6–5 Conservative and Nonconservative Forces

The work done against gravity in moving an object from one point to another does not depend on the path taken. For example, it takes the same work $(= mgh)$ to lift an object of mass m vertically a height h as to carry it up an incline of the same vertical height, as in Fig. 6–4 (see Example 6–2). Forces such as gravity, for which the work done does not depend on the path taken but only on the initial and final positions, are called **conservative forces**. The elastic force of a spring (or other elastic material), in which $F = -kx$, is also a conservative force. An object that starts at a given point and returns to that same point under the action of a conservative force has no net work done on it because the potential energy is the same at the start and the finish of such a round trip.

Many forces, such as friction and a push or pull exerted by a person, are **nonconservative forces** since any work they do depends on the path. For example, if you push a crate across a floor from one point to another, the work you do depends on whether the path taken is straight or is curved. As shown in Fig. 6–16, if a crate is pushed slowly from point 1 to point 2 along the longer semicircular path, you do more work against friction than if you push it along the straight path.

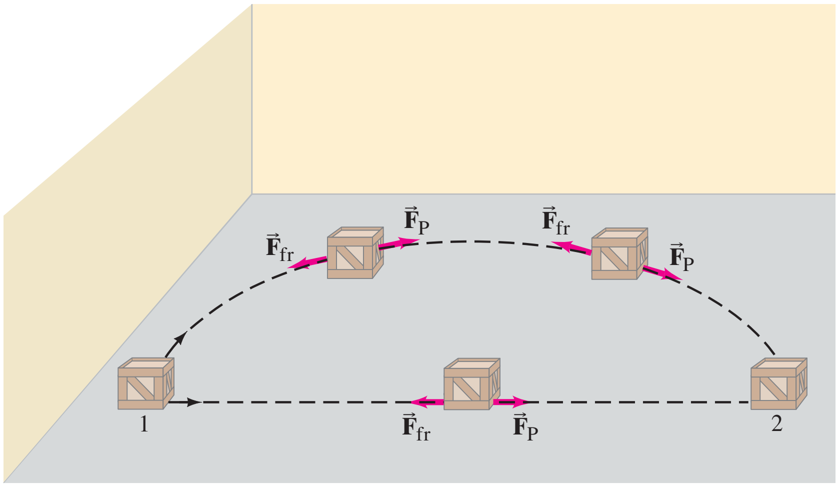


FIGURE 6–16 A crate is pushed slowly at constant speed across a rough floor from position 1 to position 2 via two paths, one straight and one curved. The pushing force $\vec{\mathbf{F}}_P$ is in the direction of motion at each point. (The friction force opposes the motion.) Hence for a constant magnitude pushing force, the work it does is $W = F_P d$, so if the distance traveled d is greater (as for the curved path), then W is greater. The work done does not depend only on points 1 and 2; it also depends on the path taken.

You do more work on the curved path because the distance is greater and, unlike the gravitational force, the pushing force $\vec{\mathbf{F}}_P$ is in the direction of motion at each point. Thus the work done by the person in Fig. 6–16 does not depend *only* on points 1 and 2; it depends also on the path taken. The force of kinetic friction, also shown in Fig. 6–16, always opposes the motion; it too is a nonconservative force, and we discuss how to treat it later in this Chapter (Section 6–9). Table 6–1 lists a few conservative and nonconservative forces.

Because potential energy is energy associated with the position or configuration of objects, potential energy can only make sense if it can be stated uniquely for a given point. This cannot be done with nonconservative forces because the work done depends on the path taken (as in Fig. 6–16). Hence, *potential energy can be defined only for a conservative force*. Thus, although potential energy is always associated with a force, not all forces have a potential energy. For example, there is no potential energy for friction.

TABLE 6–1 Conservative and Nonconservative Forces

Conservative Forces	Nonconservative Forces
Gravitational	Friction
Elastic	Air resistance
Electric	Tension in cord
	Motor or rocket propulsion
	Push or pull by a person

EXERCISE E An object acted on by a constant force F moves from point 1 to point 2 and back again. The work done by the force F in this round trip is 60 J. Can you determine from this information if F is a conservative or nonconservative force?

Work-Energy Extended

We can extend the **work-energy principle** (discussed in Section 6–3) to include potential energy. Suppose several forces act on an object which can undergo translational motion. And suppose only some of these forces are conservative. We write the total (net) work W_{net} as a sum of the work done by conservative forces, W_{C}, and the work done by nonconservative forces, W_{NC}:

$$W_{\text{net}} = W_{\text{C}} + W_{\text{NC}}.$$

Then, from the work-energy principle, Eq. 6–4, we have

$$W_{\text{net}} = \Delta\text{KE}$$
$$W_{\text{C}} + W_{\text{NC}} = \Delta\text{KE}$$

where $\Delta\text{KE} = \text{KE}_2 - \text{KE}_1$. Then

$$W_{\text{NC}} = \Delta\text{KE} - W_{\text{C}}.$$

Work done by a conservative force can be written in terms of potential energy, as we saw in Eq. 6–7b for gravitational potential energy:

$$W_{\text{C}} = -\Delta\text{PE}.$$

We combine these last two equations:

$$W_{\text{NC}} = \Delta\text{KE} + \Delta\text{PE}. \qquad \textbf{(6–10)}$$

Thus, *the work W_{NC} done by the nonconservative forces acting on an object is equal to the total change in kinetic and potential energies.*

It must be emphasized that *all* the forces acting on an object must be included in Eq. 6–10, either in the potential energy term on the right (if it is a conservative force), or in the work term on the left (but not in both!).

