```
In[•]:= threterms[a_, b_, c_] := ToExpression[StringJoin[ToString[a], ToString[b]]] * c +
          ToExpression[StringJoin[ToString[a], ToString[c]]] * b +
          a * ToExpression[StringJoin[ToString[b], ToString[c]]] - 2 a * b * c;

In[•]:= fourterms[a_, b_, c_, d_] := a * threterms[b, c, d] + b * threterms[a, c, d] + c * threterms[a, b, d] +
          d * threterms[a, b, c] + ToExpression[StringJoin[ToString[a], ToString[b]]] *
           ToExpression[StringJoin[ToString[c], ToString[d]]] +
          ToExpression[StringJoin[ToString[a], ToString[c]]] *
           ToExpression[StringJoin[ToString[b], ToString[d]]] +
          ToExpression[StringJoin[ToString[a], ToString[d]]] *
           ToExpression[StringJoin[ToString[b], ToString[c]]] -
          2 * (a * b * ToExpression[StringJoin[ToString[c], ToString[d]]] +
             a * c * ToExpression[StringJoin[ToString[b], ToString[d]]] +
             a * d * ToExpression[StringJoin[ToString[b], ToString[c]]] +
             ToExpression[StringJoin[ToString[a], ToString[b]]] * c * d +
             ToExpression[StringJoin[ToString[a], ToString[c]]] * b * d +
             ToExpression[StringJoin[ToString[a], ToString[d]]] * b * c) + 6 * a * b * c * d;



In[•]:=

In[•]:= fiveterms[a_, b_, c_, d_, e_] := a * fourterms[b, c, d, e] + b * fourterms[a, c, d, e] +
          c * fourterms[a, b, d, e] + d * fourterms[a, b, c, e] + e * fourterms[a, b, c, d] +
          ToExpression[StringJoin[ToString[a], ToString[b]]] * threterms[c, d, e] +
          ToExpression[StringJoin[ToString[a], ToString[c]]] * threterms[b, d, e] +
          ToExpression[StringJoin[ToString[a], ToString[d]]] * threterms[b, c, e] +
          ToExpression[StringJoin[ToString[a], ToString[e]]] * threterms[b, c, d] +
          ToExpression[StringJoin[ToString[b], ToString[c]]] * threterms[a, d, e] +
          ToExpression[StringJoin[ToString[b], ToString[d]]] * threterms[a, c, e] +
          ToExpression[StringJoin[ToString[b], ToString[e]]] * threterms[a, c, d] +
          ToExpression[StringJoin[ToString[c], ToString[d]]] * threterms[a, b, e] +
          ToExpression[StringJoin[ToString[c], ToString[e]]] * threterms[a, b, d] +
          ToExpression[StringJoin[ToString[d], ToString[e]]] * threterms[a, b, c] +
          (*términos con productos de momentos de 2,2 y 1*)
          ToExpression[StringJoin[ToString[a], ToString[b]]] *
           ToExpression[StringJoin[ToString[c], ToString[d]]] * e +
          ToExpression[StringJoin[ToString[a], ToString[b]]] *
           ToExpression[StringJoin[ToString[c], ToString[e]]] * d +
          ToExpression[StringJoin[ToString[a], ToString[b]]] *
           ToExpression[StringJoin[ToString[d], ToString[e]]] * c +
          ToExpression[StringJoin[ToString[a], ToString[c]]] *
           ToExpression[StringJoin[ToString[b], ToString[d]]] * e +
          ToExpression[StringJoin[ToString[a], ToString[c]]] *
           ToExpression[StringJoin[ToString[b], ToString[e]]] * d +
          ToExpression[StringJoin[ToString[a], ToString[c]]] *
```

```
 ToExpression[StringJoin[ToString[d], ToString[e]]] * b +
ToExpression[StringJoin[ToString[a], ToString[d]]] *
 ToExpression[StringJoin[ToString[b], ToString[c]]] * e +
ToExpression[StringJoin[ToString[a], ToString[d]]] *
 ToExpression[StringJoin[ToString[b], ToString[e]]] * c +
ToExpression[StringJoin[ToString[a], ToString[d]]] *
 ToExpression[StringJoin[ToString[c], ToString[e]]] * b +
ToExpression[StringJoin[ToString[a], ToString[e]]] *
 ToExpression[StringJoin[ToString[b], ToString[c]]] * d +
ToExpression[StringJoin[ToString[a], ToString[e]]] *
 ToExpression[StringJoin[ToString[b], ToString[d]]] * c +
ToExpression[StringJoin[ToString[a], ToString[e]]] *
 ToExpression[StringJoin[ToString[c], ToString[d]]] * b +
ToExpression[StringJoin[ToString[b], ToString[c]]] *
 ToExpression[StringJoin[ToString[d], ToString[e]]] * a +
ToExpression[StringJoin[ToString[b], ToString[d]]] *
 ToExpression[StringJoin[ToString[c], ToString[e]]] * a +
ToExpression[StringJoin[ToString[b], ToString[e]]] *
 ToExpression[StringJoin[ToString[c], ToString[d]]] * a -
2 * (a * b * threterms[c, d, e] + a * c * threterms[b, d, e] + a * d * threterms[b, c, e] +
   a * e * threterms[b, c, d] + b * c * threterms[a, d, e] + b * d * threterms[a, c, e] +
   b * e * threterms[a, c, d] + c * d * threterms[a, b, e] + c * e * threterms[a, b, d] +
   d * e * threterms[a, b, c]) - 2 * (ToExpression[StringJoin[ToString[a], ToString[b]]] * c * d * e +
   ToExpression[StringJoin[ToString[a], ToString[c]]] * b * d * e +
   ToExpression[StringJoin[ToString[a], ToString[d]]] * b * c * e +
   ToExpression[StringJoin[ToString[a], ToString[e]]] * b * c * d +
   ToExpression[StringJoin[ToString[b], ToString[c]]] * a * d * e +
   ToExpression[StringJoin[ToString[b], ToString[d]]] * a * c * e +
   ToExpression[StringJoin[ToString[b], ToString[e]]] * a * c * d +
   ToExpression[StringJoin[ToString[c], ToString[d]]] * a * b * e +
   ToExpression[StringJoin[ToString[c], ToString[e]]] * a * b * d +
   ToExpression[StringJoin[ToString[d], ToString[e]]] * a * b * c) +
6 * (a * b * c * ToExpression[StringJoin[ToString[d], ToString[e]]] +
   a * b * d * ToExpression[StringJoin[ToString[c], ToString[e]]] +
   a * b * e * ToExpression[StringJoin[ToString[c], ToString[d]]] +
   a * c * d * ToExpression[StringJoin[ToString[b], ToString[e]]] +
   a * c * e * ToExpression[StringJoin[ToString[b], ToString[d]]] +
   a * d * e * ToExpression[StringJoin[ToString[b], ToString[c]]] +
   b * c * d * ToExpression[StringJoin[ToString[a], ToString[e]]] +
   b * c * e * ToExpression[StringJoin[ToString[a], ToString[d]]] +
   b * d * e * ToExpression[StringJoin[ToString[a], ToString[c]]] +
   c * d * e * ToExpression[StringJoin[ToString[a], ToString[b]]]) - 24 * a * b * c * d * e;
```

