\pi f)^2\ell = (2\pi)^2(0.8190\ \mathrm{s^{-1}})^2(0.3710\ \mathrm{m}) = 9.824\ \mathrm{m/s^2}.$$

11–5 Damped Harmonic Motion

The amplitude of any real oscillating spring or swinging pendulum slowly decreases in time until the oscillations stop altogether. Figure 11–14 shows a typical graph of the displacement as a function of time. This is called **damped harmonic motion**. The damping† is generally due to the resistance of air and to internal friction within the oscillating system. The energy that is dissipated to thermal energy results in a decreased amplitude of oscillation.

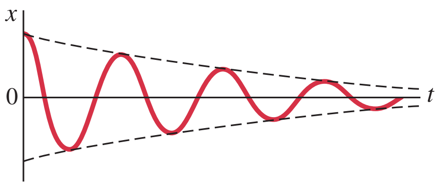
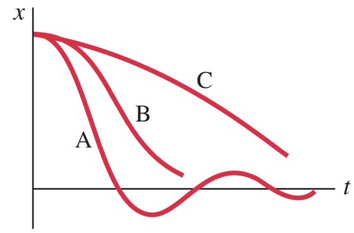


FIGURE 11–14 Damped harmonic motion.

Since natural oscillating systems are damped in general, why do we even talk about (undamped) simple harmonic motion? The answer is that SHM is much easier to deal with mathematically. And if the damping is not large, the oscillations can be thought of as simple harmonic motion on which the damping is superposed, as represented by the dashed curves in Fig. 11–14. Although damping does alter the frequency of vibration, the effect can be small if the damping is small; then Eqs. 11–6 can still be useful approximations.

Sometimes the damping is so large, however, that the motion no longer resembles simple harmonic motion. Three common cases of *heavily damped* systems are shown in Fig. 11–15. Curve A represents an **underdamped** situation, in which the system makes several oscillations before coming to rest; it corresponds to a more heavily damped version of Fig. 11–14. Curve C represents the **overdamped** situation, when the damping is so large that there is no oscillation and the system takes a long time to come to rest (equilibrium). Curve B represents **critical damping**: in this case the displacement reaches zero in the shortest time. These terms all derive from the use of practical damped systems such as door-closing mechanisms and **shock absorbers** in a car (Fig. 11–16), which are usually designed to give critical damping. But as they wear out, underdamping occurs: the door of a room slams and a car bounces up and down several times when it hits a bump.

FIGURE 11–15 Graphs that represent (A) underdamped, (B) critically damped, and (C) overdamped oscillatory motion.



In many systems, the oscillatory motion is what counts, as in clocks and musical instruments, and damping may need to be minimized. In other systems, oscillations are the problem, such as a car's springs, so a proper amount of damping (i.e., critical) is desired. Well-designed damping is needed for all kinds of applications. Large buildings, especially in California, are now built (or retrofitted) with huge dampers to reduce possible earthquake damage (Fig. 11–17).

PHYSICS APPLIED
Shock absorbers and building dampers

†To "damp" means to diminish, restrain, or extinguish, as to "dampen one's spirits."

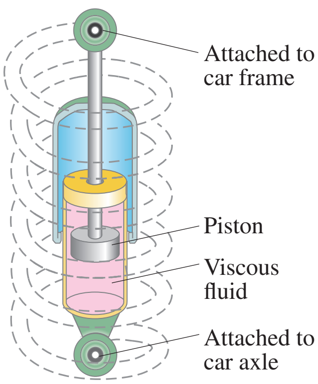


FIGURE 11–16 Automobile spring and shock absorber provide damping so that a car won't bounce up and down so much.

FIGURE 11–17 These huge dampers placed in a building look a lot like huge automobile shock absorbers, and they serve a similar purpose—to reduce the amplitude and the acceleration of movement when the shock of an earthquake hits.

11–6 Forced Oscillations; Resonance

When an oscillating system is set into motion, it oscillates at its natural frequency (Eqs. 11–6b and 11–11b). However, a system may have an external force applied to it that has its own particular frequency. Then we have a **forced oscillation**.

For example, we might pull the mass on the spring of Fig. 11–1 back and forth at an externally applied frequency f. The mass then oscillates at the external frequency f of the external force, even if this frequency is different from the **natural frequency** of the spring, which we will now denote by f_0, where (see Eq. 11–6b)

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}.$$

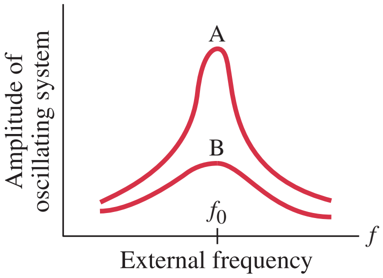


FIGURE 11–18 Amplitude as a function of driving frequency f, showing resonance for lightly damped (A) and heavily damped (B) systems.

For a forced oscillation with only light damping, the amplitude of oscillation is found to depend on the difference between f and f_0, and is a maximum when the frequency of the external force equals the natural frequency of the system—that is, when $f = f_0$. The amplitude is plotted in Fig. 11–18 as a function of the external frequency f. Curve A represents light damping and curve B heavy damping. When the external driving frequency f is near the natural frequency, $f \approx f_0$, the amplitude can become large if the damping is small. This effect of increased amplitude at $f = f_0$ is known as **resonance**. The natural oscillation frequency f_0 of a system is also called its **resonant frequency**.

PHYSICS APPLIED
Child on a swing

A simple illustration of resonance is pushing a child on a swing. A swing, like any pendulum, has a natural frequency of oscillation. If you push on the swing at a random frequency, the swing bounces around and reaches no great amplitude. But if you push with a frequency equal to the natural frequency of the swing, the amplitude increases greatly. At resonance, relatively little effort is required to obtain and maintain a large amplitude.

PHYSICS APPLIED
Shattering glass via resonance

The great tenor Enrico Caruso was said to be able to shatter a crystal goblet by singing a note of just the right frequency at full voice. This is an example of resonance, for the sound waves emitted by the voice act as a forced oscillation on the glass. At resonance, the resulting oscillation of the goblet may be large enough in amplitude that the glass exceeds its elastic limit and breaks (Fig. 11–19).

FIGURE 11–19 This goblet breaks as it vibrates in resonance to a trumpet call.

PHYSICS APPLIED
Resonant collapse

Since material objects are, in general, elastic, resonance is an important phenomenon in a variety of situations. It is particularly important in construction, although the effects are not always foreseen. For example, it has been reported that a railway bridge collapsed because a nick in one of the wheels of a crossing train set up a resonant oscillation in the bridge. Marching soldiers break step when crossing a bridge to avoid the possibility that their rhythmic march might match a resonant frequency of the bridge. The famous collapse of the Tacoma Narrows Bridge (Fig. 11–20a) in 1940 occurred as a result of strong gusting winds driving the span into large-amplitude oscillatory motion. Bridges and tall buildings are now designed with more inherent damping. The Oakland freeway collapse in the 1989 California earthquake (Fig. 11–20b) involved resonant oscillation of a section built on mudfill that readily transmitted that frequency.

Resonance can be very useful, too, and we will meet important examples later, such as in musical instruments and tuning a radio. We will also see that vibrating objects often have not one, but many resonant frequencies.


(a)


(b)

FIGURE 11–20 (a) Large-amplitude oscillations of the Tacoma Narrows Bridge, due to gusty winds, led to its collapse (November 7, 1940). (b) Collapse of a freeway in California, due to the 1989 earthquake.

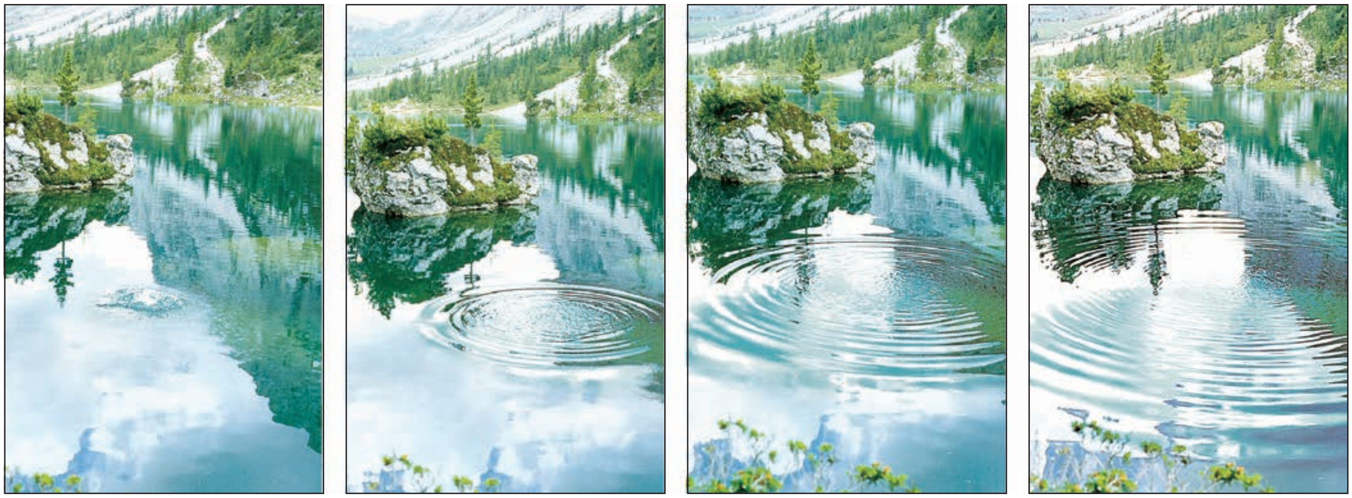


FIGURE 11–21 Water waves spreading outward from a source. In this case the source is a small spot of water oscillating up and down briefly where a rock hit (left photo).

11–7 Wave Motion

When you throw a stone into a lake or pool of water, circular waves form and move outward, Fig. 11–21. Waves will also travel along a rope that is stretched out straight on a table if you vibrate one end back and forth as shown in Fig. 11–22. Water waves and waves on a rope or cord are two common examples of **mechanical waves**, which propagate as oscillations of matter. We will discuss other kinds of waves in later Chapters, including electromagnetic waves and light.

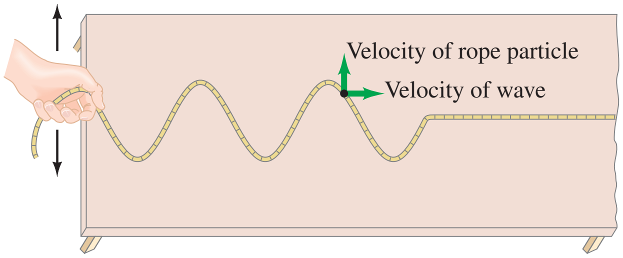


FIGURE 11–22 Wave traveling on a rope or cord. The wave travels to the right along the rope. Particles of the rope oscillate back and forth on the tabletop.

If you have ever watched ocean waves moving toward shore before they break, you may have wondered if the waves were carrying water from far out at sea onto the beach. They don't.[†] Water waves move with a recognizable velocity. But each particle (or molecule) of the water itself merely oscillates about an equilibrium point. This is clearly demonstrated by observing leaves on a pond as waves move by. The leaves (or a cork) are not carried forward by the waves, but oscillate more or less up and down about an equilibrium point because this is the motion of the water itself.

CONCEPTUAL EXAMPLE 11–10 **Wave vs. particle velocity.** Is the velocity of a wave moving along a rope the same as the velocity of a particle of the rope? See Fig. 11–22.

RESPONSE No. The two velocities are different, both in magnitude and direction. The wave on the rope of Fig. 11–22 moves to the right along the tabletop, but each piece of the rope only vibrates to and fro, perpendicular to the traveling wave. (The rope clearly does not travel in the direction that the wave on it does.)

Waves can move over large distances, but the medium (the water or the rope) itself has only a limited movement, oscillating about an equilibrium point as in simple harmonic motion. Thus, although a wave is not itself matter, the wave pattern can travel in matter. A wave consists of oscillations that move without carrying matter with them.

[†]Do not be confused by the "breaking" of ocean waves, which occurs when a wave interacts with the ground in shallow water and hence is no longer a simple wave.

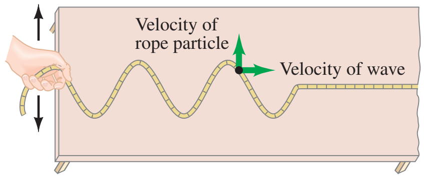
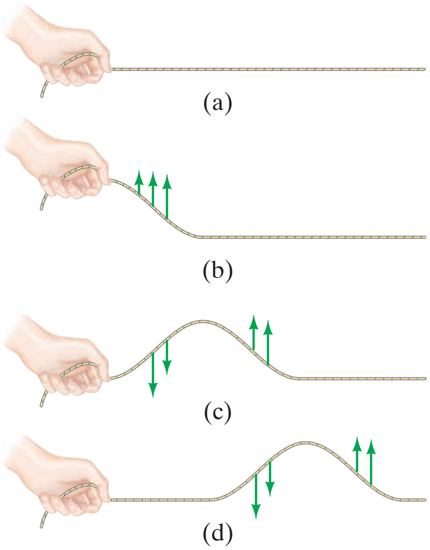


FIGURE 11–22 (Repeated.) Wave traveling on a rope or cord. The wave travels to the right along the rope. Particles of the rope oscillate back and forth on the tabletop.

FIGURE 11–23 A wave pulse is generated by a hand holding the end of a cord and moving up and down once. Motion of the wave pulse is to the right. Arrows indicate velocity of cord particles.