6–6 Mechanical Energy and Its Conservation

If we can ignore friction and other nonconservative forces, or if only conservative forces do work on a system, we arrive at a particularly simple and beautiful relation involving energy.

When no nonconservative forces do work, then $W_{\text{NC}} = 0$ in the general form of the work-energy principle (Eq. 6–10). Then we have

$$\Delta\text{KE} + \Delta\text{PE} = 0 \qquad \begin{bmatrix}\text{conservative}\\ \text{forces only}\end{bmatrix} \qquad \textbf{(6–11a)}$$

or

$$(\text{KE}_2 - \text{KE}_1) + (\text{PE}_2 - \text{PE}_1) = 0. \qquad \begin{bmatrix}\text{conservative}\\ \text{forces only}\end{bmatrix} \qquad \textbf{(6–11b)}$$

We now define a quantity E, called the **total mechanical energy** of our system, as the sum of the kinetic and potential energies at any moment:

$$E = \text{KE} + \text{PE}.$$

Now we can rewrite Eq. 6–11b as

CONSERVATION OF MECHANICAL ENERGY

$$\text{KE}_2 + \text{PE}_2 = \text{KE}_1 + \text{PE}_1 \qquad \begin{bmatrix}\text{conservative}\\ \text{forces only}\end{bmatrix} \qquad \textbf{(6–12a)}$$

or

$$E_2 = E_1 = \text{constant}. \qquad \begin{bmatrix}\text{conservative}\\ \text{forces only}\end{bmatrix} \qquad \textbf{(6–12b)}$$

Equations 6–12 express a useful and profound principle regarding the total mechanical energy of a system—namely, that it is a **conserved quantity**. The total mechanical energy E remains constant as long as no nonconservative forces do work: $\text{KE} + \text{PE}$ at some initial time 1 is equal to the $\text{KE} + \text{PE}$ at any later time 2.

To say it another way, consider Eq. 6–11a which tells us $\Delta\text{PE} = -\Delta\text{KE}$; that is, if the kinetic energy KE of a system increases, then the potential energy PE must decrease by an equivalent amount to compensate. Thus, the total, KE + PE, remains constant:

If only conservative forces do work, the total mechanical energy of a system neither increases nor decreases in any process. It stays constant—it is conserved.

CONSERVATION OF MECHANICAL ENERGY

This is the **principle of conservation of mechanical energy** for conservative forces.

In the next Section we shall see the great usefulness of the conservation of mechanical energy principle in a variety of situations, and how it is often easier to use than the kinematic equations or Newton's laws. After that we will discuss how other forms of energy can be included in the general conservation of energy law, such as energy associated with friction.

6–7 Problem Solving Using Conservation of Mechanical Energy

A simple example of the conservation of mechanical energy (neglecting air resistance) is a rock allowed to fall due to Earth's gravity from a height h above the ground, as shown in Fig. 6–17. If the rock starts from rest, all of the initial energy is potential energy. As the rock falls, the potential energy mgy decreases (because the rock's height above the ground y decreases), but the rock's kinetic energy increases to compensate, so that the sum of the two remains constant. At any point along the path, the total mechanical energy is given by

$$E = \text{KE} + \text{PE} = \tfrac{1}{2}mv^2 + mgy$$

where v is its speed at that point. If we let the subscript 1 represent the rock at one point along its path (for example, the initial point), and the subscript 2 represent it at some other point, then we can write

total mechanical energy at point 1 = total mechanical energy at point 2

or (see also Eq. 6–12a)

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2. \qquad \text{[gravity only]} \qquad \textbf{(6–13)}$$

Just before the rock hits the ground, where we chose $y = 0$, all of the initial potential energy will have been transformed into kinetic energy.

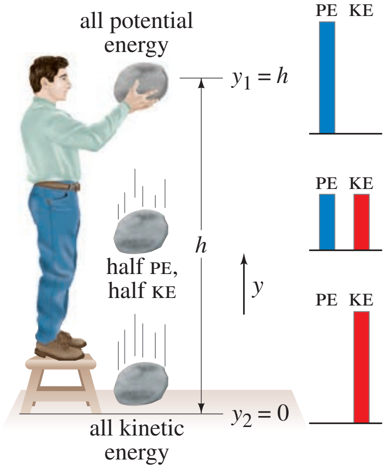


FIGURE 6–17 The rock's potential energy changes to kinetic energy as it falls. Note bar graphs representing potential energy PE and kinetic energy KE for the three different positions.

