2–8 Graphical Analysis of Linear Motion

Velocity as Slope

Analysis of motion using graphs can give us additional insight into kinematics. Let us draw a graph of x vs. t, making the choice that at $t = 0$, the position of an object is $x = 0$, and the object is moving at a constant velocity, $v = \bar{v} = 11\ \mathrm{m/s}$ $(40\ \mathrm{km/h})$. Our graph starts at $x = 0$, $t = 0$ (the origin). The graph of the position increases linearly in time because, by Eq. 2–2, $\Delta x = \bar{v}\,\Delta t$ and $\bar{v}$ is a constant. So the graph of x vs. t is a straight line, as shown in Fig. 2–26. The small (shaded) triangle on the graph indicates the **slope** of the straight line:

$$\text{slope} = \frac{\Delta x}{\Delta t}.$$

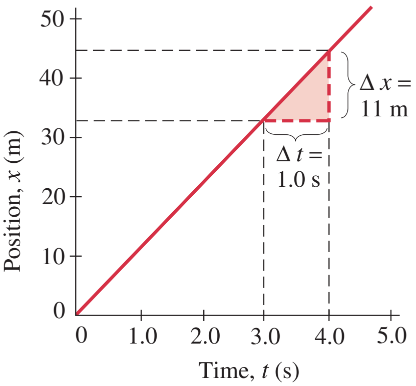
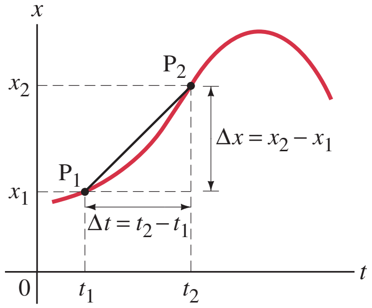


FIGURE 2–26 Graph of position vs. time for an object moving at a constant velocity of 11 m/s.

We see, using the definition of average velocity (Eq. 2–2), that the *slope of the x vs. t graph is equal to the velocity*. And, as can be seen from the small triangle on the graph, $\Delta x/\Delta t = (11\ \mathrm{m})/(1.0\ \mathrm{s}) = 11\ \mathrm{m/s}$, which is the given velocity.

If the object's velocity changes in time, we might have an x vs. t graph like that shown in Fig. 2–27. (Note that this graph is different from showing the "path" of an object on an x vs. y plot.) Suppose the object is at position x_1 at time t_1, and at position x_2 at time t_2. P_1 and P_2 represent these two points on the graph. A straight line drawn from point $P_1(x_1, t_1)$ to point $P_2(x_2, t_2)$ forms the hypotenuse of a right triangle whose sides are Δx and Δt. The ratio $\Delta x/\Delta t$ is the **slope** of the straight line P_1P_2. But $\Delta x/\Delta t$ is also the average velocity of the object during the time interval $\Delta t = t_2 - t_1$. Therefore, we conclude that the *average velocity of an object during any time interval $\Delta t = t_2 - t_1$ is equal to the slope of the straight line* (or **chord**) connecting the two points (x_1, t_1) and (x_2, t_2) on an x vs. t graph.

FIGURE 2–27 Graph of an object's position x vs. time t. The slope of the straight line P_1P_2 represents the average velocity of the object during the time interval $\Delta t = t_2 - t_1$.



Consider now a time intermediate between t_1 and t_2, call it t_3, at which moment the object is at x_3 (Fig. 2–28). The slope of the straight line P_1P_3 is less than the slope of P_1P_2. Thus the average velocity during the time interval $t_3 - t_1$ is less than during the time interval $t_2 - t_1$.