Waves carry energy from one place to another. Energy is given to a water wave, for example, by a rock thrown into the water, or by wind far out at sea. The energy is transported by waves to the shore. The oscillating hand in Fig. 11–22 transfers energy to the rope, and that energy is transported down the rope and can be transferred to an object at the other end. All forms of traveling waves transport energy.

EXERCISE G Return to Chapter-Opening Question 2, page 292, and answer it again now. Try to explain why you may have answered differently the first time.

Let us look more closely at how a wave is formed and how it comes to "travel." We first look at a single wave bump, or **pulse**. A single pulse can be formed on a cord by a quick up-and-down motion of the hand, Fig. 11–23. The hand pulls up on one end of the cord. Because the end section is attached to adjacent sections, these also feel an upward force and they too begin to move upward. As each succeeding section of cord moves upward, the wave crest moves outward along the cord. Meanwhile, the end section of cord has been returned to its original position by the hand. As each succeeding section of cord reaches its peak position, it too is pulled back down again by tension from the adjacent section of cord. Thus the source of a traveling wave pulse is a disturbance (or vibration), and cohesive forces between adjacent sections of cord cause the pulse to travel. Waves in other media are created and propagate outward in a similar fashion. A dramatic example of a wave pulse is a tsunami or tidal wave that is created by an earthquake in the Earth's crust under the ocean. The bang you hear when a door slams is a sound wave pulse.

A **continuous** or **periodic wave**, such as that shown in Fig. 11–22, has as its source a disturbance that is continuous and oscillating; that is, the source is a *vibration* or *oscillation*. In Fig. 11–22, a hand oscillates one end of the rope. Water waves may be produced by any vibrating object at the surface, such as your hand; or the water itself is made to vibrate when wind blows across it or a rock is thrown into it. A vibrating tuning fork or drum membrane gives rise to sound waves in air. We will see later that oscillating electric charges give rise to light waves. Indeed, almost any vibrating object sends out waves.

The source of any wave, then, is a vibration. And it is a *vibration* that propagates outward and thus constitutes the wave. If the source vibrates sinusoidally in SHM, then the wave itself—if the medium is elastic—will have a sinusoidal shape both in space and in time. (1) In space: if you take a picture of the wave in space at a given instant of time, the wave will have the shape of a sine or cosine as a function of position. (2) In time: if you look at the motion of the medium at one place over a long period of time—for example, if you look between two closely spaced posts of a pier or out of a ship's porthole as water waves pass by—the up-and-down motion of that small segment of water will be simple harmonic motion. The water moves up and down sinusoidally in time.

Some of the important quantities used to describe a periodic sinusoidal wave are shown in Fig. 11–24. The high points on a wave are called *crests*; the low points, *troughs*. The **amplitude**, A, is the maximum height of a crest, or depth of a trough, relative to the normal (or equilibrium) level. The total swing from a crest to a trough is $2A$ (twice the amplitude). The distance between two successive crests is the **wavelength**, λ (the Greek letter lambda). The wavelength is also equal to the distance between *any* two successive identical points on the wave. The **frequency**, f, is the number of crests—or complete cycles—that pass a given point per unit time. The **period**, T, equals $1/f$ and is the time elapsed between two successive crests passing by the same point in space.

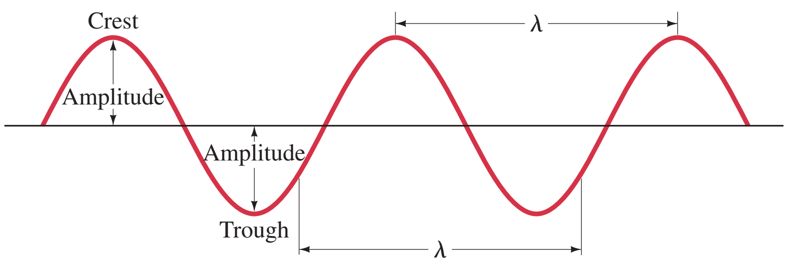


FIGURE 11–24 Characteristics of a single-frequency continuous wave moving through space.

The **wave speed**, v, is the speed at which wave crests (or any other fixed point on the wave shape) move forward. The wave speed must be distinguished from the speed of a particle of the medium itself as we saw in Example 11–10.

A wave crest travels a distance of one wavelength, λ, in a time equal to one period, T. Thus the wave speed is $v = \lambda/T$. Then, since $1/T = f$,

$$v = \lambda f. \tag{11–12}$$

For example, suppose a wave has a wavelength of 5 m and a frequency of 3 Hz. Since three crests pass a given point per second, and the crests are 5 m apart, the first crest (or any other part of the wave) must travel a distance of 15 m during the 1 s. So the wave speed is 15 m/s.

EXERCISE H You notice a water wave pass by the end of a pier, with about 0.5 s between crests. Therefore (*a*) the frequency is 0.5 Hz; (*b*) the velocity is 0.5 m/s; (*c*) the wavelength is 0.5 m; (*d*) the period is 0.5 s.

11–8 Types of Waves and Their Speeds: Transverse and Longitudinal

When a wave travels down a cord—say, from left to right as in Fig. 11–22—the particles of the cord vibrate back and forth in a direction transverse (that is, perpendicular) to the motion of the wave itself. Such a wave is called a **transverse wave** (Fig. 11–25a). There exists another type of wave known as a **longitudinal wave**. In a longitudinal wave, the vibration of the particles of the medium is *along* the direction of the wave's motion. Longitudinal waves are readily formed on a stretched spring or Slinky by alternately compressing and expanding one end. This is shown in Fig. 11–25b, and can be compared to the transverse wave in Fig. 11–25a. A series of compressions and expansions travel along the spring. The *compressions* are those areas where the coils are momentarily close together. *Expansions* (sometimes called *rarefactions*) are regions where the coils are momentarily far apart. Compressions and expansions correspond to the crests and troughs of a transverse wave.

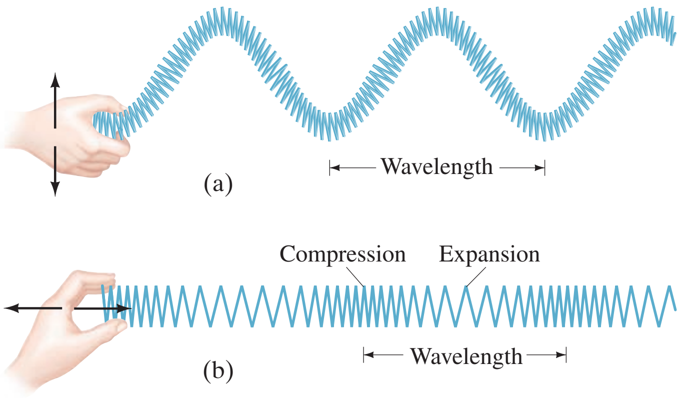


FIGURE 11–25 (a) Transverse wave; (b) longitudinal wave.

An important example of a longitudinal wave is a sound wave in air. A vibrating drumhead, for instance, alternately compresses and expands the air in contact with it, producing a longitudinal wave that travels outward in the air, as shown in Fig. 11–26.

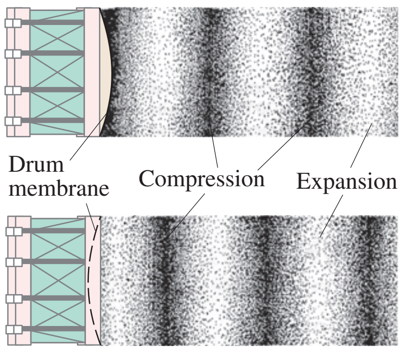


FIGURE 11–26 Production of a sound wave, which is longitudinal, shown at two moments in time about a half period $\left(\frac{1}{2}T\right)$ apart.

As in the case of transverse waves, each section of the medium in which a longitudinal wave passes oscillates over a very small distance, whereas the wave itself can travel large distances. Wavelength, frequency, and wave speed all have meaning for a longitudinal wave. The wavelength is the distance between successive compressions (or between successive expansions), and frequency is the number of compressions that pass a given point per second. The wave speed is the speed with which each compression appears to move; it is equal to the product of wavelength and frequency, $v = \lambda f$ (Eq. 11–12).

A longitudinal wave can be represented graphically by plotting the density of air molecules (or coils of a Slinky) versus position at a given instant, as shown in Fig. 11–27. Such a graphical representation makes it easy to illustrate what is happening. Note that the graph looks much like a transverse wave.

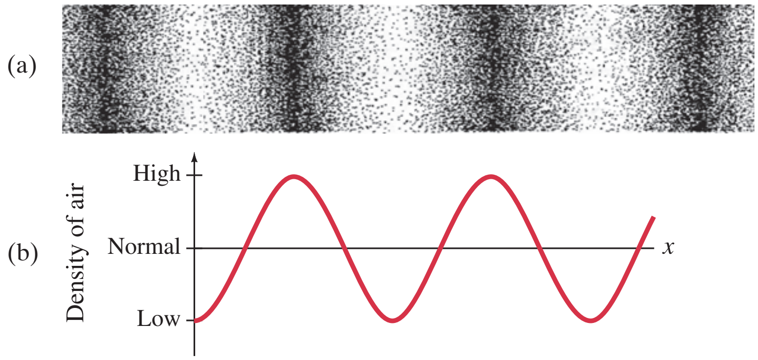


FIGURE 11–27 (a) A longitudinal wave in air, with (b) its graphical representation at a particular instant in time.

Speed of Transverse Waves

The speed of a wave depends on the properties of the medium in which it travels. The speed of a transverse wave on a stretched string or cord, for example, depends on the tension in the cord, F_T, and on the mass per unit length of the cord, μ (the Greek letter mu). If m is the mass of a length ℓ of wire, $\mu = m/\ell$. For waves of small amplitude, the wave speed is

$$v = \sqrt{\frac{F_T}{\mu}}. \quad \begin{bmatrix}\text{transverse wave}\\ \text{on a cord}\end{bmatrix} \quad \textbf{(11–13)}$$

This formula makes sense qualitatively on the basis of Newtonian mechanics. That is, we do expect the tension to be in the numerator and the mass per unit length in the denominator. Why? Because when the tension is greater, we expect the speed to be greater since each segment of cord is in tighter contact with its neighbor. Also, the greater the mass per unit length, the more inertia the cord has and the more slowly the wave would be expected to propagate.

EXAMPLE 11–11 **Wave along a wire.** A wave whose wavelength is 0.30 m is traveling down a 300-m-long wire whose total mass is 15 kg. If the wire is under a tension of 1000 N, what are the speed and frequency of this wave?

APPROACH We assume the velocity of this wave on a wire is given by Eq. 11–13. We get the frequency from Eq. 11–12, $f = v/\lambda$.

SOLUTION From Eq. 11–13, the velocity is

$$v = \sqrt{\frac{1000\ \text{N}}{(15\ \text{kg})/(300\ \text{m})}} = \sqrt{\frac{1000\ \text{N}}{(0.050\ \text{kg/m})}} = 140\ \text{m/s}.$$

The frequency is

$$f = \frac{v}{\lambda} = \frac{140\ \text{m/s}}{0.30\ \text{m}} = 470\ \text{Hz}.$$

NOTE A higher tension would increase both v and f, whereas a thicker, denser wire would reduce v and f.

Speed of Longitudinal Waves

The speed of a longitudinal wave has a form similar to that for a transverse wave on a cord (Eq. 11–13); that is,

$$v = \sqrt{\frac{\text{elastic force factor}}{\text{inertia factor}}}.$$

In particular, for a longitudinal wave traveling down a long solid rod,

$$v = \sqrt{\frac{E}{\rho}}, \quad \begin{bmatrix}\text{longitudinal wave}\\ \text{in a long rod}\end{bmatrix} \quad \textbf{(11–14a)}$$

where E is the elastic modulus (Section 9–5) of the material and ρ is its density.

For a longitudinal wave traveling in a liquid or gas,

$$v = \sqrt{\frac{B}{\rho}}, \qquad \begin{bmatrix}\text{longitudinal wave} \\ \text{in a fluid}\end{bmatrix} \qquad \textbf{(11–14b)}$$

where B is the bulk modulus (Section 9–5) and ρ again is the density.

EXAMPLE 11–12 **Echolocation.** Echolocation is a form of sensory perception used by animals such as bats, dolphins, and toothed whales (Fig. 11–28). The animal emits a pulse of sound (a longitudinal wave) which, after reflection from objects, returns and is detected by the animal. Echolocation waves can have frequencies of about 100,000 Hz. (*a*) Estimate the wavelength of a sea animal's echolocation wave. (*b*) If an obstacle is 100 m from the animal, how long after the animal emits a wave is its reflection detected?

APPROACH We first compute the speed of longitudinal (sound) waves in sea water, using Eq. 11–14b and Tables 9–1 and 10–1. The wavelength is $\lambda = v/f$.

SOLUTION (*a*) The speed of longitudinal waves in sea water, which is slightly more dense than pure water, is (Tables 9–1 and 10–1)

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2.0 \times 10^9\,\mathrm{N/m^2}}{1.025 \times 10^3\,\mathrm{kg/m^3}}} = 1.4 \times 10^3\,\mathrm{m/s}.$$

Then, using Eq. 11–12, we find

$$\lambda = \frac{v}{f} = \frac{(1.4 \times 10^3\,\mathrm{m/s})}{(1.0 \times 10^5\,\mathrm{Hz})} = 14\,\mathrm{mm}.$$

(*b*) The time required for the round trip between the animal and the object is

$$t = \frac{\text{distance}}{\text{speed}} = \frac{2(100\,\mathrm{m})}{1.4 \times 10^3\,\mathrm{m/s}} = 0.14\,\mathrm{s}.$$

NOTE We shall see later that waves can be used to "resolve" (or detect) objects whose size is comparable to or larger than the wavelength. Thus, a dolphin can resolve objects on the order of a centimeter or larger in size.

