

C3 linearization properties

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Notation Convention

We adopt the following convention throughout the article:

- Variables with a tilde (e.g., \tilde{C}) represent **sequences** (finite ordered lists of elements).
- Variables with a double tilde (e.g., $\tilde{\tilde{C}}$) represent **sequence of sequences**.

Ingredients

\mathcal{C} of classes $C_0, C_1, C_2, \dots, C_n$.

Set \mathcal{D} of pairs (class, set of classes) $\mathcal{C} \times \tilde{\mathcal{C}}$.

$\text{MRO}_D : \mathcal{C} \Rightarrow \tilde{\mathcal{C}}$.

Remove

$\text{remove} : \tilde{\mathcal{C}} \times \mathcal{C} \Rightarrow \tilde{\mathcal{C}}$

Let $\tilde{L} = [\tilde{L}_1; \dots; \tilde{L}_n]$, $\tilde{L} \in \tilde{\mathcal{C}}$

Let $C \in \mathcal{C}$

$$\frac{C \quad \tilde{L} = [\tilde{L}_1 \setminus \{C\}, \dots, \tilde{L}_n \setminus \{C\}]}{\text{remove}(\tilde{L}, C) = \tilde{\tilde{L}}}$$

Merge

$merge : \tilde{\tilde{\mathcal{C}}} \Rightarrow \tilde{\mathcal{C}}$

Let $\tilde{\tilde{L}} = [\tilde{\tilde{L}}_1; \dots; \tilde{\tilde{L}}_n]$, $\tilde{\tilde{L}} \in \tilde{\tilde{\mathcal{C}}}$

$$merge(\tilde{\tilde{L}}) = \begin{cases} C \cdot (merge(remove(\tilde{\tilde{L}}), C)), & \text{if } (\exists k \in \mathbb{N}, C \in \tilde{\tilde{L}}_k \wedge \tilde{\tilde{L}}_k \in \tilde{\tilde{L}} \wedge C = head(\tilde{\tilde{L}}_k)) \wedge \\ & (\forall k \in \mathbb{N}, C \notin \tilde{\tilde{L}}_k) \\ fail & otherwise \end{cases}$$

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