Magnets produce magnetic fields, but so do electric currents. Compass needles are magnets, and they align along the direction of any magnetic field present. Here, the compasses show the presence (and direction) of a magnetic field near a current-carrying wire. We shall see in this Chapter how magnetic field is defined, and how magnetic fields exert forces on electric currents and on charged particles. We also discuss useful applications of the interaction between magnetic fields and electric currents and moving electric charges, such as motors and loudspeakers.

CHAPTER 20

Magnetism

CONTENTS

CHAPTER-OPENING QUESTION—Guess now!

Which of the following can experience a force when placed in the magnetic field of a magnet?

(a) An electric charge at rest.
(b) An electric charge moving.
(c) An electric current in a wire.
(d) Another magnet.

The history of magnetism began thousands of years ago, when in a region of Asia Minor known as Magnesia, rocks were found that could attract each other. These rocks were called "magnets" after their place of discovery.

Not until the nineteenth century, however, was it seen that magnetism and electricity are closely related. A crucial discovery was that electric currents produce magnetic effects (we will say "magnetic fields") like magnets do. All kinds of practical devices depend on magnetism, from compasses to motors, loudspeakers, computer memory, and electric generators.

20–1 Magnets and Magnetic Fields

You probably have observed a magnet attract paper clips, nails, and other objects made of iron, as in Fig. 20–1. Any magnet, whether it is in the shape of a bar or a horseshoe, has two ends or faces, called **poles**, which is where the magnetic effect is strongest. If a bar magnet is suspended from a fine thread, it is found that one pole of the magnet will always point toward the north. It is not known for sure when this fact was discovered, but it is known that the Chinese were making use of it as an aid to navigation by the eleventh century and perhaps earlier.

This is the principle of a compass. A compass needle is simply a bar magnet which is supported at its center of gravity so that it can rotate freely. The pole of a freely suspended magnet that points toward geographic north is called the **north pole** of the magnet. The other pole points toward the south and is called the **south pole**.

It is a familiar observation that when two magnets are brought near one another, each exerts a force on the other. The force can be either attractive or repulsive and can be felt even when the magnets don't touch. If the north pole of one bar magnet is brought near the north pole of a second magnet, the force is repulsive. Similarly, if the south poles are brought close, the force is repulsive. But when the north pole of one magnet is brought near the south pole of another magnet, the force is attractive. These results are shown in Fig. 20–2, and are reminiscent of the forces between electric charges: like poles repel, and unlike poles attract. But *do not confuse magnetic poles with electric charge*. They are very different. One important difference is that a positive or negative electric charge can easily be isolated. But an isolated single magnetic pole has never been observed. If a bar magnet is cut in half, you do not obtain isolated north and south poles. Instead, two new magnets are produced, Fig. 20–3, each with north (N) and south (S) poles. If the cutting operation is repeated, more magnets are produced, each with a north and a south pole. Physicists have searched for isolated single magnetic poles (monopoles), but no **magnetic monopole** has ever been observed.

FIGURE 20–1 A horseshoe magnet attracts pins made of iron.

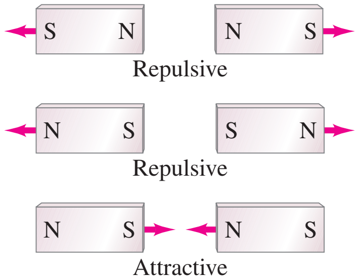


FIGURE 20–2 Like poles of two magnets repel; unlike poles attract.

CAUTION
Magnets do not attract all metals

Besides iron, a few other materials, such as cobalt, nickel, gadolinium, and some of their oxides and alloys, show strong magnetic effects. They are said to be **ferromagnetic** (from the Latin word *ferrum* for iron). Other materials show some slight magnetic effect, but it is very weak and can be detected only with delicate instruments. We will look in more detail at ferromagnetism in Section 20–12.

In Chapter 16, we used the concept of an electric field surrounding an electric charge. In a similar way, we can picture a **magnetic field** surrounding a magnet. The force one magnet exerts on another can then be described as the interaction between one magnet and the magnetic field of the other. Just as we drew electric field lines, we can also draw **magnetic field lines**. They can be drawn, as for electric field lines, so that

1. the direction of the magnetic field is tangent to a field line at any point, and
2. the number of lines per unit area is proportional to the strength of the magnetic field.

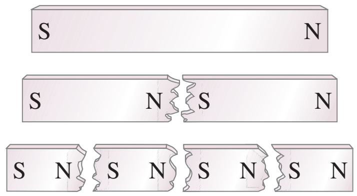
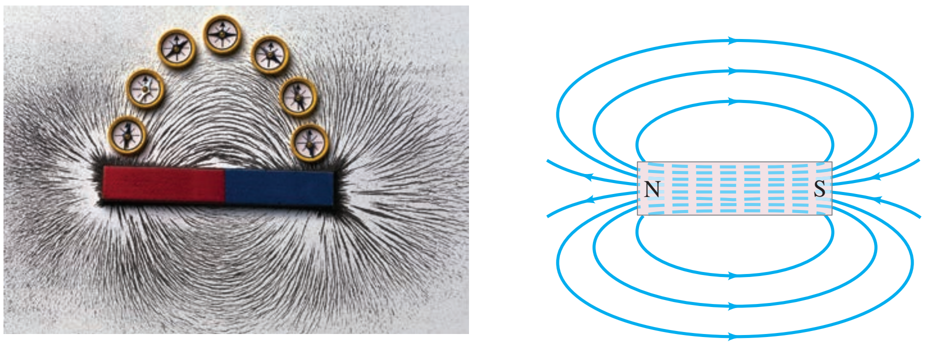


FIGURE 20–3 If you split a magnet, you won't get isolated north and south poles; instead, two new magnets are produced, each with a north and a south pole.

The *direction* of the magnetic field at a given location can be defined as the direction that the north pole of a compass needle would point if placed at that location. (We will give a more precise definition of magnetic field shortly.) Figure 20–4a shows how thin iron filings (acting like tiny magnets) reveal the magnetic field lines by lining up like the compass needles. The magnetic field determined in this way for the field surrounding a bar magnet is shown in Fig. 20–4b. Notice that because of our definition, the lines always point out from the north pole and in toward the south pole of a magnet (the north pole of a magnetic compass needle is attracted to the south pole of the magnet).

Magnetic field lines continue inside a magnet, as indicated in Fig. 20–4b. Indeed, given the lack of single magnetic poles, magnetic field lines always form closed loops, unlike electric field lines that begin on positive charges and end on negative charges.

CAUTION
Magnetic field lines form closed loops, unlike electric field lines



(a) (b)

FIGURE 20–4 (a) Visualizing magnetic field lines around a bar magnet, using iron filings and compass needles. The red end of the bar magnet is its north pole. The N pole of a nearby compass needle points away from the north pole of the magnet. (b) Diagram of magnetic field lines for a bar magnet.

Earth's Magnetic Field

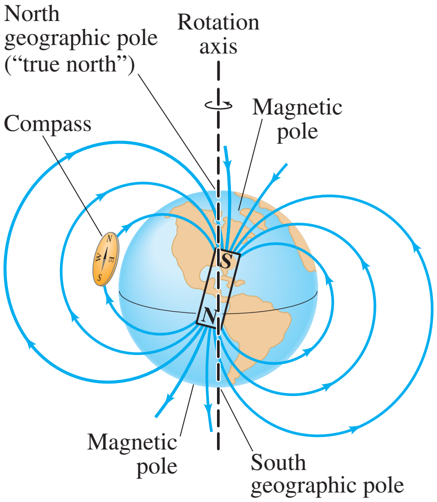


FIGURE 20–5 The Earth acts like a huge magnet. But its magnetic poles are not at the geographic poles (on the Earth's rotation axis).

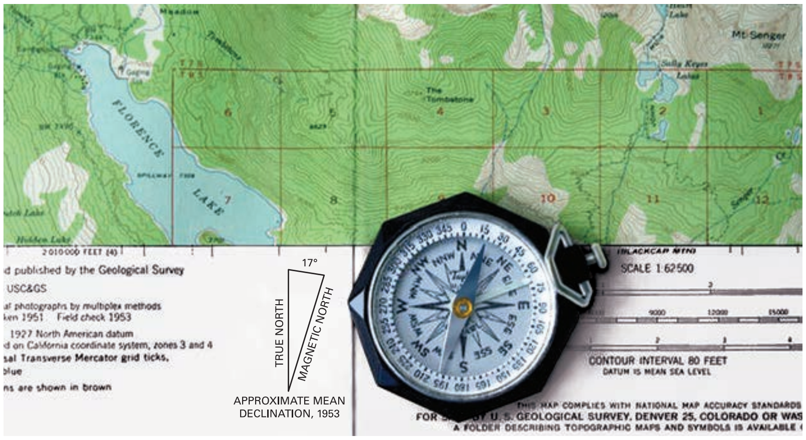
The Earth's magnetic field is shown in Fig. 20–5, and is thought to be produced by electric currents in the Earth's molten iron outer core. The pattern of field lines is almost as though there were an imaginary bar magnet inside the Earth. Since the north pole (N) of a compass needle points north, the Earth's **magnetic pole** which is in the geographic north is magnetically a south pole, as indicated in Fig. 20–5 by the S on the schematic bar magnet inside the Earth. Remember that the north pole of one magnet is attracted to the south pole of another magnet. Nonetheless, Earth's pole in the north is still often called the "north magnetic pole," or "geomagnetic north," simply because it is in the north. Similarly, the Earth's southern magnetic pole, which is near the geographic south pole, is magnetically a north pole (N). The Earth's magnetic poles do not coincide with the **geographic poles**, which are on the Earth's axis of rotation. The north magnetic pole, for example, is in the Canadian Arctic, now on the order of 1000 km† from the geographic north pole, or **true north**. This difference must be taken into account for accurate use of a compass (Fig. 20–6). The angular difference between the direction of a compass needle (which points along the magnetic field lines) at any location and true (geographical) north is called the **magnetic declination**. In the U.S. it varies from 0° to about 20°, depending on location.

Notice in Fig. 20–5 that the Earth's magnetic field at most locations is not tangent to the Earth's surface. The angle that the Earth's magnetic field makes with the horizontal at any point is referred to as the **angle of dip**, or the "inclination." It is 67° at New York, for example, and 55° at Miami.

EXERCISE A Does the Earth's magnetic field have a greater magnitude near the poles or near the equator? [How can you tell using the field lines in Fig. 20–5?]

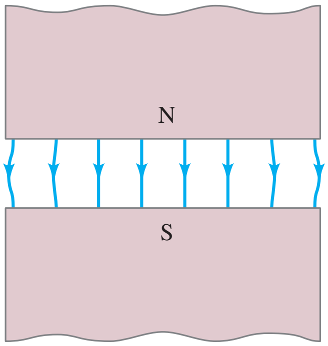
PHYSICS APPLIED
Use of a compass

FIGURE 20–6 Using a map and compass in the wilderness. First you align the compass case so the needle points away from true north (N) exactly the number of degrees of declination stated on the map (for this topographic map, it is 17° as shown just to the left of the compass). Then align the map with true north, as shown, *not* with the compass needle. [This is an old map (1953) of a part of California; on new maps (2012) the declination is only 13°, telling us the position of magnetic north has moved—see footnote below.]



Uniform Magnetic Field

FIGURE 20–7 Magnetic field between two wide poles of a magnet is nearly uniform, except near the edges.



The simplest magnetic field is one that is uniform—it doesn't change in magnitude or direction from one point to another. A perfectly uniform field over a large area is not easy to produce. But the field between two flat parallel pole pieces of a magnet is nearly uniform if the area of the pole faces is large compared to their separation, as shown in Fig. 20–7. At the edges, the field "fringes" out somewhat: the magnetic field lines are no longer quite parallel and uniform. The parallel evenly spaced field lines in the central region of the gap indicate that the field is uniform at points not too near the edges, much like the electric field between two parallel plates (Fig. 17–1).

†Earth's north magnetic pole has been moving over time, on the order of 10 km per year in recent decades. Magnetism in rocks solidified at various times in the past (age determined by radioactive dating—see Section 30–11) suggests that Earth's magnetic poles have not only moved significantly over geologic time, but have also reversed direction 400 times over the last 330 million years. Also note that a compass gives a false reading if you are standing on rock containing magnetized iron ore (as you move around, the compass needle is inconsistent).

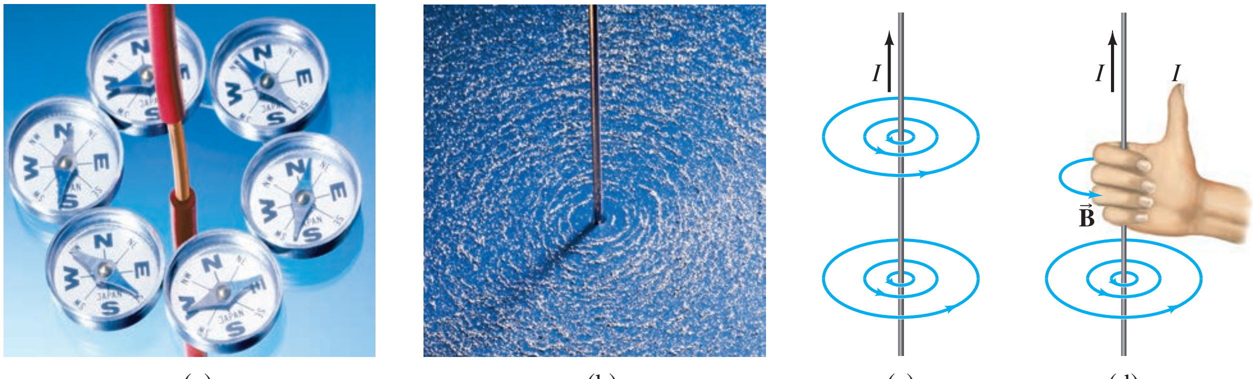


FIGURE 20–8 (a) Deflection of compass needles near a current-carrying wire, showing the presence and direction of the magnetic field. (b) Iron filings also align along the direction of the magnetic field lines near a straight current-carrying wire. (c) Diagram of the magnetic field lines around an electric current in a straight wire. (d) Right-hand rule for remembering the direction of the magnetic field: when the thumb points in the direction of the conventional current, the fingers wrapped around the wire point in the direction of the magnetic field. ($\vec{\mathbf{B}}$ is the symbol for magnetic field.)

20–2 Electric Currents Produce Magnetic Fields

During the eighteenth century, many scientists sought to find a connection between electricity and magnetism. A stationary electric charge and a magnet were shown to have no influence on each other. But in 1820, Hans Christian Oersted (1777–1851) found that when a compass is placed near a wire, the compass needle deflects if (and only if) the wire carries an electric current. As we have seen, a compass needle is deflected by a magnetic field. So Oersted's experiment showed that **an electric current produces a magnetic field**. He had found a connection between electricity and magnetism.

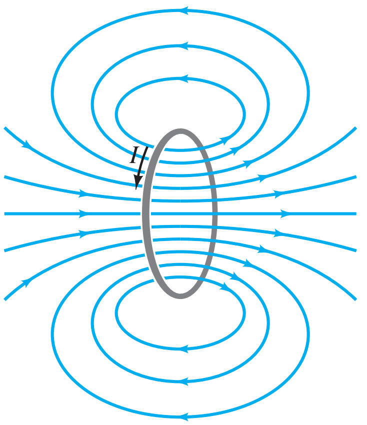
A compass needle placed near a straight section of current-carrying wire experiences a force, causing the needle to align tangent to a circle around the wire, Fig. 20–8a. Thus, the magnetic field lines produced by a current in a straight wire are in the form of circles with the wire at their center, Figs. 20–8b and c. The direction of these lines is indicated by the north pole of the compasses in Fig. 20–8a. There is a simple way to remember the direction of the magnetic field lines in this case. It is called a **right-hand rule**: grasp the wire with your right hand so that your thumb points in the direction of the conventional (positive) current; then your fingers will encircle the wire in the direction of the magnetic field, Fig. 20–8d.

Right-Hand-Rule-1: *Magnetic field direction produced by electric current*

The magnetic field lines due to a circular loop of current-carrying wire can be determined in a similar way by placing a compass at various locations near the loop. The result is shown in Fig. 20–9. Again the right-hand rule can be used, as shown in Fig. 20–10. Unlike the uniform field shown in Fig. 20–7, the magnetic fields shown in Figs. 20–8 and 20–9 are *not* uniform—the fields are different in magnitude and direction at different locations.

EXERCISE B A straight wire carries a current directly toward you. In what direction are the magnetic field lines surrounding the wire?

FIGURE 20–9 Magnetic field lines due to a circular loop of wire.



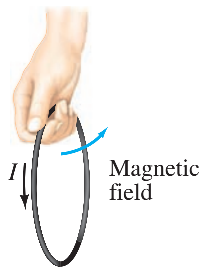


FIGURE 20–10 Right-hand rule for determining the direction of the magnetic field relative to the current in a loop of wire.