In[•]:= `fiveterms[a, ad, b, bd, b] // Simplify`

Out[•]= $5\,a\,adbd\,b^2 + 2\,a\,adbd\,bb + abd\,(4\,adb\,b + 5\,ad\,b^2 + 2\,ad\,bb) + 2\,a\,adb\,bbd + 2\,aad\,b\,bbd +$
$5\,a\,ad\,b\,bbd + 10\,a\,adb\,b\,bd + 5\,aad\,b^2\,bd - 54\,a\,ad\,b^2\,bd + 2\,aad\,bb\,bd + 5\,a\,ad\,bb\,bd +$
$2\,a\,adb\,bdb + 2\,aad\,b\,bdb + 5\,a\,ad\,b\,bdb + 2\,ab\,(2\,adbd\,b + 2\,adb\,bd + ad\,(bbd + 5\,b\,bd + bdb))$

In[•]:=

```
(*Definición de las ecuaciones de estado (RHS de las ODEs)*)(*1. d<a>/dt*)
eqa[ad_, a_, ada_, aad_, aa_, adad_, bd_, b_, bdb_, bbd_, bb_, bdbd_, ab_, abd_, adb_,
    adbd_, adaada_, ft_] := -I (ωc a + E0 d1 (bd + b) + E0 d2 (bdbd + 2 bdb + bb + 1) + ft) - κ / 2 a;

(*2. d<a^dag>/dt*)
eqad[ad_, a_, ada_, aad_, aa_, adad_, bd_, b_, bdb_, bbd_, bb_, bdbd_, ab_, abd_, adb_,
    adbd_, adaada_, ft_] := I (ωc ad + E0 d1 (bd + b) + E0 d2 (bdbd + 2 bdb + bb + 1) + ft) - κ / 2 ad;

(*3. d<b>/dt*)
eqb[ad_, a_, ada_, aad_, aa_, adad_, bd_, b_, bdb_, bbd_, bb_, bdbd_, ab_, abd_, adb_, adbd_,
    adaada_, ft_] := -I (ω0 b - 2 Ub threterms[bd, b, b] + E0 d1 (ad + a) + 2 E0 d2 (ab + abd + adb + adbd));

(*4. d<b^dag>/dt*)
eqbd[ad_, a_, ada_, aad_, aa_, adad_, bd_, b_,
    bdb_, bbd_, bb_, bdbd_, ab_, abd_, adb_, adbd_, adaada_, ft_] :=
  I (ω0 bd - 2 Ub threterms[bd, bd, b] + E0 d1 (ad + a) + 2 E0 d2 (abd + ab + adbd + adb));

(*5. d<a^dag a>/dt*)
eqada[ad_, a_, ada_, aad_, aa_, adad_, bd_, b_, bdb_, bbd_, bb_,
    bdbd_, ab_, abd_, adb_, adbd_, adaada_, ft_] := -I (E0 d1 (adbd + adb - abd - ab) +
      E0 d2 (threterms[ad, bd, bd] + 2 threterms[ad, bd, b] + threterms[ad, b, b] + ad -
        threterms[a, bd, bd] - 2 threterms[a, bd, b] - threterms[a, b, b] - a) + ft (ad - a)) - κ ada;

(*6. d<b^dag b>/dt*)
eqbdb[ad_, a_, ada_, aad_, aa_, adad_, bd_, b_, bdb_, bbd_, bb_, bdbd_,
    ab_, abd_, adb_, adbd_, adaada_, ft_] := -I (E0 d1 (adbd - adb + abd - ab) + 2 E0 d2
      (threterms[ad, bd, bd] - threterms[ad, b, b] + threterms[a, bd, bd] - threterms[a, b, b]));

(*7. d<a^2>/dt*)
eqaa[ad_, a_, ada_, aad_, aa_, adad_, bd_, b_, bdb_, bbd_, bb_,
    bdbd_, ab_, abd_, adb_, adbd_, adaada_, ft_] := -2 I (ωc aa + E0 d1 (abd + ab) +
      E0 d2 (threterms[a, bd, bd] + 2 threterms[a, bd, b] + threterms[a, b, b] + a) + ft a) - κ aa;

(*8. d<a^dag^2>/dt*)
eqadad[ad_, a_, ada_, aad_, aa_, adad_, bd_, b_, bdb_, bbd_, bb_, bdbd_,
    ab_, abd_, adb_, adbd_, adaada_, ft_] := 2 I (ωc adad + E0 d1 (adbd + adb) + E0 d2
      (threterms[ad, bd, bd] + 2 threterms[ad, bd, b] + threterms[ad, b, b] + ad) + ft ad) - κ adad;
```

```
(*9. d<b^2>/dt*)
eqbb[ad_, a_, ada_, aad_, aa_, adad_, bd_, b_,
    bdb_, bbd_, bb_, bdbd_, ab_, abd_, adb_, adbd_, adaada_, ft_] :=
  -2 I (ω0 bb - Ub (2 fourterms[bd, b, b, b] + bb) + E0 d1 (adb + ab) + E0 d2 (2 threterms[ad, bd, b] +
        ad + 2 threterms[ad, b, b] + 2 threterms[a, bd, b] + a + 2 threterms[a, b, b]));

(*10. d<b^dag^2>/dt*)
eqbdbd[ad_, a_, ada_, aad_, aa_, adad_, bd_,
    b_, bdb_, bbd_, bb_, bdbd_, ab_, abd_, adb_, adbd_, adaada_, ft_] :=
  2 I (ω0 bdbd - Ub (2 fourterms[bd, bd, bd, b] + bdbd) + E0 d1 (adbd + abd) + E0 d2 (2 threterms[ad, bd,
          bd] + ad + 2 threterms[ad, bd, b] + 2 threterms[a, bd, bd] + a + 2 threterms[a, bd, b]));

(*11. d<a b>/dt CORREGIDA*)
(*Los términos como<a^dag a b>son 3 operadores:threterms[ad,a,b]*)
eqab[ad_, a_, ada_, aad_, aa_, adad_, bd_, b_, bdb_, bbd_, bb_, bdbd_, ab_, abd_, adb_, adbd_,
    adaada_, ft_] := -I ((ωc + ω0) ab - 2 Ub fourterms[a, bd, b, b] + E0 d1 (ada + aa + bdb + bb + 1) + E0 d2
      (2 (threterms[ad, a, b] + threterms[ad, a, bd] + threterms[a, a, b] + threterms[a, a, bd]) +
        threterms[b, b, b] + 2 threterms[b, bd, b] + threterms[b, bd, bd] + b) + ft b) - κ ab;

(*12. d<a^dag b^dag>/dt*)
eqadbd[ad_, a_, ada_, aad_, aa_, adad_, bd_,
    b_, bdb_, bbd_, bb_, bdbd_, ab_, abd_, adb_, adbd_, adaada_, ft_] :=
  I ((ωc + ω0) adbd - 2 Ub fourterms[ad, bd, bd, b] + E0 d1 (ada + adad + bdb + bdbd + 1) +
      E0 d2 (2 (threterms[ad, a, bd] + threterms[ad, a, b] +
