

```
ln[5]:= threterm$a_, b_, c_] := ToExpressionStringJoin[ToStringa], ToStringb]]] * c +
  ToExpressionStringJoin[ToStringa], ToStringc]]] * b +
  a * ToExpressionStringJoin[ToStringb], ToStringc]]] - 2 a * b * c;
```

```
ln[6]:= fourterm$a_, b_, c_, d_] := a * threterm$b, c, d] + b * threterm$a, c, d] + c * threterm$a, b, d] +
  d * threterm$a, b, c] + ToExpressionStringJoin[ToStringa], ToStringb]]] *
  ToExpressionStringJoin[ToStringc], ToStringd]]] +
  ToExpressionStringJoin[ToStringa], ToStringc]]] *
  ToExpressionStringJoin[ToStringb], ToStringd]]] +
  ToExpressionStringJoin[ToStringa], ToStringd]]] *
  ToExpressionStringJoin[ToStringb], ToStringc]]] -
  2 * (a * b * ToExpressionStringJoin[ToStringc], ToStringd]]] +
  a * c * ToExpressionStringJoin[ToStringb], ToStringd]]] +
  a * d * ToExpressionStringJoin[ToStringb], ToStringc]]] +
  ToExpressionStringJoin[ToStringa], ToStringb]]] * c * d +
  ToExpressionStringJoin[ToStringa], ToStringc]]] * b * d +
  ToExpressionStringJoin[ToStringa], ToStringd]]] * b * c) + 6 * a * b * c * d;
```

```
ln[7]:=
```

```
ln[8]:= fiveterm$a_, b_, c_, d_, e_] := a * fourterm$b, c, d, e] + b * fourterm$a, c, d, e] +
  c * fourterm$a, b, d, e] + d * fourterm$a, b, c, e] + e * fourterm$a, b, c, d] +
  ToExpressionStringJoin[ToStringa], ToStringb]]] * threterm$c, d, e] +
  ToExpressionStringJoin[ToStringa], ToStringc]]] * threterm$b, d, e] +
  ToExpressionStringJoin[ToStringa], ToStringd]]] * threterm$b, c, e] +
  ToExpressionStringJoin[ToStringa], ToStringe]]] * threterm$b, c, d] +
  ToExpressionStringJoin[ToStringb], ToStringc]]] * threterm$a, d, e] +
  ToExpressionStringJoin[ToStringb], ToStringd]]] * threterm$a, c, e] +
  ToExpressionStringJoin[ToStringb], ToStringe]]] * threterm$a, c, d] +
  ToExpressionStringJoin[ToStringc], ToStringd]]] * threterm$a, b, e] +
  ToExpressionStringJoin[ToStringc], ToStringe]]] * threterm$a, b, d] +
  ToExpressionStringJoin[ToStringd], ToStringe]]] * threterm$a, b, c] +
  (*términos con productos de momentos de 2,2 y 1*)
  ToExpressionStringJoin[ToStringa], ToStringb]]] *
  ToExpressionStringJoin[ToStringc], ToStringd]]] * e +
  ToExpressionStringJoin[ToStringa], ToStringb]]] *
  ToExpressionStringJoin[ToStringc], ToStringe]]] * d +
  ToExpressionStringJoin[ToStringa], ToStringb]]] *
  ToExpressionStringJoin[ToStringd], ToStringe]]] * c +
  ToExpressionStringJoin[ToStringa], ToStringc]]] *
  ToExpressionStringJoin[ToStringb], ToStringd]]] * e +
  ToExpressionStringJoin[ToStringa], ToStringc]]] *
  ToExpressionStringJoin[ToStringb], ToStringe]]] * d +
  ToExpressionStringJoin[ToStringa], ToStringc]]] *
```

```

ToExpressionStringJoinToString[d], ToString[e]]] * b +
ToExpressionStringJoinToString[a], ToString[d]]] *
ToExpressionStringJoinToString[b], ToString[c]]] * e +
ToExpressionStringJoinToString[a], ToString[d]]] *
ToExpressionStringJoinToString[b], ToString[e]]] * c +
ToExpressionStringJoinToString[a], ToString[d]]] *
ToExpressionStringJoinToString[c], ToString[e]]] * b +
ToExpressionStringJoinToString[a], ToString[e]]] *
ToExpressionStringJoinToString[b], ToString[c]]] * d +
ToExpressionStringJoinToString[a], ToString[e]]] *
ToExpressionStringJoinToString[b], ToString[d]]] * c +
ToExpressionStringJoinToString[a], ToString[e]]] *
ToExpressionStringJoinToString[c], ToString[d]]] * b +
ToExpressionStringJoinToString[b], ToString[c]]] *
ToExpressionStringJoinToString[d], ToString[e]]] * a +
ToExpressionStringJoinToString[b], ToString[d]]] *
ToExpressionStringJoinToString[c], ToString[e]]] * a +
ToExpressionStringJoinToString[b], ToString[e]]] *
ToExpressionStringJoinToString[c], ToString[d]]] * a -
2 * (a * b * threterms[c, d, e] + a * c * threterms[b, d, e] + a * d * threterms[b, c, e] +
a * e * threterms[b, c, d] + b * c * threterms[a, d, e] + b * d * threterms[a, c, e] +
b * e * threterms[a, c, d] + c * d * threterms[a, b, e] + c * e * threterms[a, b, d] +
d * e * threterms[a, b, c]) - 2 * (ToExpressionStringJoinToString[a], ToString[b]]] * c * d * e +
ToExpressionStringJoinToString[a], ToString[c]]] * b * d * e +
ToExpressionStringJoinToString[a], ToString[d]]] * b * c * e +
ToExpressionStringJoinToString[a], ToString[e]]] * b * c * d +
ToExpressionStringJoinToString[b], ToString[c]]] * a * d * e +
ToExpressionStringJoinToString[b], ToString[d]]] * a * c * e +
ToExpressionStringJoinToString[b], ToString[e]]] * a * c * d +
ToExpressionStringJoinToString[c], ToString[d]]] * a * b * e +
ToExpressionStringJoinToString[c], ToString[e]]] * a * b * d +
ToExpressionStringJoinToString[d], ToString[e]]] * a * b * c) +
6 * (a * b * c * ToExpressionStringJoinToString[d], ToString[e]]] +
a * b * d * ToExpressionStringJoinToString[c], ToString[e]]] +
a * b * e * ToExpressionStringJoinToString[c], ToString[d]]] +
a * c * d * ToExpressionStringJoinToString[b], ToString[e]]] +
a * c * e * ToExpressionStringJoinToString[b], ToString[d]]] +
a * d * e * ToExpressionStringJoinToString[b], ToString[c]]] +
b * c * d * ToExpressionStringJoinToString[a], ToString[e]]] +
b * c * e * ToExpressionStringJoinToString[a], ToString[d]]] +
b * d * e * ToExpressionStringJoinToString[a], ToString[c]]] +
c * d * e * ToExpressionStringJoinToString[a], ToString[b]]] - 24 * a * b * c * d * e;