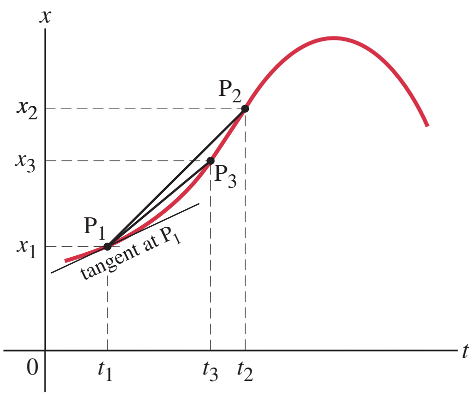


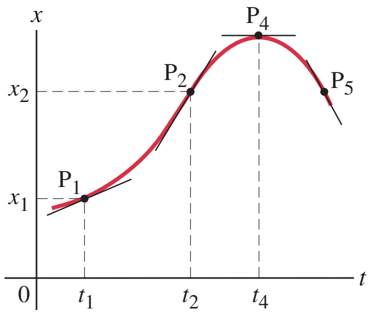
FIGURE 2–28 Same position vs. time curve as in Fig. 2–27. Note that the average velocity over the time interval $t_3 - t_1$ (which is the slope of P_1P_3) is less than the average velocity over the time interval $t_2 - t_1$. The slope of the line tangent to the curve at point P_1 equals the *instantaneous* velocity at time t_1.

Next let us take point P_3 in Fig. 2–28 to be closer and closer to point P_1. That is, we let the interval $t_3 - t_1$, which we now call Δt, to become smaller and smaller. The slope of the line connecting the two points becomes closer and closer to the slope of a line **tangent**† to the curve at point P_1. The average velocity (equal to the slope of the chord) thus approaches the slope of the tangent at point P_1. The definition of the instantaneous velocity (Eq. 2–3) is the limiting value of the average velocity as Δt approaches zero. Thus the *instantaneous velocity equals the slope of the tangent to the curve of x vs. t* at any chosen point (which we can simply call "the slope of the curve" at that point).

PROBLEM SOLVING
Velocity equals slope of x vs. t graph at any instant

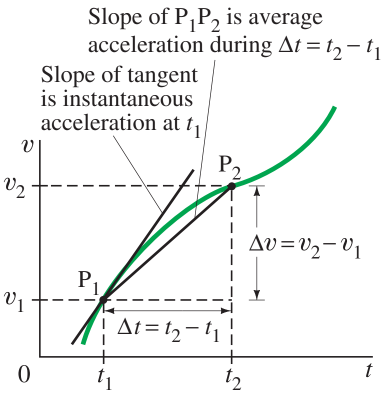
†The tangent is a straight line that touches the curve only at the one chosen point, without passing across or through the curve at that point.

FIGURE 2–29 Same x vs. t curve as in Figs. 2–27 and 2–28, but here showing the slope at four different points: At P_4, the slope is zero, so $v = 0$. At P_5 the slope is negative, so $v < 0$.



We can obtain the velocity of an object at any instant from its graph of x vs. t. For example, in Fig. 2–29 (which shows the same graph as in Figs. 2–27 and 2–28), as our object moves from x_1 to x_2, the slope continually increases, so the velocity is increasing. For times after t_2, the slope begins to decrease and reaches zero $(v = 0)$ where x has its maximum value, at point P_4 in Fig. 2–29. Beyond point P_4, the slope is negative, as for point P_5. The velocity is therefore negative, which makes sense since x is now decreasing—the particle is moving toward decreasing values of x, to the left on a standard xy plot.

FIGURE 2–30 A graph of velocity v vs. time t. The average acceleration over a time interval $\Delta t = t_2 - t_1$ is the slope of the straight line P_1P_2: $\bar{a} = \Delta v/\Delta t$. The instantaneous acceleration at time t_1 is the slope of the v vs. t curve at that instant.