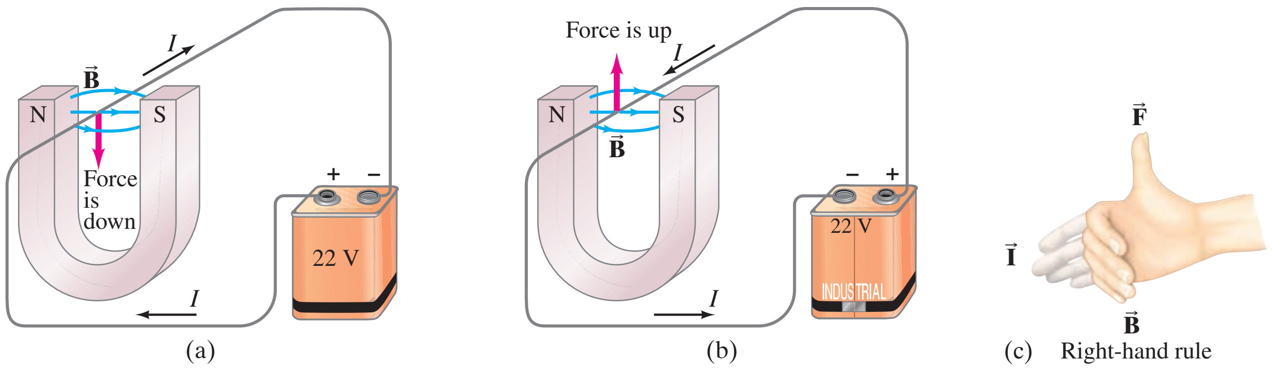


FIGURE 20–11 (a) Force on a current-carrying wire placed in a magnetic field $\vec{\mathbf{B}}$; (b) same, but current reversed; (c) right-hand rule for setup in (b), with current $\vec{\mathbf{I}}$ shown as if a vector with direction.

20–3 Force on an Electric Current in a Magnetic Field; Definition of $\vec{\mathbf{B}}$

In Section 20–2 we saw that an electric current exerts a force on a magnet, such as a compass needle. By Newton's third law, we might expect the reverse to be true as well: we should expect that *a magnet exerts a force on a current-carrying wire*. Experiments indeed confirm this effect, and it too was first observed by Oersted.

Suppose a straight wire is placed in the magnetic field between the poles of a horseshoe magnet as shown in Fig. 20–11, where the vector symbol $\vec{\mathbf{B}}$ represents the magnitude and direction of the magnetic field. When a current flows in the wire, experiment shows that a force is exerted on the wire. But this force is *not* toward one or the other pole of the magnet. Instead, the force is directed at right angles to the magnetic field direction, downward in Fig. 20–11a. If the current is reversed in direction, the force is in the opposite direction, upward as shown in Fig. 20–11b. Experiments show that *the direction of the force is always perpendicular to the direction of the current and also perpendicular to the direction of the magnetic field*, $\vec{\mathbf{B}}$.

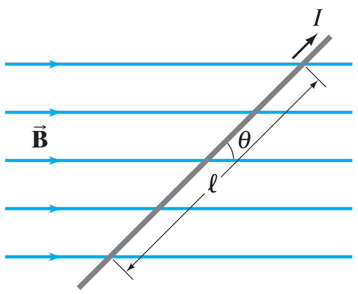
Right-Hand-Rule-2: *Force on current exerted by* $\vec{\mathbf{B}}$

The direction of the force is given by another **right-hand rule**, as illustrated in Fig. 20–11c. Orient your right hand until your outstretched fingers can point in the direction of the conventional current I, and when you bend your fingers they point in the direction of the magnetic field lines, $\vec{\mathbf{B}}$. Then your outstretched thumb will point in the direction of the force $\vec{\mathbf{F}}$ on the wire.

This right-hand rule describes the direction of the force. What about the magnitude of the force on the wire? It is found experimentally that the magnitude of the force is directly proportional to the current I in the wire, to the magnetic field B (assumed uniform), and to the length ℓ of wire exposed to the magnetic field. The force also depends on the angle θ between the current direction and the magnetic field (Fig. 20–12), being proportional to $\sin\theta$. Thus, the force on a wire carrying a current I with length ℓ in a uniform magnetic field B is given by

$$F \propto I\ell B \sin\theta.$$

FIGURE 20–12 Current-carrying wire in a magnetic field. Force on the wire is directed into the page.



When the current is perpendicular to the field lines ($\theta = 90°$ and $\sin 90° = 1$), the force is strongest. When the wire is parallel to the magnetic field lines ($\theta = 0°$), there is no force at all.

Up to now we have not defined the magnetic field strength precisely. In fact, the magnetic field B can be conveniently defined in terms of the above proportion so that the proportionality constant is precisely 1. Thus we have

$$F = I\ell B \sin\theta. \qquad \textbf{(20–1)}$$

If the current's direction is perpendicular to the field $\vec{\mathbf{B}}$ ($\theta = 90°$), then the force is

$$F_{max} = I\ell B. \qquad [\text{current} \perp \vec{\mathbf{B}}] \quad \textbf{(20–2)}$$

(If B is not uniform, then B in Eqs. 20–1 and 20–2 can be the average field over the length ℓ of the wire.)

The magnitude of $\vec{\mathbf{B}}$ can be defined using Eq. 20–2 as $B = F_{max}/I\ell$, where F_{max} is the magnitude of the force on a straight length ℓ of wire carrying a current I when the wire is perpendicular to $\vec{\mathbf{B}}$.

EXERCISE C A wire carrying current I is perpendicular to a magnetic field of strength B. Assuming a fixed length of wire, which of the following changes will result in decreasing the force on the wire by a factor of 2? (a) Decrease the angle from 90° to 45°; (b) decrease the angle from 90° to 30°; (c) decrease the current in the wire to $I/2$; (d) decrease the magnetic field to $B/2$; (e) none of these will do it.

The SI unit for magnetic field B is the **tesla** (T). From Eq. 20–1 or 20–2, we see that $1\ \text{T} = 1\ \text{N/A}\cdot\text{m}$. An older name for the tesla is the "weber per meter squared" $(1\ \text{Wb/m}^2 = 1\ \text{T})$. Another unit sometimes used to specify magnetic field is a cgs unit, the **gauss** (G): $1\ \text{G} = 10^{-4}\ \text{T}$. A field given in gauss should always be changed to teslas before using with other SI units. To get a "feel" for these units, we note that the magnetic field of the Earth at its surface is about $\frac{1}{2}$ G or 0.5×10^{-4} T. On the other hand, strong electromagnets can produce fields on the order of 2 T and superconducting magnets can produce over 10 T.

EXAMPLE 20–1 **Magnetic force on a current-carrying wire.** A wire carrying a steady (dc) 30-A current has a length $\ell = 12$ cm between the pole faces of a magnet. The wire is at an angle $\theta = 60°$ to the field (Fig. 20–13). The magnetic field is approximately uniform at 0.90 T. We ignore the field beyond the pole pieces. Determine the magnitude and direction of the force on the wire.

APPROACH We use Eq. 20–1, $F = I\ell B \sin\theta$.

SOLUTION The force F on the 12-cm length of wire within the uniform field B is

$$F = I\ell B \sin\theta = (30\ \text{A})(0.12\ \text{m})(0.90\ \text{T})(\sin 60°) = 2.8\ \text{N}.$$

We use right-hand-rule-2 to find the direction of $\vec{\mathbf{F}}$. Hold your right hand flat, pointing your fingers in the direction of the current. Then bend your fingers (maybe needing to rotate your hand) so they point along $\vec{\mathbf{B}}$, Fig. 20–13. Your thumb then points into the page, which is thus the direction of the force F.

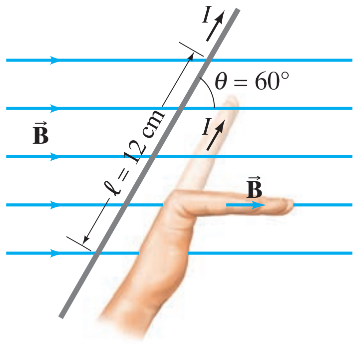


FIGURE 20–13 Example 20–1. For right-hand-rule-2, the thumb points into the page. See Fig. 20–11c.

EXERCISE D A straight power line carries 30 A and is perpendicular to the Earth's magnetic field of 0.50×10^{-4} T. What magnitude force is exerted on 100 m of this power line?

On a diagram, when we want to represent an electric current or a magnetic field that is pointing out of the page (toward us) or into the page, we use ⊙ or ×, respectively. The ⊙ is meant to resemble the tip of an arrow pointing directly toward the reader, whereas the × or ⊗ resembles the tail of an arrow pointing away. See Fig. 20–14.

EXAMPLE 20–2 **Measuring a magnetic field.** A rectangular loop of wire hangs vertically as shown in Fig. 20–14. A magnetic field $\vec{\mathbf{B}}$ is directed horizontally, perpendicular to the plane of the loop, and points out of the page as represented by the symbol ⊙. The magnetic field $\vec{\mathbf{B}}$ is very nearly uniform along the horizontal portion of wire ab (length $\ell = 10.0$ cm) which is near the center of the gap of a large magnet producing the field. The top portion of the wire loop is out of the field. The loop hangs from a balance (reads 0 when $B = 0$) which measures a downward magnetic force of $F = 3.48 \times 10^{-2}$ N when the wire carries a current $I = 0.245$ A. What is the magnitude of the magnetic field B?

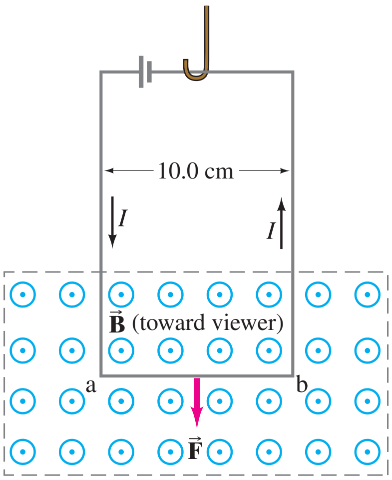
APPROACH Three straight sections of the wire loop are in the magnetic field: a horizontal section and two vertical sections. We apply Eq. 20–1 to each section and use the right-hand rule.

SOLUTION Using right-hand-rule-2 (page 564), we see that the magnetic force on the left vertical section of wire points to the left, and the force on the vertical section on the right points to the right. These two forces are equal and in opposite directions and so add up to zero. Hence, the net magnetic force on the loop is that on the horizontal section ab, whose length is $\ell = 0.100$ m. The angle θ between $\vec{\mathbf{B}}$ and the wire is $\theta = 90°$, so $\sin\theta = 1$. Thus Eq. 20–1 gives

$$B = \frac{F}{I\ell} = \frac{3.48 \times 10^{-2}\ \text{N}}{(0.245\ \text{A})(0.100\ \text{m})} = 1.42\ \text{T}.$$

NOTE This technique can be a precise means of determining magnetic field strength.

FIGURE 20–14 Measuring a magnetic field $\vec{\mathbf{B}}$. Example 20–2.



20–4 Force on an Electric Charge Moving in a Magnetic Field

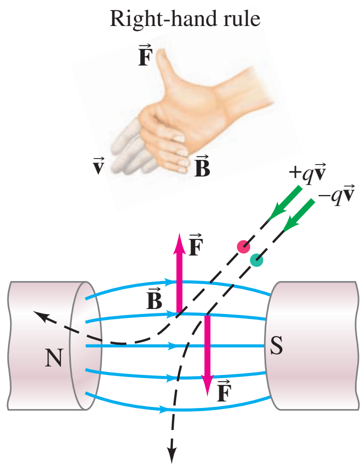


FIGURE 20–15 Force on charged particles due to a magnetic field is perpendicular to the magnetic field direction. If $\vec{v}$ is horizontal, then $\vec{F}$ is vertical. The right-hand rule is shown for the force on a positive charge, $+q$.

We have seen that a current-carrying wire experiences a force when placed in a magnetic field. Since a current in a wire consists of moving electric charges, we might expect that freely moving charged particles (not in a wire) would also experience a force when passing through a magnetic field. Free electric charges are not as easy to produce in the lab as a current in a wire, but it can be done, and experiments do show that moving electric charges experience a force in a magnetic field.

From what we already know, we can predict the force on a single electric charge moving in a magnetic field $\vec{\mathbf{B}}$. If N such particles of charge q pass by a given point in time t, they constitute a current $I = Nq/t$. We let t be the time for a charge q to travel a distance ℓ in a magnetic field $\vec{\mathbf{B}}$; then $\ell = vt$ where v is the magnitude of the velocity $\vec{\mathbf{v}}$ of the particle. Thus, the force on these N particles is, by Eq. 20–1, $F = I\ell B \sin\theta = (Nq/t)(vt)B\sin\theta = NqvB\sin\theta$. The force on *one* of the N particles is then

$$F = qvB\sin\theta. \qquad [\theta \text{ between } \vec{\mathbf{v}} \text{ and } \vec{\mathbf{B}}] \qquad \textbf{(20–3)}$$

This equation gives the magnitude of the force exerted by a magnetic field on a particle of charge q moving with velocity v at a point where the magnetic field has magnitude B. The angle between $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$ is θ. The force is greatest when the particle moves perpendicular to $\vec{\mathbf{B}}$ $(\theta = 90°)$:

$$F_{\text{max}} = qvB. \qquad [\vec{\mathbf{v}} \perp \vec{\mathbf{B}}] \qquad \textbf{(20–4)}$$

The force is *zero* if the particle moves *parallel* to the field lines $(\theta = 0°)$. The *direction* of the force is perpendicular to the magnetic field $\vec{\mathbf{B}}$ and to the velocity $\vec{\mathbf{v}}$ of the particle. For a positive charge, the force direction is given by another **right-hand rule**: you orient your right hand so that your outstretched fingers point along the direction of the particle's velocity ($\vec{\mathbf{v}}$), and when you bend your fingers they must point along the direction of $\vec{\mathbf{B}}$. Then your thumb will point in the direction of the force. This is true only for *positively* charged particles, and will be "up" for the positive particle shown in Fig. 20–15. For negatively charged particles, the force is in exactly the opposite direction, "down" in Fig. 20–15.

Right-hand-rule-3: *Force on moving charge exerted by* $\vec{\mathbf{B}}$

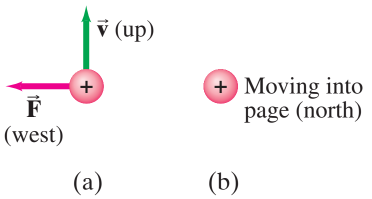
CONCEPTUAL EXAMPLE 20–3 **Negative charge near a magnet.** A negative charge $-Q$ is placed at rest near a magnet. Will the charge begin to move? Will it feel a force? What if the charge were positive, $+Q$?

RESPONSE No to all questions. A charge at rest has velocity equal to zero. Magnetic fields exert a force only on moving electric charges (Eq. 20–3).

EXERCISE E Return to the Chapter-Opening Question, page 560, and answer it again now. Try to explain why you may have answered differently the first time.

EXAMPLE 20–4 **Magnetic force on a proton.** A magnetic field exerts a force of 8.0×10^{-14} N toward the west on a proton moving vertically upward at a speed of 5.0×10^{6} m/s (Fig. 20–16a). When moving horizontally in a northerly direction, the force on the proton is zero (Fig. 20–16b). Determine the magnitude and direction of the magnetic field in this region. (The charge on a proton is $q = +e = 1.6 \times 10^{-19}$ C.)

FIGURE 20–16 Example 20–4.



APPROACH Since the force on the proton is zero when moving north, the field must be in a north–south direction $(\theta = 0°$ in Eq. 20–3). To produce a force to the west when the proton moves upward, right-hand-rule-3 tells us that $\vec{\mathbf{B}}$ must point toward the north. (Your thumb points west and the outstretched fingers of your right hand point upward only when your bent fingers point north.) The magnitude of $\vec{\mathbf{B}}$ is found using Eq. 20–3.

SOLUTION Equation 20–3 with $\theta = 90°$ gives

$$B = \frac{F}{qv} = \frac{8.0 \times 10^{-14}\,\text{N}}{(1.6 \times 10^{-19}\,\text{C})(5.0 \times 10^{6}\,\text{m/s})} = 0.10\,\text{T}.$$

EXERCISE F Determine the force on the proton of Example 20–4 if it heads horizontally south.

EXAMPLE 20–5 ESTIMATE Magnetic force on ions during a nerve pulse. Estimate the magnitude of the magnetic force due to the Earth's magnetic field on ions crossing a cell membrane during an action potential (Section 18–10). Assume the speed of the ions is 10^{-2} m/s.

APPROACH Using $F = qvB$, set the magnetic field of the Earth to be roughly $B \approx 10^{-4}$ T, and the charge $q \approx e \approx 10^{-19}$ C.

SOLUTION $F \approx (10^{-19}\,\mathrm{C})(10^{-2}\,\mathrm{m/s})(10^{-4}\,\mathrm{T}) = 10^{-25}\,\mathrm{N}.$

NOTE This is an extremely small force. Yet it is thought that migrating animals do somehow detect the Earth's magnetic field, and this is an area of active research.

The path of a charged particle moving in a plane perpendicular to a uniform magnetic field is a circle as we shall now show. In Fig. 20–17 the magnetic field is directed *into* the paper, as represented by ×'s. An electron at point P is moving to the right, and the force on it at this point is toward the bottom of the page as shown (use the right-hand rule and reverse the direction for negative charge). The electron is thus deflected toward the page bottom. A moment later, say, when it reaches point Q, the force is still perpendicular to the velocity and is in the direction shown. Because the force is always perpendicular to $\vec{\mathbf{v}}$, the magnitude of $\vec{\mathbf{v}}$ does not change—the electron moves at constant speed. We saw in Chapter 5 that if the force on a particle is always perpendicular to its velocity $\vec{\mathbf{v}}$, the particle moves in a circle and has a centripetal acceleration of magnitude $a = v^2/r$ (Eq. 5–1). Thus a charged particle moves in a circular path with a constant magnitude of centripetal acceleration in a uniform magnetic field (see Fig. 20–18). The electron moves clockwise in Fig. 20–17. A positive particle in this field would feel a force in the opposite direction and would thus move counterclockwise.

