C3 linearization algorithm

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Ingredients

Let $\mathcal{C}$ be a finite set of symbols, ranged over by $C_0, C_1, \ldots, C_n$, possibly primed, which we refer to as *classes*, and let $(\mathcal{C}, <_c)$ be an ordered set of classes. Consider $\mathcal{P} \subseteq (\mathcal{C}, <_c)$ and $\mathcal{D} \subseteq \mathcal{C} \times \wp(\mathcal{P})$.

The purpose of this document is to present the `MRO` algorithm[1], which maps each $\mathcal{C}$ with an ordered set of classes, $\mathcal{P}$. We show first the envisaged properties, then define it, and finally prove the definition ensures those properties.

Properties

Let $D = \langle C, \{P_1, \ldots, P_n\} \rangle \in \mathcal{D}$, with $n \in \mathbb{N}_0$.

Consistency with the hierarchy of classes

This property requires that $\texttt{MRO}(\mathcal{C}) = \{C_0, \ldots, C_m\}$, with $m \in \mathbb{N}_0$ and $C_0 = C$ where $\{P_0, P_1, \ldots, P_n\} \subseteq \{C_1, \ldots, C_m\}$.

Consistency with the local precedence order

Let $\widetilde{C}$ denote a possibly empty sequence of class names and consider

DEFINIR ASSINATURA e SbStr como macro

$\texttt{chars(s)} = \{c \in \Sigma \mid \exists i \in [\![1, |s|]\!] \text{ such that } s_i = c\}$

$\mathit{SbStr}(\varepsilon, \widetilde{C})$ holds always

$\mathit{SbStr}(a \cdot s_1, s_2 \cdot a \cdot s_3)$ holds if $a \notin s_1 \cdot s_2 \cdot s_3 \wedge \text{chars}(s_1) \subseteq \text{chars}(s_3)$

This property requires that $\mathit{SbStr}(a \cdot s_1, \pi_2(D))$ and $\mathit{SbStr}(s_2 \cdot a \cdot s_3, \texttt{MRO}(C))$.

[1] Mbla Rsomething Oyada

Consistency with the local precedence graph

Let $X \in \mathcal{C}$ and $\mathrm{P}(X) = [PX_0, \ldots, PX_r]$, where $r \in \mathbb{N}_0$ is a list of superclasses.
Let D, B and A be a class.
If $D \in \mathrm{P}(B) \setminus \mathrm{P}(A)$, then $\exists p, q, 0 < p < q \leq m$, where $\mathcal{M}_p = D$, $\mathcal{M}_q = A$.

Functions

Remove

Let $(\mathcal{C}, <_c)^*$ be a sequence of ordered sets of classes.
remove : $(\mathcal{C}, <_c)^* \times C \Rightarrow (\mathcal{C}, <_c)^*$
Let $L = [L_1; \ldots; L_n]$, $L \in (\mathcal{C}, <_c)^*$
Let $C \in \mathcal{C}$

$$\mathrm{remove}(L, C) = [l/C \mid l \in L]$$

Merge

merge : $(\mathcal{C}, <_c)^* \Rightarrow (\mathcal{C}, <_c)$
Let $L = [L_1; \ldots; L_n]$, $L \in (\mathcal{C}, <_c)^*$

$$merge(L) = \begin{cases} \{C\} \cup \mathrm{merge}(\mathrm{remove}(L, C)), & \text{if } (\exists k \in [\![1, n]\!], L_k \neq \emptyset \wedge C = \mathrm{head}(L_k)) \wedge \\ & (\forall j < k, C \neq \mathrm{head}(L_j)) \wedge \\ & (\forall i \in [\![1, n]\!], C \notin \mathrm{tail}(L_i)) \\ \text{fail} & \text{otherwise} \end{cases}$$

C3 Linearization

c3linearization : $\mathcal{D} \Rightarrow (\mathcal{C}, <_c)$
Let $D = \langle C, P \rangle$ where $D \in \mathcal{D}$
$\forall P_i \in P,\ \exists \mathrm{DP}_i \in \mathcal{D}$ such that $\mathrm{DP}_i = \langle P_i, \mathrm{PP}_i \rangle$
Let $DP = [DP_i \mid i \in [\![1, |P|]\!]]$

$$\text{c3linearization}(D) = \begin{cases} \{C\} & \text{if } P = \emptyset \\ \{C\} \cup \text{merge}([\text{c3linearization}(dp) \mid dp \in DP] \cdot [P]) & \text{otherwise} \end{cases}$$