18–7 Alternating Current

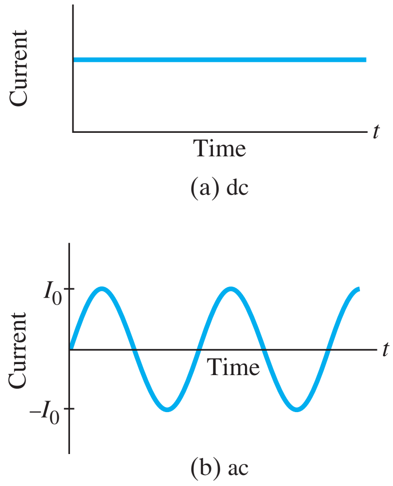


FIGURE 18–21 (a) Direct current, and (b) alternating current, as functions of time.

When a battery is connected to a circuit, the current moves steadily in one direction. This is called a **direct current**, or **dc**. Electric generators at electric power plants, however, produce **alternating current**, or **ac**. (Sometimes capital letters are used, DC and AC.) An alternating current reverses direction many times per second and is commonly sinusoidal, Fig. 18–21. The electrons in a wire first move in one direction and then in the other. The current supplied to homes and businesses by electric companies is ac throughout virtually the entire world. We will discuss and analyze ac circuits in detail in Chapter 21. But because ac circuits are so common in real life, we will discuss some of their basic aspects here.

The voltage produced by an ac electric generator is sinusoidal, as we shall see later. The current it produces is thus sinusoidal (Fig. 18–21b). We can write the voltage as a function of time as

$$V = V_0 \sin 2\pi f t = V_0 \sin \omega t. \qquad \textbf{(18–7a)}$$

The potential V oscillates between $+V_0$ and $-V_0$, and V_0 is referred to as the **peak voltage**. The frequency f is the number of complete oscillations made per second, and $\omega = 2\pi f$. In most areas of the United States and Canada, f is 60 Hz (the unit "hertz," as we saw in Chapters 8 and 11, means cycles per second). In many countries, 50 Hz is used.

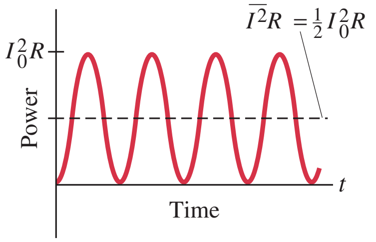
Equation 18–2, $V = IR$, works also for ac: if a voltage V exists across a resistance R, then the current I through the resistance is

$$I = \frac{V}{R} = \frac{V_0}{R}\sin \omega t = I_0 \sin \omega t. \qquad \textbf{(18–7b)}$$

The quantity $I_0 = V_0/R$ is the **peak current**. The current is considered positive when the electrons flow in one direction and negative when they flow in the opposite direction. It is clear from Fig. 18–21b that an alternating current is as often positive as it is negative. Thus, the average current is zero. This does not mean, however, that no power is needed or that no heat is produced in a resistor. Electrons do move back and forth, and do produce heat. Indeed, the power transformed in a resistance R at any instant is (Eq. 18–7b)

$$P = I^2 R = I_0^2 R \sin^2 \omega t.$$

FIGURE 18–22 Power transformed in a resistor in an ac circuit.



Because the current is squared, we see that the power is always positive, as graphed in Fig. 18–22. The quantity $\sin^2 \omega t$ varies between 0 and 1; and it is not too difficult to show[†] that its average value is $\frac{1}{2}$, as indicated in Fig. 18–22. Thus, the *average power* transformed, $\overline{P}$, is

$$\overline{P} = \tfrac{1}{2} I_0^2 R.$$

Since power can also be written $P = V^2/R = \left(V_0^2/R\right)\sin^2 \omega t$, we also have that the average power is

$$\overline{P} = \frac{1}{2}\frac{V_0^2}{R}.$$

The average or mean value of the *square* of the current or voltage is thus what is important for calculating average power: $\overline{I^2} = \frac{1}{2} I_0^2$ and $\overline{V^2} = \frac{1}{2} V_0^2$. The square root of each of these is the **rms** (root-mean-square) value of the current or voltage:

$$I_{\rm rms} = \sqrt{\overline{I^2}} = \frac{I_0}{\sqrt{2}} = 0.707\, I_0, \qquad \textbf{(18–8a)}$$

$$V_{\rm rms} = \sqrt{\overline{V^2}} = \frac{V_0}{\sqrt{2}} = 0.707\, V_0. \qquad \textbf{(18–8b)}$$

[†]A graph of $\cos^2 \omega t$ versus t is identical to that for $\sin^2 \omega t$ in Fig. 18–22, except that the points are shifted (by $\frac{1}{4}$ cycle) on the time axis. Thus the average value of $\sin^2$ and $\cos^2$, averaged over one or more full cycles, will be the same. From the trigonometric identity $\sin^2\theta + \cos^2\theta = 1$, we can write

$$\overline{(\sin^2 \omega t)} + \overline{(\cos^2 \omega t)} = 2\overline{(\sin^2 \omega t)} = 1.$$

Hence the average value of $\sin^2 \omega t$ is $\frac{1}{2}$.