PHYSICS APPLIED
Space perception by animals using sound waves

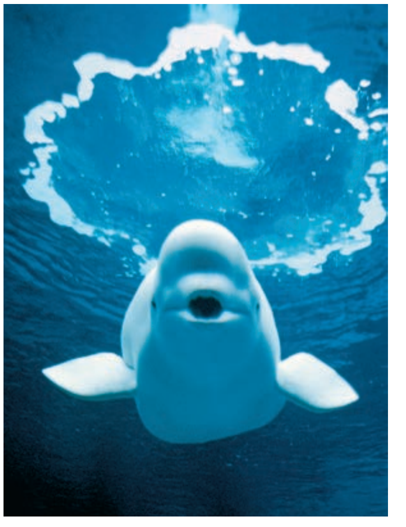


FIGURE 11–28 A toothed whale (Example 11–12).

Other Waves

Both transverse and longitudinal waves are produced when an **earthquake** occurs. The transverse waves that travel through the body of the Earth are called S waves (S for shear), and the longitudinal waves are called P waves (P for pressure) or *compression* waves. Both longitudinal and transverse waves can travel through a solid since the atoms or molecules can vibrate about their relatively fixed positions in any direction. But only longitudinal waves can propagate through a fluid, because any transverse motion would not experience any restoring force since a fluid is readily deformable. This fact was used by geophysicists to infer that a portion of the Earth's core must be liquid: after an earthquake, longitudinal waves are detected diametrically across the Earth, but not transverse waves.

Besides these two types of waves that can pass through the body of the Earth (or other substance), there can also be *surface waves* that travel along the boundary between two materials. A wave on water is actually a surface wave that moves on the boundary between water and air. The motion of each particle of water at the surface is circular or elliptical (Fig. 11–29), so it is a combination of horizontal and vertical motions. Below the surface, there is also horizontal plus vertical motion, as shown. At the bottom, the motion is only horizontal. (When a wave approaches shore, the water drags at the bottom and is slowed down, while the crests move ahead at higher speed (Fig. 11–30) and "spill" over the top.)

Surface waves are also set up on the Earth when an earthquake occurs. The waves that travel along the surface are mainly responsible for the damage caused by earthquakes.

PHYSICS APPLIED
Earthquake waves

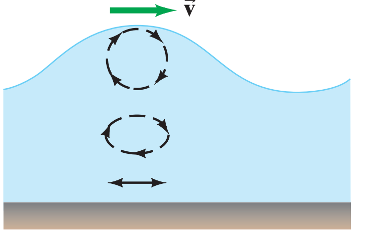
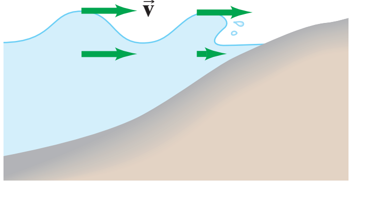


FIGURE 11–29 A shallow water wave is an example of a *surface wave*, which is a combination of transverse and longitudinal wave motions.

FIGURE 11–30 How a water wave breaks. The green arrows represent the local velocity of water molecules.



Waves which travel along a line in one dimension, such as transverse waves on a stretched string, or longitudinal waves in a rod or fluid-filled tube, are *linear* or *one-dimensional waves*. Surface waves, such as water waves (Fig. 11–21), are *two-dimensional waves*. Finally, waves that move out from a source in all directions, such as sound from a loudspeaker or earthquake waves through the Earth, are *three-dimensional waves*.

11–9 Energy Transported by Waves

Waves transport energy from one place to another. As waves travel through a medium, the energy is transferred as vibrational energy from particle to particle of the medium. For a sinusoidal wave of frequency f, the particles move in SHM as a wave passes, so each particle has an energy $E = \frac{1}{2}kA^2$, where A is the amplitude of its motion, either transversely or longitudinally. See Eq. 11–4a.

Thus, we have the important result that the **energy transported by a wave is proportional to the square of the amplitude**. The **intensity** I of a wave is defined as the power (energy per unit time) transported across unit area perpendicular to the direction of energy flow:

$$I = \frac{\text{energy/time}}{\text{area}} = \frac{\text{power}}{\text{area}}.$$

The SI unit of intensity is watts per square meter $(\mathrm{W/m^2})$. Since the energy is proportional to the wave amplitude squared, so too is the intensity:

$$I \propto A^2. \qquad \textbf{(11–15)}$$

If a wave flows out from the source in all directions, it is a three-dimensional wave. Examples are sound traveling in open air, earthquake waves, and light waves. If the medium is isotropic (same in all directions), the wave is a *spherical wave* (Fig. 11–31). As the wave moves outward, the energy it carries is spread over a larger and larger area since the surface area of a sphere of radius r is $4\pi r^2$. Thus the intensity of a spherical wave is

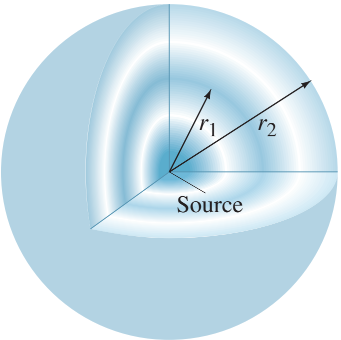


FIGURE 11–31 A wave traveling uniformly outward in three dimensions from a source is spherical. Two crests (or compressions) are shown, of radii r_1 and r_2.

$$I = \frac{\text{power}}{\text{area}} = \frac{P}{4\pi r^2}. \qquad \text{[spherical wave]} \quad \textbf{(11–16a)}$$

If the power output P of the source is constant, then the intensity decreases as the inverse square of the distance from the source:

PROBLEM SOLVING
The $1/r^2$ law

$$I \propto \frac{1}{r^2}. \qquad \text{[spherical wave]} \quad \textbf{(11–16b)}$$

This is often called the **inverse square law**, or the "one over r^2 law." If we consider two points at distances r_1 and r_2 from the source, as in Fig. 11–31, then $I_1 = P/4\pi r_1^2$ and $I_2 = P/4\pi r_2^2$, so

$$\frac{I_2}{I_1} = \frac{r_1^2}{r_2^2}. \qquad \text{[spherical wave]} \quad \textbf{(11–16c)}$$

Thus, for example, when the distance doubles $(r_2/r_1 = 2)$, the intensity is reduced to $\frac{1}{4}$ its earlier value: $I_2/I_1 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$.

The amplitude of a wave also decreases with distance. Since the intensity is proportional to the square of the amplitude (Eq. 11–15), the amplitude A must decrease as $1/r$ so that $I \propto A^2$ will be proportional to $1/r^2$ (as in Eq. 11–16b). Hence

$$A \propto \frac{1}{r}.$$

If we consider again two distances from the source, r_1 and r_2, then

$$\frac{A_2}{A_1} = \frac{r_1}{r_2}. \qquad \text{[spherical wave]}$$

When the wave is twice as far from the source, the amplitude is half as large, and so on (ignoring damping due to friction).

EXAMPLE 11–13 **Earthquake intensity.** The intensity of an earthquake P wave traveling through the Earth and detected 100 km from the source is $1.0 \times 10^6\ \mathrm{W/m^2}$. What is the intensity of that wave if detected 400 km from the source?

APPROACH We assume the wave is spherical, so the intensity decreases as the square of the distance from the source.

SOLUTION At 400 km the distance is 4 times greater than at 100 km, so the intensity will be $\left(\frac{1}{4}\right)^2 = \frac{1}{16}$ of its value at 100 km, or $(1.0 \times 10^6\ \mathrm{W/m^2})/16 = 6.3 \times 10^4\ \mathrm{W/m^2}$.

NOTE Using Eq. 11–16c directly gives:

$$I_2 = I_1 r_1^2/r_2^2 = (1.0 \times 10^6\ \mathrm{W/m^2})(100\ \mathrm{km})^2/(400\ \mathrm{km})^2 = 6.3 \times 10^4\ \mathrm{W/m^2}.$$

The situation is different for a one-dimensional wave, such as a transverse wave on a string or a longitudinal wave pulse traveling down a thin uniform metal rod. The area remains constant, so the amplitude A also remains constant (ignoring friction). Thus the amplitude and the intensity do not decrease with distance.

In practice, frictional damping is generally present, and some of the energy is transformed into thermal energy. Thus the amplitude and intensity of a one-dimensional wave will decrease with distance from the source. For a three-dimensional wave, the decrease will be greater than that discussed above, more than $1/r^2$, although the effect may often be small.

Intensity Related to Amplitude and Frequency

For a sinusoidal wave of frequency f, the particles move in SHM as a wave passes, so each particle has an energy $E = \frac{1}{2}kA^2$, where A is the amplitude of its motion. Using Eq. 11–6b, we can write k in terms of the frequency: $k = 4\pi^2 m f^2$, where m is the mass of a particle (or small volume) of the medium. Then

$$E = \tfrac{1}{2}kA^2 = 2\pi^2 m f^2 A^2.$$

The mass $m = \rho V$, where ρ is the density of the medium and V is the volume of a small slice of the medium as shown in Fig. 11–32. The volume $V = S\ell$, where S is the cross-sectional surface area through which the wave travels. (We use S instead of A for area because we are using A for amplitude.) We can write ℓ as the distance the wave travels in a time t as $\ell = vt$, where v is the speed of the wave. Thus $m = \rho V = \rho S\ell = \rho S v t$, and

$$E = 2\pi^2 \rho S v t f^2 A^2. \quad \textbf{(11–17a)}$$

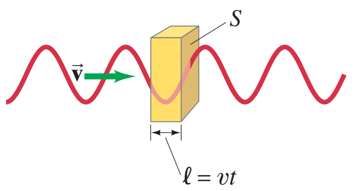


FIGURE 11–32 Calculating the energy carried by a wave moving with velocity v.

From this equation, we see again the important result that the energy transported by a wave is proportional to the square of the amplitude. The average power transported, $\overline{P} = E/t$, is

$$\overline{P} = \frac{E}{t} = 2\pi^2 \rho S v f^2 A^2. \quad \textbf{(11–17b)}$$

Finally, the **intensity** I of a wave is the average power transported across unit area perpendicular to the direction of energy flow:

$$I = \frac{\overline{P}}{S} = 2\pi^2 \rho v f^2 A^2. \quad \textbf{(11–18)}$$

This relation shows explicitly that the intensity of a wave is proportional both to the square of the wave amplitude A at any point and to the square of the frequency f.

11–10 Reflection and Transmission of Waves

When a wave strikes an obstacle, or comes to the end of the medium in which it is traveling, at least a part of the wave is reflected. You have probably seen water waves reflect off a rock or the side of a swimming pool. And you may have heard a shout reflected from a distant cliff—which we call an "echo."

A wave pulse traveling along a cord is reflected as shown in Fig. 11–33 (time increases going downward in both a and b). The reflected pulse returns inverted as in Fig. 11–33a if the end of the cord is fixed; it returns right side up if the end is free as in Fig. 11–33b. When the end is fixed to a support, as in Fig. 11–33a, the pulse reaching that fixed end exerts a force (upward) on the support. The support exerts an equal but opposite force downward on the cord (Newton's third law). This downward force on the cord is what "generates" the inverted reflected pulse.

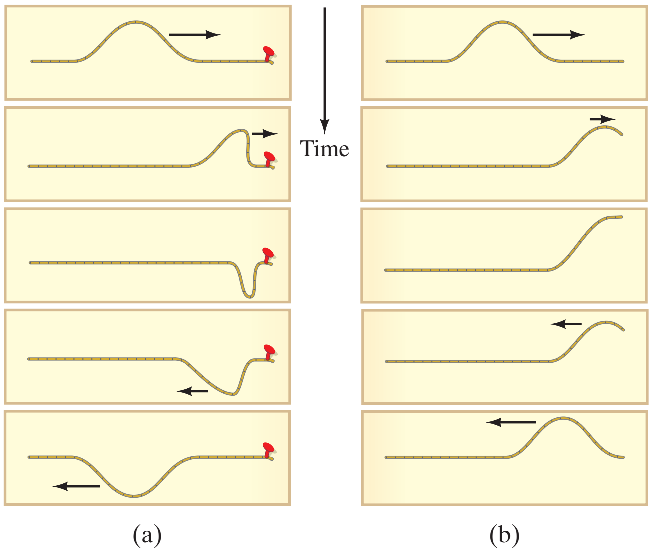
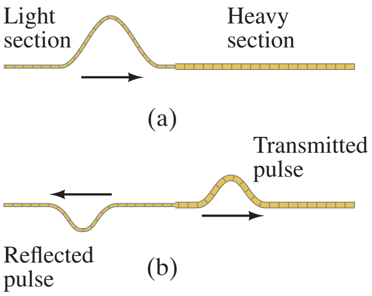


FIGURE 11–33 Reflection of a wave pulse traveling along a cord lying on a table. (Time increases going down.) (a) The end of the cord is fixed to a peg. (b) The end of the cord is free to move.

Consider next a pulse that travels along a cord which consists of a light section and a heavy section, as shown in Fig. 11–34. When the wave pulse reaches the boundary between the two sections, part of the pulse is reflected and part is transmitted, as shown. The heavier the second section of the cord, the less the energy that is transmitted. (When the second section is a wall or rigid support, very little is transmitted and most is reflected, as in Fig. 11–33a.) For a sinusoidal wave, the frequency of the transmitted wave does not change across the boundary because the boundary point oscillates at that frequency. Thus if the transmitted wave has a lower speed, its wavelength is also less $(\lambda = v/f)$.