EXAMPLE 6–7 **Falling rock.** If the initial height of the rock in Fig. 6–17 is $y_1 = h = 3.0\text{ m}$, calculate the rock's velocity when it has fallen to 1.0 m above the ground.

APPROACH We apply the principle of conservation of mechanical energy, Eq. 6–13, with only gravity acting on the rock. We choose the ground as our reference level $(y = 0)$.

SOLUTION At the moment of release (point 1) the rock's position is $y_1 = 3.0\text{ m}$ and it is at rest: $v_1 = 0$. We want to find v_2 when the rock is at position $y_2 = 1.0\text{ m}$. Equation 6–13 gives

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2.$$

The m's cancel out and $v_1 = 0$, so

$$gy_1 = \tfrac{1}{2}v_2^2 + gy_2.$$

Solving for v_2 we find

$$v_2 = \sqrt{2g(y_1 - y_2)} = \sqrt{2(9.8\text{ m/s}^2)[(3.0\text{ m}) - (1.0\text{ m})]} = 6.3\text{ m/s}.$$

The rock's velocity 1.0 m above the ground is 6.3 m/s downward.

NOTE The velocity of the rock is independent of the rock's mass.

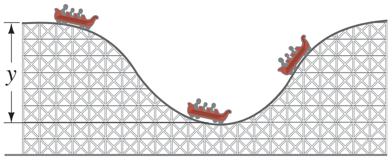


FIGURE 6–18 A roller-coaster car moving without friction illustrates the conservation of mechanical energy.

Equation 6–13 can be applied to any object moving without friction under the action of gravity. For example, Fig. 6–18 shows a roller-coaster car starting from rest at the top of a hill and coasting without friction to the bottom and up the hill on the other side. True, there is another force besides gravity acting on the car, the normal force exerted by the tracks. But the normal force acts perpendicular to the direction of motion at each point and so does zero work. We ignore rotational motion of the car's wheels and treat the car as a particle undergoing simple translation. Initially, the car has only potential energy. As it coasts down the hill, it loses potential energy and gains in kinetic energy, but the sum of the two remains constant. At the bottom of the hill it has its maximum kinetic energy, and as it climbs up the other side the kinetic energy changes back to potential energy. When the car comes to rest again at the same height from which it started, all of its energy will be potential energy. Given that the gravitational potential energy is proportional to the vertical height, energy conservation tells us that (in the absence of friction) the car comes to rest at a height equal to its original height. If the two hills are the same height, the car will just barely reach the top of the second hill when it stops. If the second hill is lower than the first, not all of the car's kinetic energy will be transformed to potential energy and the car can continue over the top and down the other side. If the second hill is higher, the car will reach a maximum height on it equal to its original height on the first hill. This is true (in the absence of friction) no matter how steep the hill is, since potential energy depends only on the vertical height (Eq. 6–6).

EXAMPLE 6–8 **Roller-coaster car speed using energy conservation.** Assuming the height of the hill in Fig. 6–18 is 40 m, and the roller-coaster car starts from rest at the top, calculate (*a*) the speed of the roller-coaster car at the bottom of the hill, and (*b*) at what height it will have half this speed. Take $y = 0$ at the bottom of the hill.

APPROACH We use conservation of mechanical energy. We choose point 1 to be where the car starts from rest $(v_1 = 0)$ at the top of the hill $(y_1 = 40\,\text{m})$. In part (*a*), point 2 is the bottom of the hill, which we choose as our reference level, so $y_2 = 0$. In part (*b*) we let y_2 be the unknown.

SOLUTION (*a*) We use Eq. 6–13 with $v_1 = 0$ and $y_2 = 0$, which gives

$$mgy_1 = \tfrac{1}{2}mv_2^2$$

or

$$\begin{aligned} v_2 &= \sqrt{2gy_1} \\ &= \sqrt{2(9.8\,\text{m/s}^2)(40\,\text{m})} = 28\,\text{m/s}. \end{aligned}$$

(*b*) Now y_2 will be an unknown. We again use conservation of energy,

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2,$$

but now $v_2 = \tfrac{1}{2}(28\,\text{m/s}) = 14\,\text{m/s}$ and $v_1 = 0$. Solving for the unknown y_2 gives

$$y_2 = y_1 - \frac{v_2^2}{2g} = 40\,\text{m} - \frac{(14\,\text{m/s})^2}{2(9.8\,\text{m/s}^2)} = 30\,\text{m}.$$

That is, the car has a speed of 14 m/s when it is 30 *vertical* meters above the lowest point, both when descending the left-hand hill and when ascending the right-hand hill.

The mathematics of the roller-coaster Example 6–8 is almost the same as in Example 6–7. But there is an important difference between them. In Example 6–7 the motion is all vertical and could have been solved using force, acceleration, and the kinematic equations (Eqs. 2–11). For the roller coaster, where the motion is not vertical, we could *not* have used Eqs. 2–11 because *a* is not constant on the curved track of Example 6–8. But energy conservation readily gives us the answer.