```

```
ln[ ]:= fiveterms[a, ad, b, bd, b] // Simplify
```

```
Out[*]:= 5 a adbd b2 + 2 a adbd bb + abd (4 adb b + 5 ad b2 + 2 ad bb) + 2 a adb bbd + 2 aad b bbd +
5 a ad b bbd + 10 a adb b bd + 5 aad b2 bd - 54 a ad b2 bd + 2 aad bb bd + 5 a ad bb bd +
2 a adb bdb + 2 aad b bdb + 5 a ad b bdb + 2 ab (2 adbd b + 2 adb bd + ad (bbd + 5 b bd + bdb))
```

```
In[*]:=
```

```
(*Definición de las ecuaciones de estado (RHS de las ODEs)*)(*1. d<a>/dt*)
eqad[ad_, a_, ada_, aad_, aa_, adad_, bd_, b_, bdb_, bbd_, bb_, bdbd_, ab_, abd_, adb_,
adbd_, adaada_, ft_] := -I (wc a + E0 d1 (bd + b) + E0 d2 (bdbd + 2 bdb + bb + 1) + ft) - κ / 2 a;

(*2. d<a^dag>/dt*)
eqad[ad_, a_, ada_, aad_, aa_, adad_, bd_, b_, bdb_, bbd_, bb_, bdbd_, ab_, abd_, adb_,
adbd_, adaada_, ft_] := I (wc ad + E0 d1 (bd + b) + E0 d2 (bdbd + 2 bdb + bb + 1) + ft) - κ / 2 ad;

(*3. d<b>/dt*)
eqb[ad_, a_, ada_, aad_, aa_, adad_, bd_, b_, bdb_, bbd_, bb_, bdbd_, ab_, abd_, adb_, adbd_,
adaada_, ft_] := -I (ω0 b - 2 Ub threterm[ad, b, b] + E0 d1 (ad + a) + 2 E0 d2 (ab + abd + adb + adbd));

(*4. d<b^dag>/dt*)
eqb[ad_, a_, ada_, aad_, aa_, adad_, bd_, b_,
bdb_, bbd_, bb_, bdbd_, ab_, abd_, adb_, adbd_, adaada_, ft_] :=
I (ω0 bd - 2 Ub threterm[ad, bd, b] + E0 d1 (ad + a) + 2 E0 d2 (abd + ab + adbd + adb));

(*5. d<a^dag a>/dt*)
eqad[ad_, a_, ada_, aad_, aa_, adad_, bd_, b_, bdb_, bbd_, bb_,
bdbd_, ab_, abd_, adb_, adbd_, adaada_, ft_] := -I (E0 d1 (adbd + adb - abd - ab) +
E0 d2 (threterm[ad, bd, bd] + 2 threterm[ad, bd, b] + threterm[ad, b, b] + ad -
threterm[ad, bd, bd] - 2 threterm[ad, bd, b] - threterm[ad, b, b] - a) + ft (ad - a)) - κ ada;

(*6. d<b^dag b>/dt*)
eqb[ad_, a_, ada_, aad_, aa_, adad_, bd_, b_, bdb_, bbd_, bb_, bdbd_,
ab_, abd_, adb_, adbd_, adaada_, ft_] := -I (E0 d1 (adbd - adb + abd - ab) + 2 E0 d2
(threterm[ad, bd, bd] - threterm[ad, b, b] + threterm[ad, bd, bd] - threterm[ad, b, b]));

(*7. d<a^2>/dt*)
eqaa[ad_, a_, ada_, aad_, aa_, adad_, bd_, b_, bdb_, bbd_, bb_,
bdbd_, ab_, abd_, adb_, adbd_, adaada_, ft_] := -2 I (wc aa + E0 d1 (abd + ab) +
E0 d2 (threterm[ad, bd, bd] + 2 threterm[ad, bd, b] + threterm[ad, b, b] + a) + ft a) - κ aa;

(*8. d<a^dag^2>/dt*)
eqada[ad_, a_, ada_, aad_, aa_, adad_, bd_, b_, bdb_, bbd_, bb_, bdbd_,
ab_, abd_, adb_, adbd_, adaada_, ft_] := 2 I (wc adad + E0 d1 (adbd + adb) + E0 d2
(threterm[ad, bd, bd] + 2 threterm[ad, bd, b] + threterm[ad, b, b] + ad) + ft ad) - κ adad;
```