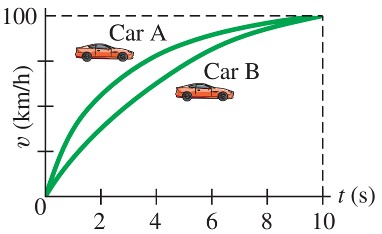
Slope and Acceleration

We can also draw a graph of the *velocity*, v, vs. time, t, as shown in Fig. 2–30. Then the average acceleration over a time interval $\Delta t = t_2 - t_1$ is represented by the slope of the straight line connecting the two points P_1 and P_2 as shown. [Compare this to the position vs. time graph of Fig. 2–27 for which the slope of the straight line represents the average velocity.] The instantaneous acceleration at any time, say t_1, is the slope of the tangent to the v vs. t curve at that time, which is also shown in Fig. 2–30. Using this fact for the situation graphed in Fig. 2–30, as we go from time t_1 to time t_2 the velocity continually increases, but the acceleration (the rate at which the velocity changes) is decreasing since the slope of the curve is decreasing.

CONCEPTUAL EXAMPLE 2–17 **Analyzing with graphs.** Figure 2–31 shows the velocity as a function of time for two cars accelerating from 0 to 100 km/h in a time of 10.0 s. Compare (a) the average acceleration; (b) the instantaneous acceleration; and (c) the total distance traveled for the two cars.

RESPONSE (a) Average acceleration is $\Delta v/\Delta t$. Both cars have the same Δv (100 km/h) over the same time interval $\Delta t = 10.0$ s, so the average acceleration is the same for both cars. (b) Instantaneous acceleration is the slope of the tangent to the v vs. t curve. For the first 4 s or so, the top curve (car A) is steeper than the bottom curve, so car A has a greater acceleration during this interval. The bottom curve is steeper during the last 6 s, so car B has the larger acceleration for this period. (c) Except at $t = 0$ and $t = 10.0$ s, car A is always going faster than car B. Since it is going faster, it will go farther in the same time.

FIGURE 2–31 (below) Example 2–17.



Summary

[*The Summary that appears at the end of each Chapter in this book gives a brief overview of the main ideas of the Chapter. The Summary* cannot *serve to give an understanding of the material, which can be accomplished only by a detailed reading of the Chapter.*]

Kinematics deals with the description of how objects move. The description of the motion of any object must always be given relative to some particular **reference frame**.

The **displacement** of an object is the change in position of the object.

Average speed is the distance traveled divided by the **elapsed time** or **time interval**, Δt (the time period over which we choose to make our observations). An object's **average velocity** over a particular time interval is

$$\bar{v} = \frac{\Delta x}{\Delta t}, \quad (2\text{–}2)$$

where Δx is the displacement during the time interval Δt.

The **instantaneous velocity**, whose magnitude is the same as the *instantaneous speed*, is defined as the average velocity taken over an infinitesimally short time interval.

Acceleration is the change of velocity per unit time. An object's **average acceleration** over a time interval Δt is

$$\bar{a} = \frac{\Delta v}{\Delta t}, \qquad \textbf{(2–4)}$$

where Δv is the change of velocity during the time interval Δt. **Instantaneous acceleration** is the average acceleration taken over an infinitesimally short time interval.

If an object has position x_0 and velocity v_0 at time $t = 0$ and moves in a straight line with **constant acceleration**, the velocity v and position x at a later time t are related to the acceleration a, the initial position x_0, and the initial velocity v_0 by Eqs. 2–11:

$$\begin{aligned} v &= v_0 + at, \\ x &= x_0 + v_0 t + \tfrac{1}{2}at^2, \\ v^2 &= v_0^2 + 2a(x - x_0), \\ \bar{v} &= \frac{v + v_0}{2}. \end{aligned} \qquad \textbf{(2–11)}$$

Objects that move vertically near the surface of the Earth, either falling or having been projected vertically up or down, move with the constant downward **acceleration due to gravity**, whose magnitude is $g = 9.80\,\mathrm{m/s^2}$ if air resistance can be ignored. We can apply Eqs. 2–11 for constant acceleration to objects that move up or down freely near the Earth's surface.

