# *23–9 Combinations of Lenses

Many optical instruments use lenses in combination. When light passes through more than one lens, we find the image formed by the first lens as if it were alone. Then this image becomes the *object* for the second lens. Next we find the image formed by this second lens using the first image as object. This second image is the final image if there are only two lenses. The total magnification will be the product of the separate magnifications of each lens. Even if the second lens intercepts the light from the first lens before it forms an image, this technique still works.

**EXAMPLE 23–15** **A two-lens system.** Two converging lenses, A and B, with focal lengths $f_A = 20.0\text{ cm}$ and $f_B = 25.0\text{ cm}$, are placed 80.0 cm apart, as shown in Fig. 23–44a. An object is placed 60.0 cm in front of the first lens as shown in Fig. 23–44b. Determine (*a*) the position, and (*b*) the magnification, of the final image formed by the combination of the two lenses.

**APPROACH** Starting at the tip of our object O, we draw rays 1, 2, and 3 for the first lens, A, and also a ray 4 which, after passing through lens A, acts for the second lens, B, as ray 3′ (through the center). We use primes now for the standard rays relative to lens B. Ray 2 for lens A exits parallel, and so is ray 1′ for lens B. To determine the position of the image $I_A$ formed by lens A, we use Eq. 23–8 with $f_A = 20.0\text{ cm}$ and $d_{oA} = 60.0\text{ cm}$. The distance of $I_A$ (lens A's image) from lens B is the object distance $d_{oB}$ for lens B. The final image is found using the thin lens equation, this time with all distances relative to lens B. For (*b*) the magnifications are found from Eq. 23–9 for each lens in turn.

**SOLUTION** (*a*) The object is a distance $d_{oA} = +60.0\text{ cm}$ from the first lens, A, and this lens forms an image whose position can be calculated using the thin lens equation:

$$\frac{1}{d_{iA}} = \frac{1}{f_A} - \frac{1}{d_{oA}} = \frac{1}{20.0\text{ cm}} - \frac{1}{60.0\text{ cm}} = \frac{3-1}{60.0\text{ cm}} = \frac{1}{30.0\text{ cm}}.$$

**CAUTION**

*Object distance for second lens is* ***not*** *equal to the image distance for first lens*

So the first image $I_A$ is at $d_{iA} = 30.0\text{ cm}$ behind the first lens. This image becomes the object for the second lens, B. It is a distance $d_{oB} = 80.0\text{ cm} - 30.0\text{ cm} = 50.0\text{ cm}$ in front of lens B (Fig. 23–44b). The image formed by lens B, again using the thin lens equation, is at a distance $d_{iB}$ from the lens B:

$$\frac{1}{d_{iB}} = \frac{1}{f_B} - \frac{1}{d_{oB}} = \frac{1}{25.0\text{ cm}} - \frac{1}{50.0\text{ cm}} = \frac{2-1}{50.0\text{ cm}} = \frac{1}{50.0\text{ cm}}.$$

Hence $d_{iB} = 50.0\text{ cm}$ behind lens B. This is the final image—see Fig. 23–44b.

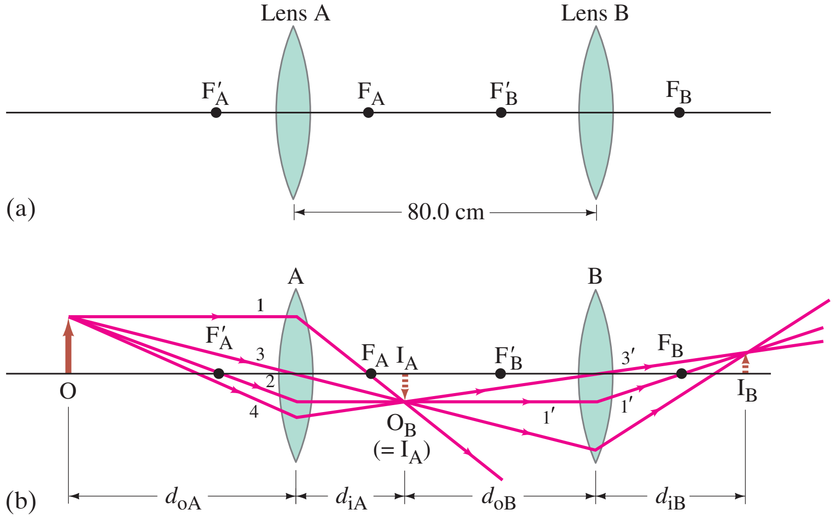


**FIGURE 23–44** Two lenses, A and B, used in combination, Example 23–15. The small numbers refer to the easily drawn rays.