          threterms[ad, ad, bd] + threterms[ad, ad, b]) + threterms[bd, bd, bd] +
        2 threterms[bd, b, bd] + threterms[b, b, bd] + bd) + ft bd) - κ adbd;

(*13. d<a^dag b>/dt *)

eqadb[ad_, a_, ada_, aad_, aa_, adad_, bd_, b_,
    bdb_, bbd_, bb_, bdbd_, ab_, abd_, adb_, adbd_, adaada_, ft_] :=
  -I ((ω0 - ωc) adb - 2 Ub fourterms[ad, bd, b, b] + E0 d1 (ada + adad - bdb - bb) - E0 d2 (threterms[b,
        b, b] + 2 threterms[b, bd, b] + threterms[b, bd, bd] + b - 2 (threterms[ad, ad, b] +
          threterms[ad, ad, bd] + threterms[a, ad, b] + threterms[a, ad, bd])) - ft b) - κ adb;

(*14. d<a b^dag>/dt*)
eqabd[ad_, a_, ada_, aad_, aa_, adad_, bd_, b_,
    bdb_, bbd_, bb_, bdbd_, ab_, abd_, adb_, adbd_, adaada_, ft_] :=
  I ((ω0 - ωc) abd - 2 Ub fourterms[a, bd, bd, b] + E0 d1 (ada + aa - bdb - bdbd) -
      E0 d2 (threterms[bd, bd, bd] + 2 threterms[bd, b, bd] +
        threterms[b, b, bd] + bd - 2 (threterms[a, a, bd] + threterms[a, a, b] +
```

```
            threterms[a, ad, bd] + threterms[a, ad, b]) - ft bd) - κ abd;
(*15. d<a^dag a a^dag a>/dt (N^2)*)

eqadaada[ad_, a_, ada_, aad_, aa_, adad_, bd_,
    b_, bdb_, bbd_, bb_, bdbd_, ab_, abd_, adb_, adbd_, adaada_, ft_] :=
  -I (ft ((ada + 1) ad + 3 ad ada) + E0 d1 (2 abd adad + (ada + 1) adb + 3 ada adb + (ada + 1) adbd +
          3 ada adbd - (ada + 1) ab - (ada + 1) abd - 3 ab ada - 3 abd ada + 2 ab adad) - 2 aa (ft ad + E0 d1 (adb + adbd)) +
      4 a^2 ad (ft + E0 d1 (b + bd)) - a ((ada + 1 + 3 ada - 2 adad) ft + 4 ad^2 (ft + (b + bd) E0 d1)) +
      E0 d2 (fiveterms[ad, a, ad, bd, bd] + 2 fiveterms[ad, a, ad, bd, b] +
          fiveterms[ad, a, ad, b, b] + threterms[ad, a, ad] + fiveterms[ad, ad, a, bd, bd] +
          2 fiveterms[ad, ad, a, bd, b] + fiveterms[ad, ad, a, b, b] + threterms[ad, ad, a] -
          fiveterms[ad, a, a, bd, bd] - 2 fiveterms[ad, a, a, bd, b] - fiveterms[ad, a, a, b, b] -
          threterms[ad, a, a] - fiveterms[a, ad, a, bd, bd] - 2 fiveterms[a, ad, a, bd, b] -
          fiveterms[a, ad, a, b, b] - threterms[a, ad, a])) + κ (ada - adaada);

(*Función HH Completa (Hamiltoniano de Control)*)


HH[ad_, a_, ada_, aad_, aa_, adad_, bd_, b_,
    bdb_, bbd_, bb_, bdbd_, ab_, abd_, adb_, adbd_, adaada_, ft_] :=
  ft * ft + Pa * eqa[ad, a, ada, aad, aa, adad, bd, b, bdb, bbd, bb, bdbd, ab, abd,
      adb, adbd, adaada, ft] + Pad * eqad[ad, a, ada, aad, aa, adad, bd, b, bdb,
      bbd, bb, bdbd, ab, abd, adb, adbd, adaada, ft] + Pb * eqb[ad, a, ada, aad,
      aa, adad, bd, b, bdb, bbd, bb, bdbd, ab, abd, adb, adbd, adaada, ft] +
    Pbd * eqbd[ad, a, ada, aad, aa, adad, bd, b, bdb, bbd, bb, bdbd, ab, abd,
      adb, adbd, adaada, ft] + Pada * eqada[ad, a, ada, aad, aa, adad,
      bd, b, bdb, bbd, bb, bdbd, ab, abd, adb, adbd, adaada, ft] +
    Pbdb * eqbdb[ad, a, ada, aad, aa, adad, bd, b, bdb, bbd, bb, bdbd, ab,
      abd, adb, adbd, adaada, ft] + Paa * eqaa[ad, a, ada, aad, aa, adad,
      bd, b, bdb, bbd, bb, bdbd, ab, abd, adb, adbd, adaada, ft] +
    Padad * eqadad[ad, a, ada, aad, aa, adad, bd, b, bdb, bbd, bb, bdbd, ab,
      abd, adb, adbd, adaada, ft] + Pbb * eqbb[ad, a, ada, aad, aa, adad,
      bd, b, bdb, bbd, bb, bdbd, ab, abd, adb, adbd, adaada, ft] +
    Pbdbd * eqbdbd[ad, a, ada, aad, aa, adad, bd, b, bdb, bbd, bb, bdbd, ab,
      abd, adb, adbd, adaada, ft] + Pab * eqab[ad, a, ada, aad, aa, adad,
      bd, b, bdb, bbd, bb, bdbd, ab, abd, adb, adbd, adaada, ft] +
    Pabd * eqabd[ad, a, ada, aad, aa, adad, bd, b, bdb, bbd, bb, bdbd, ab,
      abd, adb, adbd, adaada, ft] + Padb * eqadb[ad, a, ada, aad, aa,
      adad, bd, b, bdb, bbd, bb, bdbd, ab, abd, adb, adbd, adaada, ft] +
    Padbd * eqadbd[ad, a, ada, aad, aa, adad, bd, b, bdb, bbd, bb, bdbd, ab,
      abd, adb, adbd, adaada, ft] + Padaada * eqadaada[ad, a, ada, aad, aa,
      adad, bd, b, bdb, bbd, bb, bdbd, ab, abd, adb, adbd, adaada, ft];
```

```
costateAdad = -D[HH[ad, a, ada, aad, aa, adad, bd, b,
      bdb, bbd, bb, bdbd, ab, abd, adb, adbd, adaada, ft], adad] // Simplify
```