FIGURE 11–34 When a wave pulse traveling to the right along a thin cord (a) reaches a discontinuity where the cord becomes thicker and heavier, then part is reflected and part is transmitted (b).



For a two or three dimensional wave, such as a water wave, we are concerned with **wave fronts**, by which we mean all the points along the wave forming the wave crest (what we usually refer to simply as a "wave" at the seashore). A line drawn in the direction of wave motion, perpendicular to the wave front, is called a **ray**, as shown in Fig. 11–35. Wave fronts far from the source have lost almost all their curvature (Fig. 11–35b) and are nearly straight, as ocean waves often are. They are then called **plane waves**.

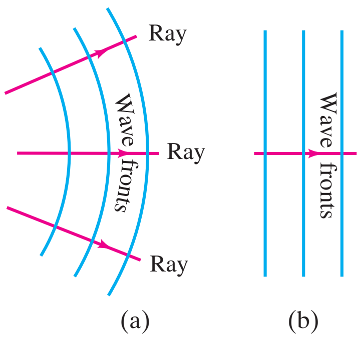


FIGURE 11–35 Rays, signifying the direction of wave motion, are always perpendicular to the wave fronts (wave crests). (a) Circular or spherical waves near the source. (b) Far from the source, the wave fronts are nearly straight or flat, and are called plane waves.

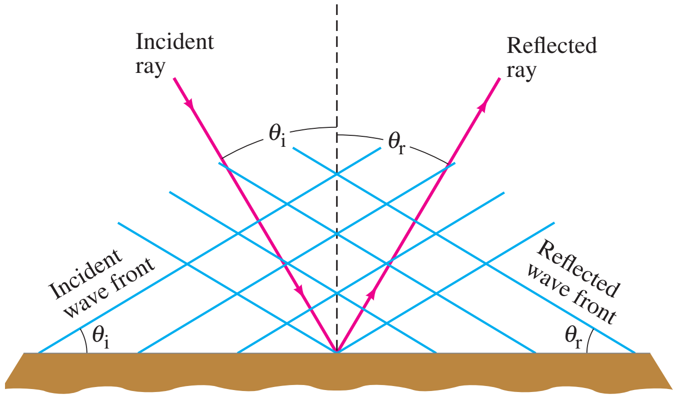


FIGURE 11–36 Law of reflection: $\theta_r = \theta_i$.

For reflection of a two or three dimensional plane wave, as shown in Fig. 11–36, the angle that the incoming or *incident wave* makes with the reflecting surface is equal to the angle made by the reflected wave. This is the **law of reflection:**

the angle of reflection equals the angle of incidence.

The **angle of incidence** is defined as the angle (θ_i) the incident ray makes with the perpendicular to the reflecting surface (or the wave front makes with the surface). The **angle of reflection** is the corresponding angle (θ_r) for the reflected wave.

11–11 Interference; Principle of Superposition

Interference refers to what happens when two waves pass through the same region of space at the same time. Consider, for example, the two wave pulses on a cord traveling toward each other as shown in Fig. 11–37 (time increases downward in both a and b). In Fig. 11–37a the two pulses have the same amplitude, but one is a crest and the other a trough; in Fig. 11–37b they are both crests. In both cases, the waves meet and pass right by each other. However, in the region where they overlap, the resultant displacement is the *algebraic sum of their separate displacements* (a crest is considered positive and a trough negative). This is the **principle of superposition**. In Fig. 11–37a, the two waves have opposite displacements at the instant they pass one another, and they add to zero. The result is called **destructive interference**. In Fig. 11–37b, at the instant the two pulses overlap, they produce a resultant displacement that is greater than the displacement of either separate pulse, and the result is **constructive interference**.

You may wonder where the energy is at the moment of destructive interference in Fig. 11–37a; the cord may be straight at this instant, but the central parts of it are still moving up or down (kinetic energy).

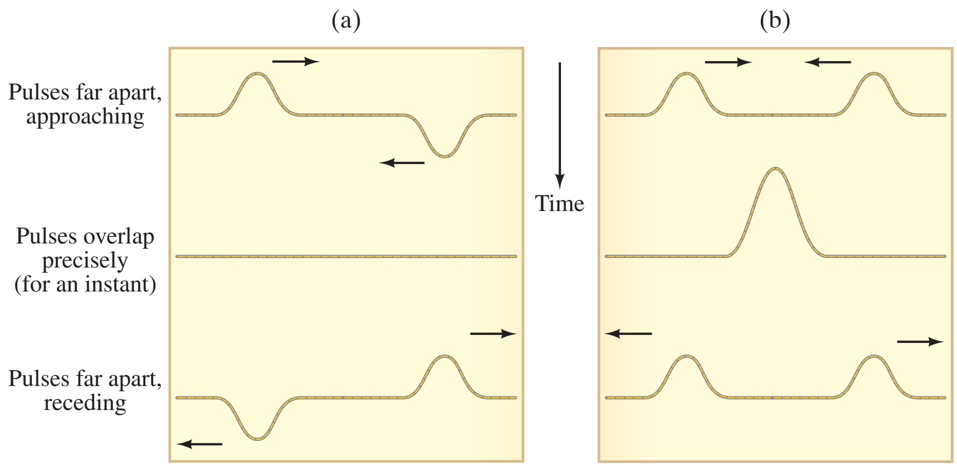


FIGURE 11–37 Two wave pulses pass each other. Where they overlap, interference occurs: (a) destructive, and (b) constructive. Read (a) and (b) downward (increasing time).

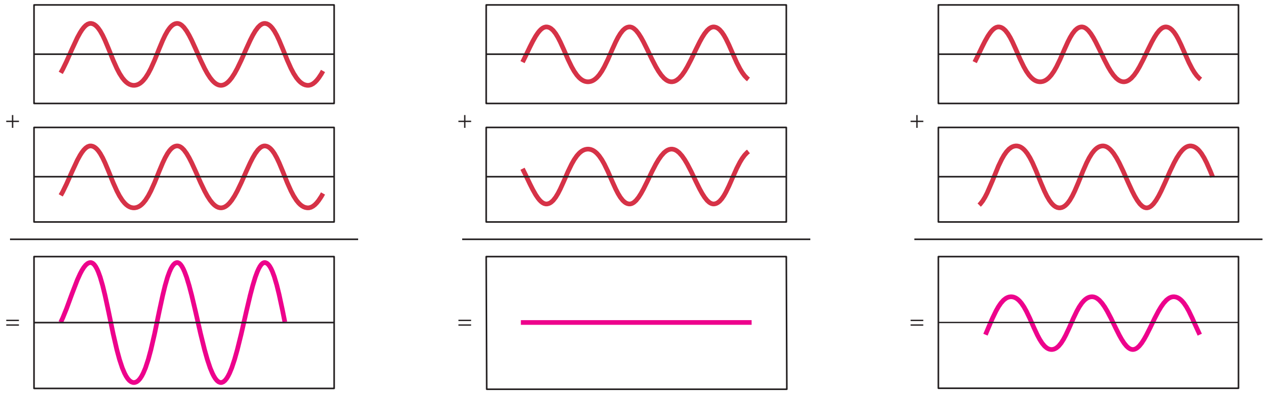
(a) (b)

FIGURE 11–38 (a) Interference of water waves. (b) Constructive interference occurs where one wave's maximum (a crest) meets the other's maximum. Destructive interference ("flat water") occurs where one wave's maximum (a crest) meets the other's miminum (a trough).

When two rocks are thrown into a pond simultaneously, the two sets of circular waves that move outward interfere with one another as shown in Fig. 11–38a. In some areas of overlap, crests of one wave repeatedly meet crests of the other (and troughs meet troughs), Fig. 11–38b. Constructive interference is occurring at these points, and the water continuously oscillates up and down with greater amplitude than either wave separately. In other areas, destructive interference occurs where the water does not move up and down at all over time. This is where crests of one wave meet troughs of the other, and vice versa. Figure 11–39a shows the displacement of two identical waves graphically as a function of time, as well as their sum, for the case of constructive interference. For any two such waves, we use the term **phase** to describe the relative positions of their crests. When the crests and troughs are aligned as in Fig. 11–39a, for constructive interference, the two waves are **in phase**. At points where destructive interference occurs (Fig. 11–39b), crests of one wave repeatedly meet troughs of the other wave and the two waves are said to be completely **out of phase** or, more precisely, out of phase by one-half wavelength (or 180°).† That is, the crests of one wave occur a half wavelength behind the crests of the other wave. The relative phase of the two water waves in Fig. 11–38 in most areas is intermediate between these two extremes, resulting in *partially* destructive interference, as illustrated in Fig. 11–39c. If the amplitudes of two interfering waves are not equal, fully destructive interference (as in Fig. 11–39b) does not occur.

†One wavelength, or one full oscillation, corresponds to 360°—see Section 11–3, just after Eq. 11–8c, and also Fig. 11–7.

FIGURE 11–39 Graphs showing two identical waves, and their sum, as a function of time at three locations. In (a) the two waves interfere constructively, in (b) destructively, and in (c) partially destructively.



(a) (b) (c)

11–12 Standing Waves; Resonance

If you shake one end of a cord and the other end is kept fixed, a continuous wave will travel down to the fixed end and be reflected back, inverted, as we saw in Fig. 11–33a. As you continue to oscillate the cord, waves will travel in both directions, and the wave traveling along the cord, away from your hand, will interfere with the reflected wave coming back. Usually there will be quite a jumble. But if you oscillate the cord at just the right frequency, the two traveling waves will interfere in such a way that a large-amplitude **standing wave** will be produced, Fig. 11–40. It is called a "standing wave" because it does not appear to be traveling. The cord simply appears to have segments that oscillate up and down in a fixed pattern. The points of destructive interference, where the cord remains still at all times, are called **nodes**. Points of constructive interference, where the cord oscillates with maximum amplitude, are called **antinodes**. The nodes and antinodes remain in fixed positions for a particular frequency.

Standing waves can occur at more than one frequency. The lowest frequency of oscillation that produces a standing wave gives rise to the pattern shown in Fig. 11–40a. The standing waves shown in Figs. 11–40b and 11–40c are produced at precisely twice and three times the lowest frequency, respectively, assuming the tension in the cord is the same. The cord can also oscillate with four loops (four antinodes) at four times the lowest frequency, and so on.

The frequencies at which standing waves are produced are the **natural frequencies** or **resonant frequencies** of the cord, and the different standing wave patterns shown in Fig. 11–40 are different "resonant modes of vibration." A standing wave on a cord is the result of the interference of two waves traveling in opposite directions. A standing wave can also be considered a vibrating object at resonance. Standing waves represent the same phenomenon as the resonance of an oscillating spring or pendulum, which we discussed in Section 11–6. However, a spring or pendulum has only one resonant frequency, whereas the cord has an infinite number of resonant frequencies, each of which is a whole-number multiple of the lowest resonant frequency.

Consider a string stretched between two supports that is plucked like a guitar or violin string, Fig. 11–41a. Waves of a great variety of frequencies will travel in both directions along the string, will be reflected at the ends, and will travel back in the opposite direction. Most of these waves interfere with each other and quickly die out. However, those waves that correspond to the resonant frequencies of the string will persist. The ends of the string, since they are fixed, will be nodes. There may be other nodes as well. Some of the possible resonant modes of vibration (standing waves) are shown in Fig. 11–41b. Generally, the motion will be a combination of these different resonant modes, but only those frequencies that correspond to a resonant frequency will be present.

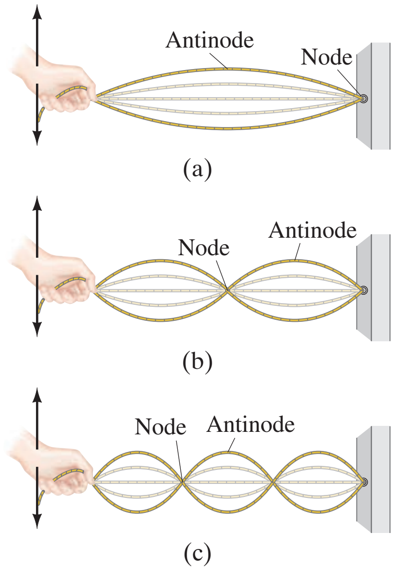
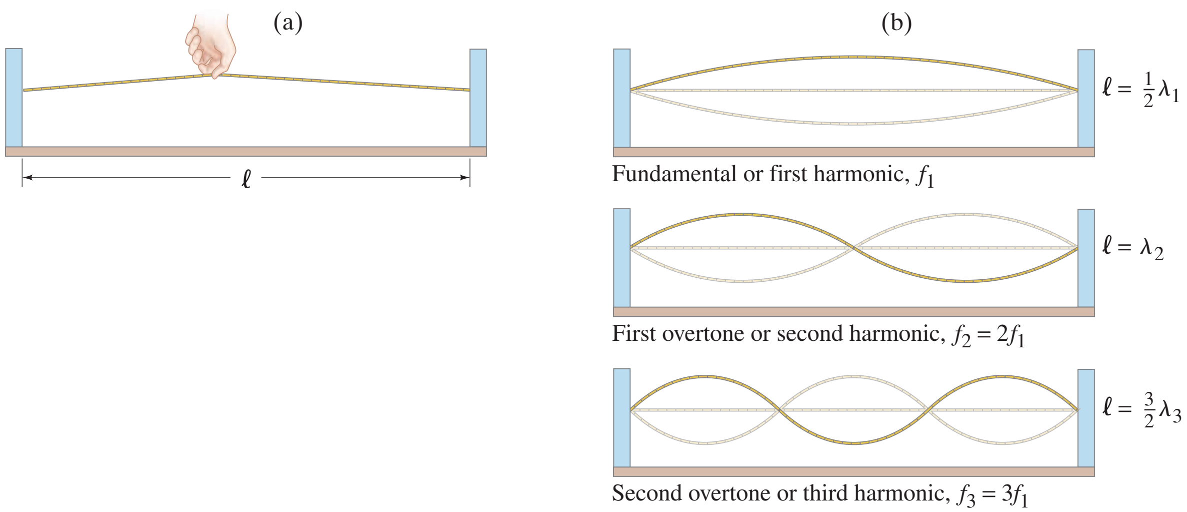


FIGURE 11–40 Standing waves corresponding to three resonant frequencies.