(*b*) Lens A has a magnification (Eq. 23–9)

$$m_A = -\frac{d_{iA}}{d_{oA}} = -\frac{30.0\text{ cm}}{60.0\text{ cm}} = -0.500.$$

Thus, the first image is inverted and is half as high as the object (again Eq. 23–9):

$$h_{iA} = m_A h_{oA} = -0.500 h_{oA}.$$

Lens B takes this first image as object and changes its height by a factor

$$m_B = -\frac{d_{iB}}{d_{oB}} = -\frac{50.0\text{ cm}}{50.0\text{ cm}} = -1.000.$$

The second lens reinverts the image (the minus sign) but doesn't change its size. The final image height is (remember $h_{oB}$ is the same as $h_{iA}$)

$$h_{iB} = m_B h_{oB} = m_B h_{iA} = m_B m_A h_{oA} = (m_{\text{total}})h_{oA}.$$

The total magnification is the product of $m_A$ and $m_B$, which here equals $m_{\text{total}} = m_A m_B = (-1.000)(-0.500) = +0.500$, or half the original height, and the final image is upright.

**PROBLEM SOLVING**
*Total magnification is* $m_{\text{total}} = m_A m_B$

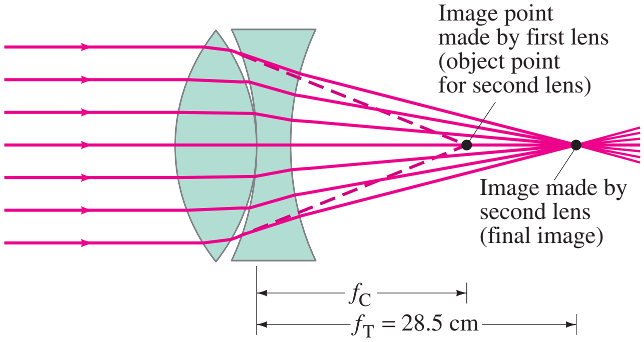


**FIGURE 23–45** Determining the focal length of a diverging lens. Example 23–16.

**EXAMPLE 23–16** **Measuring *f* for a diverging lens.** To measure the focal length of a diverging lens, a converging lens is placed in contact with it, as shown in Fig. 23–45. The Sun's rays are focused by this combination at a point 28.5 cm behind the lenses as shown. If the converging lens has a focal length $f_C$ of 16.0 cm, what is the focal length $f_D$ of the diverging lens? Assume both lenses are thin and the space between them is negligible.

**APPROACH** The image distance for the first lens equals its focal length (16.0 cm) since the object distance is infinity ($\infty$). The position of this image, even though it is never actually formed, acts as the object for the second (diverging) lens. We apply the thin lens equation to the diverging lens to find its focal length, given that the final image is at $d_i = 28.5$ cm.

**SOLUTION** Rays from the Sun are focused 28.5 cm behind the combination, so the focal length of the total combination is $f_T = 28.5$ cm. If the diverging lens was absent, the converging lens would form the image at its focal point—that is, at a distance $f_C = 16.0$ cm behind it (dashed lines in Fig. 23–45). When the diverging lens is placed next to the converging lens, we treat the image formed by the first lens as the *object* for the second lens. Since this object lies to the right of the diverging lens, this is a situation where $d_o$ is negative (see the sign conventions, page 665). Thus, for the diverging lens, the object is virtual and $d_o = -16.0$ cm. The diverging lens forms the image of this virtual object at a distance $d_i = 28.5$ cm away (given). Thus,

**CAUTION**
$d_o < 0$

$$\frac{1}{f_D} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{-16.0\text{ cm}} + \frac{1}{28.5\text{ cm}} = -0.0274\text{ cm}^{-1}.$$

We take the reciprocal to find $f_D = -1/(0.0274\text{ cm}^{-1}) = -36.5$ cm.

**NOTE** If this technique is to work, the converging lens must be "stronger" than the diverging lens—that is, it must have a focal length whose magnitude is less than that of the diverging lens.

## *23–10 Lensmaker's Equation

A useful equation, called the **lensmaker's equation**, relates the focal length of a lens to the radii of curvature $R_1$ and $R_2$ of its two surfaces and its index of refraction $n$:

*Lensmaker's equation*

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right). \tag{23–10}$$

If both surfaces are convex, $R_1$ and $R_2$ are considered positive.† For a concave surface, the radius must be considered *negative*.