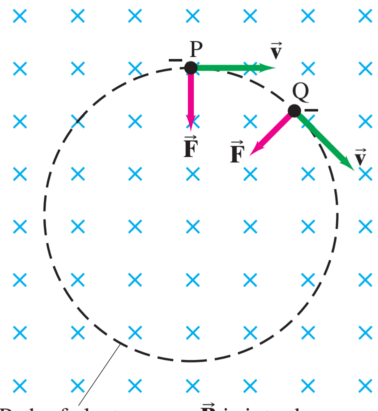
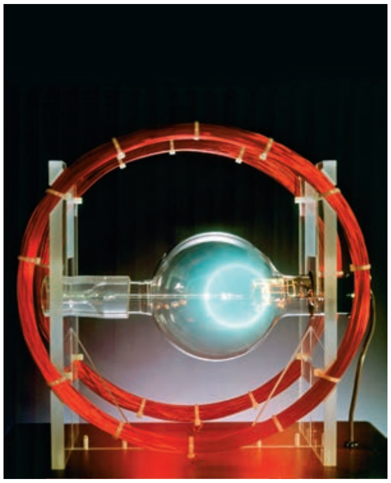


FIGURE 20–17 Force exerted by a uniform magnetic field on a moving charged particle (in this case, an electron) produces a circular path.

FIGURE 20–18 The white ring inside the glass tube is the glow of a beam of electrons that ionize the gas molecules. The red coils of current-carrying wire produce a nearly uniform magnetic field, illustrating the circular path of charged particles in a uniform magnetic field.



EXAMPLE 20–6 Electron's path in a uniform magnetic field. An electron travels at 1.5×10^7 m/s in a plane perpendicular to a uniform 0.010-T magnetic field. Describe its path quantitatively. Ignore gravity (= very small in comparison).

APPROACH The electron moves at speed v in a curved path and so must have a centripetal acceleration $a = v^2/r$ (Eq. 5–1). We find the radius of curvature using Newton's second law. The force is given by Eq. 20–3 with $\sin\theta = 1$: $F = qvB$.

SOLUTION We insert F and a into Newton's second law:

$$\Sigma F = ma$$

$$qvB = \frac{mv^2}{r}.$$

We solve for r and find

$$r = \frac{mv}{qB}.$$

Since $\vec{\mathbf{F}}$ is perpendicular to $\vec{\mathbf{v}}$, the magnitude of $\vec{\mathbf{v}}$ doesn't change. From this equation we see that if $\vec{\mathbf{B}}$ = constant, then r = constant, and the curve must be a circle as we claimed above. To get r we put in the numbers:

$$r = \frac{(9.1 \times 10^{-31}\,\mathrm{kg})(1.5 \times 10^{7}\,\mathrm{m/s})}{(1.6 \times 10^{-19}\,\mathrm{C})(0.010\,\mathrm{T})} = 0.85 \times 10^{-2}\,\mathrm{m} = 8.5\,\mathrm{mm}.$$

NOTE See Fig. 20–18. If the magnetic field B is larger, is the radius larger or smaller?

The time T required for a particle of charge q moving with constant speed v to make one circular revolution in a uniform magnetic field $\vec{\mathbf{B}}\ (\perp \vec{\mathbf{v}})$ is $T = 2\pi r/v$, where $2\pi r$ is the circumference of its circular path. From Example 20–6, $r = mv/qB$, so

$$T = \frac{2\pi m}{qB}.$$

Since T is the period of rotation, the frequency of rotation is

$$f = \frac{1}{T} = \frac{qB}{2\pi m}. \qquad \textbf{(20–5)}$$

This is often called the **cyclotron frequency** of a particle in a field because this is the frequency at which particles revolve in a cyclotron (see Problem 73).

CONCEPTUAL EXAMPLE 20–7 **Stopping charged particles.** An electric charge q moving in an electric field $\vec{\mathbf{E}}$ can be decelerated to a stop if the force $\vec{\mathbf{F}} = q\vec{\mathbf{E}}$ (Eq. 16–5) acts in the direction opposite to the charge's velocity. Can a magnetic field be used to stop a charged particle?

RESPONSE No, because the force is always *perpendicular* to the velocity of the particle and thus can only change the direction but not the magnitude of its velocity. Also the magnetic force cannot do work on the particle (force and displacement are perpendicular, Eq. 6–1) and so cannot change the kinetic energy of the particle, Eq. 6–4.

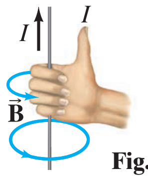
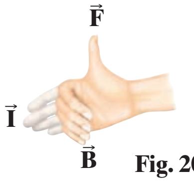
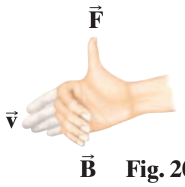
PROBLEM SOLVING

Magnetic Fields

Magnetic fields are somewhat analogous to the electric fields of Chapter 16, but there are several important differences to recall:

1. The force experienced by a charged particle moving in a magnetic field is *perpendicular* to the direction of the magnetic field (and to the direction of the velocity of the particle), whereas the force exerted by an electric field is *parallel* to the direction of the field (and independent of the velocity of the particle).
2. The *right-hand rule*, in its different forms, is intended to help you determine the directions of magnetic field, and the forces they exert, and/or the directions of electric current or charged particle velocity. The right-hand rules (Table 20–1) are designed to deal with the "perpendicular" nature of these quantities.
3. The equations in this Chapter are generally not printed as vector equations, but involve magnitudes only. Right-hand rules are to be used to find directions of vector quantities.

TABLE 20–1 Summary of Right-hand Rules (= RHR)

Physical Situation	Example	How to Orient Right Hand	Result
1. Magnetic field produced by current (RHR-1)	 **Fig. 20–8d**	Wrap fingers around wire with thumb pointing in direction of current I	Fingers curl in direction of $\vec{\mathbf{B}}$
2. Force on electric current I due to magnetic field (RHR-2)	 **Fig. 20–11c**	Fingers first point straight along current I, then bend along magnetic field $\vec{\mathbf{B}}$	Thumb points in direction of the force $\vec{\mathbf{F}}$
3. Force on electric charge $+q$ due to magnetic field (RHR-3)	 **Fig. 20–15**	Fingers point along particle's velocity $\vec{\mathbf{v}}$, then along $\vec{\mathbf{B}}$	Thumb points in direction of the force $\vec{\mathbf{F}}$

CONCEPTUAL EXAMPLE 20–8 **A helical path.** What is the path of a charged particle in a uniform magnetic field if its velocity is *not* perpendicular to the magnetic field?

RESPONSE The velocity vector can be broken down into components parallel and perpendicular to the field. The velocity component parallel to the field lines experiences no force ($\theta = 0$), so this component remains constant. The velocity component perpendicular to the field results in circular motion about the field lines. Putting these two motions together produces a helical (spiral) motion around the field lines as shown in Fig. 20–19.

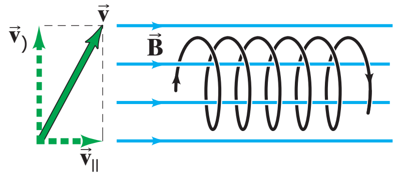


FIGURE 20–19 Example 20–8.

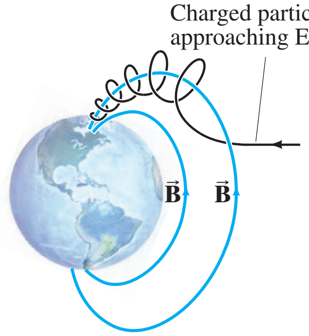
EXERCISE G What is the sign of the charge in Fig. 20–19? How would you modify the drawing if the charge had the opposite sign?

*Aurora Borealis

Charged ions approach the Earth from the Sun (the "solar wind") and enter the atmosphere mainly near the poles, sometimes causing a phenomenon called the **aurora borealis** or "northern lights" in northern latitudes. To see why, consider Example 20–8 and Fig. 20–20 (see also Fig. 20–19). In Fig. 20–20 we imagine a stream of charged particles approaching the Earth. The velocity component *perpendicular* to the field for each particle becomes a circular orbit around the field lines, whereas the velocity component *parallel* to the field carries the particle along the field lines toward the poles. As a particle approaches the Earth's North Pole, the magnetic field is stronger and the radius of the helical path becomes smaller (see Example 20–6, $r \propto 1/B$).

A high concentration of high-speed charged particles ionizes the air, and as the electrons recombine with atoms, light is emitted (Chapter 27) which is the aurora. Auroras are especially spectacular during periods of high sunspot activity when more charged particles are emitted and more come toward Earth.

FIGURE 20–20 (a) Diagram showing a charged particle that approaches the Earth and is "captured" by the magnetic field of the Earth. Such particles follow the field lines toward the poles as shown. (b) Photo of aurora borealis.



(a)

(b)

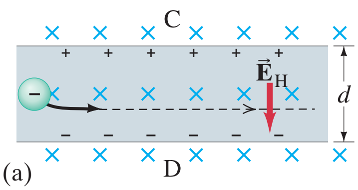
*The Hall Effect

When a current-carrying conductor is held fixed in a magnetic field, the field exerts a sideways force on the charges moving in the conductor. For example, if electrons move to the right in the rectangular conductor shown in Fig. 20–21a, the inward magnetic field will exert a downward force on the electrons of magnitude $F = ev_\mathrm{d}B$, where v_d is the drift velocity of the electrons (Section 18–8). Thus the electrons will tend to move nearer to side D than side C, causing a potential difference between sides C and D of the conductor. This potential difference builds up until the electric field $\vec{\mathbf{E}}_\mathrm{H}$ that it produces exerts a force ($= e\vec{\mathbf{E}}_\mathrm{H}$) on the moving charges that is equal and opposite to the magnetic force ($= ev_\mathrm{d}B$). This is the **Hall effect**, named after E. H. Hall who discovered it in 1879. The difference of potential produced is called the **Hall emf**. Its magnitude is $V_\mathrm{Hall} = E_\mathrm{H}d = (F/e)d = v_\mathrm{d}Bd$, where d is the width of the conductor.

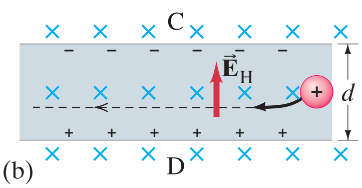
A current of negative charges moving to the right is equivalent to positive charges moving to the left, at least for most purposes. But the Hall effect can distinguish these two. As can be seen in Fig. 20–21b, positive particles moving to the left are deflected downward, so that the bottom surface is positive relative to the top surface. This is the reverse of part (a). Indeed, the direction of the emf in the Hall effect first revealed that it is negative particles that move in metal conductors, and that positive "holes" move in p-type semiconductors.

Because the Hall emf is proportional to B, the Hall effect can be used to measure magnetic fields. A device to do so is called a *Hall probe*. When B is known, the Hall emf can be used to determine the drift velocity of charge carriers.

FIGURE 20–21 The Hall effect. (a) Negative charges moving to the right as the current. (b) Positive charges moving to the left as the current.



(a)



(b)

20–5 Magnetic Field Due to a Long Straight Wire

We saw in Section 20–2, Fig. 20–8, that the magnetic field lines due to the electric current in a long straight wire form circles with the wire at the center (Fig. 20–22). You might expect that the field strength at a given point would be greater if the current flowing in the wire were greater; and that the field would be less at points farther from the wire. This is indeed the case. Careful experiments show that the magnetic field B due to the current in a long straight wire is directly proportional to the current I in the wire and inversely proportional to the distance r from the wire:

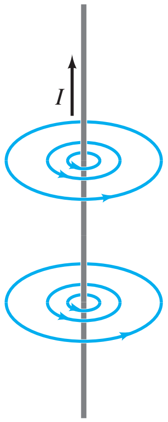


FIGURE 20–22 Same as Fig. 20–8c, magnetic field lines around a long straight wire carrying an electric current I.

$$B \propto \frac{I}{r}.$$

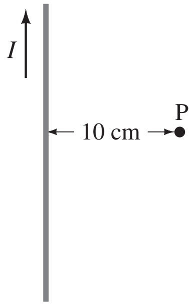
This relation is valid as long as r, the perpendicular distance to the wire, is much less than the distance to the ends of the wire (i.e., the wire is long).

The proportionality constant is written as $\mu_0/2\pi$, so

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}. \qquad \text{[near a long straight wire]} \qquad \textbf{(20–6)}$$

The value of the constant μ_0, which is called the **permeability of free space**, is† $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{T\cdot m/A}$.

FIGURE 20–23 Example 20–9.



CAUTION

A compass, near a current, may not point north

EXAMPLE 20–9 **Calculation of $\vec{B}$ near a wire.** An electric wire in the wall of a building carries a dc current of 25 A vertically upward. What is the magnetic field due to this current at a point P, 10 cm due north of the wire (Fig. 20–23)?

APPROACH We assume the wire is much longer than the 10-cm distance to the point P so we can apply Eq. 20–6.

SOLUTION According to Eq. 20–6:

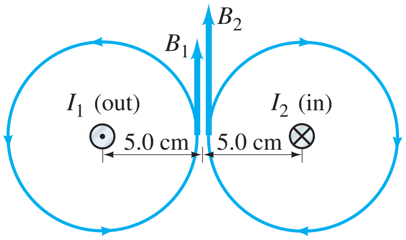
$$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(25\,\mathrm{A})}{(2\pi)(0.10\,\mathrm{m})} = 5.0 \times 10^{-5}\,\mathrm{T},$$

or 0.50 G. By right-hand-rule-1 (page 568), the field points to the west (into the page in Fig. 20–23) at point P.

NOTE The magnetic field at point P produced by the wire has about the same magnitude as Earth's, so a compass at P would not point north but to the northwest.

NOTE Most electrical wiring in buildings consists of cables with two wires in each cable. Since the two wires carry current in opposite directions, their magnetic fields cancel to a large extent, but may still affect sensitive electronic devices.

FIGURE 20–24 Example 20–10. Wire 1 carrying current I_1 out towards us, and wire 2 carrying current I_2 into the page, produce magnetic fields whose lines are circles around their respective wires.



EXAMPLE 20–10 **Magnetic field midway between two currents.** Two parallel straight wires 10.0 cm apart carry currents in opposite directions (Fig. 20–24). Current $I_1 = 5.0$ A is out of the page, and $I_2 = 7.0$ A is into the page. Determine the magnitude and direction of the magnetic field halfway between the two wires.

APPROACH The magnitude of the field produced by each wire is calculated from Eq. 20–6. The direction of *each* wire's field is determined with the right-hand rule. The total field is the vector sum of the two fields at the midway point.

SOLUTION The magnetic field lines due to current I_1 form circles around the wire of I_1, and right-hand-rule-1 (Fig. 20–8d) tells us they point counterclockwise around the wire. The field lines due to I_2 form circles around the wire of I_2 and point clockwise, Fig. 20–24. At the midpoint, both fields point upward in Fig. 20–24 as shown, and so add together. The midpoint is 0.050 m from each wire.

†The constant is chosen in this complicated way so that Ampère's law (Section 20–8), which is considered more fundamental, will have a simple and elegant form.

From Eq. 20–6 the magnitudes of B_1 and B_2 are

$$B_1 = \frac{\mu_0 I_1}{2\pi r} = \frac{(4\pi \times 10^{-7}\,\mathrm{T \cdot m/A})(5.0\,\mathrm{A})}{2\pi(0.050\,\mathrm{m})} = 2.0 \times 10^{-5}\,\mathrm{T};$$

$$B_2 = \frac{\mu_0 I_2}{2\pi r} = \frac{(4\pi \times 10^{-7}\,\mathrm{T \cdot m/A})(7.0\,\mathrm{A})}{2\pi(0.050\,\mathrm{m})} = 2.8 \times 10^{-5}\,\mathrm{T}.$$

The total field is *up* with a magnitude of

$$B = B_1 + B_2 = 4.8 \times 10^{-5}\,\mathrm{T}.$$

EXERCISE H Suppose both I_1 and I_2 point into the page in Fig. 20–24. What then is the field B midway between the wires?

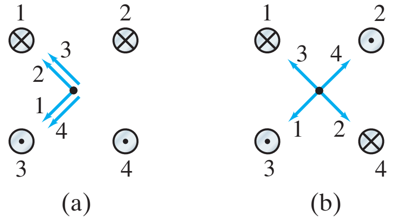


FIGURE 20–25 Example 20–11.

CONCEPTUAL EXAMPLE 20–11 **Magnetic field due to four wires.** Figure 20–25 shows four long parallel wires which carry equal currents into or out of the page as shown. In which configuration, (*a*) or (*b*), is the magnetic field greater at the center of the square?

RESPONSE It is greater in (*a*). The arrows illustrate the directions of the field produced by each wire; check it out, using the right-hand rule to confirm these results. The net field at the center is the superposition of the four fields (which are of equal magnitude), which will point to the left in (*a*) and is zero in (*b*).

20–6 Force between Two Parallel Wires

We have seen that a wire carrying a current produces a magnetic field (magnitude given by Eq. 20–6 for a long straight wire). Also, a current-carrying wire feels a force when placed in a magnetic field (Section 20–3, Eq. 20–1). Thus, we expect that two current-carrying wires will exert a force on each other.

Consider two long parallel wires separated by a distance d, as in Fig. 20–26a. They carry currents I_1 and I_2, respectively. Each current produces a magnetic field that is "felt" by the other, so each must exert a force on the other. For example, the magnetic field B_1 produced by I_1 in Fig 20–26 is given by Eq. 20–6, which at the location of wire 2 points into the page and has magnitude

$$B_1 = \frac{\mu_0}{2\pi}\frac{I_1}{d}.$$

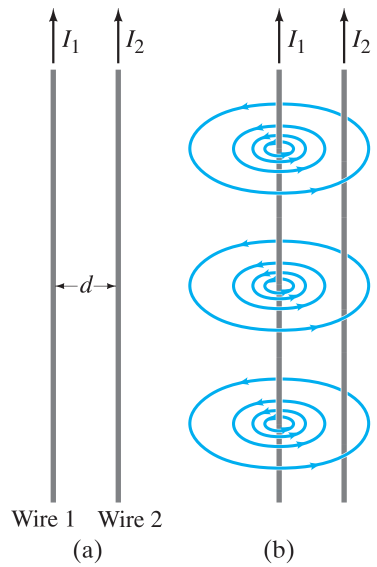


FIGURE 20–26 (a) Two parallel conductors carrying currents I_1 and I_2. (b) Magnetic field $\vec{\mathbf{B}}_1$ produced by I_1. (Field produced by I_2 is not shown.) $\vec{\mathbf{B}}_1$ points into page at position of I_2.

