C3 linearization properties

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Ingredients

Set $\mathcal{C}$ of classes $C_0, C_1, C_2, \ldots, C_n$.
Set $\mathcal{D}$ of pairs (class, set of classes) $\mathcal{C} \times P(\mathcal{C})$.
$\mathrm{MRO}_D : \mathcal{C} \Rightarrow \prod_{i \in \mathbb{N}} \mathcal{C}$.

Consistency with the hierarchy of classes

Let C be a class with superclasses $P_0, P_1, \ldots, P_n$, where $n \in \mathbb{N}$.
Let the MRO of $\mathcal{C}$ be $MRO(\mathcal{C}) = [\mathcal{M}_0, \mathcal{M}_1, \mathcal{M}_2, \ldots, \mathcal{M}_m]$, with $m \in \mathbb{N}$ and $\mathcal{M}_0 = \mathcal{C}$.

$$\{P_1, \ldots, P_n\} \subseteq \{\mathcal{M}_1, \mathcal{M}_2, \ldots, \mathcal{M}_m\}$$

Consistency with the local precedence order

Let $a \cdot s_1$ be $SbStr(a \cdot s_1, P(C))$ and $s_2 \cdot a \cdot s_3$ be $SbStr(s_2 \cdot a \cdot s_3, \mathrm{MRO}(C))$.

$$SbStr(\varepsilon, \varepsilon)$$

$$\begin{aligned} SbStr(a \cdot s_1, s_2 \cdot a \cdot s_3) \quad & \text{if } a \notin s_1 \cdot s_2 \cdot s_3 \\ & \quad \wedge\ SbStr(s_1, s_2) \end{aligned}$$

$$\begin{aligned} & SbStr : \mathcal{S} \times \mathcal{S} \Rightarrow \{\top, \bot\} \\ & \mathcal{S} = \Sigma^* \end{aligned}$$

Consistency with the local precedence graph

Let $X \in \mathcal{C}$ and $\mathrm{P}(X) = [PX_0, \dots, PX_r]$, where $r \in \mathbb{N}$ is a list of superclasses.
Let D, B and A be a class.
If $D \in \mathrm{P}(B) \setminus \mathrm{P}(A)$, then $\exists p, q, 0 < p < q \leq m$, where $\mathcal{M}_p = D$, $\mathcal{M}_q = A$.