Out[•]=
$$2\,i\,\left(ab\left(d1+8\,(b+bd)\,d2\right)E0+abd\left(d1+8\,(b+bd)\,d2\right)E0+a\left(\left(1+2\,b^2+4\,bb+4\,b\,bd+2\,bd^2+8\,bdb+4\,bdbd\right)d2\,E0+ft\right)\right)Padaada+i\left(d1+2\,(b+bd)\,d2\right)E0\,Padb-i\left(d1+2\,(b+bd)\,d2\right)E0\,Padbd+Padad\left(\kappa-2\,i\,\omega c\right)$$

In[•]:=
```
costateAda = -D[HH[ad, a, ada, aad, aa, adad, bd, b,
      bdb, bbd, bb, bdbd, ab, abd, adb, adbd, adaada, ft], ada] // Simplify
```

Out[•]=
$$-i\left(-2\,bd\,d2\,E0\,Pab+3\,a\,d2\,E0\,Padaada-3\,ad\,d2\,E0\,Padaada+6\,(a-ad)\,b^2\,d2\,E0\,Padaada+12\,a\,bb\,d2\,E0\,Padaada-12\,ad\,bb\,d2\,E0\,Padaada+24\,ab\,bd\,d2\,E0\,Padaada+24\,abd\,bd\,d2\,E0\,Padaada-24\,adb\,bd\,d2\,E0\,Padaada-24\,adbd\,bd\,d2\,E0\,Padaada+6\,a\,bd^2\,d2\,E0\,Padaada-6\,ad\,bd^2\,d2\,E0\,Padaada+24\,a\,bdb\,d2\,E0\,Padaada-24\,ad\,bdb\,d2\,E0\,Padaada+12\,a\,bdbd\,d2\,E0\,Padaada-12\,ad\,bdbd\,d2\,E0\,Padaada+4\,a\,ft\,Padaada-4\,ad\,ft\,Padaada-2\,b\,d2\,E0\left(Pab-12\,ab\,Padaada-12\,abd\,Padaada+12\,adb\,Padaada+12\,adbd\,Padaada-6\,a\,bd\,Padaada+6\,ad\,bd\,Padaada-Padbd\right)+2\,bd\,d2\,E0\,Padbd+d1\,E0\left(-Pab+Pabd+4\,ab\,Padaada+4\,abd\,Padaada-4\,adb\,Padaada-4\,adbd\,Padaada-Padb+Padbd\right)+i\,Pada\,\kappa-i\,Padaada\,\kappa\right)$$

In[•]:=
```
costateAd = -D[HH[ad, a, ada, aad, aa, adad, bd, b,
      bdb, bbd, bb, bdbd, ab, abd, adb, adbd, adaada, ft], ad] // Simplify
```