(*9. d<b^2>/dt*)

```
eqbb[ad_, a_, ada_, aad_, aa_, adad_, bd_, b_,
  bdb_, bbd_, bb_, bdbd_, ab_, abd_, adb_, adbd_, adaada_, ft_] :=
-2 I (ω0 bb - Ub (2 fourterms[bd, b, b, bj + bb] + E0 d1 (adb + ab) + E0 d2 (2 threterms[ad, bd, bj +
  ad + 2 threterms[ad, b, bj + 2 threterms[a, bd, bj + a + 2 threterms[a, b, bj]]));
```

(*10. d<b^dag^2>/dt*)

```
eqbdb[ad_, a_, ada_, aad_, aa_, adad_, bd_,
  b_, bdb_, bbd_, bb_, bdbd_, ab_, abd_, adb_, adbd_, adaada_, ft_] :=
2 I (ω0 bdbd - Ub (2 fourterms[bd, bd, bd, bj + bdbd] + E0 d1 (adb + abd) + E0 d2 (2 threterms[ad, bd,
  bdj + ad + 2 threterms[ad, bd, bj + 2 threterms[a, bd, bdj + a + 2 threterms[a, bd, bj]]));
```

(*11. d<a b>/dt CORREGIDA*)

(*Los términos como <a^dag a b> son 3 operadores: threterms[ad, a, bj]*)

```
eqab[ad_, a_, ada_, aad_, aa_, adad_, bd_, b_, bdb_, bbd_, bb_, bdbd_, ab_, abd_, adb_, adbd_,
  adaada_, ft_] := -I ((ωc + ω0) ab - 2 Ub fourterms[a, bd, b, bj + E0 d1 (ada + aa + bdb + bb + 1) + E0 d2
  (2 (threterms[ad, a, bj + threterms[ad, a, bdj + threterms[a, a, bj + threterms[a, a, bdj] +
  threterms[b, b, bj + 2 threterms[b, bd, bj + threterms[b, bd, bdj + b] + ft b) - κ ab;
```

(*12. d<a^dag b^dag>/dt CORREGIDA*)

```
eqadb[ad_, a_, ada_, aad_, aa_, adad_, bd_,
  b_, bdb_, bbd_, bb_, bdbd_, ab_, abd_, adb_, adbd_, adaada_, ft_] :=
I ((ωc + ω0) abdb - 2 Ub fourterms[ad, bd, bd, bj + E0 d1 (ada + adad + bdb + bdbd + 1) +
  E0 d2 (2 (threterms[ad, a, bdj + threterms[ad, a, bj +
  threterms[ad, ad, bdj + threterms[ad, ad, bj] + threterms[bd, bd, bdj +
  2 threterms[bd, b, bdj + threterms[b, b, bdj + bd] + ft bd) - κ abdb;
```

(*13. d<a^dag b>/dt CORREGIDA*)

(*El error estaba aquí: <a^dag^2 b> es threterms[ad, ad, bj], no fourterms[adad, bj]*)

```
eqadb[ad_, a_, ada_, aad_, aa_, adad_, bd_, b_,
  bdb_, bbd_, bb_, bdbd_, ab_, abd_, adb_, adbd_, adaada_, ft_] :=
-I ((ω0 - ωc) adb - 2 Ub fourterms[ad, bd, b, bj + E0 d1 (ada + adad - bdb - bb) - E0 d2 (threterms[b,
  b, bj + 2 threterms[b, bd, bj + threterms[b, bd, bdj + b - 2 (threterms[ad, ad, bj +
  threterms[ad, bdj + threterms[a, ad, bj + threterms[a, ad, bdj]) - ft b) - κ adb;
```