FIGURE 11–41 (a) A string is plucked. (b) Only standing waves corresponding to resonant frequencies persist for long.



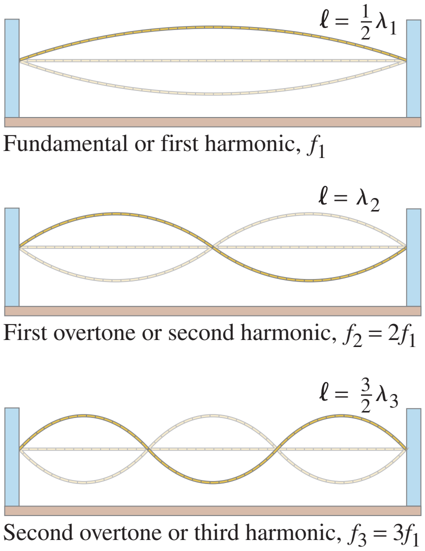


FIGURE 11–41b (Repeated.) (b) Only standing waves corresponding to resonant frequencies persist for long.

To determine the resonant frequencies, we first note that the wavelengths of the standing waves bear a simple relationship to the length ℓ of the string. The lowest frequency, called the **fundamental frequency**, corresponds to one antinode (or loop). And as can be seen in Fig. 11–41b, the whole length corresponds to one-half wavelength. Thus $\ell = \frac{1}{2}\lambda_1$, where λ_1 stands for the wavelength of the fundamental frequency. The other natural frequencies are called **overtones**; for a vibrating string they are whole-number (integral) multiples of the fundamental, and then are also called **harmonics**, with the fundamental being referred to as the **first harmonic**.† The next mode of vibration after the fundamental has two loops and is called the **second harmonic** (or first overtone), Fig. 11–41b. The length of the string ℓ at the second harmonic corresponds to one complete wavelength: $\ell = \lambda_2$. For the third and fourth harmonics, $\ell = \frac{3}{2}\lambda_3$, and $\ell = \frac{4}{2}\lambda_4 = 2\lambda_4$, respectively, and so on. In general, we can write

$$\ell = \frac{n\lambda_n}{2}, \qquad \text{where } n = 1,\ 2,\ 3, \cdots.$$

The integer n labels the number of the harmonic: $n = 1$ for the fundamental, $n = 2$ for the second harmonic, and so on. We solve for λ_n and find

$$\lambda_n = \frac{2\ell}{n}, \qquad n = 1,\ 2,\ 3, \cdots. \qquad \begin{bmatrix}\text{string fixed}\\ \text{at both ends}\end{bmatrix} \qquad \textbf{(11–19a)}$$

To find the frequency f of each vibration we use Eq. 11–12, $f = v/\lambda$, and see that

$$f_n = \frac{v}{\lambda_n} = n\frac{v}{2\ell} = nf_1, \qquad n = 1,\ 2,\ 3, \cdots, \qquad \textbf{(11–19b)}$$

where $f_1 = v/\lambda_1 = v/2\ell$ is the fundamental frequency. We see that each resonant frequency is an integer multiple of the fundamental frequency on a vibrating string.

Because a standing wave is equivalent to two traveling waves moving in opposite directions, the concept of wave velocity still makes sense and is given by Eq. 11–13 in terms of the tension F_T in the string and its mass per unit length ($\mu = m/\ell$). That is, $v = \sqrt{F_T/\mu}$ for waves traveling in either direction.

EXAMPLE 11–14 **Piano string.** A piano string 1.10 m long has mass 9.00 g. (*a*) How much tension must the string be under if it is to vibrate at a fundamental frequency of 131 Hz? (*b*) What are the frequencies of the first four harmonics?

APPROACH To determine the tension, we need to find the wave speed using Eq. 11–12 ($v = \lambda f$), and then use Eq. 11–13, solving it for F_T.

SOLUTION (*a*) The wavelength of the fundamental is $\lambda = 2\ell = 2.20\,\text{m}$ (Eq. 11–19a with $n = 1$). The speed of the wave on the string is $v = \lambda f = (2.20\,\text{m})(131\,\text{s}^{-1}) = 288\,\text{m/s}$. Then we have (Eq. 11–13)

$$F_T = \mu v^2 = \frac{m}{\ell}v^2 = \left(\frac{9.00 \times 10^{-3}\,\text{kg}}{1.10\,\text{m}}\right)(288\,\text{m/s})^2 = 679\,\text{N}.$$

(*b*) The first harmonic (the fundamental) has a frequency $f_1 = 131\,\text{Hz}$. The frequencies of the second, third, and fourth harmonics are two, three, and four times the fundamental frequency: 262, 393, and 524 Hz, respectively.

NOTE The speed of the wave on the string is *not* the same as the speed of the sound wave that the piano string produces in the air (as we shall see in Chapter 12).

A standing wave does appear to be standing in place (and a traveling wave appears to move). The term "standing" wave is also meaningful from the point of view of energy. Since the string is at rest at the nodes, no energy flows past these points. Hence the energy is not transmitted down the string but "stands" in place in the string.

Standing waves are produced not only on strings, but also on any object that is struck, such as a drum membrane or an object made of metal or wood. The resonant frequencies depend on the dimensions of the object, just as for a string they depend on its length. Large objects have lower resonant frequencies than small objects.

†The term "harmonic" comes from music, because such integral multiples of frequencies "harmonize."

All musical instruments, from stringed to wind instruments (in which a column of air oscillates as a standing wave) to drums and other percussion instruments, depend on standing waves to produce their particular musical sounds, as we shall see in Chapter 12.

* 11–13 Refraction†

When any wave strikes a boundary, some of the energy is reflected and some is transmitted or absorbed. When a two- or three-dimensional wave traveling in one medium crosses a boundary into a medium where its speed is different, the transmitted wave may move in a different direction than the incident wave, as shown in Fig. 11–42. This phenomenon is known as **refraction**. One example is a water wave; the velocity decreases in shallow water and the waves refract, as shown in Fig. 11–43. [When the wave velocity changes gradually, as in Fig. 11–43, without a sharp boundary, the waves change direction (refract) gradually.]

In Fig. 11–42, the velocity of the wave in medium 2 is less than in medium 1. In this case, the wave front bends so that it travels more nearly parallel to the boundary. That is, the *angle of refraction*, θ_r, is less than the *angle of incidence*, θ_i. To see why this is so, and to help us get a quantitative relation between θ_r and θ_i, let us think of each wave front as a row of soldiers. The soldiers are marching from firm ground (medium 1) into mud (medium 2) and hence are slowed down after the boundary. The soldiers that reach the mud first are slowed down first, and the row bends as shown in Fig. 11–44a. Let us consider the wave front (or row of soldiers) labeled A in Fig. 11–44b. In the same time t that A_1 moves a distance $\ell_1 = v_1 t$, we see that A_2 moves a distance $\ell_2 = v_2 t$. The two right triangles in Fig. 11–44b, shaded yellow and green, have the side labeled a in common. Thus

$$\sin\theta_1 = \frac{\ell_1}{a} = \frac{v_1 t}{a}$$

since a is the hypotenuse, and

$$\sin\theta_2 = \frac{\ell_2}{a} = \frac{v_2 t}{a}.$$

Dividing these two equations, we obtain the **law of refraction**:

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1}. \tag{11–20}$$

Since θ_1 is the angle of incidence (θ_i), and θ_2 is the angle of refraction (θ_r), Eq. 11–20 gives the quantitative relation between the two. If the wave were going in the opposite direction, the geometry would not change; only θ_1 and θ_2 would change roles: θ_2 would be the angle of incidence and θ_1 the angle of refraction. Thus, if the wave travels into a medium where it can move faster, it will bend the opposite way, $\theta_r > \theta_i$. We see from Eq. 11–20 that if the velocity increases, the angle increases, and vice versa.

†This Section and the next are covered in more detail in Chapters 23 and 24 on optics.

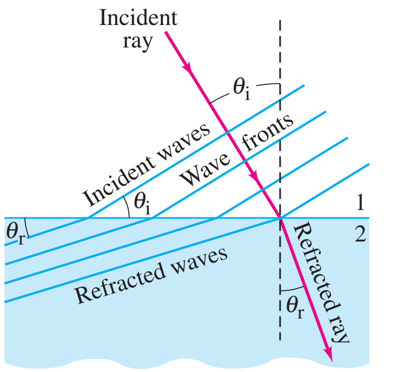


FIGURE 11–42 Refraction of waves passing a boundary.

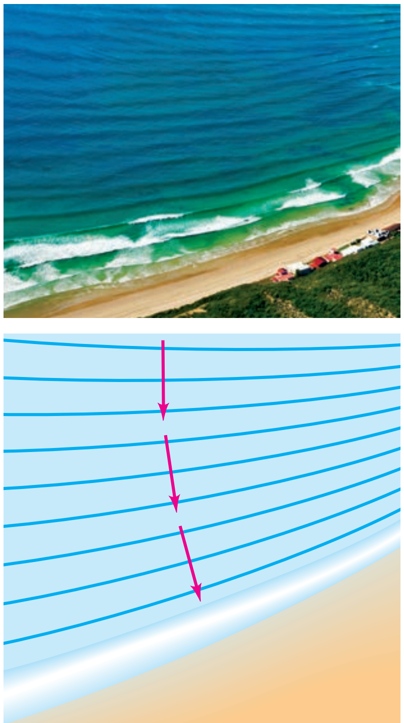


FIGURE 11–43 Water waves refract gradually as they approach the shore, as their velocity decreases. There is no distinct boundary, as in Fig. 11–42, because the wave velocity changes gradually.

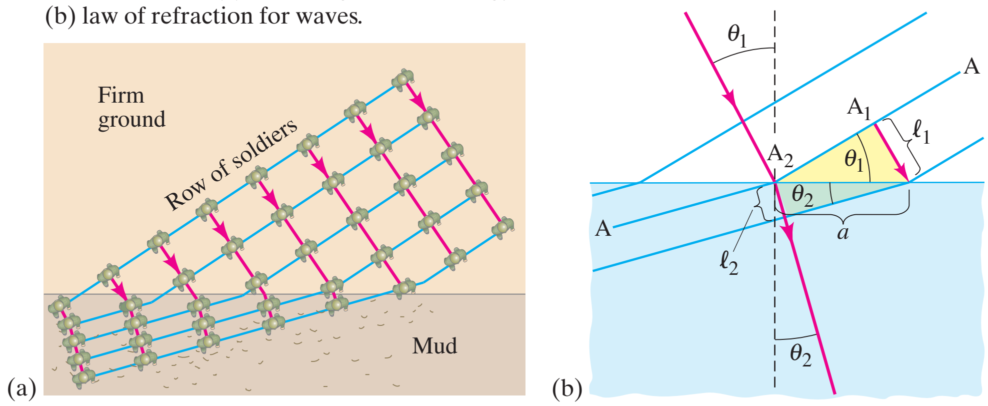
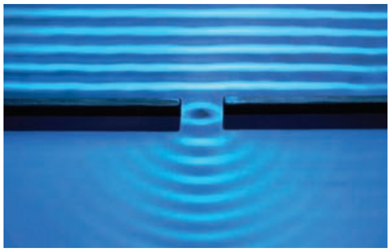


FIGURE 11–44 (a) Marching soldier analogy to derive (b) law of refraction for waves.

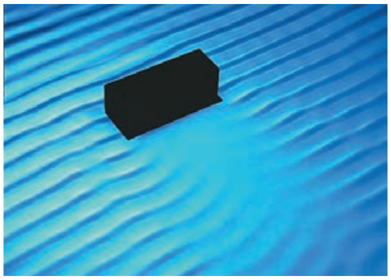
PHYSICS APPLIED
Earthquake wave refraction

Earthquake waves refract within the Earth as they travel through rock layers of different densities (which have different velocities) just as water waves do. Light waves refract as well, and when we discuss light, we shall find Eq. 11–20 very useful.

*11–14 Diffraction



(a)



(b)

FIGURE 11–45 Wave diffraction. In (a) the waves pass through a slit and into the "shadow region" behind. In (b) the waves are coming from the upper left. As they pass an obstacle, they bend around it into the shadow region behind it.

Waves spread as they travel. When waves encounter an obstacle, they bend around it somewhat and pass into the region behind it, as shown in Fig. 11–45 for water waves. This phenomenon is called **diffraction**.

The amount of diffraction depends on the wavelength of the wave and on the size of the obstacle, as shown in Fig. 11–46. If the wavelength is much larger than the object, as with the grass blades of Fig. 11–46a, the wave bends around them almost as if they are not there. For larger objects, parts (b) and (c), there is more of a "shadow" region behind the obstacle where we might not expect the waves to penetrate—but they do, at least a little. Then notice in part (d), where the obstacle is the same as in part (c) but the wavelength is longer, that there is more diffraction into the shadow region. As a rule of thumb, *only if the wavelength is smaller than the size of the object will there be a significant shadow region*. This rule applies to *reflection* from an obstacle as well. Very little of a wave is reflected unless the wavelength is smaller than the size of the obstacle.