CONCEPTUAL EXAMPLE 6–9 **Speeds on two water slides.** Two water slides at a pool are shaped differently, but start at the same height h (Fig. 6–19). Two riders start from rest at the same time on different slides. (*a*) Which rider, Paul or Corinne, is traveling faster at the bottom? (*b*) Which rider makes it to the bottom first? Ignore friction and assume both slides have the same path length.

RESPONSE (*a*) Each rider's initial potential energy mgh gets transformed to kinetic energy, so the speed v at the bottom is obtained from $\frac{1}{2}mv^2 = mgh$. The mass cancels and so the speed will be the same, regardless of the mass of the rider. Since they descend the same vertical height, they will finish with the same speed. (*b*) Note that Corinne is consistently at a lower elevation than Paul at any instant, until the end. This means she has converted her potential energy to kinetic energy earlier. Consequently, she is traveling faster than Paul for the whole trip, and because the distance is the same, Corinne gets to the bottom first.

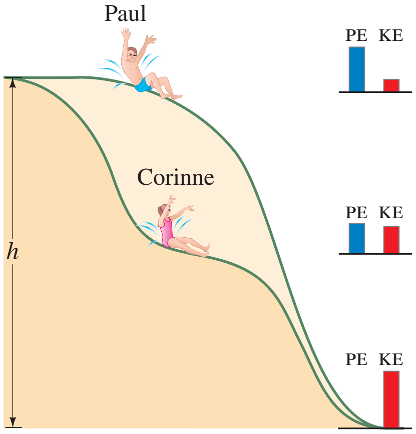


FIGURE 6–19 Example 6–9.


FIGURE 6–20 Transformation of energy during a pole vault: KE $\rightarrow$ PE_{el} $\rightarrow$ PE_G.

There are many interesting examples of the conservation of energy in sports, such as the pole vault illustrated in Fig. 6–20. We often have to make approximations, but the sequence of events in broad outline for the pole vault is as follows. The initial kinetic energy of the running athlete is transformed into elastic potential energy of the bending pole and, as the athlete leaves the ground, into gravitational potential energy. When the vaulter reaches the top and the pole has straightened out again, the energy has all been transformed into gravitational potential energy (if we ignore the vaulter's low horizontal speed over the bar). The pole does not supply any energy, but it acts as a device to *store* energy and thus aid in the transformation of kinetic energy into gravitational potential energy, which is the net result. The energy required to pass over the bar depends on how high the center of mass (CM) of the vaulter must be raised. By bending their bodies, pole vaulters keep their CM so low that it can actually pass slightly beneath the bar (Fig. 6–21), thus enabling them to cross over a higher bar than would otherwise be possible. (Center of mass is covered in Chapter 7.)

FIGURE 6–21 By bending her body, a pole vaulter can keep her center of mass so low that it may even pass below the bar.

As another example of the conservation of mechanical energy, let us consider an object of mass m connected to a compressed horizontal spring (Fig. 6–13b) whose own mass can be neglected and whose spring stiffness constant is k. When the spring is released, the mass m has speed v at any moment. The potential energy of the system (object plus spring) is $\frac{1}{2}kx^2$, where x is the displacement of the spring from its unstretched length (Eq. 6–9). If neither friction nor any other force is acting, conservation of mechanical energy tells us that

$$\frac{1}{2}mv_1^2 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2 + \frac{1}{2}kx_2^2, \qquad \text{[elastic PE only]} \quad \textbf{(6–14)}$$

where the subscripts 1 and 2 refer to the velocity and displacement at two different moments.

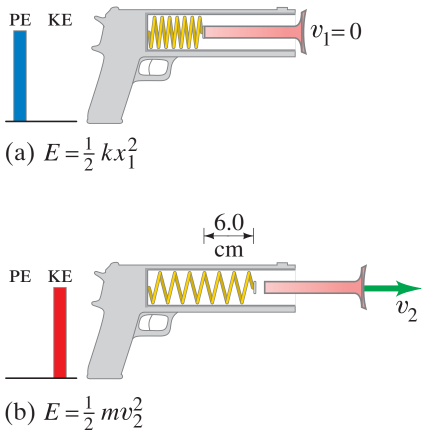


FIGURE 6–22 Example 6–10. (a) A dart is pushed against a spring, compressing it 6.0 cm. The dart is then released, and in (b) it leaves the spring at velocity v_2.

EXAMPLE 6–10 **Toy dart gun.** A dart of mass 0.100 kg is pressed against the spring of a toy dart gun as shown in Fig. 6–22a. The spring, with spring stiffness constant $k = 250\,\mathrm{N/m}$ and ignorable mass, is compressed 6.0 cm and released. If the dart detaches from the spring when the spring reaches its natural length $(x = 0)$, what speed does the dart acquire?

APPROACH The dart is initially at rest (point 1), so $\mathrm{KE}_1 = 0$. We ignore friction and use conservation of mechanical energy; the only potential energy is elastic.