(*14. d<a b^dag>/dt CORREGIDA*)

```
eqab[ad_, a_, ada_, aad_, aa_, adad_, bd_, b_,
  bdb_, bbd_, bb_, bdbd_, ab_, abd_, adb_, adbd_, adaada_, ft_] :=
I ((ω0 - ωc) abd - 2 Ub fourterms[a, bd, bd, bj + E0 d1 (ada + aa - bdb - bdbd) -
  E0 d2 (threterms[bd, bd, bdj + 2 threterms[bd, b, bdj +
  threterms[b, b, bdj + bd - 2 (threterms[a, a, bdj + threterms[a, a, bj +
```

```

thretermsa, ad, bd]+thretermsa, ad, bj))-ft bd)-κ abd;
(*15. d<a^dag a a^dag a>/dt (N^2)*)
(*Se han expandido los términos<N...>usando fiveterms y threterms expandiendo N=ad*a*)
eqadaadqad_, a_, ada_, aad_, aa_, adad_, bd_,
b_, bdb_, bbd_, bb_, bdbd_, ab_, abd_, adb_, adbd_, adaada_, ft_] :=
-I (ft ((ada+1) ad+3 ad ada)+E0 d1 (2 abd adad+(ada+1) adb+3 ada adb+(ada+1) adbd+3 ada adbd-
(ada+1) ab-(ada+1) abd-3 ab ada-3 abd ada+2 ab adad)-2 aa (ft ad+E0 d1 (adb+adbd))+
4 a^2 ad (ft+E0 d1 (b+bd))-a ((ada+1+3 ada-2 adad) ft+4 ad^2 (ft+(b+bd) E0 d1))+
E0 d2 (fivetermsad, a, ad, bd, bd]+2 fivetermsad, a, ad, bd, bj+
fivetermsad, a, ad, b, bj)+thretermsad, a, ad]+fivetermsad, ad, a, bd, bd]+
2 fivetermsad, ad, a, bd, bj]+fivetermsad, ad, a, b, bj)+thretermsad, ad, a]-
fivetermsad, a, a, bd, bd]-2 fivetermsad, a, a, bd, bj]-fivetermsad, a, a, b, bj]-
thretermsad, a, a]-fivetermsa, ad, a, bd, bd]-2 fivetermsa, ad, a, bd, bj]-
fivetermsa, ad, a, b, bj)-thretermsa, ad, a))+κ (ada-adaad);

```

(*Función HH Completa (Hamiltoniano de Control)*)

(*Se asume que las variables P (Costates) tienen
el prefijo'P' seguido del nombre de la variable*)

```

HH[ad_, a_, ada_, aad_, aa_, adad_, bd_, b_,
bdb_, bbd_, bb_, bdbd_, ab_, abd_, adb_, adbd_, adaada_, ft_] :=
ft*ft+Pa*eqa[ad, a, ada, aad, aa, adad, bd, b, bdb, bbd, bb, bdbd, ab, abd,
adb, adbd, adaada, ft]+Pad*eqad[ad, a, ada, aad, aa, adad, bd, b, bdb,
bbd, bb, bdbd, ab, abd, adb, adbd, adaada, ft]+Pb*eqb[ad, a, ada, aad,
aa, adad, bd, b, bdb, bbd, bb, bdbd, ab, abd, adb, adbd, adaada, ft]+
Pbd*eqbd[ad, a, ada, aad, aa, adad, bd, b, bdb, bbd, bb, bdbd, ab, abd,
adb, adbd, adaada, ft]+Pada*eqada[ad, a, ada, aad, aa, adad,
bd, b, bdb, bbd, bb, bdbd, ab, abd, adb, adbd, adaada, ft]+
Pbdb*eqbdb[ad, a, ada, aad, aa, adad, bd, b, bdb, bbd, bb, bdbd, ab,
abd, adb, adbd, adaada, ft]+Paa*eqaa[ad, a, ada, aad, aa, adad,
bd, b, bdb, bbd, bb, bdbd, ab, abd, adb, adbd, adaada, ft]+
Padad*eqada[ad, a, ada, aad, aa, adad, bd, b, bdb, bbd, bb, bdbd, ab,
abd, adb, adbd, adaada, ft]+Pbb*eqbb[ad, a, ada, aad, aa, adad,
bd, b, bdb, bbd, bb, bdbd, ab, abd, adb, adbd, adaada, ft]+
Pbdbd*eqbdb[ad, a, ada, aad, aa, adad, bd, b, bdb, bbd, bb, bdbd, ab,
abd, adb, adbd, adaada, ft]+Pab*eqab[ad, a, ada, aad, aa, adad,
bd, b, bdb, bbd, bb, bdbd, ab, abd, adb, adbd, adaada, ft]+
Pabd*eqabd[ad, a, ada, aad, aa, adad, bd, b, bdb, bbd, bb, bdbd, ab,
abd, adb, adbd, adaada, ft]+Padb*eqadb[ad, a, ada, aad, aa,
adad, bd, b, bdb, bbd, bb, bdbd, ab, abd, adb, adbd, adaada, ft]+
Padbd*eqadb[ad, a, ada, aad, aa, adad, bd, b, bdb, bbd, bb, bdbd, ab,
abd, adb, adbd, adaada, ft]+Padaada*eqadaad[ad, a, ada, aad, aa,
adad, bd, b, bdb, bbd, bb, bdbd, ab, abd, adb, adbd, adaada, ft];

```

In[*]:= (*Asegúrate de haber evaluado primero todas las definiciones de eq... y HH*)

costateAdad= -D[HH[ad, a, ada, aad, aa, adad, bd, b,
bdb, bbd, bb, bdbd, ab, abd, adb, adbd, adaada, ft], adad] // Simplify