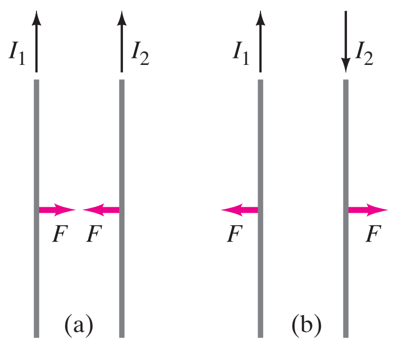
See Fig. 20–26b, where the field due *only* to I_1 is shown. According to Eq. 20–2, the force F_2 exerted by B_1 on a length ℓ_2 of wire 2, carrying current I_2, has magnitude

$$F_2 = I_2 B_1 \ell_2.$$

Note that the force on I_2 is due only to the field produced by I_1. Of course, I_2 also produces a field, but it does not exert a force on itself. We substitute B_1 into the formula for F_2 and find that the force on a length ℓ_2 of wire 2 is

$$F_2 = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}\ell_2. \qquad \text{[parallel wires]} \quad \textbf{(20–7)}$$

FIGURE 20–27 (a) Parallel currents in the same direction exert an attractive force on each other. (b) Antiparallel currents (in opposite directions) exert a repulsive force on each other.



If we use right-hand-rule-1 of Fig. 20–8d, we see that the lines of B_1 are as shown in Fig. 20–26b. Then using right-hand-rule-2 of Fig. 20–11c, we see that the force exerted on I_2 will be to the left in Fig. 20–26b. That is, I_1 exerts an attractive force on I_2 (Fig. 20–27a). This is true as long as the currents are in the same direction. If I_2 is in the opposite direction from I_1, right-hand-rule-2 indicates that the force is in the opposite direction. That is, I_1 exerts a repulsive force on I_2 (Fig. 20–27b).

Reasoning similar to that above shows that the magnetic field produced by I_2 exerts an equal but opposite force on I_1. We expect this to be true also from Newton's third law. Thus, as shown in Fig. 20–27, parallel currents in the same direction attract each other, whereas parallel currents in opposite directions repel.

EXAMPLE 20–12 **Force between two current-carrying wires.** The two wires of a 2.0-m-long appliance cord are 3.0 mm apart and carry a current of 8.0 A. Calculate the force one wire exerts on the other.

APPROACH Each wire is in the magnetic field of the other when the current is on, so we can apply Eq. 20–7.

SOLUTION Equation 20–7 gives

$$F = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}\ell_2 = \frac{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(8.0\,\mathrm{A})^2(2.0\,\mathrm{m})}{(2\pi)(3.0 \times 10^{-3}\,\mathrm{m})} = 8.5 \times 10^{-3}\,\mathrm{N}.$$

The currents are in opposite directions (one toward the appliance, the other away from it), so the force would be repulsive and tend to spread the wires apart.

Definition of the Ampere and the Coulomb

You may have wondered how the constant μ_0 in Eq. 20–6 could be exactly $4\pi \times 10^{-7}\,\mathrm{T\cdot m/A}$. Here is how it happened. With an older definition of the ampere, μ_0 was measured experimentally to be very close to this value. Today, μ_0 is *defined* to be exactly $4\pi \times 10^{-7}\,\mathrm{T\cdot m/A}$. This could not be done if the ampere were defined independently. The ampere, the unit of current, is now defined in terms of the magnetic field it produces using the defined value of μ_0.

In particular, we use the force between two parallel current-carrying wires, Eq. 20–7, to define the ampere precisely. If $I_1 = I_2 = 1\,\mathrm{A}$ exactly, and the two wires are exactly 1 m apart, then

$$\frac{F}{\ell} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d} = \frac{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})}{(2\pi)}\frac{(1\,\mathrm{A})(1\,\mathrm{A})}{(1\,\mathrm{m})} = 2 \times 10^{-7}\,\mathrm{N/m}.$$

Thus, *one* **ampere** *is defined as that current flowing in each of two long parallel wires, 1 m apart, which results in a force of exactly 2×10^{-7} N per meter of length of each wire.*

This is the precise definition of the ampere, and because it is readily reproducible, is called an **operational definition**. The **coulomb** is defined in terms of the ampere as being *exactly* one ampere-second: $1\,\mathrm{C} = 1\,\mathrm{A\cdot s}$.

20–7 Solenoids and Electromagnets

PHYSICS APPLIED
Solenoids and electromagnets

A long coil of wire consisting of many loops (or turns) of wire is called a **solenoid**. The current in each loop produces a magnetic field, as we saw in Fig. 20–9. The magnetic field within a solenoid can be fairly large because it is the sum of the fields due to the current in each loop (Fig. 20–28). A solenoid acts like a magnet; one end can be considered the north pole and the other the south pole, depending on the direction of the current in the loops (use the right-hand rule). Since the magnetic field lines leave the north pole of a magnet, the north pole of the solenoid in Fig. 20–28 is on the right. As we will see in the next Section, the magnetic field inside a tightly wrapped solenoid with N turns of wire in a length ℓ, each carrying current I, is

$$B = \frac{\mu_0 N I}{\ell}. \tag{20–8}$$

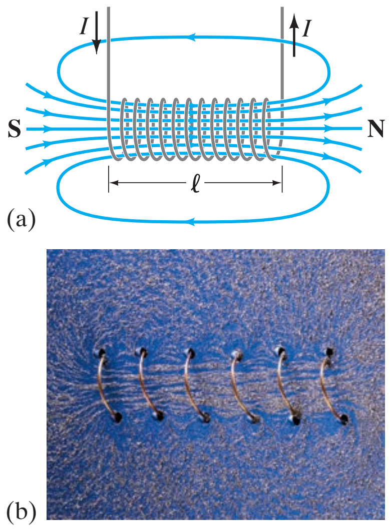


FIGURE 20–28 (a) Magnetic field of a solenoid. The north pole of this solenoid, thought of as a magnet, is on the right, and the south pole is on the left. (b) Photo of iron filings aligning along $\vec{\mathbf{B}}$ field lines of a solenoid with loosely spaced loops. The field is smoother if the loops are closely spaced.

If a piece of iron is placed inside a solenoid, the magnetic field is increased greatly because the iron becomes a magnet. The resulting magnetic field is the sum of the field due to the current and the field due to the iron, and can be hundreds or thousands of times the field due to the current alone (see Section 20–12). Such an iron-core solenoid is an **electromagnet**.

Electromagnets have many practical applications, from use in motors and generators to producing large magnetic fields for research. Sometimes an iron core is not present—the magnetic field then comes only from the current in the wire coils. A large field B in this case requires a large current I, which produces a large amount of waste heat $(P = I^2R)$. But if the current-carrying wires are made of superconducting material kept below the transition temperature (Section 18–9), very high fields can be produced, and no electric power is needed to maintain the large current in the superconducting coils. Energy is required, however, to refrigerate the coils at the low temperatures where they superconduct.

Another useful device consists of a solenoid into which a rod of iron is partially inserted. This combination is also referred to as a **solenoid**. One simple use is as a doorbell (Fig. 20–29). When the circuit is closed by pushing the button, the coil effectively becomes a magnet and exerts a force on the iron rod. The rod is pulled into the coil and strikes the bell. A large solenoid is used for the starter of a car: when you engage the starter, you are closing a circuit that not only turns the starter motor, but first activates a solenoid that moves the starter into direct contact with the gears on the engine's flywheel. Solenoids are used a lot as switches in cars and many other devices. They have the advantage of moving mechanical parts quickly and accurately.

PHYSICS APPLIED
Doorbell, car starter

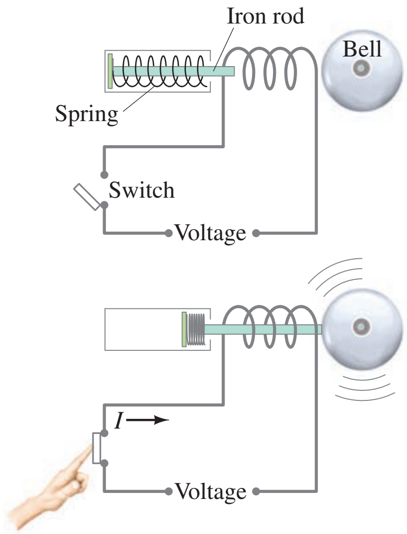


FIGURE 20–29 Solenoid used as a doorbell.

Magnetic Circuit Breakers

Modern circuit breakers that protect houses and buildings from overload and fire contain not only a "thermal" part (bimetallic strip as described in Section 18–6, Fig. 18–19) but also a magnetic sensor. If the current is above a certain level, the magnetic field the current produces pulls an iron plate that breaks the same contact points as in Figs. 18–19b and c. Magnetic circuit breakers react quickly (<10 ms), and for buildings are designed to react to the high currents of short circuits (but not shut off for the start-up surges of motors).

In more sophisticated circuit breakers, including ground fault circuit interrupters (GFCIs—discussed in Section 21–9), a solenoid is used. The iron rod of Fig. 20–29, instead of striking a bell, strikes one side of a pair of electric contact points, opening them and opening the circuit. They react very quickly (≈ 1 ms) and to very small currents (≈ 5 mA) and thus protect humans (not just property) and save lives.

PHYSICS APPLIED
Magnetic circuit breakers

20–8 Ampère's Law

The relation between the current in a long straight wire and the magnetic field it produces is given by Eq. 20–6, Section 20–5. This equation is valid *only* for a long straight wire. Is there a general relation between a current in a wire of any shape and the magnetic field around it? Yes: the French scientist André Marie Ampère (1775–1836) proposed such a relation shortly after Oersted's discovery. Consider any (arbitrary) closed path around a current, as shown in Fig. 20–30, and imagine this path as being made up of short segments each of length $\Delta\ell$. We take the product of the length of each segment times the component of magnetic field $\vec{\mathbf{B}}$ parallel to that segment. If we now sum all these terms, the result (according to Ampère) will be equal to μ_0 times the net current I_{encl} that passes through the surface *enclosed* by the path. This is known as **Ampère's law** and can be written

AMPÈRE'S LAW

$$\Sigma B_{\parallel}\,\Delta\ell = \mu_0 I_{\text{encl}}. \qquad \textbf{(20–9)}$$

The symbol Σ means "the sum of" and $B_{\parallel}$ means the component of $\vec{\mathbf{B}}$ parallel to that particular $\Delta\ell$. The lengths $\Delta\ell$ are chosen small enough so that $B_{\parallel}$ is essentially constant along each length. The sum must be made over a closed path, and I_{encl} is the total net current enclosed by this closed path.

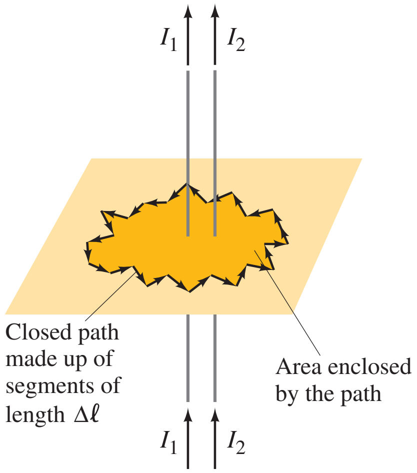


FIGURE 20–30 Arbitrary path enclosing electric currents, for Ampère's law. The path is broken down into segments of equal length $\Delta\ell$. The total current enclosed by the path shown is $I_{\text{encl}} = I_1 + I_2$.

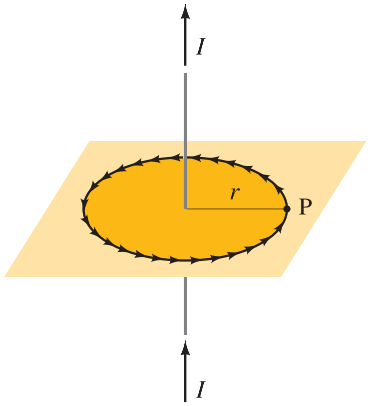


FIGURE 20–31 Circular path of radius r.

Field Due to a Straight Wire

We can check Ampère's law by applying it to the simple case of a long straight wire carrying a current I. Let us find the magnitude of B at point P, a distance r from the wire in Fig. 20–31. The magnetic field lines are circles with the wire at their center (as in Fig. 20–8). As the path to be used in Eq. 20–9, we choose a convenient one: a circle of radius r, because at any point on this path, $\vec{\mathbf{B}}$ will be tangent to this circle. For any short segment of the circle (Fig. 20–31), $\vec{\mathbf{B}}$ will be parallel to that segment, so $B_{\parallel} = B$. Suppose we break the circular path down into 100 segments.† Then Ampère's law states that

$$(B\,\Delta\ell)_1 + (B\,\Delta\ell)_2 + (B\,\Delta\ell)_3 + \cdots + (B\,\Delta\ell)_{100} = \mu_0 I.$$

The dots represent all the terms we did not write down. All the segments are the same distance from the wire, so by *symmetry* we expect B to be the same at each segment. We can then factor out B from the sum:

$$B(\Delta\ell_1 + \Delta\ell_2 + \Delta\ell_3 + \cdots + \Delta\ell_{100}) = \mu_0 I.$$

The sum of the segment lengths $\Delta\ell$ equals the circumference of the circle, $2\pi r$. Thus we have

$$B(2\pi r) = \mu_0 I,$$

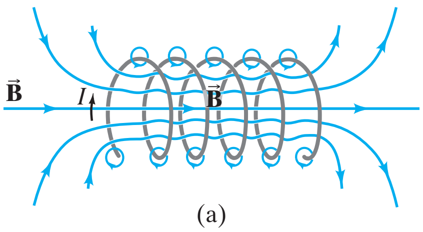
or

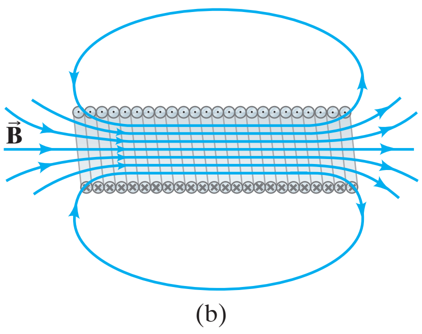
$$B = \frac{\mu_0 I}{2\pi r}.$$

This is just Eq. 20–6 for the magnetic field near a long straight wire, so Ampère's law agrees with experiment in this case.

A great many experiments indicate that Ampère's law is valid in general. Practically, it can be used to calculate the magnetic field mainly for simple or symmetric situations. Its importance is that it relates the magnetic field to the current in a direct and mathematically elegant way. Ampère's law is considered one of the basic laws of electricity and magnetism. It is valid for any situation where the currents and fields are not changing in time.

FIGURE 20–32 (a) Magnetic field due to several loops of a solenoid. (b) For many closely spaced loops, the field is very nearly uniform.





Field Inside a Solenoid

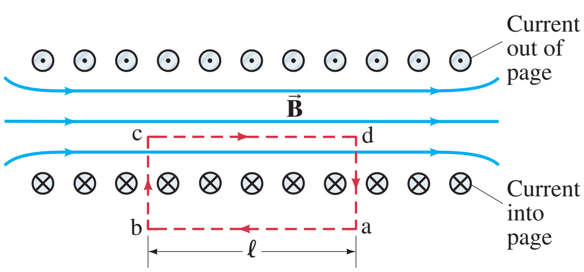
We now use Ampère's law to calculate the magnetic field inside a *solenoid* (Section 20–7), a long coil of wire with many loops or turns, Fig. 20–32. Each loop produces a magnetic field as was shown in Fig. 20–9, and the total field inside the solenoid will be the sum of the fields due to each current loop as shown in Fig. 20–32a for a few loops. If the solenoid has many loops and they are close together, the field inside will be nearly uniform and parallel to the solenoid axis except at the ends, as shown in Fig. 20–32b. Outside the solenoid, the field lines spread out in space, so the magnetic field is much weaker than inside. For applying Ampère's law, we choose the path abcd shown in Fig. 20–33 far from either end. We consider this path as made up of four straight segments, the sides of the rectangle: ab, bc, cd, da. Then Ampère's law, Eq. 20–9, becomes

$$(B_{\parallel}\,\Delta\ell)_{\rm ab} + (B_{\parallel}\,\Delta\ell)_{\rm bc} + (B_{\parallel}\,\Delta\ell)_{\rm cd} + (B_{\parallel}\,\Delta\ell)_{\rm da} = \mu_0 I_{\rm encl}.$$

The first term in the sum on the left will be (nearly) zero because the field outside the solenoid is negligible compared to the field inside. Furthermore, $\vec{\mathbf{B}}$ is perpendicular to the segments bc and da, so these terms are zero, too.

†Actually, Ampère's law is precisely accurate when there is an infinite number of infinitesimally short segments, but that leads into calculus.

FIGURE 20–33 Cross-sectional view into a solenoid. The magnetic field inside is straight except at the ends. Red dashed lines indicate the path chosen for use in Ampère's law. ⊙ and ⊗ are electric current direction (in the wire loops) out of the page and into the page.