SOLUTION We use Eq. 6–14 with point 1 being at the maximum compression of the spring, so $v_1 = 0$ (dart not yet released) and $x_1 = -0.060\,\mathrm{m}$. Point 2 we choose to be the instant the dart flies off the end of the spring (Fig. 6–22b), so $x_2 = 0$ and we want to find v_2. Thus Eq. 6–14 can be written

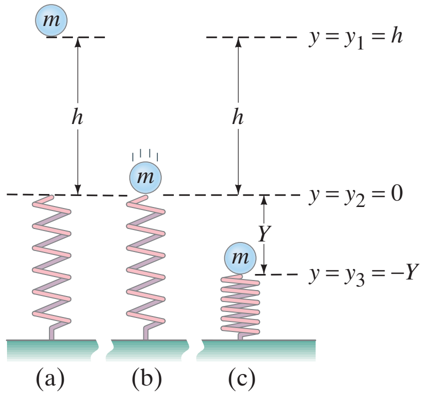
$$0 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + 0.$$

Then

$$v_2^2 = \frac{kx_1^2}{m} = \frac{(250\,\mathrm{N/m})(-0.060\,\mathrm{m})^2}{(0.100\,\mathrm{kg})} = 9.0\,\mathrm{m^2/s^2},$$

and $v_2 = \sqrt{v_2^2} = 3.0\,\mathrm{m/s}$.

FIGURE 6–23 Example 6–11. A falling ball compresses a spring.



EXAMPLE 6–11 **Two kinds of potential energy.** A ball of mass $m = 2.60\,\mathrm{kg}$, starting from rest, falls a vertical distance $h = 55.0\,\mathrm{cm}$ before striking a vertical coiled spring, which it compresses an amount $Y = 15.0\,\mathrm{cm}$ (Fig. 6–23). Determine the spring stiffness constant k of the spring. Assume the spring has negligible mass, and ignore air resistance. Measure all distances from the point where the ball first touches the uncompressed spring $(y = 0$ at this point).

APPROACH The forces acting on the ball are the gravitational pull of the Earth and the elastic force exerted by the spring. Both forces are conservative, so we can use conservation of mechanical energy, including both types of potential energy. We must be careful, however: gravity acts throughout the fall (Fig. 6–23), whereas the elastic force does not act until the ball touches the spring (Fig. 6–23b). We choose y positive upward, and $y = 0$ at the end of the spring in its natural (uncompressed) state.

SOLUTION We divide this solution into two parts. (An alternate solution follows.) *Part 1*: Let us first consider the energy changes as the ball falls from a height $y_1 = h = 0.550\,\mathrm{m}$, Fig. 6–23a, to $y_2 = 0$, just as it touches the spring, Fig. 6–23b. Our system is the ball acted on by gravity plus the spring (which up to this point doesn't do anything). Thus

$$\begin{aligned}\tfrac{1}{2}mv_1^2 + mgy_1 &= \tfrac{1}{2}mv_2^2 + mgy_2\\ 0 + mgh &= \tfrac{1}{2}mv_2^2 + 0.\end{aligned}$$

We solve for $v_2 = \sqrt{2gh} = \sqrt{2(9.80\,\mathrm{m/s^2})(0.550\,\mathrm{m})} = 3.283\,\mathrm{m/s} \approx 3.28\,\mathrm{m/s}$. This is the speed of the ball just as it touches the top of the spring, Fig. 6–23b. *Part 2*: As the ball compresses the spring, Figs. 6–23b to c, there are two conservative forces on the ball—gravity and the spring force. So our conservation of energy equation is

$$\begin{aligned}E_2\ (\text{ball touches spring}) &= E_3\ (\text{spring compressed})\\ \tfrac{1}{2}mv_2^2 + mgy_2 + \tfrac{1}{2}ky_2^2 &= \tfrac{1}{2}mv_3^2 + mgy_3 + \tfrac{1}{2}ky_3^2.\end{aligned}$$

Substituting $y_2 = 0$, $v_2 = 3.283\,\mathrm{m/s}$, $v_3 = 0$ (the ball comes to rest for an instant), and $y_3 = -Y = -0.150\,\mathrm{m}$, we have

$$\tfrac{1}{2}mv_2^2 + 0 + 0 = 0 - mgY + \tfrac{1}{2}k(-Y)^2.$$

We know m, v_2, and Y, so we can solve for k:

$$\begin{aligned}k &= \frac{2}{Y^2}\left[\tfrac{1}{2}mv_2^2 + mgY\right] = \frac{m}{Y^2}\left[v_2^2 + 2gY\right]\\ &= \frac{(2.60\,\mathrm{kg})}{(0.150\,\mathrm{m})^2}\left[(3.283\,\mathrm{m/s})^2 + 2(9.80\,\mathrm{m/s^2})(0.150\,\mathrm{m})\right] = 1590\,\mathrm{N/m}.\end{aligned}$$

