C3 linearization algorithm

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Ingredients

Let $\mathcal{C}$ be a finite set of symbols, ranged over by $C_0, C_1, \ldots, C_n$, possibly primed, which we refer to as *classes*, and let $(\mathcal{C}, <_c)$ be an ordered set of classes.

Consider $\mathcal{P} \subseteq (\mathcal{C}, <_c)$ and $\mathcal{D} \subseteq \mathcal{C} \times \wp(\mathcal{P})$.

The purpose of this document is to present the `MRO` algorithm[1], which maps each $\mathcal{C}$ with an ordered set of classes, $\mathcal{P}$. We show first the envisaged properties, then define it, and finally prove the definition ensures those properties.

Properties

Let $D = \langle C, \{P_1, \ldots, P_n\} \rangle \in \mathcal{D}$, with $n \in \mathbb{N}_0$.
Let $\texttt{MRO}(C) = \{C, C_1 \ldots, C_m\}$, with $m \in \mathbb{N}_0$.

Consistency with the extended precedence graph

This property requires that $\{P_1, \ldots, P_n\} \subseteq \{C_1, \ldots, C_m\}$.

Consistency with the local precedence order

For $m, n \geq 2$.

$$\forall\, i, j \; \big(0 \leq i < j \leq n \implies \exists\, p, q \; \big(0 \leq p < q \leq m \; \wedge \; C_p = P_i \; \wedge \; C_q = P_j\big)\big).$$

Consistency with monotonicity

Let $D', D'' \in \mathcal{D}$
If $C \in \pi_2(D') \setminus \pi_2(D'')$, then $\exists p, q.\ 0 < p < q \leq m \; \wedge \; C_p = C \; \wedge \; C_q = \pi_1(D'')$.

[1] *M*ethod *R*esolution *O*rder

Functions

Let $(\mathcal{C}, <_c)^*$ be a sequence of ordered sets of classes.
Let $L = (L_1, \dots, L_n)$, $L \in (\mathcal{C}, <_c)^*$
Let $C \in \mathcal{C}$

Remove

$\texttt{remove} : (\mathcal{C}, <_c)^* \times \mathcal{C} \Rightarrow (\mathcal{C}, <_c)^*$

$$\texttt{remove}((\ ), C) = (\ )$$
$$\texttt{remove}(l :: L, C) = l \setminus \{C\} :: \texttt{remove}(L, C)$$

Merge

$\texttt{merge} : (\mathcal{C}, <_c)^* \Rightarrow (\mathcal{C}, <_c)$

$$\texttt{merge}(L) = \begin{cases} \emptyset, & \text{if } (\forall k \in [\![1, n]\!], L_k = \emptyset) \\ \{C\} \cup \texttt{merge}(\texttt{remove}(L, C)), & \text{if } (\exists k \in [\![1, n]\!], L_k \neq \emptyset \wedge C = \texttt{head}(L_k)) \wedge \\ & (\forall j < k, C \neq \texttt{head}(L_j)) \wedge \\ & (\forall i \in [\![1, n]\!], C \notin \texttt{tail}(L_i)) \\ \text{fail}, & \text{otherwise} \end{cases}$$

C3 Linearization

$\texttt{c3linearization} : \mathcal{D} \Rightarrow (\mathcal{C}, <_c)$
Let $D = \langle C, P \rangle$ where $D \in \mathcal{D}$
Let $D' = (D_1, D_2, \dots, D_{|P|})$, such that
$\forall P_i \in P$, $\exists D_i \in \mathcal{D}$ such that $D_i = \langle P_i, P' \rangle$ where $i \in [\![1, |P|]\!]$

$$\texttt{c3linearization}(D) = \begin{cases} \{C\} & \text{if } P = \emptyset \\ \{C\} \cup \texttt{merge}\left((\texttt{c3linearization}(D_i))_{D_i \in D'} \cdot (P)\right) & \text{otherwise} \end{cases}$$