The rms values of V and I are sometimes called the *effective values*. They are useful because they can be substituted directly into the power formulas, Eqs. 18–5 and 18–6, to get the average power:

$$\overline{P} = I_{\text{rms}} V_{\text{rms}} \tag{18–9a}$$

$$\overline{P} = \tfrac{1}{2} I_0^2 R = I_{\text{rms}}^2 R \tag{18–9b}$$

$$\overline{P} = \frac{1}{2}\frac{V_0^2}{R} = \frac{V_{\text{rms}}^2}{R}. \tag{18–9c}$$

Thus, a direct current whose values of I and V equal the rms values of I and V for an alternating current will produce the same power. Hence it is usually the rms value of current and voltage that is specified or measured. For example, in the United States and Canada, standard line voltage is 120-V ac. The 120 V is V_{rms}; the peak voltage V_0 is (Eq. 18–8b)

$$V_0 = \sqrt{2}\, V_{\text{rms}} = 170\ \text{V}.$$

In much of the world (Europe, Australia, Asia) the rms voltage is 240 V, so the peak voltage is 340 V. The line voltage can vary, depending on the total load; the frequency of 60 Hz or 50 Hz, however, remains extremely steady.

EXAMPLE 18–13 **Hair dryer.** (*a*) Calculate the resistance and the peak current in a 1500-W hair dryer (Fig. 18–23) connected to a 120-V ac line. (*b*) What happens if it is connected to a 240-V ac line in Britain?

APPROACH We are given $\overline{P}$ and V_{rms}, so $I_{\text{rms}} = \overline{P}/V_{\text{rms}}$ (Eq. 18–9a or 18–5), and $I_0 = \sqrt{2}\, I_{\text{rms}}$. Then we find R from $V = IR$.

SOLUTION (*a*) We solve Eq. 18–9a for the rms current:

$$I_{\text{rms}} = \frac{\overline{P}}{V_{\text{rms}}} = \frac{1500\ \text{W}}{120\ \text{V}} = 12.5\ \text{A}.$$

Then

$$I_0 = \sqrt{2}\, I_{\text{rms}} = 17.7\ \text{A}.$$

The resistance is

$$R = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{120\ \text{V}}{12.5\ \text{A}} = 9.6\ \Omega.$$

The resistance could equally well be calculated using peak values:

$$R = \frac{V_0}{I_0} = \frac{170\ \text{V}}{17.7\ \text{A}} = 9.6\ \Omega.$$

(*b*) When connected to a 240-V line, more current would flow and the resistance would change with the increased temperature (Section 18–4). But let us make an estimate of the power transformed based on the same 9.6-Ω resistance. The average power would be

$$\begin{aligned}\overline{P} &= \frac{V_{\text{rms}}^2}{R} \\ &= \frac{(240\ \text{V})^2}{(9.6\ \Omega)} = 6000\ \text{W}.\end{aligned}$$

This is four times the dryer's power rating and would undoubtedly melt the heating element or the wire coils of the motor.

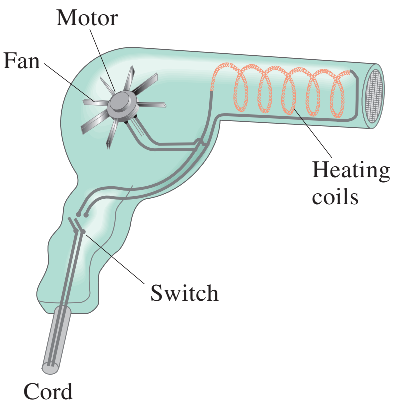


FIGURE 18–23 A hair dryer. Most of the current goes through the heating coils, a pure resistance; a small part goes to the motor to turn the fan. Example 18–13.

This Section has given a brief introduction to the simpler aspects of alternating currents. We will discuss ac circuits in more detail in Chapter 21. In Chapter 19 we will deal with the details of dc circuits only.