Out[*]= $2i \left(ab(d1 + 8(b + bd)d2)E0 + abd(d1 + 8(b + bd)d2)E0 + \right.$
 $a((1 + 2b^2 + 4bb + 4b bd + 2bd^2 + 8bdb + 4bdbd)d2E0 + ft))Padaada +$
 $i(d1 + 2(b + bd)d2)E0Padb - i(d1 + 2(b + bd)d2)E0Padbd + Padad(\kappa - 2i\omega C)$

In[*]:= costateAda= -D[HH[ad, a, ada, aad, aa, adad, bd, b,
bdb, bbd, bb, bdbd, ab, abd, adb, adbd, adaada, ft], ada] // Simplify

Out[*]= $-i(-2bd d2 E0 Pab + 3a d2 E0 Padaada - 3ad d2 E0 Padaada + 6(a - ad)b^2 d2 E0 Padaada +$
 $12abb d2 E0 Padaada - 12adb d2 E0 Padaada + 24abbd d2 E0 Padaada +$
 $24abd d2 E0 Padaada - 24adb d2 E0 Padaada - 24adbd d2 E0 Padaada +$
 $6abd^2 d2 E0 Padaada - 6adbd^2 d2 E0 Padaada + 24abdb d2 E0 Padaada -$
 $24adbdb d2 E0 Padaada + 12abdbd d2 E0 Padaada - 12adbdb d2 E0 Padaada + 4aft Padaada -$
 $4adft Padaada - 2bd2 E0(Pab - 12ab Padaada - 12abd Padaada + 12adb Padaada +$
 $12adbd Padaada - 6abd Padaada + 6adbd Padaada - Padbd) + 2bd d2 E0 Padbd +$
 $d1 E0(-Pab + Pabd + 4ab Padaada + 4abd Padaada - 4adb Padaada - 4adbd Padaada - Padb + Padbd) +$
 $i Pada \kappa - i Padaada \kappa)$

In[*]:= **costateAd** = -D[HH[ad, a, ada, aad, aa, adad, bd, b,

bdb, bbd, bb, bdbd, ab, abd, adb, adbd, adaada, ft], adj] // Simplify

Out[*] = $-\frac{1}{2} i$

$$\begin{aligned}
 & (8 a b d^2 E0 Pab + 8 a b d^2 E0 Pab - 8 a b d^2 E0 Pabd - 8 a b d^2 E0 Pabd - 2 d^2 E0 Pada + 4 b^2 d^2 E0 Pada - \\
 & 2 b b d^2 E0 Pada + 8 b b d^2 E0 Pada + 4 b d^2 d^2 E0 Pada - 4 b d b d^2 E0 Pada - 2 b d b d^2 E0 Pada - \\
 & 2 f t Pada - 8 a^2 b d^1 E0 Padaada + 16 a a d b d^1 E0 Padaada - 8 a^2 b d^1 E0 Padaada + \\
 & 16 a a d b d^1 E0 Padaada - 8 a^2 d^2 E0 Padaada + 4 a a d^2 E0 Padaada - 2 a a d^2 E0 Padaada + \\
 & 32 a b^2 d^2 E0 Padaada + 32 a b d^2 d^2 E0 Padaada + 16 a a d^2 E0 Padaada - 6 a d a d^2 E0 Padaada + \\
 & 64 a a d b d^2 E0 Padaada + 64 a a d b d^2 E0 Padaada + 24 a^2 b^2 d^2 E0 Padaada - \\
 & 16 a a b^2 d^2 E0 Padaada + 8 a a d b^2 d^2 E0 Padaada - 48 a a d b^2 d^2 E0 Padaada + \\
 & 24 a d a b^2 d^2 E0 Padaada - 16 a^2 b b d^2 E0 Padaada + 16 a a b b d^2 E0 Padaada - \\
 & 8 a a d b b d^2 E0 Padaada + 32 a a d b b d^2 E0 Padaada - 24 a d a b b d^2 E0 Padaada + \\
 & 64 a a d b b d^2 E0 Padaada + 64 a a d b b d^2 E0 Padaada + 48 a^2 b b d^2 E0 Padaada - \\
 & 32 a a b b d^2 E0 Padaada + 16 a a d b b d^2 E0 Padaada - 96 a a d b b d^2 E0 Padaada + \\
 & 48 a d a b b d^2 E0 Padaada + 24 a^2 b d^2 d^2 E0 Padaada - 16 a a b d^2 d^2 E0 Padaada + \\
 & 8 a a d b d^2 d^2 E0 Padaada - 48 a a d b d^2 d^2 E0 Padaada + 24 a d a b d^2 d^2 E0 Padaada - \\
 & 32 a^2 b d b d^2 E0 Padaada + 32 a a b d b d^2 E0 Padaada - 16 a a d b d b d^2 E0 Padaada + \\
 & 64 a a d b d b d^2 E0 Padaada - 48 a d a b d b d^2 E0 Padaada - 16 a^2 b d b d^2 E0 Padaada + \\
 & 16 a a b d b d^2 E0 Padaada - 8 a a d b d b d^2 E0 Padaada + 32 a a d b d b d^2 E0 Padaada - \\
 & 24 a d a b d b d^2 E0 Padaada - 2 f t Padaada - 8 a^2 f t Padaada + 4 a a f t Padaada + \\
 & 16 a a d f t Padaada - 8 a d a f t Padaada + 4 d^2 E0 Padad - 8 b^2 d^2 E0 Padad + 4 b b d^2 E0 Padad - \\
 & 16 b b d^2 E0 Padad - 8 b d^2 d^2 E0 Padad + 8 b d b d^2 E0 Padad + 4 b d b d^2 E0 Padad + 4 f t Padad - \\
 & 8 a d b d^2 E0 Padb - 8 a d b d^2 E0 Padb + 8 a b d^2 E0 Padb + 16 a d b d^2 E0 Padb + 8 a b d^2 E0 Padb + \\
 & 16 a d b d^2 E0 Padb - 4 a b d^2 E0 (Pab - Pabd + 16 a d b Padaada + 16 a d b d Padaada + \\
 & 16 a b Padaada - 16 a d b Padaada + 16 a b d Padaada - 16 a d b d Padaada + Padb - Padbd) - \\
 & 4 a b d^2 E0 (Pab - Pabd - 16 a b d Padaada + 16 a d b Padaada + 16 a d b d Padaada + 16 a b Padaada - \\
 & 16 a d b Padaada + 16 a b d Padaada - 16 a d b d Padaada + Padb - Padbd) + 8 a d b d^2 E0 Padbd + \\
 & 8 a d b d^2 E0 Padbd - 8 a b d^2 E0 Padbd - 16 a d b d^2 E0 Padbd - 8 a b d^2 E0 Padbd - \\
 & 16 a d b d^2 E0 Padbd - 2 d^1 E0 Pb - 4 d^2 E0 Pbb + 16 b^2 d^2 E0 Pbb - 8 b b d^2 E0 Pbb + \\
 & 16 b b d^2 E0 Pbb - 8 b d b d^2 E0 Pbb + 2 d^1 E0 Pbd - 8 b^2 d^2 E0 Pbdb + 4 b b d^2 E0 Pbdb + \\
 & 8 b d^2 d^2 E0 Pbdb - 4 b d b d^2 E0 Pbdb + 4 d^2 E0 Pbdbd - 16 b b d^2 E0 Pbdbd - 16 b d^2 d^2 E0 Pbdbd + \\
 & 8 b d b d^2 E0 Pbdbd + 8 b d b d^2 E0 Pbdbd - 8 b^2 b d Padb Ub + 8 b b d^2 Padbd Ub + i Pad \kappa + 2 Pad \omega c)
 \end{aligned}$$