Out[•]=

$$-\frac{1}{2}\,i\,\Big(8\,a\,b\,d2\,E0\,Pab+8\,a\,bd\,d2\,E0\,Pab-8\,a\,b\,d2\,E0\,Pabd-8\,a\,bd\,d2\,E0\,Pabd-2\,d2\,E0\,Pada+4\,b^2\,d2\,E0\,Pada-2\,bb\,d2\,E0\,Pada+8\,b\,bd\,d2\,E0\,Pada+4\,bd^2\,d2\,E0\,Pada-4\,bdb\,d2\,E0\,Pada-2\,bdbd\,d2\,E0\,Pada-2\,ft\,Pada-8\,a^2\,b\,d1\,E0\,Padaada+16\,a\,ad\,b\,d1\,E0\,Padaada-8\,a^2\,bd\,d1\,E0\,Padaada+16\,a\,ad\,bd\,d1\,E0\,Padaada-8\,a^2\,d2\,E0\,Padaada+4\,aa\,d2\,E0\,Padaada-2\,aad\,d2\,E0\,Padaada+32\,ab^2\,d2\,E0\,Padaada+32\,abd^2\,d2\,E0\,Padaada+16\,a\,ad\,d2\,E0\,Padaada-6\,ada\,d2\,E0\,Padaada+64\,a\,adb\,b\,d2\,E0\,Padaada+64\,a\,adbd\,b\,d2\,E0\,Padaada+24\,a^2\,b^2\,d2\,E0\,Padaada-16\,aa\,b^2\,d2\,E0\,Padaada+8\,aad\,b^2\,d2\,E0\,Padaada-48\,a\,ad\,b^2\,d2\,E0\,Padaada+24\,ada\,b^2\,d2\,E0\,Padaada-16\,a^2\,bb\,d2\,E0\,Padaada+16\,aa\,bb\,d2\,E0\,Padaada-8\,aad\,bb\,d2\,E0\,Padaada+32\,a\,ad\,bb\,d2\,E0\,Padaada-24\,ada\,bb\,d2\,E0\,Padaada+64\,a\,adb\,bd\,d2\,E0\,Padaada+64\,a\,adbd\,bd\,d2\,E0\,Padaada+48\,a^2\,b\,bd\,d2\,E0\,Padaada-32\,aa\,b\,bd\,d2\,E0\,Padaada+16\,aad\,b\,bd\,d2\,E0\,Padaada-96\,a\,ad\,b\,bd\,d2\,E0\,Padaada+48\,ada\,b\,bd\,d2\,E0\,Padaada+24\,a^2\,bd^2\,d2\,E0\,Padaada-16\,aa\,bd^2\,d2\,E0\,Padaada+8\,aad\,bd^2\,d2\,E0\,Padaada-48\,a\,ad\,bd^2\,d2\,E0\,Padaada+24\,ada\,bd^2\,d2\,E0\,Padaada-32\,a^2\,bdb\,d2\,E0\,Padaada+32\,aa\,bdb\,d2\,E0\,Padaada-16\,aad\,bdb\,d2\,E0\,Padaada+64\,a\,ad\,bdb\,d2\,E0\,Padaada-48\,ada\,bdb\,d2\,E0\,Padaada-16\,a^2\,bdbd\,d2\,E0\,Padaada+16\,aa\,bdbd\,d2\,E0\,Padaada-8\,aad\,bdbd\,d2\,E0\,Padaada+32\,a\,ad\,bdbd\,d2\,E0\,Padaada-24\,ada\,bdbd\,d2\,E0\,Padaada-2\,ft\,Padaada-8\,a^2\,ft\,Padaada+4\,aa\,ft\,Padaada+16\,a\,ad\,ft\,Padaada-8\,ada\,ft\,Padaada+4\,d2\,E0\,Padad-8\,b^2\,d2\,E0\,Padad+4\,bb\,d2\,E0\,Padad-16\,b\,bd\,d2\,E0\,Padad-8\,bd^2\,d2\,E0\,Padad+8\,bdb\,d2\,E0\,Padad+4\,bdbd\,d2\,E0\,Padad+4\,ft\,Padad-8\,adb\,d2\,E0\,Padb-8\,adbd\,d2\,E0\,Padb+8\,a\,b\,d2\,E0\,Padb+16\,ad\,b\,d2\,E0\,Padb+8\,a\,bd\,d2\,E0\,Padb+16\,ad\,bd\,d2\,E0\,Padb-4\,abd\,d2\,E0\,\big(Pab-Pabd+16\,adb\,Padaada+16\,adbd\,Padaada+16\,a\,b\,Padaada-16\,ad\,b\,Padaada+16\,a\,bd\,Padaada-16\,ad\,bd\,Padaada+Padb-Padbd\big)-4\,ab\,d2\,E0\,\big(Pab-Pabd-16\,abd\,Padaada+16\,adb\,Padaada+16\,adbd\,Padaada+16\,a\,b\,Padaada-16\,ad\,b\,Padaada+16\,a\,bd\,Padaada-16\,ad\,bd\,Padaada+Padb-Padbd\big)+8\,adb\,d2\,E0\,Padbd+8\,adbd\,d2\,E0\,Padbd-8\,a\,b\,d2\,E0\,Padbd-16\,ad\,b\,d2\,E0\,Padbd-8\,a\,bd\,d2\,E0\,Padbd-16\,ad\,bd\,d2\,E0\,Padbd-2\,d1\,E0\,Pb-4\,d2\,E0\,Pbb+16\,b^2\,d2\,E0\,Pbb-8\,bb\,d2\,E0\,Pbb+16\,b\,bd\,d2\,E0\,Pbb-8\,bdb\,d2\,E0\,Pbb+2\,d1\,E0\,Pbd-8\,b^2\,d2\,E0\,Pbdb+4\,bb\,d2\,E0\,Pbdb+8\,bd^2\,d2\,E0\,Pbdb-4\,bdbd\,d2\,E0\,Pbdb+4\,d2\,E0\,Pbdbd-16\,b\,bd\,d2\,E0\,Pbdbd-16\,bd^2\,d2\,E0\,Pbdbd+8\,bdb\,d2\,E0\,Pbdbd+8\,bdbd\,d2\,E0\,Pbdbd-8\,b^2\,bd\,Padb\,Ub+8\,b\,bd^2\,Padbd\,Ub+i\,Pad\,\kappa+2\,Pad\,\omega c\Big)$$

In[•]:=
```
costateBdb = -D[HH[ad, a, ada, aad, aa, adad, bd, b,
      bdb, bbd, bb, bdbd, ab, abd, adb, adbd, adaada, ft], bdb] // Simplify
```

Out[•]=

$$i\,\Big(d1\,E0\,(Pab+Pabd-Padb-Padbd)+2\,d2\,E0\,\Big(Pa+b\,Pab+bd\,Pabd-Pad+ad\,Pada+8\,a^2\,ad\,Padaada-8\,aa\,ad\,Padaada+4\,aad\,ad\,Padaada+12\,ad\,ada\,Padaada-2\,ad\,Padad-b\,Padb-bd\,Padbd+2\,ad\,Pbb-2\,ad\,Pbdbd+a\,\Big(2\,Paa-Pada-2\,\big(2\,aad\,Padaada+4\,ad^2\,Padaada+6\,ada\,Padaada-4\,adad\,Padaada-Pbb+Pbdbd\big)\Big)\Big)-4\,\big(ab\,Pab-abd\,Pabd+adb\,Padb-adbd\,Padbd+b\,Pb+3\,bb\,Pbb-bd\,Pbd-3\,bdbd\,Pbdbd\big)\,Ub\Big)$$

