C3 linearization properties

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Notation Convention

We adopt the following convention throughout the article:

- Variables with a tilde (e.g., $\tilde{C}$) represent **sequences** (finite ordered lists of elements).
- Variables with a double tilde (e.g., $\tilde{\tilde{C}}$) represent **sequence of sequences**.

Ingredients

$\mathcal{C}$ of classes $C_0, C_1, C_2, \ldots, C_n$.
Set $\mathcal{D}$ of pairs (class, set of classes) $\mathcal{C} \times \tilde{\mathcal{D}}$.
$\text{MRO}_D : \mathcal{C} \Rightarrow \tilde{\mathcal{C}}$.

Remove

$remove : \tilde{\tilde{\mathcal{C}}} \times C \Rightarrow \tilde{\tilde{\mathcal{C}}}$
Let $\tilde{\tilde{L}} = [\tilde{L}_1; \ldots; \tilde{L}_n]$, $\tilde{\tilde{L}} \in \tilde{\tilde{\mathcal{C}}}$
Let $C \in \mathcal{C}$

$$\frac{C \quad \tilde{\tilde{L}} = [\tilde{L}_1 \setminus \{C\}, \ldots, \tilde{L}_n \setminus \{C\}]}{\text{remove}(\tilde{\tilde{L}}, C) = \tilde{\tilde{L}}}$$

Merge

$merge : \tilde{\tilde{\mathcal{C}}} \Rightarrow \tilde{\mathcal{C}}$
Let $\tilde{\tilde{L}} = [\tilde{L}_1; \ldots; \tilde{L}_n]$, $\tilde{\tilde{L}} \in \tilde{\tilde{\mathcal{C}}}$

$$merge(\tilde{\tilde{L}}) = \begin{cases} [C] \cdot merge(remove(\tilde{\tilde{L}}, C)), & \text{if } (\exists k \in \{1, \ldots, n\}, \tilde{L}_k \neq \emptyset \wedge C = head(\tilde{L}_k)) \wedge \\ & (\forall j < k, C \neq head(\tilde{L}_j)) \wedge \\ & (\forall i \in \{1, \ldots, n\}, C \notin tail(\tilde{L}_i)) \\ fail & otherwise \end{cases}$$

Extract Classes

$classes : \tilde{\mathcal{D}} \Rightarrow \tilde{\mathcal{C}}$
Let $\tilde{D} = [(C_1, \tilde{P}_1), \ldots, (C_n, \tilde{P}_n)] \in \tilde{\mathcal{D}}$

$$classes(\tilde{D}) = [C_1, \ldots, C_n]$$

C3 Linearization

$c3linearization : \mathcal{D} \Rightarrow \tilde{\mathcal{C}}$
Let $D = (C, \tilde{P})$ where $D \in \mathcal{D}$

$$c3linearization(D) = \begin{cases} [C] & \text{if } \tilde{P} = \emptyset \\ [C] \cdot merge([c3linearization(P_1), \ldots, & \\ \qquad c3linearization(P_n), & \text{otherwise} \\ \qquad classes(\tilde{P})]) & \end{cases}$$