C3 linearization algorithm

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Ingredients

Let $\mathcal{C}$ be a finite set of symbols, ranged over by $C_0, C_1, \ldots, C_n$, possibly primed, which we refer to as *classes*, and let $(\mathcal{C}, <_c)$ be an ordered set of classes. Consider $\mathcal{P} \subseteq (\mathcal{C}, <_c)$ and $\mathcal{D} \subseteq \mathcal{C} \times \wp(\mathcal{P})$.

The purpose of this document is to present the `MRO` algorithm[1], which maps each $\mathcal{C}$ with an ordered set of classes, $\mathcal{P}$. We show first the envisaged properties, then define it, and finally prove the definition ensures those properties.

Properties

Let $D = \langle C, \{P_1, \ldots, P_n\} \rangle \in \mathcal{D}$, with $n \in \mathbb{N}_0$.
Let $\texttt{MRO}(\mathcal{C}) = \{C_0, \ldots, C_m\}$, with $m \in \mathbb{N}_0$.

Consistency with the hierarchy of classes

This property requires that $C_0 = C$ where $\{P_0, P_1, \ldots, P_n\} \subseteq \{C_1, \ldots, C_m\}$.

Consistency with the local precedence order

Let A and B be classes, for $m, n \geq 2$.

$$\forall\, i, j \ \big(0 \leq i < j \leq n \implies \exists\, p, q \ \big(0 \leq p < q \leq m \ \wedge \ C_p = P_i \ \wedge \ C_q = P_j\big)\big).$$

Consistency with the local precedence graph

Let $D', D'' \in \mathcal{D}$
If $C \in \pi_2(D') \backslash \pi_2(D'')$, then $\exists p, q$, $0 < p < q \leq m$, where $C_p = C$, $C_q = \pi_1(D'')$.

[1] *M*ethod *R*esolution *O*rder

Functions

Let $(\mathcal{C}, <_c)^*$ be a sequence of ordered sets of classes.
Let $L = [L_1; \ldots; L_n]$, $L \in (\mathcal{C}, <_c)^*$
Let $C \in \mathcal{C}$

Remove

$\texttt{remove} : (\mathcal{C}, <_c)^* \times \mathcal{C} \Rightarrow (\mathcal{C}, <_c)^*$

$$\texttt{remove}(L, C) = [l/C \mid l \in L]$$

Merge

merge : $(\mathcal{C}, <_c)^* \Rightarrow (\mathcal{C}, <_c)$

$$\texttt{merge}(L) = \begin{cases} \{C\} \cup \texttt{merge}(\texttt{remove}(L, C)), & \text{if } (\exists k \in [\![1, n]\!], L_k \neq \emptyset \wedge C = \texttt{head}(L_k)) \wedge \\ & (\forall j < k, C \neq \texttt{head}(L_j)) \wedge \\ & (\forall i \in [\![1, n]\!], C \notin \texttt{tail}(L_i)) \\ \text{fail} & \text{otherwise} \end{cases}$$

C3 Linearization

$\texttt{c3linearization} : \mathcal{D} \Rightarrow (\mathcal{C}, <_c)$
Let $D = \langle C, P \rangle$ where $D \in \mathcal{D}$
$\forall P_i \in P$, $\exists D_i \in \mathcal{D}$ such that $D_i = \langle P_i, P' \rangle$
Let $D' = [D_i \mid i \in [\![1, |P|]\!]]$

$$\texttt{c3linearization}(D) = \begin{cases} \{C\} & \text{if } P = \emptyset \\ \{C\} \cup \texttt{merge}([\texttt{c3linearization}(d) \mid d \in D'] \cdot [P]) & \text{otherwise} \end{cases}$$