In[•]:=
```
costateB = -D[HH[ad, a, ada, aad, aa, adad, bd, b,
    bdb, bbd, bb, bdbd, ab, abd, adb, adbd, adaada, ft], b] // Simplify
```

Out[•]=
$$i\,\Big(4\,\text{abd}\,\text{d2}\,\text{E0}\,\text{Paa}-8\,\text{a}\,\text{b}\,\text{d2}\,\text{E0}\,\text{Paa}-8\,\text{a}\,\text{bd}\,\text{d2}\,\text{E0}\,\text{Paa}+\text{d2}\,\text{E0}\,\text{Pab}-4\,\text{a}^2\,\text{d2}\,\text{E0}\,\text{Pab}+2\,\text{aa}\,\text{d2}\,\text{E0}\,\text{Pab}-$$
$$4\,\text{a}\,\text{ad}\,\text{d2}\,\text{E0}\,\text{Pab}+2\,\text{ada}\,\text{d2}\,\text{E0}\,\text{Pab}-6\,\text{b}^2\,\text{d2}\,\text{E0}\,\text{Pab}+3\,\text{bb}\,\text{d2}\,\text{E0}\,\text{Pab}+2\,\text{bbd}\,\text{d2}\,\text{E0}\,\text{Pab}-$$
$$8\,\text{b}\,\text{bd}\,\text{d2}\,\text{E0}\,\text{Pab}-2\,\text{bd}^2\,\text{d2}\,\text{E0}\,\text{Pab}+2\,\text{bdb}\,\text{d2}\,\text{E0}\,\text{Pab}+\text{bdbd}\,\text{d2}\,\text{E0}\,\text{Pab}+\text{ft}\,\text{Pab}+4\,\text{a}^2\,\text{d2}\,\text{E0}\,\text{Pabd}-$$
$$2\,\text{aa}\,\text{d2}\,\text{E0}\,\text{Pabd}-2\,\text{aad}\,\text{d2}\,\text{E0}\,\text{Pabd}+4\,\text{a}\,\text{ad}\,\text{d2}\,\text{E0}\,\text{Pabd}-4\,\text{bd}^2\,\text{d2}\,\text{E0}\,\text{Pabd}+2\,\text{bdbd}\,\text{d2}\,\text{E0}\,\text{Pabd}-$$
$$2\,\text{abd}\,\text{d2}\,\text{E0}\,\text{Pada}+2\,\text{adb}\,\text{d2}\,\text{E0}\,\text{Pada}+2\,\text{adbd}\,\text{d2}\,\text{E0}\,\text{Pada}+4\,\text{a}\,\text{b}\,\text{d2}\,\text{E0}\,\text{Pada}-4\,\text{ad}\,\text{b}\,\text{d2}\,\text{E0}\,\text{Pada}+$$
$$4\,\text{a}\,\text{bd}\,\text{d2}\,\text{E0}\,\text{Pada}-4\,\text{ad}\,\text{bd}\,\text{d2}\,\text{E0}\,\text{Pada}-8\,\text{aad}\,\text{abd}\,\text{d2}\,\text{E0}\,\text{Padaada}+32\,\text{a}\,\text{abd}\,\text{ad}\,\text{d2}\,\text{E0}\,\text{Padaada}-$$
$$16\,\text{abd}\,\text{ad}^2\,\text{d2}\,\text{E0}\,\text{Padaada}-24\,\text{abd}\,\text{ada}\,\text{d2}\,\text{E0}\,\text{Padaada}+16\,\text{abd}\,\text{adad}\,\text{d2}\,\text{E0}\,\text{Padaada}+$$
$$16\,\text{a}^2\,\text{adb}\,\text{d2}\,\text{E0}\,\text{Padaada}-16\,\text{aa}\,\text{adb}\,\text{d2}\,\text{E0}\,\text{Padaada}+8\,\text{aad}\,\text{adb}\,\text{d2}\,\text{E0}\,\text{Padaada}-$$
$$32\,\text{a}\,\text{ad}\,\text{adb}\,\text{d2}\,\text{E0}\,\text{Padaada}+24\,\text{ada}\,\text{adb}\,\text{d2}\,\text{E0}\,\text{Padaada}+16\,\text{a}^2\,\text{adbd}\,\text{d2}\,\text{E0}\,\text{Padaada}-$$
$$16\,\text{aa}\,\text{adbd}\,\text{d2}\,\text{E0}\,\text{Padaada}+8\,\text{aad}\,\text{adbd}\,\text{d2}\,\text{E0}\,\text{Padaada}-32\,\text{a}\,\text{ad}\,\text{adbd}\,\text{d2}\,\text{E0}\,\text{Padaada}+$$
$$24\,\text{ada}\,\text{adbd}\,\text{d2}\,\text{E0}\,\text{Padaada}+8\,\text{a}\,\text{aad}\,\text{b}\,\text{d2}\,\text{E0}\,\text{Padaada}-24\,\text{a}^2\,\text{ad}\,\text{b}\,\text{d2}\,\text{E0}\,\text{Padaada}+$$
$$16\,\text{aa}\,\text{ad}\,\text{b}\,\text{d2}\,\text{E0}\,\text{Padaada}-8\,\text{aad}\,\text{ad}\,\text{b}\,\text{d2}\,\text{E0}\,\text{Padaada}+24\,\text{a}\,\text{ad}^2\,\text{b}\,\text{d2}\,\text{E0}\,\text{Padaada}+$$
$$24\,\text{a}\,\text{ada}\,\text{b}\,\text{d2}\,\text{E0}\,\text{Padaada}-24\,\text{ad}\,\text{ada}\,\text{b}\,\text{d2}\,\text{E0}\,\text{Padaada}-16\,\text{a}\,\text{adad}\,\text{b}\,\text{d2}\,\text{E0}\,\text{Padaada}+$$