In[*]:= **costateBdb** = -D[HH[ad, a, ada, aad, aa, adad, bd, b,

bdb, bbd, bb, bdbd, ab, abd, adb, adbd, adaada, ft], bdb] // Simplify

Out[*] = $i (d^1 E0 (Pab + Pabd - Padb - Padbd) +$

$$\begin{aligned}
 & 2 d^2 E0 (Pa + b Pab + b d Pabd - Pad + ad Pada + 8 a^2 ad Padaada - 8 a a ad Padaada + 4 a a d ad Padaada + \\
 & 12 a d a d Padaada - 2 a d Padad - b Padb - b d Padbd + 2 a d Pbb - 2 a d Pbdbd + a (2 Paa - Pada - \\
 & 2 (2 a a d Padaada + 4 a d^2 Padaada + 6 a d a Padaada - 4 a d a d Padaada - Pbb + Pbdbd))) - \\
 & 4 (a b Pab - a b d Pabd + a d b Padb - a d b d Padbd + b Pb + 3 b b Pbb - b d Pbd - 3 b d b d Pbdbd) Ub)
 \end{aligned}$$

$In[*]:=$ **costateB** = -D[HH[ad, a, ada, aad, aa, adad, bd, b,

bdb, bbd, bb, bdbd, ab, abd, adb, adbd, adaada, ft], b] // Simplify

$Out[*]=$ $i \left(4 \text{abd d2 E0 Paa} - 8 \text{a b d2 E0 Paa} - 8 \text{a bd d2 E0 Paa} + \text{d2 E0 Pab} - 4 \text{a}^2 \text{ d2 E0 Pab} + 2 \text{aa d2 E0 Pab} - \right.$
 $4 \text{a ad d2 E0 Pab} + 2 \text{ada d2 E0 Pab} - 6 \text{b}^2 \text{ d2 E0 Pab} + 3 \text{bb d2 E0 Pab} + 2 \text{bbd d2 E0 Pab} -$
 $8 \text{b bd d2 E0 Pab} - 2 \text{bd}^2 \text{ d2 E0 Pab} + 2 \text{bdb d2 E0 Pab} + \text{bdbd d2 E0 Pab} + \text{ft Pab} + 4 \text{a}^2 \text{ d2 E0 Pabd} -$
 $2 \text{aa d2 E0 Pabd} - 2 \text{aad d2 E0 Pabd} + 4 \text{a ad d2 E0 Pabd} - 4 \text{bd}^2 \text{ d2 E0 Pabd} + 2 \text{bdbd d2 E0 Pabd} -$
 $2 \text{abd d2 E0 Pada} + 2 \text{adb d2 E0 Pada} + 2 \text{adbd d2 E0 Pada} + 4 \text{a b d2 E0 Pada} - 4 \text{ad b d2 E0 Pada} +$
 $4 \text{a bd d2 E0 Pada} - 4 \text{ad bd d2 E0 Pada} - 8 \text{aad abd d2 E0 Padaada} + 32 \text{a abd ad d2 E0 Padaada} -$
 $16 \text{abd ad}^2 \text{ d2 E0 Padaada} - 24 \text{abd ada d2 E0 Padaada} + 16 \text{abd adad d2 E0 Padaada} +$
 $16 \text{a}^2 \text{ adb d2 E0 Padaada} - 16 \text{aa adb d2 E0 Padaada} + 8 \text{aad adb d2 E0 Padaada} -$
 $32 \text{a ad adb d2 E0 Padaada} + 24 \text{ada adb d2 E0 Padaada} + 16 \text{a}^2 \text{ adbd d2 E0 Padaada} -$
 $16 \text{aa adbd d2 E0 Padaada} + 8 \text{aad adbd d2 E0 Padaada} - 32 \text{a ad adbd d2 E0 Padaada} +$
 $24 \text{ada adbd d2 E0 Padaada} + 8 \text{a aad b d2 E0 Padaada} - 24 \text{a}^2 \text{ ad b d2 E0 Padaada} +$