A rough guide to the amount of diffraction is

$$\theta(\text{radians}) \approx \frac{\lambda}{\ell},$$

where θ is roughly the angular spread of waves after they have passed through an opening of width ℓ or around an obstacle of width ℓ.

That waves can bend around obstacles, and thus can carry energy to areas behind obstacles, is very different from energy carried by material particles. A clear example is the following: if you are standing around a corner on one side of a building, you cannot be hit by a baseball thrown from the other side, but you can hear a shout or other sound because the sound waves diffract around the edges of the building.

(a) Water waves passing blades of grass (b) Stick in water (c) Short-wavelength waves passing log (d) Long-wavelength waves passing log

FIGURE 11–46 Water waves, coming from upper left, pass objects of various sizes. Note that the longer the wavelength compared to the size of the object, the more diffraction there is into the "shadow region."

CONCEPTUAL EXAMPLE 11–15 **Cell phones.** Cellular phones operate by radio waves with frequencies of about 1 or 2 GHz $(1 \text{ gigahertz} = 10^9 \text{ Hz})$. These waves cannot penetrate objects that conduct electricity, such as a sheet of metal or a tree trunk. The sound quality is best if the transmitting antenna is within clear view of the handset. Yet it is possible to carry on a phone conversation even if the tower is blocked by trees, or if the handset is inside a car. Why?

RESPONSE If the radio waves have a frequency of about 2 GHz, and the speed of propagation is equal to the speed of light, 3×10^8 m/s (Section 1–5), then the wavelength is $\lambda = v/f = (3 \times 10^8 \text{ m/s})/(2 \times 10^9 \text{ Hz}) = 0.15$ m. The waves can diffract readily around objects 15 cm in diameter or smaller.

* 11–15 Mathematical Representation of a Traveling Wave

A simple wave with a single frequency, as in Fig. 11–47, is sinusoidal. To express such a wave mathematically, we assume it has a particular wavelength λ and frequency f. At $t = 0$, the wave shape shown is

$$y = A \sin \frac{2\pi}{\lambda} x, \tag{11–21}$$

where y is the **displacement** of the wave (either longitudinal or transverse) at position x, λ is the wavelength, and A is the **amplitude** of the wave. [Equation 11–21 works because it repeats itself every wavelength: when $x = \lambda$, $y = \sin 2\pi = \sin 0$.]

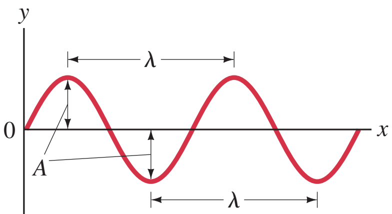
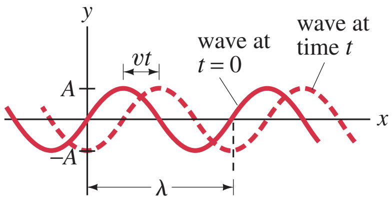


FIGURE 11–47 The characteristics of a single-frequency wave at $t = 0$ (just as in Fig. 11–24).

Suppose the wave is moving to the right with speed v. After a time t, each part of the wave (indeed, the whole wave "shape") has moved to the right a distance vt. Figure 11–48 shows the wave at $t = 0$ as a solid curve, and at a later time t as a dashed curve. Consider any point on the wave at $t = 0$: say, a crest at some position x. After a time t, that crest will have traveled a distance vt, so its new position is a distance vt greater than its old position. To describe this crest (or other point on the wave shape), the argument of the sine function must have the same numerical value, so we replace x in Eq. 11–21 by $(x - vt)$:

$$y = A \sin\left[\frac{2\pi}{\lambda}(x - vt)\right]. \tag{11–22}$$

FIGURE 11–48 A traveling wave. In time t, the wave moves a distance vt.



Said another way, if you are on a crest, as t increases, x must increase at the same rate so that $(x - vt)$ remains constant.

For a wave traveling along the x axis to the left, toward decreasing values of x, v becomes $-v$, so

$$y = A \sin\left[\frac{2\pi}{\lambda}(x + vt)\right].$$

Summary

An oscillating (or vibrating) object undergoes **simple harmonic motion** (SHM) if the restoring force is proportional to (the negative of) the displacement,

$$F = -kx. \tag{11–1}$$

The maximum displacement from equilibrium is called the **amplitude**.

The **period**, T, is the time required for one complete cycle (back and forth), and the **frequency**, f, is the number of cycles per second; they are related by

$$f = \frac{1}{T}. \tag{11–2}$$

The period of oscillation for a mass m on the end of a spring is given by

$$T = 2\pi\sqrt{\frac{m}{k}}. \tag{11–6a}$$

SHM is **sinusoidal**, which means that the displacement as a function of time follows a sine curve.

During SHM, the total energy

$$E = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 \tag{11–3}$$

is continually changing from potential to kinetic and back again.

A **simple pendulum** of length ℓ approximates SHM if its amplitude is small and friction can be ignored. For small amplitudes, its period is given by

$$T = 2\pi\sqrt{\frac{\ell}{g}}, \tag{11–11a}$$

where g is the acceleration of gravity.

When friction is present (for all real springs and pendulums), the motion is said to be **damped**. The maximum displacement decreases in time, and the mechanical energy is eventually all transformed to thermal energy.

If a varying force of frequency f is applied to a system capable of oscillating, the amplitude of oscillation can be very large if the frequency of the applied force is near the **natural** (or **resonant**) **frequency** of the oscillator. This is called **resonance**.

Vibrating objects act as sources of **waves** that travel outward from the source. Waves on water and on a cord are examples. The wave may be a **pulse** (a single crest), or it may be continuous (many crests and troughs).

The **wavelength** of a continuous sinusoidal wave is the distance between two successive crests.

The **frequency** is the number of full wavelengths (or crests) that pass a given point per unit time.

The **amplitude** of a wave is the maximum height of a crest, or depth of a trough, relative to the normal (or equilibrium) level.

The **wave speed** (how fast a crest moves) is equal to the product of wavelength and frequency,

$$v = \lambda f. \qquad \textbf{(11–12)}$$

In a **transverse wave**, the oscillations are perpendicular to the direction in which the wave travels. An example is a wave on a cord.

In a **longitudinal wave**, the oscillations are along (parallel to) the line of travel; sound is an example.

Waves carry energy from place to place without matter being carried. The **intensity** of a wave is the energy per unit time carried across unit area (in watts/m^2). For three-dimensional waves traveling outward from a point source, the intensity decreases inversely as the square of the distance from the source (ignoring damping):

$$I \propto \frac{1}{r^2}. \qquad \textbf{(11–16b)}$$

Wave intensity is proportional to the amplitude squared and to the frequency squared.

Waves reflect off objects in their path. When the **wave front** (of a two- or three-dimensional wave) strikes an object, the **angle of reflection** is equal to the **angle of incidence**. This is the **law of reflection**. When a wave strikes a boundary between two materials in which it can travel, part of the wave is reflected and part is transmitted.

When two waves pass through the same region of space at the same time, they **interfere**. The resultant displacement at any point and time is the sum of their separate displacements (= the **superposition principle**). This can result in **constructive interference**, **destructive interference**, or something in between, depending on the amplitudes and relative phases of the waves.

Waves traveling on a string of fixed length interfere with waves that have reflected off the end and are traveling back in the opposite direction. At certain frequencies, **standing waves** can be produced in which the waves seem to be standing still rather than traveling. The string (or other medium) is vibrating as a whole. This is a resonance phenomenon, and the frequencies at which standing waves occur are called **resonant frequencies**. Points of destructive interference (no oscillation) are called **nodes**. Points of constructive interference (maximum amplitude of vibration) are called **antinodes**.

[*Waves change direction, or **refract**, when traveling from one medium into a second medium where their speed is different. Waves spread, or **diffract**, as they travel and encounter obstacles. A rough guide to the amount of diffraction is $\theta \approx \lambda/\ell$, where λ is the wavelength and ℓ the width of an obstacle or opening. There is a significant "shadow region" only if the wavelength λ is smaller than the size of the obstacle.]

[*A traveling wave can be represented mathematically as $y = A \sin\{(2\pi/\lambda)(x \pm vt)\}$.]

Questions

1. Is the acceleration of a simple harmonic oscillator ever zero? If so, where?
2. Real springs have mass. Will the true period and frequency be larger or smaller than given by the equations for a mass oscillating on the end of an idealized massless spring? Explain.
3. How could you double the maximum speed of a simple harmonic oscillator (SHO)?
4. If a pendulum clock is accurate at sea level, will it gain or lose time when taken to high altitude? Why?
5. A tire swing hanging from a branch reaches nearly to the ground (Fig. 11–49). How could you estimate the height of the branch using only a stopwatch?

FIGURE 11–49 Question 5.

6. For a simple harmonic oscillator, when (if ever) are the displacement and velocity vectors in the same direction? When are the displacement and acceleration vectors in the same direction?
7. Two equal masses are attached to separate identical springs next to one another. One mass is pulled so its spring stretches 40 cm and the other is pulled so its spring stretches only 20 cm. The masses are released simultaneously. Which mass reaches the equilibrium point first?
8. What is the approximate period of your walking step?
9. What happens to the period of a playground swing if you rise up from sitting to a standing position?
10. Why can you make water slosh back and forth in a pan only if you shake the pan at a certain frequency?
11. Is the frequency of a simple periodic wave equal to the frequency of its source? Why or why not?
12. Explain the difference between the speed of a transverse wave traveling along a cord and the speed of a tiny piece of the cord.
13. What kind of waves do you think will travel along a horizontal metal rod if you strike its end (*a*) vertically from above and (*b*) horizontally parallel to its length?
14. Since the density of air decreases with an increase in temperature, but the bulk modulus B is nearly independent of temperature, how would you expect the speed of sound waves in air to vary with temperature?
15. If a rope has a free end, a pulse sent down the rope behaves differently on reflection than if the rope has that end fixed in position. What is this difference, and why does it occur?
16. How did geophysicists determine that part of the Earth's interior is liquid?

17. The speed of sound in most solids is somewhat greater than in air, yet the density of solids is much greater (10^3 to 10^4 times). Explain.

18. Give two reasons why circular water waves decrease in amplitude as they travel away from the source.

19. Two linear waves have the same amplitude and speed, and otherwise are identical, except one has half the wavelength of the other. Which transmits more energy? By what factor?

20. When a sinusoidal wave crosses the boundary between two sections of cord as in Fig. 11–34, the frequency does not change (although the wavelength and velocity do change). Explain why.

21. Is energy always conserved when two waves interfere? Explain.

22. If a string is vibrating as a standing wave in three loops, are there any places you could touch it with a knife blade without disturbing the motion?

23. Why do the strings used for the lowest-frequency notes on a piano normally have wire wrapped around them?

24. When a standing wave exists on a string, the vibrations of incident and reflected waves cancel at the nodes. Does this mean that energy was destroyed? Explain.

25. Can the amplitude of the standing waves in Fig. 11–40 be greater than the amplitude of the vibrations that cause them (up and down motion of the hand)?

26. "In a round bowl of water, waves move from the center to the rim, or from the rim to the center, depending on whether you strike at the center or at the rim." So wrote Dante Alighieri 700 years ago in his great poem *Paradiso* (Canto 14), the last part of his famous *Divine Comedy*. Try this experiment and discuss your results.

*27. AM radio signals can usually be heard behind a hill, but FM often cannot. That is, AM signals bend more than FM. Explain. (Radio signals, as we shall see, are carried by electromagnetic waves whose wavelength for AM is typically 200 to 600 m and for FM about 3 m.)

MisConceptual Questions

1. A mass on a spring in SHM (Fig. 11–1) has amplitude A and period T. At what point in the motion is the velocity zero and the acceleration zero simultaneously?
   (*a*) $x = A$.
   (*b*) $x > 0$ but $x < A$.
   (*c*) $x = 0$.
   (*d*) $x < 0$.
   (*e*) None of the above.

2. An object oscillates back and forth on the end of a spring. Which of the following statements are true at some time during the course of the motion?
   (*a*) The object can have zero velocity and, simultaneously, nonzero acceleration.
   (*b*) The object can have zero velocity and, simultaneously, zero acceleration.
   (*c*) The object can have zero acceleration and, simultaneously, nonzero velocity.
   (*d*) The object can have nonzero velocity and nonzero acceleration simultaneously.

3. An object of mass M oscillates on the end of a spring. To double the period, replace the object with one of mass:
   (*a*) $2M$.
   (*b*) $M/2$.
   (*c*) $4M$.
   (*d*) $M/4$.
   (*e*) None of the above.

4. An object of mass m rests on a frictionless surface and is attached to a horizontal ideal spring with spring constant k. The system oscillates with amplitude A. The oscillation frequency of this system can be increased by
   (*a*) decreasing k.
   (*b*) decreasing m.
   (*c*) increasing A.
   (*d*) More than one of the above.
   (*e*) None of the above will work.

5. When you use the approximation $\sin\theta \approx \theta$ for a pendulum, you must specify the angle θ in
   (*a*) radians only.
   (*b*) degrees only.
   (*c*) revolutions or radians.
   (*d*) degrees or radians.