$$8\,\text{a}\,\text{aad}\,\text{bd}\,\text{d2}\,\text{E0}\,\text{Padaada}-24\,\text{a}^2\,\text{ad}\,\text{bd}\,\text{d2}\,\text{E0}\,\text{Padaada}+16\,\text{aa}\,\text{ad}\,\text{bd}\,\text{d2}\,\text{E0}\,\text{Padaada}-$$
$$8\,\text{aad}\,\text{ad}\,\text{bd}\,\text{d2}\,\text{E0}\,\text{Padaada}+24\,\text{a}\,\text{ad}^2\,\text{bd}\,\text{d2}\,\text{E0}\,\text{Padaada}+24\,\text{a}\,\text{ada}\,\text{bd}\,\text{d2}\,\text{E0}\,\text{Padaada}-$$
$$24\,\text{ad}\,\text{ada}\,\text{bd}\,\text{d2}\,\text{E0}\,\text{Padaada}-16\,\text{a}\,\text{adad}\,\text{bd}\,\text{d2}\,\text{E0}\,\text{Padaada}+\text{d1}\,\text{E0}\,\big(\text{Pa}-\text{Pad}+4\,\text{a}\,(\text{a}-\text{ad})\,\text{ad}\,\text{Padaada}\big)-$$
$$4\,\text{adb}\,\text{d2}\,\text{E0}\,\text{Padad}-4\,\text{adbd}\,\text{d2}\,\text{E0}\,\text{Padad}+8\,\text{ad}\,\text{b}\,\text{d2}\,\text{E0}\,\text{Padad}+8\,\text{ad}\,\text{bd}\,\text{d2}\,\text{E0}\,\text{Padad}-\text{d2}\,\text{E0}\,\text{Padb}+$$
$$2\,\text{aad}\,\text{d2}\,\text{E0}\,\text{Padb}-4\,\text{a}\,\text{ad}\,\text{d2}\,\text{E0}\,\text{Padb}-4\,\text{ad}^2\,\text{d2}\,\text{E0}\,\text{Padb}+2\,\text{adad}\,\text{d2}\,\text{E0}\,\text{Padb}+6\,\text{b}^2\,\text{d2}\,\text{E0}\,\text{Padb}-$$
$$3\,\text{bb}\,\text{d2}\,\text{E0}\,\text{Padb}-2\,\text{bbd}\,\text{d2}\,\text{E0}\,\text{Padb}+8\,\text{b}\,\text{bd}\,\text{d2}\,\text{E0}\,\text{Padb}+2\,\text{bd}^2\,\text{d2}\,\text{E0}\,\text{Padb}-2\,\text{bdb}\,\text{d2}\,\text{E0}\,\text{Padb}-$$
$$\text{bdbd}\,\text{d2}\,\text{E0}\,\text{Padb}-\text{ft}\,\text{Padb}+4\,\text{a}\,\text{ad}\,\text{d2}\,\text{E0}\,\text{Padbd}+4\,\text{ad}^2\,\text{d2}\,\text{E0}\,\text{Padbd}-2\,\text{ada}\,\text{d2}\,\text{E0}\,\text{Padbd}-$$
$$2\,\text{adad}\,\text{d2}\,\text{E0}\,\text{Padbd}+4\,\text{bd}^2\,\text{d2}\,\text{E0}\,\text{Padbd}-2\,\text{bdbd}\,\text{d2}\,\text{E0}\,\text{Padbd}+4\,\text{abd}\,\text{d2}\,\text{E0}\,\text{Pbb}+8\,\text{adb}\,\text{d2}\,\text{E0}\,\text{Pbb}+$$
$$4\,\text{adbd}\,\text{d2}\,\text{E0}\,\text{Pbb}-16\,\text{a}\,\text{b}\,\text{d2}\,\text{E0}\,\text{Pbb}-16\,\text{ad}\,\text{b}\,\text{d2}\,\text{E0}\,\text{Pbb}-8\,\text{a}\,\text{bd}\,\text{d2}\,\text{E0}\,\text{Pbb}-8\,\text{ad}\,\text{bd}\,\text{d2}\,\text{E0}\,\text{Pbb}-$$
$$4\,\text{adb}\,\text{d2}\,\text{E0}\,\text{Pbdb}+8\,\text{a}\,\text{b}\,\text{d2}\,\text{E0}\,\text{Pbdb}+8\,\text{ad}\,\text{b}\,\text{d2}\,\text{E0}\,\text{Pbdb}+2\,\text{ab}\,\text{d2}\,\text{E0}\,\big(2\,\text{Paa}-\text{Pada}-2\,\big(2\,\text{aad}\,\text{Padaada}-$$
$$8\,\text{a}\,\text{ad}\,\text{Padaada}+4\,\text{ad}^2\,\text{Padaada}+6\,\text{ada}\,\text{Padaada}-4\,\text{adad}\,\text{Padaada}-2\,\text{Pbb}+\text{Pbdb}\big)\big)-$$
$$4\,\text{abd}\,\text{d2}\,\text{E0}\,\text{Pbdbd}-4\,\text{adbd}\,\text{d2}\,\text{E0}\,\text{Pbdbd}+8\,\text{a}\,\text{bd}\,\text{d2}\,\text{E0}\,\text{Pbdbd}+8\,\text{ad}\,\text{bd}\,\text{d2}\,\text{E0}\,\text{Pbdbd}+$$
$$8\,\text{a}\,\text{b}\,\text{bd}\,\text{Pab}\,\text{Ub}-4\,\text{a}\,\text{bd}^2\,\text{Pabd}\,\text{Ub}+8\,\text{ad}\,\text{b}\,\text{bd}\,\text{Padb}\,\text{Ub}-4\,\text{ad}\,\text{bd}^2\,\text{Padbd}\,\text{Ub}+8\,\text{b}\,\text{bd}\,\text{Pb}\,\text{Ub}-$$
$$4\,\text{bdb}\,\text{Pb}\,\text{Ub}+24\,\text{b}^2\,\text{bd}\,\text{Pbb}\,\text{Ub}-4\,\text{bd}^2\,\text{Pbd}\,\text{Ub}+2\,\text{bdbd}\,\text{Pbd}\,\text{Ub}-8\,\text{bd}^3\,\text{Pbdbd}\,\text{Ub}+\text{Pb}\,\omega 0\Big)$$

