C3 linearization properties

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Ingredients

Let $\mathcal{C}$ be a set of symbols, $\mathcal{C} = \{C_0, C_1, \ldots, C_n\}$, which we refer to as classes.
Assume that $|\mathcal{C}| < |\mathbb{N}|$, meaning that $\mathcal{C}$ is finite and countable.
Let $(\mathcal{C}, <_c)$ be an ordered set of classes.
Let $\mathcal{D} \subseteq \mathcal{C} \times (\mathcal{C}, <_c)$.
$\text{MRO} : \mathcal{C} \Rightarrow (\mathcal{C}, <_c)$.

Properties

Consistency with the hierarchy of classes

Let $D = \langle C, \{P_0, P_1, \ldots, P_n\} \rangle \in \mathcal{D}$, with $n \in \mathbb{N}_0$.
Let the MRO of $\mathcal{C}$ be $MRO(\mathcal{C}) = \{\mathcal{M}_0, \mathcal{M}_1, \mathcal{M}_2, \ldots, \mathcal{M}_m\}$, with $m \in \mathbb{N}_0$ and $\mathcal{M}_0 = \mathcal{C}$.

$$\{P_0, P_1, \ldots, P_n\} \subseteq \{\mathcal{M}_1, \mathcal{M}_2, \ldots, \mathcal{M}_m\}$$

Consistency with the local precedence order

Let $a \cdot s_1$ be $SbStr(a \cdot s_1, \pi_2(D))$ and $s_2 \cdot a \cdot s_3$ be $SbStr(s_2 \cdot a \cdot s_3, \mathrm{MRO}(C))$.
Let $\mathrm{chars}(s) = \{c \in \Sigma \mid \exists i \in [\![1, |s|]\!] \text{ such that } s_i = c\}$.

$$SbStr(\varepsilon, \varepsilon)$$

$$SbStr(a \cdot s_1, s_2 \cdot a \cdot s_3) \quad \text{if } a \notin s_1 \cdot s_2 \cdot s_3 \\ \wedge\ \mathrm{chars}(s_1) \subseteq \mathrm{char}(s_3)$$

$$\begin{aligned} &SbStr : \mathcal{S} \times \mathcal{S} \Rightarrow \{\top, \bot\} \\ &\mathcal{S} = \Sigma^* \end{aligned}$$

Consistency with the local precedence graph

Let $X \in \mathcal{C}$ and $\mathrm{P}(X) = [PX_0, \ldots, PX_r]$, where $r \in \mathbb{N}_0$ is a list of superclasses.
Let D, B and A be a class.
If $D \in \mathrm{P}(B) \setminus \mathrm{P}(A)$, then $\exists p, q, 0 < p < q \leq m$, where $\mathcal{M}_p = D$, $\mathcal{M}_q = A$.

Functions

Remove

Let $(\mathcal{C}, <_c)^*$ be a sequence of ordered sets of classes.
remove : $(\mathcal{C}, <_c)^* \times C \Rightarrow (\mathcal{C}, <_c)^*$
Let $L = [L_1; \ldots; L_n]$, $L \in (\mathcal{C}, <_c)^*$
Let $C \in \mathcal{C}$

$$\mathrm{remove}(L, C) = [l/C \mid l \in L]$$

Merge

merge : $(\mathcal{C}, <_c)^* \Rightarrow (\mathcal{C}, <_c)$
Let $L = [L_1; \ldots; L_n]$, $L \in (\mathcal{C}, <_c)^*$

$$merge(L) = \begin{cases} \{C\} \cup \text{merge}(\text{remove}(L, C)), & \text{if } (\exists k \in [\![1, n]\!], L_k \neq \emptyset \wedge C = \text{head}(L_k)) \wedge \\ & (\forall j < k, C \neq \text{head}(L_j)) \wedge \\ & (\forall i \in [\![1, n]\!], C \notin \text{tail}(L_i)) \\ \text{fail} & \text{otherwise} \end{cases}$$

C3 Linearization

c3linearization : $\mathcal{D} \Rightarrow (\mathcal{C}, <_c)$
Let $D = \langle C, P \rangle$ where $D \in \mathcal{D}$
$\forall P_i \in P$, $\exists \text{DP}_i \in \mathcal{D}$ such that $\text{DP}_i = \langle P_i, \text{PP}_i \rangle$
Let $DP = [DP_i \mid i \in [\![1, |P|]\!]]$

$$\text{c3linearization}(D) = \begin{cases} \{C\} & \text{if } P = \emptyset \\ \{C\} \cup \text{merge}([\text{c3linearization}(dp) \mid dp \in DP] \cdot [P]) & \text{otherwise} \end{cases}$$