Thus the left side of our Ampère equation we just wrote becomes $(B_{\parallel}\,\Delta\ell)_{\mathrm{cd}} = B\ell$, where B is the field inside the solenoid, and ℓ is the length cd. We set $B\ell$ equal to μ_0 times the current enclosed by our chosen rectangular loop: if a current I flows in the wire of the solenoid, the total current enclosed by our path abcd is NI, where N is the number of loops (or turns) our path encircles (five in Fig. 20–33). Thus Ampère's law gives us

$$B\ell = \mu_0 NI,$$

so

$$B = \frac{\mu_0 IN}{\ell}, \qquad \text{[solenoid]} \quad \textbf{(20–8 } \textit{again}\textbf{)}$$

as we quoted in the previous Section. This is the magnetic field magnitude inside a solenoid. B depends only on the number of loops per unit length, N/ℓ, and the current I. The field does not depend on the position within the solenoid, so B is uniform inside the solenoid. This is strictly true only for an infinite solenoid, but it is a good approximation for real ones at points not close to the ends.

The direction of the magnetic field inside the solenoid is found by applying right-hand-rule-1, Fig. 20–8d (see also Figs. 20–9 and 20–10), and is as shown in Fig. 20–33.

20–9 Torque on a Current Loop; Magnetic Moment

When an electric current flows in a closed loop of wire placed in an external magnetic field, as shown in Fig. 20–34, the magnetic force on the current can produce a torque. This is the principle behind a number of important practical devices, including motors and analog voltmeters and ammeters, which we discuss in the next Section.

Current flows through the rectangular loop in Fig. 20–34a, whose face we assume is parallel to $\vec{\mathbf{B}}$. $\vec{\mathbf{B}}$ exerts no force and no torque on the horizontal segments of wire because they are parallel to the field and $\sin\theta = 0$ in Eq. 20–1. But the magnetic field does exert a force on each of the vertical sections of wire as shown, $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ (see also top view, Fig. 20–34b). By right-hand-rule-2 (Fig. 20–11c or Table 20–1) the direction of the force on the upward current on the left is in the opposite direction from the equal magnitude force $\vec{\mathbf{F}}_2$ on the downward current on the right. These forces give rise to a net torque that acts to rotate the coil about its vertical axis.

Let us calculate the magnitude of this torque. From Eq. 20–2 (current $\perp \vec{\mathbf{B}}$), the force $F = IaB$, where a is the length of the vertical arm of the coil (Fig. 20–34a). The lever arm for each force is $b/2$, where b is the width of the coil and the "axis" is at the midpoint. The torques around this axis produced by $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ act in the same direction (Fig. 20–34b), so the total torque τ is the sum of the two torques:

$$\tau = IaB\frac{b}{2} + IaB\frac{b}{2} = IabB = IAB,$$

where $A = ab$ is the area of the coil. If the coil consists of N loops of wire, the current is then NI, so the torque becomes

$$\tau = NIAB.$$

If the coil makes an angle with the magnetic field, as shown in Fig. 20–34c, the forces are unchanged, but each lever arm is reduced from $\frac{1}{2}b$ to $\frac{1}{2}b\sin\theta$. Note that the angle θ is taken to be the angle between $\vec{\mathbf{B}}$ and the perpendicular to the face of the coil, Fig. 20–34c. So the torque becomes

$$\tau = NIAB\sin\theta. \qquad \textbf{(20–10)}$$

This formula, derived here for a rectangular coil, is valid for any shape of flat coil.

The quantity NIA is called the **magnetic dipole moment** of the coil:

$$M = NIA \qquad \textbf{(20–11)}$$

and is considered a vector perpendicular to the coil.

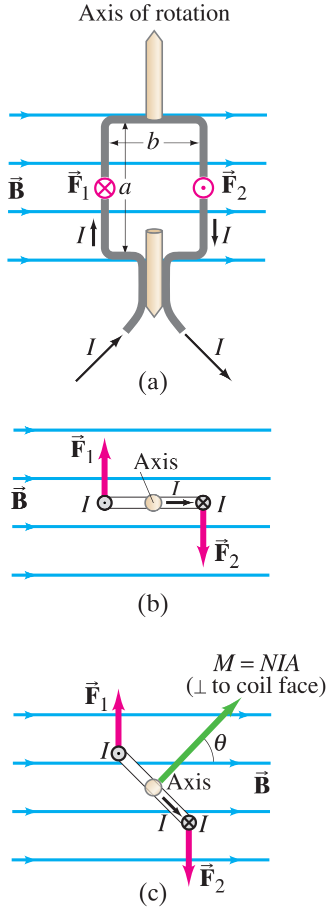


FIGURE 20–34 Calculating the torque on a current loop in a magnetic field $\vec{\mathbf{B}}$. (a) Loop face parallel to $\vec{\mathbf{B}}$ field lines; (b) top view; (c) loop makes an angle to $\vec{\mathbf{B}}$, reducing the torque since the lever arm is reduced.

EXAMPLE 20–13 **Torque on a coil.** A circular loop of wire has a diameter of 20.0 cm and contains 10 loops. The current in each loop is 3.00 A, and the coil is placed in a 2.00-T external magnetic field. Determine the maximum and minimum torque exerted on the coil by the field.

APPROACH Equation 20–10 is valid for any shape of coil, including circular loops. Maximum and minimum torque are determined by the angle θ the coil makes with the magnetic field.

SOLUTION The area of one loop of the coil is

$$A = \pi r^2 = \pi(0.100\,\mathrm{m})^2 = 3.14 \times 10^{-2}\,\mathrm{m}^2.$$

The maximum torque occurs when the coil's face is parallel to the magnetic field, so $\theta = 90°$ in Fig. 20–34c, and $\sin\theta = 1$ in Eq. 20–10:

$$\tau = NIAB\sin\theta = (10)(3.00\,\mathrm{A})\left(3.14 \times 10^{-2}\,\mathrm{m}^2\right)(2.00\,\mathrm{T})(1) = 1.88\,\mathrm{N\cdot m}.$$

The minimum torque occurs if $\sin\theta = 0$, for which $\theta = 0°$, and then $\tau = 0$ from Eq. 20–10.

NOTE If the coil is free to turn, it will rotate toward the orientation with $\theta = 0°$.

20–10 Applications: Motors, Loudspeakers, Galvanometers

There are many practical applications of the forces related to magnetism. Among the most common are motors and loudspeakers. First we look at the galvanometer, which is the easiest to explain, and which you find on the instrument panels of automobiles and other devices whose readout is via a pointer or needle.

Galvanometer

The basic component of analog meters (those with pointer and dial), including analog ammeters, voltmeters, and ohmmeters, including gauges on car dashboards, is a galvanometer. We have already seen how these meters are designed (Section 19–8), and now we can examine how the crucial element, a galvanometer, works. As shown in Fig. 20–35, a **galvanometer** consists of a coil of wire (with attached pointer) suspended in the magnetic field of a permanent magnet. When current flows through the loop of wire, the magnetic field B exerts a torque τ on the loop, as given by Eq. 20–10,

$$\tau = NIAB\sin\theta.$$

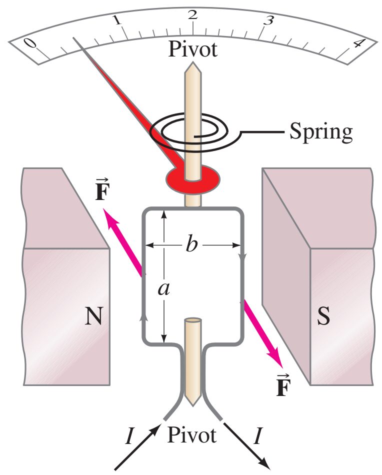


FIGURE 20–35 Galvanometer.

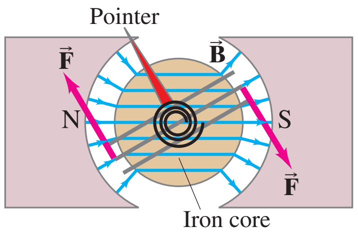
This torque is opposed by a spring which exerts a torque τ_s approximately proportional to the angle ϕ through which it is turned (Hooke's law). That is,

$$\tau_s = k\phi,$$

where k is the stiffness constant of the spring. The coil and attached pointer rotate to the angle where the torques balance. When the needle is in equilibrium at rest, the torques have equal magnitude: $k\phi = NIAB\sin\theta$, so

$$\phi = \frac{NIAB\sin\theta}{k}.$$

FIGURE 20–36 Galvanometer coil (3 loops shown) wrapped on an iron core.



The deflection of the pointer, ϕ, is directly proportional to the current I flowing in the coil, but also depends on the angle θ the coil makes with $\vec{\mathbf{B}}$. For a useful meter we need ϕ to depend only on the current I, independent of θ. To solve this problem, magnets with curved pole pieces are used and the galvanometer coil is wrapped around a cylindrical iron core as shown in Fig. 20–36. The iron tends to concentrate the magnetic field lines so that $\vec{\mathbf{B}}$ always points parallel to the face of the coil at the wire outside the core. The force is then always perpendicular to the face of the coil, and the torque will not vary with angle. Thus ϕ will be proportional to I, as required for a useful meter.

Electric Motors

An **electric motor** changes electric energy into (rotational) mechanical energy. A motor works on the same principle as a galvanometer (a torque is exerted on a current-carrying loop in a magnetic field) except that the coil must turn continuously in one direction. The coil is mounted on an iron cylinder called the **rotor** or **armature**, Fig. 20–37. Actually, there are several coils, although only one is indicated in Fig. 20–37. The armature is mounted on a shaft or axle. When the armature is in the position shown in Fig. 20–37, the magnetic field exerts forces on the current in the loop as shown (perpendicular to $\vec{\mathbf{B}}$ and to the current direction). However, when the coil, which is rotating clockwise in Fig. 20–37, passes beyond the vertical position, the forces would then act to return the coil back toward the vertical if the current remained the same. But if the current could be reversed at that critical moment, the forces would reverse, and the coil would continue rotating in the same direction. Thus, alternation of the current is necessary if a motor is to turn continuously in one direction. This can be achieved in a **dc motor** with the use of **commutators** and **brushes**: as shown in Fig. 20–38, input current passes through stationary brushes that rub against the conducting commutators mounted on the motor shaft. At every half revolution, each commutator changes its connection over to the other brush. Thus the current in the coil reverses every half revolution as required for continuous rotation.

PHYSICS APPLIED
DC motor

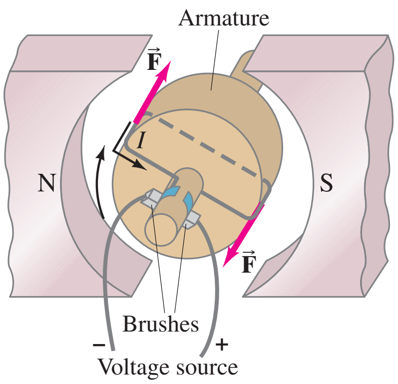


FIGURE 20–37 Diagram of a simple dc motor. (Magnetic field lines are as shown in Fig. 20–36.)

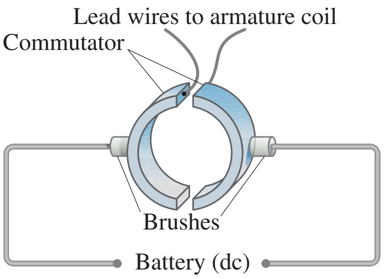


FIGURE 20–38 Commutator-brush arrangement in a dc motor ensures alternation of the current in the armature to keep rotation continuous in one direction. The commutators are attached to the motor shaft and turn with it, whereas the brushes remain stationary.

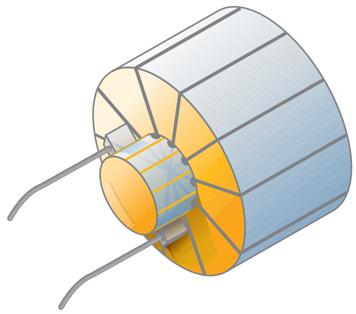


FIGURE 20–39 Motor with many windings.

Most motors contain several coils, called *windings*, each connected to a different portion of the armature, Fig. 20–39. Current flows through each coil only during a small part of a revolution, at the time when its orientation results in the maximum torque. In this way, a motor produces a much steadier torque than can be obtained from a single coil.

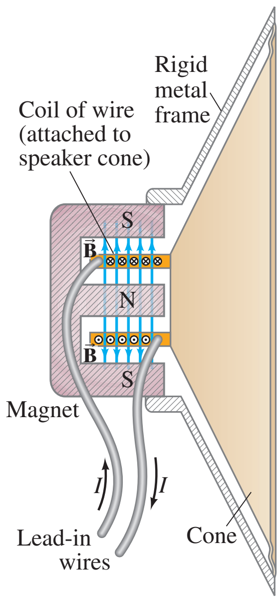
An **ac motor**, with ac current as input, can work without commutators since the current itself alternates. Many motors use wire coils to produce the magnetic field (electromagnets) instead of a permanent magnet. Indeed the design of most motors is more complex than described here, but the general principles remain the same.

PHYSICS APPLIED
AC motor

Loudspeakers and Headsets

Loudspeakers and audio headsets also work on the principle that a magnet exerts a force on a current-carrying wire. The electrical output of a stereo or TV set is connected to the wire leads of the speaker or earbuds. The speaker leads are connected internally to a coil of wire, which is itself attached to the speaker cone, Fig. 20–40. The speaker cone is usually made of stiffened cardboard and is mounted so that it can move back and forth freely (except at its attachment on the outer edges). A permanent magnet is mounted directly in line with the coil of wire. When the alternating current of an audio signal flows through the wire coil, which is free to move within the magnet, the coil experiences a force due to the magnetic field of the magnet. (The force is to the right at the instant shown in Fig. 20–40, RHR-2, page 568.) As the current alternates at the frequency of the audio signal, the coil and attached speaker cone move back and forth at the same frequency, causing alternate compressions and rarefactions of the adjacent air, and sound waves are produced. A speaker thus changes electrical energy into sound energy, and the frequencies and intensities of the emitted sound waves can be an accurate reproduction of the electrical input.

FIGURE 20–40 Loudspeaker.



*20–11 Mass Spectrometer

PHYSICS APPLIED
The mass spectrometer

A **mass spectrometer** is a device to measure masses of atoms. It is used today not only in physics but also in chemistry, geology, and medicine, often to identify atoms (and their concentration) in given samples. Ions are produced by heating the sample, or by using an electric current. As shown in Fig. 20–41, the ions (mass m, charge q) pass through slit S_1 and enter a region (before S_2) where there are crossed ($\perp$) electric and magnetic fields. Ions follow a straight-line path in this region if the electric force qE (upward on a positive ion) is just balanced by the magnetic force qvB (downward on a positive ion): that is, if $qE = qvB$, or

$$v = \frac{E}{B}.$$

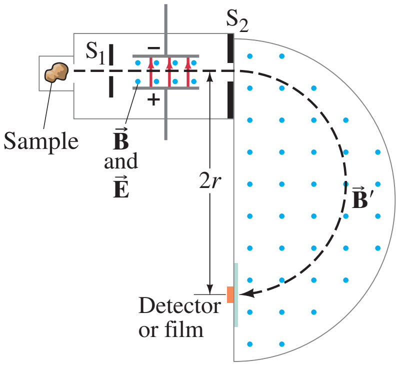


FIGURE 20–41 Bainbridge-type mass spectrometer. The magnetic fields B and B' point out of the paper (indicated by the dots).

Only those ions whose speed is $v = E/B$ will pass through undeflected and emerge through slit S_2. (This arrangement is called a **velocity selector**.) In the semicircular region, after S_2, there is only a magnetic field, B', so the ions follow a circular path. The radius of the circular path is found from their mark on film, or by detectors, if B' is fixed. If instead r is fixed by the position of a detector, then B' is varied until detection occurs. Newton's second law, $\Sigma F = ma$, applied to an ion moving in a circle under the influence only of the magnetic field B' gives $qvB' = mv^2/r$. Since $v = E/B$, we have

$$m = \frac{qB'r}{v} = \frac{qBB'r}{E}. \qquad \textbf{(20–12)}$$

All the quantities on the right side are known or can be measured, and thus m can be determined.

Historically, the masses of many atoms were measured this way. When a pure substance was used, it was sometimes found that two or more closely spaced marks would appear on the film. For example, neon produced two marks whose radii corresponded to atoms of 20 and 22 atomic mass units (u). Impurities were ruled out and it was concluded that there must be two types of neon with different masses. These different forms were called **isotopes**. It was soon found that most elements are mixtures of isotopes, and the difference in mass is due to different numbers of neutrons (discussed in Chapter 30).

EXAMPLE 20–14 **Mass spectrometry.** Carbon atoms of atomic mass 12.0 u are found to be mixed with an unknown element. In a mass spectrometer with fixed B', the carbon traverses a path of radius 22.4 cm and the unknown's path has a 26.2-cm radius. What is the unknown element? Assume the ions of both elements have the same charge.

APPROACH The carbon and unknown atoms pass through the same electric and magnetic fields. Hence their masses are proportional to the radius of their respective paths (see Eq. 20–12).

SOLUTION We write a ratio for the masses, using Eq. 20–12:

$$\frac{m_x}{m_C} = \frac{qBB'r_x/E}{qBB'r_C/E} = \frac{r_x}{r_C}$$

$$= \frac{26.2\text{ cm}}{22.4\text{ cm}} = 1.17.$$

Thus $m_x = 1.17 \times 12.0\text{ u} = 14.0\text{ u}$. The other element is probably nitrogen (see the Periodic Table, inside the back cover).

NOTE The unknown could also be an isotope such as carbon-14 $\left({}^{14}_{6}\text{C}\right)$. See Appendix B. Further physical or chemical analysis would be needed.