In[•]:=
```
costateBb = -D[HH[ad, a, ada, aad, aa, adad, bd, b,
    bdb, bbd, bb, bdbd, ab, abd, adb, adbd, adaada, ft], bb] // Simplify
```

Out[•]=
$$i\,\Big(\text{d1}\,\text{E0}\,(\text{Pab}-\text{Padb})+$$
$$\text{d2}\,\text{E0}\,\big(\text{Pa}+3\,\text{b}\,\text{Pab}+2\,\text{bd}\,\text{Pab}+\text{bd}\,\text{Pabd}-\text{Pad}+\text{ad}\,\text{Pada}+8\,\text{a}^2\,\text{ad}\,\text{Padaada}-8\,\text{aa}\,\text{ad}\,\text{Padaada}+$$
$$4\,\text{aad}\,\text{ad}\,\text{Padaada}+12\,\text{ad}\,\text{ada}\,\text{Padaada}-2\,\text{ad}\,\text{Padad}-3\,\text{b}\,\text{Padb}-2\,\text{bd}\,\text{Padb}-$$
$$\text{bd}\,\text{Padbd}+4\,\text{ad}\,\text{Pbb}-2\,\text{ad}\,\text{Pbdb}+\text{a}\,\big(2\,\text{Paa}-\text{Pada}-$$
$$2\,\big(2\,\text{aad}\,\text{Padaada}+4\,\text{ad}^2\,\text{Padaada}+6\,\text{ada}\,\text{Padaada}-4\,\text{adad}\,\text{Padaada}-2\,\text{Pbb}+\text{Pbdb}\big)\big)\big)-$$
$$2\,\big(\text{abd}\,\text{Pab}\,\text{Ub}+\text{adbd}\,\text{Padb}\,\text{Ub}+\text{bd}\,\text{Pb}\,\text{Ub}+\text{Pbb}\,\text{Ub}+6\,\text{bdb}\,\text{Pbb}\,\text{Ub}-\text{Pbb}\,\omega 0\big)\Big)$$

In[•]:=
```
costateAdbd = -D[HH[ad, a, ada, aad, aa, adad, bd, b,
    bdb, bbd, bb, bdbd, ab, abd, adb, adbd, adaada, ft], adbd] // Simplify
```

Out[•]=
$$2\,i\,a\,d2\,E0\,Pab - 2\,i\,a\,d2\,E0\,Pabd + i\,(d1 + 2\,(b + bd)\,d2)\,E0\,Pada -$$
$$i\,((-1 + 2\,aa - 4\,ada)\,d1 - 8\,(4\,ab\,ad + 4\,abd\,ad - 2\,aa\,b + aad\,b + 3\,ada\,b - 2\,aa\,bd + aad\,bd +$$
$$3\,ada\,bd + 2\,a^2\,(b + bd) - 4\,a\,(ab + abd - adb - adbd + ad\,b + ad\,bd))\,d2)\,E0\,Padaada -$$
$$2\,i\,(d1 + 2\,(b + bd)\,d2)\,E0\,Padad + 2\,i\,d2\,E0\,Pb + 4\,i\,b\,d2\,E0\,Pbb - 2\,i\,d2\,E0\,Pbd +$$
$$i\,(d1 + 4\,bd\,d2)\,E0\,Pbdb - 2\,i\,(d1 + 2\,(b + 2\,bd)\,d2)\,E0\,Pbdbd +$$
$$2\,i\,Padb\,(a\,d2\,E0 + 2\,ad\,d2\,E0 - bb\,Ub) +$$
$$Padbd\,(\kappa - i\,(2\,(a + 2\,ad)\,d2\,E0 - 4\,bdb\,Ub + \omega 0 + \omega c))$$

In[•]:=
```
costateAbd = -D[HH[ad, a, ada, aad, aa, adad, bd, b,
    bdb, bbd, bb, bdbd, ab, abd, adb, adbd, adaada, ft], abd] // Simplify
```

Out[•]=
$$2\,i\,(d1 + 2\,(b + bd)\,d2)\,E0\,Paa - i\,(d1 + 2\,(b + bd)\,d2)\,E0\,Pada -$$
$$i\,((1 + 4\,ada - 2\,adad)\,d1 + 8\,(4\,ab\,ad + 4\,abd\,ad + 4\,a\,adb - 4\,ad\,adb + 4\,a\,adbd - 4\,ad\,adbd + aad\,b - 4\,a\,ad\,b +$$
$$2\,ad^2\,b + 3\,ada\,b - 2\,adad\,b + aad\,bd - 4\,a\,ad\,bd + 2\,ad^2\,bd + 3\,ada\,bd - 2\,adad\,bd)\,d2)\,E0\,Padaada +$$
$$2\,i\,ad\,d2\,E0\,Padb - 2\,i\,ad\,d2\,E0\,Padbd + 2\,i\,d2\,E0\,Pb + 4\,i\,b\,d2\,E0\,Pbb - 2\,i\,d2\,E0\,Pbd +$$
$$i\,(d1 + 4\,bd\,d2)\,E0\,Pbdb - 2\,i\,(d1 + 2\,(b + 2\,bd)\,d2)\,E0\,Pbdbd +$$
$$2\,i\,Pab\,(2\,a\,d2\,E0 + ad\,d2\,E0 - bb\,Ub) +$$
$$Pabd\,(\kappa - i\,(2\,(2\,a + ad)\,d2\,E0 - 4\,bdb\,Ub + \omega 0 - \omega c))$$

In[•]:=
```
costateAad = -D[HH[ad, a, ada, aad, aa, adad, bd, b,
    bdb, bbd, bb, bdbd, ab, abd, adb, adbd, adaada, ft], aad] // Simplify
```

Out[•]=
$$-i\,d2\,E0$$
$$(2\,(b + bd)\,Pabd - (-8\,(ab + abd - adb - adbd)\,(b + bd) + a\,(-1 + 4\,b^2 - 4\,bb + 8\,b\,bd + 4\,bd^2 - 8\,bdb - 4\,bdbd) +$$
$$ad\,(1 - 4\,b^2 + 4\,bb - 8\,b\,bd - 4\,bd^2 + 8\,bdb + 4\,bdbd))\,Padaada - 2\,(b + bd)\,Padb)$$