 $16 \text{aa ad b d2 E0 Padaada} - 8 \text{aad ad b d2 E0 Padaada} + 24 \text{a ad}^2 \text{ b d2 E0 Padaada} +$
 $24 \text{a ada b d2 E0 Padaada} - 24 \text{ad ada b d2 E0 Padaada} - 16 \text{a adad b d2 E0 Padaada} +$
 $8 \text{a aad bd d2 E0 Padaada} - 24 \text{a}^2 \text{ ad bd d2 E0 Padaada} + 16 \text{aa ad bd d2 E0 Padaada} -$
 $8 \text{aad ad bd d2 E0 Padaada} + 24 \text{a ad}^2 \text{ bd d2 E0 Padaada} + 24 \text{a ada bd d2 E0 Padaada} -$
 $24 \text{ad ada bd d2 E0 Padaada} - 16 \text{a adad bd d2 E0 Padaada} + \text{d1 E0 (Pa - Pad} + 4 \text{a (a - ad) ad Padaada} -$
 $4 \text{adb d2 E0 Padad} - 4 \text{adbd d2 E0 Padad} + 8 \text{ad b d2 E0 Padad} + 8 \text{ad bd d2 E0 Padad} - \text{d2 E0 Padb} +$
 $2 \text{aad d2 E0 Padb} - 4 \text{a ad d2 E0 Padb} - 4 \text{ad}^2 \text{ d2 E0 Padb} + 2 \text{adad d2 E0 Padb} + 6 \text{b}^2 \text{ d2 E0 Padb} -$
 $3 \text{bb d2 E0 Padb} - 2 \text{bbd d2 E0 Padb} + 8 \text{b bd d2 E0 Padb} + 2 \text{bd}^2 \text{ d2 E0 Padb} - 2 \text{bdb d2 E0 Padb} -$
 $\text{bdbd d2 E0 Padb} - \text{ft Padb} + 4 \text{a ad d2 E0 Padbd} + 4 \text{ad}^2 \text{ d2 E0 Padbd} - 2 \text{ada d2 E0 Padbd} -$
 $2 \text{adad d2 E0 Padbd} + 4 \text{bd}^2 \text{ d2 E0 Padbd} - 2 \text{bdbd d2 E0 Padbd} + 4 \text{abd d2 E0 Pbb} + 8 \text{adb d2 E0 Pbb} +$
 $4 \text{adbd d2 E0 Pbb} - 16 \text{a b d2 E0 Pbb} - 16 \text{ad b d2 E0 Pbb} - 8 \text{a bd d2 E0 Pbb} - 8 \text{ad bd d2 E0 Pbb} -$
 $4 \text{adb d2 E0 Pbdb} + 8 \text{a b d2 E0 Pbdb} + 8 \text{ad b d2 E0 Pbdb} + 2 \text{ab d2 E0 (2 Paa - Pada} - 2 (2 \text{aad Padaada} -$
 $8 \text{a ad Padaada} + 4 \text{ad}^2 \text{ Padaada} + 6 \text{ada Padaada} - 4 \text{adad Padaada} - 2 \text{Pbb} + \text{Pbdb})) -$
 $4 \text{abd d2 E0 Pbdb} - 4 \text{adbd d2 E0 Pbdb} + 8 \text{a bd d2 E0 Pbdb} + 8 \text{ad bd d2 E0 Pbdb} +$
 $8 \text{a b bd Pab Ub} - 4 \text{a bd}^2 \text{ Pabd Ub} + 8 \text{ad b bd Padb Ub} - 4 \text{ad bd}^2 \text{ Padbd Ub} + 8 \text{b bd Pb Ub} -$
 $4 \text{bdb Pb Ub} + 24 \text{b}^2 \text{ bd Pbb Ub} - 4 \text{bd}^2 \text{ Pbd Ub} + 2 \text{bdbd Pbd Ub} - 8 \text{bd}^3 \text{ Pbdbd Ub} + \text{Pb } \omega 0 \bigg)$