*20–12 Ferromagnetism: Domains and Hysteresis

We saw in Section 20–1 that iron (and a few other materials) can be made into strong magnets. These materials are said to be **ferromagnetic**.

* Sources of Ferromagnetism

Microscopic examination reveals that a piece of iron is made up of tiny regions known as **domains**, less than 1 mm in length or width. Each domain behaves like a tiny magnet with a north and a south pole. In an unmagnetized piece of iron, the domains are arranged randomly, Fig. 20–42a. The magnetic effects of the domains cancel each other out, so this piece of iron is not a magnet. In a magnet, the domains are preferentially aligned in one direction as shown in Fig. 20–42b (downward in this case). A magnet can be made from an unmagnetized piece of iron by placing it in a strong magnetic field. (You can make a needle magnetic, for example, by stroking it with one pole of a strong magnet.) The magnetization direction of domains may actually rotate slightly to be more nearly parallel to the external field, and the borders of domains may move so domains with magnetic orientation parallel to the external field grow larger (compare Figs. 20–42a and b).

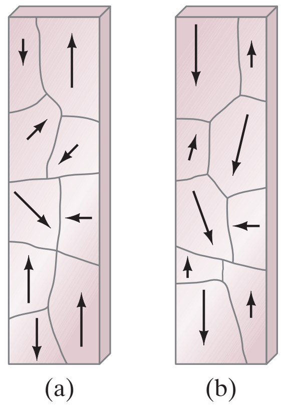
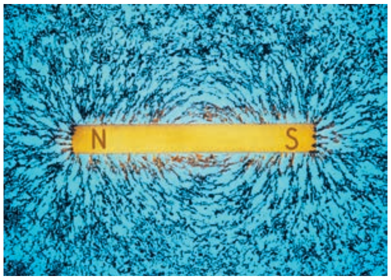


FIGURE 20–42 (a) An unmagnetized piece of iron is made up of domains that are randomly arranged. Each domain is like a tiny magnet; the arrows represent the magnetization direction, with the arrowhead being the N pole. (b) In a magnet, the domains are preferentially aligned in one direction (down in this case), and may be altered in size by the magnetization process.

We can now explain how a magnet can pick up unmagnetized pieces of iron like paper clips. The magnet's field causes a slight realignment of the domains in the unmagnetized object so that it becomes a temporary magnet with its north pole facing the south pole of the permanent magnet; thus, attraction results. Similarly, elongated iron filings in a magnetic field acquire aligned domains and align themselves to reveal the shape of the magnetic field, Fig. 20–43.

FIGURE 20–43 Iron filings line up along magnetic field lines due to a permanent magnet.



An iron magnet can remain magnetized for a long time, and is referred to as a "permanent magnet." But if you drop a magnet on the floor or strike it with a hammer, you can jar the domains into randomness and the magnet loses some or all of its magnetism. Heating a permanent magnet can also cause loss of magnetism, for raising the temperature increases the random thermal motion of atoms, which tends to randomize the domains. Above a certain temperature known as the **Curie temperature** (1043 K for iron), a magnet cannot be made at all.

The striking similarity between the fields produced by a bar magnet and by a loop of electric current (Figs. 20–4b, 20–9) offers a clue that perhaps magnetic fields produced by electric currents may have something to do with ferromagnetism. According to modern atomic theory, atoms can be roughly visualized as having electrons that orbit around a central nucleus. The electrons are charged, and so constitute an electric current and therefore produce a magnetic field. But the fields due to orbiting electrons end up adding to zero. Electrons themselves produce an additional magnetic field, almost as if they and their electric charge were spinning about their own axes. And it is this magnetic field due to electron **spin**† that is believed to produce ferromagnetism in most ferromagnetic materials.

It is believed today that *all* magnetic fields are caused by electric currents. This means that magnetic field lines always form closed loops, unlike electric field lines which begin on positive charges and end on negative charges.

CAUTION
$\vec{\mathbf{B}}$ lines form closed loops,
$\vec{\mathbf{E}}$ lines start on + and end on −

* Magnetic Permeability

If a piece of ferromagnetic material like iron is placed inside a solenoid to form an electromagnet (Section 20–7), the magnetic field increases greatly over that produced by the current in the solenoid coils alone, often by hundreds or thousands of times. This happens because the domains in the iron become aligned by the external field produced by the current in the solenoid coil.

†The name "spin" comes from an early suggestion that this intrinsic magnetic field arises from the electron "spinning" on its axis (as well as "orbiting" the nucleus) to produce the extra field. However, this view of a spinning electron is oversimplified and not valid (see Chapter 28).

The total magnetic field $\vec{\mathbf{B}}$ is then the sum of two terms,

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_0 + \vec{\mathbf{B}}_M .$$

$\vec{\mathbf{B}}_0$ is the field due to the current in the solenoid coil and $\vec{\mathbf{B}}_M$ is the additional field due to the iron. Often $B_M \gg B_0$. The total field can also be written by replacing the constant μ_0 in Eq. 20–8 ($B = \mu_0 NI/\ell$ for a solenoid) by another constant called the **magnetic permeability** μ, which is characteristic of the magnetic material inside the coil. Then $B = \mu NI/\ell$. For ferromagnetic materials, μ is much greater than μ_0. For all other materials, its value is very close to μ_0.[†] The value of μ, however, is not constant for ferromagnetic materials; it depends on the strength of the "external" field B_0, as the following experiment shows.

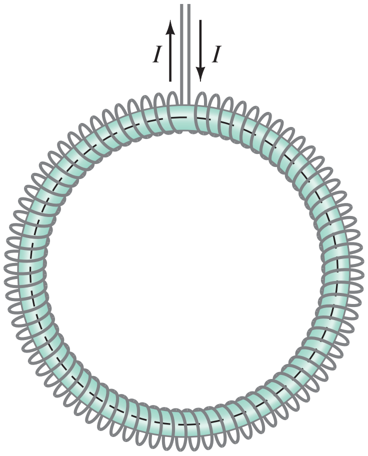


FIGURE 20–44 Iron-core toroid.

* Hysteresis

Measurements on magnetic materials often use a **torus** or **toroid**, which is like a long solenoid bent into the shape of a donut (Fig. 20–44), so practically all the lines of $\vec{\mathbf{B}}$ remain within the toroid. Consider a toroid with an iron core that is initially unmagnetized and there is no current in the wire loops. Then the current I is slowly increased, and B_0 (which is due only to I) increases linearly with I. The total field B also increases, but follows the curved line shown in Fig. 20–45 which is a graph of total B vs. B_0. Initially, point a, the domains are randomly oriented. As B_0 increases, the domains become more and more aligned until at point b, nearly all are aligned. The iron is said to be approaching **saturation**.

FIGURE 20–45 Total magnetic field B in an iron-core toroid as a function of the external field B_0 (B_0 is caused by the current I in the coil). We use gauss ($1\text{ G} = 10^{-4}\text{ T}$) so that labels are clear.

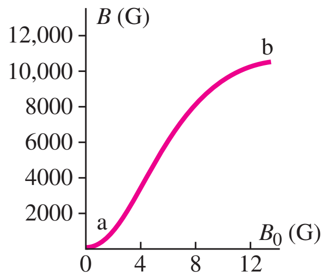
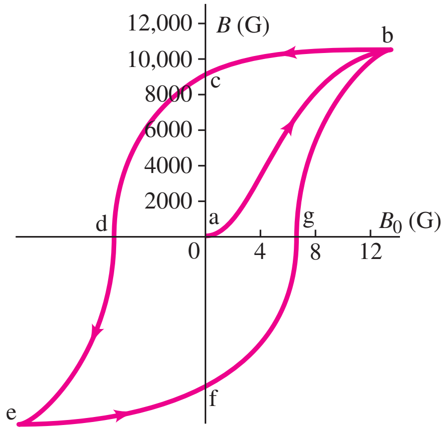


FIGURE 20–46 Hysteresis curve.



Next, suppose current in the coil is reduced, so the field B_0 decreases. If the current (and B_0) is reduced to zero, point c in Fig. 20–46, the domains do *not* become completely random. Instead, some permanent magnetism remains in the iron core. If the current is increased in the opposite direction, enough domains can be turned around so the total B becomes zero at point d. As the reverse current is increased further, the iron approaches saturation in the opposite direction, point e. Finally, if the current is again reduced to zero (point f) and then increased in the original direction, the total field follows the path efgb, again approaching saturation at point b.

Notice that the field did not pass through the origin (point a) in this cycle. The fact that the curve does not retrace itself on the same path is called **hysteresis**. The curve bcdefgb is called a **hysteresis loop**. In such a cycle, much energy is transformed to thermal energy (friction) due to realigning of the domains. Note that at points c and f, the iron core is magnetized even though there is no current in the coils. These points correspond to a permanent magnet.

[†]All materials are slightly magnetic. Nonferromagnetic materials fall into two principal classes: (1) **paramagnetic** materials consist of atoms that have a net magnetic dipole moment which can align slightly with an external field, just as the galvanometer coil in Fig. 20–35 experiences a torque that tends to align it; (2) **diamagnetic** materials have atoms with no net dipole moment, but in the presence of an external field electrons revolving in one direction increase in speed slightly whereas electrons revolving in the opposite direction are reduced in speed; the result is a slight net magnetic effect that opposes the external field.

Summary

A magnet has two **poles**, north and south. The north pole is that end which points toward geographic north when the magnet is freely suspended. Like poles of two magnets repel each other, whereas unlike poles attract.

We can picture that a **magnetic field** surrounds every magnet. The SI unit for magnetic field is the **tesla** (T).

Electric currents produce magnetic fields. For example, the lines of magnetic field due to a current in a straight wire form circles around the wire, and the field exerts a force on magnets (or currents) near it.

A magnetic field exerts a force on an electric current. For a straight wire of length ℓ carrying a current I, the force has magnitude

$$F = I\ell B\sin\theta, \tag{20–1}$$

where θ is the angle between the magnetic field $\vec{\mathbf{B}}$ and the current direction. The direction of the force is perpendicular to the current-carrying wire and to the magnetic field, and is given by a right-hand rule. Equation 20–1 serves as the definition of magnetic field $\vec{\mathbf{B}}$.

Similarly, a magnetic field exerts a force on a charge q moving with velocity v of magnitude

$$F = qvB\sin\theta, \tag{20–3}$$

where θ is the angle between $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$. The direction of $\vec{\mathbf{F}}$ is perpendicular to $\vec{\mathbf{v}}$ and to $\vec{\mathbf{B}}$ (again a right-hand rule). The path of a charged particle moving perpendicular to a uniform magnetic field is a circle.

The magnitude of the magnetic field produced by a current I in a long straight wire, at a distance r from the wire, is

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}. \tag{20–6}$$

Two currents exert a force on each other via the magnetic field each produces. Parallel currents in the same direction attract each other; currents in opposite directions repel.

The magnetic field inside a long tightly wound solenoid is

$$B = \mu_0 NI/\ell, \tag{20–8}$$

where N is the number of loops in a length ℓ of coil, and I is the current in each loop.

Ampère's law states that around any chosen closed loop path, the sum of each path segment $\Delta\ell$ times the component of $\vec{\mathbf{B}}$ parallel to the segment equals μ_0 times the current I enclosed by the closed path:

$$\Sigma B_{\parallel}\,\Delta\ell = \mu_0 I_{\text{encl}}. \tag{20–9}$$

The torque τ on N loops of current I in a magnetic field $\vec{\mathbf{B}}$ is

$$\tau = NIAB\sin\theta. \tag{20–10}$$

The force or torque exerted on a current-carrying wire by a magnetic field is the basis for operation of many devices, such as **motors**, **loudspeakers**, and **galvanometers** used in analog electric meters.

[*A **mass spectrometer** uses electric and magnetic fields to determine the mass of ions.]

[*Iron and a few other materials that are **ferromagnetic** can be made into strong permanent magnets. Ferromagnetic materials are made up of tiny **domains**—each a tiny magnet—which are preferentially aligned in a permanent magnet. When iron or another ferromagnetic material is placed in a magnetic field B_0 due to a current, the iron becomes magnetized. When the current is turned off, the material remains magnetized; when the current is increased in the opposite direction, a graph of the total field B versus B_0 is a **hysteresis loop**, and the fact that the curve does not retrace itself is called **hysteresis**.]

Questions

1. A compass needle is not always balanced parallel to the Earth's surface, but one end may dip downward. Explain.
2. Explain why the Earth's "north pole" is really a magnetic south pole. Indicate how north and south magnetic poles were defined and how we can tell experimentally that the north pole is really a south magnetic pole.
3. In what direction are the magnetic field lines surrounding a straight wire carrying a current that is moving directly away from you? Explain.
4. A horseshoe magnet is held vertically with the north pole on the left and south pole on the right. A wire passing between the poles, equidistant from them, carries a current directly away from you. In what direction is the force on the wire? Explain.
5. Will a magnet attract any metallic object, such as those made of aluminum or copper? (Try it and see.) Why is this so?
6. Two iron bars attract each other no matter which ends are placed close together. Are both magnets? Explain.
7. The magnetic field due to current in wires in your home can affect a compass. Discuss the effect in terms of currents, including if they are ac or dc.
8. If a negatively charged particle enters a region of uniform magnetic field which is perpendicular to the particle's velocity, will the kinetic energy of the particle increase, decrease, or stay the same? Explain your answer. (Neglect gravity and assume there is no electric field.)
9. In Fig. 20–47, charged particles move in the vicinity of a current-carrying wire. For each charged particle, the arrow indicates the initial direction of motion of the particle, and the + or − indicates the sign of the charge. For each of the particles, indicate the direction of the magnetic force due to the magnetic field produced by the wire. Explain.

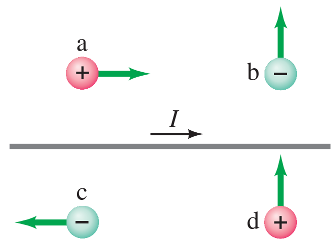


FIGURE 20–47 Question 9.

10. Three particles, a, b, and c, enter a magnetic field and follow paths as shown in Fig. 20–48. What can you say about the charge on each particle? Explain.

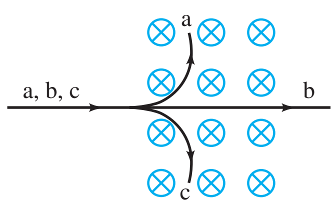


FIGURE 20–48 Question 10.

11. Can an iron rod attract a magnet? Can a magnet attract an iron rod? What must you consider to answer these questions?

12. A positively charged particle in a nonuniform magnetic field follows the trajectory shown in Fig. 20–49. Indicate the direction of the magnetic field at points near the path, assuming the path is always in the plane of the page, and indicate the relative magnitudes of the field in each region. Explain your answers.

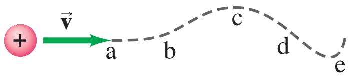


FIGURE 20–49 Question 12.

13. Explain why a strong magnet held near a CRT television screen (Section 17–11) causes the picture to become distorted. Also, explain why the picture sometimes goes completely black where the field is the strongest. [But don't risk damage to your TV by trying this.]

14. Suppose you have three iron rods, two of which are magnetized but the third is not. How would you determine which two are the magnets without using any additional objects?

15. Can you set a resting electron into motion with a magnetic field? With an electric field? Explain.

16. A charged particle is moving in a circle under the influence of a uniform magnetic field. If an electric field that points in the same direction as the magnetic field is turned on, describe the path the charged particle will take.

17. A charged particle moves in a straight line through a particular region of space. Could there be a nonzero magnetic field in this region? If so, give two possible situations.

18. If a moving charged particle is deflected sideways in some region of space, can we conclude, for certain, that $\vec{\mathbf{B}} \neq 0$ in that region? Explain.

19. Two insulated long wires carrying equal currents I cross at right angles to each other. Describe the magnetic force one exerts on the other.

20. A horizontal current-carrying wire, free to move in Earth's gravitational field, is suspended directly above a parallel, current-carrying wire. (*a*) In what direction is the current in the lower wire? (*b*) Can the lower wire be held in stable equilibrium due to the magnetic force of the upper wire? Explain.

21. What would be the effect on B inside a long solenoid if (*a*) the diameter of all the loops was doubled, (*b*) the spacing between loops was doubled, or (*c*) the solenoid's length was doubled along with a doubling in the total number of loops?

22. A type of magnetic switch similar to a solenoid is a **relay** (Fig. 20–50). A relay is an electromagnet (the iron rod inside the coil does not move) which, when activated, attracts a strip of iron on a pivot. Design a relay to close an electrical switch. A relay is used when you need to switch on a circuit carrying a very large current but do not want that large current flowing through the main switch. For example, a car's starter switch is connected to a relay so that the large current needed for the starter doesn't pass to the dashboard switch.

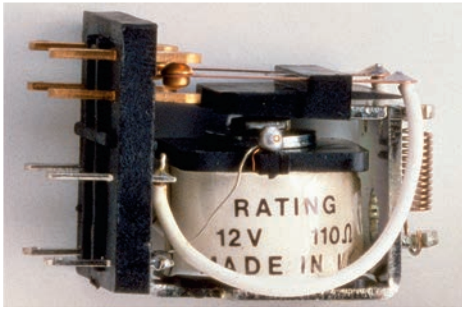


FIGURE 20–50 Question 22.

***23.** Two ions have the same mass, but one is singly ionized and the other is doubly ionized. How will their positions on the film of a mass spectrometer (Fig. 20–41) differ? Explain.

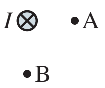
***24.** Why will either pole of a magnet attract an unmagnetized piece of iron?

***25.** An unmagnetized nail will not attract an unmagnetized paper clip. However, if one end of the nail is in contact with a magnet, the other end *will* attract a paper clip. Explain.