6. Suppose you pull a simple pendulum to one side by an angle of 5°, let go, and measure the period of oscillation that ensues. Then you stop the oscillation, pull the pendulum to an angle of 10°, and let go. The resulting oscillation will have a period about ________ the period of the first oscillation.
   (*a*) four times
   (*b*) twice
   (*c*) half
   (*d*) one-fourth
   (*e*) the same as

7. At a playground, two young children are on identical swings. One child appears to be about twice as heavy as the other. If you pull them back together the same distance and release them to start them swinging, what will you notice about the oscillations of the two children?
   (*a*) The heavier child swings with a period twice that of the lighter one.
   (*b*) The lighter child swings with a period twice that of the heavier one.
   (*c*) Both children swing with the same period.

8. A grandfather clock is "losing" time because its pendulum moves too slowly. Assume that the pendulum is a massive bob at the end of a string. The motion of this pendulum can be sped up by (list all that work):
   (*a*) shortening the string.
   (*b*) lengthening the string.
   (*c*) increasing the mass of the bob.
   (*d*) decreasing the mass of the bob.

9. Consider a wave traveling down a cord and the transverse motion of a small piece of the cord. Which of the following is true?
(*a*) The speed of the wave must be the same as the speed of a small piece of the cord.
(*b*) The frequency of the wave must be the same as the frequency of a small piece of the cord.
(*c*) The amplitude of the wave must be the same as the amplitude of a small piece of the cord.
(*d*) All of the above are true.
(*e*) Both (*b*) and (*c*) are true.

10. Two waves are traveling toward each other along a rope. When they meet, the waves
(*a*) pass through each other.
(*b*) bounce off of each other.
(*c*) disappear.

11. Which of the following increases the speed of waves in a stretched elastic cord? (More than one answer may apply.)
(*a*) Increasing the wave amplitude.
(*b*) Increasing the wave frequency.
(*c*) Increasing the wavelength.
(*d*) Stretching the elastic cord further.

12. Consider a wave on a string moving to the right, as shown in Fig. 11–50. What is the direction of the velocity of a particle of string at point B?
(*a*) →
(*b*) ↘
(*c*) ↓
(*d*) ↑
(*e*) $\vec{\mathbf{v}} = 0$, so no direction.

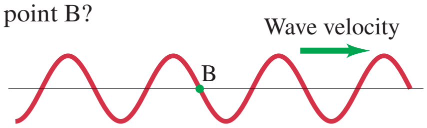


FIGURE 11–50
MisConceptual Question 12.

13. What happens when two waves, such as waves on a lake, come from different directions and run into each other?
(*a*) They cancel each other out and disappear.
(*b*) If they are the same size, they cancel each other out and disappear. If one wave is larger than the other, the smaller one disappears and the larger one shrinks but continues.
(*c*) They get larger where they run into each other; then they continue in a direction between the direction of the two original waves and larger than either original wave.
(*d*) They may have various patterns where they overlap, but each wave continues with its original pattern away from the region of overlap.
(*e*) Waves cannot run into each other; they always come from the same direction and so are parallel.

14. A student attaches one end of a Slinky to the top of a table. She holds the other end in her hand, stretches it to a length ℓ, and then moves it back and forth to send a wave down the Slinky. If she next moves her hand faster while keeping the length of the Slinky the same, how does the wavelength down the Slinky change?
(*a*) It increases.
(*b*) It stays the same.
(*c*) It decreases.

15. A wave transports
(*a*) energy but not matter.
(*b*) matter but not energy.
(*c*) both energy and matter.

For assigned homework and other learning materials, go to the MasteringPhysics website.

Problems

11–1 to 11–3 Simple Harmonic Motion

1. (I) If a particle undergoes SHM with amplitude 0.21 m, what is the total distance it travels in one period?

2. (I) The springs of a 1700-kg car compress 5.0 mm when its 66-kg driver gets into the driver's seat. If the car goes over a bump, what will be the frequency of oscillations? Ignore damping.

3. (II) An elastic cord is 61 cm long when a weight of 75 N hangs from it but is 85 cm long when a weight of 210 N hangs from it. What is the "spring" constant k of this elastic cord?

4. (II) Estimate the stiffness of the spring in a child's pogo stick if the child has a mass of 32 kg and bounces once every 2.0 seconds.

5. (II) A fisherman's scale stretches 3.6 cm when a 2.4-kg fish hangs from it. (*a*) What is the spring stiffness constant and (*b*) what will be the amplitude and frequency of oscillation if the fish is pulled down 2.1 cm more and released so that it oscillates up and down?

6. (II) A small fly of mass 0.22 g is caught in a spider's web. The web oscillates predominantly with a frequency of 4.0 Hz. (*a*) What is the value of the effective spring stiffness constant k for the web? (*b*) At what frequency would you expect the web to oscillate if an insect of mass 0.44 g were trapped?

7. (II) A mass m at the end of a spring oscillates with a frequency of 0.83 Hz. When an additional 780-g mass is added to m, the frequency is 0.60 Hz. What is the value of m?

8. (II) A vertical spring with spring stiffness constant 305 N/m oscillates with an amplitude of 28.0 cm when 0.235 kg hangs from it. The mass passes through the equilibrium point $(y = 0)$ with positive velocity at $t = 0$. (*a*) What equation describes this motion as a function of time? (*b*) At what times will the spring be longest and shortest?

9. (II) Figure 11–51 shows two examples of SHM, labeled A and B. For each, what is (*a*) the amplitude, (*b*) the frequency, and (*c*) the period?

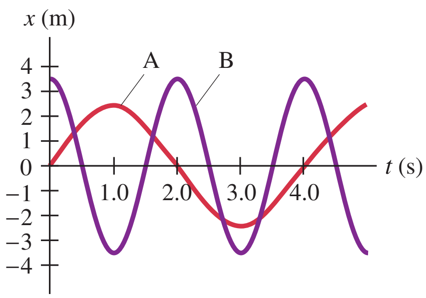


FIGURE 11–51
Problem 9.

10. (II) A balsa wood block of mass 52 g floats on a lake, bobbing up and down at a frequency of 3.0 Hz. (*a*) What is the value of the effective spring constant of the water? (*b*) A partially filled water bottle of mass 0.28 kg and almost the same size and shape of the balsa block is tossed into the water. At what frequency would you expect the bottle to bob up and down? Assume SHM.

11. (II) At what displacement of a SHO is the energy half kinetic and half potential?

12. (II) An object of unknown mass m is hung from a vertical spring of unknown spring constant k, and the object is observed to be at rest when the spring has stretched by 14 cm. The object is then given a slight push upward and executes SHM. Determine the period T of this oscillation.

13. (II) A 1.65-kg mass stretches a vertical spring 0.215 m. If the spring is stretched an additional 0.130 m and released, how long does it take to reach the (new) equilibrium position again?

14. (II) A 1.15-kg mass oscillates according to the equation $x = 0.650\cos(8.40t)$ where x is in meters and t in seconds. Determine (*a*) the amplitude, (*b*) the frequency, (*c*) the total energy, and (*d*) the kinetic energy and potential energy when $x = 0.360$ m.

15. (II) A 0.25-kg mass at the end of a spring oscillates 2.2 times per second with an amplitude of 0.15 m. Determine (*a*) the speed when it passes the equilibrium point, (*b*) the speed when it is 0.10 m from equilibrium, (*c*) the total energy of the system, and (*d*) the equation describing the motion of the mass, assuming that at $t = 0$, x was a maximum.

16. (II) It takes a force of 91.0 N to compress the spring of a toy popgun 0.175 m to "load" a 0.160-kg ball. With what speed will the ball leave the gun if fired horizontally?

17. (II) If one oscillation has 3.0 times the energy of a second one of equal frequency and mass, what is the ratio of their amplitudes?

18. (II) A mass of 240 g oscillates on a horizontal frictionless surface at a frequency of 2.5 Hz and with amplitude of 4.5 cm. (*a*) What is the effective spring constant for this motion? (*b*) How much energy is involved in this motion?

19. (II) An object with mass 2.7 kg is executing simple harmonic motion, attached to a spring with spring constant $k = 310$ N/m. When the object is 0.020 m from its equilibrium position, it is moving with a speed of 0.55 m/s. (*a*) Calculate the amplitude of the motion. (*b*) Calculate the maximum speed attained by the object.

20. (II) At $t = 0$, an 885-g mass at rest on the end of a horizontal spring ($k = 184$ N/m) is struck by a hammer which gives it an initial speed of 2.26 m/s. Determine (*a*) the period and frequency of the motion, (*b*) the amplitude, (*c*) the maximum acceleration, (*d*) the total energy, and (*e*) the kinetic energy when $x = 0.40A$ where A is the amplitude.

21. (III) Agent Arlene devised the following method of measuring the muzzle velocity of a rifle (Fig. 11–52). She fires a bullet into a 4.148-kg wooden block resting on a smooth surface, and attached to a spring of spring constant $k = 162.7$ N/m. The bullet, whose mass is 7.870 g, remains embedded in the wooden block. She measures the maximum distance that the block compresses the spring to be 9.460 cm. What is the speed v of the bullet?

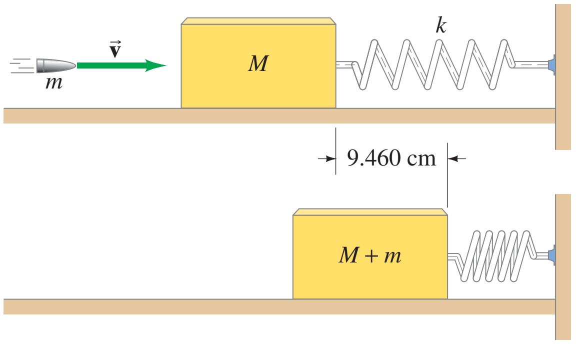


FIGURE 11–52 Problem 21.

22. (III) A bungee jumper with mass 65.0 kg jumps from a high bridge. After arriving at his lowest point, he oscillates up and down, reaching a low point seven more times in 43.0 s. He finally comes to rest 25.0 m below the level of the bridge. Estimate the spring stiffness constant and the unstretched length of the bungee cord assuming SHM.

23. (III) A block of mass m is supported by two identical parallel vertical springs, each with spring stiffness constant k (Fig. 11–53). What will be the frequency of vertical oscillation?

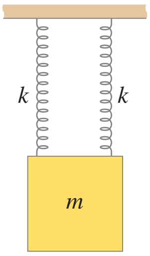


FIGURE 11–53
Problem 23.

24. (III) A 1.60-kg object oscillates at the end of a vertically hanging light spring once every 0.45 s. (*a*) Write down the equation giving its position y (+ upward) as a function of time t. Assume the object started by being compressed 16 cm from the equilibrium position (where $y = 0$), and released. (*b*) How long will it take to get to the equilibrium position for the first time? (*c*) What will be its maximum speed? (*d*) What will be the object's maximum acceleration, and where will it first be attained?

11–4 Simple Pendulum

25. (I) A pendulum has a period of 1.85 s on Earth. What is its period on Mars, where the acceleration of gravity is about 0.37 that on Earth?

26. (I) How long must a simple pendulum be if it is to make exactly one swing per second? (That is, one complete oscillation takes exactly 2.0 s.)

27. (I) A pendulum makes 28 oscillations in exactly 50 s. What is its (*a*) period and (*b*) frequency?

28. (II) What is the period of a simple pendulum 47 cm long (*a*) on the Earth, and (*b*) when it is in a freely falling elevator?

29. (II) Your grandfather clock's pendulum has a length of 0.9930 m. If the clock runs slow and loses 21 s per day, how should you adjust the length of the pendulum?

30. (II) Derive a formula for the maximum speed v_{max} of a simple pendulum bob in terms of g, the length ℓ, and the maximum angle of swing θ_{max}.

31. (III) A clock pendulum oscillates at a frequency of 2.5 Hz. At $t = 0$, it is released from rest starting at an angle of 12° to the vertical. Ignoring friction, what will be the position (angle in radians) of the pendulum at (*a*) $t = 0.25$ s, (*b*) $t = 1.60$ s, and (*c*) $t = 500$ s?

11–7 and 11–8 Waves

32. (I) A fisherman notices that wave crests pass the bow of his anchored boat every 3.0 s. He measures the distance between two crests to be 7.0 m. How fast are the waves traveling?

33. (I) A sound wave in air has a frequency of 282 Hz and travels with a speed of 343 m/s. How far apart are the wave crests (compressions)?

34. (I) Calculate the speed of longitudinal waves in (*a*) water, (*b*) granite, and (*c*) steel.

35. (I) AM radio signals have frequencies between 550 kHz and 1600 kHz (kilohertz) and travel with a speed of 3.0×10^8 m/s. What are the wavelengths of these signals? On FM the frequencies range from 88 MHz to 108 MHz (megahertz) and travel at the same speed. What are their wavelengths?

36. (II) P and S waves from an earthquake travel at different speeds, and this difference helps locate the earthquake "epicenter" (where the disturbance took place). (*a*) Assuming typical speeds of 8.5 km/s and 5.5 km/s for P and S waves, respectively, how far away did an earthquake occur if a particular seismic station detects the arrival of these two types of waves 1.5 min apart? (*b*) Is one seismic station sufficient to determine the position of the epicenter? Explain.

37. (II) A cord of mass 0.65 kg is stretched between two supports 8.0 m apart. If the tension in the cord is 120 N, how long will it take a pulse to travel from one support to the other?

38. (II) A 0.40-kg cord is stretched between two supports, 8.7 m apart. When one support is struck by a hammer, a transverse wave travels down the cord and reaches the other support in 0.85 s. What is the tension in the cord?

39. (II) A sailor strikes the side of his ship just below the surface of the sea. He hears the echo of the wave reflected from the ocean floor directly below 2.4 s later. How deep is the ocean at this point?

40. (II) Two children are sending signals along a cord of total mass 0.50 kg tied between tin cans with a tension of 35 N. It takes the vibrations in the string 0.55 s to go from one child to the other. How far apart are the children?