The slope of a curve at any point on a graph is the slope of the tangent to the curve at that point. On a graph of position vs. time, the **slope** is equal to the instantaneous velocity. On a graph of velocity vs. time, the slope is the acceleration.

Questions

1. Does a car speedometer measure speed, velocity, or both? Explain.

2. When an object moves with constant velocity, does its average velocity during any time interval differ from its instantaneous velocity at any instant? Explain.

3. If one object has a greater speed than a second object, does the first necessarily have a greater acceleration? Explain, using examples.

4. Compare the acceleration of a motorcycle that accelerates from 80 km/h to 90 km/h with the acceleration of a bicycle that accelerates from rest to 10 km/h in the same time.

5. Can an object have a northward velocity and a southward acceleration? Explain.

6. Can the velocity of an object be negative when its acceleration is positive? What about vice versa? If yes, give examples in each case.

7. Give an example where both the velocity and acceleration are negative.

8. Can an object be increasing in speed as its acceleration decreases? If so, give an example. If not, explain.

9. Two cars emerge side by side from a tunnel. Car A is traveling with a speed of 60 km/h and has an acceleration of 40 km/h/min. Car B has a speed of 40 km/h and has an acceleration of 60 km/h/min. Which car is passing the other as they come out of the tunnel? Explain your reasoning.

10. A baseball player hits a ball straight up into the air. It leaves the bat with a speed of 120 km/h. In the absence of air resistance, how fast would the ball be traveling when it is caught at the same height above the ground as it left the bat? Explain.

11. As a freely falling object speeds up, what is happening to its acceleration—does it increase, decrease, or stay the same? (*a*) Ignore air resistance. (*b*) Consider air resistance.

12. You travel from point A to point B in a car moving at a constant speed of 70 km/h. Then you travel the same distance from point B to another point C, moving at a constant speed of 90 km/h. Is your average speed for the entire trip from A to C equal to 80 km/h? Explain why or why not.

13. Can an object have zero velocity and nonzero acceleration at the same time? Give examples.

14. Can an object have zero acceleration and nonzero velocity at the same time? Give examples.

15. Which of these motions is *not* at constant acceleration: a rock falling from a cliff, an elevator moving from the second floor to the fifth floor making stops along the way, a dish resting on a table? Explain your answers.

16. Describe in words the motion plotted in Fig. 2–32 in terms of velocity, acceleration, etc. [*Hint*: First try to duplicate the motion plotted by walking or moving your hand.]

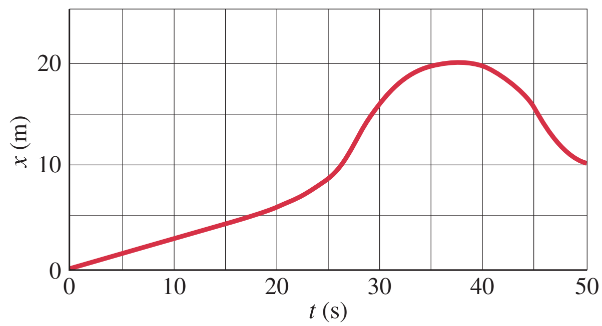


FIGURE 2–32 Question 16.

17. Describe in words the motion of the object graphed in Fig. 2–33.

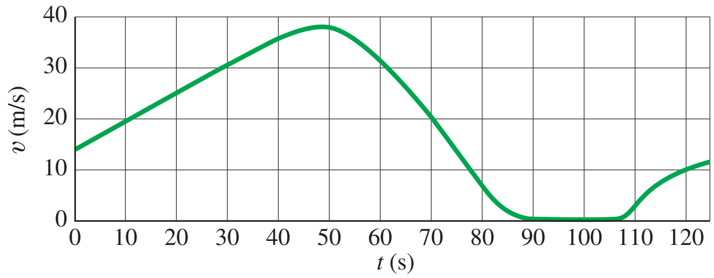


FIGURE 2–33 Question 17.