MisConceptual Questions

1. Indicate which of the following will produce a magnetic field:
(*a*) A magnet.
(*b*) The Earth.
(*c*) An electric charge at rest.
(*d*) A moving electric charge.
(*e*) An electric current.
(*f*) The voltage of a battery not connected to anything.
(*g*) An ordinary piece of iron.
(*h*) A piece of any metal.

2. A current in a wire points into the page as shown at the right. In which direction is the magnetic field at point A (choose below)?



(*a*) (*b*) (*c*) (*d*) (*e*) None of these.

3. In which direction (see above) is the magnetic field at point B?

4. When a charged particle moves parallel to the direction of a magnetic field, the particle travels in a
(*a*) straight line.
(*b*) circular path.
(*c*) helical path.
(*d*) hysteresis loop.

5. As a proton moves through space, it creates
(*a*) an electric field only.
(*b*) a magnetic field only.
(*c*) both an electric field and magnetic field.
(*d*) nothing; the electric field and magnetic fields cancel each other out.

6. Which statements about the force on a charged particle placed in a magnetic field are true?
(*a*) A magnetic force is exerted only if the particle is moving.
(*b*) The force is a maximum if the particle is moving in the direction of the field.
(*c*) The force causes the particle to gain kinetic energy.
(*d*) The direction of the force is along the magnetic field.
(*e*) A magnetic field always exerts a force on a charged particle.

7. Which of the following statements is false? The magnetic field of a current-carrying wire
(*a*) is directed circularly around the wire.
(*b*) decreases inversely with the distance from the wire.
(*c*) exists only if the current in the wire is changing.
(*d*) depends on the magnitude of the current.

8. A wire carries a current directly away from you. Which way do the magnetic field lines produced by this wire point?
(*a*) They point parallel to the wire in the direction of the current.
(*b*) They point parallel to the wire opposite the direction of the current.
(*c*) They point toward the wire.
(*d*) They point away from the wire.
(*e*) They make circles around the wire.

9. A proton enters a uniform magnetic field that is perpendicular to the proton's velocity (Fig. 20–51). What happens to the kinetic energy of the proton?
(*a*) It increases.
(*b*) It decreases.
(*c*) It stays the same.
(*d*) It depends on the velocity direction.
(*e*) It depends on the B field direction.

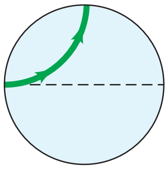


FIGURE 20–51 MisConceptual Question 9.

10. For a charged particle, a constant magnetic field can be used to change
(*a*) only the direction of the particle's velocity.
(*b*) only the magnitude of the particle's velocity.
(*c*) both the magnitude and direction of the particle's velocity.
(*d*) None of the above.

11. Which of the following statements about the force on a charged particle due to a magnetic field are not valid?
(*a*) It depends on the particle's charge.
(*b*) It depends on the particle's velocity.
(*c*) It depends on the strength of the external magnetic field.
(*d*) It acts at right angles to the direction of the particle's motion.
(*e*) None of the above; all of these statements are valid.

12. Two parallel wires are vertical. The one on the left carries a 10-A current upward. The other carries 5-A current downward. Compare the magnitude of the force that each wire exerts on the other.
(*a*) The wire on the left carries twice as much current, so it exerts twice the force on the right wire as the right one exerts on the left one.
(*b*) The wire on the left exerts a smaller force. It creates a magnetic field twice that due to the wire on the right; and therefore has less energy to cause a force on the wire on the right.
(*c*) The two wires exert the same force on each other.
(*d*) Not enough information; we need the length of the wire.

For assigned homework and other learning materials, go to the MasteringPhysics website.

Problems

20–3 Force on Electric Current in Magnetic Field

1. (I) (*a*) What is the force per meter of length on a straight wire carrying a 6.40-A current when perpendicular to a 0.90-T uniform magnetic field? (*b*) What if the angle between the wire and field is 35.0°?

2. (I) How much current is flowing in a wire 4.80 m long if the maximum force on it is 0.625 N when placed in a uniform 0.0800-T field?

3. (I) A 240-m length of wire stretches between two towers and carries a 120-A current. Determine the magnitude of the force on the wire due to the Earth's magnetic field of 5.0×10^{-5} T which makes an angle of 68° with the wire.

4. (I) A 2.6-m length of horizontal wire carries a 4.5-A current toward the south. The dip angle of the Earth's magnetic field makes an angle of 41° to the wire. Estimate the magnitude of the magnetic force on the wire due to the Earth's magnetic field of 5.5×10^{-5} T.

5. (I) The magnetic force per meter on a wire is measured to be only 45% of its maximum possible value. What is the angle between the wire and the magnetic field?

6. (II) The force on a wire carrying 6.45 A is a maximum of 1.28 N when placed between the pole faces of a magnet. If the pole faces are 55.5 cm in diameter, what is the approximate strength of the magnetic field?

7. (II) The force on a wire is a maximum of 8.50×10^{-2} N when placed between the pole faces of a magnet. The current flows horizontally to the right and the magnetic field is vertical. The wire is observed to "jump" toward the observer when the current is turned on. (*a*) What type of magnetic pole is the top pole face? (*b*) If the pole faces have a diameter of 10.0 cm, estimate the current in the wire if the field is 0.220 T. (*c*) If the wire is tipped so that it makes an angle of 10.0° with the horizontal, what force will it now feel? [*Hint*: What length of wire will now be in the field?]

8. (II) Suppose a straight 1.00-mm-diameter copper wire could just "float" horizontally in air because of the force due to the Earth's magnetic field $\vec{\mathbf{B}}$, which is horizontal, perpendicular to the wire, and of magnitude 5.0×10^{-5} T. What current would the wire carry? Does the answer seem feasible? Explain briefly.

20–4 Force on Charge Moving in Magnetic Field

9. (I) Determine the magnitude and direction of the force on an electron traveling 7.75×10^5 m/s horizontally to the east in a vertically upward magnetic field of strength 0.45 T.

10. (I) An electron is projected vertically upward with a speed of 1.70×10^6 m/s into a uniform magnetic field of 0.640 T that is directed horizontally away from the observer. Describe the electron's path in this field.

11. (I) Alpha particles (charge $q = +2e$, mass $m = 6.6 \times 10^{-27}$ kg) move at 1.6×10^6 m/s. What magnetic field strength would be required to bend them into a circular path of radius $r = 0.14$ m?

12. (I) Find the direction of the force on a negative charge for each diagram shown in Fig. 20–52, where $\vec{\mathbf{v}}$ (green) is the velocity of the charge and $\vec{\mathbf{B}}$ (blue) is the direction of the magnetic field. (⊗ means the vector points inward. ⊙ means it points outward, toward you.)

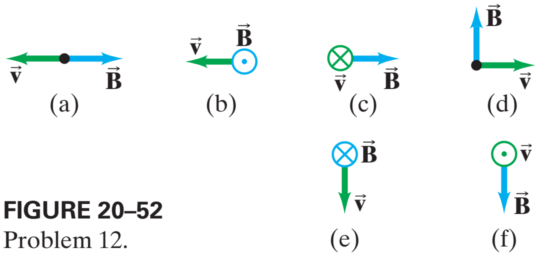


FIGURE 20–52 Problem 12.

13. (I) Determine the direction of $\vec{\mathbf{B}}$ for each case in Fig. 20–53, where $\vec{\mathbf{F}}$ represents the maximum magnetic force on a positively charged particle moving with velocity $\vec{\mathbf{v}}$.

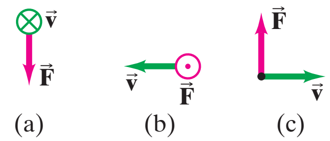


FIGURE 20–53 Problem 13.

14. (II) Determine the velocity of a beam of electrons that goes undeflected when moving perpendicular to an electric and to a magnetic field. $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ are also perpendicular to each other and have magnitudes 7.7×10^3 V/m and 7.5×10^{-3} T, respectively. What is the radius of the electron orbit if the electric field is turned off?

15. (II) A helium ion ($Q = +2e$) whose mass is 6.6×10^{-27} kg is accelerated by a voltage of 3700 V. (*a*) What is its speed? (*b*) What will be its radius of curvature if it moves in a plane perpendicular to a uniform 0.340-T field? (*c*) What is its period of revolution?

16. (II) For a particle of mass m and charge q moving in a circular path in a magnetic field B, (*a*) show that its kinetic energy is proportional to r^2, the square of the radius of curvature of its path. (*b*) Show that its angular momentum is $L = qBr^2$, around the center of the circle.

17. (II) A 1.5-MeV (kinetic energy) proton enters a 0.30-T field, in a plane perpendicular to the field. What is the radius of its path? See Section 17–4.

18. (II) An electron experiences the greatest force as it travels 2.8×10^6 m/s in a magnetic field when it is moving northward. The force is vertically upward and of magnitude 6.2×10^{-13} N. What is the magnitude and direction of the magnetic field?

19. (II) A proton and an electron have the same kinetic energy upon entering a region of constant magnetic field. What is the ratio of the radii of their circular paths?

20. (III) A proton (mass m_p), a deuteron $(m = 2m_p,\ Q = e)$, and an alpha particle $(m = 4m_p,\ Q = 2e)$ are accelerated by the same potential difference V and then enter a uniform magnetic field $\vec{\mathbf{B}}$, where they move in circular paths perpendicular to $\vec{\mathbf{B}}$. Determine the radius of the paths for the deuteron and alpha particle in terms of that for the proton.

21. (III) A 3.40-g bullet moves with a speed of 155 m/s perpendicular to the Earth's magnetic field of 5.00×10^{-5} T. If the bullet possesses a net charge of 18.5×10^{-9} C, by what distance will it be deflected from its path due to the Earth's magnetic field after it has traveled 1.50 km?

***22.** (III) A **Hall probe**, consisting of a thin rectangular slab of current-carrying material, is calibrated by placing it in a known magnetic field of magnitude 0.10 T. When the field is oriented normal to the slab's rectangular face, a Hall emf of 12 mV is measured across the slab's width. The probe is then placed in a magnetic field of unknown magnitude B, and a Hall emf of 63 mV is measured. Determine B assuming that the angle θ between the unknown field and the plane of the slab's rectangular face is (*a*) $\theta = 90°$, and (*b*) $\theta = 60°$.

***23.** (III) The Hall effect can be used to measure **blood flow rate** because the blood contains ions that constitute an electric current. (*a*) Does the sign of the ions influence the emf? Explain. (*b*) Determine the flow velocity in an artery 3.3 mm in diameter if the measured emf across the width of the artery is 0.13 mV and B is 0.070 T. (In actual practice, an alternating magnetic field is used.)

***24.** (III) A long copper strip 1.8 cm wide and 1.0 mm thick is placed in a 1.2-T magnetic field as in Fig. 20–21a. When a steady current of 15 A passes through it, the Hall emf is measured to be 1.02 μV. Determine (*a*) the drift velocity of the electrons and (*b*) the density of free (conducting) electrons (number per unit volume) in the copper. [*Hint*: See also Section 18–8.]

20–5 and 20–6 Magnetic Field of Straight Wire, Force between Two Wires

25. (I) Jumper cables used to start a stalled vehicle often carry a 65-A current. How strong is the magnetic field 4.5 cm from one cable? Compare to the Earth's magnetic field (5.0×10^{-5} T).

26. (I) If an electric wire is allowed to produce a magnetic field no larger than that of the Earth $(0.50 \times 10^{-4}\ \text{T})$ at a distance of 12 cm from the wire, what is the maximum current the wire can carry?

27. (I) Determine the magnitude and direction of the force between two parallel wires 25 m long and 4.0 cm apart, each carrying 25 A in the same direction.

28. (I) A vertical straight wire carrying an upward 28-A current exerts an attractive force per unit length of 7.8×10^{-4} N/m on a second parallel wire 9.0 cm away. What current (magnitude and direction) flows in the second wire?

29. (II) In Fig. 20–54, a long straight wire carries current I out of the page toward you. Indicate, with appropriate arrows, the direction and (relative) magnitude of $\vec{\mathbf{B}}$ at each of the points C, D, and E in the plane of the page.

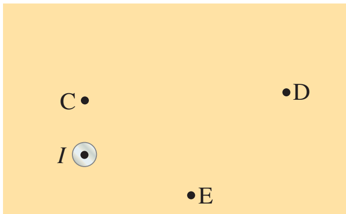


FIGURE 20–54 Problem 29.

30. (II) An experiment on the Earth's magnetic field is being carried out 1.00 m from an electric cable. What is the maximum allowable current in the cable if the experiment is to be accurate to $\pm 3.0\%$?

31. (II) A rectangular loop of wire is placed next to a straight wire, as shown in Fig. 20–55. There is a current of 3.5 A in both wires. Determine the magnitude and direction of the net force on the loop.

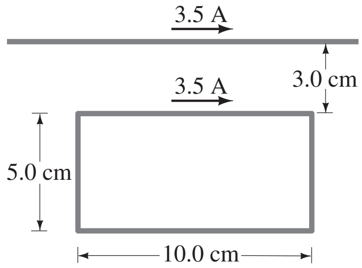


FIGURE 20–55 Problem 31.

32. (II) A horizontal compass is placed 18 cm due south from a straight vertical wire carrying a 48-A current downward. In what direction does the compass needle point at this location? Assume the horizontal component of the Earth's field at this point is 0.45×10^{-4} T and the magnetic declination is 0°.

33. (II) A long horizontal wire carries 24.0 A of current due north. What is the net magnetic field 20.0 cm due west of the wire if the Earth's field there points downward, 44° below the horizontal, and has magnitude 5.0×10^{-5} T?

34. (II) A straight stream of protons passes a given point in space at a rate of 2.5×10^9 protons/s. What magnetic field do they produce 1.5 m from the beam?

35. (II) Determine the magnetic field midway between two long straight wires 2.0 cm apart in terms of the current I in one when the other carries 25 A. Assume these currents are (*a*) in the same direction, and (*b*) in opposite directions.

36. (II) Two straight parallel wires are separated by 7.0 cm. There is a 2.0-A current flowing in the first wire. If the magnetic field strength is found to be zero between the two wires at a distance of 2.2 cm from the first wire, what is the magnitude and direction of the current in the second wire?

37. (II) Two long straight wires each carry a current I out of the page toward the viewer, Fig. 20–56. Indicate, with appropriate arrows, the direction of $\vec{\mathbf{B}}$ at each of the points 1 to 6 in the plane of the page. State if the field is zero at any of the points.

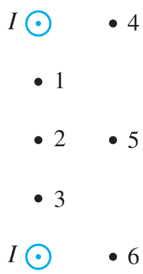


FIGURE 20–56 Problem 37.

38. (II) A power line carries a current of 95 A west along the tops of 8.5-m-high poles. (*a*) What is the magnitude and direction of the magnetic field produced by this wire at the ground directly below? How does this compare with the Earth's magnetic field of about $\frac{1}{2}$ G? (*b*) Where would the wire's magnetic field cancel the Earth's field?

39. (II) A long pair of insulated wires serves to conduct 24.5 A of dc current to and from an instrument. If the wires are of negligible diameter but are 2.8 mm apart, what is the magnetic field 10.0 cm from their midpoint, in their plane (Fig. 20–57)? Compare to the magnetic field of the Earth.

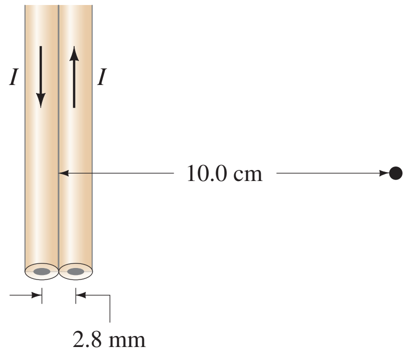


FIGURE 20–57 Problems 39 and 40.

40. (II) A third wire is placed in the plane of the two wires shown in Fig. 20–57 parallel and just to the right. If it carries 25.0 A upward, what force per meter of length does it exert on each of the other two wires? Assume it is 2.8 mm from the nearest wire, center to center.

41. (III) Two long thin parallel wires 13.0 cm apart carry 28-A currents in the same direction. Determine the magnetic field vector at a point 10.0 cm from one wire and 6.0 cm from the other (Fig. 20–58).

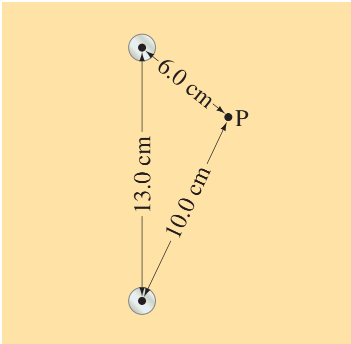


FIGURE 20–58 Problem 41.

20–7 Solenoids and Electromagnets

42. (I) A thin 12-cm-long solenoid has a total of 460 turns of wire and carries a current of 2.0 A. Calculate the field inside the solenoid near the center.

43. (I) A 30.0-cm-long solenoid 1.25 cm in diameter is to produce a field of 4.65 mT at its center. How much current should the solenoid carry if it has 935 turns of the wire?

44. (I) A 42-cm-long solenoid, 1.8 cm in diameter, is to produce a 0.030-T magnetic field at its center. If the maximum current is 4.5 A, how many turns must the solenoid have?

45. (II) A 550-turn horizontal solenoid is 15 cm long. The current in its coils is 38 A. A straight wire cuts through the center of the solenoid, along a 3.0-cm diameter. This wire carries a 22-A current downward (and is connected by other wires that don't concern us). What is the force on this wire assuming the solenoid's magnetic field points due east?

46. (III) You have 1.0 kg of copper and want to make a practical solenoid that produces the greatest possible magnetic field for a given voltage. Should you make your copper wire long and thin, short and fat, or something else? Consider other variables, such as solenoid diameter, length, and so on. Explain your reasoning.