PROBLEM SOLVING
Quicker Solution

Alternate Solution Instead of dividing the solution into two parts, we can do it all at once. After all, we get to choose what two points are used on the left and right of the energy equation. Let us write the energy equation for points 1 and 3 in Fig. 6–23. Point 1 is the initial point just before the ball starts to fall (Fig. 6–23a), so $v_1 = 0$, and $y_1 = h = 0.550\,\mathrm{m}$. Point 3 is when the spring is fully compressed (Fig. 6–23c), so $v_3 = 0$, $y_3 = -Y = -0.150\,\mathrm{m}$. The forces on the ball in this process are gravity and (at least part of the time) the spring. So conservation of energy tells us

$$\begin{aligned} \tfrac{1}{2}mv_1^2 + mgy_1 + \tfrac{1}{2}k(0)^2 &= \tfrac{1}{2}mv_3^2 + mgy_3 + \tfrac{1}{2}ky_3^2 \\ 0 + mgh + 0 &= 0 - mgY + \tfrac{1}{2}kY^2 \end{aligned}$$

where we have set $y = 0$ for the spring at point 1 because it is not acting and is not compressed or stretched. We solve for k:

$$k = \frac{2mg(h+Y)}{Y^2} = \frac{2(2.60\,\mathrm{kg})(9.80\,\mathrm{m/s^2})(0.550\,\mathrm{m} + 0.150\,\mathrm{m})}{(0.150\,\mathrm{m})^2} = 1590\,\mathrm{N/m}$$

just as in our first method of solution.

6–8 Other Forms of Energy and Energy Transformations; The Law of Conservation of Energy

FIGURE 6–24 Gravitational potential energy of water at the top of Yosemite Falls gets transformed into kinetic energy as the water falls. (Some of the energy is transformed into heat by air resistance, and some into sound.)

Besides the kinetic energy and potential energy of mechanical systems, other forms of energy can be defined as well. These include electric energy, nuclear energy, thermal energy, and the chemical energy stored in food and fuels. These other forms of energy are considered to be kinetic or potential energy at the atomic or molecular level. For example, according to atomic theory, thermal energy is the kinetic energy of rapidly moving molecules—when an object is heated, the molecules that make up the object move faster. On the other hand, the energy stored in food or in a fuel such as gasoline is regarded as potential energy stored by virtue of the relative positions of the atoms within a molecule due to electric forces between the atoms (chemical bonds). The energy in chemical bonds can be released through chemical reactions. This is analogous to a compressed spring which, when released, can do work. Electric, magnetic, and nuclear energies also can be considered examples of kinetic and potential (or stored) energies. We will deal with these other forms of energy in later Chapters.

Energy can be transformed from one form to another. For example, a rock held high in the air has potential energy; as it falls, it loses potential energy and gains in kinetic energy. Potential energy is being transformed into kinetic energy.

Often the transformation of energy involves a transfer of energy from one object to another. The potential energy stored in the spring of Fig. 6–13b is transformed into the kinetic energy of the ball, Fig. 6–13c. Water at the top of a waterfall (Fig. 6–24) or a dam has potential energy, which is transformed into kinetic energy as the water falls. At the base of a dam, the kinetic energy of the water can be transferred to turbine blades and further transformed into electric energy, as discussed later. The potential energy stored in a bent bow can be transformed into kinetic energy of the arrow (Fig. 6–25).

FIGURE 6–25 Potential energy of a bent bow about to be transformed into kinetic energy of an arrow.

In each of these examples, the transfer of energy is accompanied by the performance of work. The spring of Fig. 6–13 does work on the ball. Water does work on turbine blades. A bow does work on an arrow. This observation gives us a further insight into the relation between work and energy: *work is done when energy is transferred from one object to another.*[†]

[†] If the objects are at different temperatures, heat can flow between them instead, or in addition. See Chapters 14 and 15.

One of the great results of physics is that whenever energy is transferred or transformed, it is found that no energy is gained or lost in the process.

This is the **law of conservation of energy**, one of the most important principles in physics; it can be stated as:

LAW OF CONSERVATION OF ENERGY

The total energy is neither increased nor decreased in any process. Energy can be transformed from one form to another, and transferred from one object to another, but the total amount remains constant.

We have already discussed the conservation of energy for mechanical systems involving conservative forces, and we saw how it could be derived from Newton's laws and thus is equivalent to them. But in its full generality, the validity of the law of conservation of energy, encompassing all forms of energy including those associated with nonconservative forces like friction, rests on experimental observation. Even though Newton's laws are found to fail in the submicroscopic world of the atom, the law of conservation of energy has been found to hold in every experimental situation so far tested.

