

C3 linearization algorithm

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Ingredients

Let \mathcal{C} be a finite set of symbols, ranged over by C_0, C_1, \dots, C_n , possibly primed, which we refer to as *classes*, and let $(\mathcal{C}, <_c)$ be an ordered set of classes.

Consider $\mathcal{P} \subseteq (\mathcal{C}, <_c)$ and $\mathcal{D} \subseteq \mathcal{C} \times \wp(\mathcal{P})$.

The purpose of this document is to present the MRO algorithm¹, which maps each \mathcal{C} with an ordered set of classes, \mathcal{P} . We show first the envisaged properties, then define it, and finally prove the definition ensures those properties.

Properties

Let $D = \langle C, \{P_1, \dots, P_n\} \rangle \in \mathcal{D}$, with $n \in \mathbb{N}_0$.

Let $\text{MRO}(C) = \{C, C_1 \dots, C_m\}$, with $m \in \mathbb{N}_0$.

Consistency with the extended precedence graph

This property requires that $\{P_1, \dots, P_n\} \subseteq \{C_1, \dots, C_m\}$.

Consistency with the local precedence order

For $m, n \geq 2$.

$$\forall i, j \ (0 \leq i < j \leq n \implies \exists p, q \ (0 \leq p < q \leq m \wedge C_p = P_i \wedge C_q = P_j)).$$

Consistency with monotonicity

Let $D', D'' \in \mathcal{D}$

If $C \in \pi_2(D') \setminus \pi_2(D'')$, then $\exists p, q. 0 < p < q \leq m \wedge C_p = C \wedge C_q = \pi_1(D'')$.

¹Method Resolution Order

Functions

Let $(\mathcal{C}, <_c)^*$ be a sequence of ordered sets of classes.

Let $L = (L_1, \dots, L_n)$, $L \in (\mathcal{C}, <_c)^*$

Let $C \in \mathcal{C}$

Remove

$\text{remove} : (\mathcal{C}, <_c)^* \times \mathcal{C} \Rightarrow (\mathcal{C}, <_c)^*$

$\text{remove}((), C) = ()$

$\text{remove}(l :: L, C) = l \setminus \{C\} :: \text{remove}(L, C)$

Merge

$\text{merge} : (\mathcal{C}, <_c)^* \Rightarrow (\mathcal{C}, <_c)$

$$\text{merge}(L) = \begin{cases} \{C\} \cup \text{merge}(\text{remove}(L, C)), & \text{if } (\exists k \in \llbracket 1, n \rrbracket, L_k \neq \emptyset \wedge C = \text{head}(L_k)) \wedge \\ & (\forall j < k, C \neq \text{head}(L_j)) \wedge \\ & (\forall i \in \llbracket 1, n \rrbracket, C \notin \text{tail}(L_i)) \\ \text{fail} & \text{otherwise} \end{cases}$$

C3 Linearization

$\text{c3linearization} : \mathcal{D} \Rightarrow (\mathcal{C}, <_c)$

Let $D = \langle C, P \rangle$ where $D \in \mathcal{D}$

Let $D' = (D_1, D_2, \dots, D_{|P|})$, such that

$\forall P_i \in P, \exists D_i \in \mathcal{D}$ such that $D_i = \langle P_i, P' \rangle$ where $i \in \llbracket 1, |P| \rrbracket$

$$\text{c3linearization}(D) = \begin{cases} \{C\} & \text{if } P = \emptyset \\ \{C\} \cup \text{merge}((\text{c3linearization}(D_i))_{D_i \in D'} \cdot (P)) & \text{otherwise} \end{cases}$$