20–8 Ampère's Law

47. (III) A *toroid* is a solenoid in the shape of a donut (Fig. 20–59). Use Ampère's law along the circular paths, shown dashed in Fig. 20–59a, to determine that the magnetic field (*a*) inside the toroid is $B = \mu_0 NI/2\pi R$, where N is the total number of turns, and (*b*) outside the toroid is $B = 0$. (*c*) Is the field inside a toroid uniform like a solenoid's? If not, how does it vary?

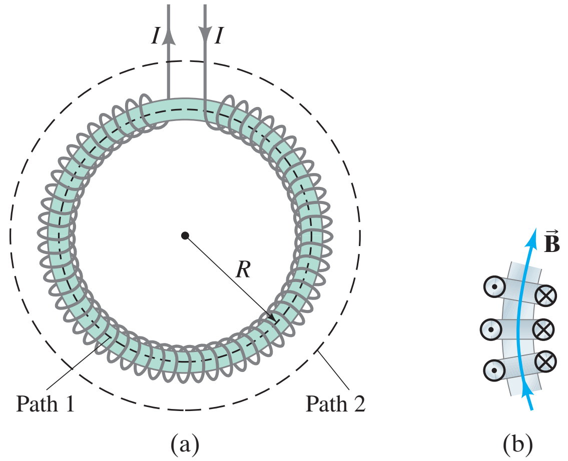


FIGURE 20–59 Problem 47. (a) A toroid or torus. (b) A section of the toroid showing direction of the current for three loops: ⊙ means current toward you, and ⊗ means current away from you.

48. (III) (*a*) Use Ampère's law to show that the magnetic field between the conductors of a **coaxial cable** (Fig. 20–60) is $B = \mu_0 I/2\pi r$ if r (distance from center) is greater than the radius of the inner wire and less than the radius of the outer cylindrical braid (= ground). (*b*) Show that $B = 0$ outside the coaxial cable.

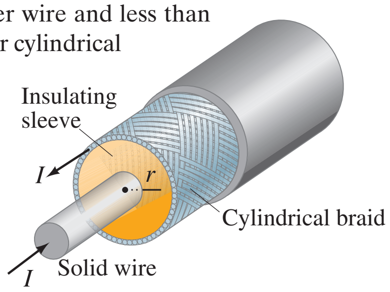


FIGURE 20–60 Coaxial cable. Problem 48.

20–9 and 20–10 Torque on Current Loop, Motors, Galvanometers

49. (I) A single square loop of wire 22.0 cm on a side is placed with its face parallel to the magnetic field as in Fig. 20–34b. When 5.70 A flows in the coil, the torque on it is 0.325 m · N. What is the magnetic field strength?

50. (I) If the current to a motor drops by 12%, by what factor does the output torque change?

51. (I) A galvanometer needle deflects full scale for a 53.0-μA current. What current will give full-scale deflection if the magnetic field weakens to 0.760 of its original value?

52. (II) A circular coil 12.0 cm in diameter and containing nine loops lies flat on the ground. The Earth's magnetic field at this location has magnitude 5.50×10^{-5} T and points into the Earth at an angle of 56.0° below a line pointing due north. If a 7.20-A clockwise current passes through the coil, (*a*) determine the torque on the coil, and (*b*) which edge of the coil rises up: north, east, south, or west?

*20–11 Mass Spectrometer

***53.** (I) Protons move in a circle of radius 6.10 cm in a 0.566-T magnetic field. What value of electric field could make their paths straight? In what direction must the electric field point?

***54.** (I) In a mass spectrometer, germanium atoms have radii of curvature equal to 21.0, 21.6, 21.9, 22.2, and 22.8 cm. The largest radius corresponds to an atomic mass of 76 u. What are the atomic masses of the other isotopes?

***55.** (II) One form of mass spectrometer accelerates ions by a voltage V before they enter a magnetic field B. The ions are assumed to start from rest. Show that the mass of an ion is $m = qB^2R^2/2V$, where R is the radius of the ions' path in the magnetic field and q is their charge.

***56.** (II) An unknown particle moves in a straight line through crossed electric and magnetic fields with $E = 1.5$ kV/m and $B = 0.034$ T. If the electric field is turned off, the particle moves in a circular path of radius $r = 2.7$ cm. What might the particle be?

***57.** (III) A mass spectrometer is monitoring air pollutants. It is difficult, however, to separate molecules of nearly equal mass such as CO (28.0106 u) and N_2 (28.0134 u). How large a radius of curvature must a spectrometer have (Fig. 20–41) if these two molecules are to be separated on the film by 0.50 mm?

*20–12 Ferromagnetism, Hysteresis

***58.** (I) A long thin iron-core solenoid has 380 loops of wire per meter, and a 350-mA current flows through the wire. If the permeability of the iron is $3000\mu_0$, what is the total field B inside the solenoid?

***59.** (II) An iron-core solenoid is 38 cm long and 1.8 cm in diameter, and has 780 turns of wire. The magnetic field inside the solenoid is 2.2 T when 48 A flows in the wire. What is the permeability μ at this high field strength?

General Problems

60. Protons with momentum $4.8 \times 10^{-21}\ \text{kg}\cdot\text{m/s}$ are magnetically steered clockwise in a circular path 2.2 m in diameter. Determine the magnitude and direction of the field in the magnets surrounding the beam pipe.

61. A small but rigid ∪-shaped wire carrying a 5.0-A current (Fig. 20–61) is placed inside a solenoid. The solenoid is 15.0 cm long and has 700 loops of wire, and the current in each loop is 7.0 A. What is the net force on the ∪-shaped wire?

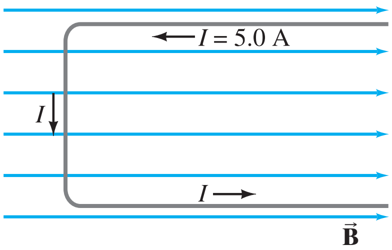


FIGURE 20–61 Problem 61.

62. The power cable for an electric trolley (Fig. 20–62) carries a horizontal current of 330 A toward the east. The Earth's magnetic field has a strength 5.0×10^{-5} T and makes an angle of dip of 22° at this location. Calculate the magnitude and direction of the magnetic force on an 18-m length of this cable.

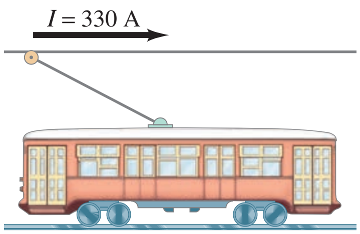


FIGURE 20–62 Problem 62.

63. A particle of charge q moves in a circular path of radius r perpendicular to a uniform magnetic field B. Determine its linear momentum in terms of the quantities given.

64. An airplane has acquired a net charge of $1280\ \mu\text{C}$. If the Earth's magnetic field of 5.0×10^{-5} T is perpendicular to the airplane's velocity of magnitude 120 m/s, determine the force on the airplane.

65. A 32-cm-long solenoid, 1.8 cm in diameter, is to produce a 0.050-T magnetic field at its center. If the maximum current is 6.4 A, how many turns must the solenoid have?

66. Near the equator, the Earth's magnetic field points almost horizontally to the north and has magnitude $B = 0.50 \times 10^{-4}$ T. What should be the magnitude and direction for the velocity of an electron if its weight is to be exactly balanced by the magnetic force?

67. A doubly charged helium atom, whose mass is 6.6×10^{-27} kg, is accelerated by a voltage of 3200 V. (*a*) What will be its radius of curvature in a uniform 0.240-T field? (*b*) What is its period of revolution?

68. Four very long straight parallel wires, located at the corners of a square of side ℓ, carry equal currents I_0 perpendicular to the page as shown in Fig. 20–63. Determine the magnitude and direction of $\vec{\mathbf{B}}$ at the center C of the square.

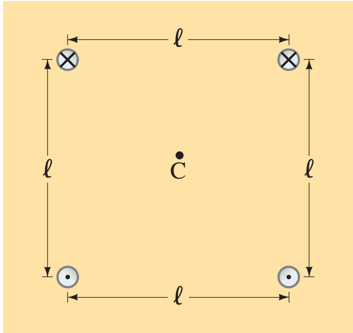


FIGURE 20–63 Problem 68.

69. (*a*) What value of magnetic field would make a beam of electrons, traveling to the west at a speed of 4.8×10^6 m/s, go undeflected through a region where there is a uniform electric field of 12,000 V/m pointing south? (*b*) What is the direction of the magnetic field if it is perpendicular to the electric field? (*c*) What is the frequency of the circular orbit of the electrons if the electric field is turned off?

70. Magnetic fields are very useful in particle accelerators for "beam steering"; that is, the magnetic fields can be used to change the direction of the beam of charged particles without altering their speed (Fig. 20–64). Show how this could work with a beam of protons. What happens to protons that are not moving with the speed for which the magnetic field was designed? If the field extends over a region 5.0 cm wide and has a magnitude of 0.41 T, by approximately what angle θ will a beam of protons traveling at 2.5×10^6 m/s be bent?

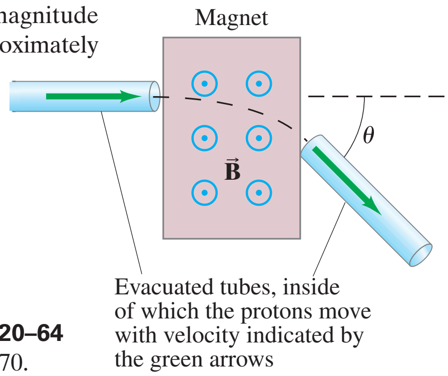


FIGURE 20–64 Problem 70.

71. The magnetic field B at the center of a circular coil of wire carrying a current I (as in Fig. 20–9) is

$$B = \frac{\mu_0 NI}{2r},$$

where N is the number of loops in the coil and r is its radius. Imagine a simple model in which the Earth's magnetic field of about 1 G $(= 1 \times 10^{-4}\ \text{T})$ near the poles is produced by a single current loop around the equator. Roughly estimate the current this loop would carry.

72. Two long straight aluminum wires, each of diameter 0.42 mm, carry the same current but in opposite directions. They are suspended by 0.50-m-long strings as shown in Fig. 20–65. If the suspension strings make an angle of 3.0° with the vertical and are hanging freely, what is the current in the wires?

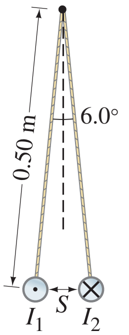


FIGURE 20–65 Problem 72.

73. A motor run by a 9.0-V battery has a 20-turn square coil with sides of length 5.0 cm and total resistance 28 Ω. When spinning, the magnetic field felt by the wire in the coil is 0.020 T. What is the maximum torque on the motor?

74. Electrons are accelerated horizontally by 2.2 kV. They then pass through a uniform magnetic field B for a distance of 3.8 cm, which deflects them upward so they reach the top of a screen 22 cm away, 11 cm above the center. Estimate the value of B.

75. A 175-g model airplane charged to 18.0 mC and traveling at 3.4 m/s passes within 8.6 cm of a wire, nearly parallel to its path, carrying a 25-A current. What acceleration (in g's) does this interaction give the airplane?

76. A uniform conducting rod of length ℓ and mass m sits atop a fulcrum, which is placed a distance $\ell/4$ from the rod's left-hand end and is immersed in a uniform magnetic field of magnitude B directed into the page (Fig. 20–66). An object whose mass M is 6.0 times greater than the rod's mass is hung from the rod's left-hand end. What current (direction and magnitude) should flow through the rod in order for it to be "balanced" (i.e., be at rest horizontally) on the fulcrum? (Flexible connecting wires which exert negligible force on the rod are not shown.)

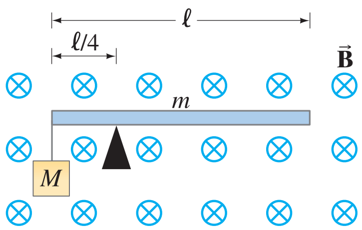


FIGURE 20–66 Problem 76.

77. Suppose the Earth's magnetic field at the equator has magnitude 0.50×10^{-4} T and a northerly direction at all points. Estimate the speed a singly ionized uranium ion ($m = 238$ u, $q = +e$) would need to circle the Earth 6.0 km above the equator. Can you ignore gravity? [Ignore relativity.]

78. A particle with charge q and momentum p, initially moving along the x axis, enters a region where a uniform magnetic field B_0 extends over a width $x = \ell$ as shown in Fig. 20–67. The particle is deflected a distance d in the $+y$ direction as it traverses the field. Determine (a) whether q is positive or negative, and (b) the magnitude of its momentum p in terms of q, B_0, ℓ, and d.

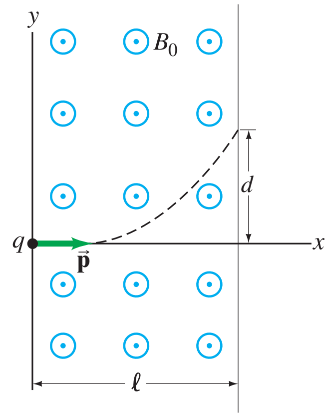


FIGURE 20–67 Problem 78.

79. A bolt of lightning strikes a metal flag pole, one end of which is anchored in the ground. Estimate the force the Earth's magnetic field can exert on the flag pole while the lightning-induced current flows. See Example 18–10.

80. The **cyclotron** (Fig. 20–68) is a device used to accelerate elementary particles such as protons to high speeds. Particles starting at point A with some initial velocity travel in semicircular orbits in the magnetic field B. The particles are accelerated to higher speeds each time they pass through the gap between the metal "dees," where there is an electric field E. (There is no electric field inside the hollow metal dees where the electrons move in circular paths.) The electric field changes direction each half-cycle, owing to an ac voltage $V = V_0 \sin 2\pi f t$, so that the particles are increased in speed at each passage through the gap. (a) Show that the frequency f of the voltage must be $f = Bq/2\pi m$, where q is the charge on the particles and m their mass. (b) Show that the kinetic energy of the particles increases by $2qV_0$ each revolution, assuming that the gap is small. (c) If the radius of the cyclotron is 2.0 m and the magnetic field strength is 0.50 T, what will be the maximum kinetic energy of accelerated protons in MeV?

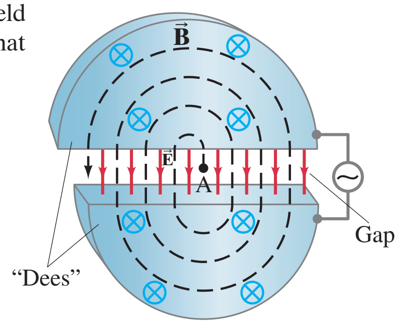


FIGURE 20–68 A cyclotron. Problem 80.

81. Three long parallel wires are 3.8 cm from one another. (Looking along them, they are at three corners of an equilateral triangle.) The current in each wire is 8.00 A, but its direction in wire M is opposite to that in wires N and P (Fig. 20–69). (*a*) Determine the magnetic force per unit length on each wire due to the other two. (*b*) In Fig. 20–70, determine the magnitude and direction of the magnetic field at the midpoint of the line between wire M and wire N.

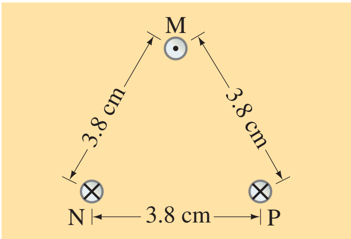


FIGURE 20–69 Problems 81 and 82.

82. In Fig. 20–69 the top wire is 1.00-mm-diameter copper wire and is suspended in air due to the two magnetic forces from the bottom two wires. The current flow through the two bottom wires is 75 A in each. Calculate the required current flow in the suspended wire (M).

83. You want to get an idea of the magnitude of magnetic fields produced by overhead power lines. You estimate that a transmission wire is about 13 m above the ground. The local power company tells you that the lines operate at 240 kV and provide a maximum power of 46 MW. Estimate the magnetic field you might experience walking under one such power line, and compare to the Earth's field.

Search and Learn

1. How many magnetic force equations are there in Chapter 20? List each one and explain when it applies. For each magnetic force equation, show how the units work out to give force in newtons.

2. (*a*) A particle of charge q moves in a circular path of radius r in a uniform magnetic field $\vec{\mathbf{B}}$. If the magnitude of the magnetic field is double, and the kinetic energy of the particle is the same, how does the angular momentum of the particle differ? (*b*) Show that the magnetic dipole moment M (Section 20–9) of an electron orbiting the proton nucleus of a hydrogen atom is related to the orbital angular momentum L of the electron by

$$M = \frac{e}{2m}L.$$

3. The force on a moving particle in a magnetic field is the idea behind **electromagnetic pumping**. It can be used to pump metallic fluids (such as sodium) and to pump blood in artificial heart machines. A basic design is shown in Fig. 20–70. For blood, an electric field is applied perpendicular to a blood vessel and to the magnetic field. Explain in detail how ions in the blood are caused to move. Do positive and negative ions feel a force in the same direction?

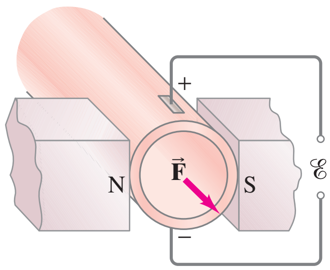


FIGURE 20–70 Electromagnetic pumping in a blood vessel. Search and Learn 3.

ANSWERS TO EXERCISES

A: Near the poles, where the field lines are closer together.
B: Circles, pointing counterclockwise.
C: (*b*), (*c*), (*d*).
D: 0.15 N.
E: (*b*), (*c*), (*d*).
F: Zero.
G: Negative; the helical path would rotate in the opposite direction (still going to the right).
H: 0.8×10^{-5} T, up.