6–9 Energy Conservation with Dissipative Forces: Solving Problems

In our applications of energy conservation in Section 6–7, we neglected friction and other nonconservative forces. But in many situations they cannot be ignored. In a real situation, the roller-coaster car in Fig. 6–18, for example, will not in fact reach the same height on the second hill as it had on the first hill because of friction. In this, and in other natural processes, the mechanical energy (sum of the kinetic and potential energies) does not remain constant but decreases. Because frictional forces reduce the mechanical energy (but *not* the total energy), they are called **dissipative forces**. Historically, the presence of dissipative forces hindered the formulation of a comprehensive conservation of energy law until well into the nineteenth century. It was only then that heat, which is always produced when there is friction (try rubbing your hands together), was interpreted in terms of energy. Quantitative studies by nineteenth-century scientists (discussed in Chapters 14 and 15) demonstrated that if heat is considered as a transfer of energy (thermal energy), then the total energy is conserved in any process. For example, if the roller-coaster car in Fig. 6–18 is subject to frictional forces, then the initial total energy of the car will be equal to the kinetic plus potential energy of the car at any subsequent point along its path plus the amount of thermal energy produced in the process (equal to the work done by friction).

Let us recall the general form of the work-energy principle, Eq. 6–10:

$$W_{\mathrm{NC}} = \Delta \mathrm{KE} + \Delta \mathrm{PE},$$

where W_{NC} is the work done by nonconservative forces such as friction. Consider an object, such as a roller-coaster car, as a particle moving under gravity with nonconservative forces like friction acting on it. When the object moves from some point 1 to another point 2, then

$$W_{\mathrm{NC}} = \mathrm{KE}_2 - \mathrm{KE}_1 + \mathrm{PE}_2 - \mathrm{PE}_1 .$$

We can rewrite this as

$$\mathrm{KE}_1 + \mathrm{PE}_1 + W_{\mathrm{NC}} = \mathrm{KE}_2 + \mathrm{PE}_2 . \qquad \textbf{(6–15)}$$

For the case of friction, $W_{\mathrm{NC}} = -F_{\mathrm{fr}}\, d$, where d is the distance over which the friction (assumed constant) acts as the object moves from point 1 to point 2. ($\vec{\mathbf{F}}$ and $\vec{\mathbf{d}}$ are in opposite directions, hence the minus sign from $\cos 180° = -1$ in Eq. 6–1.)

With $\text{KE} = \frac{1}{2}mv^2$ and $\text{PE} = mgy$, Eq. 6–15 with $W_{NC} = -F_{fr}d$ becomes

$$\tfrac{1}{2}mv_1^2 + mgy_1 - F_{fr}d = \tfrac{1}{2}mv_2^2 + mgy_2. \qquad \begin{bmatrix}\text{gravity and}\\ \text{friction acting}\end{bmatrix} \qquad \textbf{(6–16a)}$$

That is, the initial mechanical energy is reduced by the amount $F_{fr}d$. We could also write this equation as

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 + F_{fr}d$$

or

$$\text{KE}_1 + \text{PE}_1 = \text{KE}_2 + \text{PE}_2 + F_{fr}d, \qquad \begin{bmatrix}\text{gravity and}\\ \text{friction}\\ \text{acting}\end{bmatrix} \qquad \textbf{(6–16b)}$$

and state equally well that the initial mechanical energy of the car (point 1) equals the (reduced) final mechanical energy of the car plus the energy transformed by friction into thermal energy.

Equations 6–16 can be seen to be Eq. 6–13 modified to include nonconservative forces such as friction. As such, they are statements of conservation of energy. When other forms of energy are involved, such as chemical or electrical energy, the total amount of energy is always found to be conserved. Hence the law of conservation of energy is believed to be universally valid.

EXERCISE F Return to the Chapter-Opening Question, page 138, and answer it again now. Try to explain why you may have answered differently the first time.

Work-Energy versus Energy Conservation

The law of conservation of energy is more general and more powerful than the work-energy principle. Indeed, the work-energy principle should not be viewed as a statement of conservation of energy. It is nonetheless useful for mechanical problems; and whether you use it, or use the more powerful conservation of energy, can depend on your *choice of the system* under study. If you choose as your system a particle or rigid object on which external forces do work, then you can use the work-energy principle: the work done by the external forces on your object equals the change in its kinetic energy.

PROBLEM SOLVING
Choosing the system

On the other hand, if you choose a system on which no external forces do work, then you need to apply conservation of energy to that system directly.

Consider, for example, a spring connected to a block on a frictionless table (Fig. 6–26). If you choose the block as your system, then the work done on the block by the spring equals the change in kinetic energy of the block: the work-energy principle. (Energy conservation does not apply to this system—the block's energy changes.) If instead you choose the block plus the spring as your system, no external forces do work (since the spring is part of the chosen system). To this system you need to apply conservation of energy: if you compress the spring and then release it, the spring still exerts a force† on the block, but the subsequent motion can be discussed in terms of kinetic energy $\left(\frac{1}{2}mv^2\right)$ plus potential energy $\left(\frac{1}{2}kx^2\right)$, whose total remains constant.

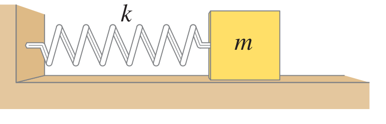


FIGURE 6–26 A spring connected to a block on a frictionless table. If you choose your system to be the block plus spring, then

$$E = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2$$

is conserved.