Notice that Eq. 23–10 is *symmetrical* in $R_1$ and $R_2$. Thus, if a lens is turned around so that light impinges on the other surface, the focal length is the same even if the two lens surfaces are different. This confirms what we said earlier: a lens' focal length is the same on both sides of the lens.

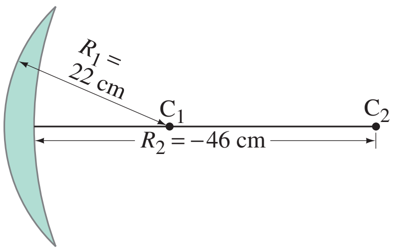


**FIGURE 23–46** Example 23–17. The left surface is convex (center bulges outward); the right surface is concave.

**EXAMPLE 23–17 Calculating $f$ for a converging lens.** A convex meniscus lens (Figs. 23–31a and 23–46) is made from glass with $n = 1.50$. The radius of curvature of the convex surface (left in Fig. 23–46) is 22 cm. The surface on the right is concave with radius of curvature 46 cm. What is the focal length?

**APPROACH** We use the lensmaker's equation, Eq. 23–10, to find $f$.

**SOLUTION** $R_1 = 0.22\text{ m}$ and $R_2 = -0.46\text{ m}$ (concave surface). Then

$$\frac{1}{f} = (1.50 - 1.00)\left(\frac{1}{0.22\text{ m}} - \frac{1}{0.46\text{ m}}\right) = 1.19\text{ m}^{-1}.$$

So

$$f = \frac{1}{1.19\text{ m}^{-1}} = 0.84\text{ m},$$

and the lens is converging since $f > 0$.

**NOTE** If we turn the lens around so that $R_1 = -46\text{ cm}$ and $R_2 = +22\text{ cm}$, we get the same result.

**NOTE** Because Eq. 23–10 gives $1/f$, it gives directly the power of a lens in diopters, Eq. 23–7. The power of this lens is about 1.2 D.

†Some books use a different convention: $R_1$ and $R_2$ may be considered positive if their centers of curvature are to the right of the lens; then a minus sign replaces the + sign in their version of Eq. 23–10.

# Summary

Light appears to travel along straight-line paths, called **rays**, through uniform transparent materials including air and glass. When light reflects from a flat surface, the *angle of reflection equals the angle of incidence*. This **law of reflection** explains why mirrors can form **images**.

In a **plane mirror**, the image is virtual, upright, the same size as the object, and as far behind the mirror as the object is in front.

A **spherical mirror** can be concave or convex. A **concave** spherical mirror focuses parallel rays of light (light from a very distant object) to a point called the **focal point**. The distance of this point from the mirror is the **focal length** $f$ of the mirror and

$$f = \frac{r}{2} \tag{23–1}$$

where $r$ is the radius of curvature of the mirror.

Parallel rays falling on a **convex mirror** reflect from the mirror as if they diverged from a common point behind the mirror. The distance of this point from the mirror is the focal length and is considered negative for a convex mirror.

For a given object, the approximate position and size of the image formed by a mirror can be found by ray tracing. Algebraically, the relation between image and object distances, $d_i$ and $d_o$, and the focal length $f$, is given by the **mirror equation**:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}. \tag{23–2}$$

The ratio of image height $h_i$ to object height $h_o$, which equals the magnification $m$ of a mirror, is

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}. \tag{23–3}$$

If the rays that converge to form an image actually pass through the image, so the image would appear on a screen or film placed there, the image is said to be a **real image**. If the light rays do not actually pass through the image, the image is a **virtual image**.

The speed of light $v$ depends on the **index of refraction**, $n$, of the material:

$$n = \frac{c}{v}, \tag{23–4}$$

where $c$ is the speed of light in vacuum.

When light passes from one transparent medium into another, the rays bend or refract. The **law of refraction** (**Snell's law**) states that

$$n_1 \sin\theta_1 = n_2 \sin\theta_2, \tag{23–5}$$

where $n_1$ and $\theta_1$ are the index of refraction and angle with the normal (perpendicular) to the surface for the incident ray, and $n_2$ and $\theta_2$ are for the refracted ray.

When light rays reach the boundary of a material where the index of refraction decreases, the rays will be **totally internally reflected** if the incident angle, $\theta_1$, is such that Snell's law would