$In[*]:=$ **costateBb** = -D[HH[ad, a, ada, aad, aa, adad, bd, b,

bdb, bbd, bb, bdbd, ab, abd, adb, adbd, adaada, ft], bb] // Simplify

$Out[*]=$ $i \left(\text{d1 E0 (Pab - Padb)} + \right.$
 $\text{d2 E0 (Pa} + 3 \text{b Pab} + 2 \text{bd Pab} + \text{bd Pabd} - \text{Pad} + \text{ad Pada} + 8 \text{a}^2 \text{ ad Padaada} - 8 \text{aa ad Padaada} +$
 $4 \text{aad ad Padaada} + 12 \text{ad ada Padaada} - 2 \text{ad Padad} - 3 \text{b Padb} - 2 \text{bd Padb} -$
 $\text{bd Padbd} + 4 \text{ad Pbb} - 2 \text{ad Pbdb} + \text{a (2 Paa - Pada} -$
 $2 (2 \text{aad Padaada} + 4 \text{ad}^2 \text{ Padaada} + 6 \text{ada Padaada} - 4 \text{adad Padaada} - 2 \text{Pbb} + \text{Pbdb})) -$
 $2 (\text{abd Pab Ub} + \text{adbd Padb Ub} + \text{bd Pb Ub} + \text{Pbb Ub} + 6 \text{bdb Pbb Ub} - \text{Pbb } \omega 0 \bigg)$

In[*]:= **costateAbdbd** = -D[HH[ad, a, ada, aad, aa, adad, bd, b,
bdb, bbd, bb, bdbd, ab, abd, adb, adbd, adaada, ft], adbd] // Simplify

Out[*]= $2i a d_2 E_0 P_{ab} - 2i a d_2 E_0 P_{abd} + i (d_1 + 2(b + bd) d_2) E_0 P_{ada} -$
 $i \left((-1 + 2aa - 4ada) d_1 - 8(4ab ad + 4abd ad - 2aa b + aad b + 3ada b - 2aa bd + aad bd +$
 $3ada bd + 2a^2(b + bd) - 4a(ab + abd - adb - adbd + ad b + ad bd) \right) d_2 E_0 P_{adaada} -$
 $2i (d_1 + 2(b + bd) d_2) E_0 P_{adad} + 2i d_2 E_0 P_b + 4i b d_2 E_0 P_{bb} - 2i d_2 E_0 P_{bd} +$
 $i (d_1 + 4bd d_2) E_0 P_{bdb} - 2i (d_1 + 2(b + 2bd) d_2) E_0 P_{bdbd} +$
 $2i P_{adb} (a d_2 E_0 + 2ad d_2 E_0 - bb U_b) +$
 $P_{adb} (\kappa - i (2(a + 2ad) d_2 E_0 - 4bdb U_b + \omega_0 + \omega_c))$

In[*]:= **costateAbd** = -D[HH[ad, a, ada, aad, aa, adad, bd, b,
bdb, bbd, bb, bdbd, ab, abd, adb, adbd, adaada, ft], abd] // Simplify

Out[*]= $2i (d_1 + 2(b + bd) d_2) E_0 P_{aa} - i (d_1 + 2(b + bd) d_2) E_0 P_{ada} -$
 $i \left((1 + 4ada - 2adad) d_1 + 8(4ab ad + 4abd ad + 4a adb - 4ad adb + 4a adbd - 4ad adbd + aad b - 4a ad b +$
 $2ad^2 b + 3ada b - 2adad b + aad bd - 4a ad bd + 2ad^2 bd + 3ada bd - 2adad bd) d_2 \right) E_0 P_{adaada} +$
 $2i ad d_2 E_0 P_{adb} - 2i ad d_2 E_0 P_{adb} + 2i d_2 E_0 P_b + 4i b d_2 E_0 P_{bb} - 2i d_2 E_0 P_{bd} +$
 $i (d_1 + 4bd d_2) E_0 P_{bdb} - 2i (d_1 + 2(b + 2bd) d_2) E_0 P_{bdbd} +$
 $2i P_{ab} (2a d_2 E_0 + ad d_2 E_0 - bb U_b) +$
 $P_{abd} (\kappa - i (2(2a + ad) d_2 E_0 - 4bdb U_b + \omega_0 - \omega_c))$

In[*]:= **costateAad** = -D[HH[ad, a, ada, aad, aa, adad, bd, b,
bdb, bbd, bb, bdbd, ab, abd, adb, adbd, adaada, ft], aad] // Simplify

Out[*]= $-i d_2 E_0$
 $(2(b + bd) P_{abd} - (-8(ab + abd - adb - adbd)(b + bd) + a(-1 + 4b^2 - 4bb + 8b bd + 4bd^2 - 8bdb - 4bdbd) +$
 $ad(1 - 4b^2 + 4bb - 8b bd - 4bd^2 + 8bdb + 4bdbd)) P_{adaada} - 2(b + bd) P_{adb})$