You may also wonder sometimes whether to approach a problem using work and energy, or instead to use Newton's laws. As a rough guideline, if the force(s) involved are constant, either approach may succeed. If the forces are not constant, and/or the path is not simple, energy may be the better approach because it is a scalar.

PROBLEM SOLVING
Use energy, or Newton's laws?

Problem solving is not a process that can be done by simply following a set of rules. The Problem Solving Strategy on the next page, like all others, is thus *not* a prescription, but is a summary to help you get started solving problems involving energy.

†The force the spring exerts on the block, and the force the block exerts back on the spring, are not "external" forces—they are within the system.

PROBLEM SOLVING

Conservation of Energy

1. **Draw a picture** of the physical situation.
2. Determine **the system** for which you will apply energy conservation: the object or objects and the forces acting.
3. Ask yourself what quantity you are looking for, and **choose initial** (point 1) **and final** (point 2) **positions**.
4. If the object under investigation changes its height during the problem, then **choose a reference frame** with a convenient $y = 0$ level for gravitational potential energy; the lowest point in the situation is often a good choice.

   If springs are involved, choose the unstretched spring position to be x (or y) $= 0$.
5. **Is mechanical energy conserved?** If no friction or other nonconservative forces act, then conservation of mechanical energy holds:
$$\text{KE}_1 + \text{PE}_1 = \text{KE}_2 + \text{PE}_2. \qquad \textbf{(6–12a)}$$
6. **Apply conservation of energy.** If friction (or other nonconservative forces) are present, then an additional term (W_{NC}) will be needed:
$$W_{\text{NC}} = \Delta\text{KE} + \Delta\text{PE}. \qquad \textbf{(6–10)}$$
   For a constant friction force acting over a distance d
$$\text{KE}_1 + \text{PE}_1 = \text{KE}_2 + \text{PE}_2 + F_{\text{fr}}\, d. \qquad \textbf{(6–16b)}$$
   For other nonconservative forces use your intuition for the sign of W_{NC}: is the total mechanical energy increased or decreased in the process?
7. Use the equation(s) you develop to **solve** for the unknown quantity.

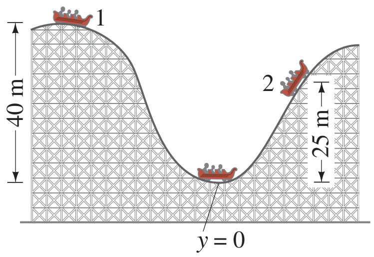


FIGURE 6–27 Example 6–12. Because of friction, a roller-coaster car does not reach the original height on the second hill. (Not to scale.)

EXAMPLE 6–12 **ESTIMATE** **Friction on the roller-coaster car.** The roller-coaster car in Example 6–8 reaches a vertical height of only 25 m on the second hill, where it slows to a momentary stop, Fig. 6–27. It traveled a total distance of 400 m. Determine the thermal energy produced and estimate the average friction force (assume it is roughly constant) on the car, whose mass is 1000 kg.

APPROACH We explicitly follow the Problem Solving Strategy above.

SOLUTION

1. **Draw a picture**. See Fig. 6–27.
2. **The system**. The system is the roller-coaster car and the Earth (which exerts the gravitational force). The forces acting on the car are gravity and friction. (The normal force also acts on the car, but does no work, so it does not affect the energy.) Gravity is accounted for as potential energy, and friction as a term $F_{\text{fr}}\, d$.
3. **Choose initial and final positions**. We take point 1 to be the instant when the car started coasting (at the top of the first hill), and point 2 to be the instant it stopped at a height of 25 m up the second hill.
4. **Choose a reference frame**. We choose the lowest point in the motion to be $y = 0$ for the gravitational potential energy.
5. **Is mechanical energy conserved**? No. Friction is present.
6. **Apply conservation of energy**. There is friction acting on the car, so we use conservation of energy in the form of Eq. 6–16b, with $v_1 = 0$, $y_1 = 40\text{ m}$, $v_2 = 0$, $y_2 = 25\text{ m}$, and $d = 400\text{ m}$. Thus
$$0 + (1000\text{ kg})(9.8\text{ m/s}^2)(40\text{ m}) = 0 + (1000\text{ kg})(9.8\text{ m/s}^2)(25\text{ m}) + F_{\text{fr}}\, d.$$
7. **Solve**. We solve the above equation for $F_{\text{fr}}\, d$, the energy dissipated to thermal energy:
$$F_{\text{fr}}\, d = mg\,\Delta h = (1000\text{ kg})(9.8\text{ m/s}^2)(40\text{ m} - 25\text{ m}) = 147{,}000\text{ J}.$$
   The friction force, which acts over a distance of 400 m, averages out to be
$$F_{\text{fr}} = (1.47 \times 10^5\text{ J})/400\text{ m} = 370\text{ N}.$$

NOTE This result is only a rough average: the friction force at various points depends on the normal force, which varies with slope.