11–9 Energy Transported by Waves

41. (II) What is the ratio of (*a*) the intensities, and (*b*) the amplitudes, of an earthquake P wave passing through the Earth and detected at two points 15 km and 45 km from the source?

42. (II) The intensity of an earthquake wave passing through the Earth is measured to be 3.0×10^6 J/m^2·s at a distance of 54 km from the source. (*a*) What was its intensity when it passed a point only 1.0 km from the source? (*b*) At what rate did energy pass through an area of 2.0 m^2 at 1.0 km?

43. (II) A bug on the surface of a pond is observed to move up and down a total vertical distance of 7.0 cm, from the lowest to the highest point, as a wave passes. If the ripples decrease to 4.5 cm, by what factor does the bug's maximum KE change?

11–11 Interference

44. (I) The two pulses shown in Fig. 11–54 are moving toward each other. (*a*) Sketch the shape of the string at the moment they directly overlap. (*b*) Sketch the shape of the string a few moments later. (*c*) In Fig. 11–37a, at the moment the pulses pass each other, the string is straight. What has happened to the energy at this moment?

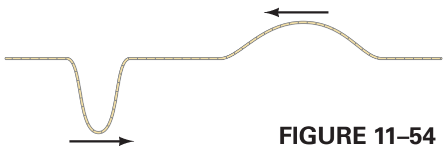


FIGURE 11–54 Problem 44.

11–12 Standing Waves; Resonance

45. (I) If a violin string vibrates at 440 Hz as its fundamental frequency, what are the frequencies of the first four harmonics?

46. (I) A violin string vibrates at 294 Hz when unfingered. At what frequency will it vibrate if it is fingered one-third of the way down from the end? (That is, only two-thirds of the string vibrates as a standing wave.)

47. (I) A particular string resonates in four loops at a frequency of 240 Hz. Give at least three other frequencies at which it will resonate. What is each called?

48. (II) The speed of waves on a string is 97 m/s. If the frequency of standing waves is 475 Hz, how far apart are two adjacent nodes?

49. (II) If two successive overtones of a vibrating string are 280 Hz and 350 Hz, what is the frequency of the fundamental?

50. (II) One end of a horizontal string is attached to a small-amplitude mechanical 60.0-Hz oscillator. The string's mass per unit length is 3.5×10^{-4} kg/m. The string passes over a pulley, a distance $\ell = 1.50$ m away, and weights are hung from this end, Fig. 11–55. What mass m must be hung from this end of the string to produce (*a*) one loop, (*b*) two loops, and (*c*) five loops of a standing wave? Assume the string at the oscillator is a node, which is nearly true.

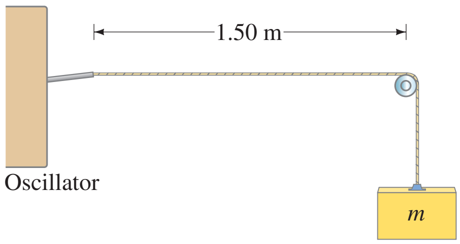


FIGURE 11–55 Problems 50 and 51.

51. (II) In Problem 50 (Fig. 11–55), the length ℓ of the string may be adjusted by moving the pulley. If the hanging mass m is fixed at 0.080 kg, how many different standing wave patterns may be achieved by varying ℓ between 10 cm and 1.5 m?

52. (II) When you slosh the water back and forth in a tub at just the right frequency, the water alternately rises and falls at each end, remaining relatively calm at the center. Suppose the frequency to produce such a standing wave in a 75-cm-wide tub is 0.85 Hz. What is the speed of the water wave?

*11–13 Refraction

*53. (I) An earthquake P wave traveling at 8.0 km/s strikes a boundary within the Earth between two kinds of material. If it approaches the boundary at an incident angle of 44° and the angle of refraction is 33°, what is the speed in the second medium?

*54. (II) A sound wave is traveling in warm air when it hits a layer of cold, dense air. If the sound wave hits the cold air interface at an angle of 25°, what is the angle of refraction? Assume that the cold air temperature is −15°C and the warm air temperature is +15°C. The speed of sound as a function of temperature can be approximated by $v = (331 + 0.60\,T)$ m/s, where T is in °C.

*11–14 Diffraction

*55. (II) What frequency of sound would have a wavelength the same size as a 0.75-m-wide window? (The speed of sound is 344 m/s at 20°C.) What frequencies would diffract through the window?

General Problems

56. A 62-kg person jumps from a window to a fire net 20.0 m directly below, which stretches the net 1.4 m. Assume that the net behaves like a simple spring. (*a*) Calculate how much it would stretch if the same person were lying in it. (*b*) How much would it stretch if the person jumped from 38 m?

57. An energy-absorbing car bumper has a spring constant of 410 kN/m. Find the maximum compression of the bumper if the car, with mass 1300 kg, collides with a wall at a speed of 2.0 m/s (approximately 5 mi/h).

58. The length of a simple pendulum is 0.72 m, the pendulum bob has a mass of 295 g, and it is released at an angle of 12° to the vertical. Assume SHM. (*a*) With what frequency does it oscillate? (*b*) What is the pendulum bob's speed when it passes through the lowest point of the swing? (*c*) What is the total energy stored in this oscillation assuming no losses?

59. A block with mass $M = 6.0$ kg rests on a frictionless table and is attached by a horizontal spring $(k = 130\text{ N/m})$ to a wall. A second block, of mass $m = 1.25$ kg, rests on top of M. The coefficient of static friction between the two blocks is 0.30. What is the maximum possible amplitude of oscillation such that m will not slip off M?

60. A simple pendulum oscillates with frequency f. What is its frequency if the entire pendulum accelerates at $0.35\,g$ (*a*) upward, and (*b*) downward?

61. A 0.650-kg mass oscillates according to the equation $x = 0.25\sin(4.70\,t)$ where x is in meters and t is in seconds. Determine (*a*) the amplitude, (*b*) the frequency, (*c*) the period, (*d*) the total energy, and (*e*) the kinetic energy and potential energy when x is 15 cm.

62. An oxygen atom at a particular site within a DNA molecule can be made to execute simple harmonic motion when illuminated by infrared light. The oxygen atom is bound with a spring-like chemical bond to a phosphorus atom, which is rigidly attached to the DNA backbone. The oscillation of the oxygen atom occurs with frequency $f = 3.7 \times 10^{13}$ Hz. If the oxygen atom at this site is chemically replaced with a sulfur atom, the spring constant of the bond is unchanged (sulfur is just below oxygen in the Periodic Table). Predict the frequency after the sulfur substitution.

63. A diving board oscillates with simple harmonic motion of frequency 2.8 cycles per second. What is the maximum amplitude with which the end of the board can oscillate in order that a pebble placed there (Fig. 11–56) does not lose contact with the board during the oscillation?

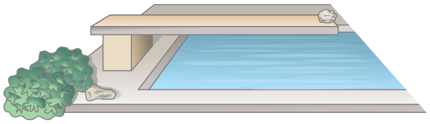


FIGURE 11–56 Problem 63.

64. A 950-kg car strikes a huge spring at a speed of 25 m/s (Fig. 11–57), compressing the spring 4.0 m. (*a*) What is the spring stiffness constant of the spring? (*b*) How long is the car in contact with the spring before it bounces off in the opposite direction?

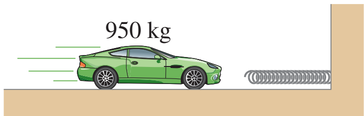


FIGURE 11–57 Problem 64.

65. Carbon dioxide is a linear molecule. The carbon–oxygen bonds in this molecule act very much like springs. Figure 11–58 shows one possible way the oxygen atoms in this molecule can oscillate: the oxygen atoms oscillate symmetrically in and out, while the central carbon atom remains at rest. Hence each oxygen atom acts like a simple harmonic oscillator with a mass equal to the mass of an oxygen atom. It is observed that this oscillation occurs at a frequency $f = 2.83 \times 10^{13}$ Hz. What is the spring constant of the C—O bond?

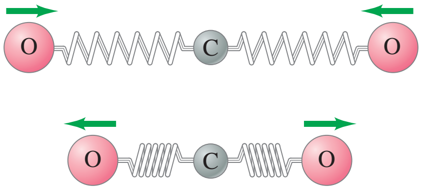


FIGURE 11–58 Problem 65, the CO_2 molecule.

66. Tall buildings are designed to sway in the wind. In a 100-km/h wind, suppose the top of a 110-story building oscillates horizontally with an amplitude of 15 cm at its natural frequency, which corresponds to a period of 7.0 s. Assuming SHM, find the maximum horizontal velocity and acceleration experienced by an employee as she sits working at her desk located on the top floor. Compare the maximum acceleration (as a percentage) with the acceleration due to gravity.

67. When you walk with a cup of coffee (diameter 8 cm) at just the right pace of about one step per second, the coffee sloshes higher and higher in your cup until eventually it starts to spill over the top, Fig 11–59. Estimate the speed of the waves in the coffee.

FIGURE 11–59 Problem 67.

68. A bug on the surface of a pond is observed to move up and down a total vertical distance of 0.12 m, lowest to highest point, as a wave passes. (*a*) What is the amplitude of the wave? (*b*) If the amplitude increases to 0.16 m, by what factor does the bug's maximum kinetic energy change?

69. An earthquake-produced surface wave can be approximated by a sinusoidal transverse wave. Assuming a frequency of 0.60 Hz (typical of earthquakes, which actually include a mixture of frequencies), what amplitude is needed so that objects begin to leave contact with the ground? [*Hint*: Set the acceleration $a > g$.]

70. Two strings on a musical instrument are tuned to play at 392 Hz (G) and 494 Hz (B). (*a*) What are the frequencies of the first two overtones for each string? (*b*) If the two strings have the same length and are under the same tension, what must be the ratio of their masses (m_G/m_B)? (*c*) If the strings, instead, have the same mass per unit length and are under the same tension, what is the ratio of their lengths (ℓ_G/ℓ_B)? (*d*) If their masses and lengths are the same, what must be the ratio of the tensions in the two strings?

71. A wave with a frequency of 180 Hz and a wavelength of 10.0 cm is traveling along a cord. The maximum speed of particles on the cord is the same as the wave speed. What is the amplitude of the wave?

72. Estimate the average power of a moving water wave that strikes the chest of an adult standing in the water at the seashore. Assume that the amplitude of the wave is 0.50 m, the wavelength is 2.5 m, and the period is 4.0 s.

73. A tsunami is a sort of pulse or "wave packet" consisting of several crests and troughs that become dramatically large as they enter shallow water at the shore. Suppose a tsunami of wavelength 235 km and velocity 550 km/h travels across the Pacific Ocean. As it approaches Hawaii, people observe an unusual decrease of sea level in the harbors. Approximately how much time do they have to run to safety? (In the absence of knowledge and warning, people have died during tsunamis, some of them attracted to the shore to see stranded fishes and boats.)

***74.** For any type of wave that reaches a boundary beyond which its speed is increased, there is a maximum incident angle if there is to be a transmitted refracted wave. This maximum incident angle θ_{iM} corresponds to an angle of refraction equal to 90°. If $\theta_i > \theta_{iM}$, all the wave is reflected at the boundary and none is refracted, because refraction would correspond to $\sin\theta_r > 1$ (where θ_r is the angle of refraction), which is impossible. This phenomenon is referred to as *total internal reflection*. (*a*) Find a formula for θ_{iM} using the law of refraction, Eq. 11–20. (*b*) How far from the bank should a trout fisherman stand (Fig. 11–60) so trout won't be frightened by his voice (1.8 m above the ground)? The speed of sound is about 343 m/s in air and 1440 m/s in water.

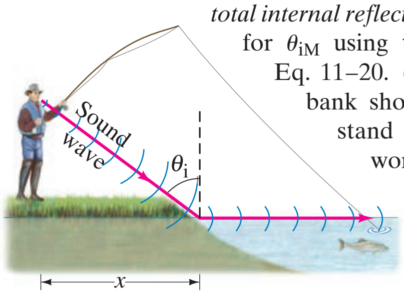


FIGURE 11–60 Problem 74b.

Search and Learn

1. Describe a procedure to measure the spring constant k of a car's springs. Assume that the owner's manual gives the car's mass M and that the shock absorbers are worn out so that the springs are underdamped. (See Sections 11–3 and 11–5.)

2. A particular unbalanced wheel of a car shakes when the car moves at 90.0 km/h. The wheel plus tire has mass 17.0 kg and diameter 0.58 m. By how much will the springs of this car compress when it is loaded with 280 kg? (Assume the 280 kg is split evenly among all four springs, which are identical.) [*Hint*: Reread Sections 11–1, 11–3, 11–6, and 8–3.]

3. Sometimes a car develops a pronounced rattle or vibration at a particular speed, especially if the road is hot enough that the tar between concrete slabs bumps up at regularly spaced intervals. Reread Sections 11–5 and 11–6, and decide whether each of the following is a factor and, if so, how: underdamping, overdamping, critical damping, and forced resonance.

4. Destructive interference occurs where two overlapping waves are $\frac{1}{2}$ wavelength or 180° out of phase. Explain why 180° is equivalent to $\frac{1}{2}$ wavelength.

5. Estimate the effective spring constant of a trampoline. [*Hint*: Go and jump, or watch, and give your data.]

ANSWERS TO EXERCISES

A: (*b*).
B: (*c*).
C: (*a*) Increases; (*b*) increases; (*c*) increases.
D: (*c*).
E: (*c*).
F: (*a*).
G: (*c*).
H: (*d*).